The Optimal Design of a Fiscal Union*

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We study cooperative and non-cooperative fiscal policy in an open economy model where cross-country risk sharing is imperfect and countries face terms of trade externalities. We show that the optimal form of fiscal cooperation, or fiscal union, is defined by one parameter: the Armington elasticity of substitution between goods from different countries. We prove that members of a fiscal union should: (1) harmonize steady state tax rates when the Armington elasticity is low in order to ameliorate terms of trade externalities; and (2) send fiscal transfers across countries when the Armington elasticity is high in order to improve risk sharing. Crucially, these predictions hold both outside of and within currency unions. For standard calibrations, we find that the welfare gain from the optimal fiscal union is as high as 5% of permanent consumption when countries are able to trade safe government bonds, and can approach 20% when countries lose access to international financial markets. We also find that labor mobility significantly improves welfare and alleviates the need for a transfer union entirely.

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1 Introduction

The recent crisis in the euro area has prompted much debate about the need for greater fiscal cooperation across member countries. Although not to the same extent, fiscal crises at the state and municipal level have begun to raise similar questions in federal currency unions like the United States.[1] This debate has longstanding roots in the economics profession, dating back to Kenen (1969), who emphasized the importance of fiscal integration in a currency union.

While the concept of a fiscal union has received a great deal of attention in policy circles, there is considerable uncertainty about how to design such a union. We study the optimal design of a fiscal union in an open economy model where countries face three distortions: nominal rigidities, incomplete financial markets, and terms of trade externalities. We show that the negative welfare impact of these distortions is highly sensitive to the elasticity of substitution between goods produced in different countries. As such, we find that the magnitude of this elasticity — the Armington elasticity — governs the optimal design of a fiscal union.

When the Armington elasticity is equal to one, a common assumption in the literature known as the Cole-Obstfeld specification, terms of trade movements provide complete international risk-sharing through offsetting income and substitution effects, even in financial autarky.[2] If a country produces less output and exports fewer goods under unitary elasticity, its terms of trade will improve and exactly offset the decline in quantity produced so that export revenues are constant. In this case, there is no need for a fiscal union to improve international risk-sharing. At the same time, when goods are imperfect substitutes countries are exposed to a relatively high degree of monopoly power at the export level. Domestic fiscal policymakers use this monopoly power to impose a large markup on their exports — what the literature refers to as a terms of trade externality. The optimal fiscal union will force domestic fiscal policymakers to internalize this externality and prevent countries from exploiting their monopoly power and manipulating their terms of trade. We find that when the elasticity is close to one countries should cooperate in setting steady state domestic income tax rates to ameliorate large terms of trade externalities — what we call a *tax union*.

As the Armington elasticity increases and exports become closer substitutes, the degree of international risk sharing provided by terms of trade movements declines, as does each country's monopoly power. Because country-level monopoly power falls, the welfare gains from a tax union fall as well. On the other hand, there is now a role for a fiscal union to improve international risk-sharing. We find that when the elasticity is high and goods are close

[1] For example, the $14 billion-plus bankruptcy of the city of Detroit is on par with the roughly €10 billion bank bailout of Cyprus by the euro area governments.

[2] Cole and Obstfeld (1991) were the first to demonstrate the provision of complete international risk-sharing via terms of trade movements under unitary elasticity.

substitutes countries should organize a contingent cross-country transfer scheme to provide international risk-sharing — what we call a *transfer union*.

The Armington elasticity thus governs the optimal design of a fiscal union: the welfare gains from a tax union *decrease* in the Armington elasticity; the welfare gains from a transfer union *increase* in the Armington elasticity. We prove that this is true for countries outside of and within currency unions, in financial autarky and incomplete markets.

Empirical estimates of the Armington elasticity range from one to twelve or higher, depending on the estimation method, country and time period being examined.[3] In Section 8 we compute the welfare gains from a fiscal union for a wide range of elasticities, including country-specific estimates for European countries from Corbo and Osbat (2013). For standard calibrations, the welfare gains from a tax union are as high as 3% of permanent consumption when the elasticity is close to one; the welfare gains from a transfer union are as high as 5% of permanent consumption when the elasticity is five or higher, and can approach 20% when countries lose access to international financial markets. These gains are much larger than Lucas' (2003) estimates of the welfare cost of business cycle fluctuations (0.05% of permanent consumption), which we replicate in a flexible wage environment. The large losses we find under wage rigidity demonstrate that a single monetary policy for a wide range of countries in a currency union with asymmetric shocks is problematic from a welfare perspective.

Although a tax union removes terms of trade externalities and a transfer union improves international risk-sharing, the distortive effects of nominal rigidities remain unchecked by international fiscal cooperation. If countries control their own monetary policy this is not a problem, as the central bank can move the economy toward the efficient level of output. However within a currency union, the union-wide central bank cannot eliminate nominal rigidities in the presence of asymmetric shocks across countries, which prevents efficient adjustment of the economy through changes in relative prices.[4] National fiscal authorities therefore have a role to play in implementing contingent policies that move the economy toward the efficient level of output and eliminate nominal rigidities. Importantly, such policies do not require international fiscal cooperation. We show that the negative welfare impact of nominal rigidities and hence

[3]Using highly disaggregated data, Eaton and Kortum (2002) estimate the elasticity to be 9.28, Broda and Weinstein (2006) find an unweighted median of 3.1 and mean of 12.6, while Romalis finds a range of 4 to 13. Imbs and Majean (2011) find a mean of 6.7 with a standard deviaion of 4.9, and a median of 5.1. Lai and Trefler (2002) estimate a range between 5 and 8. More recently, Simonovska and Waugh (2011) find a range between 3.38 and 5.42, while Feenstra, Obstfeld and Russ (2012) find a median estimate of the elasticity between foreign countries of 3.1 for the U.S. In a survey of the literature on elasticity estimates, Anderson and Van Wincoop (2004) conclude a range of five to ten is reasonable. Ruhl (2008) explains why the international macro and trade literatures have quite different estimates of the Armington elasticity.

[4]This role is fulfilled by national central banks under flexible exchange rates, but a common union-wide central bank has only one instrument to fight many idiosyncratic shocks. Note that if shocks are symmetric across countries, the union wide central bank is able to eliminate nominal rigidities and mimic the flexible price equilibrium.

the necessity of contingent domestic fiscal policy is increasing in the Armington elasticity.

Given our findings on the cost of ceding monetary independence to a union-wide central bank, we also study the welfare implications of joining a currency union. One of the arguments advanced by Mundell (1961, 1973) and others in favor of a currency union is that countries who join such a union experience deeper financial integration. We thus compare the welfare of a country outside of a currency union with no access to international financial markets versus a country in a currency union with full access to international financial markets. Even in this extreme scenario, the welfare benefits of entering a currency union are not large enough to overcome the loss of independent monetary policy for standard calibrations. Although these losses can be partially offset by contingent domestic fiscal policy or labor mobility, it is telling that from the perspective of deeper financial integration, the benefits of a currency union are relatively small compared with the costs of losing monetary independence.

In addition to studying optimal fiscal policies and the costs of joining a currency union, we examine the implications of labor mobility. We show that labor mobility alleviates the negative welfare impact of wage rigidity by moving workers from depressed regions to booming regions and also facilitates international risk-sharing. In so doing, labor mobility eliminates the need for contingent domestic fiscal policy as well as the need for a transfer union. For standard calibrations, the welfare gains resulting from labor mobility are as high as 10% of permanent consumption when the elasticity is five or higher. While the construction of a fiscal union is fraught with political and legal hurdles, labor mobility is already guaranteed as one of the four pillars of the European Union. However, in the data (shown in Fig 3), labor mobility is quite low within the euro area, despite its constitutional guarantees. So from a policy perspective, a fiscal union is even more important within the euro area where labor mobility is low than within the United States, Canada or other federal currency unions where labor mobility is much higher. In addition, countries like the United States and Canada have higher labor mobility *and* deeper fiscal integration than the euro area, and are thus better suited to withstand asymmetric shocks across regions.

How We Differ From the Literature: Non-Cooperative Fiscal Policy, Imperfect Risk-Sharing and a Global Closed-Form Solution

This paper is related to the literature on the conduct of optimal monetary and fiscal policy among interdependent economies, particularly within a currency union.[5] Beetsma and Jensen

[5]Early non-microfounded contributions in this area include Canzoneri and Henderson (1990) and Eichengreen and Ghironi (2002). Another strand of the literature focuses on the division of seignorage within a currency union. Sims (1999) and Bottazzi and Manasse (2002) examine the interaction between monetary and fiscal policy when seignorage is distributed by a common central bank. While this is an important issue, we abstract from the role of seignorage and focus on the potential for fiscal policy cooperation to improve welfare. Benigno and De Paoli (2010) emphasize the international dimension of fiscal policy for the case of

(2005), Gali and Monacelli (2008) and Ferrero (2009) focus primarily on the case of cooperative policy with internationally complete asset markets. These papers show that monetary policy should stabilize inflation at the union level and that cooperative fiscal policy in the form of government spending has a country-specific stabilization role. Farhi and Werning (2012) study cooperative fiscal policy in a transfer union in the aforementioned Cole-Obstfeld specification with unitary elasticity and log utility.[6] They demonstrate that even when private asset markets are complete internationally, there is a role for contingent cross country transfers to provide consumption insurance. We differ from all of these papers in two important ways. First, we analyze non-cooperative equilibria between fiscal policymakers in a currency union. Second, we relax the assumption of perfect risk-sharing via complete markets or unitary elasticity. Our model thus introduces two additional distortions that are empirically relevant: terms of trade externalities and incomplete international risk sharing. We also analyze the welfare implications of a tax union and of labor mobility, elements which are absent in the literature.

Our methodology provides us with a unique perspective on the role of currency unions, fiscal unions and labor mobility. We obtain a novel global closed-form solution for an open economy model that does not restrict the elasticity of substitution between the goods of different countries to one. This provides a tractable framework to analyze optimal policy and enables us to accurately compare welfare across a variety of risk-sharing regimes, which is not possible under unitary elasticity. In the closed-form model, we consider two financial market regimes: internationally complete asset markets with cross-border trade in bonds and equities (perfect risk sharing) and financial autarky (no risk sharing).

Our closed-form model requires two simplifying assumptions: complete openness in consumption for all economies in the model as well as one period in advance nominal rigidities. We relax both of these assumptions in Section 8 and evaluate the welfare gains from a fiscal union in a model with consumption home bias and Calvo wage rigidities. As in the closed-form case, we solve the extended model away from the Cole-Obstfeld calibration so that financial market structure matters. In the extended model, we consider incomplete markets with cross-border trade in safe government bonds (incomplete risk sharing) as well as financial autarky. In this setup, we find that home bias *increases* the welfare gains from a transfer union, while home

a small open economy in a flexible exchange rate regime, abstracting from the role of strategic interactions between countries as well as from the role of a currency union, both focuses of our analysis. Evers (2012) takes a more quantitative approach and estimates the welfare gains from a variety of "transfer rules", in essence running a horse race between different types of fiscal regimes within a currency union. Our exact analytical solution for optimal policy and our consideration of flexible exchange rate allocations stand in contrast with this approach.

[6]We nest the Cole-Obstfeld calibration as a special case in the closed-form version of our model in Section 2, as well as in our extended model in Section 8. We show that the welfare gains from a transfer union are extremely small in the Cole-Obstfeld specification due to the provision of complete cross-country risk-sharing via terms of trade movements.

bias *decreases* the welfare gains from a tax union. We also find that Calvo rigidities do not impact the results, yielding similar welfare consequences as one period in advance rigidities. As such, our closed-form results *underestimate* the welfare benefits of a transfer union, contingent fiscal policy and labor mobility, but *overestimate* the welfare benefits of a tax union. We focus on the closed-form model for the first half of the paper as it generates analytically tractable and intuitive results, and then shift to the extended model.

2 The Model in Closed-Form

We consider a continuum of small open economies represented by the unit interval, as popularized in the literature by Gali and Monacelli (2005, 2008). Our model is based on Dmitriev and Hoddenbagh (2013), although here we consider wage rigidity rather than price rigidity and extend the closed-form solution for flexible exchange rates to the case of a currency union.

Each economy consists of a representative household and a representative firm. All countries are identical ex-ante: they have the same preferences, technology, and wage-setting. Ex-post, economies will differ depending on the realization of their technology shock. Households are immobile across countries, however goods can move freely across borders. Each economy produces one final good, over which it exercises a degree of monopoly power. This is crucially important: countries are able to manipulate their terms of trade even though they are measure zero. As in Corsetti and Pesenti (2001, 2005) and Obstfeld and Rogoff (2000, 2002), we use one-period-in-advance wage setting to introduce nominal rigidities. Workers set next period's nominal wages, in terms of domestic currency, prior to next-period's production and consumption decisions. Given this preset wage, workers supply as much labor as demanded by firms. We lay out a general framework below, and then hone in on the specific case of complete markets and financial autarky. To avoid additional notation, we ignore time subindices unless absolutely necessary. When time subindices are absent, we are implicitly referring to period t.

Production Each economy i produces a final good, which requires technology, Z_i, and aggregated labor, N_i. We assume that technology is independent across time and across countries. We need not impose any particular distributional requirement on technology at this point. The production function of each economy will be:

$$Y_i = Z_i N_i. \tag{1}$$

Households, indexed by h, each have monopoly power over their differentiated labor input, which will lead to a markup on wages. A perfectly competitive, representative final goods producer aggregates differentiated labor inputs from households in CES fashion into a final good for export. Production of the representative final goods firm in a specific country is:

$$N_i = \left(\int_0^1 N_i(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

where ε is the elasticity of substitution between different types of labor, and $\mu_\varepsilon = \frac{\varepsilon}{\varepsilon-1}$ is the markup on labor.

The aggregate labor cost index, W, defined as the minimum cost to produce one unit of output, will be a function of the nominal wage for household h, $W(h)$:

$$W_i = \left(\int_0^1 W_i(h)^{1-\varepsilon} dh \right)^{\frac{1}{1-\varepsilon}}.$$

Cost minimization by the firm leads to demand for labor from household h:

$$N_i(h) = \left(\frac{W_i(h)}{W_i} \right)^{-\varepsilon} N_i. \tag{3}$$

In the open economy, monopoly power is exercised at both the household and the country level: at the household level because of differentiated labor, and at the country level because each economy produces a unique good. We show in Section 4 that optimizing non-cooperative policymakers will remove the household markup on labor but will introduce a terms of trade markup through the income tax rate. Just to be clear, firms have no monopoly power and are perfectly competitive.

Households In each economy, there is a household, h, with lifetime expected utility

$$\mathbb{E}_{t-1} \left\{ \sum_{k=0}^{\infty} \beta^k \left(\frac{C_{it+k}(h)^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it+k}(h)^{1+\varphi}}{1+\varphi} \right) \right\} \tag{4}$$

where $\beta < 1$ is the household discount factor, $C(h)$ is the consumption basket or index, $N(h)$ is household labor effort (think of this as hours worked). Households face a general budget constraint that nests both complete markets and financial autarky; we will discuss the differences between the two in subsequent sections. For now, it is sufficient to simply write out the most general form of the budget constraint:

$$C_{it}(h) = (1-\tau_i) \left(\frac{W_{it}(h)}{P_{it}(h)} \right) N_{it}(h) + \mathcal{D}_{it}(h) + \mathcal{T}_{it}(h) + \Gamma_{it}(h). \tag{5}$$

The distortionary tax rate on household labor income in country i is denoted by τ_i, while Γ_{it} is a domestic lump-sum tax rebate to households. $\mathcal{T}$ refers to lump-sum cross-country transfers. In the absence of a fiscal union, these cross-country transfers will equal zero ($\mathcal{T} = 0$). Net taxes equal zero in the model, as any amount of government revenue is rebated lump-sum to households. The consumer price index corresponds to P_{it}, while the nominal wage is W_{it}. $\mathcal{D}_{it}$ denotes state-contingent portfolio payments expressed in real consumption units, and can be written in more detail as:

$$\mathcal{D}_{it} P_{it} = \int_0^1 \mathcal{E}_{ijt} B_{ijt} dj, \tag{6}$$

where B_{ijt} is a state-contingent payment in currency j.[7] $\mathcal{E}_{ijt}$ is the exchange rate in units of currency i per one unit of currency j; an increase in $\mathcal{E}_{ijt}$ signals a depreciation of currency i relative to currency j. In a currency union, $\mathcal{E}_{ijt} = 1$ for all i, j, t. When international asset markets are complete, households perform all cross-border trades in contingent claims in period 0, insuring against all possible states in all future periods. The transverality condition simply states that all period 0 transactions must be balanced: payment for claims issued must equal payment for claims received. Leaving the details in the appendix, we use the following relationship as the transversality condition for complete markets:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t C_{it}^{-\sigma} \mathcal{D}_{it} \right\} = 0, \tag{7}$$

while in financial autarky

$$\mathcal{D}_{it} = 0.$$

Intuitively, the transversality condition (7) stipulates that the present discounted value of future earnings should be equal to the present discounted value of future consumption flows. Under complete markets, consumers choose a state contingent plan for consumption, labor supply and portfolio holdings in period 0.

Consumption and Price Indices Households in each country consume a basket of imported goods. This consumption basket is an aggregate of all of the varieties produced by different countries. The consumption basket for a representative small open economy i, which is common

[7] Equation (6) holds in all possible states in all periods. Details are provided in Appendix A.1.

across countries, is defined as follows:

$$C_i = \left(\int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma-1}} \tag{8}$$

where lower case c_{ij} is the consumption by country i of the final good produced by country j, and γ is the elasticity of substitution between domestic and foreign goods (the Armington elasticity). Because there is no home bias in consumption, countries will export all of the output of their unique variety, and import varieties from other countries to assemble the consumption basket.

Prices are defined as follows: lower case p_{ij} denotes the price in country i (in currency i) of the unique final good produced in country j, while upper case P_i is the aggregate consumer price index in country i. Given the above consumption index, the consumer price index will be:

$$P_i = \left(\int_0^1 p_{ij}^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}}. \tag{9}$$

Consumption by country i of the unique variety produced by country j is:

$$c_{ij} = \left(\frac{p_{ij}}{P_i} \right)^{-\gamma} C_i. \tag{10}$$

We assume that producer currency pricing (PCP) holds, and that the law of one price (LOP) holds, so that the price of the same good is equal across countries when converted into a common currency. We define the nominal bilateral exchange rate between countries i and j, $\mathcal{E}_{ij}$, as units of currency i per one unit of currency j. LOP requires that:

$$p_{ij} = \mathcal{E}_{ij} p_{jj}. \tag{11}$$

Given LOP and identical preferences across countries, PPP will also hold for all i, j country pairs:

$$P_i = \mathcal{E}_{ij} P_j, \tag{12}$$

The terms of trade for country j will be:

$$TOT_j = \frac{p_{jj}}{P_j}, \tag{13}$$

where TOT_j is defined as the home currency price of exports over the home currency price of imports. Now we can take (10), and using (11) and (12), solve for demand for country j's unique variety:

$$Y_j = \int_0^1 c_{ij} di = \int_0^1 \left(\frac{p_{ij}}{P_i}\right)^{-\gamma} C_i di \overset{(11)+(12)}{=} \left(\frac{p_{jj}}{P_j}\right)^{-\gamma} \int_0^1 C_i di = TOT_j^{-\gamma} C_w. \tag{14}$$

where C_w is defined as the average world consumption across all i economies, $C_w = \int_0^1 C_i di$.

Labor Market Clearing Households maximize (4) subject to (5). The first order condition for labor will give the optimal preset wage (that is, the labor supply condition):

$$W_{it} = \left(\frac{\chi\mu_\varepsilon}{1-\tau_i}\right) \frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{\frac{C_{it}^{-\sigma} N_{it}}{P_{it}}\right\}}. \tag{15}$$

The optimization problem of the representative firm in country i is standard. It maximizes profit choosing the appropriate amount of aggregate labor.

$$\max_{N_i} Y_i p_i - W_i N_i \Rightarrow \frac{W_i}{p_i} = \frac{Y_i}{N_i} = Z_i \tag{16}$$

This labor demand condition equates the real wage at time t with the marginal product of labor, Z_{it}. Using the labor demand condition ($N_{it} = Y_{it} p_{it} / W_{it}$) from (16), and the fact that the wage is preset at time $t-1$, the labor market clearing condition will be:

$$1 = \left(\frac{\chi\mu_\varepsilon}{1-\tau}\right) \frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma} Y_{it} \frac{p_{it}}{P_{it}}\right\}}. \tag{17}$$

This is the general labor market clearing condition; it holds for the closed economy and in the open economy for producer currency pricing and local currency pricing. Under producer currency pricing, our focus in this paper, the demand for the unique variety (14) will give the following labor market clearing condition:

$$1 = \left(\frac{\chi\mu_\varepsilon}{1-\tau}\right) \frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma} Y_{it}^{\frac{\gamma-1}{\gamma}} C_{wt}^{\frac{1}{\gamma}}\right\}}. \tag{18}$$

Taking the expectations operator out of (18) will give the flexible wage equilibrium.

We now turn our attention to the difference between complete markets and financial autarky.

2.1 Complete Markets

In complete markets, agents in each economy have access to a full set of domestic and foreign state-contingent assets. Households in all countries will maximize (4), choosing consumption, leisure, money holdings, and a complete set of state-contingent nominal bonds, subject to (5). Complete markets and PPP imply the following risk-sharing condition:

$$\frac{C_{it}^{-\sigma}}{C_{it+1}^{-\sigma}} = \frac{C_{jt}^{-\sigma}}{C_{jt+1}^{-\sigma}} \quad \forall i, j \tag{19}$$

which states that the ratio of the marginal utility of consumption at time t and $t+1$ must be equal across all countries. Importantly, this condition does not imply that consumption is equal across countries. Consumption in country i will depend on the initial asset position, fiscal and monetary policy, the distribution of country-specific shocks, the covariance of global and local shocks, and other factors.[8]

When (7),(17), and (19) hold, consumption in country i can be expressed as a function of world consumption:

$$C_{it} = \frac{\mathbb{E}_{t-1}\left\{\sum \beta^s \left[Y_{it+s} C_{wt+s}^{-\sigma} TOT_{it+s}\right]\right\}}{\mathbb{E}_{t-1}\left\{\sum \beta^s C_{wt+s}^{1-\sigma}\right\}} C_{wt}. \tag{20}$$

This defines the optimal consumption allocation for country i in complete markets.[9] Using the fact that Z_{it} is independent across time and across countries, and wages are preset, (20) is equivalent to

$$C_{it} = \mathbb{E}_{t-1}\left\{Y_{it} TOT_{it}\right\} + \mathcal{T}_{it} = C_w^{\frac{1}{\gamma}} \mathbb{E}_{t-1}\left\{Y_{it}^{\frac{\gamma-1}{\gamma}}\right\} + \mathcal{T}_{it} \tag{21}$$

where transfers will equal zero in all states of the world because risk-sharing is provided by trade in contingent claims.

2.2 Financial Autarky

The aggregate resource constraint under financial autarky specifies that the nominal value of output in the home country (exports) must equal the nominal of consumption in the home country (imports). That is, trade in goods must be balanced. In a model with cross-border lending, bonds would also show up in this condition, but in financial autarky they are obviously absent. The primary departure from complete markets lies in the household and economy-wide budget constraints,

$$\underbrace{P_i \cdot C_i}_{\text{Imports}} = \underbrace{p_{ii} \cdot Y_i}_{\text{Exports}} + \underbrace{\mathcal{T}_{it}}_{\text{Transfers}} \tag{22}$$

[8] A policy change in economy i may lead to a change in consumption. For example, monetary policy affects the covariance between home production and world consumption, which in turn influences home consumption, even in complete markets. Fiscal policy can tax consumption and cause a lower level of consumption in the long-run relative to the rest of the world. In spite of this, it is still possible to characterize an optimal consumption plan that is robust to changes in monetary and fiscal policy.

[9] Details are found in the appendix of Dmitriev and Hoddenbagh (2013).

where transfers will be zero unless countries form a transfer union, in which case transfers will provide risk-sharing. Using the fact that (14) holds under both complete markets and financial autarky, and substituting this into (22), one can show that demand for country i's good in financial autarky will be

$$C_{it} = C_w^{\frac{1}{\gamma}} Y_{it}^{\frac{\gamma-1}{\gamma}} + \mathcal{T}_{it}. \tag{23}$$

Complete markets and autarky differ only by goods market clearing. In complete markets consumption is equal to expected domestic output expressed in consumption baskets; in autarky consumption is equal to realized domestic output expressed in consumption baskets.

3 Global Social Planner

We begin by describing the maximization problem faced by a benevolent global social planner who has complete control over the monetary and fiscal policies of each country. Since the economies in our model are identical ex-ante, the global social planner will maximize a weighted utility function over all i countries,

$$\int_0^1 \left[\frac{C_i^{1-\sigma}}{1-\sigma} - \chi \frac{N_i^{1+\varphi}}{(1+\varphi)} \right] di, \tag{24}$$

subject to the consumption basket and the aggregate resource constraint:

$$C_i = \left(\int_0^1 c_{ij}^{\frac{\gamma-1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma-1}}, \tag{25}$$

$$Y_i = N_i Z_i = \int_0^1 c_{ji} dj. \tag{26}$$

Proposition 1 *The global social planner will maximize (24), subject to (25) and (26). The solution to the global social planner problem is:*

$$C_i = \left(\frac{1}{\chi} \right)^{\frac{1}{\sigma+\varphi}} Z_w^{\frac{1+\varphi}{\sigma+\varphi}}, \tag{27a}$$

$$N_i = \left(\frac{1}{\chi} \right)^{\frac{1}{\sigma+\varphi}} Z_w^{\frac{(1-\gamma\sigma)(1+\varphi)}{(1+\gamma\varphi)(\sigma+\varphi)}} Z_i^{\frac{\gamma-1}{1+\gamma\varphi}}, \tag{27b}$$

$$Z_w = \left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} di \right)^{\frac{1+\gamma\varphi}{(\gamma-1)(1+\varphi)}}. \tag{27c}$$

Proof See Appendix C in Dmitriev and Hoddenbagh (2013). ■

The global social planner solution characterizes the Pareto efficient allocation. From (27a), we see that domestic consumption depends on average world technology Z_w, which is a constant because technology shocks are identically and independently distributed. Consumption is thus stabilized at the country level, insuring risk averse households from consumption risk. On the other hand, (27b) shows that labor will fluctuate with technology shocks, increasing in booms and decreasing in recessions. There are no distortions in the efficient allocation: wage ridigity, incomplete risk-sharing, and the terms of trade externality are absent.

The efficient allocation provides a natural benchmark to evaluate different policy regimes. In Sections 4 and 5 we look closely at optimal monetary and fiscal policy in non-cooperative and cooperative settings and see what conditions are necessary to replicate the Pareto efficient allocation outside of and within a currency union. In Section 6 we study the effect of labor mobility and see if it can replicate the Pareto efficient allocation.

4 Non-Cooperative Policy

In order to study the benefits of international policy cooperation, we must first understand the non-cooperative Nash equilibrium. What outcomes naturally arise when policymakers do not cooperate? Our goal in this section is to illuminate the various distortions that are present in the non-cooperative Nash equilibrium, and to compare and contrast with the global social planner equilibrium defined in Proposition 1. We can then pinpoint specific areas of policy cooperation that ameliorate welfare decreasing distortions, leading us to the optimal design of a fiscal union. We begin with the Nash equilibrium under flexible exchange rates and then move to the case of a currency union.

4.1 Flexible Exchange Rates

When exchange rates are flexible, each country has its own central bank and its own fiscal authority. Before any shocks are realized, national fiscal authorities declare non state-contingent taxes, and then national central banks declare monetary policy for all states of the world. With this knowledge in hand, households lay out a state-contingent plan for consumption and labor as well asset holdings when markets are complete. After that, shocks hit the economy. A detailed timeline is provided in Figure 1.

Without loss of generality, we assume a cashless limiting economy.[10] Central banks set monetary policy in each period by optimally choosing the amount of labor. Although central banks optimize by choosing labor instead of money or an interest rate, the three are equivalent in this model.[11] We can write down an interest rate rule that gives the exact same allocation. Domestic fiscal authorities choose the optimal labor tax rate τ_i. The objective function for

[10]Benigno and Benigno (2003) describe a cashless-limiting economy in detail in their appendix, pp.756-758.

[11]We demonstrate the equivalence in Appendix F of Dmitriev and Hoddenbagh (2013).

non-cooperative domestic policymakers will be

$$\max_{N_{it}} \max_{\tau_i} \mathbb{E}_{t-1}\left\{\frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi\frac{N_{it}^{1+\varphi}}{1+\varphi}\right\}, \tag{28}$$

where the fiscal authority acts first and chooses τ_i and the central bank then chooses N_{it}.

We first examine the Nash equilibrium for non-cooperative policymakers when international asset markets are complete. Policymakers in complete markets will maximize their objective function subject to the labor market clearing (29a) and goods market clearing (29b) constraints, and production (29c) and aggregate world consumption (29d):

$$1 = \left(\frac{\chi\mu_\varepsilon}{1-\tau_i}\right)\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma}Y_{it}^{\frac{\gamma-1}{\gamma}}C_{wt}^{\frac{1}{\gamma}}\right\}}, \tag{29a}$$

$$C_{it} = C_{wt}^{\frac{1}{\gamma}}\mathbb{E}_{t-1}\left\{Y_{it}^{\frac{\gamma-1}{\gamma}}\right\} \tag{29b}$$

$$Y_{it} = Z_{it}N_{it}, \tag{29c}$$

$$C_{wt} = \left(\int_0^1 Y_{it}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}. \tag{29d}$$

Proposition 2 ***Flexible Exchange Rates + Complete Markets*** *When international asset markets are complete and exchange rates are flexible, non-cooperative policymakers will maximize (28) subject to (29a), (29b), (29c) and (29d). The solution under commitment for non-cooperative policymakers in complete markets is:*

$$C_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}}\left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}}di\right)^{\frac{1+\gamma\varphi}{(\gamma-1)(\sigma+\varphi)}}, \tag{30a}$$

$$N_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}}\left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}}di\right)^{\frac{1-\gamma\sigma}{(\gamma-1)(\sigma+\varphi)}} Z_i^{\frac{\gamma-1}{1+\gamma\varphi}}. \tag{30b}$$

It is optimal for non-cooperative central banks under commitment to mimic the flexible wage allocation. The optimal tax rate for non-cooperative fiscal authorities is $\tau_i = 1 - \frac{\mu_\varepsilon}{\mu_\gamma}$.

Proof See Appendix A. ■

The above allocation replicates the global social planner allocation with the addition of a terms of trade markup, $\mu_\gamma = \frac{\gamma}{\gamma-1}$, that lowers consumption and output. It is optimal for central banks to mimic the flexible wage allocation through a policy of price stability.[12] Optimizing

[12]In a related paper (Dmitriev and Hoddenbagh 2013), we prove that mimicking the flexible price allocation is a dominant strategy for small open economy central banks. This result is robust to changes in elasticity between domestic and foreign goods, the degree of cooperation between policymakers in different countries,

fiscal authorities internalize the negative welfare impact of the domestic markup on differentiated labor inputs (μ_ε), and thus choose an income tax rate that cancels out the labor markup. However, fiscal authorities also want to use their country-level monopoly power. Because each country in the continuum is measure zero, policymakers do not internalize the impact of charging a higher markup for their export good on the welfare of other countries. This leads fiscal authorities to set an income tax rate which reduces hours worked and restricts production, $\tau_i = 1 - \frac{\mu_\varepsilon}{\mu_\gamma}$, so that exports from each country are subject to a terms of trade markup (μ_γ). The terms of trade externality leads to lower welfare outcomes because households in each country must pay a higher price on each import good in the consumption basket. Even though asset markets are complete, the non-cooperative allocation under flexible exchange rates yields lower welfare than the global social planner allocation due to the imposition of the terms of trade markup. The need for some sort of international fiscal cooperation that would force domestic fiscal authorities to internalize this externality is clear.

Now that we've examined the complete markets equilibrium when exchange rates are flexible, we turn our attention to the case of financial autarky. The objective function in financial autarky will be identical to the complete markets case. Domestic fiscal authorities will first choose the optimal tax rate, and then central banks will set the optimal monetary policy by choosing labor. However, there is a slight difference in the constraints faced by policymakers in complete markets and financial autarky. In complete markets, home consumption is a function of expected output (29b), while in autarky home consumption is a function of actual output

$$C_{it} = C_{wt}^{\frac{1}{\gamma}} Y_{it}^{\frac{\gamma-1}{\gamma}}. \tag{31}$$

Aside from (31), all other constraints are identical in complete markets and financial autarky.

Proposition 3 ***Flexible Exchange Rates + Financial Autarky*** *Non-cooperative policymakers in financial autarky will maximize (28) subject to (29a), (29c), (29d) and (31). The solution under commitment for non-cooperative policymakers in financial autarky is:*

$$C_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1-\sigma+\gamma(\sigma+\varphi)}} di\right)^{\frac{(1+\varphi)}{(\gamma-1)(\sigma+\varphi)}} Z_i^{\frac{(\gamma-1)(1+\varphi)}{1-\sigma+\gamma(\sigma+\varphi)}}, \tag{32a}$$

$$N_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1-\sigma+\gamma(\sigma+\varphi)}} di\right)^{\frac{(1-\sigma)}{(\gamma-1)(\sigma+\varphi)}} Z_i^{\frac{(\gamma-1)(1-\sigma)}{1-\sigma+\gamma(\varphi+\sigma)}}. \tag{32b}$$

It is optimal for non-cooperative central banks to mimic the flexible wage allocation. The optimal tax rate for non-cooperative fiscal authorities is $\tau_i = 1 - \frac{\mu_\varepsilon}{\mu_\gamma}$.

and the degree of financial integration across countries.

Proof See Appendix A. ■

As in complete markets, central banks find it optimal to mimic the flexible wage equilibrium through a policy of price stability in financial autarky. On the fiscal side, policymakers again eliminate the domestic markup μ_ε, but impose a terms of trade markup on their unique export good μ_γ via the steady state income tax rate. Financial autarky removes cross-country consumption insurance, as households no longer have the ability to trade in international contingent claims. This can be seen most clearly in (32a), where equilibrium consumption is a function of idiosyncratic productivity, Z_i, and will fluctuate with country-specific shocks to technology.

4.2 Currency Union

Within a currency union, a single central bank sets monetary policy for the union as a whole. Countries no longer control their domestic monetary policy as they do when exchange rates are flexible. In the presence of aggregate shocks, the union-wide central bank will stabilize inflation at the union level, a result shown in Gali and Monacelli (2008). However, for tractability we assume no aggregate shocks, only asymmetric country-specific shocks. With only one policy instrument, the union-wide central bank cannot eliminate wage rigidity at the country level in the presence of asymmetric shocks. As a result, the union-wide central bank does nothing in our model. None of our results change if we add aggregate shocks: these shocks would simply be counteracted by the union-wide central bank.

In a currency union each country retains control over it's own fiscal policy. The objective function for non-cooperative fiscal policymakers in a currency union is:

$$\max_{\tau_i} \mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\} \tag{33}$$

The constraints faced by policymakers within a currency union are identical to those faced by policymakers under flexible exchange rates, with the addition of a fifth constraint unique to currency unions. Thus, relative to the optimization problem under flexible exchange rates, we add one constraint and subtract one FOC. We know that demand for country i's good is $Y_i = TOT_i^{-\gamma} C_w = \left(\frac{p_{ii}}{P_i}\right)^{-\gamma} C_w$ from (14) and that $p_{ii} = \frac{W_i}{Z_i}$ from (16). Plugging (16) into (14) gives:

$$Y_{it} = \underbrace{\left(\frac{W_{it}}{P_{it}}\right)^{-\gamma} C_w}_{A} Z_{it}^{\gamma} = A Z_{it}^{\gamma} \tag{34}$$

where A is a constant. (34) is the additional constraint faced by the policymaker in a currency union.

Proposition 4 ***Currency Union + Complete Markets*** *Non-cooperative policymakers in a currency union will maximize (33) subject to (29a), (29b), (29c), (29d) and (34). The solution under commitment for non-cooperative policymakers within a currency union in complete markets is:*

$$C_i = C_w = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left[\frac{\left(\int_0^1 Z_i^{\gamma-1} di\right)^{\frac{\gamma(1+\varphi)}{\gamma-1}}}{\int_0^1 Z_i^{(\gamma-1)(1+\varphi)} di}\right]^{\frac{1}{\sigma+\varphi}}, \tag{35a}$$

$$N_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left[\frac{\left(\int_0^1 Z_i^{\gamma-1} di\right)^{\frac{\gamma(1-\sigma)}{\gamma-1}}}{\int_0^1 Z_i^{(\gamma-1)(1+\varphi)} di}\right]^{\frac{1}{\sigma+\varphi}} Z_i^{\gamma-1}. \tag{35b}$$

The resulting equilibrium allocation does not replicate the flexible wage equilibrium. The optimal tax rate for non-cooperative fiscal authorities is $\tau_i = 1 - \frac{\mu_\varepsilon}{\mu_\gamma}$.

Proof See Appendix B. ■

Within a currency union, the inability of the union-wide central bank to alleviate asymmetric shocks across countries leads to the presence of wage rigidity in the optimal allocation. In addition, non-cooperative fiscal authorities exploit their country-level monopoly power and impose a terms of trade markup via income tax policy. We thus see the presence of two distortions in the equilibrium allocation: wage rigidity and a terms of trade markup. As in the flexible exchange rate allocation, there is no idiosyncratic technology risk in consumption under complete markets, so consumption will be equalized across countries in equilibrium. However, welfare will be lower when wages are rigid than when they are flexible, as one can notice by comparing the above allocation with the Pareto efficient allocation.[13]

Proposition 5 ***Currency Union + Financial Autarky*** *In financial autarky, non-cooperative policymakers in a currency union will maximize (33) subject to (29a), (29c), (29d), (31) and (34). The optimal allocation in financial autarky given by a non-contingent policymaker in a*

[13]We calculate explicit welfare differences between allocations in Section 6.

currency union is:

$$C_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left[\frac{\left(\int_0^1 Z_i^{(\gamma-1)(1-\sigma)}di\right)\left(\int_0^1 Z_i^{\gamma-1}di\right)^{\frac{1+\varphi}{\gamma-1}}}{\int_0^1 Z_i^{(\gamma-1)(1+\varphi)}di}\right]^{\frac{1}{\sigma+\varphi}} Z_i^{\gamma-1}, \tag{36a}$$

$$N_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left[\frac{\left(\int_0^1 Z_i^{(\gamma-1)(1-\sigma)}di\right)\left(\int_0^1 Z_i^{\gamma-1}di\right)^{\frac{1-\sigma}{\gamma-1}}}{\int_0^1 Z_i^{(\gamma-1)(1+\varphi)}di}\right]^{\frac{1}{\sigma+\varphi}} Z_i^{\gamma-1}. \tag{36b}$$

The resulting equilibrium allocation does not replicate the flexible wage allocation. The optimal tax rate for non-cooperative fiscal authorities is $\tau_i = 1 - \frac{\mu_\varepsilon}{\mu_\gamma}$.

Proof See Appendix B. ■

In the autarky Nash equilibrium described in Proposition 5, members of a currency union face three welfare decreasing distortions: wage rigidity resulting from the absence of country-specific monetary policy; idiosyncratic consumption risk, caused by lack of access to international financial markets; and a terms of trade markup, imposed by non-cooperative fiscal authorities in other countries. The potential for cooperative measures to ameliorate these distortions is evident, and will be the focus of Section 5. Before we broach the topic of a fiscal union however, we study the implications of contingent fiscal policy in the non-cooperative setup.

4.3 Contingent Fiscal Policy

Up to this point we have assumed that fiscal policy is non-contingent, so that fiscal authorities can only set a constant income tax rate. If we relax this assumption so that fiscal policymakers can adjust tax rates over the business cycle, the objective function under flexible exchange rates is

$$\max_{N_{it}} \max_{\tau_{it}} \mathbb{E}_{t-1}\left\{\frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi\frac{N_{it}^{1+\varphi}}{1+\varphi}\right\}, \tag{37}$$

and within a currency union is

$$\max_{\tau_{it}} \mathbb{E}_{t-1}\left\{\frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi\frac{N_{it}^{1+\varphi}}{1+\varphi}\right\}. \tag{38}$$

As we showed in Proposition 2 and 3, when exchange rates are flexible national central banks will mimic the flexible wage allocation and a constant labor tax rate will be optimal for both contingent and non-contingent fiscal policymakers. The role of fiscal policy under flexible exchange rates is simply to ameliorate the monopolistic markup on differentiated labor inputs and impose a terms of trade markup in the non-cooperative case. In other words, contingent fiscal policy is redundant when exchange rates are flexible because national central banks adjust monetary policy over the business cycle to counteract wage rigidity.

In contrast, the role of fiscal policy in a currency union is twofold: to impose a terms of trade markup, but also to eliminate wage rigidity at the national level resulting from asymmetric shocks. Optimal contingent fiscal policy within a currency union will not set a constant tax rate. Within a currency union the union-wide central bank has only one policy instrument at its disposal and cannot offset the effect of asymmetric shocks across countries. Contingent national fiscal policy can fill the void, setting domestic tax rates in each period to remove domestic wage rigidity and mimic the flexible wage equilibrium. One already begins to see that fiscal policy is more important within a currency union than outside of one. Given that contingent fiscal policy is only necessary in a currency union, we ignore flexible exchange rate allocations in Proposition 6.

Proposition 6 ***Contingent Fiscal Policy*** *Contingent non-cooperative policymakers within a currency union will maximize (38), subject to (29a), (29b), (29c), (29d), and (34) in complete markets and subject to (29a), (29c), (29d), (31) and (34) in autarky. The optimal allocation will exactly coincide with (30a) in complete markets and (32a) in autarky, replicating the flexible wage allocation with a terms of trade markup.*

Proof See Appendix C. ■

The optimal contingent labor tax within a currency union will be equal to $\tau_{it} = 1 - \mu_{it}$. The realized markup μ_{it} is defined by

$$MPL_{it} = \mu_{it} \cdot MRS_{it},$$

where $MPL_{it} = Z_{it}$ is the marginal product of labor and MRS_{it} is the marginal rate of substitution. A positive productivity shock increases MPL_{it}, but since wages remain fixed, there will be a rise in demand for labor. The rise in demand for labor will induce households to work more hours and cause an even larger increase in MRS_{it}. As a result, the markup will be countercylical and the optimal contingent labor tax will be procyclical: taxes will increase when productivity shocks are positive and decline when productivity shocks are negative.

Note that international policy cooperation is not necessary to eliminate the distortionary impact of wage rigidity: non-cooperative contingent fiscal policy is all that is required. For the remainder of the paper, we will assume that fiscal policy is non-contingent. However, keep in mind that contingent fiscal policy within a currency union can play the same role as monetary policy outside of a currency union, eliminating nominal rigidities and mimicking the flexible-wage allocation. While the ability of contingent fiscal policy to eliminate nominal rigidities has been shown in other closed and open economy studies, we are the first to emphasize that the distortionary impact of wage rigidity and hence the importance of contingent fiscal policy is increasing in the Armington elasticity γ.[14]

4.4 Summary of Non-Cooperative Policy

In sections 4.1 and 4.2 we solved for Nash equilibria under flexible exchange rates and within a currency union, for complete markets and financial autarky. Policymakers maximized the welfare of their domestic households without internalizing the impact of their policy decisions on other countries. The non-cooperative allocations featured three distortions: wage rigidity, lack of access to international financial markets, and terms of trade externalities. Proposition 2 and 3 proved that domestic monetary policy is sufficient to eliminate wage rigidity outside of a currency union, while Proposition 6 proved that contingent fiscal policy is sufficient to eliminate wage rigidity within a currency union. In both cases, policy cooperation was unnecessary. In Section 5, we will show that the remaining two distortions — incomplete risk-sharing and terms of trade externalities — can be remedied with appropriate international policy cooperation via the construction of a tax union and a transfer union.

5 Cooperative Policy in a Fiscal Union

In this section we analyze fiscal policy cooperation. Although the concept of fiscal cooperation may be quite broad, we focus here on two specific types — a *tax union* and a *transfer union*. In a tax union, fiscal policymakers in each country cooperatively set steady state income tax rates to maximize the welfare of the union as a whole. A tax union may be viewed as a cross-country agreement on income tax setting between domestic fiscal authorities, or as a set of tax rates chosen by a supranational (or federal) fiscal authority. In a transfer union, fiscal policymakers in each country arrange contingent cross-country transfers to maximize the welfare of the union as a whole. The transfer scheme may derive from an agreement between national fiscal authorities or from a supranational (or federal) fiscal authority.

Each arrow in Figure 2 denotes cooperation between countries in a fiscal union. Complete cooperation (all arrows), no cooperation (no arrows), or cooperation along only one axis are all possible.[15]

The objective functions for the various types of fiscal union described in Figure 2 are below.

[14] A non-exhaustive list of papers demonstrating the importance of contingent fiscal policy includes Chugh (2006), Correia et al (2013), Schmitt-Grohe and Uribe (2004) and Siu (2004).

[15] Because our focus is on the optimal design of a fiscal union, we ignore the implications of monetary policy cooperation. With one common central bank, monetary cooperation within a currency union is not possible. In another paper (Dmitriev and Hoddenbagh (2013)), we show that in a continuum of small open economies monetary cooperation yields no welfare gains, as non-cooperative and cooperative equilibria exactly coincide.

$$\max_{\forall \tau_i} \int_0^1 \left[\max_{N_{it}} \mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\} \right] di \tag{39a}$$

$$\max_{\forall \tau_i} \int_0^1 \left[\mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\} \right] di \tag{39b}$$

Objective functions (39a) and (39b) refer to a tax union outside of and within a currency union, respectively. Here, the fiscal authorities in each country jointly maximize the welfare of all countries in the union by choosing the same steady state income tax rate.

$$\max_{\forall \mathcal{T}_{it}} \int_0^1 \left[\max_{N_{it}} \max_{\tau_i} \mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\} \right] di \tag{39c}$$

$$\max_{\forall \mathcal{T}_{it}} \int_0^1 \left[\max_{\tau_i} \mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\} \right] di \tag{39d}$$

Objective functions (39c) and (39d) refer to a transfer union outside of and within a currency union, respectively. Here, a supranational (or federal) fiscal body optimally chooses cross-country transfers in order to maximize union-wide welfare.

$$\max_{\forall \tau_i, \mathcal{T}_{it}} \int_0^1 \left[\max_{N_{it}} \mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\} \right] di \tag{39e}$$

$$\max_{\forall \tau_i, \mathcal{T}_{it}} \int_0^1 \left[\mathbb{E}_{t-1} \left\{ \frac{C_{it}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{it}^{1+\varphi}}{1+\varphi} \right\} \right] di \tag{39f}$$

Finally, (39e) and (39f) refer to a tax *and* transfer union outside of and within a currency union, respectively. Here, countries not only agree on income tax rates, but also agree to send contingent cash transfers across countries.

Proposition 7 ***Tax Unions*** *Policymakers in a tax union will internalize the impact of their income tax rate on all union members. As a result, a tax union will remove the incentive for policymakers to manipulate their terms of trade. The optimal tax rate in a tax union is* $\tau_i = 1 - \mu_\varepsilon$*, which will remove the markup on domestic production in each country,* μ_ε*, while preventing the imposition of a terms of trade markup on exports,* μ_γ*, from all equilibrium allocations.*

Proof See Appendix D. ■

A tax union forces domestic fiscal authorities to internalize the impact of their terms of trade externality on union wide welfare. As a result, policymakers will not impose a terms of trade markup on the export of their country's unique good in a tax union. This improves welfare for the entire union as well as for each individual country, particularly for low values of elasticity when countries have a high degree of monopoly power. The distortive impact of the terms of trade externality increases as the degree of substitutability decreases, which will become clear in Section 7 when we calculate the welfare gains from a tax union. In other words, the benefits of a tax union are increasing in the degree of country-level monopoly power.

Members of a transfer union agree to send contingent cash transfers across countries in order to insure against idiosyncratic consumption risk. In complete markets the presence of cross-country transfers will alter the goods market clearing constraint, so that (29b) is replaced by the following two conditions:

$$C_{it} = C_{wt}^{\frac{1}{\gamma}} \mathbb{E}_{t-1} \left\{ Y_{it}^{\frac{\gamma-1}{\gamma}} \right\} + \mathcal{T}_{it}, \tag{40}$$

$$\text{where } \int_0^1 \mathcal{T}_{it} di = 0. \tag{41}$$

In financial autarky the presence of cross-country transfers will alter the goods market clearing constraint, so that (31) is replaced by the following two conditions:

$$C_{it} = C_{wt}^{\frac{1}{\gamma}} Y_{it}^{\frac{\gamma-1}{\gamma}} + \mathcal{T}_{it}, \tag{42}$$

$$\text{where } \int_0^1 \mathcal{T}_{it} di = 0. \tag{43}$$

Proposition 8 ***Transfer Unions*** *Policymakers in a transfer union agree to send contingent cash transfers across countries in order to insure against idiosyncratic consumption risk. The equilibrium allocation within a transfer union will be identical with the equilibrium allocation under complete markets. As a result, transfer unions are redundant when international asset markets are complete or when substitutability is one, but yield large welfare gains when markets are incomplete.*

Proof See Appendix E. ■

As Proposition 8 states, a transfer union guarantees complete cross-country consumption insurance and thus replicates the effect of complete markets. The welfare benefits of a transfer union are increasing in the Armington elasticity: as goods become closer substitutes, the natural risk-sharing role played by the terms of trade begins to disappear. This will be seen more clearly in Section 7 when we calculate the welfare gains from a transfer union.

As shown in Figure 2, it is also possible to have a tax *and* a transfer union. If countries agree to both, they will enjoy complete risk-sharing and eliminate the distortive impact of the terms of trade externality. Proposition 7 and 8 show that a tax union, a transfer union or a combination of the two will move countries toward the Pareto efficient allocation.

Proposition 9 ***Pareto Optimum*** *The Pareto efficient allocation is achieved through a combination of: (1) independent monetary policy outside of a currency union or contingent fiscal policy within a currency union; (2) internationally complete asset markets or a transfer union; and (3) a tax union.*

Proof (1) eliminates the distortionary impact of wage rigidity, (2) provides cross-country risk-sharing, and (3) prevents terms of trade manipulation. Any combination of (1), (2) and (3), for example a tax and transfer union whose members control their own monetary policy outside of a currency union, will yield the Pareto efficient allocation. ■

Although each of these ingredients is necessary to achieve the Pareto efficient allocation, the *relative importance* of these ingredients is highly sensitive to the elasticity of substitution between products from different countries. In Section 7, we show that the optimal design of a fiscal union, and the emphasis given to a tax versus a transfer union, will depend on the value of the elasticity parameter.

6 Labor Mobility

While a fiscal union provides large economic benefits, it may be politically difficult to achieve in practice. This is especially true within the euro area, where the construction of a fiscal union would require sovereign governments to cede at least partial control over national fiscal policy. If deeper fiscal integration is not possible, what should governments do? Our answer, heralding back to James Meade (1957), is to pursue policies that increase labor mobility. Discussing the creation of a common currency area in Western Europe, Meade argued that without the free movement of goods, capital and labor, the idea was doomed to failure. Meade didn't prove the necessity of labor mobility rigorously, but we do so here using our analytical closed-form model.

Although we've assumed that labor is immobile up to this point, it is quite plausible that labor will be mobile across borders within a currency union. When labor is mobile, non-cooperative policymakers in a currency union in financial autarky maximize the familiar objective function, but face a new set of constraints (found in Appendix F). As each economy in the currency union is hit with idiosyncratic shocks, labor will shift from low demand bust countries to high demand boom countries, equalizing real wages across countries and acting as a natural shock absorber that enables efficient adjustment of the economy without any policy actions taken by the monetary or fiscal authority.

Proposition 10 ***Labor Mobility*** *Labor mobility will eliminate two distortions: wage rigidity and the lack of risk-sharing in financial autarky. The resulting equilibrium allocations yield* *higher welfare than the flexible wage allocations in complete markets and financial autarky. When labor can move freely across borders, the solution under commitment for non-cooperative policymakers outside of or within a currency union, in complete markets or financial autarky, is:*

$$C_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_i^{\gamma-1} di\right)^{\frac{1+\varphi}{(\sigma+\varphi)(\gamma-1)}} \tag{44a}$$

$$N_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_j^{\gamma-1} dj\right)^{\frac{1+\varphi-\gamma(\sigma+\varphi)}{(\gamma-1)(\sigma+\varphi)}} Z_j^{\gamma-1}. \tag{44b}$$

The optimal tax rate for non-cooperative fiscal authorities is $\tau_i = 1 - \frac{\mu_\varepsilon}{\mu_\gamma}$.

Proof See Appendix F. ■

Labor mobility achieves three goals from a policy perspective. First, households whose members can move across countries diversify their source of income even in the absence of internationally complete asset markets, which yields perfect cross-country risk-sharing and results in the complete markets allocation. Second, shifting labor hours across countries generates labor risk-sharing, so that hours worked do not fluctuate with country-specific shocks. Third, the stabilizing influence of labor mobility on both consumption and labor eliminates the distortionary impact of wage rigidity. So the fixed wage allocation under labor mobility will mimic the flexible wage allocation. Although the distortive effect of the terms of trade externality remains so there is still a need for a tax union, the benefits of labor mobility are potentially massive and ease the burden on fiscal policy greatly. We quantify these gains in Sections 7 and 8.

Legally, labor mobility is guaranteed as one of the four pillars of economic integration within the European Union. EU citizens are free to migrate to any other EU country to seek employment (Kahanec 2012, Zimmermann 2005). Workers are also free to move across state borders in the U.S., as well as provincial borders in Canada and state borders in Australia. Despite similar legal guarantees for labor mobility, actual mobility is much higher within the U.S., Canada and Australia than within the EU.

Figure 3 plots the extent of labor mobility across countries in the EU as well as across U.S. states, Canadian provinces, and Australian states and territories. Over and above regulatory and legal barriers to mobility, language seems to rule: linguistic and cultural differences across countries make emigration much more difficult. For example, notice the high degree of labor mobility within unilingual currency unions (Australia and the U.S.), the slightly lower degree of labor mobility in a bilingual currency union (Canada), and the much lower degree of labor mobility in a multilingual currency union (euro area). One can see the importance of language most clearly by focusing on the much higher degree of mobility within EU countries (0.95%) where languages are uniform than across EU countries (0.29%) where they differ, as well as the the high mobility across Canada as a whole (0.98%) versus the low degree of mobility between French-speaking Quebec and the English-speaking provinces (0.39%). These data reinforce the notion that labor mobility is vital to the sound functioning of a currency union: high mobility in the US, Australia and Canada dampens the distortionary impact of internal wage rigidity, lowers unemployment and improves risk sharing. On the other hand, low mobility in the EU leads to high unemployment and overvalued wages in areas that are hit by large negative shocks. Although achieving a fiscal union may prove politically difficult, the low degree of labor mobility across countries within Europe suggests that there is a strong need for a fiscal union within the euro area.

7 Welfare Analysis in the Closed-Form Model

In this section we analyze the welfare gains resulting from a tax union, a transfer union, labor mobility, and from eliminating wage rigidity via flexible exchange rates or contingent fiscal policy within a currency union. The advantage of the closed-form solution is most apparent here. Rather than approximating a quadratic welfare function around a particular steady state, we can calculate welfare explicitly at any steady state, whether distorted or otherwise. This is particularly important when we focus on the welfare gains from a tax union, which eliminates the terms of trade markup from the steady state allocation. In a log-linear model, comparing welfare between the tax union and no tax union cases is infeasible because of the different steady states.

To begin, we take consumption and labor from the five allocations: flexible exchange rates with complete risk-sharing (Proposition 2); flexible exchange rates in financial autarky (Proposition 3); currency union in complete markets (Proposition 4); currency union in financial autarky (Proposition 5); and labor mobility (Proposition 10). Using consumption and labor from each allocation, we then derive the expected utility. Allocations that eliminate wage rigidity (via flexible exchange rates or contingent fiscal policy) are denoted by $flex$, while those that do not are denoted by $fixed$. We assume that there is no contingent fiscal policy in the currency union allocations, so that wages are rigid. Similarly, allocations with complete international risk-sharing (via complete markets or a transfer union) are denoted by *complete*, while autarky allocations with no risk-sharing are denoted by *autarky*.

Below we calculate the log of expected utility for the five allocations. We ignore the constant terms and focus only on the exponents of Z. Details on how we compute welfare analytically for each allocation are found in Appendix G. We assume technology is log-normally distributed in all countries, $\log(Z_i) \sim N(0, \sigma_Z^2)$, and is independent across time and across countries.

$$\log \mathbb{E}\left\{U_{flex,complete}\right\} = \frac{(\gamma-1)(1-\sigma)(1+\varphi)^2}{(1+\gamma\varphi)(\sigma+\varphi)}\sigma_Z^2 \tag{45a}$$

$$\log \mathbb{E}\left\{U_{fixed,complete}\right\} = \frac{(\gamma-1)(1-\sigma)(1+\varphi)(1+\varphi-\gamma\varphi)}{(\sigma+\varphi)}\sigma_Z^2 \tag{45b}$$

$$\log \mathbb{E}\left\{U_{flex,autarky}\right\} = \frac{(\gamma-1)(1-\sigma)(1+\varphi)^2\left[1+\varphi+(\gamma-1)(1-\sigma)(\sigma+\varphi)\right]}{(\sigma+\varphi)[1-\sigma+\gamma(\sigma+\varphi)]^2}\sigma_Z^2 \tag{45c}$$

$$\log \mathbb{E}\left\{U_{fixed,autarky}\right\} = \frac{(\gamma-1)(1-\sigma)(1+\varphi)\left[1-(\gamma-1)(\sigma+\varphi)\right]}{\sigma+\varphi}\sigma_Z^2 \tag{45d}$$

$$\log \mathbb{E}\left\{U_{labor\ mobility}\right\} = \frac{(\gamma-1)(1+\varphi)}{\sigma+\varphi}\sigma_Z^2 \tag{45e}$$

Using these expected utilities, and the fact that any constant terms will cancel out when subtracted from each other, we calculate the welfare differences for four scenarios: (1) complete markets vs. autarky for flexible wages; (2) complete markets vs. autarky for fixed wages; (3) flexible vs. fixed wages for complete markets; and (4) flexible vs. fixed wages for autarky. When comparing welfare across different allocations, it is important to keep in mind that as risk-aversion decreases, (i.e. as $\sigma \to 1$), the welfare differences expressed in logarithms also decrease but the *absolute values* of utility increase. In other words, when risk aversion is low, the welfare differences shown in (46a) – (46d) will shrink, but this does not mean that the welfare differences are decreasing in absolute value.

$$\log \mathbb{E}\left\{U_{flex,complete}\right\} - \log \mathbb{E}\left\{U_{flex,autarky}\right\} = \frac{\sigma(\gamma-1)^2(1-\sigma)(1+\varphi)^2}{(\sigma+\varphi)(1+\gamma\varphi)[1-\sigma+\gamma(\sigma+\varphi)]}\sigma_Z^2 \tag{46a}$$

$$\log \mathbb{E}\left\{U_{fixed,complete}\right\} - \log \mathbb{E}\left\{U_{fixed,autarky}\right\} = \frac{\sigma(\gamma-1)^2(1-\sigma)(1+\varphi)}{\sigma+\varphi}\sigma_Z^2 \tag{46b}$$

$$\log \mathbb{E}\left\{U_{flex,complete}\right\} - \log \mathbb{E}\left\{U_{fixed,complete}\right\} = \frac{\gamma\varphi^2(\gamma-1)^2(1-\sigma)(1+\varphi)}{(1+\gamma\varphi)(\sigma+\varphi)}\sigma_Z^2 \tag{46c}$$

$$\log \mathbb{E}\left\{U_{flex,autarky}\right\} - \log \mathbb{E}\left\{U_{fixed,autarky}\right\} = \frac{(\gamma-1)^2(1-\sigma)(1+\varphi)[\gamma(\sigma+\varphi)-\sigma]}{1+\gamma(\sigma+\varphi)-\sigma}\sigma_Z^2 \tag{46d}$$

Not surprisingly, equations (46a) – (46d) prove that for non-unitary Armington elasticity: (1) improved risk-sharing always has positive welfare consequences; and (2) moving from fixed to flexible exchange rates always has positive welfare consequences. We also see that in the special case of unitary elasticity the expected utility for all policy coalitions is identical. Under this special assumption, there is no difference in welfare between a fixed and flexible exchange rate, nor is there any benefit from improved risk-sharing across countries. Equations (46a) – (46d) thus demonstrate the restrictive nature of assuming unitary elasticity. In particular, as we've mentioned above, unitary elasticity of substitution between home and foreign goods: (i) leads to complete risk-sharing, eliminating any difference between allocations in complete markets and financial autarky *and* (ii) eliminates wage rigidities, removing the difference between allocations under flexible exchange rates and within a currency union as well as between non-contingent and contingent domestic fiscal policy in a currency union. In both cases, risk-sharing and the elimination of nominal rigidities occur via movements in the terms of trade. This explains why Obstfeld and Rogoff (2002) and others found such small gains from cooperation: when the elasticity is unitary, there are simply no gains from cooperation available as movements in the terms of trade fill the role of cross-country risk-sharing and negate the influence of nominal rigidities.

One can see this explicitly by comparing consumption in all allocations when $\gamma = 1$:

$$C_{flex,complete|\gamma=1} = C_{fixed,complete|\gamma=1} = C_{flex,autarky|\gamma=1} = C_{fixed,autarky|\gamma=1} = C_{labor\ mobility|\gamma=1}.$$

Under unitary elasticity, consumption is equalized across all risk-sharing regimes, and across all exchange rate regimes. This demonstrates the importance of considering welfare away from the unitary elasticity case. The closed-form solution allows us to evaluate the optimal design of a fiscal union under non-unitary elasticity and isolate the impact of risk sharing, wage rigidity and labor mobility from a welfare perspective.

Another interesting welfare comparison concerns the gains from improved risk-sharing outside of and within currency unions. Using (46a) – (46d), one can easily show that the gains from improved risk sharing are higher within a currency union than outside of one (which matches the results of Farhi and Werning (2012)),

$$\log \mathbb{E}\left\{U_{fixed,complete}\right\} - \log \mathbb{E}\left\{U_{fixed,autarky}\right\} \geq \log \mathbb{E}\left\{U_{flex,complete}\right\} - \log \mathbb{E}\left\{U_{flex,autarky}\right\},$$

and that the gains from a flexible exchange rate are higher in autarky than under complete risk-sharing

$$\log \mathbb{E}\left\{U_{flex,autarky}\right\} - \log \mathbb{E}\left\{U_{fixed,autarky}\right\} \geq \log \mathbb{E}\left\{U_{flex,complete}\right\} - \log \mathbb{E}\left\{U_{fixed,complete}\right\}.$$

In other words, the benefits of a transfer union are higher in a currency union, and the benefits of independent monetary policy are higher in financial autarky.

7.1 Calibration in the Closed-Form Model

After deriving analytical expressions for welfare, we examine the benefits of a tax union, a transfer union, a flexible exchange rate, contingent fiscal policy and labor mobility. Our calibration for the closed-form model is reported in Table 1. We calibrate our parameters at quarterly frequency according to standard benchmarks given in Gali and Monacelli (2005, 2008), Rotemberg and Woodford (1997) and others.

In our welfare analysis, we allow the Armington elasticity to vary while fixing the other parameters of the model. Our aim is to calculate welfare exactly for a wide range of Armington elasticity values. Each plot below shows the loss in consumption as a percentage of the Pareto optimal allocation for a range of γ.

7.2 Tax Union

We begin by comparing the welfare of a country in a tax union (denoted by *tax*) with the welfare of a country outside of a tax union (denoted by *notax*), assuming that the two countries are identical in all other respects. Aside from the terms of trade externality, both countries are subject to the same distortions, whether that be incomplete risk-sharing or wage rigidity. They differ only in the fact that one country faces a terms of trade markup in it's steady state allocation (the country outside of a tax union) and the other does not (the country within a tax union). The log difference in expected utility between a country inside a tax union and a country outside of a tax union is:

$$\log \mathbb{E}\left\{U_{tax}\right\} - \mathbb{E}\left\{U_{notax}\right\} = \left(\frac{1-\sigma}{\sigma+\varphi}\right)\log \mu_\gamma = \left(\frac{1-\sigma}{\sigma+\varphi}\right)\log\left(\frac{\gamma}{\gamma-1}\right). \tag{47}$$

Equation (47) shows that the welfare gains from a tax union are decreasing in γ, the degree of substitutability between products across countries. As goods become closer substitutes, country level monopoly power falls and the distortionary impact of the terms of trade externality decreases due to the declining markup on exports. In the limit, as $\gamma \to \infty$ and goods become perfect substitutes, the terms of trade markup will go to zero, and a tax union will be unnecessary. On the other hand, as the Armington elasticity decreases, countries gain a higher degree of monopoly power over their export good and thus increase their terms of trade markup μ_γ. In the limit, as $\gamma \to 1$, the terms of trade markup approaches infinity and the benefits of a tax union dwarf the gains from other policy measures.

Figure 4 illustrates the importance of a tax union for low values of the Armington elasticity and plots the loss in consumption from the terms of trade distortion relative to the Armington elasticity. The figure makes it clear that as $\gamma \to 1$, a tax union becomes imperative. In the absence of a tax union, optimizing non-cooperative fiscal authorities charge extremely large markups on their export good, leading to a dismal equilibrium for all countries. This is one of the most important insights we glean from the closed-form model: non-cooperative fiscal policy inflicts extremely large welfare losses on other countries under low Armington elasticity. Because there are very few papers that consider non-cooperative fiscal policy, this dynamic is generally absent in the literature. As we've mentioned before, it is important to point out that the benefits of a tax union occur in the steady state because the terms of trade markup is present in the steady state. Business cycle fluctuations and shocks have no bearing on the welfare gains from a tax union. The gains from a tax union are a steady state phenomenon, and as such, cannot be analyzed in a log-linear model.

7.3 Transfer Union, Flexible Wages and Labor Mobility

Having established the relative importance of a tax union with respect to the Armington elasticity, we now turn to the welfare gains achieved through improved risk-sharing (via a transfer union or deeper financial integration), the elimination of wage rigidity (via optimal monetary policy outside of a currency union or contingent fiscal policy within a currency union) and labor mobility. In what follows, we assume the presence of a tax union, which removes the constant terms of trade markup.

Figure 5 plots the consumption in each allocation as a percentage of the Pareto optimum. The negative impact of the wage rigidity distortion dominates the negative impact of financial autarky, as both fixed wage allocations perform quite poorly relative to the flexible wage allocations, particularly as the degree of substitutability increases. The relative similarity of all flexible wage allocations (Flex Complete, Flex Autarky, and Labor Mobility) is quite striking. Even in financial autarky, when wages are flexible households are able to stabilize consumption with small movements in their labor hours. As a result, the benefit of consumption risk-sharing is very small when wages are flexible. The gains from flexible wages approach 2% of permanent consumption under complete risk-sharing and 4% of permanent consumption under financial autarky for $\gamma = 10$. On the other hand, the welfare gains from perfect risk-sharing via a transfer union or complete markets equal 2% of permanent consumption within a currency union when $\gamma = 10$.

7.4 Are countries better off in a currency union?

One of the arguments in support of a currency union, advanced by Mundell (1961, 1973) among others, is that the formation of such a union will lead to deeper financial integration and improve cross-country risk sharing. Using this logic, we conduct a thought experiment on the potential benefits of a currency union. We compare the welfare of a country outside a currency union in financial autarky with a member of a currency union in complete markets. Is a country better off with a flexible exchange rate and no risk sharing, or in a currency union with perfect risk sharing? The answer depends on the degree of risk aversion as defined by σ, the inverse Frisch elasticity of labor supply φ, as well as the Armington elasticity γ. Details are found in Appendix H. Here we focus on the intuition.

When households are completely risk neutral ($\sigma = 0$), they prefer a country with independent monetary policy in financial autarky if $\gamma > 1$. As households become more risk averse, they prefer a country that is a member of a currency union with full risk-sharing for a wider range of values of γ. In the limit, as $\sigma \to \infty$, households prefer a currency union if $\gamma \in \left(1, \frac{1+2\varphi}{\varphi}\right)$. For standard calibrations of φ, this means households will prefer a currency union when γ is between 1 and 2. So there is a very small range of γ for which households are better off in a currency union in complete markets than outside of one in financial autarky. Even under extreme risk aversion, deeper financial integration is not worth the loss of independent monetary policy.

As export goods become closer substitutes, the welfare losses from financial autarky fall relative to the gains from independent monetary policy. What causes this? Assume country i is hit with a negative productivity shock. If country i is a member of a currency union, wage rigidity will force its producers to charge a higher price. With a flexible exchange rate, the higher domestic price would be offset by a depreciated currency, but in a currency union this effect is absent. Given the higher price, consumers in country i and in other countries will switch to cheaper substitutes. If the elasticity of substitution is very high, demand for country i's good will collapse, and country i will produce almost nothing. If markets within the currency union are complete or a transfer union is in place, consumption must be equal across countries. However only a few countries will produce any output, and households in those few countries will have to work long hours to supply goods for the whole currency union. As a result, average consumption and welfare will fall. This effect is exacerbated as goods become closer substitutes. In the limit, when goods are perfect substitutes ($\gamma = \infty$) and shocks are asymmetric, only one country in the currency union will produce any output, and consumption and welfare will equal zero for all countries in the union. This illustrates the challenges that arise within a currency union when countries produce similar goods and face wage rigidity.

On the other hand, when substitutability is close to one, the welfare losses from wage rigidity and the gains from risk-sharing go to zero. Terms of trade movements will provide risk-sharing and insulate economies from the negative impact of asymmetric productivity shocks and nominal rigidities. In this case, a country will be indifferent between remaining outside a currency union in financial autarky and joining a currency union with full risk-sharing.

In reality of course, membership in a currency union does not guarantee perfect risk sharing through access to complete markets, nor does lack of membership in a currency union prevent countries from accessing international financial markets. Whether countries enjoy some degree of cross-border risk sharing seems to be largely unrelated to their membership in a currency union, although it is true that the introduction of the euro led to an increase in cross-border lending within the euro area, as well as an initial convergence of borrowing rates within the union. However, from a theoretical standpoint it is hard to argue that the potential risk sharing benefits of a currency union outweigh the loss of independent monetary policy.

8 Extended Model: Home Bias, Calvo Rigidities and Incomplete Markets

In this section we relax the assumption of complete openness and conduct welfare analyses in a model with home bias. We also employ Calvo wage rigidities in place of one period in advance wage setting and consider incomplete markets where cross-border trading in safe government bonds is permitted. The model presented here, which we refer to as the extended model, is identical in all other respects to the closed-form model described in Section 2. The extended model is laid out in full detail in Appendix I.

In each country i, the consumption basket consists of home (C_{it}^H) and foreign (C_{it}^F) goods,

$$C_{it} = \left[(1-\alpha)^{\frac{1}{\eta}}(C_{it}^H)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{it}^F)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{48}$$

where C_{it}^F and C_{it}^H are defined as

$$C_{it}^F = \left(\int (C_{ijt}^F)^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma}{\gamma-1}} \quad \text{and} \quad C_{it}^H = \left(\int (C_{it}^H(h))^{\frac{\varepsilon-1}{\varepsilon}} dh\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{49}$$

C_{ijt}^F denotes consumption by households in country i of the variety produced by country j, while $C_{it}^H(h)$ denotes consumption by households in country i of the domestic variety produced by intermediate firm h. The elasticity of substitution between home and foreign products is defined by η, while the elasticity of substitution between the goods of different countries remains γ. The relative weight of these goods in the consumption basket is defined by the degree of home bias, $1-\alpha$. When $\alpha = 0$, home bias is complete and households only consume domestic goods. In the opposite extreme, when $\alpha = 1$, the economy is fully open and households will consume a basket made up entirely of imports from all other countries in the world.

Calvo wage setting can be expressed as

$$W_t^{1-\varepsilon} = (1-\theta_W)\tilde{W}_t^{1-\varepsilon} + \theta_W W_{t-1}^{1-\varepsilon} \tag{50}$$

where W_t is the actual wage, $\tilde{W}$ is the optimal reset wage and θ_W is the fraction of households who are able to reset wages in each period.

There is a nominal government bond that pays in units of the import basket C_F. Households will maximize utility from (4) subject to the following budget constraint:

$$C_{it}(h) + \frac{B_{it}(h)}{P_{it}} = (1-\tau_i)\left(\frac{W_{it}(h)}{P_{it}(h)}\right)N_{it}(h) + \mathcal{D}_{it}(h) + \mathcal{T}_{it}(h) + \Gamma_{it}(h) + (1+i_{t-1})\left(\frac{B_{it-1}(h)}{P_{it}}\right). \tag{51}$$

The domestic interest rate i_t equals the world interest rate plus a country specific interest rate premium $p()$ that is strictly increasing in the amount of debt B_t:

$$i_t = i^* + p(B_t). \tag{52}$$

Financial autarky is the case for which p goes to infinity. The interest rate premium is necessary to ensure stationarity, a feature demonstrated in Schmitt-Grohe and Uribe (2003).

8.1 Calibration in the Extended Model

As in the closed-form case, we calibrate the extended model at a quarterly frequency. All parameter values are found in Table 2. We conduct a number of experiments to measure the robustness of our closed-form results for varying degrees of openness (α) and substitutability (γ). We calibrate $\eta = 1$, the elasticity between home and foreign goods, to match Gali and Monacelli (2005, 2008) and Farhi and Werning (2012), although later we consider cases where $\eta > 1$. The assumption of $\eta = 1$ leads to a Cobb-Douglas consumption basket: $C_{it} = C_{H,it}^{1-\alpha} C_{F,it}^{\alpha}$. We still allow for non-unitary elasticity between the products of different countries defined by γ. We consider three settings for Calvo wage rigidity: flexible wages ($\theta_W = 0$), low wage rigidity ($\theta_W = 0.75$) which implies that the average household resets wages once every four quarters, and high wage rigidity $\theta_W = 0.93$ which implies that the average household resets wages every three years. High wage rigidity ($\theta_W = 0.93$) is actually a conservative parameterization relative to recent estimates by Schmitt-Grohe and Uribe (2011 and 2012), who estimate the degree of downward wage rigidity for a number of economies in Europe from 2008-2011. They find complete downward wage rigidity in a number of countries, including Greece, Portugal and Spain. Although wages are more flexible in the upward direction, our focus here is on the negative effect of downward wage rigidity and the large welfare losses that accrue in a currency union under this scenario. We also consider the impact of the full range of wage rigidity parameterizations in Figure 9.

8.2 Welfare Analysis in the Extended Model

We first analyze the welfare gains from a tax union. We compare the difference in steady state consumption between a set of countries outside of a tax union with those inside a tax union. Figure 6 plots the loss in permanent consumption from the terms of trade externality as a function of openness and the Armington elasticity. The distortionary impact of the terms of trade markup is increasing in both openness and the Armington elasticity. One can see this by examining the optimal non-cooperative income tax rate in the extended model, $\tau_i = 1 - \mu_\varepsilon \left(\frac{\gamma - 1 + (1-\alpha)\eta}{\gamma - (1-\alpha)(1-\eta)} \right)$, which we derive in Appendix J. This tax rate is increasing in openness and both elasticities, η and γ.

The intuition is as follows. As the degree of home bias increases, optimizing non-cooperative fiscal policymakers find it less desirable to impose a large terms of trade markup on their export good because home consumers pay the markup when they consume home products. This is a result of the law of one price: if the fiscal authority taxes workers in order to reduce supply and increase the price of its unique final good, households in all countries will suffer but home households will suffer more because the home good makes up (1-α) fraction of the consumption basket. In contrast, when economies are completely open, home households consume measure zero of their domestic good, and the non-cooperative fiscal authority is no longer concerned about reducing home welfare through the terms of trade markup. As we saw earlier in the closed-form version of the model, the optimal income tax rate converges to $\tau_i = 1 - \frac{\gamma}{\gamma-1}$ when economies are completely open. Within a tax union, the optimal tax rate remains $\tau_i = 1 - \mu_\varepsilon$.

Why does the welfare loss from the terms of trade externality increase in the degree of openness? When an economy is more open the household consumption basket consists of a larger percentage of imports. These imports are subject to a terms of trade markup. As a result, the more a country needs to import, the greater the negative welfare impact of the terms of trade externality. In addition, as goods become less substitutable (as the Armington elasticity decreases), fiscal authorities have an incentive to impose larger terms of trade markups on their exports through income tax setting. Country-level monopoly power is thus increasing in both openness and the Armington elasticity.

Next, we calculate the welfare losses from business cycle fluctuations in financial autarky and incomplete markets. We follow Lucas (2003) and estimate the utility from a deterministic consumption path and a risky consumption path with the same mean. We then calculate the amount of consumption necessary to make a risk averse household indifferent between the deterministic and risky consumption streams.

Figure 7 plots the loss in permanent consumption from business cycle fluctuations in financial autarky when wage rigidity $\theta_W = 0.93$. We ignore the impact of the terms of trade externality and assume that there is no steady state terms of trade markup here. Figure 7 shows that home bias lowers welfare for every value of the Armington elasticity. In other words, home bias exacerbates the negative welfare impact of wage rigidity in the absence of risk-sharing, demonstrating that our closed-form results actually provide a *lower bound* estimate of the welfare benefits of a transfer union under financial autarky. The losses in permanent consumption in financial autarky in a currency union are as high as 25% when economies are completely open ($\alpha = 1$) and 26% under full home bias ($\alpha \to 0$). Home bias thus adds about 1% to the loss in permanent consumption from rigid wages. In contrast, depending on the value of the Armington elasticity, the losses in permanent consumption can be as low as 0% ($\gamma = 1$) and as high as 26% ($\gamma = 20$). The Armington elasticity is far more important than the degree of openness in determining welfare losses.

Why does home bias increase the welfare losses from business cycle fluctuations in financial autarky? To better understand the intuition, consider the following thought experiment. Assume that wages are completely rigid, that $\alpha = 0.01$ so the economy is almost closed, and that home and foreign consumption baskets are perfect complements. In this case, one percent of consumption always goes to imports. Under financial autarky in a currency union, the cash value of imports must equal the cash value of exports. Therefore, total consumption is equal to the value of exports multiplied by 100. Fluctuations in total household consumption are thus equal to fluctuations in export revenues when the home and foreign consumption baskets are perfect complements. This effect is only strengthened when home and foreign goods are imperfect complements or substitutes. A negative shock raises the price of the home good, which will lead domestic households to substitute home goods for foreign goods in their consumption basket. The share of imports in total consumption will thus increase. But the value of imports must equal the value of exports in financial autarky, and exports are now uncompetitive on the world market due to the rise in price caused by the negative shock. As a result, total home consumption must fall.

Figure 8 plots the loss in permanent consumption from business cycle fluctuations in incomplete markets. The ability to trade bonds greatly improves welfare for countries in a currency union who are exposed to asymmetric shocks. Different from financial autarky, an increase in home bias improves the ability of countries to stabilize business cycles when markets are incomplete. If a country is completely open, bonds allow households to stabilize consumption but not labor, because exports are not competitive following a negative technology shock and wages are rigid. When home bias increases, stabilization of both consumption and labor is possible, because firms supply goods mainly to the home market. On the other hand, under financial autarky consumption is never stabilized, so that an increase in home bias has the opposite effect.

As a robustness check, we plot the loss in permanent consumption from business cycle fluctuations for varying levels of Calvo wage rigidity in Figure 9. We set openness equal to the euro area average ($\alpha = 0.35$). As in Figure 7, we ignore the effects of the terms of trade externality. Again, it is not simply wage rigidity that leads to large welfare losses, but rather the combination of wage rigidity *and* a high Armington elasticity. Simply put, when a country in a currency union produces exports that are easily substitutable, the welfare consequences are dramatic. When wages are completely rigid, as the evidence in Schmitt-Grohe and Uribe (2011) suggests for some European countries, the losses in permanent consumption are small for low values of elasticity, but approach 80% for high values of elasticity. Even for conservative estimates of wage rigidity, the losses in permanent consumption are quite large when the

Armington elasticity is high.

Overall our findings in this section confirm our closed-form results. The Armington elasticity remains an essential parameter that governs the optimal design of a fiscal union. A tax union is more important when exports are imperfect substitutes, while a transfer union is more important when exports are close substitutes. We also find that home bias increases the distortionary impact of imperfect risk sharing, but decreases the impact of the terms of trade externality because monopoly power at the export level declines when domestic consumption baskets consist of a significant portion of domestically produced goods. In the limit, when countries only consume domestic goods, there are no terms of trade externalities. Home bias thus strengthens the need for a transfer union but decreases the need for a tax union. In addition, home bias strengthens the distortionary impact of wage rigidity, raising the importance of contingent fiscal policy or labor mobility within a currency union. Our welfare analysis also demonstrates that under the commonly assumed Cole-Obstfeld calibration, which sets $\sigma = \eta = \gamma = 1$, the welfare losses from business cycle fluctuations are extremely small. We nest the Cole-Obstfeld calibration as a special case, and show that the the distortionary impact of imperfect risk-sharing and wage rigidity is much larger as the Armington elasticity moves away from unity and goods become more substitutable.

8.3 Country-Specific Elasticity Estimates and Welfare Losses

In this section we move from general welfare analysis to country-specific analysis. In Table 3 and 4 we compute the welfare losses under financial autarky and incomplete markets resulting from 1% technology shocks for a number of European countries using the elasticity estimates calculated by Corbo and Osbat (2013). Their paper is one of the few to estimate the aggregate elasticity of substitution between home and foreign products (η) and the aggregate elasticity of substitution between the products of different countries (γ). Most trade papers focus only on sector-specific estimates. We also consider three different values for Calvo wage rigidity in our computations: high wage rigidity ($\theta_W = 0.93$), low wage rigidity ($\theta_W = 0.75$) and flexible wages ($\theta_W = 0$).

Table 3 reports the loss in permanent consumption for the mean elasticity estimates. In each country, the welfare losses increase with the degree of wage rigidity in both financial autarky and incomplete markets, from a low of 0.03%-0.04% of permanent consumption in all countries when wages are flexible, to a range of 1.06%-5.48% under low wage rigidity, and a range of 3.71%-33.67% under high wage rigidity. The results are particularly striking for countries in financial autarky. The losses in permanent consumption from financial autarky are as high as 33% for Greece under high wage rigidity. Large losses in financial autarky under high wage rigidity also occur in Austria (24.78%), France (23.25%), Germany (23.90%) and Sweden

(31.86%). The benefits of incomplete markets become clear here, as the losses drop to a range of 3.71% (Italy) to 17.98% (Greece) under high wage rigidity when countries are able to trade a safe government bond. The importance of maintaining access to international financial markets is especially apparent in the "Gain from Fin. Mkts" column, where we calculate the welfare gain when a country moves from financial autarky to incomplete markets. The values in this column can be viewed as a lower bound estimate of the welfare gain from the optimal transfer union, which enables perfect risk-sharing.

Table 4 reports the loss in permanent consumption for the median elasticity estimates of Corbo Osbat (2013), which are lower than the mean estimates. Again, the losses increase in the degree of wage rigidity, from a low of 0.03% for most countries under flexible wages, to a range of 0.68%-3.38% under low wage rigidity, to a range of 2.80%-13.89% under high wage rigidity. Under financial autarky and high wage rigidity, the losses range from 8.95% (UK) to 13.89% (Sweden). These losses fall to a range of 0.68% (Italy) to 1.54% (Sweden) in incomplete markets, where governments can borrow to offset the recessionary influence of negative shocks. These results confirm that the potential gains from international fiscal cooperation are economically significant for a number of countries in the euro area, but particularly for countries that lose access to international financial markets and have high trade elasticities. Greece is a prime example of both.

9 Conclusion

In this paper we provide a unique perspective on the welfare gains from international fiscal policy cooperation and labor mobility in a currency union. We first derive a global closed-form solution for an open economy model in which countries are subject to three distortions: nominal rigidities, imperfect risk sharing and terms of trade externalities. Using this global closed-form solution, we study the benefits of a fiscal union in complete markets and financial autarky, for varying degrees of substitutability between traded goods produced in different countries, both within and outside of currency unions. This setup allows us to examine the optimal structure of a fiscal union and analytically calculate the exact welfare gains from cooperation among national policymakers for a broad set of scenarios. We then compute the welfare gains from the optimal fiscal union in a larger model that incorporates home bias and Calvo wage rigidities.

We show that the optimal design of a fiscal union depends crucially on the Armington elasticity, which defines the degree of substitutability between the products of different countries. When substitutability is low (around one), risk-sharing occurs naturally via terms of trade movements so that a transfer union is unnecessary. Terms of trade externalities are large however, and optimal policy will implement a tax union to prevent terms of trade manipulation. The welfare gains from a tax union can be as high as 3% of permanent consumption for standard calibrations. When substitutability is high (above one), risk-sharing no longer occurs naturally via terms of trade movements. If financial markets do not provide complete risk-sharing across countries, there is a role for a transfer union to insure against idiosyncratic shocks. The relative importance of a transfer union increases as goods become more substitutable. The welfare gains from a transfer union are as high as 5% of permanent consumption when countries are able to trade safe government bonds, and approach 20% of consumption when countries lose access to international financial markets. We also show that contingent domestic fiscal policy can make up for the lack of national monetary policy within a currency union and eliminate nominal rigidities. Finally, we prove that labor mobility can eliminate the distortionary impact of nominal rigidities and enhance international risk-sharing.

Our results illustrate why federal currency unions such as the U.S., Canada and Australia, with relatively high labor mobility, tax harmonization and built-in fiscal transfer arrangements, can withstand asymmetric shocks across regions much better than the euro area, which lacks many of these ingredients at the moment. The potential welfare gain from implementing such policies in the euro area is quite large, particularly for countries that produce highly substitutable export goods and that cannot raise funds on international financial markets to insure against downside risk.

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Table 1: Calibration of the Closed-Form Model

Parameter	Value	Description
σ	2	Risk aversion parameter
φ	3	Inverse labor supply elasticity (Gali and Monacelli (2005, 2008))
χ	1	Following Gali and Monacelli (2005, 2008)
ε	6	Elasticity between different types of labor
σ_Z	0.01	Standard deviation of technology
β	0.99	Household discount factor
γ	Varies	Armington elasticity

Table 2: Calibration of the Extended Model with Home Bias and Calvo Wage Rigidity

Parameter	Value	Description
σ	2	Risk aversion parameter
φ	3	Inverse labor supply elasticity (Gali and Monacelli (2005, 2008))
χ	1	Following Gali and Monacelli (2005, 2008)
ε	6	Elasticity between different types of labor
θ_W	Varies	Calvo parameter for wage rigidity
α	0.25	Openness (Gali and Monacelli (2005, 2008))
η	1	Elasticity between home and foreign goods
ρ_Z	0.95	Persistence of technology shock
σ_Z	0.01	Standard deviation of technology
β	0.99	Household discount factor
γ	Varies	Armington elasticity

Table 3: Losses in Permanent Consumption from Business Cycle Fluctuations for Mean Country-Specific Elasticity Estimates

	Parameters			**High Wage Rigidity** ($\theta_W = 0.93$)			**Low Wage Rigidity** ($\theta_W = 0.75$)			**Flex Wages** ($\theta_W = 0$)		
	α	η	γ	Fin. Autarky	Incomp. Mkts	Gain from Fin. Mkts	Fin. Autarky	Incomp. Mkts	Gain from Fin. Mkts	Fin. Autarky	Incomp. Mkts	Gain from Fin. Mkts
Austria	0.55	24.5	5.3	24.78	12.27	12.51	4.88	3.04	1.84	0.0367	0.0114	0.0253
Czech	0.70	13.3	9.5	23.29	10.82	12.47	4.53	3.09	1.44	0.0357	0.0102	0.0255
Denmark	0.40	8.6	4.2	15.03	6.58	8.45	3.72	1.71	2.01	0.0352	0.0133	0.0219
Finland	0.32	10.9	4.6	18.16	7.03	11.13	4.09	1.91	2.18	0.0361	0.0137	0.0224
France	0.31	13.2	8.9	23.25	9.85	13.40	4.72	2.54	2.18	0.0371	0.0129	0.0242
Germany	0.34	8.2	13.3	23.90	10.54	13.36	4.79	2.69	2.10	0.0373	0.0123	0.0250
Greece	0.35	28.5	22.0	33.67	17.98	15.69	5.60	3.99	1.61	0.0380	0.0115	0.0265
Hungary	0.49	9.6	7.7	19.55	9.28	10.27	4.28	2.44	1.84	0.0360	0.0118	0.0242
Italy	0.22	4.6	4.7	13.57	3.71	9.86	3.38	1.06	2.32	0.0359	0.0167	0.0192
Netherlands	0.62	9.5	5.3	16.48	8.04	8.44	3.85	2.17	1.68	0.0345	0.0108	0.0237
Portugal	0.41	6.8	5.4	15.67	6.32	9.35	3.72	1.75	1.97	0.0354	0.0130	0.0224
Slovakia	0.30	8.5	7.8	19.87	7.67	12.20	4.32	2.02	2.30	0.0368	0.0136	0.0232
Spain	0.30	5.5	6.7	16.67	5.88	10.79	3.88	1.64	2.24	0.0363	0.0142	0.0221
Sweden	0.42	15.8	25.5	31.86	17.74	14.12	5.48	3.96	1.52	0.0378	0.0112	0.0266
UK	0.37	4.4	6.0	14.64	5.51	9.13	3.56	1.55	2.01	0.0355	0.0135	0.0220
European Avg	0.35	11.5	9.1	22.39	9.73	12.66	4.62	2.52	2.10	0.0370	0.0127	0.0243

All losses are in percent. The column "Gain from Fin. Mkts" calculates the gain in permanent consumption in percent when a country moves from financial autarky to incomplete markets. Openness (α) is taken from Balta and Delgado (2009). The elasticity of substitution between home and foreign products (η) and the elasticity of substitution between the products of different countries (γ) for European countries is taken from Table 4 and 5 of Corbo and Osbat (2013).

Table 4: Losses in Permanent Consumption from Business Cycle Fluctuations for Median Country-Specific Elasticity Estimates

	Parameters			**High Wage Rigidity** ($\theta_W = 0.93$)			**Low Wage Rigidity** ($\theta_W = 0.75$)			**Flex Wages** ($\theta_W = 0$)		
	α	η	γ	Fin. Autarky	Incomp. Mkts	Gain from Fin. Mkts	Fin. Autarky	Incomp. Mkts	Gain from Fin. Mkts	Fin. Autarky	Incomp. Mkts	Gain from Fin. Mkts
Austria	0.55	5.7	4.0	12.23	4.77	7.46	3.14	1.50	1.64	0.0331	0.0116	0.0215
Czech	0.70	3.8	4.3	11.46	4.12	7.34	2.76	1.53	1.23	0.0308	0.0092	0.0216
Denmark	0.40	3.9	3.6	9.83	4.05	5.78	2.78	1.04	1.74	0.0334	0.0140	0.0194
Finland	0.32	4.0	3.5	10.59	3.23	7.36	2.83	0.94	1.89	0.0342	0.0156	0.0186
France	0.31	4.3	4.4	12.47	3.97	8.50	3.18	1.14	2.04	0.0351	0.0152	0.0199
Germany	0.34	4.2	5.3	13.61	4.73	8.88	3.38	1.34	2.04	0.0354	0.0143	0.0211
Greece	0.35	3.1	4.6	11.38	3.75	7.63	2.98	1.08	1.90	0.0346	0.0146	0.0200
Hungary	0.49	3.8	4.5	11.46	4.72	6.74	2.99	1.36	1.63	0.0334	0.0122	0.0212
Italy	0.22	3.4	3.4	10.03	2.36	7.67	2.71	0.68	2.03	0.0350	0.0184	0.0166
Netherlands	0.62	4.1	3.7	9.87	4.47	5.40	2.68	1.30	1.38	0.0312	0.0104	0.0208
Portugal	0.41	3.6	4.1	10.87	3.90	6.97	2.88	1.13	1.75	0.0337	0.0137	0.0200
Slovakia	0.30	4.1	4.3	12.13	3.75	8.38	3.12	1.07	2.05	0.0351	0.0155	0.0196
Spain	0.30	3.8	3.5	10.43	3.04	7.39	2.79	0.88	1.91	0.0343	0.0161	0.0182
Sweden	0.42	5.0	5.2	13.89	5.49	8.40	3.43	1.54	1.89	0.0348	0.0130	0.0218
UK	0.37	3.1	3.3	8.95	2.80	6.15	2.49	0.82	1.67	0.0330	0.0149	0.0181
Euro Avg	0.35	3.9	4.1	11.35	3.74	7.61	2.98	1.09	1.89	0.0344	0.0147	0.0197

All losses are in percent. The column “Gain from Fin. Mkts” calculates the gain in permanent consumption in percent when a country moves from financial autarky to incomplete markets. Openness (α) is taken from Balta and Delgado (2009). The elasticity of substitution between home and foreign products (η) and the elasticity of substitution between the products of different countries (γ) for European countries is taken from Table 4 and 5 of Corbo and Osbat (2013).

Figure 1: Model Timeline

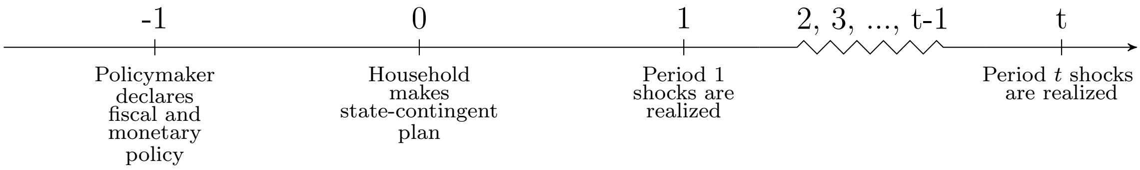


Figure 2: Types of Fiscal Union

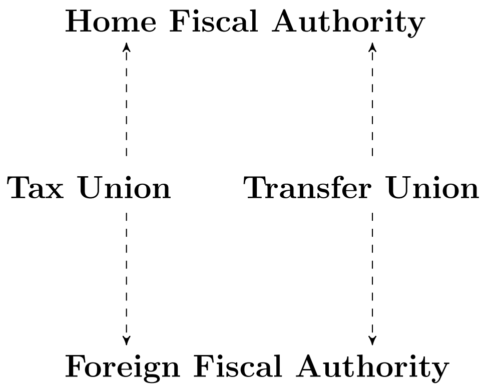
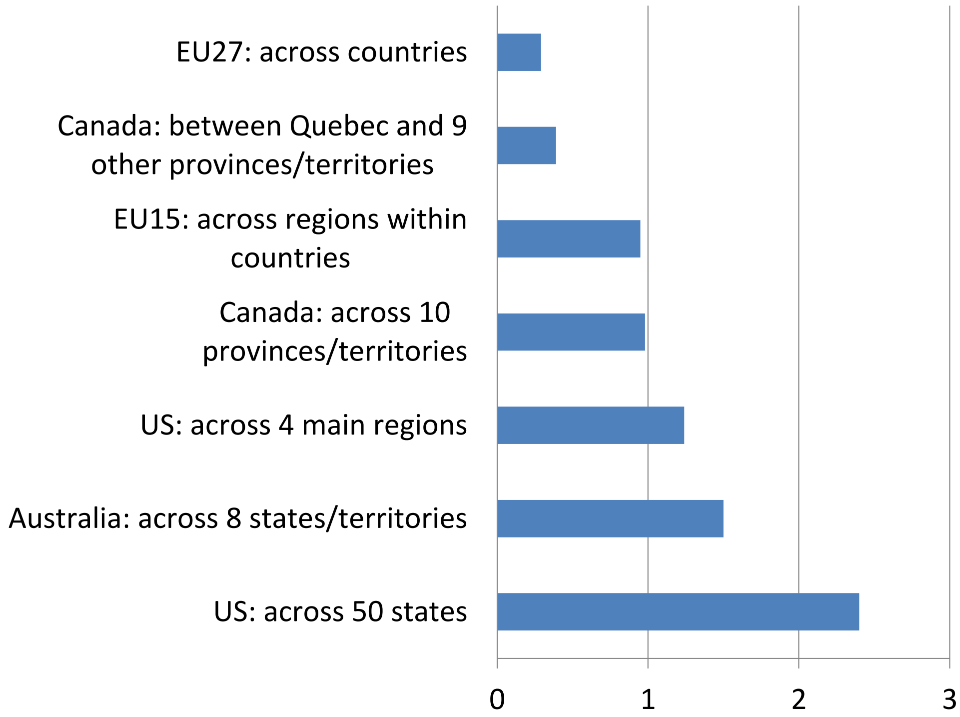


Figure 3: Annual Cross-Border Mobility, % of Population (2010)



Source: OECD. This figure plots the percentage of the population that moved across national borders (for the EU27), or across regions within the same country (all others).

Figure 4: Welfare Losses From Terms of Trade Externalities in the Closed-Form Model

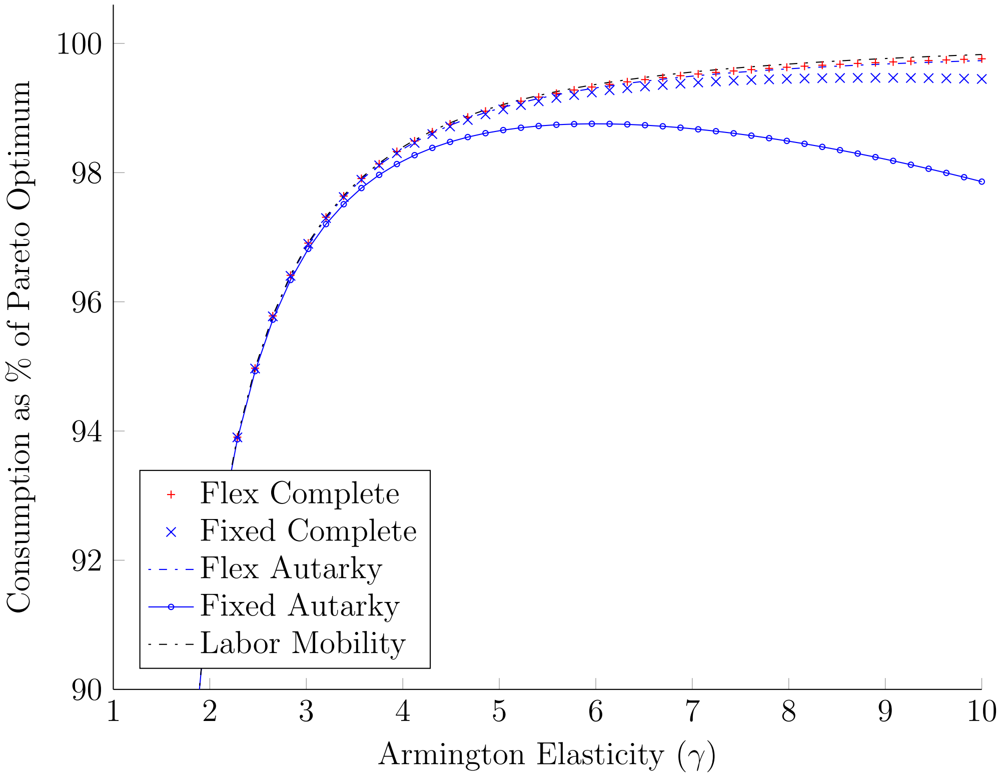


Figure 5: Welfare Losses from Business Cycle Fluctuations in the Closed-Form Model

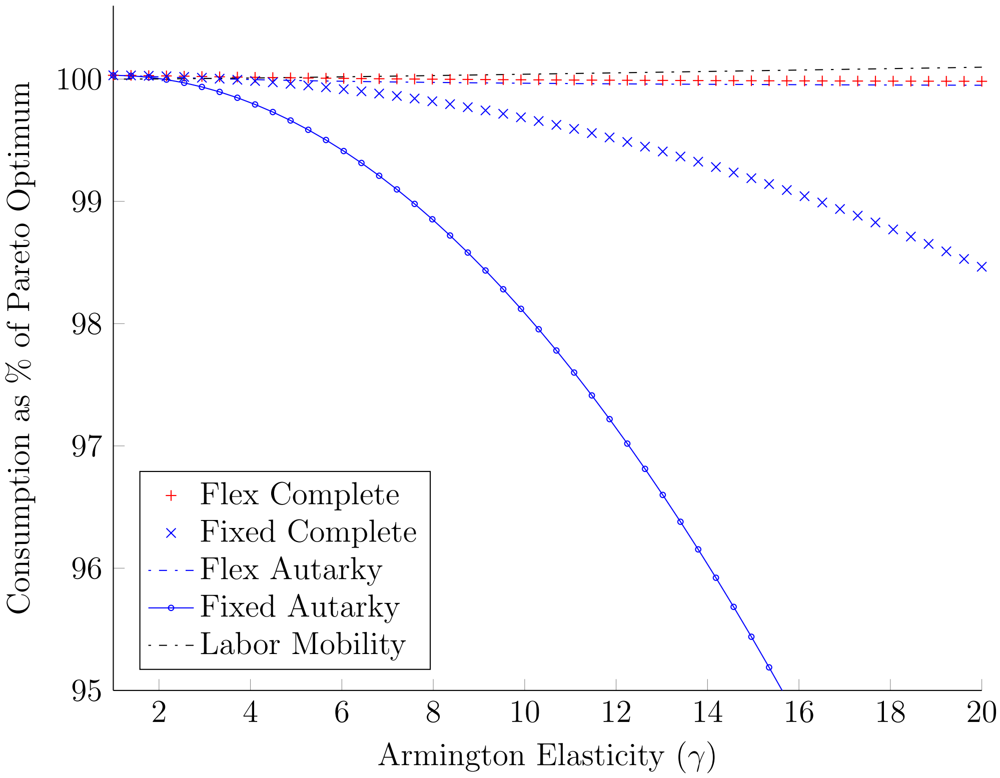


Figure 6: Welfare Losses from Terms of Trade Externalities

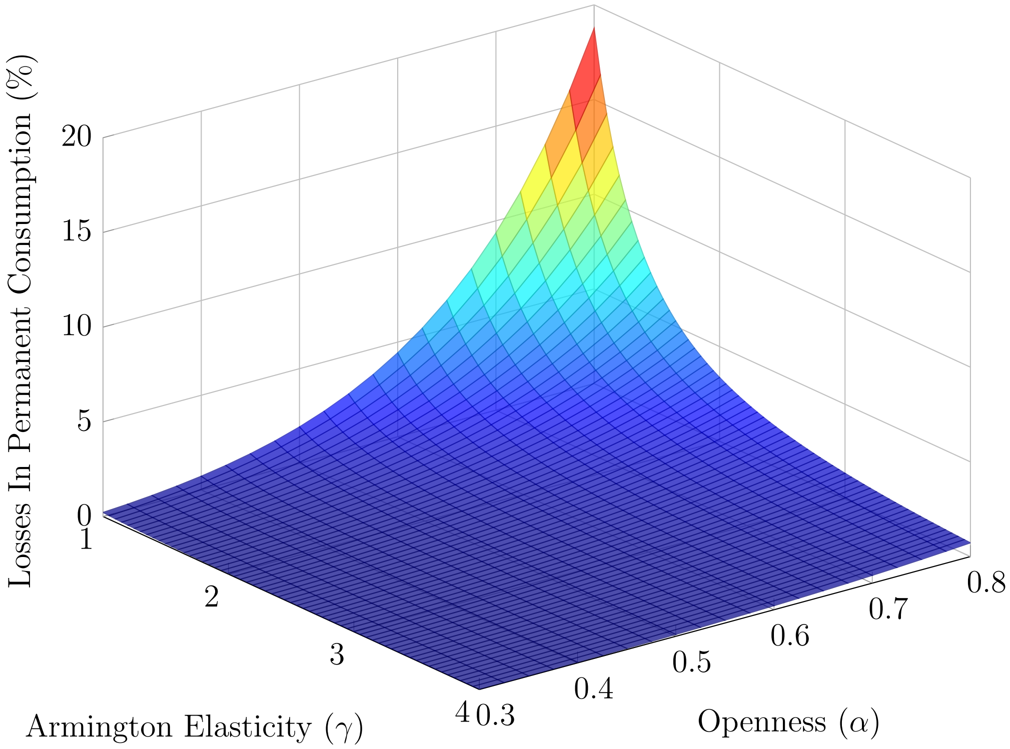


Figure 7: Welfare Losses from Business Cycle Fluctuations in Financial Autarky

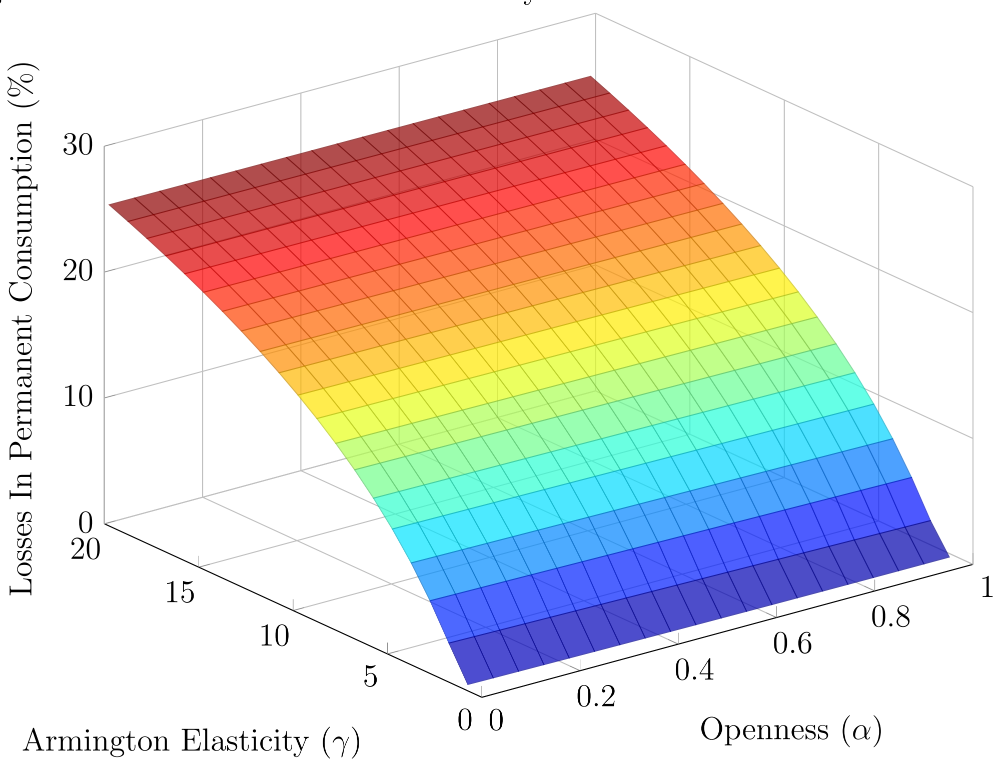


Figure 8: Welfare Losses from Business Cycle Fluctuations in Incomplete Markets

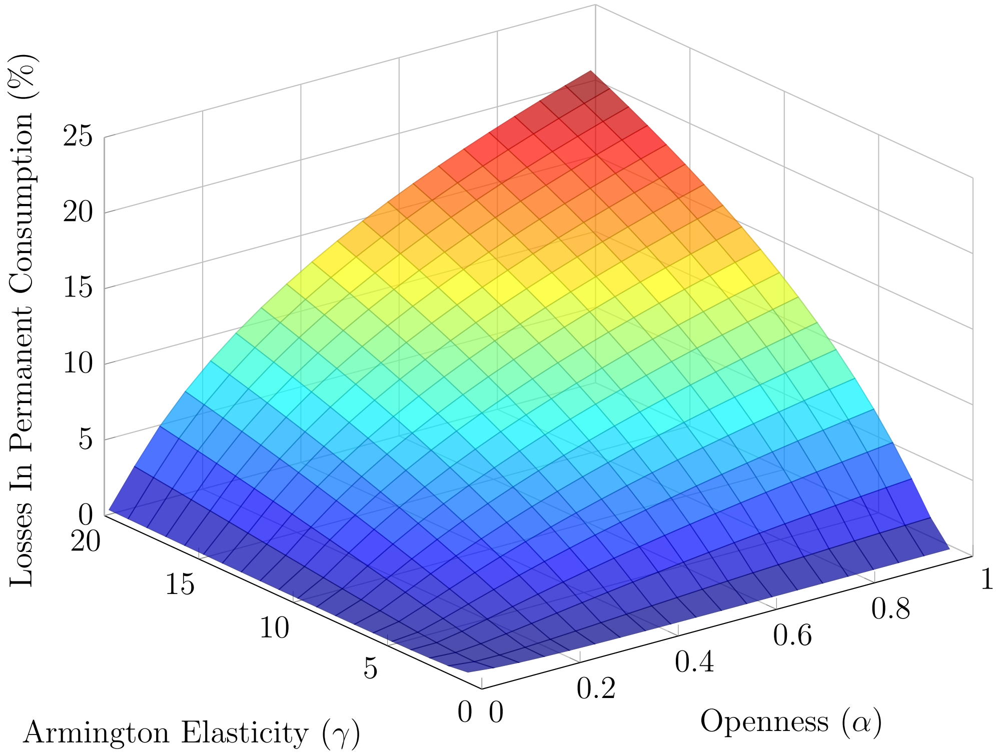
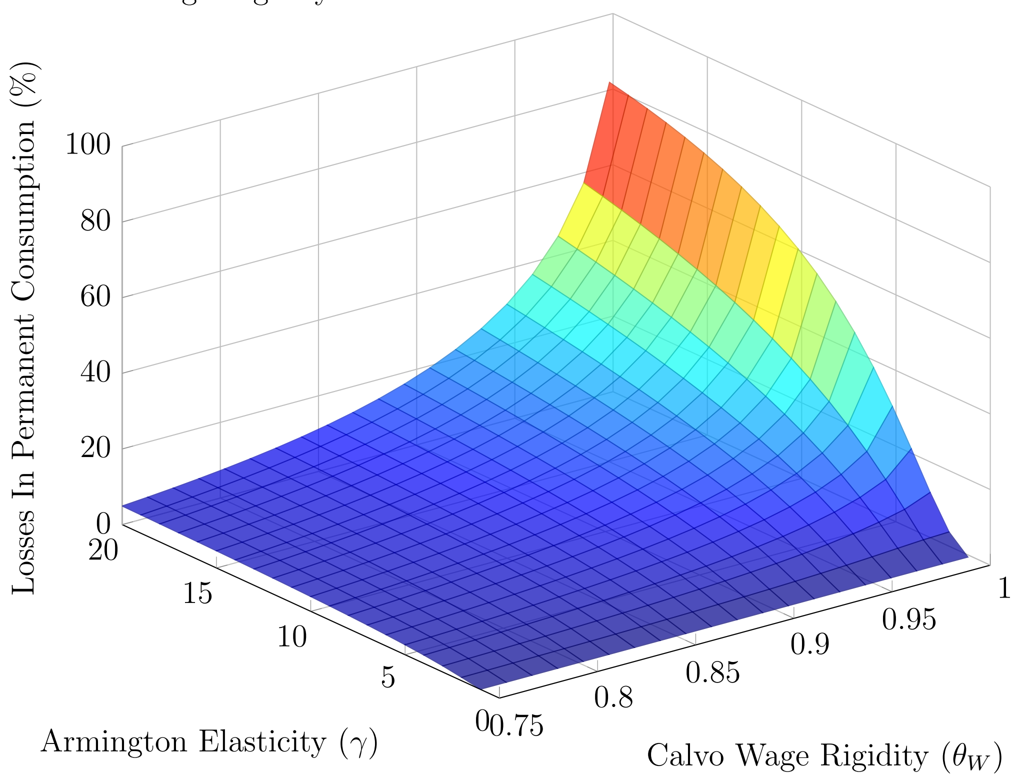


Figure 9: Welfare Losses from Business Cycle Fluctuations in Financial Autarky for Different Levels of Wage Rigidity



Technical Appendix

A Proof of Proposition 2 and 3: Flexible Exchange Rate Allocations

Non-cooperative central banks will maximize their objective function (28) subject to (29a), (29b), (29c) and (29d) in complete markets and (29a), (29c), (29d) and (31) in financial autarky. The Lagrangian for the non-cooperative and cooperative cases is:

$$\mathcal{L} = \frac{\mathbb{E}_{t-1}\left\{C_{it}^{1-\sigma}\right\}}{1-\sigma} - \chi\frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{1+\varphi} + \lambda_i\left(\mathbb{E}_{t-1}\left\{C_{it}^{1-\sigma}\right\} - \frac{\chi\mu_\varepsilon}{1-\tau_i}\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}\right)$$

Using $C_{it} = C_{wt}^{\frac{1}{\gamma}}\mathbb{E}_{t-1}\left\{N_{it}^{\frac{\gamma-1}{\gamma}}Z_{it}^{\frac{\gamma-1}{\gamma}}\right\}$ for complete markets, or $C_{it} = C_{wt}^{\frac{1}{\gamma}}N_{it}^{\frac{\gamma-1}{\gamma}}Z_{it}^{\frac{\gamma-1}{\gamma}}$ for financial autarky, we can take the first order condition with respect to N_{it}.[16] The FOC will be identical in both cases.

$$\frac{\partial\mathcal{L}}{\partial N_{it}} = C_{it}^{-\sigma}\left(1+\lambda_i(1-\sigma)\right)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{Y_{it}^{\frac{\gamma-1}{\gamma}}C_{wt}^{\frac{1}{\gamma}}}{N_{it}}\right) - \chi\left(1+\lambda\frac{\mu_\varepsilon}{1-\tau_i}(1+\varphi)\right)\frac{1}{N_{it}}N_{it}^{1+\varphi} = 0$$

In equilibrium, this equals:

$$1 = \underbrace{\chi\left(\frac{1+\frac{\lambda_i\mu_\varepsilon(1+\varphi)}{1-\tau_i}}{1+\frac{\lambda_i(1-\sigma)(\gamma-1)}{\gamma}}\right)}_{\text{Constant}}\left(\frac{N_{it}^{1+\varphi}}{C_{it}^{-\sigma}Y_{it}^{\frac{\gamma-1}{\gamma}}C_{wt}^{\frac{1}{\gamma}}}\right). \tag{A.1}$$

This equation holds in both complete markets and financial autarky, and differs from the flexible price equilibrium only by the constant term. However, subject to labor market clearing, this constant will coincide with the flexible price equilibrium. The flexible wage equilibrium in complete markets and financial autarky is found by taking expectations out of the labor market clearing condition (18) and substituting in goods market clearing (29b):

$$1 = \left(\frac{\chi\mu_\varepsilon}{1-\tau_i}\right)\frac{Y_{it}^{\frac{1+\varphi\gamma}{\gamma}}}{C_{it}^{-\sigma}C_{wt}^{\frac{1}{\gamma}}Z_{it}^{1+\varphi}}. \tag{A.2}$$

For complete markets, we can express output as a function of technology and a constant term by substituting (29b) into (A.2): $Y_{it} = A_iZ_{it}^{\frac{\gamma(1+\varphi)}{1+\gamma\varphi}}$. (We can do the same for exercise for autarky by substituting (31) into (A.2), but leave that to the reader). Using this expression for output, consumption in complete markets in country i can be expressed as

$$C_{it} = A_i^{\frac{\gamma-1}{\gamma}}C_{wt}^{\frac{1}{\gamma}}\mathbb{E}_{t-1}\left\{Z_{it}^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}}\right\}. \tag{A.3}$$

[16] Remember that we are optimizing given the fact that state s_t is realized. Expectations in our context thus refer to a summation over all possible states multiplied by the probability of each state occuring. For example, $\mathbb{E}_{t-1}\{C_{it}^{1-\sigma}\} = \sum_{s_t}C_i^{1-\sigma}(s_t)\Pr(s_t)$.

Now substitute (A.3) back into the flexible price equilibrium (A.2)

$$1 = \left(\frac{\chi\mu_\varepsilon}{1-\tau_i}\right) C_{wt}^{\frac{\sigma}{\gamma}} A_i^{\frac{(\gamma-1)\sigma}{\gamma}} \mathbb{E}_{t-1}\left\{Z_{it}^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}}\right\}^{\sigma} A_i^{\frac{1+\varphi\gamma}{\gamma}} C_{wt}^{-\frac{1}{\gamma}}, \tag{A.4}$$

and rearrange and solve for A_i:

$$A_i = \left[\left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right)^{\gamma} C_{wt}^{1-\sigma} \mathbb{E}_{t-1}\left\{Z_{it}^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}}\right\}^{-\sigma\gamma}\right]^{\frac{1}{1-\sigma+\gamma(\varphi+\sigma)}}. \tag{A.5}$$

Now, substitute the solution (A.5) into (A.3):

$$C_{it} = \left[\left(\frac{\chi\mu_\varepsilon}{1-\tau_i}\right)^{1-\gamma} C_{wt}^{\varphi+1} \mathbb{E}_{t-1}\left\{Z_{it}^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}}\right\}^{1+\gamma\varphi}\right]^{\frac{1}{1-\sigma+\gamma(\varphi+\sigma)}}. \tag{A.6}$$

Using the fact that $C_{wt} = \int_0^1 C_{it} di = C_{it}$ in equilibrium, and setting $Z_w = \left(\int_0^1 Z_{it}^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} di\right)^{\frac{1+\gamma\varphi}{(\gamma-1)(1+\varphi)}}$, integrate (A.6) over all i and solve for consumption for country i in complete markets:

$$C_{it} = \left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right)^{\frac{1}{\sigma+\varphi}} Z_w^{\frac{1+\varphi}{\sigma+\varphi}}. \tag{A.7}$$

Solving for labor and output using (A.7) is a straightforward exercise. The solution to the central bank's problem in complete markets and financial autarky for cooperative and non-cooperative equilibria coincides exactly with the flexible wage allocation.

Non-cooperative fiscal authorities will set a labor tax rate of $\tau_i = 1 - \frac{\mu_\varepsilon}{\mu_\gamma}$, introducing a terms of trade markup to exploit their country-level monopoly power. ■

B Proof of Proposition 4 and 5: Currency Union Allocations

Non-cooperative policymakers in a currency union in complete markets will maximize (33) by choosing a non state contingent income tax rate, subject to (29a), (29b), (29c), (29d) and (34). From (34) we can compute labor using $Y_{it} = Z_{it}N_{it}$:

$$N_{it} = AZ_{it}^{\gamma-1}. \tag{B.1}$$

Given the above, consumption will be

$$C_{it} = C_{wt} = A\left(\int Z_{it}^{\gamma-1} di\right)^{\frac{\gamma}{\gamma-1}}. \tag{B.2}$$

Using labor market clearing (18), and substituting in Y_{it}, C_{it}, N_{it} expressed as functions of A and Z_{it} from above, we find:

$$1 = \left(\frac{\chi\mu_\varepsilon}{1-\tau_i}\right) \frac{A^{1+\varphi}\int_0^1 Z_{it}^{(\gamma-1)(1+\varphi)} di}{A^{1-\sigma}\left(\int_0^1 Z_{it}^{(\gamma-1)} di\right)^{\frac{\gamma(1-\sigma)}{\gamma-1}}} \tag{B.3}$$

Now we can solve for A:

$$A = \left(\frac{\chi\mu_\varepsilon}{1-\tau_i}\right)^{\frac{-1}{\sigma+\varphi}} \left(\int_0^1 Z_{it}^{(\gamma-1)(1+\varphi)} di\right)^{\frac{-1}{\sigma+\varphi}} \left(\int_0^1 Z_{it}^{\gamma-1} di\right)^{\frac{\gamma}{\gamma-1}\frac{(1-\sigma)}{\sigma+\varphi}}. \tag{B.4}$$

Given this solution for the constant A, one can solve for C_{it} and N_{it} by substituting A into the expressions above, resulting in (35a) for C_{it} and (35b) for N_{it}. The same exercise in financial autarky will yield (36a) for C_{it} and (36b) for N_{it}. ■

C Proof of Proposition 6: Contingent Fiscal Policy Within A Currency Union

We now assume that fiscal policymakers can choose their income tax rate in each period. In both complete markets and financial autarky, the fiscal authority in country i will maximize utility in each period, choosing the optimal contingent labor tax τ_{it}, given the realization of Z_{it} in that period. To solve for the optimal allocation, follow the exact same steps as in Appendix B, but use τ_{it} as the policy instrument rather than N_{it}. The optimal allocations will remove the wage rigidity distortion and mimic the flexible exchange rate allocations, given by (30a) in complete markets and (32a) in financial autarky. ■

D Proof of Proposition 7: Tax Union

Under flexible exchange rates, policymakers in a tax union will maximize (39a) if they are not in a transfer union or (39e) if they are in a transfer union. In a currency union, policymakers in a tax union will maximize (39b) if they are not in a transfer union or (39f) if they are in a transfer union. Policymakers face the constraints outlined in Propositions 2 (flexible exchange rates and complete markets), 3 (flexible exchange rates and financial autarky), 4 (currency union and complete markets) or 5 (currency union and financial autarky), respectively.

In the non-cooperative case, policymakers do not internalize the impact of their tax rate on other countries. As a result, non-cooperative policymakers set τ_i to eliminate the markup on domestic intermediates, μ_ε, but also introduce a terms of trade markup, μ_γ, to take advantage of the monopoly power they exercise over their unique export good. The solution to the non-cooperative problem is $\tau_i = 1 - \frac{\mu_\varepsilon}{\mu_\gamma}$.

In a tax union policymakers do internalize the impact of their tax rate on other countries. The solution to the cooperative problem defines the optimal tax rate within a tax union: $\tau_i = 1 - \mu_\varepsilon$. The optimal tax rate in a tax union eliminates the markup on domestic intermediate goods without imposing a terms of trade markup.

■

E Proof of Proposition 8: Transfer Union

In a transfer union, policymakers agree to send contingent cash payments across countries. Under flexible exchange rates, policymakers in a transfer union will maximize (39c) if they are not in a tax union or (39e) if they are in a tax union. In a currency union, policymakers in a transfer union will maximize (39d) if they are not in a tax union or (39f) if they are in a tax union. Policymakers face the constraints outlined in Propositions 2 (flexible exchange rates and complete markets), 3 (flexible exchange rates and financial autarky), 4 (currency union and complete markets) or 5 (currency union and financial autarky), respectively. Because the transfers are state contingent, countries jointly agree on a state contingent plan to insure households against idiosyncratic consumption risk resulting from asymmetric shocks. Outside of a tax union, the solution to the transfer union optimization problem under flexible exchange rates will replicate the complete markets allocation detailed in Proposition 2. Outside of a tax union, the solution to the optimization problem in a currency union will replicate the complete markets allocation detailed in Proposition 4. Within a tax union, the solution to the transfer union optimization problem under flexible exchange rates will replicate the complete markets allocation detailed in Proposition 2 without a terms of trade markup. Outside of a tax union, the solution to the optimization problem in a currency union will replicate the complete markets allocation detailed in Proposition 4 without a terms of trade markup.

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F Proof of Proposition 10: Labor Mobility

In the presence of labor mobility, non-cooperative policymakers in a currency union in financial autarky will face the following problem

$$\max_{\tau_i} \frac{C^{1-\sigma}}{1-\sigma} - \chi \frac{N^{1+\varphi}}{1+\varphi} \tag{F.1}$$

s.t.

$$WN(h) = CP \tag{F.2a}$$

$$C = \left(\int_0^1 c_j^{\frac{\gamma-1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma-1}} \tag{F.2b}$$

$$\frac{W}{p_j} = Z_j \tag{F.2c}$$

$$P = \left(\int_0^1 p_j^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}} \tag{F.2d}$$

$$c_j = Z_j N_j \tag{F.2e}$$

$$N(h) = \int_0^1 N_j(h) dj \tag{F.2f}$$

$$N_j = \left[\int_0^1 N_j(h)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{F.2g}$$

where W is equalized across countries because of labor mobility. As each economy in the currency union is hit with idiosyncratic shocks, labor will shift from low demand bust countries to high demand boom countries.

FOCs:

$$c_j = \left(\frac{p_j}{P} \right)^{-\gamma} C \tag{F.3}$$

$$\frac{W}{P} = \left(\frac{\chi \mu_\varepsilon}{1-\tau_i} \right) C^\sigma N^\varphi \tag{F.4}$$

To solve for the optimal allocation, begin with (F.2d)

$$P = \left(\int_0^1 p_j^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}} = \left(\int_0^1 \left(\frac{W}{Z_j} \right)^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}} = W \left(\int_0^1 Z_j^{\gamma-1} dj \right)^{\frac{1}{1-\gamma}}$$

and solve for the real wage

$$\frac{W}{P} = \left(\int_0^1 Z_j^{\gamma-1} dj \right)^{\frac{1}{\gamma-1}}. \tag{F.5}$$

Now substitute this expression for the real wage into (F.2a):

$$C = N(h) \frac{W}{P} = N(h) \left(\int_0^1 Z_j^{\gamma-1} dj \right)^{\frac{1}{\gamma-1}} = \int_0^1 N_j(h) dj \left(\int_0^1 Z_j^{\gamma-1} dj \right)^{\frac{1}{\gamma-1}}.$$

Take (F.3) and substitute in the expression for the real wage from (F.2c):

$$c_j = \left(\frac{p_j}{P} \right)^{-\gamma} C = \left(\frac{Z_j}{\frac{W}{P}} \right)^{\gamma} C, \tag{F.6}$$

and then substitute $c_j = Z_j N_j$ in F.6

$$N_j = \left(\frac{W}{P} \right)^{-\gamma} Z_j^{\gamma-1} C. \tag{F.7}$$

Integrating N_j over j will yield

$$\int_0^1 N_j dj = N = C \left(\frac{W}{P} \right)^{-\gamma} \int_0^1 Z_j^{\gamma-1} dj = C \left(\int_0^1 Z_j^{\gamma-1} dj \right)^{-\frac{1}{\gamma-1}}. \tag{F.8}$$

Using (F.4), we can substitute in our expression for N from (F.8) and our expression for W/P from (F.5):

$$\frac{W}{P} = \left(\int_0^1 Z_j^{\gamma-1} dj \right)^{\frac{1}{\gamma-1}} = \left(\frac{\chi \mu_\varepsilon}{1-\tau_i} \right) C^\sigma N^\varphi.$$

Solving for C, we find:

$$C = \left(\frac{1-\tau_i}{\chi \mu_\varepsilon} \right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_j^{\gamma-1} dj \right)^{\frac{1+\varphi}{(\sigma+\varphi)(\gamma-1)}}. \tag{F.9}$$

To solve for N_j simply substitute (F.9) into (F.7):

$$N_j = \left(\frac{1-\tau_i}{\chi \mu_\varepsilon} \right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_j^{\gamma-1} dj \right)^{\frac{1+\varphi-\gamma(\sigma+\varphi)}{(\gamma-1)(\sigma+\varphi)}} Z_j^{\gamma-1}. \tag{F.10}$$

As we've mentioned a number of times, the optimal tax rate outside of a tax union in the decentralized Nash equilibrium will be $\tau_i = 1 - \frac{\mu_\varepsilon}{\mu_\gamma}$.

When labor can move freely across borders, the equilibrium allocation outside of a tax union will be:

$$C_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_i^{\gamma-1} di\right)^{\frac{1+\varphi}{(\sigma+\varphi)(\gamma-1)}}$$

$$N_i = \left(\frac{1}{\chi\mu_\gamma}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_j^{\gamma-1} dj\right)^{\frac{1+\varphi-\gamma(\sigma+\varphi)}{(\gamma-1)(\sigma+\varphi)}} Z_j^{\gamma-1}.$$

This allocation holds under flexible exchange rates and within a currency union, in both complete markets and financial autarky. ■

G Welfare Derivations

Below, we outline the steps necessary to derive the expected utility functions contained in Section 7 of the paper. Here we only conduct the exercise for flexible exchange rates in complete markets, but following the steps presented here will also yield the expected utility functions for the other allocations.

$$C_{flex,complete} = \left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right)^{\frac{1}{\sigma+\varphi}} \left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} di\right)^{\frac{1+\gamma\varphi}{(\gamma-1)(\sigma+\varphi)}}$$

$$\mathbb{E}\left\{U_{flex,complete}\right\} = \left[\frac{1}{1-\sigma} - \frac{1-\tau_i}{\mu_\varepsilon(1+\varphi)}\right] \mathbb{E}\left\{C_{flex,complete}^{1-\sigma}\right\}$$

$$= \left[\frac{1}{1-\sigma} - \frac{1-\tau_i}{\mu_\varepsilon(1+\varphi)}\right] \left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right)^{\frac{1-\sigma}{\sigma+\varphi}} \mathbb{E}\left\{\left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} di\right)^{\frac{(1+\gamma\varphi)(1-\sigma)}{(\gamma-1)(\sigma+\varphi)}}\right\}$$

For normative analysis, we assume that technology is log-normally distributed and is independent across time and across countries: $\log Z_{it} \sim N(0, \sigma_Z^2)$. The expectation above can then be rewritten as:

$$\mathbb{E}\left\{\left(\int_0^1 Z_i^{\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}} di\right)^{\frac{(1+\gamma\varphi)(1-\sigma)}{(\gamma-1)(\sigma+\varphi)}}\right\} = e^{\left[\frac{(\gamma-1)(1+\varphi)}{1+\gamma\varphi}\right]^2 \frac{(1+\gamma\varphi)(1-\sigma)}{(\gamma-1)(\sigma+\varphi)}\sigma_Z^2}$$

$$= e^{\frac{(\gamma-1)(1+\varphi)^2(1-\sigma)}{(1+\gamma\varphi)(\sigma+\varphi)}\sigma_Z^2}.$$

Now, we insert this expression back into the original equation and get:

$$\mathbb{E}\left\{U_{flex,complete}\right\} = \left[\frac{1}{1-\sigma} - \frac{1-\tau_i}{\mu_\varepsilon(1+\varphi)}\right] \left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right)^{\frac{1-\sigma}{\sigma+\varphi}} e^{\frac{(\gamma-1)(1+\varphi)^2(1-\sigma)}{(1+\gamma\varphi)(\sigma+\varphi)}\sigma_Z^2}.$$

Taking logarithms, we can rewrite the log of expected utility as:

$$\log \mathbb{E}\left\{U_{flex,complete}\right\} = \log\left[\frac{1}{1-\sigma} - \frac{1-\tau_i}{\mu_\varepsilon(1+\varphi)}\right] + \frac{1-\sigma}{\sigma+\varphi}\log\left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right) + \frac{(\gamma-1)(1+\varphi)^2(1-\sigma)}{(1+\gamma\varphi)(\sigma+\varphi)}\sigma_Z^2. \tag{G.1}$$

Calculating the expected utility for the other coalitions simply requires that one follow the steps outlined here. Notice that when we calculate welfare differences between allocations, the first and second terms on the right hand side of equation (G.1) will cancel out, leaving only the difference between the remaining term on the right hand side.

H Are countries better off in a currency union?

A country with a flexible exchange rate and no risk-sharing is better off than a country in a currency union with perfect risk-sharing whenever

$$(1+\varphi)\left[1+\varphi+(\gamma-1)(1-\sigma)(\sigma+\varphi)\right] - (1+\varphi-\gamma\varphi)\left[1-\sigma+\gamma(\sigma+\varphi)\right]^2 \geq 0, \tag{H.1}$$

which can be rewritten in cubic form as

$$\begin{aligned}&\Big(\gamma-1\Big)\Big\{\varphi(\sigma+\varphi)^2(\gamma-1)^2+(\gamma-1)\Big[2\varphi(\sigma+\varphi)(1+\varphi)-(\sigma+\varphi)^2\Big]\\&+\varphi(1+\varphi)^2+(1-\sigma)(1+\varphi)(\sigma+\varphi)-2(\sigma+\varphi)(1+\varphi)\Big\}\geq 0.\end{aligned} \tag{H.2}$$

The roots to this cubic equation are:

$$\gamma = \begin{cases} \frac{\sigma^2+2\sigma^2\varphi-\varphi^2+2\sigma\varphi^2-(\sigma+\varphi)^{\frac{3}{2}}\sqrt{\sigma+\varphi+4\sigma\varphi+4\sigma\varphi^2}}{2(\sigma^2\varphi+2\sigma\varphi^2+\varphi^2)} \\ 1 \\ \frac{\sigma^2+2\sigma^2\varphi-\varphi^2+2\sigma\varphi^2+(\sigma+\varphi)^{\frac{3}{2}}\sqrt{\sigma+\varphi+4\sigma\varphi+4\sigma\varphi^2}}{2(\sigma^2\varphi+2\sigma\varphi^2+\varphi^2)} \end{cases} \tag{H.3}$$

where the first root is less than one, and the third root is greater than one. When γ is less than one or greater than the third root expressed in (H.3), a country will be better off outside of a currency union in financial autarky than as a member of a currency union in complete markets.

In the limiting case as households become completely risk neutral ($\sigma \to 0$), the roots of (H.2) will be

$$\gamma = \begin{cases} -1 \\ 0 \\ 1 \end{cases} \tag{H.4}$$

while in the opposite limiting case, as households become extremely risk averse ($\sigma \to \infty$), the roots of (H.2) will be

$$\gamma = \begin{cases} 0 \\ 1 \\ \frac{1+2\varphi}{\varphi} \end{cases} \tag{H.5}$$

Internationally traded goods are perfect complements as γ approaches zero and perfect substitutes as γ approaches infinity, so γ must be non-negative. We can safely ignore any negative roots and focus only on positive roots. There is a very small window for which countries are better off as members of a currency union than as non-members. In a standard calibration with $\sigma = 2$ and $\varphi = 3$, countries are better off as members of a currency union only when $1 \leq \gamma \leq 1.12$. For all other values of γ outside this narrow range, households prefer to be outside of a currency union in financial autarky.

I Extended Model with Home Bias and Calvo Wage Rigidity

The consumption basket in the home country is given by (48), and consists of both home goods C^H and an import basket defined by C^F. Similarly the price index will consist of goods prices of both home and foreign products:

$$P_{it} = \left[(1-\alpha)(P_{it}^H)^{1-\eta} + \alpha(P_{it}^F)^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{I.1}$$

Relative demand for home and foreign products is given by

$$C_{it}^H = (1-\alpha)\left(\frac{P_{it}^H}{P_{it}}\right)^{-\eta} C_{it}, \tag{I.2}$$

$$C_{it}^F = \alpha\left(\frac{P_{it}^F}{P_{it}}\right)^{-\eta} C_{it}, \tag{I.3}$$

where C_{it}^F and C_{it}^H are defined as

$$C_{it}^F = \left(\int (C_{ijt}^F)^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma}{\gamma-1}}, \tag{I.4}$$

$$C_{it}^H = \left(\int (C_{it}^H(h))^{\frac{\epsilon-1}{\epsilon}} dh\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{I.5}$$

C_{ijt}^F denotes consumption by households in country i of the variety produced by country j. $C_{it}^H(h)$ denotes consumption by households in country i of the domestic variety produced by intermediate firm h.

Production in each country i and the demand for country $i's$ goods are given by:

$$Y_{it} = Z_{it}N_{it} \tag{I.6}$$

$$Y_{it} = C_{it}^H + \left(P_{it}^H/P_{it}^F\right)^{-\gamma} C_t^{H*} \tag{I.7}$$

where C_t^{H*} represents foreign consumption of the home good.

$$C_t = \frac{\mathbb{E}_0\left\{\sum \beta^t Y_{it}\frac{P_{H,it}}{P_{it}^F}\right\}}{\mathbb{E}_0\left\{\sum \beta^t \left(\frac{P_{it}}{P_{it}^F}\right)^{\frac{\sigma-1}{\sigma}}\right\}}\left(\frac{P_t^F}{P_t}\right)^{\frac{1}{\sigma}} \tag{I.8}$$

The equations describing Calvo wage setting are:

$$V_{W,t} = N_t^{1+\varphi} + \beta\theta_W \mathbb{E}_t V_{W,t+1} \tag{I.9}$$

$$\tilde{V}_{W,t} = C_t^{-\sigma}\frac{N_t}{P_t} + \beta\theta_W \tilde{V}_{W,t+1} \tag{I.10}$$

$$\tilde{W} = \chi\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{V_{W,t}}{\tilde{V}_{W,t}} \tag{I.11}$$

$$W_t^{1-\varepsilon} = (1-\theta_W)\tilde{W}_t^{1-\varepsilon} + \theta_W W_{t-1}^{1-\varepsilon} \tag{I.12}$$

where $\tilde{W}$ is the flexible wage and θ_W is the fraction of households who are able to reset wages in each period.

J Non-cooperative Tax Policy in the Extended Model

In this section we solve for the optimal steady state income tax rate for non-cooperative fiscal authorities in the extended model with home bias. Because the optimal income tax rate is a steady state object, we will drop all time subscripts in this section. Using the demand equation from (I.2) and the fact that $C = YP^H/P$, we can write

$$C^H = (1-\alpha)C^{1-\eta}Y^{\eta}. \tag{J.1}$$

We can also rearrange the price index (I.1) to get

$$\left(\frac{P_H}{P}\right)^{\eta-1} + \alpha - 1 = \alpha\left(\frac{P_H}{P_F}\right)^{\eta-1}. \tag{J.2}$$

Substitute $C/Y = P^H/P$ into (J.2) and rearrange so that

$$\frac{P^H}{P^F} = \left[\frac{(C/Y)^{\eta-1} + \alpha - 1}{\alpha}\right]^{\frac{1}{\eta-1}}. \tag{J.3}$$

Now we plug this expression into the demand equation for home products (I.7):

$$Y = (1-\alpha)C^{1-\eta}Y^{\eta} + \left[\frac{(C/Y)^{\eta-1} + \alpha - 1}{\alpha}\right]^{\frac{-\gamma}{\eta-1}} C^{H*} \tag{J.4}$$

Using the implicit function theorem, we define $C = f(Y)$=(J.4), so that the non-cooperative policymaker's objective function becomes

$$\max_{Y} \frac{f(Y)^{1-\sigma}}{1-\sigma} - \chi\frac{Y^{1+\varphi}}{1+\varphi}. \tag{J.5}$$

The policymaker's first order conditions with respect to Y are

$$f'(Y)f(Y)^{-\sigma} - \chi Y^{\varphi} = 0 \tag{J.6}$$

where we've used $Y = N$ because we are in steady state, and $C = f(Y)$. From the implicit function theorem, we know that $f'(Y) = -g_Y/g_C$, where

$$g(C,Y) = -Y + (1-\alpha)C^{1-\eta}Y^{\eta} + \left[\frac{C^{\eta-1}Y^{1-\eta}+\alpha-1}{\alpha}\right]^{\frac{-\gamma}{\eta-1}} C^{H*}. \tag{J.7}$$

Solving for g_C gives

$$g_C = (1-\eta)(1-\alpha)C^{-\eta}Y^{\eta} - \frac{\gamma}{\alpha}\left[\frac{C^{\eta-1}Y^{1-\eta}+\alpha-1}{\alpha}\right]^{\frac{-\gamma}{\eta-1}-1} C^{H*}C^{\eta-2}Y^{1-\eta}. \tag{J.8}$$

In steady state, we know that $\alpha C = \alpha Y = C^{H*}$, so that $g_C = (1-\eta)(1-\alpha) - \gamma$. Similarly, we solve for g_Y:

$$g_Y = -1 + (1-\alpha)\eta C^{1-\eta}Y^{\eta-1} + \frac{\gamma}{\alpha}\left[\frac{C^{\eta-1}Y^{1-\eta}+\alpha-1}{\alpha}\right]^{\frac{-\gamma}{\eta-1}-1} C^{H*}C^{\eta-1}Y^{-\eta}, \tag{J.9}$$

which becomes $g_Y = -1 + (1-\alpha)\eta + \gamma$ in steady state. Using these two simplified expressions for g_C and g_Y, we can rewrite the FOC from (J.6):

$$f'(Y)f(Y)^{-\sigma} - \chi Y^{\varphi} = -\frac{g_Y}{g_C}C^{-\sigma} - \chi Y^{\varphi} = \frac{1-(1-\alpha)\eta-\gamma}{(1-\eta)(1-\alpha)-\gamma}Y^{-\sigma} - \chi Y^{\varphi} = 0. \tag{J.10}$$

Using the implicit function theorem and solving for steady state Y yields:

$$Y = \left(\frac{1-\tau_i}{\chi\mu_\varepsilon}\right)^{\frac{1}{\sigma+\varphi}} = \left[\frac{1}{\chi}\left(\frac{\gamma-1+(1-\alpha)\eta}{\gamma-(1-\alpha)(1-\eta)}\right)\right]^{-\frac{1}{\sigma+\varphi}} \tag{J.11}$$

where the optimal tax rate for non-cooperative fiscal authorities is $\tau_i = 1 - \mu_\varepsilon\left(\frac{\gamma-1+(1-\alpha)\eta}{\gamma-(1-\alpha)(1-\eta)}\right)$. As in the closed-form model, the optimal tax rate in a tax union will be $\tau_i = 1 - \mu_\varepsilon$.