Income Inequality and Sovereign Debt Default*

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Abstract

Emerging markets are characterized by higher rate of sovereign default and higher income inequality compared to advanced economies. In this paper we try to show how income inequality plays a role in government's borrowing and default decisions. We introduce inequality shocks and heterogenous agents to a standard endogenous sovereign debt default model. Inequality shocks generate another source of uncertainty that is correlated with income uncertainty, such that the dispersion of income across individuals increases during a recession and decreases in an expansion. Our model is able to generate a probability of default that is around 3 percent. Compared to a model with only output shocks this is about ten times larger and matches the default probability observed in the data. Our model can also generate business cycle statistics observed in emerging economies such as countercyclical interest rates, high volatility in trade balance and consumption and trade balance that is positively correlated with the interest rates.

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1 Introduction

Emerging markets tend to experience higher rate of sovereign default and high volatility of output compared to advanced economies. Even though recessions may trigger defaults, countries do not experience defaults only when output is below the trend. Tomz and Wright (2007) empirically show that only 60 percent of the defaults are at a time when output is below the trend. Another important characteristic of the emerging economies is that they are characterized by high income inequality. According to the OECD report (OECD, 2011), income inequality in some of the largest emerging economies (Argentina, Brazil, China, India, Indonesia, the Russian Federation and South Africa) is about three times higher than the OECD average. So far the literature has mostly focused on the effects of output shocks on government's borrowing and default decision. These models of sovereign default are usually representative agent models with incomplete debt markets and they generate defaults when there is a bad output shock. However, based on these two empirical facts it is important to analyze, whether income inequality plays a role in sovereign borrowing and default decisions. Is income inequality an important source of risk that increases the probability of default?

In this paper we show that inequality shocks can be an important source of risk that increases the probability of default in a small open economy. We introduce inequality shocks and heterogeneous agents in a standard endogenous sovereign debt default model that follows the seminal paper of Eaton and Gersovitz (1981). Models based on Eaton and Bersovitz have a benevolent government that borrows from foreign lenders using one period bonds and distributes the proceeds from the debt payments to households as transfers. Since the government shares the preferences of the households, the government uses foreign loans to insure against uncertainty in the future income of households. The government always has the option to default on its debt and the penalty of default is autarky. However these models generate a low probability of default, since the default penalty is not a severe punishment. Therefore in order to match the empirical probability of 2-3 percent, either large shocks or severe punishment schemes are necessary.[1] In our model inequality shocks generate another source of uncertainty that is correlated with income uncertainty such that the dispersion of income across individuals in the economy increases during a recession and decreases during an expansion. In addition to the standard risk sharing motive due

[1] Aguiar and Gopinath (2006) and Yue (2010) use shocks to the trend of output and Arellano (2008) uses asymmetric drops in output to generate a reasonable default rate.

to income uncertainty, in our model the government also tries to eliminate the dispersion in consumption by borrowing. To highlight the importance of inequality shocks, we use a quantitative exercise to compare the effects of output shocks with and without inequality shocks. We find that a model with inequality shocks is able to generate a default rate of 3 percent, whereas for a model with only output shocks the default rate is around 0.4 percent.

The success of the model with inequality shocks in generating large default probabilities comes from its price schedule not being very steep over the range of risky borrowing compared to the model where there are only output shocks. In the latter case, the price schedule is very sensitive to changes in borrowing. Since the welfare gain in smoothing consumption due to output shocks is small, the lenders provide fewer loans and there is minimal borrowing in equilibrium and small rate of default. On the other hand in a model with inequality shocks, the borrowing can be constrained even in expansions, if there is high inequality in the economy. Since both of these shocks are persistent, and there is a negative relationship between output and inequality, such that high inequality today reduces the output tomorrow, default becomes more preferable in high inequality states.

To show that inequality plays an important role, we first document empirical evidence between default decisions and Gini coefficients using a panel dataset that covers 40 years and 67 countries. We then estimate this relationship controlling for several macro variables and find that high inequality significantly increases the probability of default. We also show that over the business cycle, the inequality index is counter-cyclical for the average country in our sample, which is in line with Krueger et al. (2010) who show that inequality at the bottom of the distribution increases sharply during recessions.

For a quantitative analysis of the model, we estimate a VAR(1) process for output and income inequality using quarterly Argentine data. Since we only have income inequality data at annual frequency, we employ the Boots-Feibes-Lisman method to disaggregate the series into quarterly frequency. Then, we generate income and inequality shocks that come from the estimated processes and feed them into our model to compute business cycle statistics. The model is calibrated using Argentinian data and we simulate the model to generate business cycle statistics. In order to shed light on the role of inequality shocks, we simulate the model with output shocks and with both inequality and output shocks. We find that the model can generate a default probability of 3 percent, as well as consumption that is more volatile than output, counter-cyclical interest rate spread and trade balance that is positively correlated with the interest rates.

Our paper is related to various strands of literature on sovereign debt default and inequality. Endogenous sovereign default literature starts with the seminal paper of Eaton and Gersovitz (1981).[2] Our model is influenced by the model setup presented by Arellano (2008) and Aguiar and Gopinath (2006) where they try to explain counter-cyclical spreads and deep recessions that are observed in emerging market defaults. However, Arellano (2008) assumes homogeneous agents that face only aggregate income shocks and Aguiar and Gopinath (2006) use a setup that relies on the observation that emerging markets are characterized as having a volatile trend. D'Erasmo and Mendoza (2013) study the distributional effects of sovereign debt default in a heterogeneous agents model, in which agents differ in their initial wealth, and they show that default is more likely when wealth inequality is higher. However, they assume that the government only borrows from the households in a closed economy model. Since their model is only over two periods, it has limitations such that the effects of self insurance motives by using public debt markets are minimized. One stream of literature that is related to income inequality tries to analyze the determinants of the changes in inequality and often uses the Gini coefficient as a measure of inequality. They state that income inequality can be subject to shocks as well. In our model we abstract from other determinants of income inequality and model it as an exogenous variable.[3]

The rest of the paper is organized as follows: We provide empirical results, showing the relationship between income inequality and default probabilities using panel data in section 2. Section 3 presents the model and defines the recursive equilibrium. Section 4 discusses the calibration, the quantitative analysis of the model and the simulation results. Section 5 concludes.

[2] Also see papers in this literature such as, Yue (2010), Wright (2011), and Aguiar and Amador (2013)

[3] The inequality shocks can be also interpreted as natural events that affect the income inequality as in Reardon and Taylor (1996) or they can be interpreted as the shocks that increases the cross-sectional dispersion of individual income in the economy. One of the papers that studies the determinants of inequality is Banerjee and Duflo (2003) and they estimate the correlation between the Gini coefficient and the economic growth.

2 Empirical Motivation

We begin by presenting the empirical evidence that shows a positive correlation between income inequality and default episodes that countries experienced between 1969-2009. In order to present this relationship, we estimate default probabilities controlling for income inequality and several macro variables. There are several papers in the literature that try to estimate the conditional probability of default and our specification in this empirical analysis is similar to the ones used in these papers.[4] We estimate the following specification

$$P(\text{Default}_{i,t} = 1) = (1 + \exp(-\alpha_0 - \alpha_1 \text{Inequality}_{i,t-1} - \text{Macro Controls} - \text{error}_{i,t}))^{-1} \quad (1)$$

where Default is a dummy variable that represents the first year of default for a country i, i.e. it is equal to one, if country i entered into a default episode in year t. The macro variables are external debt-to-GDP ratio, 3-month U.S. treasury bill (annual data), growth rate of GDP per capita, and reserves-to-GDP ratio.[5] Income inequality and macro control variables are lagged by one period. We also control for time fixed effects in the estimations.

Table 1 shows the summary statistics for the variables used for the regression sample that covers the period 1969-2009. In the sample there are 67 countries, and a total of 35 default episodes experienced by 24 different countries. A couple of differences stand out when we compare observations with default and without default. First, before going into default, countries experience higher inequality and a lower GDP growth rate. The latter observation supports the fact that it is more likely for countries to default in bad times, which is also shown by Tomz and Wright (2007). Second, we observe that going into a default period is associated with higher T-bill rates. Hatchondo et al. (2007) state that higher cost of borrowing can trigger default episodes and there is a positive correlation between the interest rates paid by the borrowing countries and T-bill rates. Third, both reserves-to-GDP and debt-to-GDP ratios are less favorable when countries default, which are in the same lines with the statistics presented by Hilscher and Nosbusch (2010).[6]

[4] For instance, Hilscher and Nosbusch (2010) predict default probabilities across countries controlling for various macro variables and show that terms of trade volatility and reserves-to-GDP ratio have a predictive power on default probability in a sample of 31 countries over the period 1994-2007. Reinhart and Rogoff (2011) estimate how the share of countries in default or restructuring depends on public debt and external debt for advanced countries between 1880 and 2009.

[5] See the Appendix for the extended definitions of the variables and data sources.

[6] Even though the mean and median values for debt-to-GDP are higher in case of default, no default observations also entail a quite high standard deviation in the sample, which might result in insignificant estimates for the variable.

Table 1: Summary statistics for the regression sample

	Inequality	GDP growth	T-bill	Reserves/GDP	Debt/GDP
Default					
Mean	44.53	-6.33%	8.59	8.16%	59.88%
Median	44.19	-4.54%	8.61	6.63%	58.80%
Std. Dev.	3.49	14.57%	3.43	6.15%	18.83%
Number of Obs.	35				
No Default					
Mean	40.97	4.78%	6.03	11.01%	56.45%
Median	40.80	3.74%	5.49	7.54%	45.70%
Std. Dev.	6.21	12.89%	2.71	12.16%	43.71%
Number of Obs.	1773				

We estimate equation 1 using logit estimation, since default is a rare event. We expect to get a positive estimate of α_1 (the coefficient for income inequality) which supports our theory that higher inequality increases the probability of going into default. Moreover, a negative coefficient for GDP-growth implies that it is less likely for countries to experience default during good times. T-bill stands for the cost of borrowing, so we expect to get a positive coefficient for T-bill variable. Also higher external debt-to-GDP ratio or lower reserves-to-GDP ratio increases the probability of default.

Table 2 shows the estimation results. In all of the specifications we find that inequality has a positive and strongly significant coefficient. The coefficients of other variables are also in the line with previous empirical studies and our predictions. We find that GDP growth and default are negatively correlated and the coefficient is significant. Default is more likely if reserves-to-GDP ratio is low and debt-to-GDP ratio is high, but these coefficients are not statistically significant. When we introduce year fixed effects, the results for inequality remain robust, but the growth rate of GDP is no longer statistically sinificant.[7]

In order to support our theory that income inequality plays a role in default decisions, we also need to determine whether there is countercyclical inequality over the business cycle.

[7] Since default is a rare event, when we include year fixed effects, the years without any variation are dropped out of the estimation, which reduces the sample size approximately by half and maybe altering the significance.

Table 2: Logit estimation results

	Dependent Variable : First year in default dummy						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Inequality	0.098***	0.089***	0.116***	0.112***	0.127***	0.120***	0.115***
	[0.032]	[0.032]	[0.035]	[0.036]	[0.036]	[0.036]	[0.037]
GDP-growth	-	-4.447***	-3.715***	-2.971***	-	-2.091	-1.951
		[1.068]	[0.999]	[1.039]		[1.323]	[1.378]
T-bill	-	-	0.265***	0.272***	-	-	-
			[0.055]	[0.057]			
Reserves/GDP	-	-	-	-1.918	-	-	-1.613
				[2.922]			[2.942]
Debt/GDP	-	-	-	0.004	-		0.002
				[0.005]	-		[0.006]
constant	-8.176***	-7.782***	-9.102***	-8.536***	-8.668***	-8.403***	-8.112***
	[0.887]	[1.415]	[1.074]	[1.056]	[1.878]	[1.872]	[1.992]
Year fixed effects	No	No	No	No	Yes	Yes	Yes
Observations	1808	1808	1808	1808	901	901	901

Standard errors in brackets
*** p<0.01, ** p<0.05, * p<0.1

Notes: All right hand side variables are lagged by one year and estimated using random effects.

Using household level data from several countries, Krueger et al. (2010) show that during recessions earnings inequality increases.[8] We perform a similar exercise using our country level data. We use the countries that have continuous series for inequality and GDP. This leaves us with 46 countries. We compute the correlation between detrended GDP and inequality. We find that on average inequality is countercyclical over the business cycle and the mean correlation is equal to -0.03.

3 Model

We consider a small open economy that has heterogeneous agents in its stream of income. There are two types of households indexed by $i = 1, 2$ and their preferences are CRRA such

[8]They have several inequality measures such as Gini coefficient, variance of logs, 50/10 and 90/50 percentile ratios and the countries they study are Canada, Germany, Italy, Mexico, Russia, Spain, Sweden and the USA.

that

$$u(c_t^i) = \frac{c_t^{i,1-\sigma}}{1-\sigma} \tag{2}$$

The household of type 1 receives a stochastic stream of a tradable good, γy and type 2 receives $(1-\gamma)y$ where y and γ are the stochastic components. The output shock y and the inequality shock γ follow a Markov process with a transition function $f((y', \gamma'), (y, \gamma))$. Households also receive an equal amount of transfer of goods from the benevolent government in a lump sum fashion.

The government of the economy can trade bonds with foreign lenders that are risk free and competitive. The government can choose to default fully on its debt anytime because contracts are not enforceable. As in a standard default model, when the government defaults the economy faces two types of default penalties; direct output costs and exclusion from borrowing in the debt markets. The government's goal is to maximize the social utility, which is the sum of utilities of both types with equal weights. The government bonds are one-period non-state contingent bonds. If the government chooses to buy B' discount bonds, the price it needs to pay is given as $q(B', y, \gamma)$. The government also decides whether to repay or default on its debt. The bond price function $q(B', y, \gamma)$ is endogenous to the government's incentives to default, so it reflects the default probability. It also depends on the size of the bond B', the output shock y and the inequality shock γ.

When the government chooses to repay its debts, the resource constraint for agent 1 is

$$c^1 = \gamma y + \frac{B - q(B', y, \gamma)B'}{2} \tag{3}$$

and the resource constraint for agent 2 is

$$c^2 = (1-\gamma)y + \frac{B - q(B', y, \gamma)B'}{2} \tag{4}$$

The economy faces three types of uncertainty that cannot be insured away with the non-state contingent bonds. The first one is the dispersion in incomes induced by shocks to γ. The second one is the output shock y that affects the aggregate output in the economy. The third one is the endogenous default risk. When the government chooses to default, consumption of the types are:

$$c^1 = \gamma y^{def} \tag{5}$$

$$c^2 = (1-\gamma)y^{def} \tag{6}$$

where $y^{def} < y$. The government can borrow or lend to foreign creditors. Foreign creditors can receive loans as much as needed from international credit markets at a constant interest rate $r > 0$. They also have perfect information regarding the inequality and income processes and they can observe the level of inequality and output every period. Creditors price defaultable bonds in a risk neutral manner. Taking the price q as given, creditors choose loans B' to maximize expected profits ϕ

$$\phi = qB' - \frac{1-\delta}{1+r}B' \tag{7}$$

The probability of default, δ, is endogenous to the model and depends on the government's incentives to repay debt.

The timing of the model is as follows. The government starts with initial assets B and observes the aggregate shock y and the individual income shock γ, and decides whether to repay its debt obligations or default. If the government decides to repay, then taking as given the bond price schedule $q(B', y, \gamma)$, the government chooses B' subject to the resource constraint. Then creditors taking q as given choose B'. Finally agents consume c^1 and c^2 with respective to their types.

3.1 Recursive Equilibrium

We focus on a recursive equilibrium in which the government does not have commitment. Households simply consume their income and the transfers from the proceeds of government's foreign credit operations. Foreign creditors are risk neutral and lend the amount of debt demanded by the government as long as the gross return on the bond equals $(1 + r)$. Given loan size B', inequality state γ and income state y, the bond price is

$$q(B', y, \gamma) = \frac{1 - \delta(B', y, \gamma)}{1+r} \tag{8}$$

The value function for the government that has the option to default or pay its debt is given as $v^o(B, y, \gamma)$. Government chooses the option that maximizes the welfare of agents. The default option will be optimal only if the government has debt. The value of default is denoted by the function $v^d(y,\gamma)$ and the value of no default is denoted by $v^c(B,y,\gamma)$.

$$v^o(B,y,\gamma) = \max_{c,d}\{v^c(B,y,\gamma), v^d(y,\gamma)\} \tag{9}$$

The value of default is expressed by

$$\begin{aligned} v^d(y,\gamma) &= u(\gamma y^{def}) + u((1-\gamma)y^{def}) \\ &+ \beta \int_{\gamma'} [\theta v^o(0,y',\gamma') + (1-\theta)v^d(y',\gamma')] f((y',\gamma'),(y,\gamma)) d(\gamma',y') \end{aligned} \tag{10}$$

Under default, individuals only consume their income. The government can gain access to debt markets with probability θ and the economy stays in autarky with probability $1-\theta$. The transition probabilities are given by the joint density function, f. Similarly the value of staying in contract is

$$\begin{aligned} v^c(B,y,\gamma) &= \max_{B'} u\left(\gamma y - \frac{q(B',y,\gamma)B' - B}{2}\right) + u\left((1-\gamma)y - \frac{q(B',y,\gamma)B' - B}{2}\right) \\ &+ \beta \int_{y',\gamma'} v^o(B',y',\gamma') f((y',\gamma'),(y,\gamma)) d(\gamma',y') \end{aligned} \tag{11}$$

If the government chooses to repay its debt, the value function for this choice reflects the future options for default and staying in contract.

The government default policy can be characterized by default sets and repayment sets. Let $A(B)$ be the set of y and γ for which repayment is optimal when assets are B, such that

$$A(B) = \left\{(y,\gamma) \in (\mathbb{Y},\Gamma) : v^c(B,y,\gamma) \geq v^d(y,\gamma)\right\} \tag{12}$$

and let $D(B) = \tilde{A}(B)$ be the set of y,γ for which default is optimal for a level of assets B:

$$D(B) = \left\{(y,\gamma) \in (\mathbb{Y},\Gamma) : v^c(B,y,\gamma) < v^d(y,\gamma)\right\} \tag{13}$$

Now that we have developed the problem for each of the agents in the economy the equilibrium is defined. Let $s = \{B,y,\gamma\}$ be the aggregate states for the economy.

Definition 1 *The recursive equilibrium for this economy is defined as a set of policy functions for (i) consumptions $c^1(s)$, $c^2(s)$; (ii) government's asset holdings $B'(s)$, repayment sets $A(B)$, and default sets $D(B)$; and (iii) the price function for bonds $q(B',y,\gamma)$ such that:*

1. *Taking as given the government policies, agents' consumption* $c^1(s)$ *and* $c^2(s)$ *satisfy the resource constraints.*

2. *Taking as given the bond price function* $q(B', y, \gamma)$*, the government's policy functions* $B'(s)$*, repayment sets* $A(B)$*, and default sets* $D(B)$ *satisfy the government optimization problem.*

3. *Bonds prices* $q(B', y, \gamma)$ *reflect the government's default probabilities and are consistent with creditors' expected zero profits.*

The equilibrium bond price function q has to be consistent with the government's optimization and with expected zero profits for lenders, such that the price correctly assesses the probability of default of the government. Default probabilities δ and default sets D are then related in the following way:

$$\delta(B', y, \gamma) = \int_{D(B')} f((y', \gamma'), (y, \gamma)) d(y', \gamma') \tag{14}$$

When default sets are empty, equilibrium default probabilities are equal to zero with assets B', the government never chooses to default for all realizations of endowment shocks. When $D(B') = (\mathbb{Y}, \Gamma)$, default probabilities are equal to one. Also default sets are shrinking in assets.

4 Quantitative Analysis and Simulation

4.1 Quantitative Analysis

In this section we calibrate the model to analyze the debt dynamics quantitatively using Argentine data.

Data

The data pertaining to interest rates, trade balance, consumption and output are taken from the dataset in Arellano (2008). For the inequality data of Argentina, we use annual income inequality data starting 1993 provided by World Bank. In order to match the annual income data with the quarterly data, we adopt the Boots-Feibes-Lisman method

Table 3: **Business Cycle Statistics for Argentina**

	Default episode x: Q1-2002	std(x)	corr(x, y)	corr(x, r^c)
Interest rates spread	28.60	5.58	-0.67	
Trade balance	9.90	1.75	-0.58	0.70
Consumption	-12.86	4.94	0.97	-0.67
Output	-12.01	4.16		-0.67
Inequality	33.11	1.33	0.42	0.00

to disaggregate the annual data into quarterly data.[9] Table 3 presents the business cycle statistics of all the data available up to the default episode that started in December 26, 2001. Output and consumption data starting from 1980 are log and filtered with a linear trend. The trade balance data are a ratio of output and start in 1993. The interest rate data start in the third quarter of 1983. The interest rate spread is the difference between the interest rate for Argentina and the yield of the five-year US treasury bond. Consumption and output in the first column of Table 3 show the deviations from trend of the variables and the other variables present their values in the first quarter of 2002. The second column shows the standard deviations up to the default episode. The third and the fourth column present the correlation of each variable with the output and the interest rate spread, respectively. Emerging market economies are characterized by counter-cyclical spread rates and net exports. We see similar empirical results for Argentina in column three. Interest rate spread is negatively correlated with consumption and output and positively correlated with trade balance. The results also show that consumption is more volatile than output. In addition to these empirical regularities, inequality shows positive correlation with output, but no correlation with interest rate spread.[10]

Calibration and Functional Forms

To compare the quantitative predictions of the model with the business cycle statistics and to analyze the effects of inequality shocks, we quantitatively solve the model assuming two different types of shocks. In the benchmark model, we impose both inequality and output shocks that are correlated. In the second model, we keep the inequality constant and only

[9] Please see Appendix for further details on tha data.

[10] Barro (2000) shows that Gini and log GDP show an inverted U relationship using a panel data that cover 1960-1990. The fact that we obtain a positive correlation between inequality and output can imply that Argentina lies on the left side of the inverted U.

impose output shocks. These two models help us asses the importance of inequality shocks in matching the high volatilities and particularly high default rates observed in emerging economies. As in standard models of default, in both specifications we assume the following utility function

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

with the constant relative risk aversion parameter (σ) equal to 2.

For the stochastic process for output and inequality shocks used in the benchmark model, we assume that log output and the inequality follow a VAR(1) process,

$$\begin{bmatrix} log(y_t) \\ \gamma_t \end{bmatrix} = \begin{bmatrix} c_y \\ c_\gamma \end{bmatrix} + \begin{bmatrix} \rho_{yy} & \rho_{y\gamma} \\ \rho_{\gamma y} & \rho_{\gamma\gamma} \end{bmatrix} \begin{bmatrix} log(y_{t-1}) \\ \gamma_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{\gamma t} \end{bmatrix}$$

where

$$\begin{bmatrix} c_y \\ c_\gamma \end{bmatrix} = \left[\boldsymbol{I} - \begin{bmatrix} \rho_{yy} & \rho_{y\gamma} \\ \rho_{\gamma y} & \rho_{\gamma\gamma} \end{bmatrix} \right] \begin{bmatrix} \mu_y \\ \mu_\gamma \end{bmatrix}$$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{\gamma t} \end{bmatrix}$$

$$E\left[\boldsymbol{\varepsilon}\right] = \mathbf{0} \quad \text{and} \quad Var\left[\boldsymbol{\varepsilon}\right] = \begin{bmatrix} \sigma_y^2 & \sigma_{y\gamma} \\ \sigma_{\gamma y} & \sigma_\gamma^2 \end{bmatrix}$$

The estimated values are derived from Argentina's GDP and income inequality data. Both output shock and the inequality shock are then discretized into a 21-state Markov chain, using Tauchen (1986). The discount factor β, probability of reentry θ and the output cost are jointly calibrated to target debt-to-GDP ratio of 5.53 percent and trade balance volatility of 1.75.[11]

Table 4 shows the parameters that we use for the benchmark model's calibration. Note that the correlation of the output at t and the inequality at $t-1$, $\rho_{y\gamma}$, is negative. This means that high inequality generates low output in the next period. Similarly, since $\rho_{\gamma y}$ is equal to zero, the output in the previous quarter does not affect the inequality in the current period.

[11] The calibrated values for β, θ and $\hat{y}$ are close to the values used in the default literature. For instance, Yue (2010) assumes $\beta = 0.72$ and Aguiar and Gopinath (2006) assume $\beta = 0.80$. Both of these papers assume that output drop is 2 percent. Arellano (2008) sets θ to 0.282.

Table 4: Parameters for the Benchmark Model

Name	Parameters	Description
Risk-free interest rate	$r = 1.7\%$	US 5-year bond quarterly yield
Risk aversion	$\sigma = 2$	
Discount factor	$\beta = 0.75$	
Probability of reentry	$\theta = 0.20$	
Output cost	$\hat{y} = 0.95E(y)$	
Stochastic structure	$\begin{bmatrix} \rho_{yy} & \rho_{y\gamma} \\ \rho_{\gamma y} & \rho_{\gamma\gamma} \end{bmatrix} = \begin{bmatrix} 0.95 & -0.38 \\ 0.00 & 0.95 \end{bmatrix}$	Argentina's GDP
	$\begin{bmatrix} \sigma_y^2 & \sigma_{y\gamma} \\ \sigma_{\gamma y} & \sigma_\gamma^2 \end{bmatrix} = \begin{bmatrix} 0.0003 & -0.0001 \\ -0.0001 & 0.0001 \end{bmatrix}$	and income inequality
	$\begin{bmatrix} c_y \\ c_\gamma \end{bmatrix} = \begin{bmatrix} 0.12 \\ 0.01 \end{bmatrix}$	

Table 5: Parameters for the Model with Only Output Shocks

Name	Parameters	Description
Risk-free interest rate	$r = 1.7\%$	US 5-year bond quarterly yield
Risk aversion	$\sigma = 2$	
Discount factor	$\beta = 0.75$	
Probability of reentry	$\theta = 0.30$	
Output cost	$\hat{y} = 0.98E(y)$	
Stochastic structure	$\rho = 0.9351$	Argentina's GDP
	$\eta = 0.0190$	

For the second model, we remove the effect of inequality by setting the level of inequality as 0.33 which is the mean of inequality up to the default episode. The stochastic process for output is assumed to be a log-normal AR(1) process such that

$$log(y_t) = \rho log(y_{t-1}) + \epsilon_t \tag{15}$$

where $E[\epsilon_t] = 0$ and $E[\epsilon_t^2] = \eta^2$, which are estimated from Argentina's GDP. We discretize the output process into a 21-state Markov chain. Similar to the benchmark model, β, θ and $\hat{y}$ are targeted to the volatility of trade balance and debt-to-GDP ratio. The parameters for the second model are presented in Table 5.

4.2 Simulation Results

To solve the model numerically, we use the discrete state-space method. We discretize the asset space into 600 equal grid points, making sure that the minimum and the maximum points on the grid do not bind when we compute the optimal debt decision. Our solution algorithm for the benchmark model is the following:[12]

1. Guess that initial price is the reciprocal of the risk free interest rate and initial value function is equal to the autarky value.

2. Given a price $q(B', y, \gamma)$ and $v^o(B, y, \gamma)$, solve for optimal policy functions and update value of option given as equation (9) by comparing $v^c(B, y, \gamma)$ and $v^d(y, \gamma)$.

3. Given the price function, compute default probabilities.

4. Update the price function using equation (8).

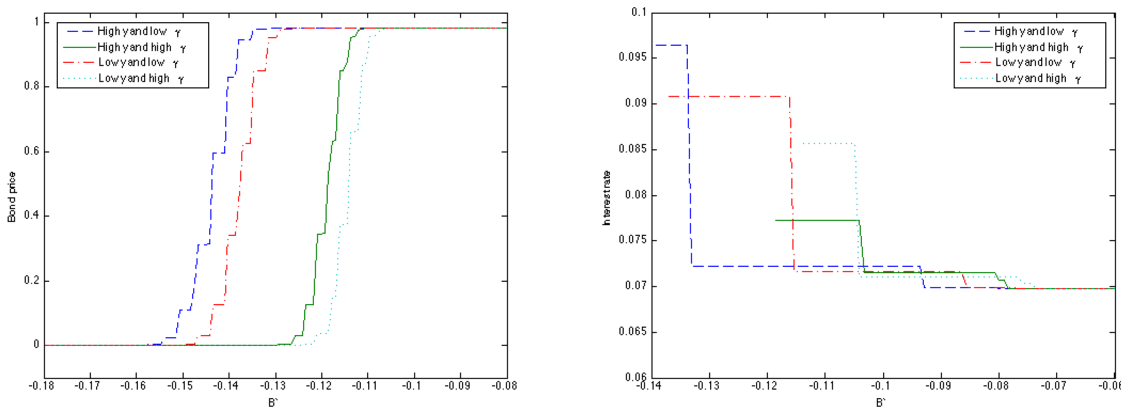
5. We simultaneously check if the initial guesses for price and value of option are close enough to their updated values. If not we update the initial values and iterate steps 2-4 until both bond price and the value of option functions converge.

First we analyze our results related to policy functions and value functions in the benchmark model. We report the results based on four different combinations of output and inequality shocks. A low (high) shock is 5 percent below (above) its mean for each type of shocks. The level of assets is denoted as a fraction of GDP. Then, we look at the business cycle statistics generated by the model.

In our model the benevolent government has two policies, which are borrowing and the decision to default. These policies are used to overcome the uncertainty generated by income and inequality shocks. The government tries to smooth consumption intertemporally as well as equate marginal utilities of consumption across agents. Also, since the government is impatient, i.e. its discount parameter is lower than the inverse of risk free interest rate, it tries to pull future consumption to present by borrowing more. The level of optimal debt depends on the current assets and the state of the world. Since, lenders have full information about the state of the world and contracts are not state dependent, borrowing

[12] We use the same algorithm to solve the model with only output shocks. Since there are only output shocks, the price function is denoted as $q(B', y)$ and value of option for default or repayment is denoted as $v^o(B, y)$ in this model.

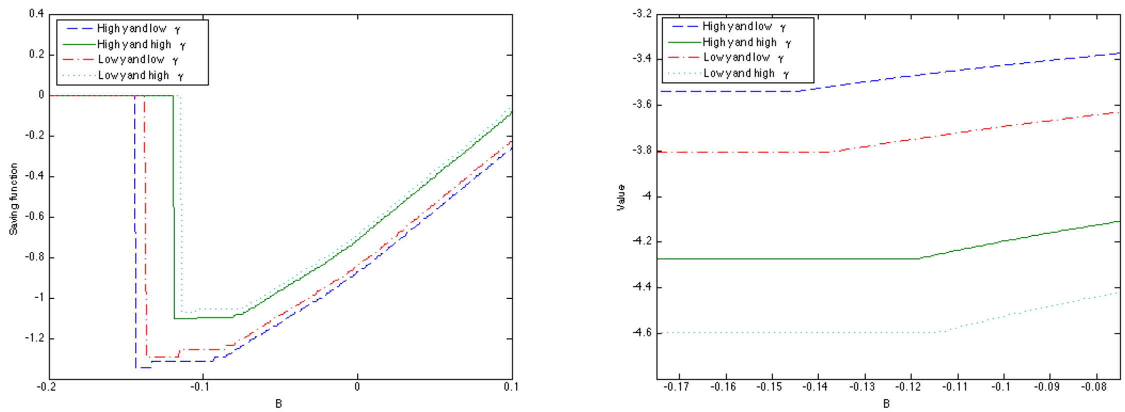
Figure 1: Bond prices and interest rate (Model I)



constraints can bind for the government particularly in bad states of the world. Therefore, we observe that the bond prices depend on the level of borrowing and the types of the shocks that the economy is subject to. Figure 1 shows the bond price schedule and the interest rate generated by the model. Similar to the results presented in standard default literature such as Arellano (2008) and Aguiar and Gopinath (2006), we observe that bond prices are an increasing function of assets, such that high levels of debt entails low bond price and high interest rates. Fixing the level of inequality shocks, we observe that it is easier to borrow during expansions than recessions. However, the results also show that the effect of a high output shock can be dominated by the effect of a high inequality shock. In other words, an economy that is subject to both high output and inequality shocks can have a bond price that is lower than the one when there are low output and low inequality shocks. This result is due to the fact that high inequality shocks increase the consumption dispersion across agents and the government tries to eliminate it by borrowing more from the lenders. Since the shocks are persistent and there is a negative relationship between output and inequality such that high inequality today reduces the output tomorrow, bond prices are lower and default becomes more preferable in high inequality states.

The right panel in Figure 1 shows the annual interest rates generated by the model. Inequality shocks generate another source of risk that is reflected in interest rates. The highest level of borrowing is possible when there is high output and low inequality in the economy. Government borrowing is subject to higher interest rates even for small amounts of debt that is above the level of default in high inequality and or low output states.

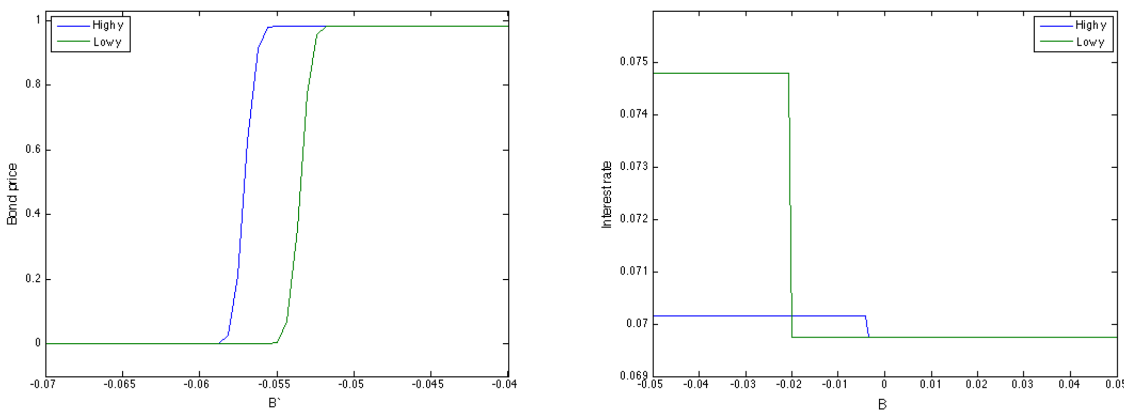
Figure 2: Savings and value functions (Model I)



The left panel in Figure 2 shows the saving policy function conditional on not defaulting. Our results show that the government borrows more in expansions and when there is low inequality. This result is consistent with the countercyclical interest rates, since it becomes more costly to borrow in bad states of the world. The right panel of Figure 2 is the value function for the option to default or repay as a function of assets for four different realizations of output and inequality. Again, the inequality plays a significant role in default decision. The value functions show that the highest debt can be supported when there is high output and low inequality. Default becomes optimal for larger debts when there is low output and high inequality.

Next we move to the business cycle statistics generated by the benchmark model and we evaluate the performance of the policy functions with Argentine data. We feed into the model the time series of Argentina's GDP and inequality starting in 1993. Even though the model fails to predict the default in the last quarter of 2001, it does well in terms of generating a default probability of 2.95. High volatility of interest rates is a consequence of high default probability. The model also generates large drops in consumption and output during default episodes. It can also generate high volatility in consumption and output. In terms of correlations with output, the simulations can generate a positive correlation with consumption and a negative correlation with the interest rate spread. However, it fails to generate a negative correlation between output and trade balance, which is shown as an empirical regularity of the emerging markets in the literature. Our model can generate more defaults thanks to the presence of inequality shocks that increase the dispersion of consumption across agents. In standard sovereign default models, large trade balances can

Figure 3: Bond prices and interest rate (Model II)



occur in recessions because interest rate is high and borrowing is constrained. In our model borrowing can be constrained even in expansions, if there is high inequality in the economy. So unlike standard models, there may be large trade balances in expansions due to high inequality, which generates a weak and positive correlation between tradebalance and output in our model. On the other hand, we see a positive correlation between the spread and the trade balance. Since the spread reflects both the risk due to inequality and output shocks, it is more correlated with the bad states of the world, where it is more likely to face borrowing constraints and experience large trade balances.

The ability of the benchmark model in generating large default probabilities comes from its price schedule being not very steep over the range of borrowing compared to the model where there is only output shocks. Figure 3 shows the bond price schedule and interest rate generated by the model. The risky borrowing range is quite narrow compared to the one in benchmark model. Also the interest rates do not show much variation. Since the price schedule is very sensitive to changes in borrowing, there is minimal borrowing in equilibrium and small rate of default.[13]

The simulation results obtained from the second model are presented in the lower panel in Table 6. Even though the model can predict the default in 2001, the default probability is only 0.4 percent. Unlike the benchmark model, the trade balance has very small volatility, which is due to the steep price function. Also the correlation between the trade balance

[13] See Aguiar and Gopinath (2006) and Wright (2011) for a detailed discussion of the shortcomings of standard default models with output shocks in generating high default rates.

and the interest rates is almost zero. On the other hand, since it is more likely to be borrowing constrained in recessions and the economy can experience large exports during these periods, we see a negative correlation between trade balance and output.

Table 6: Simulation Results

	Default episodes	std(x)	corr(x,y)	$corr(x, r^c)$
Model I: Shocks to output and inequality				
Interest rate spread	1.13	0.79	−0.33	-
Trade balance	−0.004	0.46	0.04	0.03
Total Consumption	−21.62	9.56	0.99	−0.33
Output	−5.81	7.53	–	−0.33
Other Statistics				
Mean debt (percent output)	15.21	Mean spread		1.02
Default probability	2.95	Output deviation in default		–
Model II: Shocks to only output				
Interest rate spread	0.25	0.38	−0.30	-
Trade balance	−0.001	0.05	−0.29	0.00
Total Consumption	−10.59	4.39	0.99	−0.29
Output	−7.82	4.44	–	−0.30
Other Statistics				
Mean debt (percent output)	5.35	Mean spread		0.44
Default probability	0.40	Output deviation in default		−15.84

5 Conclusion

This paper studies the role of income inequality in sovereign borrowing and default decision using a stochastic general equilibrium model in a small open economy model with endogenous default risk. To show that inequality plays an important role, the paper presents the empirical evidence between default decision and Gini coefficient using a panel data set that covers 67 countries for a period of 40 years. The results show that high inequality increases the probability of default significantly. Next the paper shows analytically that inequality shocks can generate higher probability of default when the markets are incomplete. In addition to the standard risk sharing motive due to income uncertainty, in this model the government also tries to eliminate the dispersion in consumption due to inequality. Using the Argentine data, the model predicts a default probability of 3 percent and can also match the business cycle characteristics observed in the data, such as high volatility of consumption, counter-cyclical interest rates, and a positive correlation between trade balance and interest rates. Our contribution to the literature is that the model can explain why countries do not always default in bad times using inequality shocks. To generate a plausible default rate, there is also no need for asymmetric default penalties or large shocks that are commonly used in the literature.

Rising income inequality is a general problem that has been experienced by many countries. Therefore it is important to understand how inequality induces economic crises and sovereign defaults that last several years and cause large losses. Even though our paper provides a first step to analyze the role of income inequality, we abstract from the determinants of income inequality and model inequality as exogenous shocks. We think that it is also important to study what drives high income inequality and how it affects agents' welfare and government's decision to default. Furthermore, we also think that defaults have different effects on agents from different age groups and so far endogenous default models have not taken into account age heterogeneity. We leave these issues for future study.

6 Appendix

6.1 Data Description and sources, etc.

Output and consumption data are in log and filtered with a linear trend starting in 1980. The trade balance data are reported as a percentage of output starting in 1993. The interest rate spread is the difference between the interest rate for Argentina and the yield of the five-year US treasury bond and the series starts in 1983. The inequality is the additional share of top 50% in income distribution. That is, if the inequality is 30%, top 50% of the population in income distribution have 80% of the total income in the economy.

References

Aguiar, Mark and Gita Gopinath, "Defaultable debt, interest rates and the current account," *Journal of international Economics*, 2006, *69* (1), 64–83.

_ **and Manuel Amador**, "Sovereign Debt: A Review," Technical Report, National Bureau of Economic Research 2013.

Arellano, Cristina, "Default risk and income fluctuations in emerging economies," *The American Economic Review*, 2008, pp. 690–712.

Banerjee, Abhijit V and Esther Duflo, "Inequality and growth: What can the data say?," *Journal of economic growth*, 2003, *8* (3), 267–299.

Barro, Robert J, "Inequality and Growth in a Panel of Countries," *Journal of economic growth*, 2000, *5* (1), 5–32.

D'Erasmo, Pablo and Enrique G Mendoza, "Distributional Incentives in an Equilibrium Model of Domestic Sovereign Default," Technical Report, National Bureau of Economic Research 2013.

Eaton, Jonathan and Mark Gersovitz, "Debt with potential repudiation: Theoretical and empirical analysis," *Review of Economic Studies*, 1981, *48* (2), 289–309.

Hatchondo, Juan Carlos, Leonardo Martinez, and Horacio Sapriza, "The Economics of Sovereign Defaults," *FRB Richmond Economic Quarterly*, 2007, *93* (2), 163–187.

Hilscher, Jens and Yves Nosbusch, "Determinants of sovereign risk: Macroeconomic fundamentals and the pricing of sovereign debt," *Review of Finance*, 2010, *14* (2), 235–262.

Krueger, Dirk, Fabrizio Perri, Luigi Pistaferri, and Giovanni L Violante, "Cross-sectional facts for macroeconomists," *Review of Economic Dynamics*, 2010, *13* (1), 1–14.

OECD, "Special Focus: Inequality in Emerging Economies (EEs)," 2011.

Reardon, Thomas and J Edward Taylor, "Agroclimatic shock, income inequality, and poverty: Evidence from Burkina Faso," *World Development*, 1996, *24* (5), 901–914.

Reinhart, Carmen M and Kenneth S Rogoff, "From Financial Crash to Debt Crisis," *American Economic Review*, 2011, *101* (5), 1676–1706.

Tauchen, George, "Finite state Markov-chain approximations to univariate and vector autoregressions," *Economics letters*, 1986, *20* (2), 177–181.

Tomz, Michael and Mark LJ Wright, "Do countries default in bad times?," *Journal of the European Economic Association*, 2007, *5* (2-3), 352–360.

Wright, Mark LJ, "The theory of sovereign debt and default," *Encyclopedia of Financial Globalization*, 2011.

Yue, Vivian Z, "Sovereign default and debt renegotiation," *Journal of international Economics*, 2010, *80* (2), 176–187.