Imitation, Innovation, and Growth*

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Abstract

This paper uses an endogenous growth model to explain how a less developed country can transition from imitating an advanced country's products to inventing new products. This process of learning to innovate occurs as technology diffuses from the more advanced country to the less developed country and increases the recipient country's ability to innovate. The main determinant of learning how to innovate is the efficiency of labor in the R&D sector of the economy. The model predicts that developing countries that have a healthier and more educated workforce can benefit the most from technology diffusion and are more likely to transition from imitating to innovating. On the other hand, countries that have workers who lack the skills necessary to learn how to innovate will likely opt to continue to imitate products from more developed countries instead of inventing new products. Both the level and growth rate of output and TFP increases if a country manages to transition from imitation to innovation. The system-GMM estimator developed by Arellano and Bover (1995) and Blundell and Bond (1998) is used to test the main conclusions of the theoretical model. The empirical results suggest that in the context of a developing country, innovation and imitation complement each other.

Keywords: Technology Diffusion; R&D; Innovation; Imitation

JEL classification: O1, O3, O4

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1. Introduction

Current research suggests that the growth rate of a country depends on how effectively it has overcome barriers to human development and to technological innovation (Howitt and Mayer-Folkes 2004). Feyrer (2008) asserts that differences in technology are what cause differences in growth rates and dynamics across countries. Romer (1990) concludes that technological change is the main determinant of economic growth. Klenow and Rodriguez Clare (1997) study the growth rates of productivity across countries and conclude that productivity differences account for 91% of the difference in growth rates of output per worker. Both authors assert that models that emphasize the accumulation of human capital as a catalyst for growth overstate its importance. They also conclude that endogenous growth models should incorporate technology diffusion and policies that affect productivity in order to research differences in growth rates across countries. The research done by Easterly and Levine (2001) further substantiate the findings of Klenow and Rodriguez Clare. They conclude that factor accumulation is not sufficient for explaining why countries have different growth rates of GDP per capita. Easterly and Levine use the Penn-World Tables to assert that the growth of total factor productivity (TFP) accounts for 60% of the growth of output per worker. They state that future research needs to focus not only on studying why productivity differences across countries exist, but also on defining TFP more precisely and modeling how technology diffuses from one country to another country.

This paper expands the theory of technology diffusion and endogenous growth by showing how imitation can be a catalyst for innovation in developing countries. The model I develop is based on the traditional Grossman and Helpman (G-H) variety expansion model. I extend their model to include a learning to innovate effect that increases the efficiency level of R&D workers in the innovative sector of the economy. My research incorporates the current notion proposed by van Elkan (1994), Connolly (1997), Connolly and Valderrama (2005), Currie et al. (1999), and Glass (2010) in the endogenous growth literature that the opportunity to copy an advanced country's products and its embedded technology actually helps workers in less developed countries acquire skills that will allow them to become more productive at innovating and creating new products. This is a new strand of thought in the diffusion of technology literature at the macroeconomic level, and while similar to learning by doing, the two concepts are not identical.

Michelle Connolly and Diego Valderrama explain the difference as follows: "learning to learn (innovate) differs from the more common notion of learning by doing in that the skills gained are applicable to different types of research as opposed to being limited to the exact task in which the learning occurs (2)." They further explain the process of learning to innovate by using the analogy of students in graduate school. When graduate students first begin graduate school, they imitate and dissect existing knowledge in their respective fields. Once students acquire knowledge that has already been disseminated, they can then invent new knowledge by conducting their own research i.e. the students have learned how to innovate. Development economists have researched technology diffusion within the newly industrializing economies (NIEs)[1] and concluded that there has been a definite transition from imitation to innovation. Nelson and Pack (1999) suggest that South Korea, Taiwan, Singapore, and Hong Kong used the knowledge acquired through absorbing knowledge from abroad to learn how to innovate. Both authors assert that technology diffusion was instrumental in increasing the productivity of labor and providing workers with the skills needed to invent new products. Hobday (1995) also proposes that a learning to innovate process exists, and its success (firms begin to innovate) depends on the resources firms allocate to the learning process. He contends that not only do firms absorb advanced knowledge from abroad in order to increase productivity, but they also use the acquired knowledge to learn how to invent new products. Lall

[1] Kim and Nelson (2000) contains articles that provide an excellent overview about how firms in NIEs have managed to transition form imitation to innovation. Kim (1997) investigates how firms in South Korea managed to learn how to innovate.

(1987) also investigates the learning to innovate[2] process in India.

At the macroeconomic level, current research is starting to address the issue that some countries (that used to be technology imitators) are now technology innovators, and new models are being developed to explain how this process occurred. The general consensus is that the catalyst for innovation was the diffusion of technology, and the learning process associated with it. The endogenous growth model of learning to innovate that I present in this paper is tractable and can be tested empirically. The empirical results suggest that in the context of a developing country, innovation and imitation complement each other. This paper is organized as follows: Section 2 presents the theoretical model, Section 3 includes an analysis of the aggregate dynamics of the economy and explains the general equilibrium results, Section 4 explains the estimation strategy and empirical results obtained from using the system-GMM estimator developed by Arellano and Bover (1995) and Blundell and Bond (1996) to test the main conclusions of the theoretical model, section 5 concludes.

2 The Model

2.1 Overview and Important Assumptions

The model presented in this paper characterizes how technology diffuses from a leader country to a follower country. A leader country is advanced and conducts cutting edge R&D. A follower country is less developed and uses R&D to imitate products from country A. It is assumed that technology can flow from one country to another without trade. An R&D firm (inventor) in country B buys a patent from an R&D firm (inventor) in country A and produces country A's product in its own country. Even though international trade is clearly an important channel for technology diffusion, it is not the

[2] Lall refers to learning to innovate as learning to industrialize. He also refers to technology diffusion as the acquisition of technological capability. He focuses on India's acquisition of technological capability in three industries.

only channel. Eaton and Kortum (1996,1999), Xu and Chiang (2005), and Guellec and van Pottelsberghe (2003) assume that technology can be transferred from one country to another without trade. They use data on patents and the stock of R&D capital to study technology diffusion. Xu and Chiang show that international patents are a channel whereby technology diffuses, and they find that middle-income and low-income countries benefit substantially from foreign patents. Guellec and van Pottelsberghe (2003) also assume that technology can be transferred from one country to another without trade.

Tacit knowledge is transferred to R&D workers in a developing country during the process of using the patent to produce a specific product. During this process, researchers have to figure out how to adapt the products for use in their country. They also learn how to correctly assemble each product and its embedded technology. As R&D workers in the less developed country apply the patents from country A's more technologically sophisticated products, they increase their analytical and critical thinking skills as well. If they increase their skill set sufficiently enough, they learn how to innovate new products.

Households in country B are analogous to the set-up in G-H (1991c). There are L identical households that supply labor and consumption loans in competitive markets. Labor is supplied inelastically by each household, and each household is endowed with one unit of labor. There is no population growth. The representative household maximizes lifetime utility

$$U = \int_0^\infty e^{-(\rho)t} log C_t dt, \; \rho > 0 \tag{1}$$

subject to the flow budget constraint

$$\dot{A} = Ar + w - E$$

ρ is the individual discount rate, A is assets holding, r is the rate of return on assets, w is the wage rate, and E is consumption expenditure[3]. Profits are distributed equally to consumers, and all variables are in per capita terms. To simplify the notation, time

[3]Detailed derivations for the theoretical model can be found in the Appendix

subscripts for endogenous variables are suppressed whenever confusion does not arise. As usual, the functional form used for utility is increasing in C and is concave i.e. $U'(C) > 0$ and $U''(C) < 0$. The utility function also satisfies the Inada conditions: $U'(C) \to \infty$ as $C \to 0$, and $U'(C) \to 0$ as $C \to \infty$. The consumption index C:

$$C = \left[\int_0^{N_B} C_i^{(\eta-1)/\eta} d_i\right]^{\eta/(\eta-1)}, \eta > 1 \tag{2}$$

is interpreted by using Ethier's (1982) definition and reflects the fact that total factor productivity (TFP)[4] rises with the number of available varieties (G-H 1991). C represents a final good that is consumed by each household. C_i denotes the input of intermediate good or service i into production of the final good, so every C_i is a differentiated intermediate good. $\eta > 1$ is the elasticity of product substitution. N_B is the number of goods (firms) existing at time t. It also represents the knowledge stock that is available at time t. This means that

$$C_i = E\left[P_{iX}^{-\eta} / \int_0^{N_B} P_{jX}^{1-\eta} d_j\right] \tag{3}$$

where C_i represents the optimal input of good i into the production of the final good C. The optimal aggregate demand of $C_i \equiv X_i = L \cdot C_i$ is

$$X_i = LE\left[P_{iX}^{-\eta} / \int_0^{N_B} P_{jX}^{1-\eta} d_j\right] \tag{4}$$

Consumers demand[5] equal amounts of each good because they love product variety, but they are indifferent as to how they obtain this variety. Consumers view imitated goods as imperfect substitutes for innovated goods. The fact that an increase in product variety

[4] $TFP = N_B^{\frac{1}{(\eta-1)}}$

[5] While some models assume that new products are introduced into the economy because of changes in consumer demand, I assume that new products are developed when entrepreneurs decide that it is profitable for them to do so. Benavente (2006) concludes that in Chile "technology push indicators are much stronger than demand pull elements" on the decision of firms to innovate. He also suggests that this should be typical for most LDCs.

also increases utility, leads to symmetric demand functions for each good. Because of this, the choice of which type of good (imitated or innovated) to produce will be made at the firm level as managers seek to maximize profit and obtain the highest return on their investments. Using the fact that $C = \frac{E}{P_X}$ the FOCs for $\frac{\partial L}{\partial C}$ and $\frac{\partial L}{\partial A}$ the optimal expenditure plan is the well known Euler equation

$$\dot{E}/E = (r - \rho) \tag{5}$$

Prices are normalized in order to make nominal spending constant. This means that $E_t = 1$, and $r_t = \rho$ for all t.

2.2 Production and Market Entry in Country B

Production and market entry in country B are similar to the G-H (1991c) and Peretto and Connolly (2007) models. Producers produce one differentiated consumption good according to the following production function:

$$X_i = L_{xi} \tag{6}$$

X_i is one unit of output, and L_{xi} is labor employment in production, and a symmetric equilibrium is assumed. An inventor in country B has to choose between either using R&D to develop a new, differentiated product and manufacturing process or to purchase a patent and imitate a differentiated product and manufacturing process from country A. Firms in country B buy these patents from inventors in country B and enter the market as the monopoly producer of the good. This is the market entry decision. The main reason why imitation is so appealing at first is the fact that with imitation, there is an increase in productivity as R&D workers, who now have access to the technology of country A, can increase their own skill set. As the workers increase their skill set, they learn how to think and invent new products and technologies themselves. The incentive to invent new products exists because imitation lowers the entry cost of innovation, and it increases the return to R&D aimed at inventing new goods. The entry technology[6] is characterized by two different equations based on whether a good is imitated or invented:

$$\dot{M} = \alpha(\bar{S}, \bar{H}) N_B L_M, \ \alpha > 0 \tag{7}$$

$$\dot{N} = \beta(M, \bar{S}, \bar{H}) N_B L_N, \ \beta(M) > 0 \tag{8}$$

$$\frac{\dot{N_B}}{N_B} = \frac{\dot{M}}{N_B} + \frac{\dot{N}}{N_B} \tag{9}$$

L_M is the aggregate amount of labor devoted to R&D aimed at applying the patents and adapting country A's products, and L_N is the aggregate amount of labor devoted to R&D aimed at inventing new products. The total number of product varieties available in country B is $N_B = N + M$. In the beginning, $\alpha > \beta$, and represents the fact that imitation is easier than innovation for the initial stock of knowledge capital. The variable β is a function of $M, \bar{S}, \bar{H}$ and evolves as follows:

$$\beta(M) = a - be^{-M} \tag{10}$$

$$\dot{\beta} = \dot{M} be^{-M} \tag{11}$$

$\beta(M)N_B$ and αN_B represent the productivity of labor in R&D. Based on equations (7) and (8), it can be noted that the knowledge spillover effect is not the same for invented and imitated goods. The reason for this is that the model has an inherent learning to innovate effect which implies that imitation not only increases the stock of public knowledge (N_B), but it also enhances the ability of R&D workers who innovate[7] (β). The functional form for β ($\beta \equiv a - be^{-M}$) not only represents how far country B is from the technology frontier of country A, but it also represents how efficient R&D workers can be at innovating if they are exposed to the new technology that is embedded in imitated goods. Unlike the

[6] N $\equiv$invented technology that is new to the world and M $\equiv$imitated technology that is only new to country B.

[7] for ease of exposition, the (M) is dropped from $\beta(M)$.

increase in productivity that results from an increase in public knowledge, β increases as country B approaches the technology frontier of country A. The increase in β occurs because there is a second spillover effect of the knowledge capital that is private and only helps to increase the ability of R&D workers to innovate. While the dissemination of ideas and methods makes future imitation and innovation easier, the actual increase in skills that occurs because of the reverse engineering process is only beneficial to R&D workers in the innovation sector of the economy because it increases each worker's tacit knowledge of how to innovate. In essence, labor productivity is composed of two parts: technology and efficiency (tacit knowledge).

Technology[8] in my model is represented by N_B and encompasses the knowledge about how to produce output, while β and α are variables that represent the efficiency of researchers. The distinction between technology and efficiency is clarified by David Weil. Weil defines technology[9] as "the knowledge about how factors of production can be combined to produce output," and efficiency as measuring "how effectively given technology and factors of production are actually used" (276). Both variables depend on the mean education level of the workforce ($\bar{S}$) and the life expectancy at birth of the workforce ($\bar{H}$). β can differ across countries and increases as the number of imitated varieties increases. The value of b reflects how much imitating country A's products can help increase the ability of workers to innovate, while the value of a reflects the rate at which researchers in country B benefit from imitation and learning to learn and how far away country B is from the technology frontier of country A. It is assumed that $\bar{S}$ and $\overline{H}$ are constant.

This means that (10) illustrates how much buying a patent, imitating it, and adapting it to the domestic market helps workers learn how to innovate given the initial characteristics of the workforce and the initial level of technology that is available in the economy.

[8] It is assumed that when the economy starts out, $M(0) \geq 1$ and $N(0) = 0$. In keeping with Barro and Sala-i-Martin (2004), it is just accepted, but not explicitly explained how country B learned how to imitate at least one good.

[9] For the purpose of this paper, technology is embedded in capital goods and is represented by each new product variety.

The fact that the ability of R&D workers (to innovate) increases as the number of imitated goods increases, is the key to understanding how the model works. This increase in worker ability increases the productivity of innovation based R&D, lowers the cost of innovation, and can make it possible for a country to transition from imitation to innovation.[10] In my model, the transition from imitation to innovation occurs because of a learning process that increases productivity and lowers the cost of innovation. As innovation becomes cheaper, firms have an incentive to innovate.

3 Aggregate Dynamics

3.1 The Investment Decision

The number of imitated and innovated goods that are available is decided by inventors and firms as managers choose which type of patents they want to invest in. Firms know that once they purchase a patent and enter a market, they become the monopoly provider of that product. Each firm wants to maximize profit and will devote resources to purchasing patents for imitated goods or invented goods based on the rate of returns to both types of R&D. R&D firms follow the same logic and will choose to imitate country A's goods or innovate based on the returns to R&D. My model permits arbitrage in the assets market with respect to the returns to R&D. This is what determines whether R&D firms invest in imitation or innovation. The rate of return to imitation and innovation[11] respectively is

$$r_M = \frac{\alpha LE}{\eta w} + \frac{w}{w} - \frac{\dot{N}_B}{N_B} = \rho \quad (12)$$

$$r_N = \frac{\beta LE}{\eta w} + \frac{w}{w} - \frac{\dot{N}_B}{N_B} - \frac{\dot{\beta}}{\beta} = \rho \quad (13)$$

Where (12) represents the rate of return to R&D directed at imitating country A's goods,

[10] Because of this, there are three possible steady state results. A country can either become a perpetual innovator. It can become a perpetual imitator, or it can converge to a steady state where it does a combination of both innovation and imitation. The conditions that lead to each result are discussed in more detail in Section 3

[11] Detailed calculations for the aggregate dynamics and general equilibrium results are presented in the appendix.

and (13) represents the rate of return to R&D directed at innovating new goods. Increases in productivity cause the rate of return to R&D to increase because it lowers entry costs. The returns to both types of R&D are decreasing with the growth rate of the number of varieties that are invented and imitated. As entry increases, the pool of knowledge available to all the R&D firms in the market increases; therefore decreasing the returns to both forms of knowledge. This decrease in profits, which occurs as more firms enter the market and the average firm size decreases, is exactly offset by the decrease in the cost of variety expanding innovation due to the knowledge spill-over effect. The no arbitrage condition requires that the rate of return on a riskless asset (r) be equal to the rate of return to R&D. This means that if innovation is more profitable than imitation, then $r = r_N$. If imitation is more profitable than innovation, then $r = r_M$.

3.2. Value of the Firm and Free Entry

R&D in this economy is done by private inventors who sell their patents to firms. After purchasing the patent, a firm becomes the monopoly provider of that product. An inventor will either buy a patent from an inventor in country A, or invent a new product altogether. The purchase/sell price of a patent using imitated knowledge is $P_{Mi} = \frac{w}{\alpha N_B}$, and $P_{Ni} = \frac{w}{\beta N_B}$ for a patent that uses invented knowledge. Therefore, the stock market value of the firm that purchases the patent at time t is equal to the net present value of profit at time t and is given by:

$$v_i(t) = \int_t^{\infty} \exp^{\int_t^{\tau} r(s)ds} \pi_i(\tau) d\tau \tag{14}$$

$$v_{Mi} = \frac{w}{\alpha N_B} \tag{15}$$

$$v_{Ni} = \frac{w}{\beta N_B} \tag{16}$$

where equation (15) equates the value of the firm that purchases a patent for an imitated product to its cost, and equation (16) equates the value of the firm that purchases a patent for an innovated product to its cost[12] In the beginning, $v_{Ni} > v_{Mi}$, but as M increases, β and N_B increase and v_{Ni} and v_{Mi} decrease. The changing value of β will determine whether an inventor decides to invest in R&D aimed at innovating, imitating, or both.

3.3. General Equilibrium Results

As long as the efficiency of workers at innovating (β) evolves over time and is always less than the efficiency of workers at imitating (α), the rate of return to R&D that is aimed at imitating goods is higher than the rate of return that is aimed at inventing new goods so firms choose to enter the market with imitated products only in the steady state. If $\alpha = \beta$ asymptotically, $\frac{\dot{\beta}}{\beta} = 0$, and the rate of return to both types of R&D are equal so inventors will invest in both types of R&D in the steady state. Firms choose to enter the market with invented products once

$$\alpha + \frac{\delta \eta w}{LE} = \beta \tag{17}$$

δ is the value of $\frac{\dot{\beta}}{\beta}$ that occurs right before inventors decide to shut down investment in imitation. This means that $\beta > \alpha$ for innovation to take place in the steady state. It is apparent that there are three possible growth regimes: firms enter the market with imitated products only in the steady state; firms enter the market with innovated products only in the steady state; firms enter the market with both innovated and imitated products in the steady state. The conditions that lead to each growth regime are given in table 1.

Table 1 Conditions for Each Regime

[12] This means that only interior solutions for the growth rate of the number of varieties are considered. The possibility for there to be no R&D or unbounded R&D in the economy are ruled out. Which type of R&D firms decide to invest in is explained in section 1.4.1 and the appendix.

	Regime 1	Regime 2	Regime 3
Condition	$\alpha > \beta$	$\alpha + \frac{\delta \eta w}{LE} = \beta$	$\alpha = \beta$
Rate of Return	$r_M > r_N$	$r_M = r_N$	$r_M = r_N$
Imitation	turned on; $\frac{\dot{M}}{M} > 0$	turned off; $\frac{\dot{M}}{M} = 0$	turned on; $\frac{\dot{M}}{M} > 0$
Innovation	turned off; $\frac{\dot{N}}{N} = 0$	turned on; $\frac{\dot{N}}{N} > 0$	turned on; $\frac{\dot{N}}{N} > 0$

General Equilibrium is defined by

$$L = L_X + L_M + L_N = \frac{LE}{P_{iX}} + \frac{\dot{M}}{\alpha N_B} + \frac{\dot{N}}{\beta N_B} \tag{18}$$

When $\alpha > \beta$, firms don't enter the market with innovated products because it's not profitable. A developing country can end up in regime 1 if the learning to innovate effect does not increase the productivity of R&D workers sufficiently enough to make innovation feasible. This occurs if a country is either too far away from the technology frontier of country A, or if the country's initial quality of its workforce hinders the learning to innovate effect. [13]When this situation occurs, The economy jumps to its stable equilibrium growth rate

$$g_M = \frac{\alpha L}{\eta} - \frac{(\eta-1)\rho}{\eta} \tag{19}$$

The number of imitated varieties grows at this constant rate along the BGP. Continued imitation is possible because the cost of market entry decreases with the number of firms (the available knowledge stock), offsetting the decrease in profits that occurs as the number of firms increases. A country's steady state growth rate depends on the productivity of its R&D workers, the subjective discount rate, the size of its labor force, and the elasticity of product substitution. Having a more productive workforce increases the growth rate of imitated goods while having a less productive workforce decreases the growth rate. A higher subjective discount rate leads to a lower growth rate of imitated goods because consumers value present consumption more than future consumption. A

[13]This would be the equivalent to having a high b and a low a.

lower subjective discount rate leads to a higher growth rate as consumers are more willing to wait to consume goods. A lower subjective discount rate means that consumers save more, the cost of capital is lower, and more entrepreneurs are willing to conduct R&D, and more firms are willing to enter the market with imitated products. A larger labor force increases the growth rate since more resources can be devoted to manufacturing intermediate goods and conducting research. A smaller labor force decreases the growth rate. A higher value of the elasticity of product substitution leads to a lower growth rate as consumers value product variety less, so the market power of market entrants is smaller; thereby, leading to less entry by entrepreneurs. A lower value of the elasticity of product substitution has the opposite effect on the growth rate. The growth rate of TFP is

$$g_{TFP} = \frac{1}{(\eta-1)}\frac{\dot{N_B}}{N_B} \tag{20}$$

When $\alpha + \frac{\delta\eta w}{LE} = \beta$, firms enter the market with innovated products only in the steady state because profits are the highest. The transition from imitation to innovation occurs if the initial quality of a country's workforce is high enough so that the ability of R&D workers to innovate is sufficiently augmented by the learning to innovate effect. [14] When countries specialize in innovation only, the number of varieties grows at the constant rate g_N.

$$g_N = \frac{\beta L}{\eta} - \frac{(\eta-1)\rho}{\eta} \tag{21}$$

Since $\beta > \alpha$ in regime 2, the growth rate of income and the growth rate of TFP will be higher in regime 2 than in regime 1. When $\alpha = \beta$ asymptotically, firms enter the market with both imitated and innovated goods.. The growth rates of imitated and innovated goods adjust until the growth rate of total product varieties is constant at

[14]This would be the equivalent to having a high a and a low/medium b.

$$g_{N_B} = \frac{\alpha L}{\eta} - \frac{(\eta-1)\rho}{\eta} \quad (22)$$

Since $\beta = \alpha$ in regime 3, this means that the growth rate of income and the growth rate of TFP will be the same as in regime 1.

4. Empirical Analysis and Results

4.1. Data

The main conclusion that can be drawn from the theoretical model is the fact that innovation and imitation are either complements or substitutes. Because innovation and imitation occur through the acquisition of patents, it is straightforward to test the implications of the model using royalty payments as a proxy for imitation and patent applications as well as the number of scientific and technical articles published within a year as proxies for innovation. From 1984-1990[15], the Mexican government collected detailed plant-level data on royalty payments. The source of the plant-level data are confidential annual surveys that were carried out by the National Institute of Statistics and Geography (INEGI) in Mexico. The data from these surveys was made available by Mexico's Secretariat of Commerce and Industrial Development (SECOFI). The data covers the years 1984-1990 and provides information on almost 100 variables such as revenues, costs, employment, wages, exports, imports, ownership, etc. The data was collected at the plant-level, and each establishment was given a unique number, so that it can be identified and tracked over time. The government also assigns each plant to a manufacturing industry that is very similar to the 4-digit International Standard Industrial Classification (ISIC). According to INEGI, plants were instructed to include the dollar amount that they spent on royalty payments during the year. Royalty payments were defined as payments made to a third party for the legal right to use patents, licenses, copyrights, or trademarks. Plants are advised that they should exclude any investments or payments they made toward their own in-house R&D.

[15] 1984 was dropped because there wasn't any data available for scientific publications

The panel is unbalanced and was originally used by Tybout and Westbrook (1995). The data excludes small plants and maquiladoras. The maquiladora plants were excluded because they provide a subcontracting service and do not provide information regarding the value of gross output or intermediate inputs. Plants that did not report the value of sales, did not make royalty payments or did not provide information concerning employees were eliminated; leaving a sample of 586 plants. All monetary variables were converted into millions of 1980 Mexican pesos using the appropriate price[16] deflators. All of the data with respect to the 4-digit industry input tariffs[17] and input license coverage was calculated and provided by Adrian Ten Kate of Mexico's Secretariat of Commerce and Industrial Development (SECOFI) and Tybout and Westbrook. The tariff rates and license coverage rates are aggregate rates for each industry instead of plant specific rates. The patent data includes the total number of patent applications that were filled within a given year for Mexico. This data was collected from the World Development Indicators. The scientifc articles data was also collected from the WDI and includes the number of scientific articles that were published by Mexican researchers within a specific year.

4.2. Estimation Strategy and Results

Because of endogeneity issues and the fact that I use an unbalanced panel, the system-GMM estimator developed by Arellano and Bover (1995) and Blundell and Bond (1998) is used to test if imitation and innovation are complements or substitutes. The following model is estimated:

$$LRPayments_{it} = \alpha LRPayments_{i,t-1} + x_{it}'\beta + \epsilon_{it} \tag{24}$$

$$\epsilon_{it} = \mu_i + \nu_{it} \tag{25}$$

Where $LRPayments_{it}$ is the log amount a specific plant i paid in royalty payments in year t normalized by the total number of employees. The explanatory variables that are

[16]For more on this, please see Tybout and Westbrook.
[17]The overview of the data source is taken from Kandilov and Leblebicioğlu.

used in the baseline model are NWBen which is the amount that was spent on non-wage benefits; SalesC is the amount that was spent on sales commissions; AdvC is the amount that was spent on advertising. All three variables are normalized[18] by the total number of employees. LNPatents is the log amount of: $TotalPatents * \frac{TNE}{TEM}$ where totalpatents is equal to the total number of patent applications that were filed within a given year in Mexico; TNE is equal to the total number of employees that worked within a given plant during the year, and TEM is equal to the total number of workers that were employed by all the plants in the sample in a given year. LNScientificPubs is the log amount of: $ScientificPub * \frac{TNE}{TEM}$ where scientificpub is equal to the total number of scientifc and technical articles that were published by Mexican researchers within a given year; TNE is equal to the total number of employees that worked within a given plant during the year, and TEM is equal to the total number of workers that were employed by all the plants in the sample in a given year. SalWC is a quality measure of the workforce that is calculated by dividing total spending on white collar workers by total spending on both white collar and blue collar workers. The baseline model is extended to take into account the major trade liberalization period that the Mexican economy experienced from 1984-1990 in order to see if the results are robust to trade policy. The terms InputT and InputLC represent the input tariff and license measures and are expressed as a percent. The lags of the regressors from t-2 and t-3 for the dependent variable are used as instruments. The tariff and license coverage measures and the advertising costs are assumed to be exogenous. Both the Hansen J test, the Sargan test, and the difference in Hansen test of exogeneity support the fact that the chosen instruments are valid. μ_i is the plant-level effect, ν_{it} are the idiosyncratic shocks and ϵ_{it} is the disturbance term i.e. idiosyncratic error term. Table 1 reports the summary statistics for all the variables.

Table 2 reports the estimation results. There is a positive and significant coefficient

[18]For ease of exposition, the per employees is left out. This means that it is taken as given that whenever it's written royalty payments, non-wage benefits, etc. it means royalty payments per employee, non-wage benefits per employee, etc.

on LNPatents which suggests that innovation and imitation are complements. This result is robust to the trade liberalization indicators. Column 1 presents the results from the baseline model. The point estimate indicates that a 10 percent increase in domestic patent applications leads to a 3.45 percent increase in royalty payments. The result remains positive and significant even when import tariffs and license coverage are added. The point estimator indicates that a 10 percent increase in domestic patent applications causes a 3.49 percent increase in royalty payments. The results are also robust to using scientific publications as a proxy for innovation. Column 2 reports the results from the baseline model. The results show that a 10 percent increase in scientific and technical articles published within a given year increases royalty payments by 3.65 percent. When import tariffs and license coverage are added, the point estimate shows that a 10 percent increase in publications leads to a 3.74 percent increase in royalty payments. In the case of Mexico, innovation and innovation complement each other as predicted by the theoretical model.

5. Concluding Remarks

This paper presents a variety expansion model that has an endogenous process for learning to innovate that is tractable and can be tested empirically. With the learning to innovate process, there are three possible steady state results. A country can either become a perpetual innovator. It can become a perpetual imitator, or it can converge to a steady state where it does a combination of both innovation and imitation. Productivity differences between researchers in different countries is the main determinant of whether a country transitions from imitating and reverse engineering products to inventing products that are new to the world. The results show how important technology diffusion is to developing countries that want to learn how to innovate their own products. By assuming that the productivity of labor depends on both the level of technology that is available within a country and how efficient workers are, it becomes apparent that labor efficiency and tacit knowledge are very important for technology diffusion, product imitation, and possible subsequent innovation by firms.

Labor efficiency is what determines whether a developing country can transition from reverse engineering and imitating a more developed country's products to inventing its own products. My research predicts that developing countries that have a healthier and more educated workforce can benefit the most from reverse engineering and learning to learn and are more likely to transition from imitating to innovating. On the other hand, countries that have workers, who lack the skills necessary to learn how to innovate from reverse engineering more technologically advanced products from abroad, will likely opt to continue to imitate products instead of inventing new products. If a developing country manages to use the knowledge it accumulates through technology diffusion to learn how to invent its own products, it will be rewarded for its efforts by having a higher level (growth rate) of both output and TFP. The theoretical implications of the model are tested, and the results suggest that imitation and innovation are complements.

Table 1: Summary Statistics

VARIABLE	N	Mean	SD	Min	Max
InputT	3,022	16.90	7.790	0.700	53.50
InputLC	3,022	24.93	31.76	0.200	99.50
SalesC	3,022	0.900	4.342	0	99.72
AdvC	3,022	1.187	8.117	0	397.2
NWBen	3,022	1.902	2.782	0	60.45
SalWC	3,022	0.548	0.195	0	1
LNPatents	3,022	0.778	1.088	-3.355	5.000
Number of plants	586	586	586	586	586

Table 2: Results from the Two-Step System-GMM Estimation

Dependent Variable LRPayments	(1)	(2)	(3)	(4)
L.LRPayments	0.782***	0.776***	0.778***	0.772***
	(0.094)	(0.093)	(0.094)	(0.092)
NWBen	-0.008	-0.009	-0.006	-0.008
	(0.019)	(0.018)	(0.019)	(0.019)
SalesC	0.007	0.007	0.006	0.006
	(0.007)	(0.007)	(0.007)	(0.007)
AdvC	0.015**	0.016**	0.011*	0.011*
	(0.007)	(0.006)	(0.006)	(0.006)
SalWC	0.049	0.037	0.111	0.108
	(0.635)	(0.619)	(0.655)	(0.643)
LPatentsIndex	0.345***		0.349***	
	(0.122)		(0.121)	
InputT			-0.013*	-0.013**
			(0.007)	(0.006)
InputLC			0.006***	0.006***
			(0.002)	(0.002)
LScientificIndex		0.365***		0.374***
		(0.125)		(0.125)
Plant FE	YES	YES	YES	YES
Year Dummies	YES	YES	YES	YES
1st Order Serial Correlation (p-value)	0.000	0.000	0.000	0.000
2nd Order Serial Correlation (p-value)	.425	.432	.406	.414
Number of Instruments	71	71	73	73
F-Statistic (p-value)	0.000	0.000	0.000	0.000
Observations	1,963	1,963	1,963	1,963
Hansen J Statistic (p-value)	.925	.931	.895	.904

Windmeijer's (2005) corrected standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.10

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Appendix: Overview of the Model

A.1 Calculating the Optimal Aggregate Demand of Intermediate Goods

There are L identical households that supply labor and consumption loans in competitive markets. Labor is supplied inelastically by each household, and each household is endowed with one unit of labor. There is no population growth. The representative household maximizes lifetime utility

$U = \int_0^\infty e^{-(\rho)t} log C_t dt$, $\rho > 0$

subject to the flow budget constraint

$$\dot{A} = Ar + w - E$$

ρ is the individual discount rate, A is assets holding, r is the rate of return on assets, w is the wage rate, and E is consumption expenditure. Profits are distributed equally to consumers, and all variables are in per capita terms. To simplify the notation, time subscripts for endogenous variables are suppressed whenever confusion does not arise. As usual, the functional form used for utility is increasing in C and is concave. The consumption index C:

$$C = \left[\int_0^{N_B} C_i^{(\eta-1)/\eta} d_i\right]^{\eta/(\eta-1)}, \ \eta > 1$$

is interpreted by using Ethier's (1982) definition and reflects the fact that total factor productivity (TFP)[19] rises with the number of available varieties (G-H 1991). C represents a final good that is consumed by each household. C_i denotes the input of intermediate good or service i into production of the final good, so every C_i is a differentiated intermediate good. $\eta > 1$ is the elasticity of product substitution. N_B is the number of goods (firms) existing at time t. It also represents the knowledge stock that is available at time t. The optimal input of good i into the production of the final good C is

$$C_i = E\left[P_{iX}^{-\eta} / \int_0^{N_B} P_{jX}^{1-\eta} d_j\right]$$

Using the fact that $C_i \equiv X_i = L \cdot C_i$, at each time t, individuals maximize

$$C = \left[\int_0^{N_B} C_i^{(\eta-1)/\eta} d_i\right]^{\eta/(\eta-1)}$$

$$\text{s.t. } E = \int_0^{N_B} P_i C_i d_i$$

The Lagrangian for the consumer's maximization problem is

$$L = \left[\int_0^{N_B} C_i^{(\eta-1)/\eta} d_i\right]^{\eta/(\eta-1)} + \lambda\left[E - \int_0^{N_B} P_i C_i d_i\right]$$

Setting $\frac{\partial L=}{\partial C_i} = 0$ yields $\left[\int_0^{N_B} C_i^{(\eta-1)/\eta} d_i\right]^{1/(\eta-1)} C_i^{-\frac{1}{\eta}} = \lambda P_i$

Solving for C_i yields

$$C_i = \lambda^{-\eta} P_i^{1-\eta}\left[\int_0^{N_B} C_i^{(\eta-1)/\eta} d_i\right]^{\eta/(\eta-1)} \quad \text{(A.1)}$$

The relationship between C and E is that

[19] $TFP = N_B^{\frac{1}{(\eta-1)}}$

$E = \int_0^N P_i C_i d_i$

Using this relationship and substituting for C_i using (A.1) yields

$$\lambda^{-\eta} = \frac{E}{\int_0^{N_B} P_i^{1-\eta} \left[\int_0^{N_B} C_i^{(\eta-1)/\eta} d_i \right]^{\eta/(\eta-1)}} \quad (A.2)$$

Substituting for λ in equation (A.1) using (A.2) yields

$$C_i = E\left[P_i^{-\eta} / \int_0^{N_B} P_j^{1-\eta} d_j \right]$$

where C_i represents the optimal input of good i into the production of the final good C. The optimal aggregate demand of $C_i \equiv X_i = L \cdot C_i$ is

$$X_i = LE\left[P_i^{-\eta} / \int_0^{N_B} P_j^{1-\eta} d_j \right] \quad (A.3)$$

A.2 Calculating the Cost of Entry

The amount of labor required to produce one unit of $\dot{M}$ is $\frac{L_M}{\dot{M}} = \frac{1}{\alpha N_B}$. This means that the cost of one unit of $\dot{M}$ is $\frac{w}{\alpha N_B}$. The standard free-entry condition for the sector of the economy that conducts R&D that is aimed at imitating goods is $P_{Mi} = \frac{w}{\alpha N_B}$. In order for firms to be willing to conduct R&D that is aimed at imitating goods, then it must also hold that $v_{Mi} = \frac{w}{\alpha N_B}$. This means that the cost of a patent for an imitated good equals its value and its price. The amount of labor required to produce one unit of $\dot{N}$ is $\frac{L_N}{\dot{N}} = \frac{1}{\beta N_B}$. This means that the cost of one unit of $\dot{N}$ is $\frac{w}{\beta N_B}$. The standard free-entry condition for the sector of the economy that conducts R&D that is aimed at imitating goods is $P_{Ni} = \frac{w}{\beta N_B}$. In order for firms to be willing to conduct R&D that is aimed at imitating goods, then it must also hold that $v_{Ni} = \frac{w}{\beta N_B}$. This means that the cost of a patent for an imitated good equals its value and its price.

A.3 Calculating the Optimal Price that Maximizes Profit

Per firm profit equals $\pi_i = P_{iX}X_i - wX_i$. Maximizing profit w.r.t. price yields

$$\frac{LE(1-\eta)P_{iX}^{-\eta}}{\int_0^N P_j^{1-\eta}d_j} = \frac{\eta LEP_{iX}^{-\eta-1}w}{\int_0^N P_j^{1-\eta}d_j}$$

Rearranging terms and solving for price, yields that the associated pricing strategy is the mark-up rule $P_{iX} = \frac{w\eta}{(\eta-1)}$.

A.4 The Rate of Return to Imitation and Innovation and the Investment Decision

The standard asset pricing equation for the rate of return to R&D is $r = \frac{\pi_i}{v_i} + \frac{\dot{v}_i}{v_i}$. Using the fact that $\pi_i = P_{iX}X_i - wX_i$ and $P_{iX} = \frac{w\eta}{(\eta-1)}$, profit reduces to $\pi_i = \frac{LE}{\eta N_B}$. Substituting for $v_{Mi} = \frac{w}{\alpha N_B}$ and $v_{Ni} = \frac{w}{\beta N_B}$the rate of return to imitation and innovation respectively is

$$r_M = \frac{\alpha LE}{\eta w} + \frac{w}{w} - \frac{\dot{N}_B}{N_B} = \rho \tag{A.4}$$

$$r_N = \frac{\beta LE}{\eta w} + \frac{w}{w} - \frac{\dot{N}_B}{N_B} - \frac{\dot{\beta}}{\beta} = \rho \tag{A.5}$$

As can be seen from (A.4) and (A.5), as long as $\beta < \alpha$,the rate of return to R&D that is aimed at imitating goods is higher than the rate of return that is aimed at inventing new goods. This means that entrepreneurs choose to enter the market as imitators in the steady state. If $\alpha = \beta$ asymptotically, $\frac{\dot{\beta}}{\beta} = 0$, so the rate of return to both types of R&D are equal. This means that firms will invest in both types of R&D in the steady state. Firms choose to enter the market as inventors once $\beta = \alpha + \frac{\delta \eta w}{LE}$. δ is the value of $\frac{\dot{\beta}}{\beta}$ that occurs right before firms decide to shut down investment in imitation. This means that $\beta > \alpha$ for innovation to take place in the steady state.

A.5 Steady State Growth Rates

Solving for the steady state for each regime follows the method of G-H. In regime 1, $\dot{N} = 0$, so $N_B = M$ and $\frac{\dot{N_B}}{N_B} = \frac{\dot{M}}{N_B} = \frac{\dot{M}}{M}$. Using the resource constraint $L = L_X + L_M + L_N = \frac{LE}{P_{iX}} + \frac{\dot{M}}{\alpha N_B} + \frac{\dot{N}}{\beta N_B}$ to solve for $\frac{\dot{M}}{N_B}$ yields

$$\frac{\dot{N_B}}{N_B} = \alpha L - \frac{\alpha LE}{P_{iX}} \tag{A.6}$$

Using the fact that $P_{iX} = \frac{w\eta}{(\eta-1)}$, and $v_{Mi} = \frac{w}{\alpha N_B}$ and plugging both into (A.6) yields

$$\frac{\dot{N_B}}{N_B} = \alpha L - \frac{LE(\eta-1)}{\eta v N_B}$$

Following the method of G-H, in order to simplify the analysis, I define the rate at which new product varieties are being introduced into the economy as $\frac{\dot{N_B}}{N_B} \equiv g_M$, and I define the inverse of the economy's aggregate equity value as $V \equiv \frac{1}{vN_B}$. I then rewrite (A.6) as

$$g_M = \alpha L - \frac{LE(\eta-1)V}{\eta} \tag{A.7}$$

Using the definitions of V and g_M, it is apparent that $\frac{\dot{V}}{V} = -\frac{\dot{v}}{v} - g_M$ and $\frac{\dot{v}}{v} = -g_M - \frac{\dot{V}}{V}$. Using the fact that $r = \rho$, I calculate

$$\frac{\dot{v}}{v} = \rho - \frac{LE}{\eta v N_B}$$

This means that

$$-\frac{\dot{V}}{V} - g_M = \rho - \frac{LE}{\eta v N_B}, \text{ and } \frac{\dot{V}}{V} = \frac{LE}{\eta v N_B} - \rho - g_M$$

Substituting for V, this just becomes $\frac{\dot{V}}{V} = \frac{LEV}{\eta} - \rho - g_M$. In the steady state $\frac{\dot{V}}{V} = 0$, so that $\frac{LEV}{\eta} - \rho = g_M$, and $V = \frac{\eta g_M + \eta\rho}{LE}$. I then plug $V = \frac{\eta g_M + \eta\rho}{LE}$ into (A.7). This yields

$$g_M = \alpha L - \frac{LE(\eta-1)(\eta g_M + \eta\rho)}{\eta LE} = \alpha L - (\eta-1)g_M - (\eta-1)\rho$$

A bit of algebra yields

$$g_M = \frac{\alpha L}{\eta} - \frac{(\eta - 1)\rho}{\eta} \tag{A.8}$$

Which is the growth rate for regime 1 and regime 3. Solving for the growth rate of regime 2 follows the same logic and yields

$$g_N = \frac{\beta L}{\eta} - \frac{(\eta - 1)\rho}{\eta} \tag{A.9}$$