A Central Bank's Escape from an Indeterminacy Trap

Nelson Lind

University of California, San Diego

November 6, 2014

Motivation

- Equilibrium uniqueness often assumed when estimating DSGE models.
  - Simplifies the statistical model.
  - Often good reason to select a specific equilibrium.
- But, suppose an analyst wants to answer:

  What policies ensure a unique equilibrium?

  - Often a primary policy objective
  - E.g. the Taylor principle
- What are the implications of the usual uniqueness assumption on ultimate policy recommendations?

Implications of Assuming a Unique Equilibrium?

- The determinacy assumption acts to restrict the parameter space.
- The analyst effectively ignores a source of identification failure.
  - Occurs when some shocks may be excluded.
  - Which shocks are truly structural?
- Can lead to an *indeterminacy trap*:
  1. The assumption of determinacy can implicitly exclude the true parameter value.
  2. Leading to false conclusions about prospective policies.
  3. Believe that current policies support a unique equilibrium when they actually lead to indeterminacy.

When Can This Happen?

- Suppose the analyst is using the New-Keynesian model:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa(y_t - y_t^n) \qquad \text{(Aggregate Supply)}$$

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\tau}(i_t - \mathbb{E}_t \pi_{t+1}) \qquad \text{(IS Curve)}$$

$$i_t = \phi_\pi \pi_t + \phi_y (y_t - y_t^n) \qquad \text{(Monetary Policy)}$$

  - Policy parameters: $\theta^P = (\phi_\pi, \phi_y)$.
  - Structural parameters: $\theta^S = (\beta, \kappa, \tau)$.
- Recommendation for θ^P doesn't require inference on θ^S.
  - Taylor principle ($\phi_\pi > 1$).
- But, typical quantitative models include additional frictions.

Adding Frictions

- Cost channel due to financial frictions (Ravenna and Walsh (2006)):

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa(y_t - y_t^n + \delta i_t) \quad \text{(AS)}$$

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\tau}(i_t - \mathbb{E}_t \pi_{t+1}) \quad \text{(IS)}$$

$$i_t = \phi_\pi \pi_t + \phi_y (y_t - y_t^n) \quad \text{(MP)}$$

- Determinacy characterized by Surico (2008). Result
  - Any $\theta^P = (\phi_\pi, \phi_y)$ can support multiple equilibria.
  - Depends on economic structure: $\theta^S = (\beta, \kappa, \tau, \delta)$.
  - Inference on θ^S essential for policy design.
- Adding one friction makes an indeterminacy trap possible.
  - Reason for caution when using large DSGE models to inform policy.

Today

1. Focus on flexible price limit – a neoclassical economy.
2. Examine determinacy condition.
3. Observational equivalence between determinacy and indeterminacy.
4. An indeterminacy trap.
    - In the paper: show how to engineer an escape.

Literature

- Identification Issues in DSGE Models:
  - Canova and Sala (2009), Fernández-Villaverde (2010)
- Indeterminacy in DSGE models:
  - Benhabib and Farmer (1994), Benhabib and Farmer (1999), Benhabib and Farmer (2000), Beyer and Farmer (2004).
- Cost-Channel
  - Benhabib and Farmer (2000), Barth and Ramey (2002), Christiano et al. (2005), Ravenna and Walsh (2006), Surico (2008), Christiano et al. (2010)
- Local Identification:
  - Iskrev (2010), Komunjer and Ng (2011), Qu and Tkachenko (2010)
- Global Identification:
  - Qu and Tkachenko (2013), Morris (2013), Kociecki and Kolasa (2014)

Neoclassical Economy

- Limiting case of a New-Keynesian economy with a cost channel.
    - Clarifies the story.
    - Conclusions robust to re-incorporating sticky prices.
- Key: determinacy condition depends on structural parameters.
- Mechanism: cost channel can make any policy de-stabilizing.
    - Benhabib and Farmer (2000), Surico (2008), Christiano et al. (2010)
- A policy rule can support either determinacy or indeterminacy, depending on the strength of the cost channel.

Helicopter Tour

- **Household** consumes output, supplies labor, invests nominal equity.
- **Firm** uses equity as working capital during production.
  - Consistent with evidence on firm money demand: Mulligan (1997), Bover and Watson (2005).
  - Alternatively, a short cut for cost channel due to financial frictions.
- Firm marginal costs depend on the opportunity cost of nominal funds.
- **Central bank** controls the nominal interest rate.
- Markets are perfectly competitive.

Incorporating a Cost Channel

- Cobb-Douglas technology in intermediate inputs and labor.
  - Basu (1995), Christiano et al. (2010)
- Output can supply either:
  - Final goods market: Y_t.
  - Intermediate input market: X_t^s.
- Firm's problem:

$$\max_{Y_t, X_t^s, X_t^d, N_t^d} P_t(Y_t + X_t^s) - P_t X_t^d - W_t N_t$$

$$\text{s.t.} \quad Y_t + X_t^s \leq (X_t^d)^\alpha (A_t N_t^d)^{1-\alpha}$$

$$\vartheta(P_t X_t^d + W_t N_t^d) \leq K_t$$

- K_t is a within-period cash loan to the firm.

Equilibrium Conditions

- Standard representative household.
- Intermediate input market clearing ($X_t^s = X_t^d$) implies:

$$\frac{W_t}{P_t} = A_t(1+\vartheta R_t)^{-\frac{1}{1-\alpha}} \qquad \text{(Labor Demand)}$$

$$Y_t = \left(\frac{\alpha}{1-\alpha}\right)^{\alpha} \frac{1-\alpha+\vartheta R_t}{(1+\vartheta R_t)^{\frac{1}{1-\alpha}}} A_t N_t^d \qquad \text{(Final Output)}$$

$$\frac{W_t}{P_t} = C_t^{\tau}(N_t^s)^{\psi} \qquad \text{(Labor Supply)}$$

$$1 = \beta(1+i_t)\mathbb{E}_t \frac{D_{t+1}}{D_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\tau}\frac{1}{\Pi_{t+1}} \qquad \text{(Euler Equation)}$$

$$R_t = 1 - \frac{1}{1+i_t} \qquad \text{(Cost of Capital)}$$

Log-Linear Equilibrium System

- Clear labor, capital, and goods markets.
- Take log-linear approximation near a constant inflation steady state:

$$y_t = s_t - \delta i_t \qquad \text{(Aggregate Supply)}$$

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\tau}(i_t - \mathbb{E}_t \pi_{t+1} - r_t) \qquad \text{(IS Curve)}$$

$$r_t = d_t - \mathbb{E}_t d_{t+1} \qquad \text{(Rate of Time Preference)}$$

- s_t, an aggregate supply shock due to technology
- d_t, an aggregate demand shock due to preferences
- $\delta > 0$ the elasticity of aggregate supply to i_t
- $\tau > 0$ the inverse elasticity of aggregate demand to i_t

Flexible Price Limit

What Does the Cost Channel Do?

- AS and AD when $i_t = \phi\pi_t$:

$$y_t = s_t - \delta\phi\pi_t$$

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\tau}\left(\underbrace{\phi\pi_t - \mathbb{E}_t\pi_{t+1}}_{\text{real rate}} - r_t\right)$$

- Controlling outcomes requires controlling expectations.
- Equilibrium:

$$(1-\delta\tau)\pi_t = (\phi^{-1} - \delta\tau)\mathbb{E}_t\pi_{t+1} + \phi^{-1}r_t + \phi^{-1}\tau\mathbb{E}_t[s_{t+1} - s_t]$$

What Does the Cost Channel Do?

- AS and AD when $i_t = \phi\pi_t$:

$$y_t = s_t - 0\phi\pi_t$$

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\tau}\left(\underbrace{\phi\pi_t - \mathbb{E}_t\pi_{t+1}}_{\text{real rate}} - r_t\right)$$

- Controlling outcomes requires controlling expectations.
- Equilibrium:

$$(1 - 0\tau)\pi_t = (\phi^{-1} - 0\tau)\mathbb{E}_t\pi_{t+1} + \phi^{-1}r_t + \phi^{-1}\tau\mathbb{E}_t[s_{t+1} - s_t]$$

What Does the Cost Channel Do?

- AS and AD when $i_t = \phi\pi_t$:

$$y_t = s_t$$

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\tau}\left(\underbrace{\phi\pi_t - \mathbb{E}_t\pi_{t+1}}_{\text{real rate}} - r_t\right)$$

- Controlling outcomes requires controlling expectations.
- Equilibrium:

$$\pi_t = \phi^{-1}\mathbb{E}_t\pi_{t+1} + \phi^{-1}r_t + \phi^{-1}\tau\mathbb{E}_t[s_{t+1} - s_t]$$

What Does the Cost Channel Do?

- AS and AD when $i_t = \phi\pi_t$:

$$y_t = s_t - \delta\phi\pi_t$$

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\tau}\left(\underbrace{\phi\pi_t - \mathbb{E}_t\pi_{t+1}}_{\text{real rate}} - r_t\right)$$

- Controlling outcomes requires controlling expectations.
- Equilibrium:

$$(1 - \delta\tau)\pi_t = (\phi^{-1} - \delta\tau)\mathbb{E}_t\pi_{t+1} + r_t + \tau\mathbb{E}_t[s_{t+1} - s_t]$$

Reduced Equilibrium System

- Monetary policy follows:

$$i_t = \rho_i i_{t-1} + \phi[\pi_t + \omega(y_t - y_{t-1})]$$

- AR(1) supply and demand shocks:

$$s_t = \rho_s s_{t-1} + u_{s,t} \quad \text{(Supply Shock)}$$
$$d_t = \rho_d d_{t-1} + u_{d,t} \quad \text{(Demand Shock)}$$

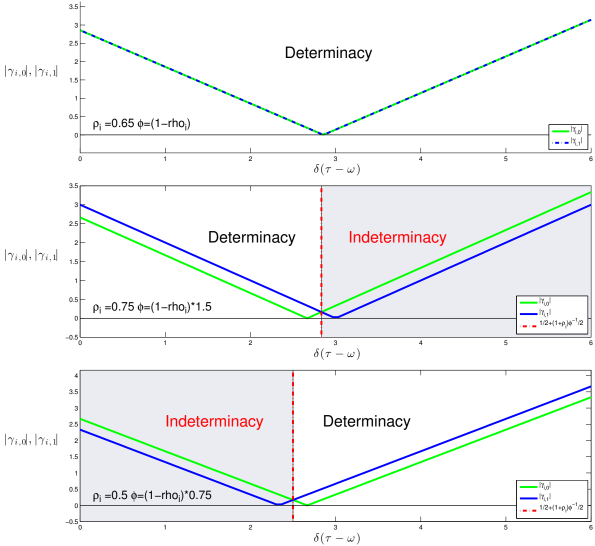
- Eliminate π_t and y_t to get

$$\gamma_{i,0}\mathbb{E}_t i_{t+1} = \gamma_{i,1} i_t + (1-\rho_s)(\tau-\omega)s_t - (1-\rho_d)d_t \quad \text{(Reduced Model)}$$

where $\gamma_{i,0} \equiv \phi^{-1} - \delta(\tau - \omega)$ and $\gamma_{i,1} \equiv 1 + \rho_i\phi^{-1} - \delta(\tau - \omega)$.

Determinacy Condition – Three Cases

$$|\gamma_{i,1}| \geq |\gamma_{i,0}| \quad \Leftrightarrow \quad |1 + \phi^{-1}\rho_i - \delta(\tau - \omega)| \geq |\phi^{-1} - \delta(\tau - \omega)|$$



Determinacy Condition

Proposition

For given policy parameters of (ρ_i, ϕ, ω)*, equilibrium is determinate if and only if* $\phi \neq 0$ *and one of:*

1. $\phi^{-1} = 1 + \phi^{-1}\rho_i$.
2. $1 + \rho_i\phi^{-1} > \phi^{-1}$ *and* $\delta(\tau - \omega) \leq \frac{1}{2} + \frac{1}{2}(1 + \rho_i)\phi^{-1}$.
3. $1 + \rho_i\phi^{-1} < \phi^{-1}$ *and* $\delta(\tau - \omega) \geq \frac{1}{2} + \frac{1}{2}(1 + \rho_i)\phi^{-1}$.

Proof.

Work through cases for the signs of $\gamma_{i,0}$ and $\gamma_{i,1}$ in $|\gamma_{i,1}| \geq |\gamma_{i,0}|$. □

- $\delta\tau$ is the elasticity of AS to i_t relative to the elasticity of AD to i_t.

Structural inference essential for policy design.

Indeterminacy Trap

- Lack of identification when there is indeterminacy.
  - Self-fulfilling phenomena generate "endogenous shocks" which look like demand shocks.
  - Indeterminacy with only supply shocks creates outcomes identical to under determinacy driven by both supply and demand shocks.
- The analyst implicitly ignores this possibility, and gets faulty inference.
- Recommends a policy supporting indeterminacy, but believes that the policy ensures a unique equilibrium.

What Data Does the Analyst Observe?

- Under determinacy must have:

$$i_t = \kappa_s s_t + \kappa_d d_t \implies \eta_t \equiv i_t - \mathbb{E}_{t-1} i_t = \kappa_s u_t^s + \kappa_d u_t^d$$

where

$$\kappa_s \equiv \frac{(1-\rho_s)(\tau-\omega)}{\gamma_{i,0}\rho_s - \gamma_{i,1}} \quad \text{and} \quad \kappa_d \equiv -\frac{1-\rho_d}{\gamma_{i,0}\rho_d - \gamma_{i,1}}$$

- Indeterminacy leaves η_t free:

$$\eta_t = \lambda_s u_{s,t} + \lambda_d u_{d,t} + v_t, \quad \mathbb{E} v_t^2 = \sigma_v^2$$

- Non-fundamental volatility, $\{v_t\}$, uncorrelated with $u_{s,t}$ and $u_{d,t}$.

Equilibrium Data Generating Process

- State space representation for equilibrium:

$$\begin{bmatrix} i_t \\ s_t \\ d_t \end{bmatrix} = \begin{bmatrix} \gamma_{i,1}/\gamma_{i,0} & (1-\rho_s)(\tau-\omega)/\gamma_{i,0} & -(1-\rho_d)/\gamma_{i,0} \\ 0 & \rho_s & 0 \\ 0 & 0 & \rho_d \end{bmatrix} \begin{bmatrix} i_{t-1} \\ s_{t-1} \\ d_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{s,t} \\ u_{d,t} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \eta_t$$

$$\begin{bmatrix} i_t \\ y_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\delta & 1 & 0 \end{bmatrix} \begin{bmatrix} i_t \\ s_t \\ d_t \end{bmatrix}$$

where

$$\eta_t = \begin{cases} \kappa_s u_{s,t} + \kappa_d u_{d,t} & \text{if } |\gamma_{i,1}| \geq |\gamma_{i,0}| \\ \lambda_s u_t^s + \lambda_d u_t^d + v_t & \text{if } |\gamma_{i,1}| < |\gamma_{i,0}| \end{cases} \quad \text{and} \quad i_0 = \begin{cases} \kappa_s s_0 + \kappa_d d_0 & \text{if } |\gamma_{i,1}| \geq |\gamma_{i,0}| \\ \text{unrestricted} & \text{if } |\gamma_{i,1}| < |\gamma_{i,0}| \end{cases}$$

- Up to three state variables drive two observable outcomes.
- The analyst must infer $(\delta, \tau, \rho_s, \rho_d)$ and the innovation variance-covariance structure from observing this system.

Identification Failure Due to Indeterminacy

- Determinacy with demand shocks ($\sigma_d^2 > 0$), two latent factors:

$$\begin{bmatrix} i_t \\ y_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\delta & 1 \end{bmatrix} \begin{bmatrix} \kappa_s & \kappa_d \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \rho_s & 0 \\ 0 & \rho_d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1/\kappa_d & -\kappa_s/\kappa_d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \delta & 1 \end{bmatrix} \begin{bmatrix} i_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -\delta & 1 \end{bmatrix} \begin{bmatrix} \kappa_s & \kappa_d \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_{s,t} \\ u_{d,t} \end{bmatrix}$$

- Indeterminacy without demand shocks ($\sigma_d^2 = 0$), two latent factors:

$$\begin{bmatrix} i_t \\ y_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\delta & 1 \end{bmatrix} \begin{bmatrix} \gamma_{i,1}/\gamma_{i,0} & (1-\rho_s)(\tau-\omega)/\gamma_{i,0} \\ 0 & \rho_s \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \delta & 1 \end{bmatrix} \begin{bmatrix} i_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -\delta & 1 \end{bmatrix} \begin{bmatrix} 1 & \lambda_s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_t \\ u_{s,t} \end{bmatrix}$$

- Observational equivalence between:
  - Determinacy with demand shocks ($|\gamma_{i,1}| < |\gamma_{i,0}|$ and $\sigma_d^2 > 0$).
  - Indeterminacy without demand shocks ($|\gamma_{i,1}| \geq |\gamma_{i,0}|$ and $\sigma_d^2 = 0$).

Unidentified Structure Despite Known Policy Rule

Let $\theta^S \equiv (\delta, \tau, \rho_s, \rho_d, \sigma_s^2, \sigma_d^2, \sigma_{s,d})$, $\theta^P \equiv (\rho_i, \phi, \omega)$, and $\lambda \equiv (\lambda_s, \lambda_d, \sigma_v^2)$.

Proposition

For each structure and policy pair (θ_0^S, θ_0^P) in the indeterminacy region where only supply shocks drive the business cycle and any equilibrium selection parameter λ_0, there exists a pair (θ_1^S, θ_1^P) in the determinacy region which generates the exact same stochastic process for nominal interest rates and output. Moreover, this holds even if policy is identical between the two scenarios: $\theta_0^P = \theta_1^P$.

Proof.

Compare the VAR(1) coefficients between the case with $|\gamma_{i,1}| < |\gamma_{i,0}|$ and $\sigma_d^2 > 0$ and the case with $|\gamma_{i,1}| \geq |\gamma_{i,0}|$ and $\sigma_d^2 = 0$. □

The Indeterminacy Trap

- The analyst knows $\theta^P = \theta_0^P$, infers θ^S, and makes a policy recommendation.
- Suppose that
  - Only supply shocks drive the business cycle.
  - θ_0^S supports multiple equilibria given θ_0^P.
- Then
  - There exists a $\theta_1^S \neq \theta_0^S$ which supports a unique equilibrium and rationalizes the data given $\theta^P = \theta_0^P$.
  - The analyst estimates the DSGE model assuming uniqueness and infers that $\theta^S = \theta_1^S$.
  - The analyst concludes that θ_0^P supports determinacy.
  - θ_0^P is maintained due to inference that $\theta^S = \theta_1^S$.
- Given true structure θ_0^S choice of θ_0^P perpetuates indeterminacy.

Escape

Conclusion

- Assuming uniqueness during DSGE estimation is not innocuous.
- An indeterminacy trap
  - An analyst can always rationalize the data under the assumption of determinacy.
  - Can get stuck with policies supporting multiple equilibria.
- A cautionary tale for analysts using DSGE models to inform policy.
- A concern when adding shocks solely to fit the data.
- Robust policies rely on global identification analysis which accounts for the possibility of multiple equilibria.

Determinacy Condition (Surico (2008))

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa(y_t - y_t^n + \delta i_t)$$
$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\tau}(i_t - \mathbb{E}_t \pi_{t+1} - r)$$
$$i_t = \rho i_{t-1} + (1-\rho)[r + \phi_\pi \pi_t + \phi_y (y_t - y_t^n)]$$

There is a unique equilibrium if and only if:

- When $2\delta\tau \leq 1$,

$$\max\left\{1 + \delta\phi_y - \left(\frac{1-\beta}{\kappa}\right)\phi_y, \delta\phi_y\right\} < \phi_\pi$$

- When $2\delta\tau > 1$,

$$\max\left\{1 + \delta\phi_y - \left(\frac{1-\beta}{\kappa}\right)\phi_y, \delta\phi_y\right\} < \phi_\pi < \frac{\phi_y(1+\beta+\kappa\delta)}{\tau\kappa(2\delta - 1/\tau)} + \frac{(1+\rho)(2+2\beta+\kappa/\tau)}{\kappa(1-\rho)(2\delta-1/\tau)}$$

Return

Flexible Price Limiting Case

- New-Keynesian Model + cost channel (Ravenna and Walsh (2006)):

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa(y_t - s_t + \delta i_t) \quad \text{(AS)}$$

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\tau}(i_t - \mathbb{E}_t \pi_{t+1}) \quad \text{(IS)}$$

$$i_t = \phi_\pi \pi_t + \phi_y (y_t - s_t) \quad \text{(MP)}$$

- As prices become flexible, the slope of AS becomes vertical: $\kappa \to \infty$.
- Re-write AS as: $y_t = y_t^n - \delta i_t + \kappa^{-1}(\pi_t - \beta \mathbb{E}_t \pi_{t+1} - r_t)$.
- A $\kappa \to \infty$,

$$y_t = s_t - \delta i_t + \kappa^{-1}(\pi_t - \beta \mathbb{E}_t \pi_{t+1}) \xrightarrow{p} s_t - \delta i_t$$

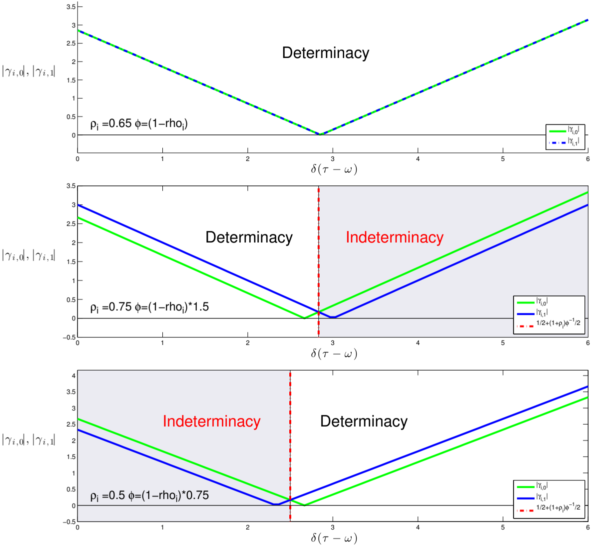
when π_t is mean-square stable (or bounded).

Return

Determinacy Condition – Three Cases

Return

$$|\gamma_{i,1}| \geq |\gamma_{i,0}| \quad \Leftrightarrow \quad |1 + \phi^{-1}\rho_i - \delta(\tau - \omega)| \geq |\phi^{-1} - \delta(\tau - \omega)|$$



Escape? Return

- Find a policy rule which implies determinacy across all plausible economic structures.
- Need to characterize the full identified set.
  - With a VAR representation can use global results in Morris (2013).
- Incorporate both the determinacy and indeterminacy regions during identification analysis.

Full Identified Set

Return

Can characterize the full identified set using parameters from a reduced form VAR(1):

$$\begin{bmatrix} i_t \\ y_t \end{bmatrix} = A \begin{bmatrix} i_{t-1} \\ y_{t-1} \end{bmatrix} + u_t, \quad \mathbb{E}u_t = 0, \quad \mathbb{E}\begin{bmatrix} i_{t-1} \\ y_{t-1} \end{bmatrix} u_t' = 0, \quad \mathbb{E}u_t u_t' = \Omega\Omega'$$

where

$$A = \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} \\ \alpha_{2,1} & \alpha_{2,2} \end{bmatrix}, \quad \Omega = \begin{bmatrix} \omega_{1,1} & 0 \\ \omega_{1,2} & \omega_{2,2} \end{bmatrix}$$

Full Identified Set ▸ Return

Lemma

For given $\theta^P = (\rho_i, \phi, \omega)$*, A* $(\sigma(A) < 1)$*, and* Ω*:*

1. *If* $(\alpha_{1,1} - \alpha_{2,2})^2 + 4\alpha_{1,2}\alpha_{2,1} < 0$ *then* **the identified set is empty**.
2. *If* $(\alpha_{1,1} - \alpha_{2,2})^2 + 4\alpha_{1,2}\alpha_{2,1} \geq 0$ *and* $\alpha_{1,1} \neq \phi + \rho_i - \frac{\alpha_{1,2}\alpha_{2,1}}{1-\alpha_{2,2}}$*, then* **equilibrium is determinate and the identified set contains up to two points***, one per* $\delta > 0$ *s.t.* $\alpha_{1,2}\delta^2 + (\alpha_{2,2} - \alpha_{1,1})\delta - \alpha_{2,1} = 0$*.*
3. *If* $(\alpha_{1,1} - \alpha_{2,2})^2 + 4\alpha_{1,2}\alpha_{2,1} \geq 0$ *and* $\alpha_{1,1} = \phi + \rho_i - \frac{\alpha_{1,2}\alpha_{2,1}}{1-\alpha_{2,2}}$*, then* **indeterminacy is possible and the identified set contains up to four points***. For each root* $\delta > 0$ *from case 2, there is the same point in the determinacy region and another point in the indeterminacy region.*

Proof.

Following Morris (2013), match VAR(1) coefficients and solve. □

Robust Policy

Return

Proposition

Given historical $\tilde{\theta}^P = (\tilde{\rho}_i, \tilde{\phi}, \tilde{\omega})$, if indeterminacy is historically plausible then $\theta^P = (\rho_i, \phi, \omega)$ rules out indeterminacy going forward if and only if $\phi \neq 0$ and one of:

1. $1 + \rho_i \phi^{-1} = \phi^{-1}$
2. $1 + \rho_i \phi^{-1} > \phi^{-1}$ *and* $\delta(\tilde{\omega} - \omega) + \chi_L(\tilde{\theta}^S, \tilde{\theta}^P) \leq \frac{1}{2} + \frac{1}{2}(1 + \rho_i)\phi^{-1}$.
3. $1 + \rho_i \phi^{-1} < \phi^{-1}$ *and* $\delta(\tilde{\omega} - \omega) + \chi_U(\tilde{\theta}^S, \tilde{\theta}^P) \geq \frac{1}{2} + \frac{1}{2}(1 + \rho_i)\phi^{-1}$.

for each of the (finite) $\tilde{\theta}^S$ in the identified set.

Proof.

The result follows from previous lemma and the determinacy condition.

Determinacy Graphs

□

Robust Policy

Return

Corollary

For any known historical policy $\tilde{\theta}^P$, unknown true structure θ^S, and chosen output growth weight ω, there exists a sufficiently large value for ρ_i and small value for $\phi > 0$ satisfying the modified Taylor principle, $\phi + \rho_i > 1$, such that $\theta^P = (\rho_i, \phi, \omega)$ is a robust policy. Knowledge of the full set of identified values for δ and τ is necessary to choose appropriate parameter values.

- Price level targeting ($\rho_i = 1$) robust for sufficiently small ϕ.
- Choice of ϕ depends on the *whole* identified set.

M. J. Barth and V. A. Ramey. The cost channel of monetary transmission. In *NBER Macroeconomics Annual 2001, Volume 16*, NBER Chapters, pages 199–256. National Bureau of Economic Research, Inc, 2002. URL http://ideas.repec.org/h/nbr/nberch/11066.html.

S. Basu. Intermediate Goods and Business Cycles: Implications for Productivity and Welfare. *American Economic Review*, 85 (3):512–31, June 1995. URL http://ideas.repec.org/a/aea/aecrev/v85y1995i3p512-31.html.

J. Benhabib and R. E. Farmer. Indeterminacy and sunspots in macroeconomics. In J. B. Taylor and M. Woodford, editors, *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics*, chapter 6, pages 387–448. Elsevier, 1999. URL http://ideas.repec.org/h/eee/macchp/1-06.html.

J. Benhabib and R. E. Farmer. The monetary transmission mechanism. *Review of Economic Dynamics*, 3(3):523–550, July 2000. URL http://ideas.repec.org/a/red/issued/v3y2000i3p523-550.html.

J. Benhabib and R. E. A. Farmer. Indeterminacy and Increasing Returns. *Journal of Economic Theory*, 63(1):19–41, June 1994. URL http://ideas.repec.org/a/eee/jetheo/v63y1994i1p19-41.html.

A. Beyer and R. E. A. Farmer. On the indeterminacy of new-Keynesian economics. Working Paper Series 0323, European Central Bank, Mar. 2004. URL http://ideas.repec.org/p/ecb/ecbwps/20040323.html.

O. Bover and N. Watson. Are there economies of scale in the demand for money by firms? some panel data estimates. *Journal of Monetary Economics*, 52(8):1569–1589, November 2005. URL http://ideas.repec.org/a/eee/moneco/v52y2005i8p1569-1589.html.

F. Canova and L. Sala. Back to square one: identification issues in dsge models. *Journal of Monetary Economics*, 56(4): 431–449, 2009.

L. J. Christiano, M. Eichenbaum, and C. L. Evans. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of political Economy*, 113(1):1–45, 2005.

L. J. Christiano, M. Trabandt, and K. Walentin. DSGE Models for Monetary Policy Analysis. In B. M. Friedman and M. Woodford, editors, *Handbook of Monetary Economics*, volume 3 of *Handbook of Monetary Economics*, chapter 7, pages 285–367. Elsevier, January 2010. URL http://ideas.repec.org/h/eee/monchp/3-07.html.

J. Fernández-Villaverde. The econometrics of dsge models. *SERIEs*, 1(1-2):3–49, 2010.

N. Iskrev. Local identification in dsge models. *Journal of Monetary Economics*, 57(2):189–202, March 2010. URL http://ideas.repec.org/a/eee/moneco/v57y2010i2p189-202.html.

A. Kociecki and M. Kolasa. Global identification of linearized dsge models. 2014.

I. Komunjer and S. Ng. Dynamic identification of dynamic stochastic general equilibrium models. *Econometrica*, 79(6): 1995–2032, 2011. ISSN 1468-0262. doi: 10.3982/ECTA8916. URL http://dx.doi.org/10.3982/ECTA8916.

S. Morris. Global identification of dsge models. Technical report, University of California, San Diego, 2013.

C. B. Mulligan. Scale economies, the value of time, and the demand for money: Longitudinal evidence from firms. *Journal of Political Economy*, 105(5):1061–79, October 1997. URL http://ideas.repec.org/a/ucp/jpolec/v105y1997i5p1061-79.html.

Z. Qu and D. Tkachenko. Identification and frequency domain qml estimation of linearized dsge models, 2010.

Z. Qu and D. Tkachenko. Local and global parameter identification in dsge models allowing for indeterminacy. Technical report, 2013.

F. Ravenna and C. E. Walsh. Optimal monetary policy with the cost channel. *Journal of Monetary Economics*, 53(2):199–216, March 2006. URL http://ideas.repec.org/a/eee/moneco/v53y2006i2p199-216.html.

P. Surico. The Cost Channel Of Monetary Policy And Indeterminacy. *Macroeconomic Dynamics*, 12(05):724–735, November 2008. URL http://ideas.repec.org/a/cup/macdyn/v12y2008i05p724-735_07.html.