Need for the Right Speed: Public Debt Deleveraging and the Timing of Austerity

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Abstract

I analyze what is the optimal public debt deleveraging timing in a context of heterogenous agents and nominal rigidities. I find that when government creates a demand shock, such as cutting public expenditure, agents' wealth position is a key variable in defining their preferences. I also find that when government creates a supply shock, such as by increasing income taxation, employment status is important for agents' preferences. I also find that in a context of public debt deleveraging nominal rigidities have non-negligible redistributional effects.

1 Introduction

Many countries have experienced in recent years a significant increase in the size of their public debt. While in some countries public debt increased largely as a consequence of the financial crisis, notably in the United States, in others, debt was already significantly high at the onset of the crisis. As some Eurozone economies struggled to refinance their debt, those who received assistance packages from institutions such as the International Monetary Fund or the European Central Bank were asked to implement plans to reduce their stock of liabilities. Even economies that did not receive assistance are now facing the question of how to deleverage. Remarkably, this episode of public debt deleveraging is set to occur after an unprecedented recession, whose effects have been felt differently according to agents' position in the wealth distribution.

This chapter studies the optimal deleveraging path a fiscal authority should undertake when forced to reduce public debt in a context of heterogenous agents and incomplete markets.

In my model, the government can use public expenditure or distortionary income taxation to deleverage. Agents differ with respect to their possibility of accessing financial markets: Only a subset of consumers can borrow or lend to smooth their consumption. In this context, public sector deleveraging has strong redistributional effects. Indeed, a large public debt implies high taxes (or low public expenditure) for all agents, while interest payments only accrue to those holding public bonds. High government debt is then a net transfer of wealth from the consumers who cannot trade financial assets to the ones who can. The timing of deleveraging and the fiscal instrument chosen to perform it are therefore not inconsequential in this context.

The choice of instrument through which the fiscal authority will reduce debt is important to determine dynamics of aggregate variables and agents' welfare. A reduction in public expenditure or an increase in distortionary taxation will lead to different types of recessions, with different effects on interest rates. In the first case, the real interest rate prevailing in the economy will fall. The government and agents who do not participate in financial markets will gain from low real interest rates. When the fiscal authority uses taxes to reduce the debt, instead, interest rates rise significantly. Consumers who hold public bonds will be the ones to benefit from this situation.

The presence of nominal rigidities and the Zero Lower Bound interact asymmetrically with the different types of recessions considered. Also, their effects on redistribution are non-trivial. When deleveraging occurs through a reduction in public expenditure a liquidity trap may occur, while downward wage rigidities are helpful for agents who have no access to financial markets. When, instead, deleveraging is brought about via taxation, presence of this kind of rigidities exerts an upward pressure on the real interest rate, worsening welfare of the economy as a whole.

2 Literature Reviews

My research is closely related to two different strands of the literature: one that has focused on private debt deleveraging and its interaction with monetary policy, and another one on optimal fiscal policy under commitment.

Papers in the literature on private debt deleveraging typically model such

an event as an exogenous shock then analyzing the impact of different monetary policies in this context. Some recent papers, such as Guerrieri and Lorenzoni (2010), Krugman and Eggertsson (2012) or Philippon and Midrigan (2011) have studied debt deleveraging in a closed economy. Others, among which Fornaro (2012) Cook and Devereux (2012) and Benigno and Romei (2012), have focused on the consequences of private debt deleveraging in an international context. I depart from this literature in two way: I analyze a public debt deleveraging and I study how the deleveraging path impacts on agents welfare.

A second strand of literature analyzes optimal fiscal policy. In their seminal paper, Lucas and Stokey (1983) show how the public authority should react to a shock when it is possible to issue a state contingent debt, in a representative consumer framework. One of the main results is that the public instrument inherits the persistence of the shock. Aiyagari, Marcet, Sargent and Seppala (2002) analyze the same problem when the public authority cannot issue state contingent debt. They show that, under this circumstance, the public authority instrument is more persistent than the shock. Werning (2007) and Karantounias (2013) analyze the same problem, the former departing from the representative agent assumption, while the latter considering recursive preferences. Lastly Bandhari, Evans, Golosov and Sargent (2013) study optimal taxation with heterogenous agents and aggregate uncertainty. This literature aims to understand how a government can react optimally to a shock. This chapter, instead, aims to understand, what is the optimal government-induced shock on the economy.

My paper is close to the work of Röhrs and Winter (2014). They analyze what are the consequences of a public debt reduction under heterogenous agents, market incompleteness under flexible prices on aggregate welfare . My research differ in that I focus on the optimal deleveraging speed and on its interaction with nominal rigidities.

3 Model

3.1 Consumers

I consider a closed economy inhabited by two types of agents, that I call Savers and Hand-to-Mouth. There is a continuum of measure $1-\chi$ of Savers and a continuum of measure χ of Hand-to-Mouth. There is no uncertainty,

so all agents have perfect foresight. Following Weil (1992), Savers differ with respect to the type of financial markets they can have access to. Hand-to-Mouth are endowed at birth with one unity of equity in all the firms in the economy. They cannot hold any other type of financial assets. Savers, instead, are not only endowed with the same amount of equity in firms that borrowers have, but they are also able to trade in riskless, one-period, non-contingent bonds. Access to financial markets allows Savers to choose how to optimally allocate their consumption intertemporally. Hand-to-Mouth agents, on the other hand, are forced to solve a static problem in each period.

Guiso, Haliassos and Jappelli (2000), among others, show that the degree of participation in financial markets increases in wealth. Accordingly, I consider Hand-to-Mouth agents in this model to proxy for the lower quantiles of the wealth distribution in the economy. Savers, on the other hand, proxy for richer consumers who hold financial assets in their portfolio.

Savers and Hand-to-Mouth have identical preferences over streams of consumpion, C and government-provided services, G. Moreover, they enjoy leisure and they supply hours of labor l to firms in the economy.

$$\sum_{t=0}^{\infty} \beta^t U(C_t^j, G_t, l_t^j) \qquad for \quad j = \{H, S\} \tag{1}$$

where $U(.)$ is concave, twice differentiable and satisfies the Inada condition.

I assume that C and G are bundles of goods

$$C = \left[\int_0^1 c(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

and

$$G = \left[\int_0^1 g(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}},$$

where $c(i)$ and $g(i)$ are private and public consumption of a generic good i produced in the economy. I assume the presence of a continuum of measure one of goods, which are imperfect substitute with an elasticity of intratemporal substitution equal to ϵ in both aggregators.

Savers and Hand-to-Mouth to firms two different varieties of labor. Every agent supplies labor to all the firms. Total labor supply of each agent is then:

$$l_H = \int_0^1 l_H(i) di \qquad l_S = \int_0^1 l_S(i) di$$

where $l_H(i)$ ($l_S(i)$) is the amount of hours supplied to firm i by the agent of type H (S). Since agents are indiffernt across firms to which they supply labor, an agent of type H (S) will receive the same wage W_H (W_S) from all firms.

Savers trade one-period riskless bond, b_t^s in unit of consumption good that pays a real interest rate, r_t. Hand-to-Mouth, instead, consume profits distributed by firms and their labor income in each period. Their respective budget constraints are as follows:

$$C_t^S = W_{S,t} l_{S,t}(1-\tau_t) - b_t^S + \frac{b_{t+1}^S}{1+r_t} + \varpi_t \tag{2}$$

and

$$C_t^H = W_{H,t} l_{H,t}(1-\tau_t) + \varpi_t \tag{3}$$

where ϖ are profits paid by each firm and τ_t is a proportional income tax charged by the fiscal authority.

Real interest rate is determined by the Fisher equation:

$$(1+r_t) = \frac{(1+i_t)}{\Pi_{t+1}}$$

where i_t is the nominal interest rate at time t and Π_{t+1} is the gross inflation at time $t+1$. Since the nominal interest rate cannot be negative, the real rate must be greater than the inverse of inflation, i.e.:

$$(1+r_t) \geq \frac{1}{\Pi_{t+1}} \tag{4}$$

Hand-to-Mouth do not face any intertemporal decision problem. In every period they decide consumption and labor supply, according to the first order condition:

$$\frac{U_l(c_t^H, G_t, l_{H,t})}{U_c(c_t^H, G_t, l_{H,t})} = -W_{H,t}(1-\tau_t) \tag{5}$$

Savers, on the other hand, decide how much to save as well as consumption and hours supplied, maximizing (1) subject to (2):

$$U_c(c_t^S, G_t, l_{S,t}) = \beta(1+r_t) U_c(c_{t+t}^S, G_{t+1}, l_{S,t+1}) \tag{6}$$

$$\frac{U_l(c_t^S, G_t, l_{S,t})}{U_c(c_t^S, G_t, l_{S,t})} = -W_{S,t}(1-\tau_t) \tag{7}$$

Savers smooth consumption intertemporally according to their Euler equation. Note that since Savers are the only type of agent that can optimally allocate their intertemporal pattern of consumption, their behavior will critically affect the price of the bond and the interest rate.

3.2 Firms

The economy is populated by a continuum of identical firms of measure one. Each firm has access to the following technology

$$y(i) = L(i)^{\omega} = (L_S(i)^{\alpha_S} L_H(i)^{\alpha_H})^{\omega} \tag{8}$$

allowing them to transform a mix of two different types of labor inputs, L_S and L_H into output of a differentiated good.

Total labor hired by each firm is a Cobb-Douglas aggregator of labor supplied by each type of agent, with $\alpha_S + \alpha_H = 1$ and $\alpha_S > \alpha_H$ to capture positive correlation between the distribution of wealth and the distribution of skills in the economy. Firms opereate under decreasing return to scale, so that $\omega < 1$.

Every firm

$$\min_{L_S(i), L_H(i)} W_S L_S(i) + W_H L_H(i) \tag{9}$$

subject to

$$y(i) \leq \bar{y}$$

From the optimization problem above, I derive

$$\frac{(1-\chi) W_S L_S(i)}{\alpha_S} = \frac{(1-\chi) W_H L_H(i)}{\alpha_H} = W L(i)$$

where the aggregate real wage W is defined as:

$$W(i) = (k W_S^{\alpha_S} W_H^{\alpha_H})$$

where $k \equiv \left(\frac{1}{\alpha_S}\right)^{\alpha_S} \left(\frac{1}{\alpha_H}\right)^{\alpha_H}$.

Each firm i, competing under monopolistic competition, faces a demand schedule of the type:

$$y(i) = \left(\frac{p(i)}{P}\right)^{-\epsilon} [C + G] = \left(\frac{p(i)}{P}\right)^{-\epsilon} Y \tag{10}$$

where $p(i)$ is the price set by firm i and P is the price aggregator defined as:

$$P = \left(\int_0^1 p(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$$

Every firm faces a quadratic cost of adjusting nominal prices a-la Rotemberg which is measured in units of aggregate final output, i.e.:

$$\phi(p_t(i), p_{t-1}(i), Y_t) = \frac{\varphi}{2} \left(\frac{p_t(i)}{p_{t-1}(i)\bar{\Pi}} - 1 \right)^2 Y_t.$$

where $\bar{\Pi}$ is steady state inflation and $\phi > 0$ determines the degree of nominal rigidity.

Firms maximize the present discounted sum of their profits:

$$\varpi(i)_t = \sum_{t=0}^{\infty} \beta^t \lambda_t \left[\frac{p_t(i)}{P_t} y_t(i) - W_t(i) L_t(i) - \phi(p_t(i), p_{t-1}(i), Y_t) \right] \tag{11}$$

subject to (8) and (10), where $\lambda_t \equiv (\chi U_c(c_t^H, G_t, l_t^H) + (1-\chi) U_c(c_t^S, G_t, l_t^S))$ is a weighted average of agents' specific stochastic discount factors. I can exploit the fact that every firm faces the same cost as well as the same demand schedule to drop the i index. First order condition for profits is as follows:

$$W_t = \frac{\omega}{\mu} Y_t^{1-\frac{1}{\omega}} + \frac{\phi\omega}{\epsilon}(\Pi_t - \bar{\Pi})\Pi_t Y_t^{1-\frac{1}{\omega}} - \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\phi\omega}{\epsilon}(\Pi_{t+1} - \bar{\Pi})\Pi_{t+1} Y_{t+1} Y_t^{-\frac{1}{\omega}} \tag{12}$$

where $\mu \equiv \frac{\epsilon}{\epsilon-1}$ is the markup and $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate. Output , Y_t, increases if either aggregate real costs decrease, W_t, if current inflation increases or if future inflation will decrease. From this equation it is possible to derive the standard positively sloped AS curve, since firms will supply more output whenever prices increase.

Additionally, firms are subject to a second type of rigidity: nominal wages are downwardly rigid for both types of labor. A generic firm i is a wage taker and does not consider this rigidity when maximizing their profits. Similarly, workers supply labor in a perfect competitive market and, as a consequence they also do not take into account the presence of this rigidity.

This friction is expressed as follows:

$$W_{S,t} \geq \psi W_{S,t-1} \Pi_t \qquad W_{H,t} \geq \psi W_{H,t-1} \Pi_t \tag{13}$$

where $\psi \in [0,1]$ is a measure of wage rigidity: $\psi = 0$ means that wages are free to move, while $\psi = 1$ leads to completely rigid nominal wage. I assume that the parameter ψ is identical for both labor markets.

Labor markets, whenever condition (13) is binding, will determine hours worked by the demand side, only. This will generate some involuntary unemployment due to high cost in that labor market. To summarize, when wages are free to move labor markets will clear, otherwise there will be an excessive supply of labor hours.

$$(L_{H,t} - \chi_{H,t})\left(W_{H,t} \ - \psi W_{H,t-1}\Pi_t\right) = 0 \tag{14}$$

and

$$(L_{S,t} - (1-\chi) l_{S,t})\left(W_{S,t} \ - \psi W_{S,t-1}\Pi_t\right) = 0 \tag{15}$$

Notice that the economy can be in four different situations: either Savers' labor market or Hand-to-Mouth labor market or both or none can experience involuntary unemployment. The introduction of two different labor markets allows me to consider the redistributional effect of downward wage rigidities in this context.

3.3 Government and Central Bank

The government provides G_t units of non-rival consumption good in every period. Public expenditure is financed either by charging taxes on labor income or by issuing a bond in term of consumption good, B_t^G whose return, also denominated in units of consumption good, is the real interest rate, r_t. Government budget constraint is :

$$\tau_t(W_t^S l_t^S + W_t^H l_t^H) = G_t + \frac{B_{t+1}^G}{(1+r_t)} - B_t^G \tag{16}$$

Since a fraction of the population is not able to optimally smooth consumption, the fiscal authority, by borrowing and saving, can positively affect the welfare of such agents.

The objective of the Central Bank is to maintain inflation on target whenever possible. The Central Bank keeps the nominal interest rate at zero whenever the desired nominal interest is negative.

$$\begin{cases} \Pi_t = \bar{\Pi} & \text{if } i_t \geq 0 \\ i_t = 0 & \text{Otherwise} \end{cases} \tag{17}$$

3.4 Market Clearing Condition

Goods and financial markets clear, i.e.:

$$(1-\chi)C_t^S + \chi C_t^H + G_t = Y_t \tag{18}$$

$$B_t^G + (1-\chi)b_t^S = 0 \tag{19}$$

Labor markets clear only if the constraint (13) is not binding. Hence:

$$(L_{H,t} - l_{H,t})\left(W_{H,t} \ - \psi W_{H,t-1}\Pi_t\right) = 0$$

and

$$(L_{S,t} - l_{S,t})\left(W_{S,t} \ - \psi W_{S,t-1}\Pi_t\right) = 0$$

3.5 Equilibrium

Given a sequence of taxes, public expenditure and nominal interest rates $\{\tau_t, G_t\}_{t=0}^{\infty}$ *an equilibrium is a sequence of prices* $\{r_t, \Pi_t, W_t, W_t^S, W_t^H\}_{t=0}^{\infty}$ *and allocations,* $\{C_t^S, C_t^H, L_t^S, L_t^H, l_t^S, l_t^H, b_t^s, Y_t\}$ *such that:*

- *Given prices, Savers maximize* (1) *subject to* (2) *and Hand-to-Mouth maximize* (1) *subject to* (3) *;*
- *Every firm maximizes* (11) *subject to* (8) *and* (10) *;*
- *Goods, financial and labor markets clear,* (18)*,* (18)*,* (14) *and* (15)*;*
- *Government Budget Constraint is satisfied,* (16)*;*
- *Central Bank targets the inflation,* (17)*;*

4 Calibration

4.1 Quantitative Results

This section analyzes the effects of a public debt reduction on agents' welfare. The assumption made in this model that the economy is populated by two types of agents helps to capture unequal responses to fiscal shocks, shown to be important by Kanishka and Surico (2011), among others. These authors, highlight, in particular, that a variable determining different effects of fiscal

changes is tightness of borrowing constraints, captured in this paper by the inability to borrow of Hand-to-Mouth agents.

I consider a scenario under which that the fiscal authority is forced to bring down its debt from an high level, B^H, to a low one, B^L in a determined time span. Such a debt reduction can be rationalized in several ways. For example, this path may be imposed by the existence of a supranational authority or by international investors willing to charge an infinite cost to the economy if at time T debt is not at the target, B^L.

Note that infinte paths are available to the fiscal authority to converge to the new steady state. I restrict my analysis to the class of monotonic decreasing deleveraging paths. This seems consistent with casual empirical evidence: as Southern European economies implemented austerity measures in response to the recent sovereign debt crisis, the proposed plans for public borrowing generally implied a mononotonically decreasing path for public debt.

I model the path of deleveraging as:

$$B_t^G = B_t^H + (B_t^L - B_t^H)\left(\frac{t}{T}\right)^{JB}$$

where I assume a strictly positive JB. Figure 1 show that JB is a measure of the speed of the public deleverage. A $JB < 1$ determines a convex deleveraging path (fast) while with a $JB > 1$ deleveraging is a concave process (slow).

Throughout the exercise, I remain agnostic about aggregate the welfare measure in this economy. I will proceed by analyzing separately the impact of different policies on the Hand-to-Mouth and Savers' welfare. This allows me to avoid imposition of Pareto weights on the heterogeneous agents that populate this economy.

The following proposition shows how, under certain conditions on agents' value functions, studying individual consumers' welfare allows us to exclude a given set of Pareto dominated policies. This will allow us, on the other hand, to state that the optimal policy for the economy as a whole will belong to the complement of this set.

I define the welfare function as:

$$We(JB) = \sum_{t=0}^{\infty} \beta^t \left\{ aWe_H(JB) + (1-a)We_S(JB) \right\}$$

where $We_H(JB)$ and $We_S(JB)$ are the Hand-to-Mouth and Savers value function, respectively, as a function of JB and a is a generic weight that can take any value in the interval $[0, 1]$. I define JB^H and JB^S, respectively as

$$JB^k = \arg\max_{JB} We_k(JB) \quad k = \{S, H\}.$$

Moreover, I define JB^W as:

$$JB^W = \arg\max_{JB} We(JB)$$

Proposition 1. *If agents' value functions are continuous, twice differentiable and concave in JB, with $JB^S < JB^H$ ($JB^H < JB^S$), then the JB^W that maximize W for a generic $a \in [0, 1]$ will lie in the interval $[JB^S, JB^H]$ ($[JB^H, JB^S]$). All the JB who do not belong to this interval are Pareto dominated.*

Proof is in appendix A.

I will restrict the set of possible actions undertaken by the government to reduce debt to include either reductions in government expenditure or increases in tax rates, but not both. Obviously, fiscal authorities in the real world may adopt a combination of these. However, this restriction allows to consider separately the effects of different types of fiscal contractions.

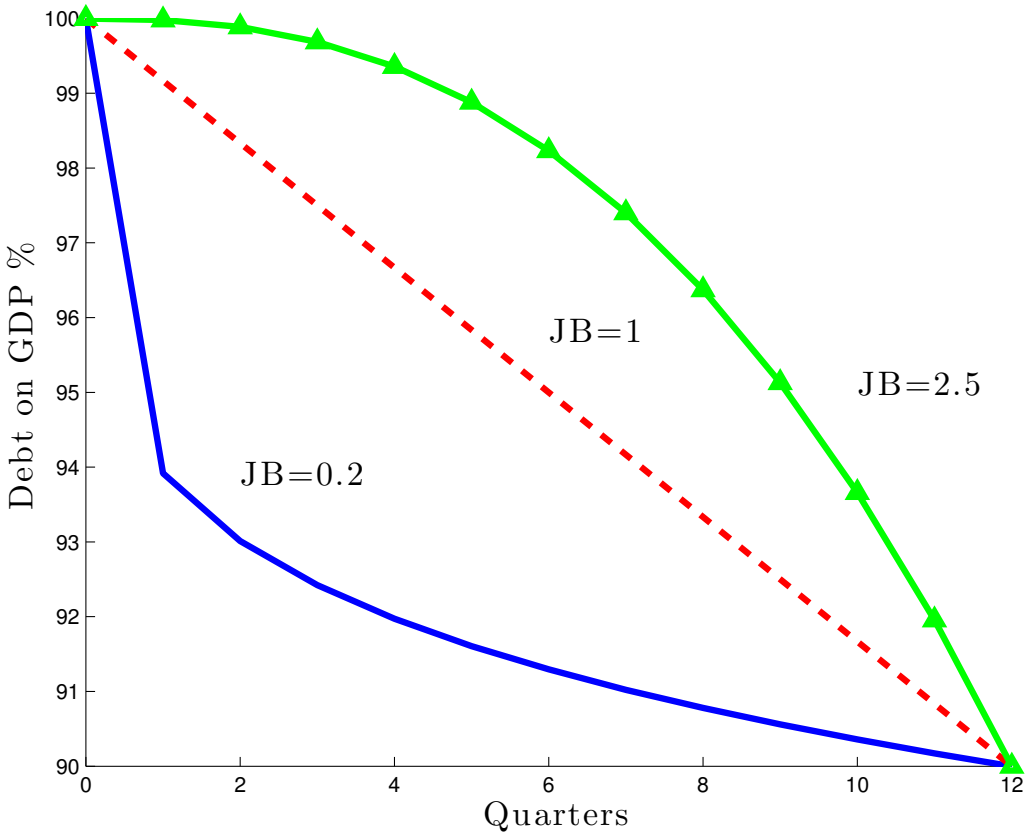

Figure 1: Figure shows different deleveraging path according to different JB. A $JB < 1$ (blue continuos line) represents a convex (fast) deleverage, a $JB = 1$ (red dotted line) stands for a smooth deleverage while a $JB > 1$ (green line) represents a concave (slow) deleverage.

4.2 Calibration

The model is calibrated quarterly. I use a global method to take into account non linearities that may arise from the Zero Lower Bound and downward nominal wage rigidities. Preferences take the following functional form:

$$U(c^j, G, l^j) = \frac{(c^{j\,\psi} G^{1-\psi})^{1-\rho}}{1-\rho} - \frac{l^{j\,(1+\eta)}}{(1+\eta)} \quad for\ j = \{S, H\}$$

where I set $\psi = .9$ and $\rho = \eta = 2$ in line with the literature[1]. I assume that 80% of consumers are Savers - $\chi = .2$. I set α_S equal to 0.8 such that individual labor income is identical for the Savers and for t Hand-to-Mouth[2]. I set ω, the parameter governing decreasing return to scale to equal 0.9.

I set real rate in line with literature at 2.5% and the steady state gross inflation equal to 1. I assume that the elasticity of substitution among the different varieties, ϵ, is equal to 8 such that the markup is 1.14.

I set tax in initial and final steady state to match the median payroll tax in US, $\tau = .127$. In 2013 US debt to GDP held by the public but not by the Federal Reserve System equalled 55%. More than half was held by foreign investors. Hence, I set the debt to GDP at yearly basis at 27.5% and I decrease this to 25% during the exercise.

In all the exercises, I set public expenditure as a share of GDP, s_g, to equal .931 in the initial steady state and equal to .938 in the final steady state. When debt reduction occurs through income tax I move in the first quarter s_g from .931 to .938. The time span of deleveraging, T, is equal to 4 quarters.

5 Benchmark

In this section I assume that $\phi = \psi = 0$, so that prices are perfectly flexible, and that the nominal interest rate can go below zero, so that the constraint (17) does not hold. This formulation is helpful to understand the main mechanism behind the model and it will be the benchmark for all other exercises.

[1] De Walque, Smets and Wouters (2005)

[2] From firms' cost minimization $\frac{(1-\chi)}{\alpha_S} W^S l^S = \frac{\chi}{\alpha_H} W^H l^H$.

Parameters	Flexible Prices	Rotemberg Costs (RC)	RC and Wage Rigidities
η	2	2	2
ρ	2	2	2
χ	.2	.2	.2
α_S	.8	.8	.8
ω	.9	.9	.9
ϵ	8	8	8
τ	.127	.127	.127
$D_{GDP}^{initial}$	.275	.275	.275
D_{GDP}^{final}	.25	.25	.25
$s_g^{initial}$	.931	.931	.931
s_g^{final}	.938	.938	.938
ϕ	0	77	77
ψ	0	0	.97

Table 1: This table shows parameters values used for calibration under different the exercises of the chapter/paper.

5.1 Public Expenditure

In this section I will consider the effects of of a reduction in public debt that is achieved by reducing public expenditure. The fiscal authority will create a shock on the demand side that will lead, independently on the speed of the deleverage, to an output recession. Savers, during the transition, are forced to consume more, as government is cutting back debt. Consequently, the real interest rate falls when deleveraging occurs. This is a key result since the government as well as Hand-to-Mouth consumers will benefit from a low rate.

It is important to note that, the less gradual the process of debt reduction, the more sizable will be the drop in the real rate. As a result, Hand-to-Mouth prefer a debt reduction that occurs in as few quarters as possible. Moreover, they would like deleverage to take place as soon as possible. The explanation is straightforward: assume that most of the debt reduction occurs in the last quarter. Knowing this, Savers will be willing to move resources from the future to the present. This will put an upward pressure on the real rate before deleveraging has been undertaken. The government would then pay a high interest rate on a large stock of debt. As a consequence, Hand-to-Mouth

consumers dislike such a deleveraging path and they would prefer the debt reduction to occur as soon as possible.

As shown in Figure (2) , under this public expenditure experiment, Hand-to-Mouth prefer an extremely fast deleverage. The fiscal authority severely reduces public expenditure in the first quarter. Output collapses since firms face a drop in demand. Savers, being forced to save less, experience a boom consumption. The real rate decreases, reducing financing costs for the government. Note that, despite the aggressive policy stance, debt to GDP do not fall immediately due to the recession.

Movements in the real rate will also drive Savers' choice of optimal deleveraging path. Since they want to avoid an extreme drop in the real interst rate, they prefer, as Figure (2) shows, a relatively slow and smooth reduction of debt. Savers, knowing that they will experience a consumption boom in the last quarters, are willing to dissave in the first quarters. The real rate falls in the last quarter but, in the preceding periods, it will be relatively high. The economy will experience a prolonged output recession, mostly due to the fall in Hand-to-Mouth consumption and government expenditure. Debt to GDP will converge slowly to the new steady state.

Note that in both cases, in general equilibrium, a big fall in the real interest rate prevents large fluctuations in real variables.

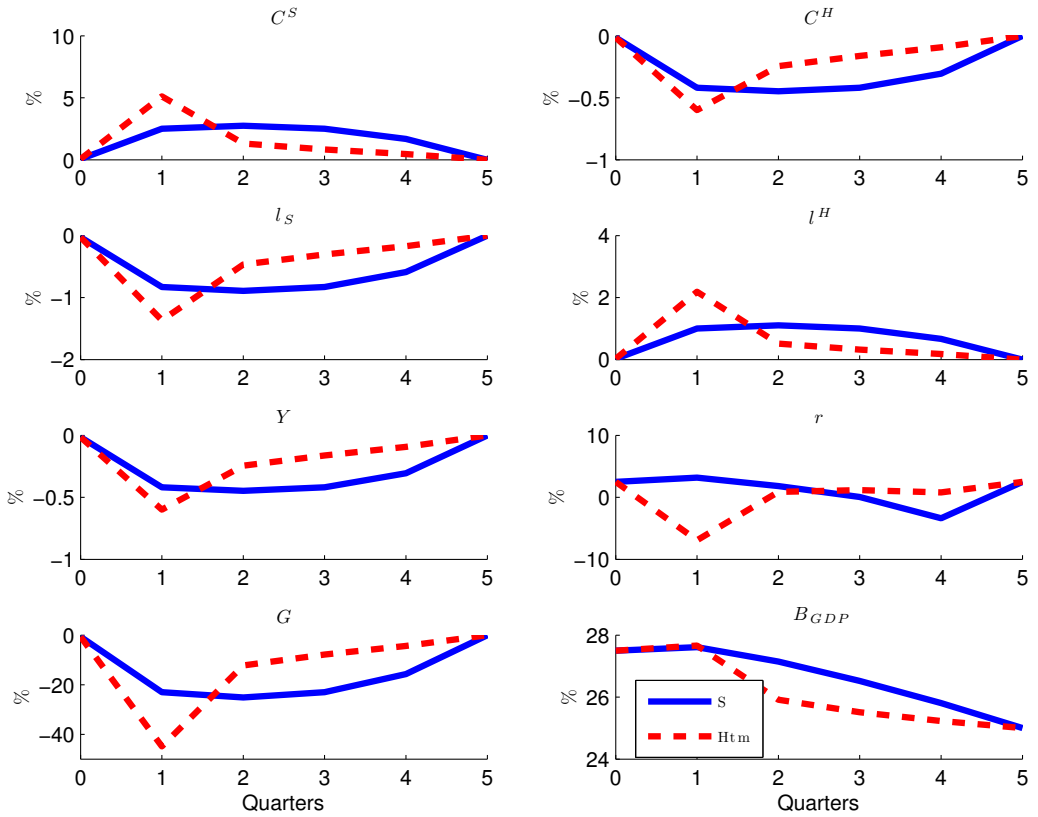

Figure 2: Figure show the optimal i.r.f. for the Hand-to-Mouth consumers (red dotted line) and for the Savers (blue continuos line) when fiscal authority reduces his debt using public expenditure. Real rate, r, and debt on GDP, B_{gdp} are in percentage levels, while the other variables are in percentage deviations from the final steady state.

5.2 Taxation

I consider now an experiment in which the fiscal authority raises income tax to finance reduction in debt.

As before, independently on the speed of deleverage, the economy will experience an output recession. Differently from before, however, the negative demand shock, will be only indirectly created by the government. Agents, facing high distortionary taxes, are poorer. As they consume less, output is depressed.

On the other hand, during the debt reduction, the real interest rate will now increase. Savers, indeed, face a reduction in their after-tax labor income. Having access to the financial market, they are then willing to borrow, putting upward pressure on the real rate. The government then has to deleverage when financing costs are high. Tax pressure on the agents needs to increase further to finance the debt reduction. Consumption falls by more, further increasing the pressure on real rate. The economy enters a vicious circle of deep output recession and high interest rate.

Note that, despite the fall in own private consumption and public expenditure, Savers gain during the transition, independently on the deleveraging speed. Their financial income increases, allowing them to enjoy more leisure when their wage is lowered by distortionary taxation.

Differently from the previous experiment, it is Savers who want to affect the interest rate the most, as this increases during the transition. Hence, they will prefer a very fast deleveraging. As figure (3) shows, in their preferred path, the fiscal authority performs the whole debt reduction in the first quarter causing a deep, but short, recession. Debt to GDP will initially increase.

Hand-to-Mouth, under this experiment would prefer instead a slow debt reduction, as they want to reduce upward pressure on the real rate. As shown by Figure (3), the economy enters a long output recession. The real rate is close to the steady state, spiking in the last quarter. Compared to the previous case, Hand-to-Mouth consumption decreases by more, while labor supply increases due to the high real interest rate.

Under this experiment, again, debt to GDP initially increases due to the endogenous recession.

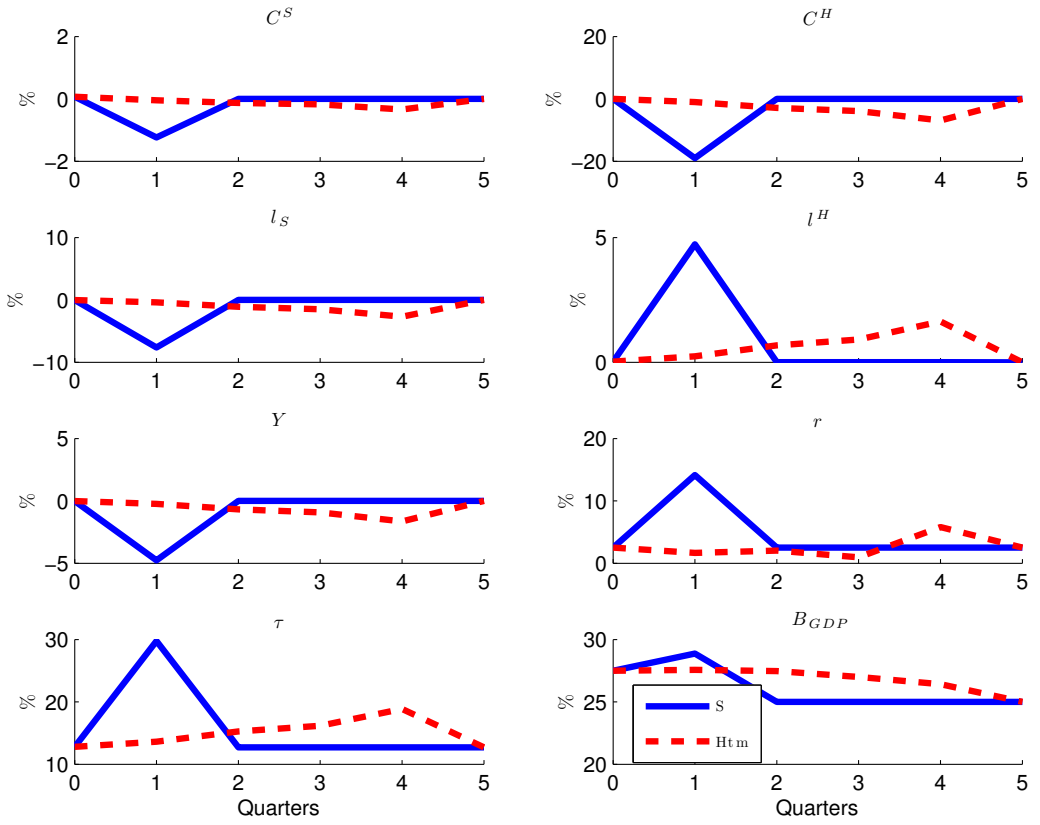

Figure 3: Figure show the optimal i.r.f. for the Hand-to-Mouth consumers (red dotted line) and for the Savers (blue continuos line) when fiscal authority reduces his debt using tax on labor. Real rate, r, debt on GDP, B_{GDP} and tax share, τ are in percentage levels, while the other variables are in percentage deviations from the final steady state.

5.3 Consumption Equivalent

In order to compare welfare under the two fiscal experiments I compute the consumption equivalent, c_e as:

$$\sum_{t=0}^{\infty} \beta^t U(C_t^j, G_t, l_t^j) = \frac{U(C^j - c_e^j, G^j, l^j)}{1-\beta} \quad j = \{S, H\}$$

where the right hand side is utility for consumer j computed along the simulations and C^j, G and l_j are consumption, public expenditure and labor, respectively, computed at the final steady state for consumer of type j. Following Lucas (1987), I define consumption equivalent as the decrease in steady-state consumption that makes agents indifferent between a constant consumption path and the time-varying one achieved in the simulation. Under this definition, a positive consumption equivalent amounts to a welfare cost while a negative consumption equivalent amounts to a welfare gain.

In Figure 4 I plot consumption equivalent as a percentage share of final steady state consumption for Hand-to-Mouth (first panel) and Savers (second panel) as a function of the speed of deleveraging, JB. Solid line refers to deleveraging achieved via a reduction in public expenditure, while the solid line refers to the increase in taxes experiment. [3]

In this flexible price setup, Hand-to-Mouth lose during the debt reduction, independently of the speed of deleveraging. On the other hand, Savers may gain or lose depending on the speed. The Figure shows how some deleveraging paths are Pareto dominated by others: for example, under tax experiment for which $JB > 1.75$, where 1.75 is the minimum consumption equivalent achievable by the Hand-to-Mouth, welfare of both agents would improve by choosing a lower JB .

Limiting the analysis to the interval of debt reduction speeds JB that are not Pareto dominated, Hand-to-Mouth lose less, in term of welfare, from a deleveraging achieved by a fall in public expenditure than from one achieved by an increase in taxes. The main difference between the two experiments is in the behavior of the real interest rate. Hand-to-Mouth, once they internalize the government budget constraint, are net borrowers. As a consequence, they dislike high real interest rates. Moreover, under the tax experiment,

[3]Note that the consumption equivalent is a continuos and convex function in the debt reduction speed JB. It follows that agents' value functions are continuos and concave in JB. The assumptions of Proposition 1 are then satisfied.

the economy enters a deeper recession than under the public expenditure case. Hand-to-Mouth agents, having no access to the financial market, will be more exposed to the cost of the recession.

On the other hand, Savers have strong preferences for deleveraging via taxation. Indeed, oppositely than Hand-to-Mouth, they hold public bonds, thus benefitting from high real interest rates.

Recently, in Southern European countries, there has been a large debate on how to implement austerity measures. One of the positions suggests to heavily tax savers, in order to achieve a more equal distribution. In light of this model, under flexible prices, this is detrimental for agents who have no access to financial markets. This result crucially depends on general equilibrium effects: Abstracting from debt reduction, let us consider the effects of policies that aim to subsidize Hand-to-Mouth agents by taxing the wealthy. Savers will work less in response, consuming their financial wealth instead. The real interest rate increases and output falls. A government who is a borrower will pay a high interest rate on his debt while tax revenues increase less due to the output recession. It is likely that, due to the recession and to the real interest rate rise, the government will have a hard time in achieving its objective.

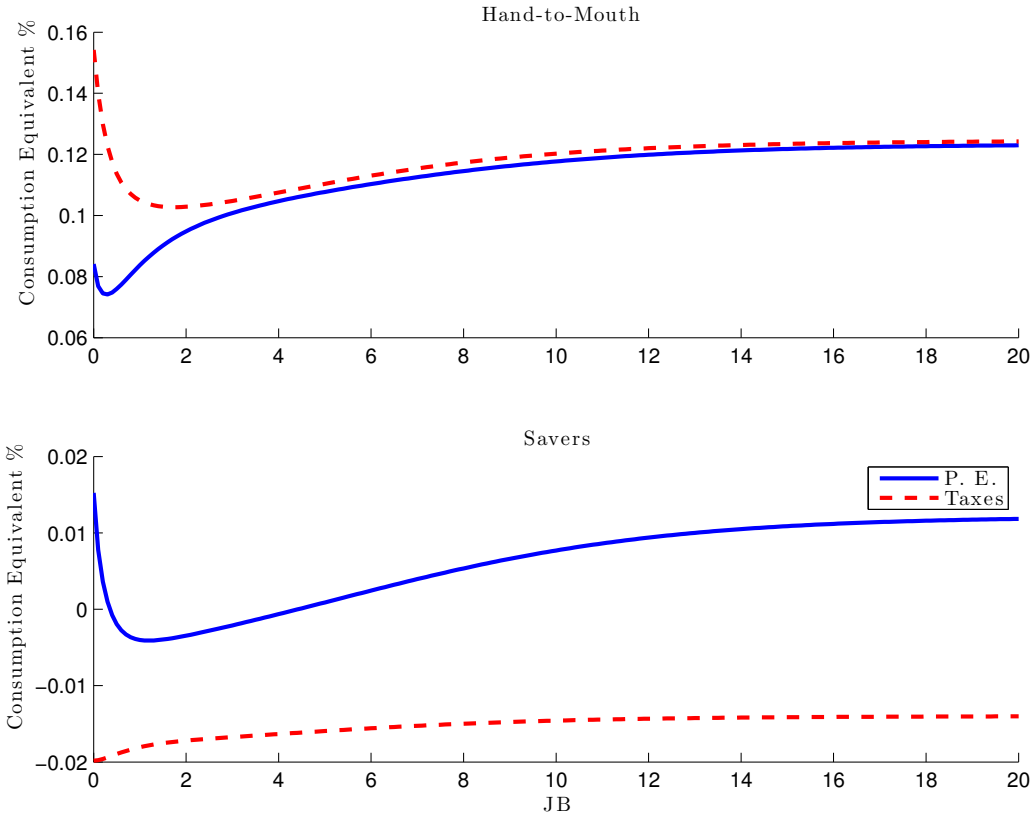

Figure 4: Figure shows consumption equivalent as a percentage of final steady state consumption for Hand-to-Mouth (first panel) and Savers (second panel) both under public expenditure (continuos blue line) and taxation (red dotted line). In both figures consumption equivalent is a function of deleveraging speed, JB.

6 Nominal Rigidities

Some of the results in the previous section hinge fundamentally on the assumption of perfect price flexibility. Figure 2 shows that the real rate needs to go below zero whenever the government reduces his public expenditure in order to deleverage. Negative real interest rates prevent real variables from experiencing large fluctuations. While negative real interest rates have been observed in reality, this is generally not the case for nominal interest rates, placing a lower bound on real rates too. I assume then in this section that the Central Bank is constrained by the presence of the Zero Lower Bound on nominal interest rates (eq. (17)).

In the previous formulation, additionally, firms were able to absorb negative demand shocks by cutting nominal wages. Again, this might not be feasible since nominal wage are downwardly rigid in the data. To capture such features in the context of my experiment, I consider now positive Rotemberg adjustment costs and occasionally binding downward wage rigidities. In subsection 6.1 I assume the presence of Rotemberg costs without wage rigidities. Under this circumstance, I will analyze only how simulations under public expenditure change, since this is the only instance in which the real rate goes below zero. In subsection 6.2 I assume the presence of both Rotemberg costs and nominal wage rigidities.

6.1 Rotemberg costs and the Zero Lower Bound

In this section and the following, I introduce the presence of a sizeable price rigidity[4].

Under this formulation, the Central Bank sets a nominal interest rate that is equal to zero whenever it had to be negative in the flexible price case. This will affect government expenditure in two ways. On the one hand, the fiscal authority needs to pay a higher real interest rate on public debt and, as a consequence, public expenditure will have to decrease more. Moreover, the fall in public expenditure will further depress output. Tax revenues fall and public expenditure decrease once more. Under the Zero Lower Bound, the economy enters a vicious cycle of low public expenditure and low output since, as shown by a strand of the literature[5], the public expenditure multiplier may be greater than one in this setup.

[4]Rotemberg cost $\phi = 77$

[5]Eggertsson(2009).

Note that, as Figure 5 shows, preferences of Savers and Hand-to-Mouth converge. As explained in the sections above, under flexible prices, extreme deleveraging paths - very slow or very fast - are the ones in which real interest rates fall more to clear the market. Consequently, if the Zero Lower Bound binds, these will be the costliest combinations for the economy. The Pareto optimal interval of values of JB shrinks substantially, as extreme choices have become very unfavourable. Due to the larger fall of productive government expenditure, Hand-to-Mouth consumers experience a welfare loss that is higher than the one under flexible prices. Savers switch from a welfare gain under flexible prices to a welfare loss.

It is now ambiguous whether Hand-to-Mouth prefers a debt reduction through taxation or public expenditure. For deleveraging paths of intermediate speed, as before, Hand-to-Mouth prefers a reduction in public expenditure. For extreme cases, however, as the Zero Lower Bound kicks in, he prefers the adjustment to take place through taxes. As deleveraging that takes place via expenditure becomes more costly, Savers, who preferred a taxation driven adjustment already, have their relative preferences unchanged.

Inroducing the Zero Lower Bound, then, has a significant effect on agents' welfare and their choices. A fiscal authority who wishes to contract public expenditure to reduce debt needs to consider the possibility of entering a detrimental liquidity trap. Hence, depending on deleveraging speed, it may be the case that distortionary taxation is a more suitable policy instrument to achieve the desired result.

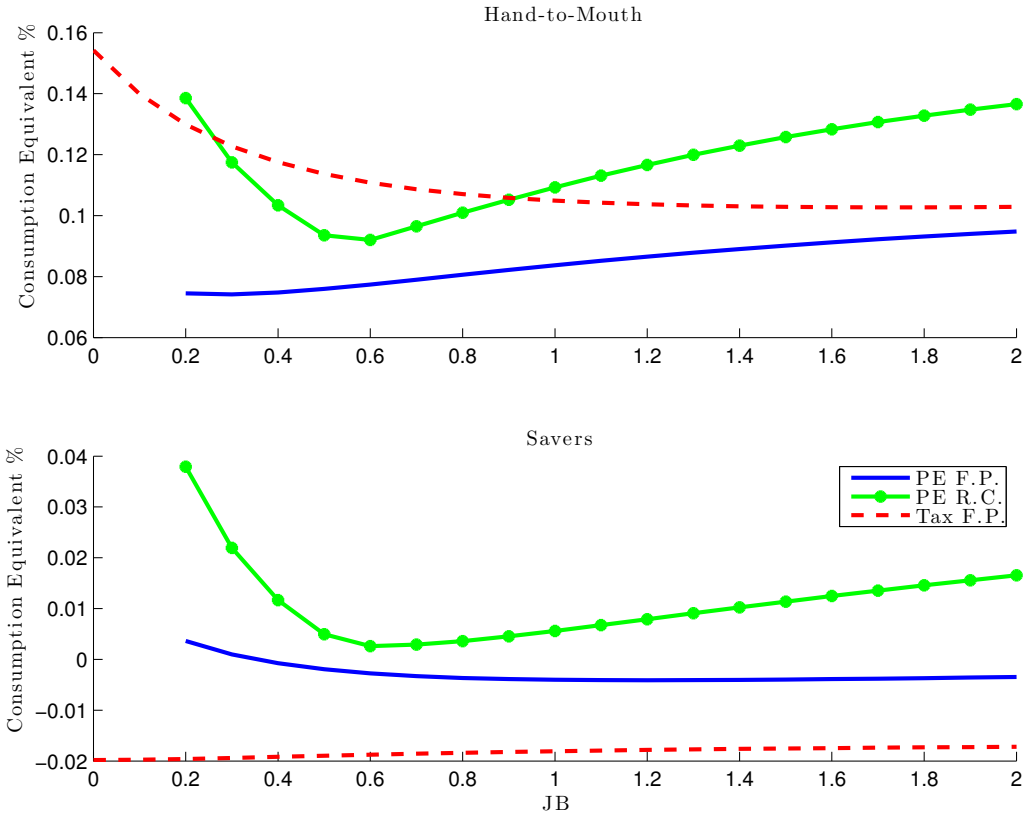

Figure 5: Figure shows consumption equivalent as percentage of final steady state consumption for Hand-to-Mouth consumers (first panel) and Savers (second panel) under public expenditure and flexible prices (blue line), under taxation and flexible prices (red dotted line) and under public expenditure and Rotemberg costs (green line). In both figures consumption equivalent is a function of deleveraging speed, JB.

6.2 Wage Rigidities

Recently, a big strand of the literature, like Benigno and Ricci (2011), Schmitt-Grohé and Uríbe(2011) and Calvo et al. (2013) among others, has analyzed how nominal downward wage rigidities affect economies both in the short and in the long run. Downward wage rigidities, under certain condition, can magnify recessions : Since firms are not able to decrease their costs, they will decrease their production. In the next subsections I will study how the presence of downward wage rigidities together with Rotemberg costs, changes agents' welfare and preferences. In equation (13) I set ψ, the parameter governing the extent of wage rigidity, to equal 0.97. As I will show, downward nominal wage rigidities have a different impact according to the type of recession the economy is facing.

6.2.1 Public Expenditure

To understand results under downward wage rigidities, it is useful to consider how labor reacts during deleveraging under flexible prices. In this circumstance, when debt reduction occurs, output falls and firms demand less labor. Savers, on one hand, experiencing a boom in consumption, will enjoy more leisure. On the other hand, Hand-to-Mouth, experiencing a negative shock on their consumption and public expenditure, increase their labor supply. In equilibrium, then, Hand-to-Mouth wage falls more than Savers' one. Adding Rotemberg costs does not change labor markets dynamics but, as I explained above, the economy is lead into a liquidity trap.

The introduction of downward wage rigidities directly affects the Hand-to-Mouth labor market, since it is the one where wages fall the most. Firms, as a result of the increase in wages, will hire less Hand-to-Mouth and more Savers. As Figures (6) and (7) show, adding downward wage rigidities only affects significantly labor market variables while aggregate variables remain largely unchanged due to the liquidity trap. High labor market costs, indeed, exert an upward pressure on nominal prices, making deflation less severe. Nominal interest rate will be still at zero. Consequently, the real interest rate will either remain unchanged or slightly fall, either leaving unaffected or mildly dampening the extent of the recession.

Downward wage rigidities under a liquidity trap, then, will be a transfer of resources from Savers - who work more- to Hand-to-Mouth - who work less. This result critically depends on the presence of the liquidity trap.

As detailed by Eggertsson (2010), when the Zero Lower Bound binds, the "paradox of flexibility" may hold. This is a situation in which flexible prices are more detrimental for the economy than rigid ones. Hence, in this case, nominal rigidities, especially regarding wages, will not exacerbate the cost of deleveraging on the economy.

As Figure (9) shows, downward wage rigidities reduce welfare costs for Hand-to-Mouth agents, while increasing costs for Savers independently on the speed of debt reduction. It is still ambiguous whether Hand-to-Mouth prefer public expenditure or taxation. Indeed, for a fast deleveraging, they bear a lower welfare cost under public expenditure, while, for a slow one, they are better off in the case of increase taxation. Agents' preferences regarding the speed of deleveraging under public expenditure remain, instead, unchanged.

A government willing to reduce debt through public expenditure needs to consider the possibility of entering a liquidity trap. If this is case, there are few deleveraging paths that are Pareto dominant, namely those that minimize the fall in public expenditure. Moreover, a fiscal authority who wants to redistribute resources from the rich to the poor may consider enforcing policies that increase the extent of wage rigidities. Note that, in this analysis, I do not consider idiosyncratic effects of unemployment. It could be the case, in a more accurate analysis, that, ex-post, unemployed Hand-to-mouth would be worse off under nominal wage rigidities. Indeed, the presence of these kind of rigidities, may decrease the probability to become employed once a worker is unemployed. This analysis is left for future research.

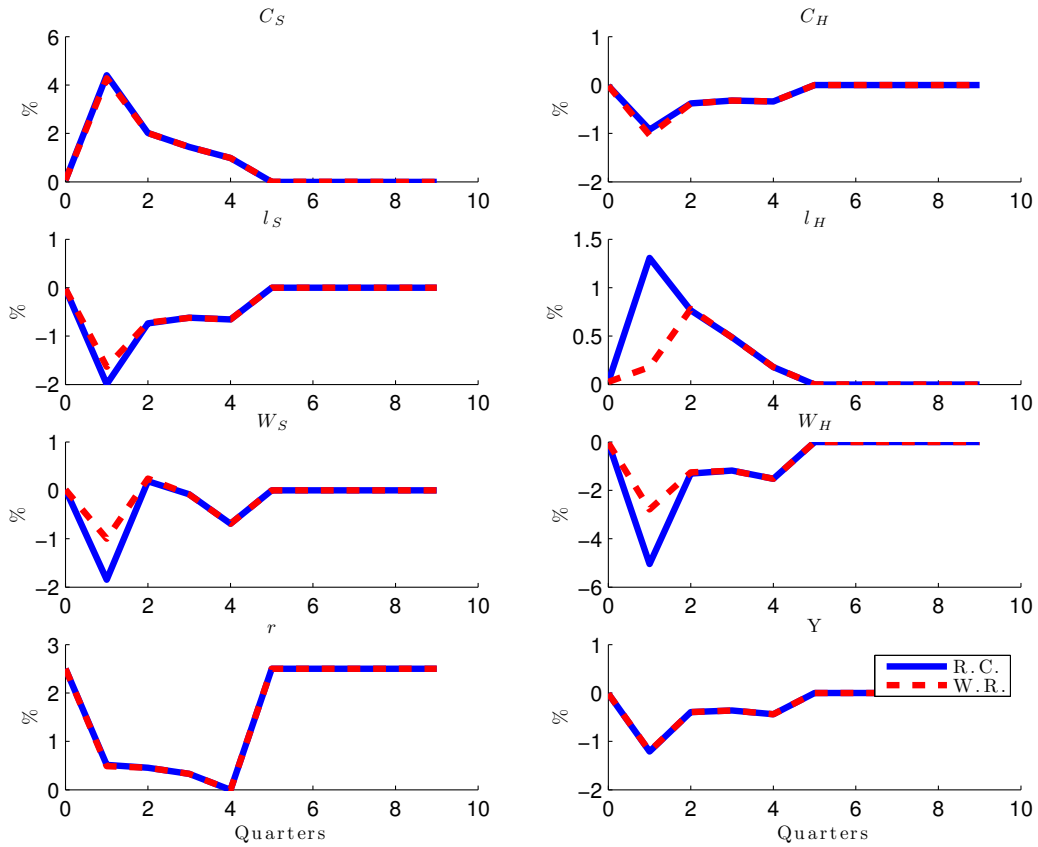

Figure 6: Figure shows i.r.f. for the Savers and Hand-to-Mouth optimal deleveraging path under Rotemberg costs and public expenditure experiment. Blue line are simulations under Rotemberg costs, while red dotted line under Rotemberg cost and wage rigidities. All the variable, except r, are in percentage deviation from the final steady state. Real rate, r, is in percentage level.

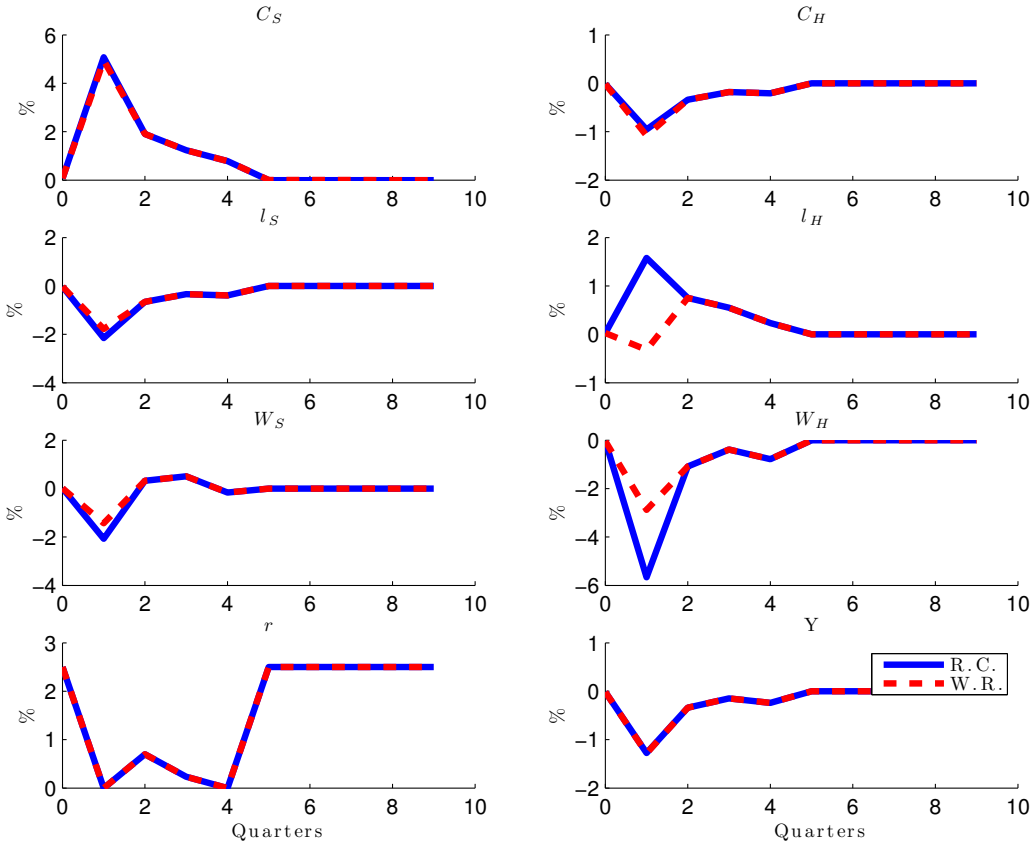

Figure 7: Figure shows the optimal deleveraging path for the Hand-to-Mouth under public expenditure experiment and wage rigidities. Blue line are simulations under Rotemberg costs, while red dotted line under Rotemberg cost and wage rigidities. All the variable, except r, are in percentage deviation from the final steady state. Real rate, r, is in percentage level.

6.2.2 Taxation

When a government is deleveraging through taxation, downward wage rigidities, whenever binding, are welfare decreasing. Since nominal wages increase, prices increases too, exerting upward pressure on the inflation rate. As a consequence, the Central Bank needs to raise the nominal interest rate and the government, paying an higher real interest rate,has to raise taxes. The recession is, then, deeper than under flexible prices. Differently than under the public expenditure experiment, aggregate variables dynamics change significantly with the introduction of nominal wage rigidities, as Figure 13 shows. There Savers optimal deleveraging path under flexible prices is compared with the same deleveraging path under wage rigidities.

Note that downward wage rigidities bind whenever the deleveraging speed is too slow or too fast, as shown in Figure 9. Under these combinations, indeed, real interest rates are very high in equilibrium, as explained in section 5.2. The higher interest rates, the higher will be the shock on Hand-to-Mouth consumption and, as a result, the higher will be their increase in labor supply. Consequently, under these debt reduction paths, nominal wages fall substantially. Under flexible prices Savers were the ones who preferred extreme debt deleveraging combinations, as they gained from high real interest rates. This beneficial effect is now dampened by the cost introduced by downward wage rigidities. They will prefer, then, the fastest debt reduction path under which the downward wage rigidity does not bind. Hand-to-Mouth, on the other hand, still find optimal the same debt reduction path they chose under flexible prices.

Despite the deep recession, Savers still gain in welfare term from a deleveraging that occurs via taxation. Hence they keep preferring this experiment to the public expenditure one . Hand-to-Mouth preferences change accordingly to the speed of debt reduction. If deleveraging is fast they prefer public expenditure, if instead it is slow they prefer taxation.

Finally, it is important to highlight that, after the introduction of downward wage rigidities, Savers' optimal path is closer to the Hand-to-Mouth one. If the government plays a Pareto dominant strategy, Hand-to-Mouth gain from the introduction of downward wage rigidities. Indeed, the Pareto dominant interval shrinks, excluding the combinations of deleveraging paths that decreased the most Hand-to-Mouth welfare under flexible prices. Then, even under the taxation experiment, they will gain from the introduction of downward wage rigidities.

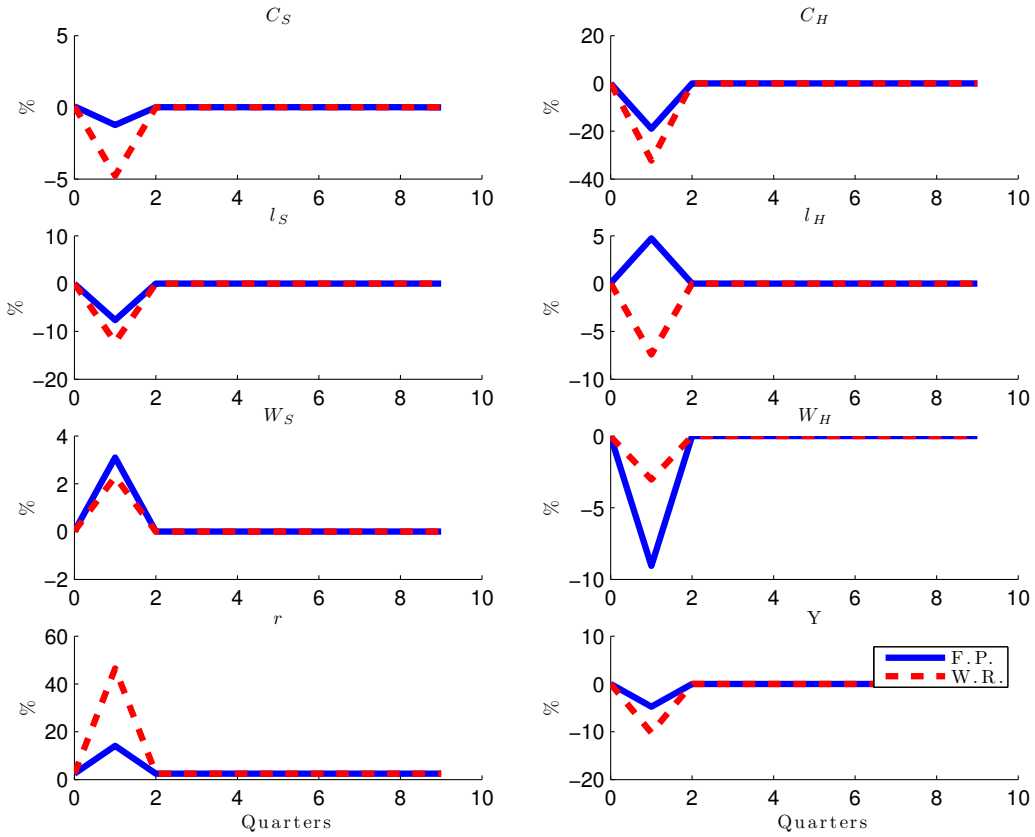

Figure 8: Figure shows i.r.f. under optimal Savers' deleveraging path in the flexible prices model (blue line). Red dotted line represents i.r.f under the same deleveraging when downward wages rigidities bind . All the variables, except r, are in percentage deviation from the final steady state. Real interest rate, r, is in percentage level.

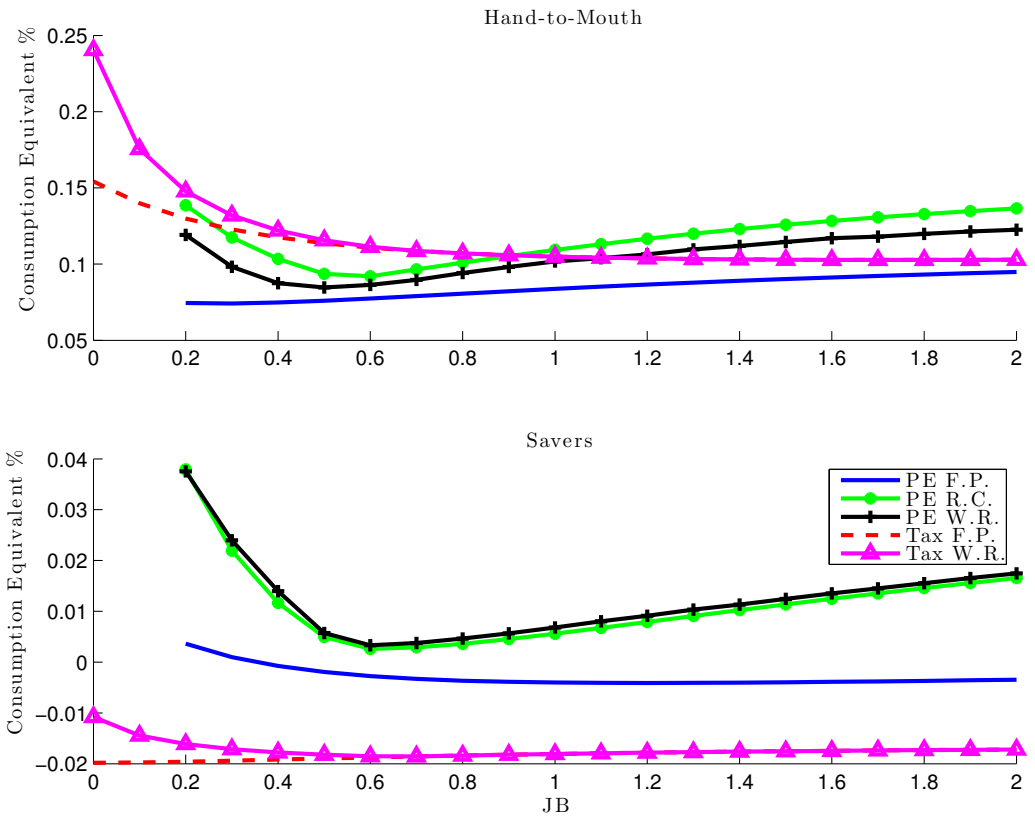

Figure 9: Figure shows consumption equivalent as percentage of final steady state consumption for Hand-to-Mouth (fist panel) and Savers (second panel) under different exercises: public expenditure (blue) and taxation (red) under flexible prices , public expenditure under Rotemberg cost (green), public expenditure (black) and taxation (magenta) under Rotemberg cost and Downward wage rigidities. Both figures shows consumption equivalent as a function of deleveraging speed, JB.

7 Time Span

Lastly I analyze whether and how the debt reduction time span changes agents' welfare. On one hand a longer time span decreases the size of the per-quarter shock, on the other hand it takes more time for agents to achieve a constant level of consumption. To address this question I focus on the Pareto dominant interval of deleveraging paths. As all this combinations are Pareto optimal, the maximum achievable by one agent corresponds to the minimum for the other. Figures 10 and 11 show the minimum and maximum consumption equivalent[6] achievable by each type of agent as function of debt deleveraging time span, T. Figure 10 shows, for the public expenditure experiment, the relevant Pareto intervals under flexible prices, Rotemberg costs and downward wage rigidities (with Rotemberg costs). Figure 11 corresponds to the taxation experiment.

Under public expenditure Savers gain slightly from an increase in the time span. Nevertheless, the magnitude of the gain is negligible. The effect of increasing the time span on Hand-to-mouth welfare is instead significant and counterintuitive. As T increases, Hand-to-Mouth welfare worsens, in both best and worst case scenarios. Indeed, with a shorter time span, as the shock on output demand is larger, real interest rates tend to be lower. The higher will be the time span, the higher will be the real interest rate and the lower will be Hand-to-Mouth welfare. Moreover, Figure 10 also shows that the Pareto interval widens, meaning that the higher the time span, the more Savers and Hand-to-Mouth preferences will differ. Then, as T increases, the variability of outcomes increases for the Hand-to-Mouth.

Under the taxation experiment, Savers are, again, indifferent to the time span, both under flexible prices and under downward wage rigidities. Hand-to-Mouth, on the other hand , experience an improvement in their welfare, as the time span increases . When a government reduces debt through taxation, interest rate are higher. As T increases, the per-quarter-shock is smaller and, as a consequence, interest rates increase less. Hand-to-Mouth, then, experience a welfare gain with an higher T.

Concluding, Savers are largely unaffected by the time span while this is not the case for Hand-to-Mouth. Under taxation they would prefer a longer time span of deleveraging while, under public expenditure, they want the fiscal authority to reduce debt in a short time span. Since this variable is

[6]Remember that the consumption equivalent is decreasing in welfare.

unimportant from the Savers' point of view, once chosen an instrument, the government, can adjust T to maximize Hand-to-Mouth welfare.

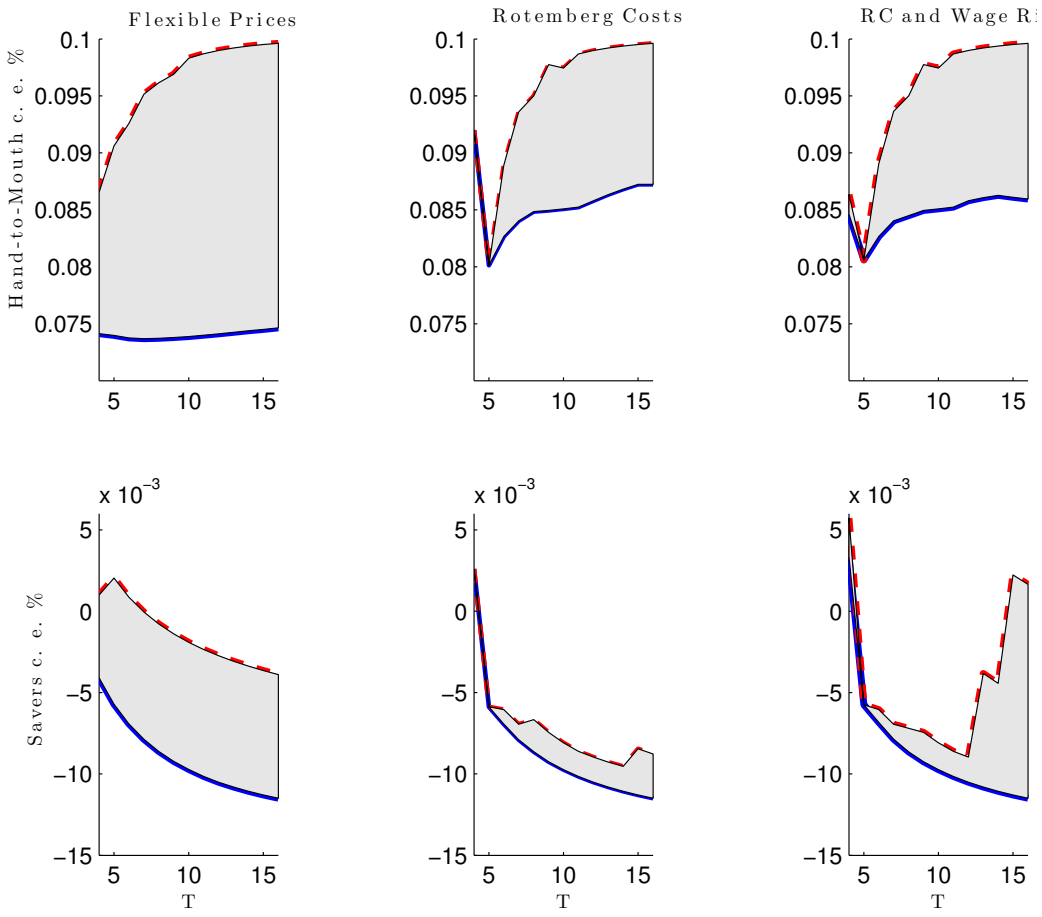

Figure 10: Figures show the minimum (blue continuos line) and maximum (red dotted line) consumption equivalent as percentage of final steady state consumption as a function of time span, T under the public expenditure experiment. The first row refers to Hand-to-Mouth while the second to the Savers. First column refer to Flexible Prices case, second to Rotemberg costs and the third to Rotemberg costs plus downward wage rigidities

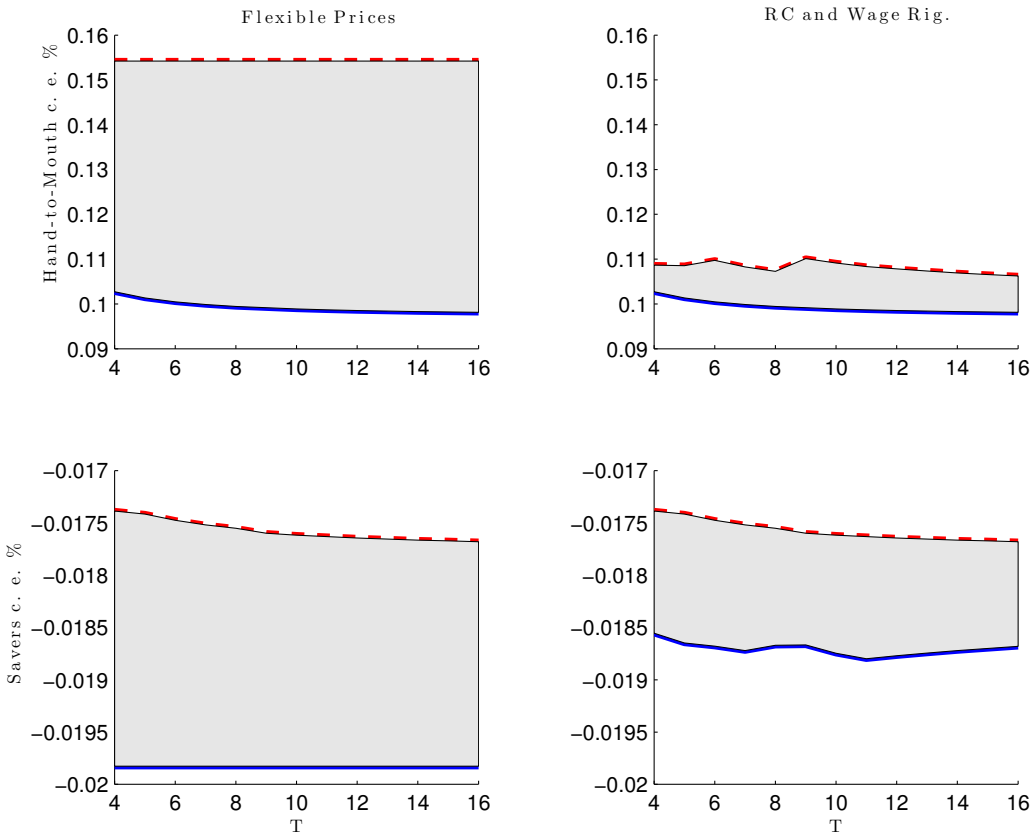

Figure 11: Figures show the minimum (blue continuos line) and maximum (red dotted line) consumption equivalent as percentage of final steady state consumption as a function of time span, T under the public expenditure experiment. The first row refers to Hand-to-Mouth while the second to the Savers. First column refer to Flexible Prices case, second to Rotemberg costs and the third to Rotemberg costs plus downward wage rigidities

8 Conclusions

I analyze welfare implications of different paths of public deleveraging on different types of agents. I find that, in a context of heterogenous agents and incomplete markets, the speed of deleveraging is not inconsequential for agents' welfare.

Deleveraging through income taxation leads to a recession with high real interest rates. In this situation, agents, who hold public debt prefer extreme deleveraging paths, while consumers who do not participate in the financial markets prefer slower one. An economy choosing, instead, to reduce public debt using public expenditure faces the risk to enter a liquidity trap. Under this circumstance, independently of their participation in financial markets, agents prefer the path that minimizes the fall in public expenditure.

Moreover, I find that downward wage rigidities are always detrimental for the agents who participate in financial market. These rigidities, acting as a sort of insurance in bad times, on the other hand, are helpful for the financial constrained.

The time span of deleveraging affects agents who do not participate in financial markets only, with little or no consequence for the others. Hence, the fiscal authority, should adjust this variable to maximize aggregate welfare, if unconstrained otherwise. Interesting results may emerge by relaxing some critical assumptions in this paper, namely the presence of exogenous wealth distribution and perfect foresight. Firstly, if poor agents were able to borrow, movements in the interest rate would directly affect them too. Moreover, the introduction of uncertainty can also affect the conclusions of this paper. For example, introducing uninsurable unemployed risk among the Hand-to-Mouth, I expect their preferences for nominal wage rigidities to change. Exploration of these avenues is left for future research.

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A

Proposition 2. *If agents' value functions are continuos, twice differentiable and concave in JB with $JB^S < JB^H$ ($JB^H < JB^S$), then the JB^W that maximize W for a generic $a \in [0,1]$, will lie in the interval $[JB^S, JB^H]$ ($[JB^H, JB^S]$). All the JB those do not belong to this interval are Pareto dominated.*

Proof. Being an argmax JB^W satisfies the following condition:

$$We'(JB^W) = aWe'_H(JB^W) + (1-a)We'_S(JB^W) = 0 \tag{20}$$

where $We'(.)$,$We'_H(.)$ and $We'_S(.)$ are the first derivatives of $We(.)$, $We_H(.)$ and$We_S(.)$ with respect to JB.

By contradiction assume that $JB^W < JB^S$, then $We'_H(JB^W) > 0$ and $We'_S(JB^W) > 0$, then condition (20) will not be met. Again by contradiction assume that $JB^W > JB^H$, then $We'_H(JB^W) < 0$ and $We'_S(JB^W) < 0$, then condition (20) will not be met. If an argmax exists it should lie in the interval $[JB^S, JB^H]$. □

B

Agents	Flexible Prices	Rotemberg Costs (RC)	RC and Wage Rigidities
Taxation			
Hand-to-Mouth	1.75	1.75	1.75
Savers	0	0	0.7
Public Expenditure			
Hand-to-Mouth	0.3	0.6	0.5
Savers	1.2	0.6	0.6

Table 2: This table shows optimal JB for Hand-to-Mouth and Savers both under public expenditure and taxation experiment.