Fiscal Multipliers at the Zero Lower Bound: The Role of Policy Inertia*

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Abstract

The presence of the lagged shadow policy rate in the interest-rate feedback rule reduces the government spending multiplier non-trivially when the policy rate is constrained at the zero lower bound (ZLB). In the economy with policy inertia, increased inflation and output due to the government spending shock during the recession speed up the return of the policy rate to the steady-state *after* the recession ends. This in turn dampens the expansionary effects of the government spending *during* the recession via expectations. In our baseline calibration, the output multiplier at the ZLB is 2.5 when the weight on the lagged shadow rate is zero, while it is 1.1 when the weight is 0.9.

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1 Introduction

Recent theoretical literature on the positive effects of fiscal policy has emphasized that the government spending multiplier on output can be quite large when the nominal interest rate is constrained at the zero lower bound (ZLB). For example, seminal papers by Christiano, Eichenbaum, and Rebelo (2011), Eggertsson (2011), and Woodford (2011) report the multiplier in the range of 2 to 4 in plausibly parameterized standard New Keynesian models.

As reviewed in details shortly, the literature has mostly examined this issue in the model where the policy rate is given by a truncated Taylor rule without policy inertia—interest rate smoothing term. The goal of this paper is to examine the sensitivity of the government spending multiplier at the ZLB to the degree of policy inertia in the interest rate feedback rule. Our main focus is a version of the inertial policy rule in which the shadow policy rate—a hypothetical policy rate that would prevail were it not for the ZLB constraint—today depends on the lagged *shadow* policy rate. However, we will also examine an alternative version of the inertial Taylor rule in which the shadow policy rate today depends on the lagged *actual* policy rate.

Our focus on the inertial Taylor rule is motivated by two considerations. First, as reviewed in detail in Section 2, there is strong empirical evidence for the presence of the lagged policy rate in the nominal interest rate rule. When the policy rule is estimated in the context of structural models, the weight on the lagged policy rate is often estimated to be positive and large. Estimation of the reduced-form policy rule also lends support to the presence of policy inertia. Second, the presence of policy-inertia is consistent with recent FOMC statements indicating that the Committee intends to keep the federal funds rate near zero for a considerable period of time after the economic recovery strengthens. Introducing the lagged policy rate to the policy rule is one way, albeit not the only way, to characterize this policy of an extended period of low nominal interest rate. Our view is that the inertia in the policy rule is an important feature of the U.S. monetary policy during the recent ZLB episode, more so than during the pre-crisis period.

The main result of our paper is that the presence of the lagged shadow policy rate in the interest-rate feedback rule reduces the government spending multiplier at the ZLB in a quantitatively important way. Under the baseline parametrization of the model intended to mimic the Great Depression, the output mutliplier at the ZLB is 1.1 with an inertial parameter of 0.9 versus 2.5 without any inertia. The reason for the smaller multiplier under the inertial Taylor rule is as follows. Independent of policy inertia, an increase in government spending during a recession increases inflation and output during the recession. In a model with policy inertia, this improved allocation during the recession speeds up the return of the policy rate to the steady-state *after* the recession ends, which in turn dampens the expansionary effects of the government spending *during* the recession via expectations. The multiplier at the ZLB approaches from above to the multiplier away from the ZLB as the inertia approaches unity and can go below one with a sufficiently

high inertial parameter. However, for a plausible range of weights on the lagged policy rate, the ZLB multiplier remains above one.

When the nominal interest rate is away from the ZLB, the policy inertia increases the fiscal multiplier. In the presence of policy inertia, the nominal interest rate is slow to adjust in response to an exogenous change in government spending. Thus, the expansionary effects of government spending is larger. However, the effects of the policy inertia is quantitatively much smaller away from the ZLB than at the ZLB, and the multiplier remains below one. The claim in the existing literature that the government spending multiplier is larger at the ZLB than away from the ZLB is thus robust to policy inertia.

With the version of the inertial policy rule where the shadow rate today depends on the lagged *actual* policy rate, the degree of inertia does not affect the government spending multiplier at the ZLB at all in our baseline environment with a two-state crisis shock. In this version of the model, what matters for the policy path after the recession is the actual policy rate at the end of the recession. Since the actual policy rate at the end of the recession is zero regardless of the government spending shock, the policy path, and thus the paths of inflation and output, after the recession is unaltered by the government spending shock during the recession. Accordingly, the inertial parameter does not affect the expansionary effects of the government spending shock during the recession in the economy with this version of the inertial policy rule.

Our result narrows the gap between theoretical multipliers of the New Keynesian model and recent empirical estimates of the multiplier at the ZLB. Using a dataset for the U.S. extending back to 1889, Ramey and Zubairy (2014) find no evidence that the government spending multipliers on output are larger when the policy rate is constrained at the ZLB than when it is not and find that the multiplier is less than one at the ZLB. Using a dataset for the U.K. during the 1930s, Crafts and Mills (2012) find the multipler below unity when the interest rate is constrained at the ZLB. Our analyses show that the policy inertia can bring the prediction of the New Keynesian model closer to these empirical estimates. The policy inertia reduces the multiplier by reducing the effects of government spending shocks on the expected inflation, and this reduced response of inflation expectations is also consistent with the empirical evidence from Dupor and Li (2013) who find no evidence that the American Recovery and Reinvestment Acts pushed up the inflation expectations in the Survey of Professional Forecasters.

This paper is related to a set of papers on the government spending multiplier that emphasizes the role of ZLB constraint. Earlier work by Christiano, Eichenbaum, and Rebelo (2011), Eggertsson (2011), and Woodford (2011) have emphasized that the multiplier can be substantially larger than one in plausibly parameterized New Keynesian models. Since then, many authors have examined the government spending multiplier at the ZLB in various setups. Examples are Albertini, Poirier, and Roulleau-Pasdeloup (2014), Braun, Korber, and Waki (2013), Braun and Waki (2010), Corsetti, Kuester, Meier, and Muller (2010), Carlstrom, Fuerst, and Paustian (2014), Denes, Eggertsson, and Gilbukh (2013), Bouakez, Guillard, and Roulleau-Pasdeloup (2014), Fernandez-Villaverde, Gor-

don, Guerron-Quintana, and Rubio-Ramirez (2012), Mertens and Ravn (2014), Roulleau-Pasdeloup (2014), among others. All of these papers only consider the truncated Taylor rule without any inertia.

While the majority of the literature focuses on the non-inertial policy rule, several authors have considered policy rules with an interest-rate smoothing term. Examples include Aruoba, Cuba-Borda, and Schorfheide (2014) and Erceg and Linde (2014) who consider an inertial policy rule with the lagged actual interest rate, and Cogan, Cwik, Taylor, and Wieland (2010) and Drautzburg and Uhlig (2013) who briefly consider the model with inertial policy rules with the lagged shadow policy rate.[1] Coenen, Erceg, Freedman, Furceri, Kumhof, Lalonde, Laxton, Lind, Mourougane, Muir, Mursula, de Resende, Roberts, Roeger, Snudden, Trabandt, and in't Veld (2012) examine the effects of various fiscal shocks—including the government spending shock—in several structural models used at policy institutions. Some of the models considered in their paper have either lagged actual or shadow policy rate in the interest-rate feedback rule. However, none of these papers have analyzed how the degree and type of policy inertia affect the government spending multiplier at the ZLB.

One exception is a brief sensitivity analysis of Carrillo and Poilly (2013) that reports a smaller government spending multiplier with smaller weight on the lagged shadow policy rate in a model with financial frictions. Our analyses not only show the generality of their result in a wide range of models and parameter configurations, but also clarify the mechanism by which the lagged shadow rate reduces the multiplier by showing that the *type* of the policy inertia matters. Another analysis close to ours is a sensitivity analysis by Erceg and Linde (2014) that considers the government spending multiplier when the nominal interest rate is determined according to price-level targeting (PLT). They find that the government spending multiplier is smaller under PLT than under a truncated (non-inertial) Taylor rule. In our setup and theirs, the key feature of the policy rule that leads to a smaller fiscal multiplier is that the policy rate after the recession depends on the government spending shock during the recession. While the way the government spending shock alters the post-recession policy rates is slightly different between our model with policy inertia and their model with PLT, our in-depth analysis of the fiscal multiplier in the presence of policy inertia nevertheless sheds light on the mechanism behind their result.

Several authors have recently emphasized that the multiplier can be small or modest even at the ZLB. Kiley (2014) finds the government spending multiplier at the ZLB below unity in the sticky information model. Cochrane (2014) argues that the multiplier can be small in alternative non-recursive equilibria, and Mertens and Ravn (2014) find the multiplier below unity when the liquidity trap event is triggered by the belief shock. Braun, Korber, and Waki (2013) demonstrate that the multiplier is only modestly above one under a variety of plausible parameter configurations, and Albertini, Poirier, and

[1]They assume an exogenous ZLB duration for most of the analyses.

Roulleau-Pasdeloup (2014) obtain small multipliers in the model with productive government spending. Our paper finds that a simple, and empirically plausible, modification to the standard setup in this literature—introducing the policy inertia into the truncated Taylor rule—goes a long way in reducing the government spending multiplier at the ZLB.

The rest of the paper is organized as follows. Section 2 reviews the empirical literature on policy inertia. Section 3 describes the model and section 4 defines the government spending multiplier. Sections 5 and 6 present the results. Section 7 discusses the sensitivity of the main results and a final section concludes.

2 Empirical Evidence for Policy Inertia

2.1 Evidence from structural estimation

Structural estimation of DSGE models provides substantial evidence toward an intrinsic adjustment of nominal interest rates in the central bank's reaction function. The level of inertia suggested by these estimates is even more remarkable. Posterior estimates of postwar, pre-Great Recession US data provided by Christiano, Trabandt, and Walentin (2010) suggest a persistence parameter between 0.85 and 0.91, Negro, Schorfheide, Smets, and Wouters (2007) place estimates between 0.7 and 0.86, Justiniano, Primiceri, and Tambalotti (2010) suggest a value between 0.757 to 0.819, and Smets and Wouters (2007) estimate a range from 0.77 to 0.85. Smets and Wouters (2003) also show that for the Euro area, smoothing levels can be particularly high with parameters ranging from 0.93 to 0.97, suggesting that partial adjustment of nominal interest rates is inherent in other advanced economies as well.

Gust, Lopez-Salido, and Smith (2012) estimate a model with a truncated inertial Taylor rule with US data including the periods in which the nominal interest rates were at the effective lower bound. They find that the level of smoothed nominal adjustments is still high with an estimated inertial parameter between 0.78 and 0.92. This and the formerly listed findings strongly suggest that inertia may very well play an important role in policy rate adjustments. Not only do the estimates suggest the presence of policy inertia, but the degree of this smoothing is quite high.

2.2 Evidence from reduced-form estimation

The reduced-form estimation of the interest-rate feedback rule also suggests evidence in favor of an interest-rate smoothing component. Clarida, Gali, and Gertler (2000) show that the weights of inertia have increased over time since pre-Volcker times and that levels as high as 0.91 are plausible. Orphanides (2003) also finds that Greenbook and SPF survey forecasts suggest inertial parameters in the range of 0.75 to 0.91. Clarida, Gali, and Gertler (1998) show that the weights on the lagged policy rate is in the range of 0.87 to 0.97 for the Fed and other advanced economies, with the majority of estimates

suggesting a high level of inertia above 0.9. These international estimates again suggest that intrinsic policy smoothing is not a phenomenon unique to the US alone.

However, the interpretaion of the reduced-form estimates has been open to debate. Rudebusch (2006) has argued that policy rates are contingent on both data that is incoming and changes in the economic outlook. Under this argument, policy is defined to be more of an extrinsic reaction rather than an intrinsic adjustment–that inertia is more likely a reflection of omitted variables in the Fed's reaction function rather than intrinsic decisions made on the part of the policy makers. Taking into account the debate surrounding the source of perceived policy inertia, Coibion and Gorodnichenko (2012) have argued that a formal statistical test favors the interest-rate smoothing hypothesis once one allows for higher order smoothing, as opposed to the commonly used restriction of a first order autoregressive process. Allowing for this generalization led Coibion and Gorodnichenko (2012) to find that autoregressive parameters in the policy rate error term to become insignificant or even negative, thus providing evidence against the argument that inertia is merely a mirage of persistent shocks.

2.3 Consistency with the Forward-Guidance Policy

An interest-rate feedback rule including the lagged policy rate is consistent with the recent forward guidance policy in the U.S. in which the Federal Reserve has repeatedly stated that the policy rate will be kept at zero for an extended period of time even after the economic recovery strengthens. If the central bank adjusts its policy rate only based on the current economic conditions, then it would raise the policy rate *as* the economic recovery strengthes. On the other hand, if the policy rule features a large weight on the lagged policy rate, then the central bank would raise the policy rate slowly.

Policy inertia is certainly not the only way to model an extended period of low nominal interest rates. For example, larger response coefficients on the economic conditions are also consistent with staying at the ZLB for long, as they would lower the nominal interest rate when inflation and output gaps are negative. Modelling the central bank as following optimal commitment policy is another way to make the ZLB bind for a prolonged period. We do not argue that our modelling approach is better than the others in capturing the recent monetary policy in the U.S. However, we do argue that the consistency of the policy inertia with the forward guidance policy provides further justification to our analyses, in addition to the empirical evidence reviewed earlier.

2.4 Lagged shadow versus actual policy rates

These empirical considerations point to the validity of introducing the lagged policy rate into the truncated Taylor rule, but do not say anything about which version of the inertial policy rules—the one with lagged shadow rate or the other with the lagged actual rate—fits the data better. In the model without the ZLB constraint, these two policy rules would imply indentical dynamics. In the model with the ZLB constraint, they imply

drastically different dynamics.

As we will see shortly, it is easier to generate persistent ZLB episodes with the shadow rate version than with the actual rate version. Thus, our conjecture is that the inertial Taylor rule with the lagged shadow rate is likely to better fit the recent prolonged liquidity trap episode in the U.S. Also, some policymakers have characterized the Federal Reserve's "low-for-long" policy as an attempt to "compensate" for the fact that the policy rate was constrained at the ZLB in the past, which is also consistent with the shadow rate version of the policy rule. However, making a convincing case for one version of the truncated inertial policy rule over the other requires formal econometric analyses, a nontrivial task we leave for future research. We simply note that our result—these two versions imply substantially different fiscal multipliers—suggests the importance of such econometric analyses.

3 Model

3.1 Model Description

We use a standard New Keynesian model with Rotemberg price adjustment. The economy is formulated in discrete time with an infinite horizon, and is populated with a representative household, a final good producer, a continuum of intermediate goods producers with unit measure, and the government.

3.1.1 Household

The representative household chooses its consumption level, amount of labor, and bond holdings so as to maximize the expected discounted sum of utility in future periods. As is common in the literature, the household enjoys consumption and dislikes labor. Assuming that period utility is separable, the household problem can be defined by

$$\max_{\{C_t,N_t,B_t\}_{t=1}^{\infty}} \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[\prod_{s=0}^{t-1} \delta_s \right] \left[\frac{C_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n} \right] \tag{1}$$

subject to the budget constraint

$$P_t C_t + R_t^{-1} B_t \leq W_t N_t + B_{t-1} + P_t \Phi_t + P_t T_t \tag{2}$$

or equivalently

$$C_t + \frac{B_t}{R_t P_t} \leq w_t N_t + \frac{B_{t-1}}{P_t} + \Phi_t + T_t \tag{3}$$

where C_t is consumption, N_t is the labor supply, P_t is the price of the consumption good, W_t (w_t) is the nominal (real) wage, Φ_t is the profit share (dividends) of the household from the intermediate goods producers, B_t is a one-period risk free bond that pays one unit of money at period t+1, T_t is lump-sum taxes, and R_t^{-1} is the price of the bond.

The discount rate at time t is given by $\beta\delta_t$, where δ_t is the discount factor shock altering the weight of future utility at time t+1 relative to the period utility at time t. Following Eggertsson (2011) and Woodford (2011) among many others, we assume that δ_t is governed by an exogenous state s_t, which follows a two-state Markov shock process. s_t takes two values, H and L, and the transition probability is given by

$$\text{Prob}(s_{t+1} = L | s_t = L) = 1 \tag{4}$$

$$\text{Prob}(s_{t+1} = H | s_t = H) = \mu \tag{5}$$

The value of δ_t depends on the realization of s_t as follows.

$$\delta_t = \begin{cases} \delta_{ss} = 1 & \text{if } s_t = L \\ \delta_H & \text{if } s_t = H \end{cases} \tag{6}$$

An increase in δ_t increases the relative valuation of future utility flows, making the household more willing to save for tomorrow and less willing to consume today. While we work with this two-state Markov structure in the baseline, we will consider a case in which this shock follows an AR(1) process.

3.1.2 Firms

There is a final good producer and a continuum of intermediate goods producers indexed by $i \in [0,1]$. The final good producer purchases the intermediate goods $Y_{i,t}$ at the intermediate price $P_{i,t}$ and aggregates them using CES technology to produce and sell the final good Y_t to the household and government at price P_t. Its problem is then summarized as

$$\max_{Y_{i,t}, i\in[0,1]} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di \tag{7}$$

subject to the CES production function

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}. \tag{8}$$

Intermediate goods producers use labor to produce the imperfectly substitutable intermediate goods according to a linear production function ($Y_{i,t} = N_{i,t}$) and then sell the product to the final good producer. Each firm maximizes its expected discounted sum of future profits[2] by setting the price of its own good. We further assume that any price changes are subject to quadratic adjustment costs.

$$\max_{\{P_{i,t}}\}_{t=1}^{\infty} \text{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[\prod_{s=0}^{t-1} \delta_s \right] \lambda_t \left[P_{i,t} Y_{i,t} - W_t N_{i,t} - P_t \frac{\varphi}{2} \left[\frac{P_{i,t}}{P_{i,t-1}} - 1 \right]^2 Y_t \right] \tag{9}$$

[2]Each period, as it is written below, is in *nominal* terms. However, we want each period's profits in *real* terms so the profits in each period will be divided by that period's price level P_t.

such that

$$Y_{i,t} = \left[\frac{P_{i,t}}{P_t}\right]^{-\theta} Y_t.^{3} \tag{10}$$

λ_t is the Lagrange multiplier on the household's budget constraint at time t and $\beta^{t-1}\left[\prod_{s=0}^{t-1}\delta_s\right]\lambda_t$ is the marginal value of an additional profit to the household. The positive time zero price is the same across firms (i.e. $P_{i,0} = P_0 > 0$).

3.1.3 Government

We assume that the nominal interest rate is determined according to the following *truncated inertial Taylor rule*:

$$R_t = \max\left[1, R_t^*\right] \tag{11}$$

$$R_t^* = \frac{1}{\beta}\left(\frac{R_{t-1}^*}{R_{ss}}\right)^{\rho_r}\left(\Pi_t^{\phi_\pi}\left(\frac{Y_t}{Y_{ss}}\right)^{\phi_y}\right)^{1-\rho_r} \tag{12}$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ and R_{ss} and Y_{ss} are the steady state levels of the nominal interest rate and output. R_t^* is the hypothetical policy rate that would prevail were it not for the ZLB constraint at time t, and will be referred to as the *shadow*, *notional*, or *recommended nominal interest rate*. In Section 6, we will also consider three alternative assumptions about the determination of the nominal interest rate: (i) a version of the inertial Taylor rule in which today's shadow rate depends on the lagged actual policy rate, (ii) price-level targeting, and (iii) the policy rates are chosen optimally by the government with commitment.

In the baseline exercise, we assume that the supply of the one-period risk free bond is zero and that the lump-sum taxes are available to finance the government spending. Thus, the government budget constraint is given by

$$G_t = T_t. \tag{13}$$

Further on, we will consider cases in which a distortionary tax is available as well as allowing the supply of the government bond to be non-zero.

The government spending G_t is governed by the state variable s_t introduced earlier.

$$G_t = \begin{cases} G_{ss} & \text{if } s_t = L \\ G_H & \text{if } s_t = H \end{cases} \tag{14}$$

[3] This expression is derived from the profit maximizing input demand schedule when solving for the final good producer's problem above. Plugging this expression back into the CES production function implies that the final good producer will set the price of the final good $P_t = \left[\int_0^1 P_{i,t}^{1-\theta} di\right]^{\frac{1}{1-\theta}}$.

Our assumption of perfect correlation between the preference shock and government spending shock follows the existing literature. We will later consider a case in which the government spending shock follows an AR(1) process.

We will use γ to denote the steady-state share of government spending to output (i.e., $\gamma \equiv \frac{G_{ss}}{Y_{ss}}$). We will use g to denote the log deviation of G_H from G_{ss} (i.e., $g \equiv \log(\frac{G_H}{G_{ss}})$).

3.1.4 Market Clearing Conditions

The market clearing conditions for the final good, labor and government bond are given by

$$Y_t = C_t + \int_0^1 \frac{\varphi}{2}\left[\frac{P_{i,t}}{P_{i,t-1}} - 1\right]^2 Y_t di + G_t \tag{15}$$

$$N_t = \int_0^1 N_{i,t} di \tag{16}$$

and

$$B_t = 0. \tag{17}$$

3.1.5 Nonlinear Equilibrium Conditions

Given P_0 and a two-state Markov shock process s_t governing the evolution of δ_t and G_t, an equilibrium is defined as $\{C_t, N_t, N_{i,t}, Y_t, Y_{i,t}, G_t\}_{t=1}^{\infty}$, prices $\{W_t, P_t, P_{i,t}\}_{t=1}^{\infty}$, and a policy instrument $\{R_t\}_{t=1}^{\infty}$ such that (i) given the determined prices and policies, allocations solve the household problem, (ii) $P_{i,t}$ solves the problem of firm i, (iii) $P_{i,t} = P_{j,t} \quad \forall i \neq j$, (iv) R_t follows a specified rule, (v) all markets clear.

Combining all of the results from (i)-(v), a symmetric equilibrium can be characterized recursively by $\{C_t, N_t, Y_t, w_t, \Pi_t, R_t\}_{t=1}^{\infty}$ satisfying the following equilibrium conditions:

$$C_t^{-\chi_c} = \beta\delta_t R_t \mathrm{E}_t C_{t+1}^{-\chi_c}\Pi_{t+1}^{-1} \tag{18}$$

$$w_t = N_t^{\chi_n} C_t^{\chi_c} \tag{19}$$

$$\frac{Y_t}{C_t^{\chi_c}}\left[\varphi(\Pi_t - 1)\Pi_t - (1-\theta) - \theta w_t\right] = \beta\delta_t \mathrm{E}_t \frac{Y_{t+1}}{C_{t+1}^{\chi_c}}\varphi(\Pi_{t+1} - 1)\Pi_{t+1} \tag{20}$$

$$Y_t = C_t + \frac{\varphi}{2}[\Pi_t - 1]^2 Y_t + G_t \tag{21}$$

$$Y_t = N_t \tag{22}$$

and equations (11) and (12). Equation (18) is the consumption Euler Equation, equation (19) is the intratemporal optimality condition of the household, equation (20) is the optimal condition of the intermediate good producing firms (forward-looking Phillips Curve) relating today's inflation to real marginal cost today and expected inflation tomorrow, equation (21) is the aggregate resource constraint capturing the resource cost of price adjustment, and equation (22) is the aggregate production function.

3.2 Semi-Loglinear Equilibrium Conditions

Following the majority of the existing literature, we will mainly work with a semi-loglinear version of the economy in which all the equilibrium conditions are log-linearized except for the ZLB constraint on the policy rate. An equilibrium in the semi-loglinear economy is characterized by $\{\hat{C}_t, \hat{Y}_t, \hat{\Pi}_t, i_t, i_t^*\}_{t=1}^{\infty}$ satifying the following conditions:

$$\hat{Y}_t = \mathrm{E}_t\hat{Y}_{t+1} - \sigma(i_t - \mathrm{E}_t\hat{\Pi}_{t+1} + \hat{\delta}_t - \bar{r}) + \gamma(\hat{G}_t - \mathrm{E}_t\hat{G}_{t+1}) \tag{23}$$

$$\hat{\Pi}_t = \kappa\hat{Y}_t - \kappa\psi\sigma^{-1}\gamma\hat{G}_t + \beta\mathrm{E}_t\hat{\Pi}_{t+1} \tag{24}$$

$$\hat{Y}_t = (1-\gamma)\hat{C}_t + \gamma\hat{G}_t \tag{25}$$

$$i_t = \max(0, i_t^*) \tag{26}$$

$$i_t^* = \bar{r} + \rho_r(i_{t-1}^* - \bar{r}) + (1-\rho_r)(\phi_\pi\hat{\Pi}_t + \phi_y\hat{Y}_t) \tag{27}$$

where $\sigma \equiv \frac{1-\gamma}{\chi_c}$, $\kappa \equiv \frac{(\theta-1)(\chi_n+\sigma^{-1})}{\varphi}$, $\psi \equiv \frac{1}{\chi_n+\sigma^{-1}}$, $\bar{r} \equiv 1-\beta$, and $i_t \equiv \hat{R}_t + \bar{r}$,

$$\hat{\delta}_t = \begin{cases} 0 & \text{if } s_t = L \\ \hat{\delta}_H & \text{if } s_t = H \end{cases} \tag{28}$$

and

$$\hat{G}_t = \begin{cases} 0 & \text{if } s_t = L \\ g \equiv \log(\frac{G_H}{G_{ss}}) & \text{if } s_t = H \end{cases} \tag{29}$$

A recursive competitive equilibrium of this semi-loglinear economy is given by a set of policy functions, $\{\hat{Y}(\cdot,\cdot),\ \hat{C}(\cdot,\cdot),\ \hat{\Pi}(\cdot,\cdot),\ i(\cdot,\cdot),\ i^*(\cdot,\cdot)\}$ that satisfies the functional equations above. These policy functions are functions of the lagged shadow nominal interest rate, i_{t-1}^*, and the state, $s_t \in \{H, L\}$. We use a time-iteration method from Coleman (1991) to find them numerically. The details of the solution method is described in the Appendix.

Some have argued that the fiscal multipliers computed in the log-linear approximation may be a poor approximation to those in the fully nonlinear economy. Accordingly, we will examine the robustness of our results to solving the model fully nonlinearly in Section 7.

3.3 Parameterization

As our baseline, we use the parameter values consistent with Denes, Eggertsson, and Gilbukh (2013) to match U.S. data during the Great Depression. The values are listed in Table 1. There are a few parameters that are not in the model of Denes, Eggertsson, and Gilbukh (2013), but are present in our model. For the steady-state ratio of government spending to output, we choose $\gamma \equiv \frac{G_{ss}}{Y_{ss}} = 0.2$. For the coefficients on inflation and output in the inertial Taylor rule, we choose $(\phi_\pi, \phi_y) = (1.5, 0.25)$. Our main exercise

is to vary the weight on the lagged shadow interest rate. We consider 100 weights that are multiples of 0.01 from 0 to 0.99.[4] We will examine the robustness of our result to alternative parameter values in Section 7.

Table 1: Great Depression (Baseline) Parameterization

β	χ_c	χ_n	γ	θ	φ	ϕ_π	ϕ_y	μ
0.997	0.9224	1.53	0.20	12.70	3444.8512	1.50	0.25	0.902

4 Definition of Government Spending Multipliers

We define *the government spending multiplier function*, $GM(i^*_{-1}, g, \hat{\delta}_H)$, as follows.

$$GM(i^*_{-1}, g, \hat{\delta}_H) := \frac{\hat{Y}(i^*_{-1}, H; g, \hat{\delta}_H) - \hat{Y}(i^*_{-1}, H; 0, \hat{\delta}_H)}{\gamma g} \tag{30}$$

where $y(\cdot, \cdot; g, \hat{\delta}_H)$ is a policy function for output indexed by $(g, \hat{\delta}_H)$. This function measures the average increase in output in response to an increase in the government spending of size g in the high (crisis) state when the lagged shadow rate is $\bar{r}$ and the state and the size of the discount factor shock is $\hat{\delta}_H$.

We call the government spending multiplier function evaluated at $(i^*_{-1}, g, \hat{\delta}_H) = (\bar{r}$, 0.01, 0) as the non-crisis multiplier. This measures the average effect of one percent increase in government spending on output in the economy without any preference shocks. We will also refer to the non-crisis multiplier as the government spending multiplier away from the ZLB or non-ZLB multiplier.

We call the government spending multiplier function evaluated at $(i^*_{-1}, g, \hat{\delta}_H) = (\bar{r}$, 0.01, $\hat{\delta}_H)$ as the crisis multiplier when $\hat{\delta}_H > 0$. This measures the average effect of one percent increase in government spending on output in the economy when the preference shock hits. We will be mostly interested in a case in which $\hat{\delta}_H$ is sufficiently large so that $i(i^*_{-1}, H; 0, \hat{\delta}_H) = 0$. Our definitions are consistent with those in the literature (for example, Eggertsson (2011) and Woodford (2011)). We will also refer to the crisis multiplier as the government spending multiplier at the ZLB or the ZLB multiplier.[5]

The main exercise of this paper is to compare the fiscal multipliers in economies with various degrees of policy inertia. When comparing the effects of policy inertia on the fiscal multiplier, we adjust the size of the preference shock $\hat{\delta}_H$ so that the initial declines in output are the same across different values of the inertia parameter. That is, the government spending multiplier in the economy with ρ_r is computed as

[4] We will not consider the case with $\rho_r = 1$ because this case violates the Taylor principle.

[5] When the weight on the nominal interest rate is sufficiently large, the ZLB does not necessarily bind at time one in response to the preference shock. Thus, in this sense, using the terms *the crisis multiplier* and *the ZLB multiplier* interchangeably can be misleading. We will be clear about this whenever this happens in order to avoid any misunderstandings.

$$GM(i^*_{-1}, g, h(\rho_r)) := \frac{\hat{Y}(i^*_{-1}, H; g, h(\rho_r)) - \hat{Y}(i^*_{-1}, H; 0, h(\rho_r))}{\gamma g} \quad (31)$$

where, for any $\rho_r > 0$, the adjusted shock size, $h(\rho_r)$, is computed so that $\hat{Y}(i^*_{-1}, H; 0, h(\rho_r))$ in the economy with policy inertia is the same as $\hat{Y}(i^*_{-1}, H; 0, \hat{\delta}_H)$ in the economy with $\rho_r = 0$. We make this adjustment because we want to understand the effects of the fiscal multiplier in a comparable situation. In the absence of this adjustment, the initial decline in output will be smaller in the economy with policy inertia than in the economy without inertia, and one would have difficulty understanding whether the difference in the fiscal multipliers across two economies is driven by the difference in the policy rule or by the differences in the severity of the recession across two economies. A similar adjustment is made when conducting sensitivity analyses by Braun, Korber, and Waki (2013) and Erceg and Linde (2014).

5 Results

Figure 1 shows how the fiscal multiplier varies with the policy inertia parameter. The black and red lines are for the government spending multipliers when the nominal interest rate is constrained at the ZLB and when it is not, respectively.

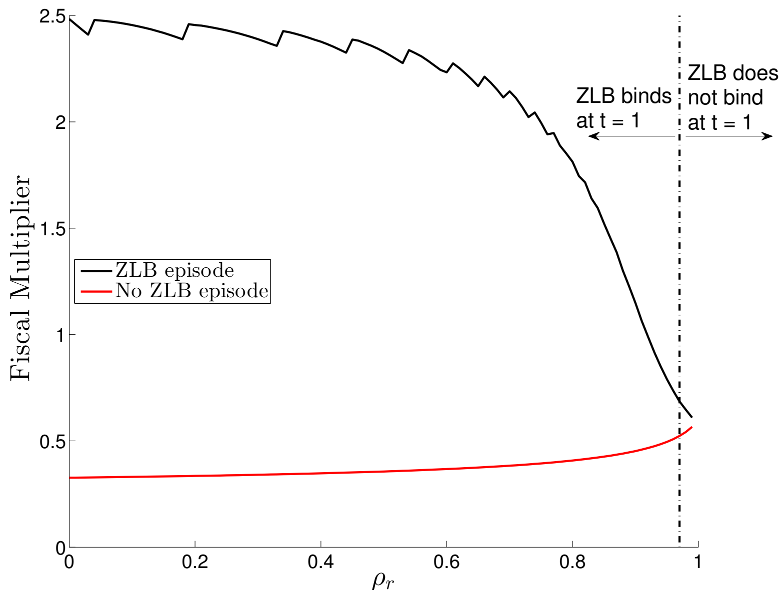
5.1 Policy Inertia and Multipliers at the ZLB

The black line shows two important features of the fiscal multipliers at the ZLB with inertia. First, the fiscal multilplier is smaller in the economy with policy inertia than in the economy without policy inertia. While the multiplier without inertia is 2.5, the multiplier with inertia of 0.9 is 1.1. As the inertia increases, the ZLB multiplier approaches the non-ZLB multiplier from above. The multiplier is substantially above those away from the ZLB and remains above one for a plausible range of the weights. The second key feature is that the policy inertia affects the fiscal multiplier in non-monotonic ways. As the inertia increases from 0 to 0.03, the multiplier decreases monotonically. However, at an inertia level of 0.04, the multiplier jumps up; after the jump, the multiplier declines monotonically until it jumps up again.

Smaller multiplier

To understand the first result, it is useful to examine how differently the government spending shock affects the economy at the ZLB with and without inertia. The left column of Figure 2 show realizations of consumption, output, inflation and the nominal interest rate in the economy without inertia when the preference shock hits the economy at time one and lasts for eight quarters. The right column of Figure 2 shows the same set of

Figure 1: Policy Inertia and the Government Spending Multiplier



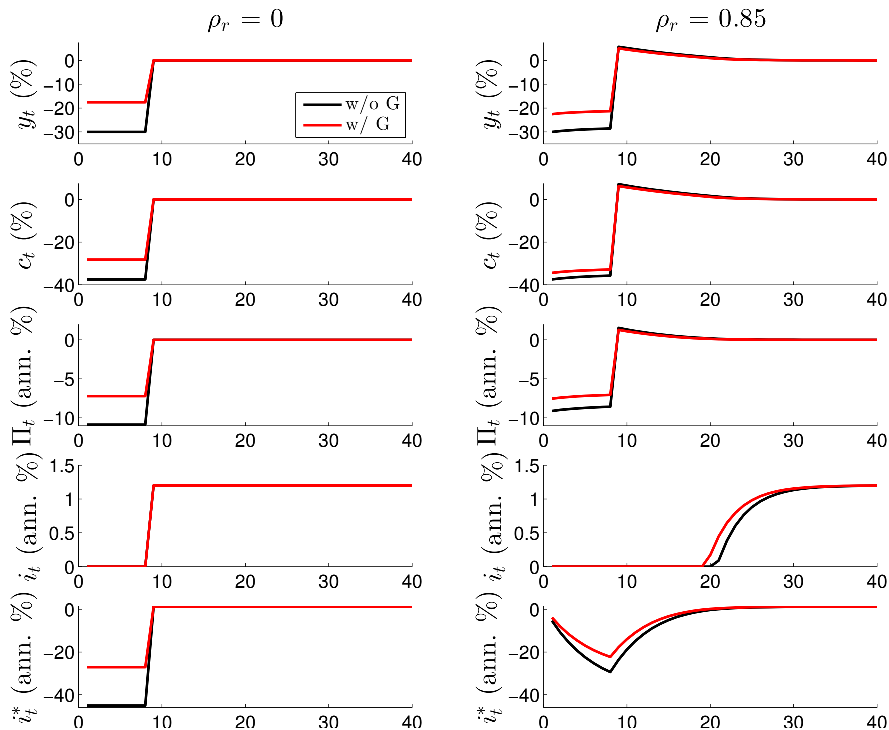
impulse response functions for the economy with $\rho_r = 0.85$.[6] In each panel, the the black line is for the case without an increase in government spending, and the red line is for the case with a 5 percent increase in government spending. Readers should bear in mind that we set the size of the government spending shock to be 5 percent in the figure so that the effects are visually clear and that the government spending multiplier is computed based on a one-percent increase in government spending as discussed in Section 4. In both economies with and without policy inertia, output, consumption, and inflation decline substantially and the nominal interest rate is at the ZLB during the crisis period.

In the economy without policy inertia, the economy reverts back to the steady-state as soon as the shock disappears. This is true regardless of whether or not the economy is hit by the government spending shock. In the economy with policy inertia, the nominal interest rate stays at the ZLB even after the shock disappears. This is because the lagged shadow rate is negative right after the shock disappears due to the fact that inflation is negative during the recession. Eventually, the nominal interest rate lifts off and gradually returns to the steady-state level. The nominal interest rate path is thus lower relative to the path under the economy without inertia after the shock disappears. This extended period of low nominal interest rates creates an overshooting of consumption, inflation, and output. This feature of the economy is true regardless of whether or not the government spending shock is present or not.

In the economy without inertia, an exogenous increase in government spending during a recession raises the demand for the final goods, which in turn increases output and inflation. With the nominal interest rate stuck at the ZLB, an increase in inflation reduces

[6]This value is close to the posterior mode of this parameter from Gust, Lopez-Salido, and Smith (2012) who estimate a sticky-price model using the U.S. data over the sample including the recent ZLB episode.

Figure 2: IRFs at the ZLB—with and without policy inertia—



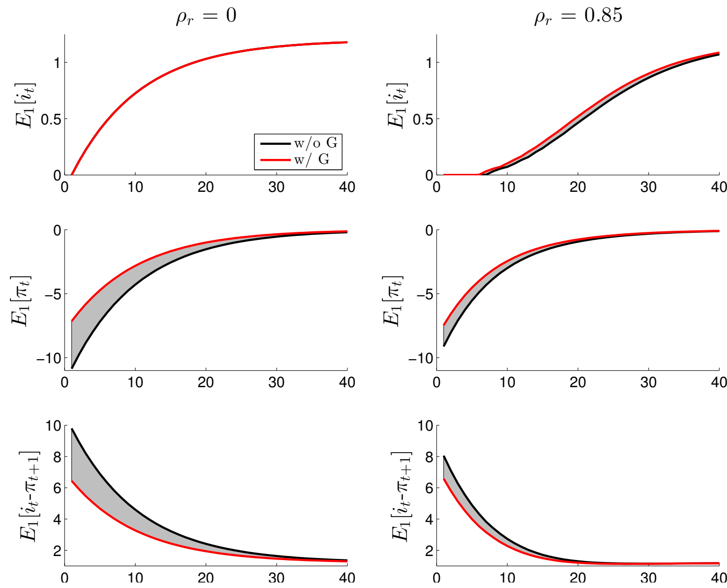
the real interest rate, which in turn boosts consumption. With a positive consumption response to the government spending shock, the multiplier on output is above one. Notice that, in this economy without inertia, the government spending shock during the recession has no implications for the economy after the recession. The economy is back at the steady-state after the shock dissipates regardless of the government spending shock.

In the economy with policy inertia, an exogenous increase in government spending *during* the crisis has effects on the economy both *during* and *after* the recession. An exogenous increase in the government spending *during* the recession increases consumption, inflation, and output *during* the recession, as it does so in the economy with inertia. In the inertial economy, a higher inflation path during the recession implies a higher path of shadow policy rates relative to the path in the absence of the government spending shock. As a result, the nominal interest rate returns to the steady-state more quickly in the presence of the government spending shock than in the absence of it. As a result, the overshooting of inflation and output is smaller after the recession with the fiscal policy than without. A lower inflation and a higher nominal interest rate *after* the recession reduces consumption during the crisis since the household is forward-looking and cares about the expected future real rate. A lower output *after* the recession reduces prices during the crisis since firms are forward-looking and care about the expected future marginal costs of production.

Since consumption is ultimately determined by the undiscounted sum of future expected real interest rates, we can better understand the effect of government spending

shocks on consumption, and thus output, by examining the effects of government spending shocks on the path of expected real interest rates. For that purpose, Figure 3 shows expected nominal interest rates, inflation, and real interest rates with and without government spending shocks. Left panels are for the economy without policy inertia while right panels are for the economy with policy inertia. By inspecting the differential effects of government spending on the path of expected nominal interest rates, inflation, and real interest rates, the differences in the government spending multipliers across economies with and without policy inertia become clearer.

Figure 3: Time-one mean forecasts at the ZLB—with and without policy inertia—



As shown in the left panels of Figure 3, and consistent with the previous discussion, the government spending shock does not affect the expected path of nominal interest rates in the economy without policy inertia. However, an increase in government spending increases inflation expectations at all forecast horizons because government spending is expected to remain elevated as long as the preference shock lasts and an elevated level of government spending is associated with smaller deflation. Accordingly, the expected real interest rates are lower with government spending than without it at all forecast horizons.

In the economy with policy inertia, an increase in government spending during the recession pushes up the path of shadow nominal interest rates and the nominal interest rate moves back to the steady-state level more quickly *for any realizations of preference shocks*. Thus, the expected nominal interest rate is higher with government spending shocks than without them at all forecast horizons, as shown in the top-left panel. The tighter monetary policy implies that the expansionary effects of the government spending shock on inflation is mitigated. As shown in the middle-left panel, the expected path of inflation is higher with government spending, but not by as much as in the economy without policy inertia. A higher path of expected nominal interest rates and subdued

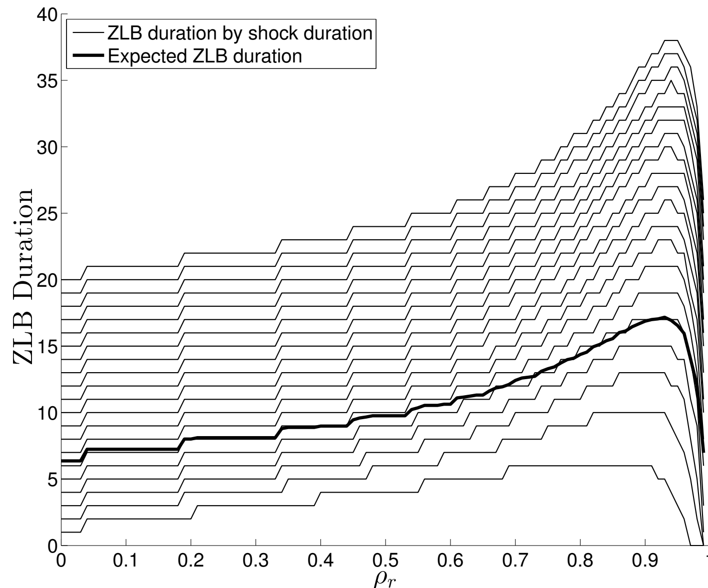
increases in the inflation expectations mean that the government spending shock reduces expected real interest rate by less in the economy with policy inertia than in the economy without it. According to the bottom panels, the effects of government spending shock on expected real interest rates, as captured by the grey area, is smaller in the inertial economy than in the non-inertial economy. As a result, consumption rises less in the inertial economy, leading to a smaller output multiplier.

Non-monotonicity

While the multiplier is smaller in the economy with policy inertia than in the economy without policy inertia, the effects of policy inertia on the multiplier is not monotonic. In particular, there are several points in the $\rho_r = [0, 1)$ where the multiplier increases in response to a marginal increase in ρ_r.

To understand where this non-monotonicity comes from, notice that the duration of the ZLB depends on the inertia parameter. In particular, an increase in the policy inertia can increase the duration of the ZLB episode given a certain path of the crisis shock and therefore the expected duration of the ZLB. When it does, it does so in a discrete way since the duration of the ZLB is a discrete variable, as shown in Figure 4. The fiscal multiplier depends on the duration of the ZLB episode since the duration determines the periods during which the expansionary effects of the government spending is not offset by a corresponding increase in the nominal interest rate, as emphasized in the work of Eggertsson (2011), Woodford (2011), and Erceg and Linde (2014). The longer the ZLB duration, the larger the multiplier. Those jumps in the fiscal multiplier corresponds to the jumps in the expected duration of the ZLB in Figure 4.

Figure 4: Policy Inertia and Expected ZLB Duration

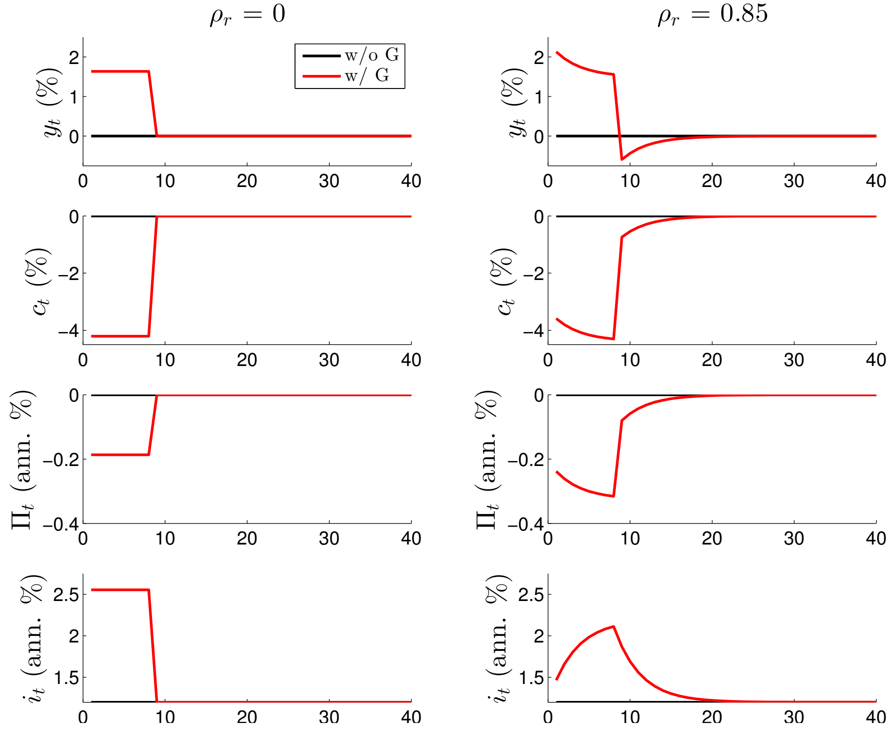


5.2 Policy Inertia and Multipliers Away from the ZLB

The effect of policy inertia on the government spending multiplier is qualitatively different when the nominal interest rate is away from the ZLB. The red line in Figure 5 shows that the government spending multiplier increases with the weight on the lagged policy rate if the nominal interest rate is away from the ZLB. The effect of inertia on the mutliplier is much smaller when the nominal interest rate is away from the ZLB than when it is at the ZLB. Away from the ZLB, the multiplier increases by 0.1 (from 0.8 to 0.9) as the inertia increases from zero to 0.85.

In the economy with inertia, the nominal interest rate adjusts slowly to the government spending shock. Thus, output expands by more in the economy with inertia than in the economy without inertia. Later on, as the policy rate gradually increases, output and inflation decline. Overall, the effects of the government spending shock on the expected real interest rate is smaller and consumption increases by more at time one in the inertial economy than in the non-inertial economy.

Figure 5: IRFs away from the ZLB—with and without policy inertia—



6 Additional results

6.1 Alternative Policy Rules

An alternative version of the inertial Taylor rule

Thus far, we have focused on the version of the inertial policy rule in which the shadow policy rate today depends on the lagged shadow policy rate. An alternative formulation would be an inertial rule in which the shadow policy rate depends on the actual policy rate as follows.

$$R_t = \max\left[1, R_t^*\right] \tag{32}$$

$$R_t^* = \frac{1}{\beta}\left(\frac{R_{t-1}}{R_{ss}}\right)^{\rho_r}\left(\Pi_t^{\phi_\pi}\left(\frac{Y_t}{Y_{ss}}\right)^{\phi_y}\right)^{1-\rho_r} \tag{33}$$

The left-hand side of Figure 6 shows how the government spending multiplier varies with the policy inertia parameter in the economy with this version of the inertial Taylor rule. The multiplier does not vary with the inertial parameter unless the inertia is very close to one. The reason for this insensitivity is that, in this specification of the policy rule, the policy rate after the recession depends on the the actual policy rate at the last quarter of the recession, which is zero no matter how much inflation and output decline during the recession. Thus, increased inflation and output during the recession due to the government spending shock do not have any influence on the path of policy rate after the recession, unless the policy inertia is very close to zero. This can be seen in the left panels of Figure 7 that show the evolution of output, consumption, inflation, and the nominal interest rate with and without government spending shocks. The figure shows that the government spending shock does not alter the path of policy rates after the shock disappears at all. Accordingly, the government spending multiplier is unchanged from that in the economy without policy inertia.

Price-Level Targeting

A key mechanism in which the inertia in the policy rule reduces the government spending multiplier is that the government spending today affects the evolution of the economy after the shock disappears. This mechanism is not a unique feature of the model with the inertial policy rule studied above; it is also present in the economies where the nominal interest rate is determined according to other history dependent-policy rules. To make this point, we examine the fiscal multipliers in the economy in which the nominal interest rate is determined according to price-level targeting. The nominal interest rate in the price-level targeting regime is determined by

$$i_t^{PT} = \max(0, \bar{r} + \phi_p p_t + \phi_\pi \hat{\Pi}_t + \phi_y \hat{Y}_t) \tag{34}$$

Figure 6: Alternative History-Dependent Policy Rules

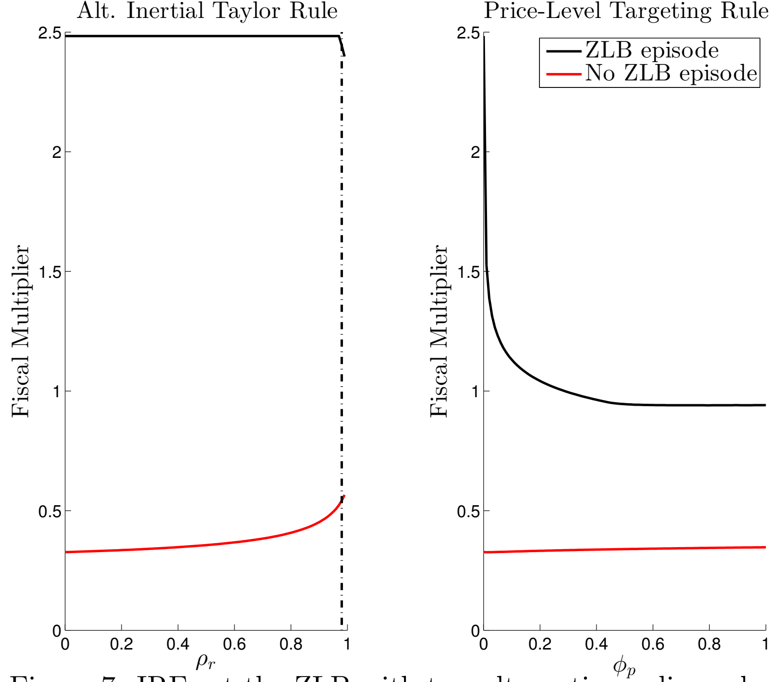
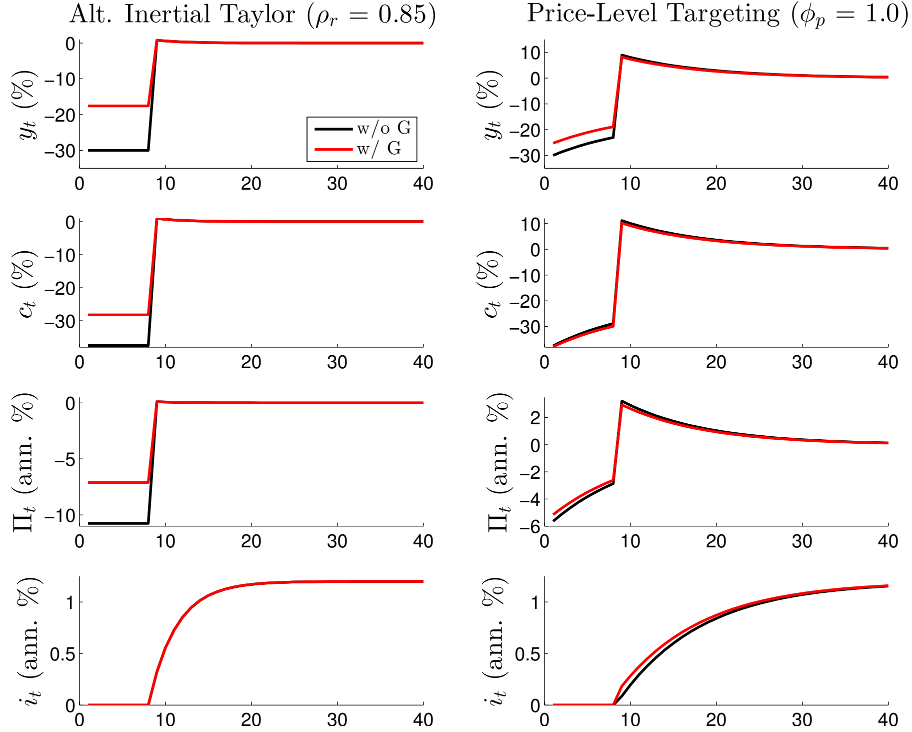


Figure 7: IRFs at the ZLB with two alternative policy rules



where p_t is the log-deviation of the price level P_t from a target level $P^* > 0$. The evolution of p_t is given by

$$p_t = p_{t-1} + \hat{\Pi}_t. \tag{35}$$

The right panel of Figure 6 shows the government spending multipliers from the econ-

omy with price level targeting. According to the the black line, the government spending multiplier declines with the weight on the price level stabilization term in the policy rule. For a sufficiently large weight on the price level, the ZLB multiplier declines below one. As shown in the right panels of Figure 7, under the price-level targeting regime, an increase in government spending during the recession speeds up the return of the policy rate to the steady-state and reduces the overshooting of output, consumption, and inflation after the shock disappears. These developments after the recession mitigate the expansionary effects of the government spending shocks during the recession via expectations. Thus, price-level targeting lowers the government spending multiplier in the same way in which our baseline inertial policy rule reduces it.

Optimal commitment policy

Investigations of two alternative policy rules above make it clear that the key element of the model lowering the government spending multiplier is the endogeneity of the post-recession evolution of policy rates on the economic performance during the recession. Such endogeneity is a hallmark of optimal commitment policy. Under optimal commitment policy, the policy rate is kept at the ZLB even after the contractionary shock disappears, and the ZLB duration is endogenous to the outcomes of the economy during the recession. In this sense, our baseline inertial Taylor rule and the price-level targeting rule are similar to optimal commitment policy. When we compute the government spending multiplier under optimal commitment policy, we find that the multiplier is 0.85 at the ZLB. Thus, one way to interpret our main result—the government spending multiplier declines with policy inertia—is that the closer the policy rule is to optimal commitment policy, the lower the multiplier. This observation is reminiscent of the observations made by Nakata (2013) and Schmidt (2013) that the optimal increase in government spending at the ZLB is smaller in the model with commitment than in the model without commitment.

6.2 Is the government spending still self-financing?

Thus far, this paper has abstracted from debt dynamics. Denes, Eggertsson, and Gilbukh (2013) and Erceg and Linde (2014) have argued that an increase in government expenditure can be self-financing at the ZLB as an expansion in output increases tax revenue. We have shown that the multiplier can be substantially smaller in the economy with policy inertia. Thus, a natural question is whether or not the government spending increase is still self-financing.

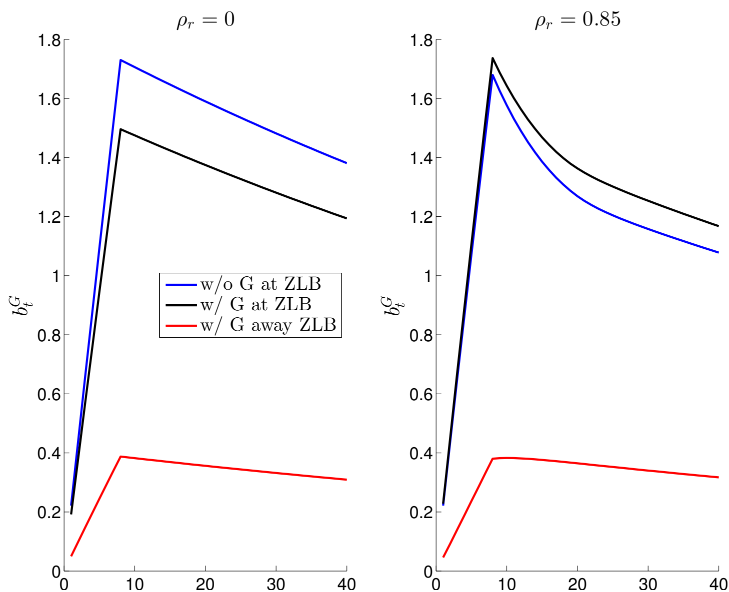
To answer this question, we follow Erceg and Linde (2014) and augment our model with a simple rule for debt-dynamics and analyze the evolution of debt in that setup. The new government budget constraint is given by

$$\gamma\hat{G}_t + b^G_{t-1} = \beta b^G_t + \gamma(\chi_n + 1)\hat{Y}_t + \gamma\chi_c\hat{C}_t + \tau_t \tag{36}$$

where $b_t^G = \frac{B_t}{P_t Y_{ss}}$ and $\tau_t = \frac{T_t}{Y_{ss}}$ are government debt and lump-sum taxes respectively as shares of nominal (real) trend in output; they are expressed as percentage point deviations from their steady state values, which are 0. Following Erceg and Linde (2014), we let $\tau_t = \phi_b b_{t-1}^G$ be the reaction function defining lump-sum tax adjustments each period and set the tax rule parameter $\phi_b = 0.01$. As is shown above, we have implicitly fixed a labor income tax τ^w in such a way so that government spending is solely financed by this labor tax in the steady state ($\gamma Y_{ss} = \frac{\tau^w}{1-\tau^w} Y_{ss}^{\chi_n+1} C_{ss}^{\chi_c} \Rightarrow \tau^w = \frac{\gamma\theta}{\theta-1}$).

Figure 8 shows how differently the debt-to-GDP ratio evolves in the economies with and without policy inertia. Under this parameterization of the fiscal policy rule, the government spending is indeed self-financing in the absence of inertia. As seen in the left panel of Figure 8, the debt ratio declines on average with respect to a small government stimulus due to the resultant higher government spending multiplier derived from an economy constrained by the ZLB. This is the same result that Erceg and Linde (2014) and Denes, Eggertsson, and Gilbukh (2013) reach in their experiments as well. However, as inertia in monetary policy increases–resulting in a decline in the government spending multiplier–the reduction in the debt share becomes small and after a certian point, as is seen in the right panel of Figure 8, the increase in the government spending increases debt. Because a higher degree of policy inertia implies a smaller fiscal multiplier, this diminishes the effect of the increased financing derived from the distortionary labor tax–which is due to an increase in output. As a result, this source of financing is unable to fully offset the upward pressure of government spending on debt. Thus we see that financing of government spending in this environment depends largely on the degree of policy inertia.

Figure 8: Evolution of the Debt-to-GDP ratio



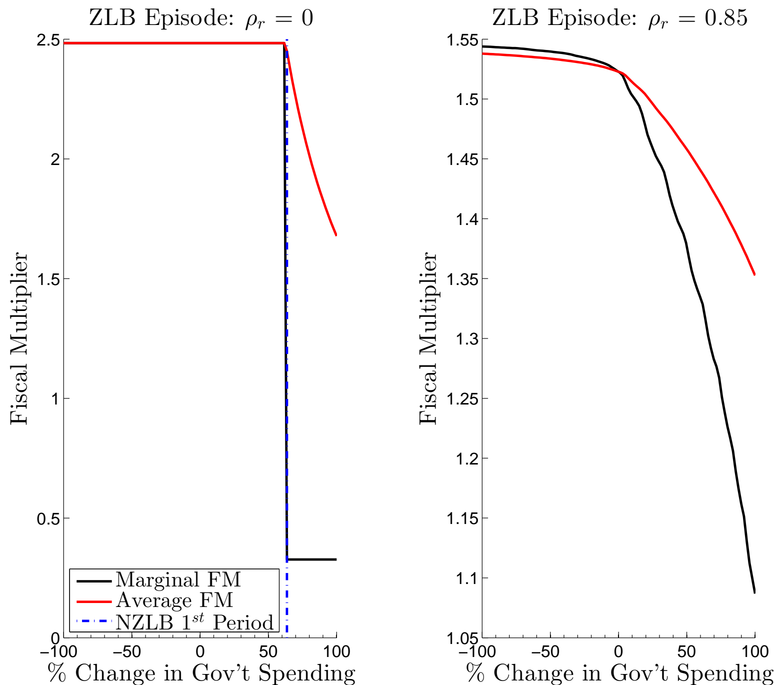
6.3 Average and Marginal Multipliers

Our baseline multipliers are what Erceg and Linde (2014) call the average multipliers; they measure the average increase in output in response to a given increase in government spending. Another interesting object would be the marginal multiplier that measures the marginal increase in output to a further ϵ-increase in government spending when government spending of size g is already in place. Formally, *the marginal government spending multiplier function*, $GM_\epsilon(i^*_{-1}, g, \hat{\delta}_H)$, is defined as follows.

$$GM_\epsilon(i^*_{-1}, g, \hat{\delta}_H) := \frac{\hat{Y}(i^*_{-1}, H; g+\epsilon, \hat{\delta}_H) - \hat{Y}(i^*_{-1}, H; g, \hat{\delta}_H)}{\gamma\epsilon} \tag{37}$$

The left panel in Figure 9 compares the average and marginal multipliers at the ZLB in the model without policy inertia, while the right panel compares those in the model with policy inertia. What is on the horizontal axis is g, the size of the government spending increase already in place when we increase government spending further by ϵ percent of G_{ss}. In this figure, ϵ is set to $\frac{0.01}{100}$. In our framework with two-state Markov shocks, the average and marginal multipliers are identical at the ZLB in the absence of policy inertia, unless the government spending shock is extremely large so that the ZLB does not bind at all with the government spending shock. However, if there is inertia in the policy rule, the average and marginal multipliers differ even in this two-state Markov-shock model.

Figure 9: Average versus Marginal Multipliers



The reason why the marginal multiplier is lower than the average multiplier in the economy with policy inertia is as follows. The effects of the government spending shock

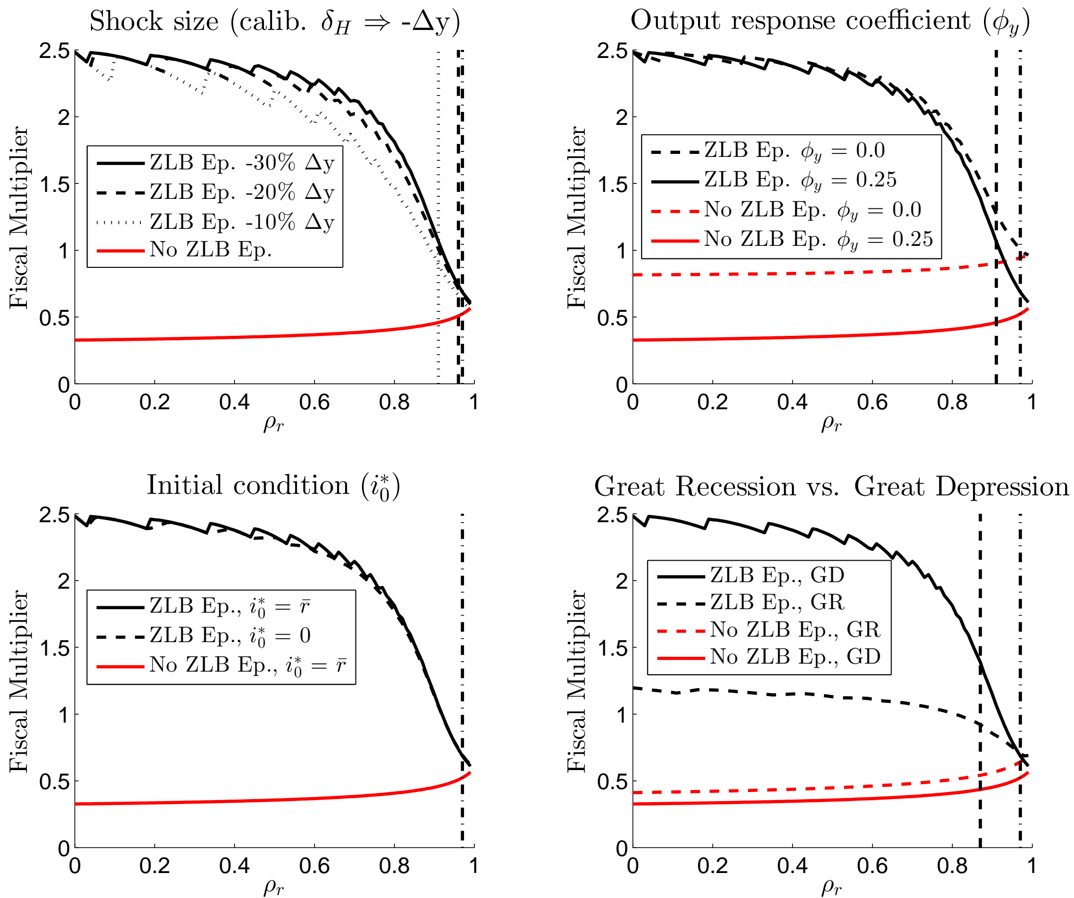
depend importantly on how long the policy rate is kept at zero after the shock disappears. The longer the policy rate is expected to remain zero, the larger the government spending multiplier is. The expected ZLB duration is shorter when there is already some government spending in place than otherwise. Thus, a marginal increase in government spending is smaller when there is already some government spending in place than otherwise. This logic will be closely related to why the multiplier is smaller when the shock size is smaller, which will be discussed in detail in Section 7.

7 Sensitivity Analyses

7.1 Shock Size

In the model without inertia, the size of the preference shock, and thus the severity of the recession, does not affect the fiscal multiplier. However, in the economy with inertia, the size of the shock does affect the fiscal multiplier. According to the top-left panel of Figure 10, the fiscal multiplier is lower when the recession is less severe.

Figure 10: Sensitivity Analyses



The fiscal multiplier is lower in the less severe recession because the duration for which the policy rate is zero after the shock disappears is shorter for any given realization of the

preference shock. With a very severe recession, the actual policy rate is kept at zero for long. Since an increase in the shadow policy rate due to a government spending shock at the end of recession gradually diminishes after the recession, the difference of shadow rates with and without government spending shocks becomes small by the time the shadow rate becomes positive. Thus, the differences in the actual policy rate path is smaller when the recession is severe and the ZLB is expected to bind for a long time. On the other hand, when the recession is not severe, the lift-off occurs soon after the shock disappears. Under this circumstance, the increase in the shadow rate at the end of the recession is still large, leading to a larger increase in the actual policy rate. A larger increase in the actual policy rate is associated with a higher path of real interest rates, making the fiscal multiplier lower when the recession is less severe.

7.2 Output response coefficient

The top-right panel of Figure 10 studies the effect of the output gap coefficient on the government spending multipliers. According to the figure, the coefficient on the output gap in the policy rule does not affect the fiscal multiplier when policy inertia parameter is zero. However, the fiscal multiplier is generally lower when the coefficient on output gap is larger.

The coefficient on the output gap does not affect the multiplier in the economy without policy inertia because it does not affect the path of actual policy rates; regardless of the coefficient, the policy rate goes back to its steady-state value once the shock disappears. In the economy with policy inertia, the output gap coefficient affects the path of actual policy rates by affecting the path of shadow rates. With a positive output gap coefficient, an increased output during the recession also contributes to the increase in the shadow rates, speeding up the return of the policy rates to the steady-state level. With tighter policy path, the effects of government spending shock is more muted with a larger response coefficent on output gap.

7.3 Initial Condition

In our baseline exercise, we compute the government spending multiplier under the scenario in which the initial lagged nominal rate is at the steady-state level, $\bar{r}$. In this environment, for a sufficiently large weight on the lagged policy rate, the ZLB does not bind at time one. An alternative assumption worth entertaining in computing the crisis multiplier is to assume that the initial lagged policy rate is zero so that the nominal interest rate is zero at time one for any degree of policy inertia. We find that the initial condition does not materially alter the government spending multiplier, as shown in the bottom-left panel of Figure 10. The reasons are that the shadow rates decline substantially during the recession and that the initial difference of $\bar{r}$ is negligible relative to the effect of government spending shocks on the path of shadow rates.

7.4 The Great Recession calibration

The size of the fiscal multiplier is of course sensitive to the choice of structural parameters. For example, in the Great Recession calibration of Denes, Eggertsson, and Gilbukh (2013) shown in Table 2, the multiplier is 1.2 as opposed to 2.5 in the Great Depression calibration in the absence of policy inertia. However, the result that the policy inertia reduces the government spending multiplier at the ZLB is robust, as seen in the bottom-right panel of Figure 10.

Table 2: Great Recession (Alternative) Parameterization

β	χ_c	χ_n	γ	θ	φ	ϕ_π	ϕ_y	μ
0.9970	0.9760	1.69	0.20	13.23	4748.7605	1.50	0.25	0.857

7.5 Distortionary Taxation

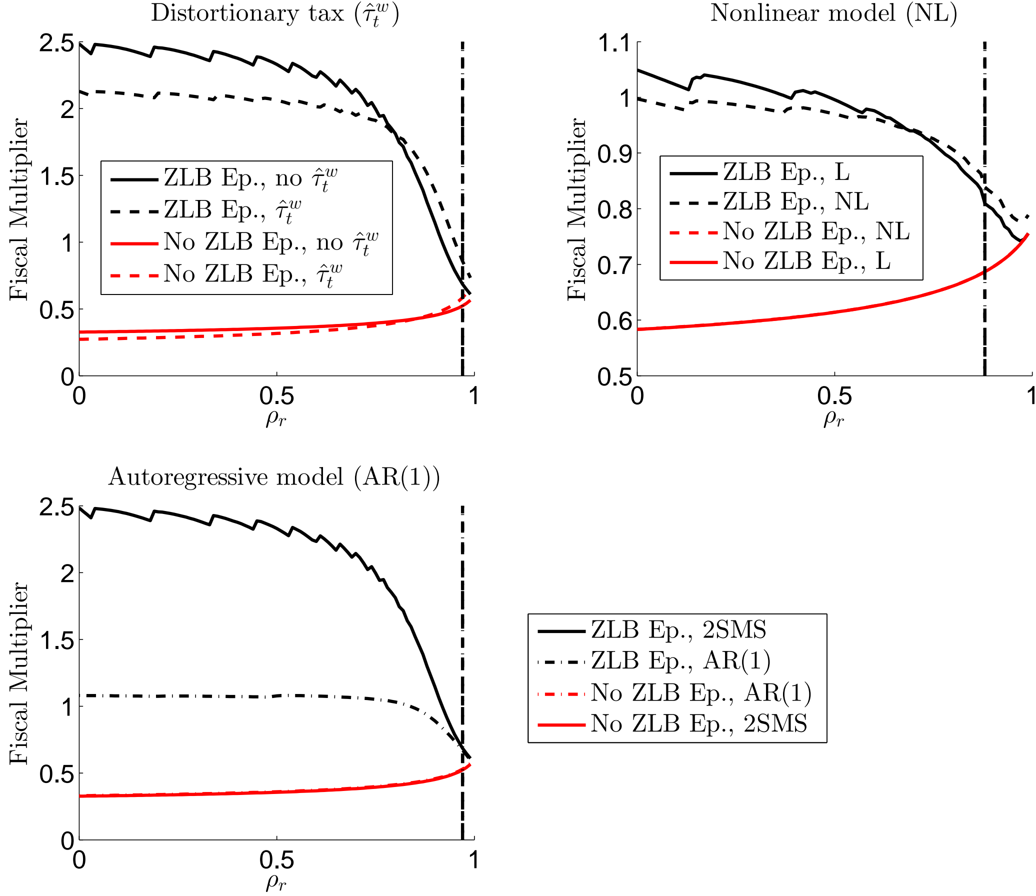
Our baseline model assumes that an increase in government spending is financed by a corresponding increase in lump-sum tax. In this section, we assess the robustness of our result to the economy in which government spending is financed by a labor income tax with no debt.

As shown in the top-left panel of figure 11, when the inertia is small, replacing the lum-sum tax with the labor income tax reduces the government spending multiplier. The reason is as follows. Since the government spending shock increases output by more than one-for-one, the labor income tax rate must decline in order to balance the budget. As pointed out by Eggertsson (2011) among others, a decline in the labor income tax reduces output when the nominal interest rate is zero, as it reduces the marginal cost for firms, exacerbates deflation, and raises the real interest rates. As a result, the government spending multiplier on output is lower in the economy with a labor income tax than in the economy with lump-sum taxes. However, in the presence of policy inertia, the government spending shock does not increase output as much and the decline in the labor income tax rate required to balance the budget is smaller. As a result, the wedge between the multipliers in the economies with lump-sum taxation and labor income taxation is smaller, and the fiscal multiplier of the economy with a labor income tax exceeds that of the economy with only lump-sum taxation with a sufficiently high degree of policy inertia.

7.6 A Fully Nonlinear Model

Some authors have shown that the commonly-used semi-loglinear approximation can be poor when the nominal interest rate is at the ZLB as the economy tends to be far away from the steady-state. In particular, Braun and Waki (2010) and Braun, Korber, and Waki (2013) have shown that the fiscal multiplier computed in the approximated economy often overstate the multiplier in the underlying fully nonlinear economy. Accordingly, we analyze how policy inertia affects the multiplier in a fully nonlinear economy. The

Figure 11: Sensitivity Analyses cont...



government spending multiplier in the fully nonlinear economy is defined similarly to those in the semi-loglinear economy. In this exercise, we mainly follow Braun, Korber, and Waki (2013) and calibrate the parameters to attain a 7 percent decline in output and 2.5 percent deflation in the low state.[7]

Table 3: Great Recession (Nonlinear) Parameterization

β	χ_c	χ_n	γ	θ	φ	ϕ_π	ϕ_y	μ
0.9970	1.00	0.2790	0.20	7.67	458.40	1.50	0.25	0.75

Top-right panel of figure 11 shows how the government spending multiplier varies with policy inertia under this calibration. Solid and dashed black lines are respectively the semi-loglinear and fully nonlinear models. As with the government spending multiplier in the semi-loglinear economy, the multiplier in the nonlinear economy declines with the policy inertia parameter. Consistent with Braun, Korber, and Waki (2013), the multiplier is modestly smaller in the nonlinear economy than in the semi-loglinear economy in the absence of policy inertia. However, the nonlinear multiplier declines by less as the policy

[7]The equilibrium does not exist under the baseline Great Depression calibration in the fully nonlinear economy.

inertia increase, and the multiplier in the nonlinear economy becomes larger than that in the semi-loglinear economy with a sufficiently high ρ_r.

7.7 AR(1) shock

Throughout the paper, we followed the majority of the literature and assumed that both the government spending and preference shocks follow two-state Markov-processes. We now modify the semi-loglinear model so that both shocks follow AR(1) processes as follows.

$$\delta_t - 1 = \rho_\delta(\delta_{t-1} - 1) \tag{38}$$

$$G_t - G_{ss} = \rho_g(G_{t-1} - G_{ss}) \tag{39}$$

where we have assumed perfect-foresight following Erceg and Linde (2014). In this exercise, we set $\rho_\delta = 0.9$ and $\rho_g = 0.9$. We initialize δ_1 so as to generate an initial 30 percent decline in output in the absence of a fiscal stimulus (i.e. $G_1 = G_{ss}$) and then consider the effects of a one percent increase in the initial government spending (i.e., $G_1 = 1.01G_{ss}$). Bottom-left panel of figure 11 demonstrate that the main result of the paper—the government spending multiplier is smaller in the economy with policy inertia—still holds under this alternative assumption on the shock process.

8 Conclusion

This paper has studied how the presence of the lagged nominal interest rate in the policy rule affects the fiscal multiplier at the ZLB. We have shown that the fiscal multiplier is non-trivially smaller in the presence of the lagged shadow rate than in the absence thereof. For the Great Depression calibration of Denes, Eggertsson, and Gilbukh (2013), the ZLB multiplier is 1.1 with an inertia parameter of 0.9, as opposed to 2.5 with an inertia parameter of zero. However, the ZLB multiplier remains above one for a plausible range of the weight on the lagged policy rate. The claim that the ZLB multiplier is larger than the non-ZLB multiplier is robust. We have also shown that the presence of the lagged actual policy rate has little or no effect on the mutliplier.

Our result shows the importance of understanding the conduct of monetary policy in understanding the effects of fiscal policy. Different rules for the nominal interest rate affect the economy differently even at the ZLB because they influence future expectations differently. While we focused on fiscal multipliers, we believe that this message is more general. For example, the specification of the nominal interest rate policy is likely to matter when one tries to understand the effects of unconventional monetary policies.[8]

[8] Nakata (2012) shows that the effect of uncertainty is smaller with a larger weight on the lagged shadow rate. Bundick (2014) shows that the effects of productivity and mark-up shocks can be even *qualitatively* different at the ZLB if the policy rate is chosen according to a rule proposed by Reifschneider and Williams (2000), which is akin to the baseline truncated inertial Taylor rule considered in this paper.

Thus, coming up with a good characterization of monetary policy in the recent ZLB episode is a high priority for future research.

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