

# International Spillovers of Large-Scale Asset Purchases\*

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FIRST DRAFT

## Abstract

This paper evaluates the international spillover effects of large-scale asset purchases (LSAPs) using a two-country dynamic stochastic general equilibrium model with nominal and real rigidities and portfolio balance effects. Portfolio balance effects arise from imperfect substitution between short and long term bonds in both domestic and foreign portfolios of each country. We find that LSAPs can stimulate both domestic and foreign activity, and the international spillover effects are higher than conventional monetary policy. The spillover effects increase when foreigners hold more long-term bonds in their foreign-asset portfolio at the steady state or if they substitute less short-term bonds for long-term bonds.

*Keywords:* Portfolio balance effects, international spillovers, preferred habitat, DSGE.

JEL Classification:

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# 1 Introduction

Following the financial turbulence in the fall of 2008, the Federal Reserve cut short-term policy rates to near-zero and announced unprecedented unconventional policy measures, such as large-scale asset purchases (LSAPs; also known as quantitative easing, “QE”), at the zero lower bound. Several studies have found significant effects from these asset purchases in terms of lowering U.S. long-term yields and strengthening economic activity (See Baumeister and Benati, 2013, D’amico et. al, 2012, and Krishnamurthy and Vissing-Jorgensen, 2011 among others). The domestic effects of these policies may have also increased the attractiveness of foreign assets, and led to portfolio re-balancing by investors. Figure 1 shows exchange rate and long-term yield movements in countries where the policy rates were not significantly binding and QE-like unconventional measures were not expected. The figure suggests that the currencies of these countries tended to appreciate during and after the announcement of LSAPs.<sup>1</sup> Long-term yields were also in declining trend during this period; on average long-term rates fell by more than 1% by mid-2013 when tapering talk started.<sup>2</sup> In their analyses, Bauer and Neely (2014), and Neely (2013) find substantial effects of LSAPs on international financial markets by lowering foreign yields and depreciating the U.S. dollar in a sample of advanced economies. Chen et al. (2011) and Fratzscher et al. (2012) also document significant spillovers of QE into financial markets in emerging economies.

In this paper, we propose a model in which foreigners hold both short-term and long-term U.S. government bonds, as well as their domestic bonds, but do not easily substitute these bonds among each other based on portfolio preferences. We show that the model can generate above-mentioned international spillovers after a QE announcement in the U.S. through the portfolio balance channel. We capture these portfolio balance effects by introducing a portfolio preference in households’ utility function with a CES structure to aggregate individual financial assets. The imperfect substitution between short-term and long-term assets, as well as between domestic and foreign assets, in one country’s portfolio could represent attitudes towards differential risk on these assets, costs of adjusting the portfolio, and institutional use of these bonds for liquidity purposes with varying degrees.<sup>3</sup> Since long-term and short-term bonds are not perfect substitutes in our set-up, long-term rates fall as a response to a drop in the relative supply of long-term bonds held by households.<sup>4</sup> Lower long-term rates then stimulates the domestic economy and generates an appreciation pressure on the currency of the rest of the world (ROW) economy. This in turn leads to current and expected policy rates to fall in the rest of the world, lowering foreign long-term yields as well. The fact that the short-term rate may be fixed in

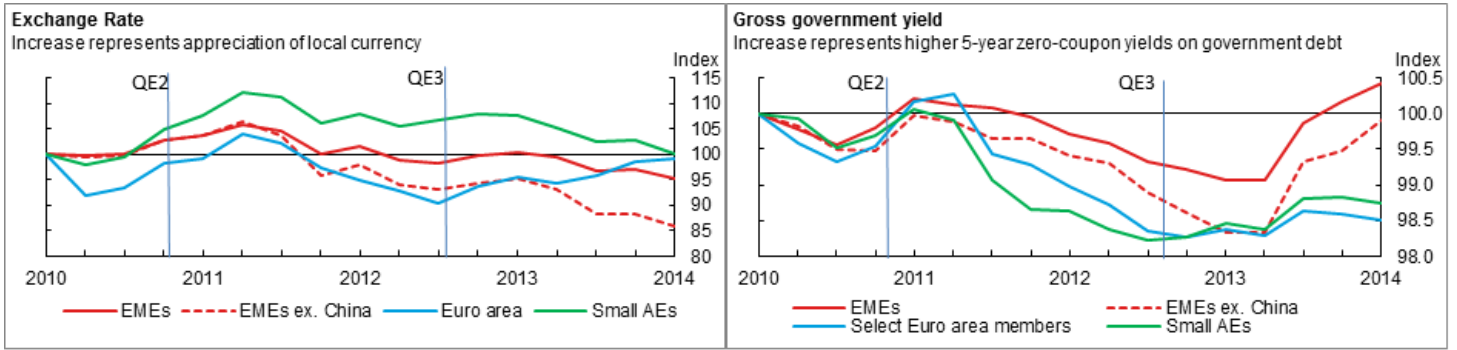
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<sup>1</sup>IMF data also indicate that the central banks of emerging market economies (EMEs) increased their U.S. dollar-denominated reserves during these episodes, partly to offset these appreciation pressures on their currencies.

<sup>2</sup>Long-term yields increased in the beginning of LSAP2 mainly because central banks in many countries hiked interest rates in expectation of higher inflation. However, this tightening cycle was short (about a year) as these expectations did not materialize. Contagious effects from the Euro crisis could have also put upward pressure on the yields during this period.

<sup>3</sup>Financial institutions, for example, use short-term money market instruments in the interbank market. Thus, they may be less willing to alter their portfolio balance when there is a change in the relative prices of short-term to long-term assets.

<sup>4</sup>When long-term and short-term bonds are perfectly substitutable, exogenous changes in the relative supply of one type of asset would have no effect on the relative price of bonds (see Curdia and Woodford, 2010).



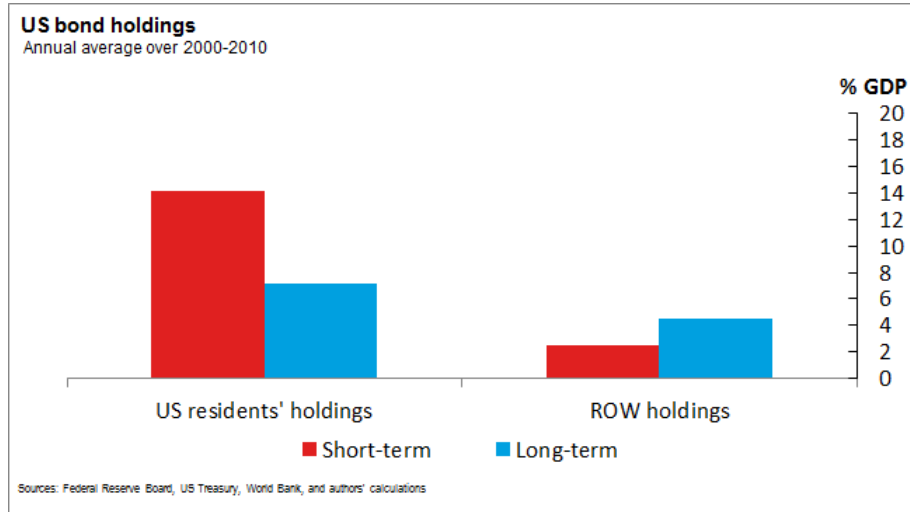
Note: EMEs include Brazil, Chile, China, Colombia, Hungary, India, Indonesia, Israel, Korea, Malaysia, Mexico, Peru, Philippines, Poland, Russia, South Africa, Taiwan, Thailand, Turkey. Small advanced economies include Australia, Canada, Denmark, Norway, Sweden, Switzerland. Select Euro members are Austria, Finland, France, Germany, Netherlands.

Figure 1: Exchange Rates and Government Yields over 2010-2014

the domestic economy does not prevent aggregate demand to increase or foreigners' currency to appreciate. This is because when household gets utility from holding financial assets, their marginal decisions between holding a short-term bond and spending, or between holding domestic short-term or foreign short-term bonds, depend not only on short-term rates but also on the quantity of short-term bonds holdings. Large increases in the U.S. short-term bond holdings following QE lowers the marginal benefit of holding these bonds, thus makes it less attractive relative to spending on consumption and investment goods or purchasing foreign bonds even when the domestic short-term rates remain constant.

Another advantage of introducing maturity structure in an open economy model is that it allows us to analyze the effects of different compositions of short-term and long-term bonds in the foreigners' portfolio. The motivation for this comes from data on foreigners' holdings of short-term and long-term U.S. government bonds. Figure 2 shows U.S. residents' and ROW holdings of U.S. government bonds as a ratio of their GDP. The picture highlights a clear difference in the maturity composition in the U.S. and ROW portfolios. In particular, U.S. residents hold twice more short-term government bonds as long-term ones. On the other hand, the ratio switches in the favor of long-term bonds over short in the ROW. This difference in the maturity composition is crucial to understand QE spillovers relative to conventional policy ones. Our results can be summarized as follows: (i) QE is effective to stimulate both domestic and foreign activity, (ii) QE spillovers are larger than standard monetary policy spillovers, (iii) spillovers increase when ROW holds more long-term U.S. bonds in their portfolio at the steady-state (conversely QE spillovers will be smaller and get close to conventional policy spillovers when ROW holds less long-term U.S. bonds similar to U.S. residents), (iv) spillovers increase if ROW substitutes less short-term bonds for long-term bonds.

Our paper is related to the DSGE literature evaluating the effects of LSAPs using a preferred habitat set-up such as Vayanos and Vila (2009), Chen et al. (2011), and Harrison (2012). Harrison (2012) analyzes the effects of asset-purchases by using a preferred short-to-long ratio in households' preferences. The difference in



Note: Short term bonds include US treasury securities with a maturity of less than one year, financial institutions' reserves at the Federal Reserve System, vault cash and currency outside banks. Long-term bonds consist of US Treasury bills with a maturity of more than one year

Figure 2: US Residents' and ROW Holdings of US Short-term and Long-term Government Bonds

our paper is that we focus on international spillovers while his is a closed-economy environment. We also use a more general CES structure for financial assets, which not only allows us a more compact way to introduce imperfect substitution among four different bonds (short-term and long-term bonds in domestic-asset and foreign-asset portfolios) but also evaluate the economic consequences of different elasticities of substitution between these bonds. Another stream of papers introduce restricted agents that hold only long-term bonds in a New-Keynesian model such as Chen et al.(2011). Dorich et al. (2012) study small open economy consequences of this type of set-up but do not study the spillover effects of QE policies to the rest of the world.

The remainder of the paper proceeds as follows. The next section introduces the model. Section 3 discusses the calibration of the model. Section 4 presents the results of the QE experiment. Section 5 conducts sensitivity analyses, and section 6 concludes.

## 2 Model

The model is a two-country large-open-economy DSGE model with real and nominal rigidities, and portfolio balance effects. The latter is achieved through modeling households' preferences on the composition of their financial portfolio, which generates imperfect substitution between short and long-term assets for both domestic and foreign sovereign debt, and therefore enables QE to have real effects on the economy.

Each country in the model is populated by households, capital producers, final goods aggregators, domestic

producers, and importers, as well as fiscal and monetary policy rules. In what follows, we focus on the agents in the domestic economy, but the foreign economy is analogous in our set-up. When variables from the foreign economy are necessary, we denote them with a (\*) superscript.

## 2.1 Households

The economy is populated by a unit measure of infinitely-lived patient households indexed by  $i$ , whose intertemporal preferences over consumption,  $c_t$ , financial asset portfolio,  $a_t$ , and labor supply,  $n_t$ , are described by the following expected utility function:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[ \log [c_{\tau}(i) - \zeta c_{\tau-1}] + \xi_a a_{\tau}(i) - \xi_n \frac{n_{\tau}(i)^{1+\vartheta}}{1+\vartheta} \right], \quad (1)$$

where  $t$  indexes time,  $\beta < 1$  is the time-discount parameter,  $\zeta$  is the external habit parameter for consumption,  $\vartheta$  is the inverse of the Frisch-elasticity of labor supply, and  $\xi_a$  and  $\xi_n$  are level parameters that determine the relative importance of financial assets and labor in the utility function.

### 2.1.1 Preferences on portfolio composition

We capture imperfect substitution across assets of different currencies and maturities by using a nested CES structure for financial assets. In particular, the asset portfolio in the utility function,  $a_t$ , is a CES aggregate of the domestic portfolio,  $a_{H,t}$ , and the foreign portfolio,  $a_{F,t}$ :

$$a_t(i) = \left[ \gamma_a^{\frac{1}{\lambda_a}} [a_{H,t}(i)]^{\frac{\lambda_a-1}{\lambda_a}} + (1-\gamma_a)^{\frac{1}{\lambda_a}} [a_{F,t}(i)]^{\frac{\lambda_a-1}{\lambda_a}} \right]^{\frac{\lambda_a}{\lambda_a-1}}, \quad (2)$$

where  $\gamma_a$  determines the share of home assets in the aggregate portfolio, and  $\lambda_a$  is the elasticity of substitution between domestic and foreign assets.

The home-asset sub-portfolio is a CES aggregate of short-term government bonds,  $B_{HS,t}$ , and long-term domestic government bonds,  $B_{HL,t}$ :

$$a_{H,t}(i) = \left[ \gamma_H^{\frac{1}{\lambda_H}} \left( \frac{B_{HS,t}(i)}{P_t} \right)^{\frac{\lambda_H-1}{\lambda_H}} + (1-\gamma_H)^{\frac{1}{\lambda_H}} \left( \frac{q_{L,t} B_{HL,t}(i)}{P_t} \right)^{\frac{\lambda_H-1}{\lambda_H}} \right]^{\frac{\lambda_H}{\lambda_H-1}}, \quad (3)$$

where  $P_t$  is the aggregate price level,  $q_{L,t}$  is the relative price of a domestic long-term bond,  $\gamma_H$  is the share of short-term bonds in the home-asset portfolio, and  $\lambda_H$  is the elasticity of substitution between short and long-term domestic bonds.

Similarly, the foreign-asset sub-portfolio is a CES aggregate of short-term foreign government bonds,

$B_{FS,t}$ , and long-term foreign government bonds,  $B_{FL,t}$ :

$$a_{F,t}(i) = \left[ \gamma_F^{\frac{1}{\lambda_F}} \left( \frac{e_t B_{FS,t}(i)}{P_t} \right)^{\frac{\lambda_F-1}{\lambda_F}} + (1-\gamma_F)^{\frac{1}{\lambda_F}} \left( \frac{e_t q_{L,t}^* B_{FL,t}(i)}{P_t} \right)^{\frac{\lambda_F-1}{\lambda_F}} \right]^{\frac{\lambda_F}{\lambda_F-1}}, \quad (4)$$

where  $e_t$  is the nominal exchange rate (in units of domestic currency per unit of foreign currency), and  $\gamma_F$  is the share of short-term bonds in the foreign-asset portfolio,  $\lambda_F$  is the elasticity of substitution between short and long-term foreign bonds, and  $q_{L,t}^*$  is the relative price of a foreign long-term bond.

### 2.1.2 Wage rigidity

Labor services are heterogenous across the patient households, and are aggregated into a homogenous labor service by perfectly-competitive labor intermediaries, who in turn rent these labor services to goods producers. The labor intermediaries use a standard Dixit-Stiglitz aggregator; therefore, the labor demand curve facing each patient household is given by

$$n_t(i) = \left( \frac{W_t(i)}{W_t} \right)^{-\eta_n} n_t, \quad (5)$$

where  $W_t$  and  $n_t$  are the aggregate nominal wage rate and labor services for patient households, respectively, and  $\eta_n$  is the elasticity of substitution between the differentiated labor services implying a mark-up of real wage over the marginal rate of substitution,  $\theta_w = \eta_n/(\eta_n - 1)$

Wage-stickiness is introduced via a quadratic cost of wage adjustment in the budget constraint similar to Rotemberg (1982),

$$\frac{\kappa_w}{2} \left( \frac{W_t(i)/W_{t-1}(i)}{\frac{\pi_{t-1}^{\varsigma_w}}{\pi_{t-1}^{1-\varsigma_w}}} - 1 \right)^2 \frac{W_t}{P_t} n_t, \quad (6)$$

where  $\kappa_w$  is a scale parameter,  $\pi_t = P_t/P_{t-1}$  is the aggregate inflation factor, and  $\varsigma_w$  determines indexation of wage adjustments to past inflation.

### 2.1.3 Budget constraint

The households' period budget constraint is given by

$$\begin{aligned} c_t(i) + q_t [k_t(i) - (1-\delta)k_{t-1}(i)] + \frac{B_{HS,t}(i)}{P_t} + \frac{e_t B_{FS,t}(i)}{P_t} + \frac{q_{L,t} B_{HL,t}(i)}{P_t} + \frac{e_t q_{L,t}^* B_{FL,t}(i)}{P_t} \\ \leq \frac{W_t(i)}{P_t} n_t(i) + r_{k,t} k_{t-1}(i) + \frac{R_{t-1} B_{HS,t-1}(i)}{P_t} + \frac{e_t R_{t-1}^* B_{FS,t-1}(i)}{P_t} + \frac{(1+\kappa q_{L,t}) B_{HL,t-1}(i)}{P_t} \\ + \frac{e_t (1+\kappa q_{L,t}^*) B_{FL,t-1}(i)}{P_t} + \frac{\Pi_{H,t}}{P_t} + \frac{\Pi_{F,t}}{P_t} - \frac{TAX_t}{P_t} - \text{wage adj. cost} \end{aligned} \quad (7)$$

where  $k_t$  is the capital stock,  $q_t$  is the relative price of capital, and  $r_{k,t}$  is the rental rate of capital.  $\Pi_{h,t}$  and  $\Pi_{f,t}$  denote the profits of monopolistically-competitive domestic producers and importers, while  $TAX_t$  is the lump-sum taxes paid to the government by the households. Short-term domestic and foreign bonds pay

a pre-determined interest rate of  $R_{t-1}$  and  $R_{t-1}^*$ , while long-term bonds are perpetuities that pay a coupon payment of 1 unit in the next period and coupon payments decaying at a rate of  $\kappa$  for each period after that as in Woodford (2001). Since these long-term bonds are tradable, we can write them in recursive form in the budget constraint above. The yields on domestic and foreign long-term bonds are defined respectively as

$$R_{L,t} = \frac{1 + \kappa q_{L,t}}{q_{L,t}} \text{ and } R_{L,t}^* = \frac{1 + \kappa q_{L,t}^*}{q_{L,t}^*}. \quad (8)$$

#### 2.1.4 Optimality conditions

The households' objective is to maximize utility subject to the budget constraint, the labor demand curve of labor intermediaries, and appropriate No-Ponzi conditions. The first-order conditions for consumption and capital are standard, and are given by

$$\frac{1}{c_t - \zeta c_{t-1}} = \lambda_t, \quad (9)$$

$$q_t = E_t \left[ \left( \beta \frac{\lambda_{t+1}}{\lambda_t} \right) [(1 - \delta) q_{t+1} + r_{k,t+1}] \right], \quad (10)$$

where  $\lambda_t$  is the Lagrange multiplier on the budget constraint. Similarly, the optimality conditions with respect to labor and wages can be combined to derive a New-Keynesian wage Phillips curve, which after log-linearization can be written as:

$$\widehat{\pi}_{w,t} - \varsigma_w \widehat{\pi}_{t-1} = \beta E_t [\widehat{\pi}_{w,t+1} - \varsigma_w \widehat{\pi}_t] - \frac{\eta_n - 1}{\kappa_w} \left( \widehat{w}_t - \vartheta \widehat{n}_t - \frac{1}{1 - \zeta} (\widehat{c}_t - \zeta \widehat{c}_{t-1}) \right), \quad (11)$$

where the nominal wage inflation,  $\widehat{\pi}_{w,t}$ , and the real wage rate,  $\widehat{w}_t$ , are related as

$$\widehat{\pi}_{w,t} - \widehat{\pi}_t = \widehat{w}_t - \widehat{w}_{t-1}. \quad (12)$$

The optimality conditions with respect to domestic short and long-term bonds are given by

$$\lambda_t = \beta E_t \left[ \lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right] + \frac{\xi_a}{a_t} \frac{\partial a_t}{\partial a_{H,t}} \frac{\partial a_{H,t}}{\partial b_{HS,t}}, \quad (13)$$

$$q_{L,t} \lambda_t = \beta E_t \left[ \lambda_{t+1} \frac{1 + \kappa q_{L,t+1}}{\pi_{t+1}} \right] + \frac{\xi_a}{a_t} \frac{\partial a_t}{\partial a_{H,t}} \frac{\partial a_{H,t}}{\partial b_{HL,t}}, \quad (14)$$

where  $b_{HS,t} = B_{HS,t}/P_t$ , and  $b_{HL,t} = B_{HL,t}/P_t$ . The two expressions above can be combined to generate the expression for the term-premium in the model; in particular, after log-linearization, the above conditions imply that the yield on long-term bonds,  $R_{L,t}$ , is a function of expected short-term rates as well as the term-premium which depends on the outstanding quantities of long versus short-term bonds:

$$\widehat{R}_{L,t} = \Omega E_t \sum_{s=0}^{\infty} \left( \beta \frac{R}{\pi} \frac{\kappa}{R_L - \kappa} \Omega \right)^s \left[ \beta \frac{R}{\pi} \widehat{R}_{t+s} + \left( 1 - \beta \frac{R}{\pi} \right) \frac{1}{\lambda_H} (\widehat{b}_{HL,t+s} - \widehat{b}_{HS,t+s}) \right], \quad (15)$$

with

$$\Omega = \frac{1}{1 - (1 - \beta \frac{R}{\pi}) \left(1 - \frac{1}{\lambda_H}\right)}. \quad (16)$$

The equation above implies that, even when short rates are kept constant (e.g. at the zero-lower-bound), the long-rate can be altered with asset purchase policies. Particularly, LSAP in the domestic economy lowers the supply of long-term bonds and, in return, increases the supply of short-term bonds through the consolidated budget constraint.<sup>5</sup> When quantities involved are large, this can lower the yields on long-term bonds, and affect aggregate demand even when short rates are constant. Note that, the portfolio preference specification in our representative agent framework is crucial for this result, and is different than standard models in the literature. For example, having lower rates by itself is not enough to stimulate the economy even if (unrestricted) households hold both types of assets in Chen et al. (2012). This is because the marginal utility of consumption depends on the short-term interest rate, which is constant at the zero lower bound. However, in our model, the representative agent's marginal utility depends not only on the short-term interest rate but also on bond quantities. To see this, observe that the first-order-condition for short-term domestic bonds (equation 13), yields the following expression after log-linearization (and setting portfolio elasticities to 1):

$$\widehat{\lambda}_t = \beta \frac{R}{\pi} \left( E_t \widehat{\lambda}_{t+1} + \widehat{R}_t - E_t \widehat{\pi}_{t+1} \right) - \left( 1 - \beta \frac{R}{\pi} \right) \widehat{b}_{HS,t}. \quad (17)$$

In the absence of the portfolio choice in preferences,  $\beta R/\pi$  would be equal to 1 at the steady-state, and the equation above becomes the standard IS curve that makes aggregate demand to depend only on the real interest rates. With our portfolio specification in preferences, the marginal benefit of holding short-term bonds diminishes as short-term bond holdings increase; this stimulates aggregate demand even when short rates are constant. Thus, our portfolio specification allows for changes in the outstanding quantity of bonds to affect demand even in a representative agent framework, unlike papers in the literature which rely on segmented markets (e.g. Chen et al., 2012, features restricted households which can only trade in long-term assets).

Note that given the calibration in Section 3, the coefficient in front of the bond quantity term in equation (17),  $1 - \beta R/\pi$  is rather small (less than 1%). Thus, for the channel emphasized in our paper to be quantitatively important, the supply of short-term bonds need to change by a significant amount. During a LSAP, a large increase in the outstanding quantity of short-term bonds will lower the willingness of agents to hold these bonds, and would stimulate aggregate demand through the "short IS" relationship in equation (17). Note that the "long IS" relationship in equation (14) is also satisfied, where lower long-term interest rates stimulate aggregate demand through this relationship.

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<sup>5</sup>This is akin to the balance sheet policies of the central bank, which buys long-term bonds by increasing its short-term liability, namely, monetary base.



The discussion above explains the domestic transmission channel of asset purchase policies. International spillovers of these policies can be illustrated through considering the optimality conditions of households with respect to foreign short and long-term bonds:

$$rer_t \lambda_t = \beta E_t \left[ \lambda_{t+1} rer_{t+1} \frac{R_t^*}{\pi_{t+1}^*} \right] + \frac{\xi_a}{a_t} \frac{\partial a_t}{\partial a_{F,t}} \frac{\partial a_{F,t}}{\partial b_{FS,t}}, \quad (18)$$

$$rer_t q_{L,t}^* \lambda_t = \beta E_t \left[ \lambda_{t+1} rer_{t+1} \frac{1 + \kappa q_{L,t+1}^*}{\pi_{t+1}^*} \right] + \frac{\xi_a}{a_t} \frac{\partial a_t}{\partial a_{F,t}} \frac{\partial a_{F,t}}{\partial b_{FL,t}}, \quad (19)$$

where  $b_{FS,t} = B_{FS,t}/P_t^*$ ,  $b_{FL,t} = B_{FL,t}/P_t^*$ , and the real exchange rate  $rer_t = e_t P_t^*/P_t$ . Note that the first-order conditions for short-term domestic and short-term foreign bonds can be combined to yield a short-term uncovered interest parity (UIP) condition. After log-linearization (and assuming unit elasticities of substitution in the nested CES for portfolio preference), this short-term UIP condition is given by

$$\widehat{R}_t - \widehat{R}_t^* = E_t \widehat{d}_{t+1} + \left( \frac{1 - \beta \frac{R}{\pi}}{\beta \frac{R}{\pi}} \right) \left[ \widehat{b}_{HS,t} - (\widehat{rer}_t + \widehat{b}_{FS,t}) \right], \quad (20)$$

where  $\widehat{d}_t = \widehat{e}_t - \widehat{e}_{t-1}$  denotes the nominal depreciation rate of domestic currency. The above condition implies that the country risk-premium is determined by the quantity of outstanding short-term home and foreign bonds. Thus, even when short-term rate differentials cannot change due to the zero lower bound, quantity policies can still affect the exchange rate through the country risk-premium. Specifically, QE in the foreign country leads to a higher supply of short-term foreign bonds, increasing domestic holdings of short-term foreign bonds relative to the short-term home bonds. Higher relative holdings of short-term foreign bonds in the home country puts downward pressure on short rates and upward pressure on the currency.

Similarly, the first-order conditions for the long-term home and foreign bonds can be combined to yield a long-term UIP condition, as follows:

$$\frac{R_L \widehat{R}_{L,t} - \kappa E_t \widehat{R}_{L,t+1}}{R_L - \kappa} - \frac{R_L \widehat{R}_{L,t}^* - \kappa E_t \widehat{R}_{L,t+1}^*}{R_L - \kappa} = E_t \widehat{d}_{t+1} + \left( \frac{1 - \beta \frac{R}{\pi}}{\beta \frac{R}{\pi}} \right) \left[ \widehat{q}_{L,t} + \widehat{b}_{HL,t} - (\widehat{rer}_t + \widehat{q}_{L,t}^* + \widehat{b}_{FL,t}) \right]$$

which suggests that the appreciation of the home currency depends on both the long-rate differential and also the relative holdings of foreign long-term bonds.<sup>6</sup> In Section 4, we show that the degree of appreciation depends on the share of long-term bonds in the foreign sub-portfolio. When the domestic country holds more long-term bonds, the percentage change in short-term bonds is larger after a LSAP in the foreign economy, which results in a larger appreciation of the domestic currency through the short-term UIP. Since the long-term bond share is large, the percentage change in foreign long-term bonds is smaller; thus, the effect of the cross-regional long-rate differentials on the exchange rate dominates the effect coming from relative bond

<sup>6</sup>Note that the expression on the left had side of the equation reflects the expected return of selling current long-term bonds in the next period.

holdings. Thus, the long-term UIP condition also validates higher appreciation of the domestic currency in equilibrium.

## 2.2 Final goods aggregators

There are two types of final goods aggregators; for consumption goods,  $c_t$ , and for investment goods,  $i_t$ . In what follows, we mainly describe the consumption goods aggregators, but investment goods aggregators are modeled in an analogous fashion.

Consumption aggregators are perfectly competitive, and they produce the final goods as a CES aggregate of home and foreign goods,  $c_{h,t}$  and  $c_{f,t}$ :

$$c_t = \left[ \gamma_c^{\frac{1}{\lambda_c}} c_{h,t}^{\frac{\lambda_c-1}{\lambda_c}} + (1-\gamma_c)^{\frac{1}{\lambda_c}} c_{f,t}^{\frac{\lambda_c-1}{\lambda_c}} \right]^{\frac{\lambda_c}{\lambda_c-1}}, \quad (21)$$

where  $\gamma_c$  denotes the share of domestic goods, and  $\lambda_c$  is the elasticity of substitution between home and foreign goods, in the consumption aggregate. For any level of output, their optimal demand for the domestic and imported goods are given by

$$c_{h,t} = \left( \frac{P_{h,t}}{P_t} \right)^{-\lambda_c} \gamma_c c_t \text{ and } c_{f,t} = \left( \frac{P_{f,t}}{P_t} \right)^{-\lambda_c} (1-\gamma_c) c_t, \quad (22)$$

where  $P_{h,t}$  and  $P_{f,t}$  are the prices of the home and foreign goods, respectively. The aggregate price index for consumption goods is given by

$$P_t = \left[ \gamma_c P_{h,t}^{1-\lambda_c} + (1-\gamma_c) P_{f,t}^{1-\lambda_c} \right]^{\frac{1}{1-\lambda_c}}. \quad (23)$$

The analogous expressions for investment goods aggregators (which are of measure  $\gamma^t$  in period  $t$ ) are given by:

$$i_t = \left[ \gamma_i^{\frac{1}{\lambda_i}} i_{h,t}^{\frac{\lambda_i-1}{\lambda_i}} + (1-\gamma_i)^{\frac{1}{\lambda_i}} i_{f,t}^{\frac{\lambda_i-1}{\lambda_i}} \right]^{\frac{\lambda_i}{\lambda_i-1}}, \quad (24)$$

$$i_{h,t} = \left( \frac{P_{h,t}}{P_t^i} \right)^{-\lambda_i} \gamma_i i_t \text{ and } i_{f,t} = \left( \frac{P_{f,t}}{P_t^i} \right)^{-\lambda_i} (1-\gamma_i) i_t, \quad (25)$$

$$P_{i,t} = \left[ \gamma_i P_{h,t}^{1-\lambda_i} + (1-\gamma_i) P_{f,t}^{1-\lambda_i} \right]^{\frac{1}{1-\lambda_i}}. \quad (26)$$

## 2.3 Domestic Firms

There is a unit measure of monopolistically competitive domestic firms indexed by  $j$ . Their technology is described by the following production function:

$$y_t(j) = [u_t(j) k_{t-1}(j)]^\alpha [n_t(j)]^{1-\alpha} - f, \quad (27)$$

where  $\alpha$  is the share of capital,  $u_t$  is the capital utilization rate, and  $f$  is a fixed cost of production.<sup>7</sup>

Domestic goods produced are heterogeneous across firms, and are aggregated into a homogenous domestic good by perfectly-competitive final goods producers using a standard Dixit-Stiglitz aggregator. The demand curve facing each firm is given by

$$y_t(j) = \left( \frac{P_{h,t}(j)}{P_{h,t}} \right)^{-\Theta_h} y_t, \quad (28)$$

where  $y_t$  is aggregate domestic output, and  $\Theta_h$  is the elasticity of substitution between differentiated goods implying a gross mark up of price over marginal cost,  $\theta_h = \Theta_h / (\Theta_h - 1)$ .

Firm  $j$ 's profits at period  $t$  is given by

$$\begin{aligned} \frac{\Pi_{h,t}(j)}{P_t} &= \frac{P_{h,t}(j)}{P_t} y_t(j) - \frac{W_t}{P_t} n_t(j) - r_{k,t} k_{t-1}(j) \\ &\quad - \frac{\kappa_u}{1 + \varpi} \left[ u_t(j)^{1+\varpi} - 1 \right] k_{t-1}(j) - \frac{\kappa_{ph}}{2} \left( \frac{P_{h,t}(j)/P_{h,t-1}(j)}{\pi_{h,t-1}^{\varsigma_h} \pi^{1-\varsigma_h}} - 1 \right)^2 \frac{P_{h,t}}{P_t} y_t, \end{aligned} \quad (29)$$

where  $\kappa_u$  and  $\varpi$  are the level and elasticity parameters for the utilization cost. Similar to wage-stickiness, price-stickiness is introduced via quadratic adjustment costs with level parameter  $\kappa_{ph}$ , and  $\varsigma_h$  captures the extent to which price adjustments are indexed to past inflation.

A domestic firm's objective is to choose the quantity of inputs and output, and the price of its output each period to maximize the present value of profits (using the households' stochastic discount factor) subject to the demand function they are facing with respect to their individual output from the aggregators. The first-order-conditions of the firm with respect to labor and capital can be combined to relate the capital-labor ratio to the relative price of inputs as

$$\widehat{w}_t - \widehat{r}_{k,t} = \widehat{u}_t + \widehat{k}_{t-1} - \widehat{n}_t. \quad (30)$$

The first-order conditions for capital and utilization can be combined to yield

$$\widehat{u}_t = \frac{1}{\varpi} \widehat{r}_{k,t}. \quad (31)$$

Finally, the first-order-condition with respect to price yields the New Keynesian Phillips curve in domestic prices as:

$$\widehat{\pi}_{h,t} = \frac{\varsigma_h}{1 + \varsigma_h \beta} \widehat{\pi}_{h,t-1} + \frac{\beta}{1 + \varsigma_h \beta} E_t \widehat{\pi}_{h,t+1} - \frac{\Theta_h - 1}{(1 + \varsigma_h \beta) \kappa_{ph}} \left[ \widehat{p}_{h,t} + z_t + \alpha \left( \widehat{u}_t + \widehat{k}_{t-1} - \widehat{n}_t \right) - \widehat{w}_t \right]. \quad (32)$$

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<sup>7</sup>The fixed cost parameter  $f$  is set equal to  $\theta_h - 1$  times the steady-state level of detrended output to ensure that pure economic profits are zero at the steady-state; hence, there is no incentive for firm entry and exit in the long-run.

## 2.4 Importers

There is a unit measure of monopolistically competitive importers indexed by  $j$ . They import foreign goods from abroad, differentiate them and mark-up their price, and then sell these heterogenous goods to perfectly competitive import aggregators, who aggregate these into a homogenous import good using a standard Dixit-Stiglitz aggregator. The demand curve facing each importer is given by

$$y_{f,t}(j) = \left( \frac{P_{f,t}(j)}{P_{f,t}} \right)^{-\Theta_f} y_{f,t}, \quad (33)$$

where  $y_{f,t}$  is aggregate imports, and  $\Theta_f$  is a time-varying elasticity of substitution between the differentiated goods implying a gross mark-up of the domestic price of imported goods over its import price  $\theta_f = \Theta_f/(\Theta_f - 1)$ .

Importers maximize the present value of profits (using the households' stochastic discount factor) subject to the demand function they are facing from the aggregators with respect to their own output. The importer's profits at period  $t$  are given by:

$$\frac{\Pi_{f,t}(j)}{P_t} = \frac{P_{f,t}(j)}{P_t} y_{f,t}(j) - \frac{e_t P_{h,t}^*}{P_t} y_{f,t}(j) - \frac{\kappa_{pf}}{2} \left( \frac{P_{f,t}(j)/P_{f,t-1}(j)}{\pi_{f,t-1}^{\zeta_f} \pi^{1-\zeta_f}} - 1 \right)^2 \frac{P_{f,t}}{P_t} y_{f,t}, \quad (34)$$

where importers face quadratic price adjustment costs, which helps generate partial exchange rate pass-through to domestic prices.  $\kappa_{pf}$  and  $\zeta_f$  are the price adjustment cost and indexation parameters respectively.

The first-order condition of importers with respect to price yields the imported-price New Keynesian Phillips curve (after log-linearization)

$$\hat{\pi}_{f,t} = \frac{\zeta_f}{1 + \zeta_f \beta} \hat{\pi}_{f,t-1} + \frac{\beta}{1 + \zeta_f \beta} E_t \hat{\pi}_{f,t+1} - \frac{\Theta_f - 1}{(1 + \zeta_f \beta) \kappa_{pf}} (\hat{p}_{f,t} - r \hat{e}r_t - \hat{p}_{h,t}^*), \quad (35)$$

where  $\pi_{f,t} = P_{f,t}/P_{f,t-1}$  is the import-price inflation factor.

The balance of payments identity in the model is given by

$$\begin{aligned} & \left( \frac{e_t B_{FS,t}}{P_t} - \frac{e_t R_{t-1}^* B_{FS,t-1}}{P_t} \right) + \left( \frac{e_t q_{L,t}^* B_{FL,t}}{P_t} - \frac{e_t R_{L,t}^* q_{L,t}^* B_{FL,t-1}}{P_t} \right) \\ & - \left( \frac{B_{FS,t}^*}{e_t P_t} - \frac{R_{t-1} B_{FS,t-1}^*}{e_t P_t} \right) - \left( \frac{q_{L,t} B_{FL,t}}{e_t P_t} - \frac{R_{L,t} q_{L,t} B_{FL,t-1}^*}{e_t P_t} \right) = \frac{P_{h,t}}{P_t} y_{f,t}^* - \frac{e_t P_{h,t}^*}{P_t} y_{f,t}. \end{aligned} \quad (36)$$

## 2.5 Capital producers

Capital producers are perfectly competitive. After goods production takes place, these firms purchase the undepreciated part of the installed capital from entrepreneurs at a relative price of  $q_t$ , and the new capital investment goods from final goods firms at a price of  $P_{i,t}$ , and produce the capital stock to be carried over to the next period. This production is subject to adjustment costs in the change in investment, and is described

by the following law-of-motion for capital:

$$k_t = (1 - \delta) k_{t-1} + \left[ 1 - \frac{\varphi}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 \right] i_t, \quad (37)$$

where  $\varphi$  is the adjustment cost parameter.

After capital production, the end-of-period installed capital stock is sold back to entrepreneurs at the installed capital price of  $q_t$ . The capital producers' objective is thus to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left[ q_t i_t - q_t \frac{\varphi}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 i_t - \frac{P_{i,t}}{P_t} i_t \right], \quad (38)$$

subject to the law-of-motion of capital, where future profits are again discounted using the patient households' stochastic discount factor. The first-order-condition of capital producers with respect to investment yields the following investment demand equation (after log-linearization):

$$\widehat{i}_t - \widehat{i}_{t-1} = \beta E_t \left[ \widehat{i}_{t+1} - \widehat{i}_t \right] + \frac{1}{\varphi} (\widehat{q}_t - \widehat{p}_{i,t}). \quad (39)$$

## 2.6 Monetary and fiscal policy

The central bank targets the nominal interest rate using a Taylor rule

$$\log R_t = \rho \log R_{t-1} + (1 - \rho) \left( \log R + r_\pi \log \frac{\pi_t}{\pi} + r_y \log \frac{y_t}{y} + r_{\Delta y} \log \frac{y_t}{y_{t-1}} \right) + \varepsilon_{r,t}, \quad (40)$$

where  $R$  is the steady-state value of the (gross) nominal policy rate,  $\rho$  determines the extent of interest rate smoothing, and the parameters  $r_\pi$ ,  $r_y$ , and  $r_{\Delta y}$  determine the importance of inflation, output gap and output growth in the Taylor rule.  $y$  is the detrended steady-state level of output, and  $\varepsilon_{r,t}$  is a monetary policy shock which follows an AR(1) process.

The consolidated government budget constraint is given by

$$g_t + \frac{R_{t-1}}{\pi_t} b_{S,t-1} + \frac{R_{L,t}}{\pi_t} q_{L,t} b_{L,t-1} = \frac{TAX_t}{P_t} + b_{S,t} + q_{L,t} b_{L,t}, \quad (41)$$

where  $b_S$  and  $b_L$  represent total short-term and long-term government debt, respectively. Lump-sum taxes adjust with the level of government debt to rule out a Ponzi scheme for the government:

$$\frac{TAX_t}{P_t} = \Xi y \left( \frac{y_t}{y} \right)^{\tau_y} \left( \frac{b_{S,t-1} + q_{L,t-1} b_{L,t-1}}{b_S + q_L b_L} \right)^{\tau_b}, \quad (42)$$

where  $\Xi$  is a level parameter,  $\tau_y$  is the automatic stabilizer component of transfers, and  $\tau_b$  determines the response of transfers to total government debt.

Finally, government controls the supply of long-term bonds in real terms following an AR(1) process:

$$\log(q_{L,t}b_{L,t}) = (1 - \rho_b) \log(q_L b_L) + \rho_b \log(q_{L,t}b_{L,t-1}) + \varepsilon_{b,t}$$

where  $\rho_b$  governs the persistence of market-value of long-term bonds and  $\varepsilon_{b,t}$  represents the unconventional monetary policy shock (i.e., QE shock) in the model.

## 2.7 Market clearing conditions

The domestic goods are used in the final goods production for consumption, investment, government expenditure, and exports:<sup>8</sup>

$$c_{h,t} + i_{h,t} + g_t + y_{f,t}^* = y_t. \quad (43)$$

Similarly, the imported goods are used only for consumption and investment; hence:

$$c_{f,t} + i_{f,t} = y_{f,t}. \quad (44)$$

The model's equilibrium is defined as prices and allocations such that households maximize discounted present value of utility, all firms maximize discounted present value of profits, subject to their constraints, and all markets clear.

## 3 Calibration

In our benchmark calibration, we set the structural parameters of both countries to equal values, except for the portfolio and labor level parameters in preferences, as well as the tax level parameter in the tax policy function.<sup>9</sup> We calibrate the parameters using steady-state relationships in the model and US data from the National Income and Product Accounts (NIPA; Bureau of Economic Analysis) and the Flow of Funds Accounts (FOF; Federal Reserve Board) averaged over the post-war period.<sup>10</sup> We first discuss the choice of the parameters governing portfolio and preferences, and then parameters related to technology and government policy. A list of parameter values is given in Table 1.

*Portfolio.* We calibrate the share of short-term assets in the domestic sub-portfolio,  $\gamma_H$ , to 0.66 based on US residents' portfolio share in short-term government liabilities (see Figure 2). Short-term bonds include privately held marketable U.S. treasury securities with a remaining maturity of less than one year and the monetary base (i.e., financial institutions' reserves at the Federal Reserve System, vault cash and currency outside banks) as in Chen et al. (2012). Monetary base is included since it is a perfect substitute for short

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<sup>8</sup>Note that utilization costs and financial costs are modeled as transfer payments to households.

<sup>9</sup>A non-zero net-foreign-asset position requires these parameters to be different across regions. See below for detailed information on these parameters.

<sup>10</sup>For portfolio parameters, we use data averaged over 2000-2010 as in Figure 2 reflecting more recent government bond supply and international developments in the financial markets.

term T-bills under the zero lower bound. The short-term asset share in the foreign asset sub-portfolio,  $\gamma_F$ , is calibrated to 0.36 based on FOF data (see table for the Rest of the World Accounts). Foreign holdings of short-term bonds include U.S. treasury securities with a remaining maturity of less than one year and U.S. dollar currency.<sup>11</sup> We assume only U.S. bonds are traded internationally, therefore, the share of the domestic sub-portfolio for the US,  $\gamma_A^*$ , is set to 1. As for the share of domestic-asset portfolio in the ROW's total portfolio,  $\gamma_A$ , we calculate it using the ratios in Figure 2. ROW holdings of both short and long-term US government liabilities is around 7% of their GDP. Their domestic government bond holdings are assumed to be the same as that of US residents, i.e. around 21% of US GDP. This implies a share of domestic-asset portfolio of 0.75 for the ROW. When calculating ROW GDP we use output time-series from all countries except the U.S. and the low-income economies in the World Bank classification.<sup>12</sup> All elasticity of substitution parameters in the nested CES specification for portfolio choice are set to unity. We do sensitivity analyses on these parameters below. The coupon rate on long-term bonds,  $\kappa$ , is calibrated to imply a duration of 30 quarters for long-term bonds, similar to the average duration of 10-year US Treasury bills in the secondary market.

*Preferences.* We calibrate the time discount factor,  $\beta$ , to match a target capital-output ratio,  $k/y$ , of 10 using the the optimality condition for household's capital decision at the steady-state. Traditionally, the discount factor is calibrated to match steady-state interest rates using the short-term bond first-order condition. However, we use this condition to calibrate the portfolio level coefficient,  $\xi_a$ , in preferences using  $a/y$  ratios both in the US and the ROW: 0.05 and 0.04 in the ROW and in the US, respectively. Since ROW holds a higher level of government assets as a proportion of its output, its portfolio level coefficient is calculated to be larger than in the US. We set the habit parameter,  $\zeta$ , to 0.70, close to values found in Smets and Wouters (2007) and Adolfson et al. (2008). The Frisch elasticity of labor supply,  $1/\vartheta$ , is set to 1. This value is in line with the estimates presented in Blundell and Macurdy (1999), and represents a compromise between the estimates in the Real Business Cycle and New Keynesian literatures (Smets and Wouters, 2007).

Labor coefficient,  $\xi_n$ , is calibrated to 32% to match working hours as a ratio of non-sleeping hours of the economically active population. Note that a non-zero trade balance requires consumption-output ratio or investment-output ratio to be different in the two regions when the same home bias parameters are assumed in both regions' aggregator functions. We choose,  $c/y$ , to be different across regions, which implies a different labor level coefficient in preferences based on the optimal labor supply at the steady state.

*Technology.* We calibrate the capital share in home goods production,  $\alpha$ , to 0.34 in order to match a labor income share of 66%. Depreciation rate,  $\delta$ , is calibrated to match an investment-output ratio,  $i/y$ , of 19%.

<sup>11</sup>FOF data reports holdings of US treasury securities with the *original* maturity. We adjust them so that short-term holdings include long-term securities with a remaining maturity of less than one year using the Treasury data on the maturity of privately-held treasury securities. When distributing long-term securities with a remaining maturity of less than one year over ROW and US residents, we use the weights in holdings of treasury bills (i.e., original maturity of less than one year) from FOF tables.

<sup>12</sup>We exclude low-income economies as they have limited access to international financial markets.

Home-bias parameters in the consumption and investment aggregators are both set to 0.9, implying a 10% of import (and export) to output ratio along the steady state. Elasticity of substitution parameters in these aggregators are similar to those used in New-Keynesian DSGE literature (see Gertler et al., 2007). Similarly, the mark up and indexation parameters in the labor and goods markets (both for domestic producers and importers) are set using values in the literature.<sup>13</sup>

Adjustment cost parameters on prices and wages,  $\kappa_{ph}$ ,  $\kappa_{pf}$ ,  $\kappa_w$ , are calibrated so that the resulting New Keynesian Phillips curves have slopes equivalent to assuming Calvo probabilities of 0.9 and 0.85 for wages and prices, respectively. Investment adjustment cost parameter,  $\varphi$ , is calibrated so that investment is 2.5 times more volatile than output with a standard monetary policy shock. The capacity utilization elasticity,  $\varpi$ , is set to 0.12, while the adjustment cost level parameter on utilization,  $\kappa_u$ , is calibrated to imply a unit utilization rate at the steady-state without loss of generality.

*Government Policy.* Taylor Rule parameters are set to values close to those found in the literature (see Smets and Wouters, 2007 and Adolfson et al., 2008). The interest rate smoothing parameter,  $\rho$ , is set to 0.75, and inflation sensitivity,  $r_\pi$  is set to 1.75. The literature typically finds small response coefficients for output gap and output growth. Thus, we set these to 0.05 in our benchmark calibration. We set the elasticity parameters in the tax function,  $\tau_y$  and  $\tau_b$ , large enough to ensure a sustainable debt path (see Chen et al., 2012). We make sure that debt converges within 10 years. The tax level parameters in the two countries are calibrated to match a steady state annualized interest rate of 4.4% using the government budget constraint expression. Note that these parameters have to be different across the two regions since we have different consumption-output ratios to match a non-zero trade balance. The resulting government-output ratios are 17% and 18% for the ROW and the U.S., and the tax level parameters are 0.17 and 0.19 for the ROW and the U.S., respectively.

## 4 Results

In this section, we first use our model to evaluate the impact of QE on both domestic and foreign activity. We then compare the spillover effects of QE in the U.S. (foreign economy) to the rest of the world (domestic economy) with those from conventional policy.<sup>14</sup>

### 4.1 The impact of a QE shock

The QE shock is calibrated to match \$600 billion drop in the privately-held long-term US government bonds similar to the purchase amount announced for LSAP2 in the last quarter of 2010. Following Chen et al.

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<sup>13</sup>Note that we have conducted sensitivity analysis using different values for these parameters. Their effect on key results is only modest compared to the portfolio parameters.

<sup>14</sup>We use a first-order approximation of the model to obtain our results and use IRIS routines for all simulations.



(2012), we assume no change in interest rates for four quarters after the asset purchase announcement, after which the central bank keeps its balance sheet size constant for eight quarters and then gradually sells these bonds over the next eight quarters. The assumption of no change in interest rates in the first four quarters following the LSAP announcement is consistent with survey evidence from Blue Chip in 2011 (Chen et al., 2012). Note that the whole path of the aforementioned QE policy is known by all agents at the impact period.

Figure 3 shows the impulse responses of U.S. variables after a QE shock in the US. Due to imperfect substitution between domestic short and long-term bonds, long rates in the U.S. fall by slightly more than 45 bps consistent with estimates found for LSAP2.<sup>15</sup> Short-term bond holdings of U.S. residents increase as well. Lower long rates and higher short holdings stimulate aggregate demand through the long and short IS curves, respectively. As mentioned in Section 2, we do not need restricted households in our set-up, unlike Chen et al. (2012), since the increase in the quantity of short-term holdings of the representative household lowers the marginal benefit of holding short-term bonds even when short-term rates are fixed. This leads to households switching expenditures to consumption and investment stimulating aggregate demand. GDP increases by 1.4% due to the increase in consumption, investment and the improvement in the trade balance. The trade balance improves mainly due to the increase in exports as a result of the 4.0% depreciation in the U.S. dollar, while the immediate impact on imports is insignificant as income and price effects offset each other.

The impulse responses of the ROW variables are shown in Figure 4. The international effects of the QE spill over to the other country mainly through the short and long-term UIP conditions. QE generates a cross-country differential in long-term rates at impact, which puts downward pressure on ROW long-term rates and appreciation pressures on its currency (see the long-term UIP condition). The resulting appreciation also has to satisfy the short-term UIP condition. Although short-term rates are not changing in the US, the quantities would matter for the exchange rate and the ROW short-term rate. The relative increase in ROW holdings of short-term U.S. bonds pushes the value of the ROW currency upwards, consistent with the direction implied by the long-term UIP condition. As a result, the ROW currency appreciates, which leads to lower inflation in the ROW, which in turn leads to a decline in current and expected short-term (policy) rates in the ROW economy. Long rates fall by 5.5 bps in the ROW as a result of these expectations. The declines in short and long-term rates generate increases in consumption and investment through the IS equations.

Appreciation of the ROW currency, along with an increased demand for consumption and investment, increases imports in the ROW. As mentioned earlier, U.S. demand for ROW goods does not change significantly on impact. This results in lower net exports for ROW putting a negative pressure on their GDP. However, stimulus coming from the domestic channel in the ROW dominates the fall in net exports, and generates an overall increase in output. These quantitative results highlight that QE spillover from the U.S. to the ROW

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<sup>15</sup>D'Amico et al. (2011) find a 55 bps decrease in 10-year Treasury bills due to LSAP2, while Krisnamurthy and Vissing-Jorgensen (2011) find 33 bps. See Bernanke (2012) for more on the effects of LSAP2 in the US.

economy is mainly through financial channels, and not through the trade channel (i.e., not through higher demand for ROW goods). Finally, similar to the U.S. responses, ROW starts to hold more short-term bonds and less long-term U.S. bonds.

The result of increased US short-term bond holdings in the ROW merits some discussion. Note that even though the ROW has a flexible exchange rate regime (i.e., does not conduct any foreign exchange intervention to offset the currency appreciation pressures during the U.S. QE), the imperfect substitution between assets leads the ROW agents to increase short-term U.S. bond holdings. If fixed or managed exchange rate regimes are considered, the results on short holdings would potentially be amplified. In the extreme case of fixed exchange rates, the ROW long-term rates would fall even more, which would necessitate larger decreases in current and expected short rates. As U.S. short rates are fixed, through the short UIP condition, this is only possible when there are further increases in U.S. short-term bond holdings in the ROW. The quantitative effects of such a regime, and reserve accumulation in U.S. bonds due to this type of policy, is beyond the scope of this paper and is left for future research.

## 4.2 QE shock versus interest rate shock

The direction of above-mentioned spillovers are consistent with the responses of standard interest rate cuts in the US. We now compare quantitative differences between the spillovers from a QE shock and a conventional interest rate shock. For comparison, we scale the interest rate shock to give the same output response that QE shock delivers, which is around 1.4% of steady-state GDP. Table 2 compares these shocks in our model, as well as a modified version of our model with a lower long-term bond share in ROW's foreign-asset portfolio.

In our baseline model, the QE shock implies an increase in the ROW economic activity that is twice as high as that implied from an interest rate shock. The depreciation of the U.S. dollar under the QE shock is also larger than that under the interest rate shock, and ROW long-term rates fall more under the QE shock as well. These amplified spillovers are mainly driven by the larger decrease in U.S. long-term rates after the QE shock. The interest rate shock has a smaller impact on the long-term rates since its bond quantity implications on the "long IS curve" are not as severe as QE policy. Therefore, the long-term interest rate differential between regions is higher under QE shock, which results in higher spillovers compared to interest rate shock through the long UIP condition.

As can be observed from the last two columns of Table 2, if the share of long-term bonds in the ROW's foreign portfolio were smaller, the QE shock starts to act more similar to an interest rate shock. In the next section, we conduct sensitivity analysis by considering different parameter values for the long-term bond shares in the ROW's foreign portfolio as well as different values for the elasticities between assets in the portfolio specification.

## 5 Sensitivity Analysis

### 5.1 The share of long-term bonds in the foreign-asset portfolio

Figure 5 shows the domestic effects and the international spillover effects of QE with extreme values for the share of long-term bonds in the ROW's foreign-asset portfolio. As the ROW holds more long-term bonds in their foreign portfolio in the steady-state, the effects of QE in both the U.S. and the ROW economic activities increase. This is because the same QE shock results in higher relative supply changes in long-term bonds, and thus lowers long rates more, as the long share increases. In the extreme case where the foreign portfolio consists of only long-term bonds, the ROW currency appreciates by 9%, and long-term rates fall by 12 bps, compared to 4.0% and 5.5 bps, respectively, under the baseline case. These results show that QE becomes more effective when steady-state international holdings of U.S. long-term bonds increase.

Conversely, when ROW does not hold any long-term US government bonds, QE does not significantly transmit cross-border, but only affects U.S. economic activity, albeit smaller than under the baseline case. Another interesting result is that, though very small, international spillovers are not zero in this case as the worldwide increase in the supply of short-term US bonds after QE slightly increases ROW holdings of U.S. short-term bonds as well. This makes U.S. bonds relatively less attractive, and results in the depreciation of the U.S. dollar through the short-term UIP condition. This special case can be seen as the isolated effect of portfolio in preferences without the long UIP condition present in the baseline model. The fact that international spillovers are very small in this case shows that introduction of long UIP in the model is crucial to understand the international effects of LSAPs quantitatively.

### 5.2 Elasticity of substitution parameters in the portfolio specification

We now analyze the sensitivity of results to elasticity parameters in our portfolio specification. We start with the elasticity of substitution between long-term and short-term bonds in the domestic-asset portfolio,  $\lambda_H$ . Figure 6 shows the results for different values of  $\lambda_H$ ; namely when  $\lambda = 0.25, 1, 25$ .<sup>16</sup> This figure suggests that spillovers increase when bonds in the domestic-asset portfolio are less substitutable with each other. A lower degree of substitution amplifies the effect of a change in the relative supply of bonds, resulting in a larger change in the relative prices of assets and a bigger fall in long-term rates, which in turn stimulates the domestic economy more. Foreign economic activity increases more as well through the financial channel. Conversely, a value as large as 25 for this elasticity parameter implies almost no spillovers since short and long-term bonds are almost perfectly substitutable.

Similar results are obtained when we change the elasticity of substitution between short-term and long-term bonds in the ROW's foreign-asset portfolio,  $\lambda_F$  (see Figure 7). As we get close to perfect substitution,

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<sup>16</sup>Since both regions hold a domestic portfolio, we change this parameter for both regions. However, as there is no QE in the ROW, keeping elasticity the same in the ROW's domestic portfolio implies similar results.

relative prices between assets remain unchanged due to arbitrage between these assets. Conversely, if ROW substitutes less short-term U.S. bonds for long-term ones, spillovers increase as this leads to a larger decline in U.S. long-term rates. Note that the increase in spillovers are smaller in Figure 7 relative to those in Figure 6 when we move from the baseline case to  $\lambda = 0.25$ . This result arises because the ROW holds both domestic-asset and foreign-asset sub-portfolios, and there is imperfect substitution between these sub-portfolios as well. However, results in Figure 6 mainly arises from the U.S. residents' portfolio. The fact that U.S. residents do not hold a foreign-asset portfolio makes the results more sensitive to the elasticity parameter,  $\lambda_H$ .

Finally, the elasticity parameter between domestic-asset and foreign-asset sub-portfolios in the ROW portfolio,  $\lambda_a$ , does not matter much (see Figure 8). QE changes the relative supply between short and long-term bonds, but not between U.S. and ROW total assets. When QE occurs, the supply of long-term U.S. bonds decreases while the supply of short-term U.S. bonds increases. As a result, the total supply of U.S. bonds relative to the total supply of ROW bonds do not change that much, implying a muted effect from a change in this elasticity parameter.

## 6 Concluding Remarks

[TO BE COMPLETED]

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Table 1: Parameter Values

Parameter	Value	Source	Parameter	Value	Source
<i>Portfolio</i>			<i>Preferences</i>		
Short share (dom.), $\gamma_H$	0.66	calibrated	Discount factor, $\beta$	0.98	calibrated
Short share (for.), $\gamma_F$	0.36	calibrated	Cons. Habit, $\zeta$	0.7	literature
Domestic share, $\gamma_A$	0.75	calibrated	Labor Elasticity, $\vartheta$	1	literature
Domestic share, $\gamma_A^*$	1	set	Portfolio Coeff., $\xi_a$	0.05	literature
Elasticity, $\lambda_A, \lambda_H, \lambda_F$	1	set	Portfolio Coeff., $\xi_a^*$	0.04	literature
Coupon, $\kappa$	0.98	calibrated	Labor Coefficient, $\xi_n$	22.46	calibrated
			Labor Coefficient, $\xi_n^*$	23.10	calibrated
<i>Technology</i>			<i>Taylor Rule and Gov't</i>		
Home biasness, $\gamma_c, \gamma_i$	0.90	calibrated	R persistence, $\rho$	0.80	literature
Elas. H and F cons., $\lambda_c$	1	literature	Inflation sensitivity, $r_\pi$	1.75	literature
Elas. H and F inv., $\lambda_i$	0.25	literature	Output gap sensitivity, $r_y$	0.05	literature
Mark-up, $\theta_w, \theta_h, \theta_f$	1.5	literature	Output growth sensitivity, $r_{\Delta y}$	0.05	literature
Indexation, $\varsigma_w, \varsigma_h, \varsigma_f$	0.50	literature	Tax coefficient, $\Xi$	0.17	calibrated
Price adj. cost, $\kappa_{ph}, \kappa_{pf}$	35	calibrated	Tax coefficient, $\Xi^*$	0.19	calibrated
Wage adj. cost, $\kappa_w$	80	calibrated	Elasticities in tax policy, $\tau_y, \tau_b$	0.5	set
Capital exponent, $\alpha$	0.34	calibrated	QE shock persistency, $\rho_b$	0.95	set
Depreciation rate, $\delta$	0.02	calibrated			
Inv. adj. cost, $\varphi$	2	calibrated			
Utilization elasticity, $\varpi$	0.12	literature			
Utilization adj. cost, $\kappa_u$	0.03	calibrated			

Note: The variables with star (\*) reflect different values for the US.

Table 2: Unconventional versus Conventional Policy Shocks

	Baseline		Lower long share	
	$\gamma_F = 0.36$		$\gamma_F = \gamma_H = 0.66$	
	QE Shock	R Shock	QE Shock	R Shock
US GDP	1.40	1.40	1.10	1.10
ROW GDP	0.30	0.16	0.13	0.16
Dep.(USD)	4.00	3.40	2.05	2.60
ROW $R_L$	-0.05	-0.02	-0.02	-0.03

Figure 3: Domestic (US) Responses to a QE shock

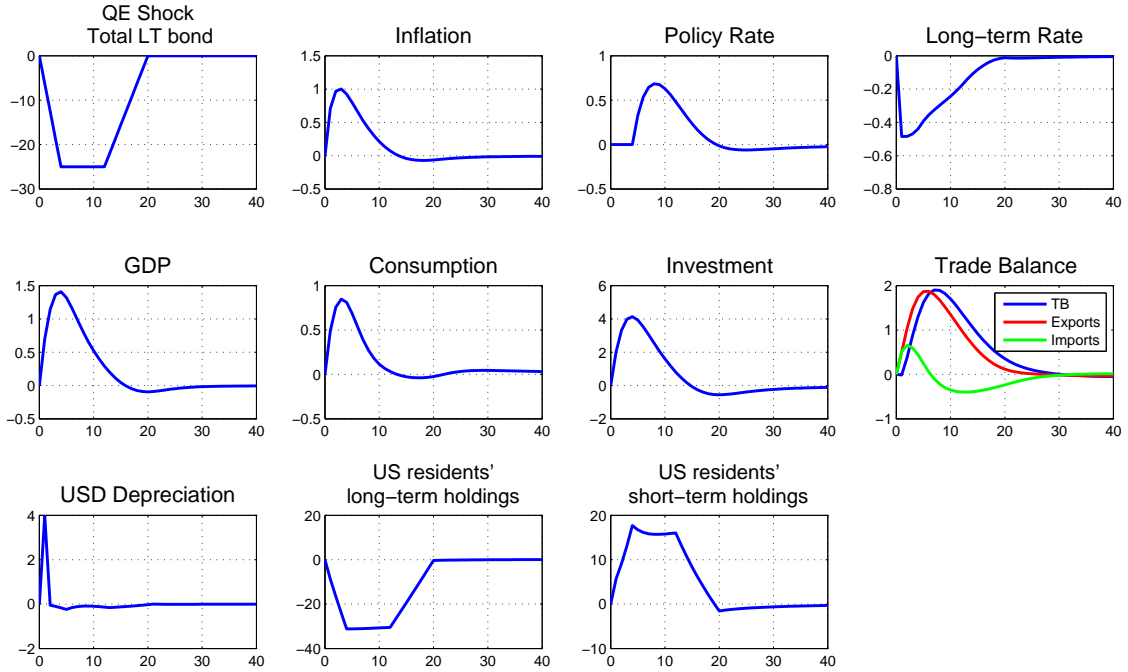


Figure 4: ROW Responses to a QE Shock

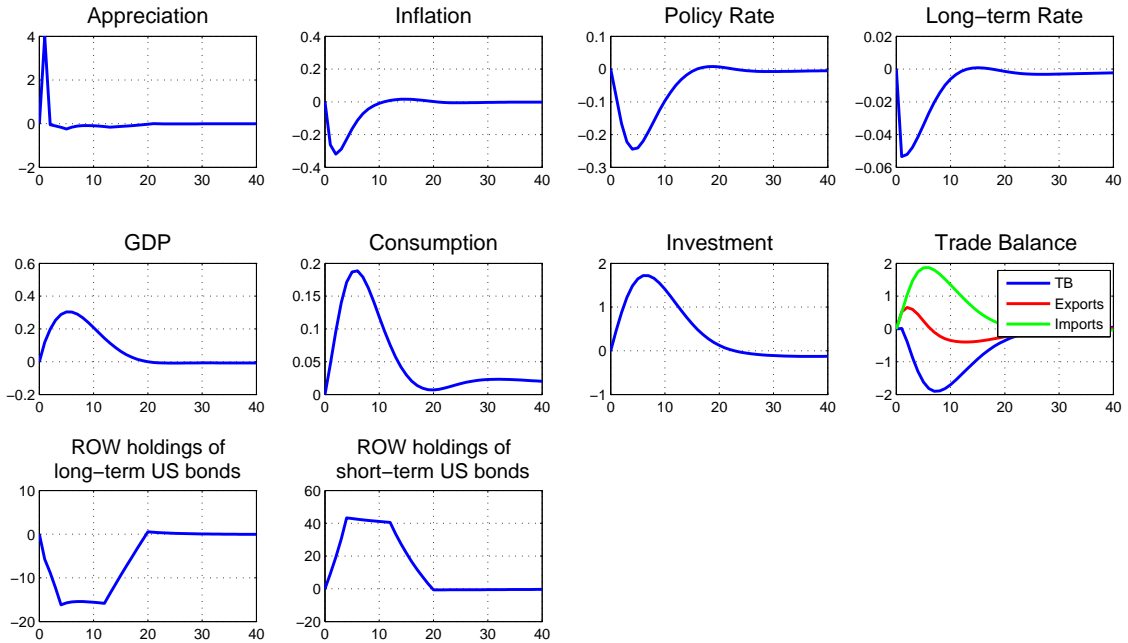




Figure 5: Results of Different Shares of US Long-term bonds in the ROW Foreign-asset portfolio

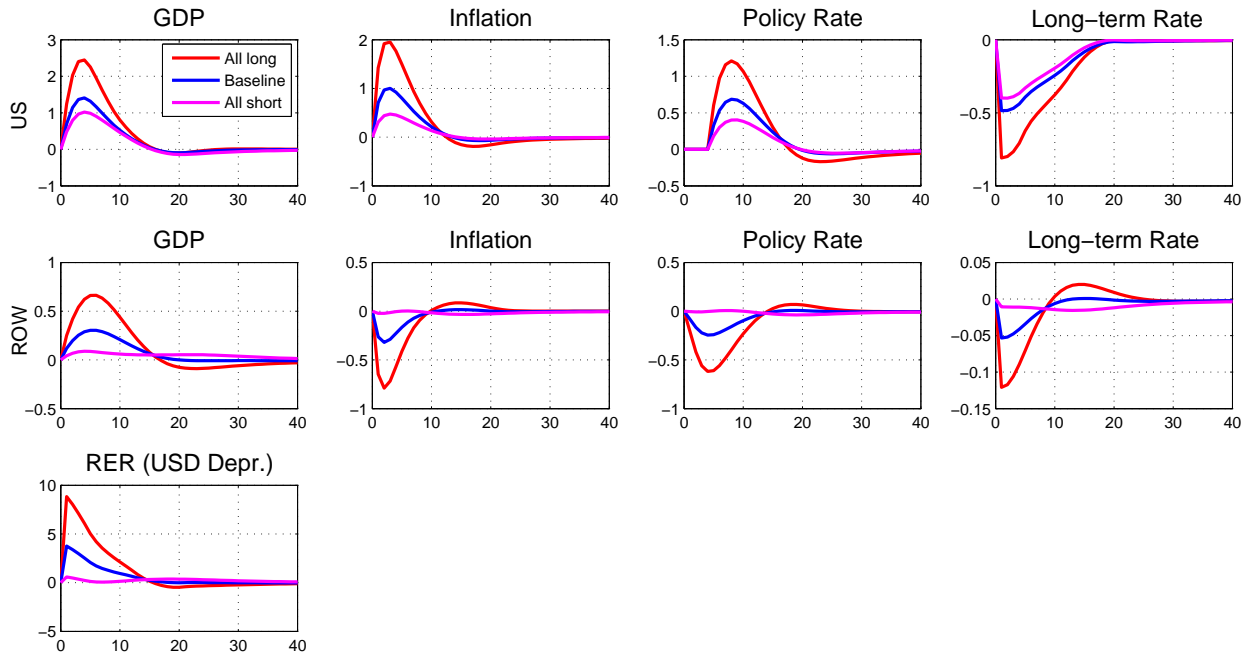


Figure 6: Results of Different Elasticity of Substitution between (domestic) ST and LT bonds

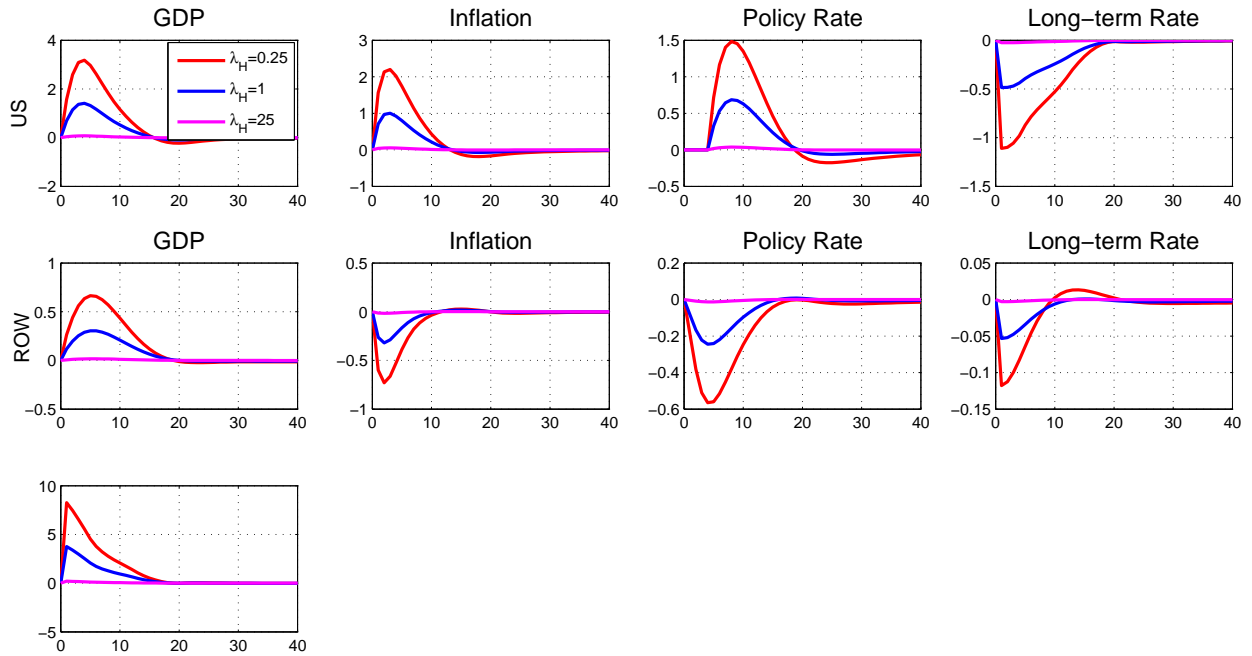


Figure 7: Results of Different Elasticity of Substitution between (foreign) ST and LT bonds

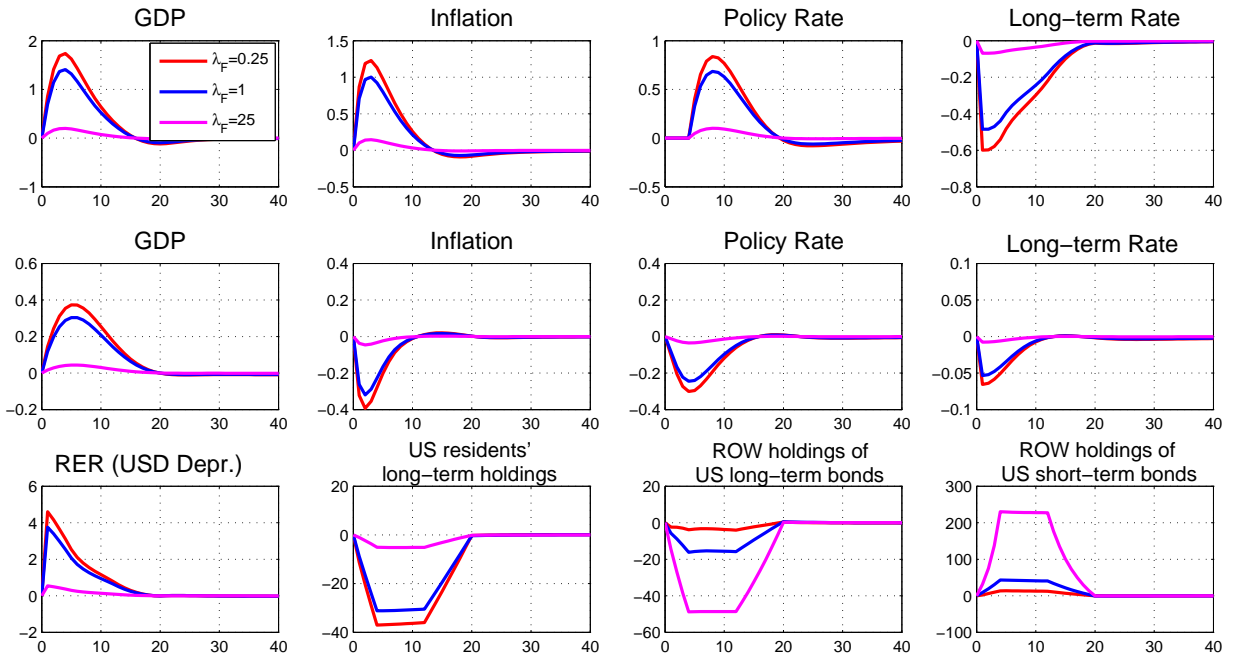


Figure 8: Results of Different Elasticity of Substitution between domestic and foreign portfolio

