Optimal Monetary Policy in Open Economies with Incomplete Markets

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Research questions

1. What are the effects of monetary policy in an international setting with real frictions (incomplete markets)?
2. What policies are optimal in such settings (maximize a global social welfare function)?

International monetary economies with nominal frictions

- Benigno and Benigno (2003), Corsetti and Pesenti (2005), and Corsetti, Dedola, and Leduc (2010)
- Optimal policies require policy makers to trade off exchange rate stabilization (to account for the nominal rigidities being imported from foreign firms) with domestic price stabilization (to simultaneously minimize the domestic output gap and the effect of domestic nominal distortions).

...and incomplete markets

- Corsetti, Dedola, and Leduc (2008b) and Devereaux and Sutherland (2008)
- Devereaux and Sutherland (2008) consider uncertainty caused by interest rate shocks.
- Without any real shocks, however, zero inflation monetary policy removes the friction caused by the nominal rigidities, while simultaneously nullifying the interest rate shocks.

"Key lessons for monetary policy analysis can be learnt from models in which asset markets do not support the efficient allocation" and the "design of monetary policy in models with explicit financial distortions [serve] as a complement."

-Corsetti, Dedola, and Leduc (2010), pg. 928

International monetary economies with real frictions

- Baxter and Crucini (1995) and Corsetti, Dedola, and Leduc (2008a)
- Baxter and Crucini (1995) examine importance of incomplete markets on international business cycles.
- Corsetti, Dedola, and Leduc (2008a) add monetary policy considerations to the analysis.

...and advocating for nominal GDP targeting

- Koenig (2013) and Sheedy (2014)
- Koenig (2013) considers a stylized 2-period setting.
- Sheedy (2014) only considers aggregate risk, and assumes a special structure on the preferences in order to ensure a stationary wealth distribution.
- Both claim nominal GDP targeting is consistent with Pareto efficiency.

This paper

- Large open economy with N countries.
- Heterogeneous households.
- Pure exchange economy, households receive stochastic endowment realizations.
- Financial friction is incomplete markets (real friction).
- Central banks use open market operations, which includes anything from targeting rules (inflation rate targeting, interest rate targeting, nominal GDP targeting) to more general policies.

Preview of results

1. Targeting rules are inconsistent with Pareto efficiency.
2. If all countries adopt policies with stationary inflation, Pareto efficiency requires number of states of uncertainty less than or equal to twice the number of countries.
3. Optimal policy under Pareto efficiency calls for (i) increases in the inflation rate when initial savings-to-debt ratio is less than 1 and (ii) decreases in the inflation rate when initial savings-to-debt ratio is greater than 1.

Model basics

- Large open economy with N countries, labeled $i \in \mathbf{I} = \{1, ..., N\}$.
- Each country contains a representative household.
- Households are infinite-lived and maximize CRRA expected utility (identical across countries).

Time and uncertainty

- Infinite time horizon.
- In each time period, a state $s \in \mathbf{S} = \{1, ..., S\}$ is realized.
- Date-events in the tree denoted by $s^t = (s_0, s_1, ..., s_t)$.

Commodity markets

- One physical commodity at each date-event, in each country.
- Commodities are perfect substitutes.

Endowments

- Households only receive endowments in terms of the domestic commodity.
- Stationary endowments $\mathbf{e}_i : \mathbf{S} \to \mathbb{R}_{++}$.
- Aggregate endowments $\mathbf{E} : \mathbf{S} \to \mathbb{R}_{++}$ such that $\mathbf{E}(s) = \sum_{i \in \mathbf{I}} \mathbf{e}_i(s)$.

Bond markets

- Each country issues a 1-period (short-term) bond.
- Nominal payout equals 1 in all date-events next period (risk-free).
- Bond price: $q_i\left(s^t\right)$.

Incomplete markets

- Number of assets equals N.
- Incomplete markets (fewer assets than states) requires $N < S$.

Household wealth

- Household wealth entering date-event s^t is $\omega_i\left(s^t\right)$.
- Initial wealth $\omega_i\left(s_0\right)$ is a parameter.

Bond and money markets

- First, the bond and money markets open.
- $\xi_i(s^t)$ is the exchange rate for country i (number of units of country 1 currency for every 1 unit of country i currency).
- Households choose bond and money positions such that:

$$\frac{1}{\xi_i(s^t)}\left(\sum_{j\in\mathbf{I}}\xi_j(s^t)\left(\hat{m}_{i,j}(s^t)+q_j(s^t)\,b_{i,j}(s^t)\right)\right)\leq\omega_i(s^t).$$

Cash-in-advance constraints

- Second, the commodity markets open.
- Households face cash-in-advance constraints:

$$p_j\left(s^t\right)c_{i,j}\left(s^t\right) \leq \hat{m}_{i,j}\left(s^t\right).$$

- Money holdings carried into next period are:

$$\begin{aligned} m_{i,j}\left(s^t\right) &= \hat{m}_{i,j}\left(s^t\right) + p_i\left(s^t\right)\mathbf{e}_i\left(s_t\right) - p_j\left(s^t\right)c_{i,j}\left(s^t\right) \text{ for } i = j. \\ m_{i,j}\left(s^t\right) &= \hat{m}_{i,j}\left(s^t\right) - p_j\left(s^t\right)c_{i,j}\left(s^t\right) \text{ for } i \neq j. \end{aligned}$$

- Cash-in-advance constraints equivalent to:

$$\begin{aligned} m_{i,j}\left(s^t\right) &\geq p_i\left(s^t\right)\mathbf{e}_i\left(s_t\right) \text{ for } i = j. \\ m_{i,j}\left(s^t\right) &\geq 0 \text{ for } i \neq j. \end{aligned}$$

Next period wealth

- Household wealth in subsequent date-events is given by:

$$\omega_i\left(s^t,\sigma\right)=\frac{1}{\xi_i\left(s^t,\sigma\right)}\left(\sum_{j\in\mathbf{I}}\xi_j\left(s^t,\sigma\right)\left(\hat{m}_{i,j}\left(s^t\right)+b_{i,j}\left(s^t\right)\right)\right).$$

Governments

- Governments hold debt positions: both domestic and foreign.
- Debt issued by fiscal authorities; debt bought and sold by monetary authorities.
- Net domestic debt positions are nonnegative.
- Foreign debt positions (+) or (-).

Government budgets

- $W_i(s_0)$ is the initial nominal obligations of country i, a parameter of the model.
- The budget constraints are Liabilities = Assets,
  - where Liabilities consist of debt positions (domestic and foreign) carried over from previous period

$$\text{Liabilities} = \frac{\sum_{j\in\mathbf{I}} \xi_j\left(s^t\right) q_j\left(s^t\right) B_{i,j}\left(s^{t-1}\right)}{\xi_i\left(s^t\right)},$$

  - and Assets consists of seigniorage plus new debt positions (domestic and foreign)

$$\text{Assets} = M_i\left(s^t\right) - M_i\left(s^{t-1}\right) + \frac{\sum_{j\in\mathbf{I}} \xi_j\left(s^t\right) q_j\left(s^t\right) B_{i,j}\left(s^t\right)}{\xi_i\left(s^t\right)}.$$

Equilibrium

An equilibrium is such that:

- Given prices, households maximize utility by choosing sequences for consumption, money, and bonds, subject to the constraints and an implicit debt constraint.
- Given prices, governments satisfy their constraints, subject to a debt constraint.
- Markets clear:
  - $\sum_{i \in \mathbf{I}} \xi_i(s_0)\, \omega_i(s_0) = \sum_{i \in \mathbf{I}} \xi_i(s_0)\, W_i(s_0)$.
  - $\sum_{i,j \in \mathbf{I}^2} c_{i,j}(s^t) = \mathbf{E}(s_t)$ for all date-events.
  - $\sum_{i \in \mathbf{I}} m_{i,j}(s^t) = M_j(s^t)$ for every j and all date-events.
  - $\sum_{i \in \mathbf{I}} b_{i,j}(s^t) = \sum_{i \in \mathbf{I}} B_{i,j}(s^t)$ for every j and all date-events.

Equilibrium properties

1. Law of One Price

$$\xi_i\left(s^t\right) = \frac{p_1\left(s^t\right)}{p_i\left(s^t\right)} \quad \forall i \in \mathbf{I}.$$

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$$M_i\left(s^t\right) = p_i\left(s^t\right)\mathbf{e}_i\left(s_t\right).$$

3. Existence of a Markov equilibrium.

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  - Government chooses ϕ_i such that $\frac{p_i(s^t)}{p_i(s^t,\sigma)} = \phi_i$ for all date-events (inflation rate equals $\frac{1}{\phi_i} - 1$).

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  - Government chooses κ_i such that $q_i\left(s^t\right) = \kappa_i$ for all date-events (nominal interest rate equals $\frac{1}{\kappa_i} - 1$).

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  - Government chooses κ_i such that $q_i\left(s^t\right) = \kappa_i$ for all date-events (nominal interest rate equals $\frac{1}{\kappa_i} - 1$).
- Nominal GDP targeting
  - Government chooses μ_i such that $\frac{p_i(s^t)}{p_i(s^t,\sigma)} = \mu_i \frac{\mathbf{e}_i(\sigma)}{\mathbf{e}_i(s)}$ for all date-events (from Quantity Theory of Money, monetary growth rate equals $\frac{1}{\mu_i} - 1$).

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Theorem

Over a generic subset of household endowments, policies in which each country adopts a targeting rule (of some form) result in a Pareto inefficient equilibrium allocation.

Result 2

- Country i adopts a policy with stationary inflation provided that there exists $\left(\boldsymbol{\pi}_i(\sigma)\right)_{\sigma\in\mathbf{S}}$ such that the price ratio $\frac{p_i(s^t,\sigma)}{p_i(s^t)} = \boldsymbol{\pi}_i(\sigma)$ for all date-events.

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Theorem

If $S > 2N$, then over a generic subset of household endowments, policies with stationary inflation result in a Pareto inefficient equilibrium allocation.

Initial savings-to-debt ratio

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- If $\omega_i(s_0) > W_i(s_0)$, initial savings-to-debt ratio greater than 1.
- If $\omega_i(s_0) < W_i(s_0)$, initial savings-to-debt ratio less than 1.

Optimal policy under Pareto efficiency

- Pareto efficiency: $\exists\left(\theta_i\right)_{i\in\mathbf{I}}$ such that $\sum_{j\in\mathbf{I}} c_{i,j}\left(s^t\right)=\theta_i\mathbf{E}\left(s_t\right)$ for all i and for all date-events.

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- For an economy with $N = 2$ countries where country 1 adopts targeting rule and country 2 adopts policy with stationary inflation, necessary condition for Pareto efficiency is $S \leq 3$.

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- Country 1 adopts inflation rate targeting and must choose an inflation target.

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- For an economy with $N = 2$ countries where country 1 adopts targeting rule and country 2 adopts policy with stationary inflation, necessary condition for Pareto efficiency is $S \leq 3$.
- Country 1 adopts inflation rate targeting and must choose an inflation target.
- How does the choice of target affect θ_1, the fraction of domestic consumption?

Economy

- $N = 2$ countries and $S = 3$ states of uncertainty.
- Discount factor $\beta = 0.8$, relative risk aversion $\rho = 3$, and iid transition matrix.
- The household endowments are given in the following table:

	$\mathbf{e}_1(s)$	$\mathbf{e}_2(s)$	$\mathbf{E}(s)$
State $s = 1$	14	12	26
State $s = 2$	12	15	27
State $s = 3$	10	18	28

- The initial period is $s_0 = 1$ and the initial period wealth parameter is $\omega_1(s_0) = 10$.

Initial savings-to-debt ratio equals 1

- $W_1(s_0) = 10$.
- Regardless of policy, the fraction θ_1 does not change.

Initial savings-to-debt ratio greater than 1

- $W_1(s_0) = 8$.

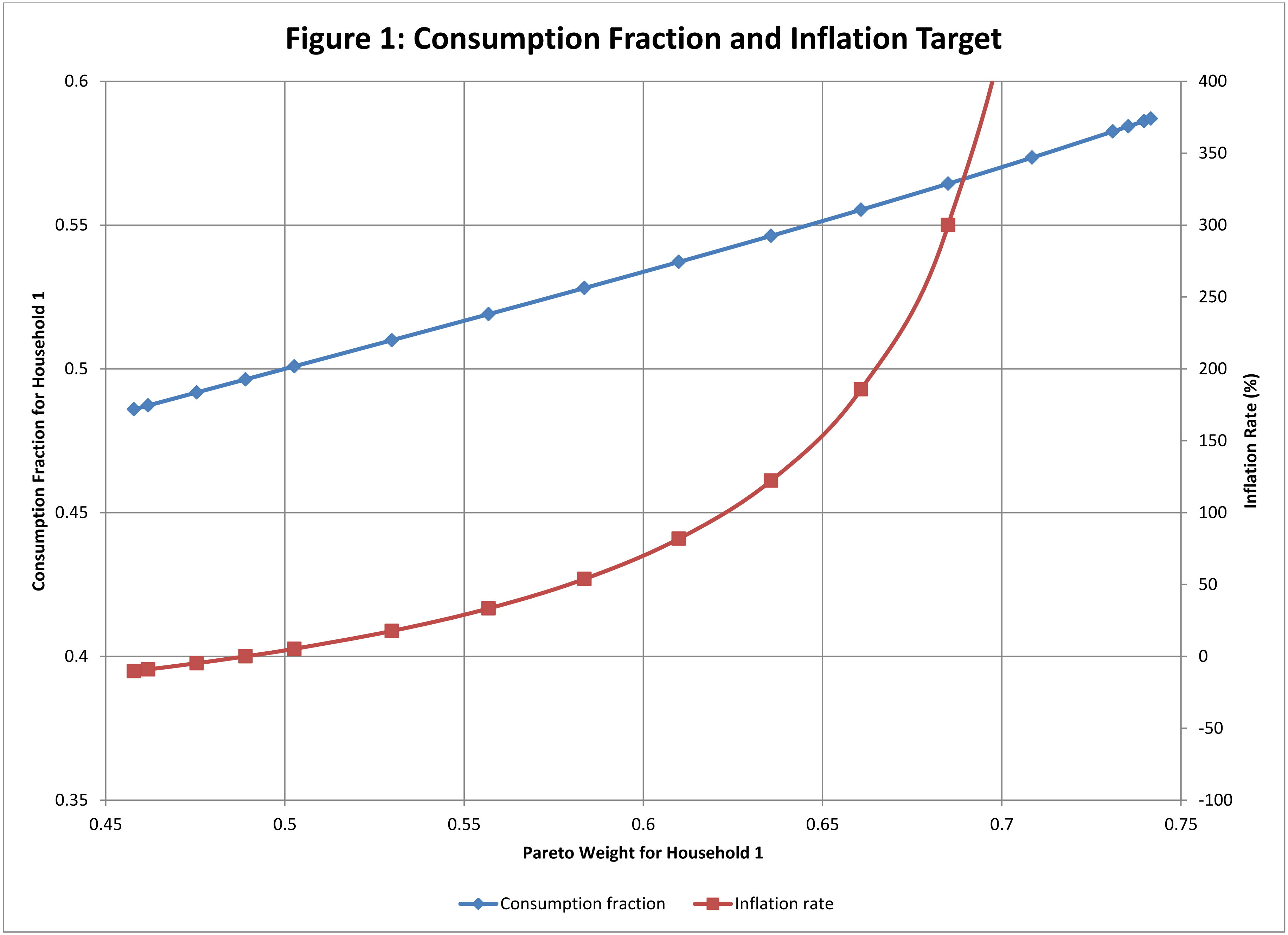

Figure 1: Consumption Fraction and Inflation Target
Consumption Fraction for Household 1
Inflation Rate (%)
Pareto Weight for Household 1
0.6
0.55
0.5
0.45
0.4
0.35
400
350
300
250
200
150
100
50
0
-50
-100
0.45
0.5
0.55
0.6
0.65
0.7
0.75
Consumption fraction
Inflation rate

Summary of Figure 1

- If $\omega_1(s_0) > W_1(s_0)$, increasing inflation target increases domestic consumption.

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- If $\omega_1(s_0) > W_1(s_0)$, increasing inflation target increases domestic consumption.
- Result holds for any of the targeting rules:

$\omega_1(s_0) > W_1(s_0)$
Increase inflation rate
Increase interest rate
Increase monetary growth rate

Initial savings-to-debt ratio less than 1

- $W_1(s_0) = 12.$

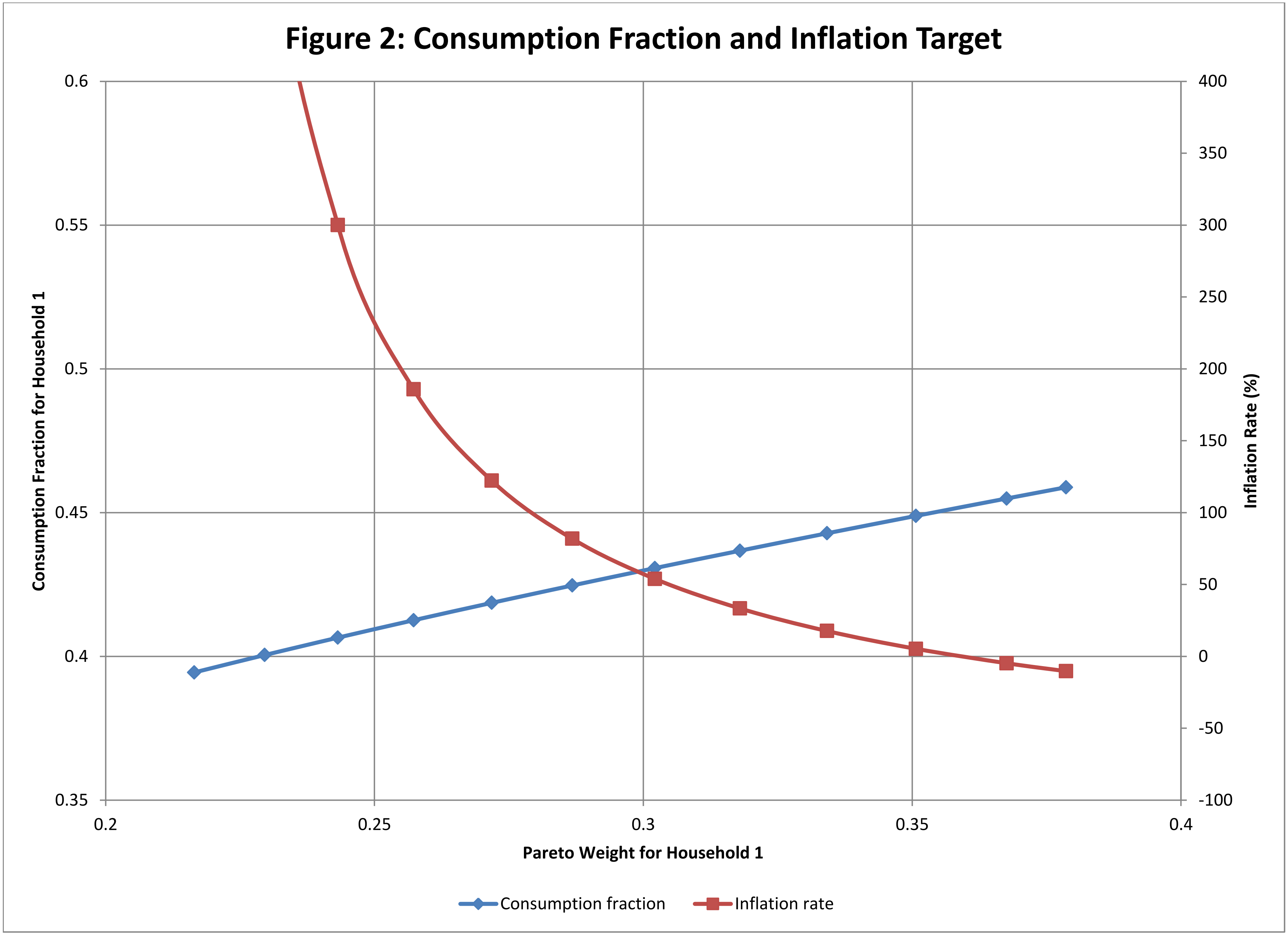

Figure 2: Consumption Fraction and Inflation Target
Consumption Fraction for Household 1
Inflation Rate (%)
Pareto Weight for Household 1
0.6
0.55
0.5
0.45
0.4
0.35
400
350
300
250
200
150
100
50
0
-50
-100
0.2
0.25
0.3
0.35
0.4
Consumption fraction
Inflation rate

Summary of Figure 2

- If $\omega_1(s_0) < W_1(s_0)$, decreasing inflation target increases domestic consumption.

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- If $\omega_1(s_0) < W_1(s_0)$, decreasing inflation target increases domestic consumption.
- Result holds for any of the targeting rules:

$\omega_1(s_0) < W_1(s_0)$
Decrease inflation rate
Decrease interest rate
Decrease monetary growth rate

Conclusions

This paper has...

- shown that targeting rules are inconsistent with Pareto efficiency.
- demonstrated that optimal policy under Pareto efficiency calls for (i) an increase in the inflation rate when the initial savings-to-debt ratio is less than 1 and (ii) a decrease in the inflation rate when the initial savings-to-debt ratio is greater than 1.

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Future work will:

Analyze the welfare implications of targeting rules. We already know this leads to Pareto inefficiency, but which countries might gain or lose from such policies?

Real variables

- Define the real debt positions for the government and the real bond positions for the households as $\hat{B}_{i,j}\left(s^t\right) = \frac{B_{i,j}(s^t)}{p_j(s^t)}$ and $\hat{b}_{i,j}\left(s^t\right) = \frac{b_{i,j}(s^t)}{p_j(s^t)}$ for all $(i,j) \in \mathbf{I}^2$.

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- Define the stochastic price ratios $\nu_i(s^t) = \frac{p_i(s^{t-1})}{p_i(s^t)}$ for $i \in \mathbf{I}$.

Real variables

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- Define the stochastic price ratios $\nu_i\left(s^t\right) = \frac{p_i\left(s^{t-1}\right)}{p_i(s^t)}$ for $i \in \mathbf{I}$.
- Define the real wealth for household h entering date-event s^t

$$\hat{\omega}_i(s^t) = \frac{\omega_i(s^t)}{p_i\left(s^t\right)} = \nu_i\left(s^t\right)\mathbf{e}_i\left(s_{t-1}\right) + \sum_{j\in\mathbf{I}} \nu_j\left(s^t\right)\hat{b}_{i,j}\left(s^{t-1}\right).$$

Determinacy

- Given the policy choice $\left\{\hat{B}_i\left(s^t\right)\right\}_{i\in\mathbf{I}}$ and matrix definition $\hat{B}\left(s^t\right)=\left[\hat{B}_1\left(s^t\right) \quad \ldots \quad \hat{B}_N\left(s^t\right)\right]$, there exists a unique sequence $\left\{\nu_i\left(s^t\right)\right\}_{i\in\mathbf{I}}$ satisfying the government constraints:

$$\left(\nu_i\left(s^t\right)\right)_{i\in\mathbf{I}}^T = \left[diag\left(\mathbf{e}_i\left(s_{t-1}\right)\right)_{i\in\mathbf{I}}+\hat{B}\left(s^{t-1}\right)\right]^{-1} \left[\left(\mathbf{e}_i\left(s_t\right)\right)_{i\in\mathbf{I}}^T+\left(q_i\left(s^t\right)\right)_{i\in\mathbf{I}}^T\left(\hat{B}\left(s^t\right)\right)\right].$$

Stationary wealth

Theorem

If the Markov equilibrium allocation is Pareto efficient and the policy rules $\{q_i(s^t)\}_{i\in\mathbf{I}}$ are stationary, then the real wealth vectors $\hat{\omega}_i(s^t)$ are stationary, meaning that there exists $(\hat{\boldsymbol{\omega}}_i(s))_{(i,s)\in\mathbf{I}\times\mathbf{S}}$ such that the real wealth $\hat{\omega}_i(s^t) = \hat{\boldsymbol{\omega}}_i(s_t)$ for all i and all date-events.

Proof.

Add up the conditional expectations of all budget constraints, using the household Euler equations and transversality conditions, to obtain the discounted present value constraint in all date-events:

$$\hat{\omega}_i(s^t) = \sum_{k=0}^{\infty} \beta^k E_t \left[\left(\frac{\mathbf{E}(s_{t+k})}{\mathbf{E}(s_t)} \right)^{-\rho} (\theta_i \mathbf{E}(s_{t+k}) - \mathbf{q}_i(s_{t+k}) \mathbf{e}_i(s_{t+k})) \right].$$

□

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- Necessary condition for system of budget constraints to hold (over a generic subset of household endowments): $S \leq 2$.
- Same result when all countries adopt interest rate targeting (or exchange rate stabilization).
- When all countries adopt nominal GDP targeting, necessary condition is $S \leq N + 1$.

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- Consider the discounted present value constraints for households and governments (derived from budget constraints and optimality conditions).
- Equilibrium must be consistent with initial wealth parameters $\omega_i(s_0)$ and $W_i(s_0)$.
- Necessary condition when each country adopts a targeting rule is $S \leq N$.
- Stationary inflation policy allows necessary condition to expand to $S \leq 2N + \frac{1}{N-1}$.

Price level determination

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- The discounted present value constraint for the government in country $i \in \mathbf{I}$ in the initial period s_0 is given by:

$$\frac{W_i(s_0)}{p_i(s_0)} = \sum_{k=0}^{\infty} \beta^k E_0 \left[\left(\frac{\mathbf{E}(s_k)}{\mathbf{E}(s_0)} \right)^{-\rho} \mathbf{e}_i(s_k)\left(1 - \mathbf{q}_i(s_k)\right) \right].$$

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- Policy choice $\{q_i(s^t)\}_{i \in \mathbf{I}}$ uniquely determines the initial period price level $p_i(s_0)$ ($W_i(s_0)$ is a parameter).

Real effects of policy

- The discounted present value constraint for the household in country $i \in \mathbf{I}$ in the initial period s_0 is given by:

$$\frac{\omega_i(s_0)}{p_i(s_0)} = \sum_{k=0}^{\infty} \beta^k E_0 \left[\left(\frac{\mathbf{E}(s_k)}{\mathbf{E}(s_0)} \right)^{-\rho} \left(\theta^h \mathbf{E}(s_k) - \mathbf{q}_i(s_k)\,\mathbf{e}_i(s_k) \right) \right].$$

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- Stochastic discount factor matrix $\hat{\Pi}$ with elements

$$\hat{\Pi}(s, \sigma) = \beta \pi(s, \sigma) \left(\frac{\mathbf{E}(\sigma)}{\mathbf{E}(s)} \right)^{-\rho}.$$

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$$\hat{\Pi}(s,\sigma) = \beta\pi(s,\sigma) \left(\frac{\mathbf{E}(\sigma)}{\mathbf{E}(s)} \right)^{-\rho}.$$

- Consumption fraction:

$$\theta_i = \frac{\left(I - \hat{\Pi}\right)^{-1}_{(s_0)} \left((\mathbf{e}_i(s))_{s \in \mathbf{S}} + \left(1 - \frac{\omega_i(s_0)}{W_i(s_0)}\right) (\mathbf{q}_i(s)\mathbf{e}_i(s))_{s \in \mathbf{S}} \right)}{\left(I - \hat{\Pi}\right)^{-1}_{(s_0)} (\mathbf{E}(s))_{s \in \mathbf{S}}}.$$

Initial savings-to-debt ratio

- If $\left(1 - \frac{\omega_i(s_0)}{W_i(s_0)}\right) < 0$, a decrease in price targets $(\mathbf{q}_i(s))_{s \in \mathbf{S}}$ increases θ_i.

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  - If $\omega_i(s_0) > W_i(s_0)$, increase inflation rate.
- If $\left(1 - \frac{\omega_i(s_0)}{W_i(s_0)}\right) > 0$, an increase in price targets $(\mathbf{q}_i(s))_{s \in \mathbf{S}}$ increases θ_i.
  - If $\omega_i(s_0) < W_i(s_0)$, decrease inflation rate.
- If $\left(1 - \frac{\omega_i(s_0)}{W_i(s_0)}\right) = 0$, then

$$\theta_i = \frac{\left(I - \hat{\Pi}\right)^{-1}_{(s_0)} (\mathbf{e}_i(s))_{s \in \mathbf{S}}}{\left(I - \hat{\Pi}\right)^{-1}_{(s_0)} (\mathbf{E}(s))_{s \in \mathbf{S}}}$$

and policy has no effect.