Signaling in Credit and Insurance Markets

Shreemoy Mishra*
Department of Economics
Oberlin College
Email: shreemoy.mishra@oberlin.edu

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Abstract

This paper presents a signaling game between consumers, banks and insurance firms. Insurance firms use credit history to set premiums. There are no direct costs of default. In the current environment, loan repayment improves credit score and acts as a signal of low actuarial risk. This paper makes the following points: (i) Welfare implications of better information on borrowers derived in purely credit market settings can be misleading; full information about insurance risk leads to complete credit market failure, (ii) The sub-prime loan market is very different (due to randomization in debt repayment) from the prime market, (iii) There are no separating equilibria and belief refinements have no bite; the multiplicity of equilibria is problematic for both theory and empirical analysis, (iv) Strategic borrowing (by college students for instance) motivated by an incentive to build a ‘good’ credit record (rather than consumption smoothing) creates externalities which can be positive or negative, (v) The potential to manipulate credit history by existing consumers leads to unfair treatment of new consumers; they are treated as the worst possible type, and (vi) Rational borrowers, who are initially uninformed about the link between credit scores and insurance, can endogenously learn about the link. Learning is driven by surprising changes in the menu of insurance contracts.

Keywords: Indeterminacy of Equilibria, Belief Refinement, Unsecured Credit, Default, Credit Score, Insurance, Sub-prime Loans

JEL: C7, D8, E5

1 Introduction

Motivation

Consumer credit rating agencies such as Equifax, Experian and Transunion maintain records of individual credit market behavior. For some consumers these records go as far back as the late 1970’s. Credit rating agencies use past credit market behavior to make inferences about a borrower’s ability to repay future loans. The likelihood of repayment is most commonly reflected in a credit

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score. Examples include the commonly used FICO score, which is a number between 300 and 850, or the more recent VantageScore which in addition to a numerical figure also provides letter grade ratings (A-F). The higher the score the lower is the likelihood of default. Almost all consumer credit institutions purchase credit histories and credit scores to decide contract terms such as loan limits and interest rates. Applications range from home and auto loans to unsecured consumer credit cards. Over the past several years credit histories and scores have found applications outside credit markets; auto insurance and home insurance firms use credit history to decide insurance premiums. These practices are the result of empirical evidence that suggest strong correlation between the outcomes in these separate markets. Specifically there is a strong negative correlation between credit scores and insurance claims. While the underlying reason for this correlation is currently not well understood recent work points towards the same set of psychological underpinnings.

"Through a detailed literature review concerning the biological, psychological, and behavioral attributes of risky automobile drivers and insured losses, ..., we delineate that basic chemical and psychobehavioral characteristics (e.g., a sensation-seeking personality type) are common to individuals exhibiting both higher insured automobile loss costs and poorer credit scores, and thus provide a connection which can be used to understand why credit scoring works." - Brockett and Golden (2007)

This paper takes the correlation between credit and insurance market outcomes as given and attempts to answer the following questions: (1) What equilibrium outcomes can arise when consumers try to signal their private information?, (2) What is the impact of credit and insurance markets on each other?, (3) What is the impact on equilibrium outcomes when consumers have the ability to manipulate their credit history?, (4) How should new arrivals (consumers without any history) be treated?, and (5) How can uninformed consumers learn about the information link between the two markets?

Economists are still trying to fully comprehend the underlying causes and implications of the sub-prime lending and banking crisis that surfaced in 2007. By definition, sub-prime is related to consumer credit history. While the financial crisis is related to consumer credit history, it is not entirely based on it. In fact the crisis was partly driven by credit ratings for firms and asset classes such as mortgage based securities.

The use of credit scores really took off with the information technology revolution in the mid 1990’s. In 2003, the United States Congress passed the Fair and Accurate Credit Transactions (FACT) Act. Under Section 215 of the FACT Act, Congress asked the Federal Trade Commission\(^2\) and the Board of Governors of the Federal Reserve System\(^3\) to study the implications of

\(^1\)Some employers run credit checks on new potential recruits. Interestingly credit agencies are also encouraging individuals to check out a potential partner’s credit history before getting married (Source: www.FreeCreditReport.com)
\(^3\)http://www.federalreserve.gov/boarddocs/RptCongress/creditscore/creditscore.pdf
using numerical summaries of consumer credit histories for insurance pricing and credit pricing respectively. One concern of consumer groups as well as legislators was the potential of using an applicant’s credit score as a proxy for race or ethnic group\textsuperscript{4}. Both agencies submitted their separate reports to congress and found no evidence for discrimination in either credit or credit-based insurance markets\textsuperscript{5}.

At present 48 state legislatures have adopted some restrictions on the use of credit history in insurance pricing\textsuperscript{6}. Hawaii is the only state that expressly forbids any use of credit history in insurance pricing. Pennsylvania and Vermont are the only states that have no legislations at all on credit-based insurance pricing. Across states, where there has been legislative action, there is significant variation in laws. Some states such as Alaska allow credit history to be used for new contracts but restrict use for contract renewals. In some cases (Texas, Louisiana and New Mexico) states forbid the use of particularly drastic entries under \textit{extraordinary life circumstances} exclusions. These include medical crises, death of spouse or identity theft. Laws also vary depending on the type of insurance: using credit based for auto insurance only (Kentucky) or both auto and home (most states). Colorado recently voted down a bill designed to ban credit-based insurance\textsuperscript{7}.

\textbf{Findings}

This paper uses a simple yet rich environment to analyze the questions raised above. To generate some intuition, I point out the main assumptions and features that drive the results. In terms of timing, consumers first participate in credit markets. The insurance market opens after the credit market is over, that is, after loan amounts, default and repayment decisions are observed. Consumers belong to one of two types based on \textit{actuarial risk}: high and low risk. High actuarial-risk consumers are more likely to find themselves in situations that lead to default. There are no direct costs of default. Credit and insurance firms are separate and cannot write joint contracts\textsuperscript{8}.

\textbf{R1)} There are no fully separating equilibria. Should there be any perfect separation of types through different borrowing amounts, then each risk type is identified and receives full insurance at an actuarially fair premium. Further, this premium is independent of default decisions. This implies that there are no incentives to repay the loans. Creditors foresee this and undo any possible revelation through loan amounts.

\textbf{R2)} A related issue is the failure of equilibrium refinements. Like most signaling games there are lots of Bayesian equilibria here as well. Belief refinements operate (perhaps with too much

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\textsuperscript{4}For instance, on average, Black and Hispanic consumers have lower credit scores than Whites and Asians.

\textsuperscript{5}For a critique of the FTC report’s procedures, particularly data collection see the dissent statement by Harbour (2007).

\textsuperscript{6}Source: National Association of Mutual Insurance Companies, \url{http://www.namic.org}

\textsuperscript{7}Colorado House Bill 1143, January 2008.

\textsuperscript{8}The implications of consolidation in credit and insurance market is presented in a working paper, Mishra (2008). The study is motivated by passage of the Gramm-Leach-Bliley Financial Modernization Act of 1999 and related mergers such as Citigroup and Travelers.
R3) A market with mostly high-risk (sub-prime) consumers is very different from one with mostly low-risk (prime) consumers. In the prime market equilibria involve pure strategies over default decisions. Conditional on default decision low-risk consumers always end up with at least the same credit score as high-risk consumers. In the sub-prime market however, equilibria involve mixed strategies over default. Only high-risk consumers use mixed strategies and it is possible for a high-risk consumer to end up with a better credit score than a low-risk consumer.

R4) There exist equilibria in which consumers who have no (consumption-based) need to borrow end up borrowing nevertheless to obtain a more favorable credit score. I term this behavior strategic borrowing. Parents often encourage their college going children to start building a ‘good’ credit history. A simple way of doing this is applying for and using a credit card. Using debit cards, which are equally convenient for payments, has no impact on credit scores. Note that if balances are not paid in full, then using credit cards is a rather expensive way of building a credit history. The original equilibrium derived by assuming that no strategic borrowing occurs is not robust to this change (of allowing strategic borrowing). A large set of interesting outcomes can arise in equilibrium including one in which all agents are better off because of strategic borrowing.

R5) As another robustness check, I allow consumers to delete their credit history. The original equilibrium is robust to this change if consumers without any history are treated as the worst possible type but is not robust if unknown consumers are treated as if they represented ‘average’ risk. One implication is that recent arrivals such as Canadian citizens with good credit history in Canada face discrimination when applying for consumer loans (higher interest) or utility and cell phone service (higher deposits) in the United States.

R6) If consumers are initially unaware of the future implications of credit scores, then there are opportunities for them to learn about them endogenously. Learning however requires a high degree of sophistication (rationality) and supports an educational role for both governments and financial institutions.

In a qualitative sense, the model’s predictions for variation in interest rates with credit scores are consistent with observations from three separate markets: auto loans, home loans and credit cards. See the discussion following the corollary to Proposition 2 and figures 3, 4, 5 and 6. Specifically, the model predicts a structural change in the interest rates around a threshold value for

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9One example of ‘deletion’ or manipulation is a consumer with a low score applying for a loan using their spouse’s credit record.
credit scores. The threshold value is what separates equilibrium outcomes involving pure strategies from those involving mixed strategies over the decision to default.

Relation to Literature

The current environment is similar to that in Chatterjee, Corbae and Rios-Rull (2008a). To my knowledge their’s is the first paper to explicitly model the interaction between credit and insurance markets. The question in their paper is not specific to insurance markets. Their focus is on explaining how unsecured debt can exist in an environment where there are no *exogenous* costs of default, such as stigma, and time is finite, which excludes reputation and trigger strategies based explanations. They use the insurance market as a source of *endogenous* cost of default. Our focus instead is on the details of the signaling game and the interaction between the two separate markets (credit and insurance). Both Chatterjee, Corbae and Rios-Rull (2008a) as well as the current paper use prior and posterior beliefs about consumer type as analogous to credit scores. High scores result in better terms of insurance whereas low scores result in more expensive premiums. Scores are updated based on loan amounts and default decisions on unsecured debt. In terms of the environment there are two main differences from their framework. The first is in the range of credit scores. Chatterjee et al. restrict their attention to a range of scores that result in pooling insurance contracts. This only happens when credit scores are particularly high. I look at the entire range of possible credit scores which is more general and results in a larger set of equilibria and specifically includes the subprime market. The second difference is that I regard assets as unobservable whereas they consider savings to be observable. Our assumption of unobservable savings is driven by the fact that credit reports do not list assets, just liabilities\textsuperscript{10}.

Apart from Chatterjee, Corbae and Rios-Rull (2008a) and the FTC’s 2007 report mentioned above, the literature on the link between credit scoring and insurance markets is virtually nonexistent. There is however a rapidly growing literature on consumer debt, bankruptcy and the use of improved reporting technologies. Some recent contributions include Athreya, Tam and Young (2008), Chatterjee, Corbae and Rios-Rull (2008b), Livshits, MacGee and Tertilt (2007), Sanchez (2007), and Narajabad (2008). These authors develop quantitative models of unsecured credit to explain empirical findings presented in Edelberg (2006)\textsuperscript{11}. Athreya, Tam and Young (2008) present a life-cycle model with asymmetric information and no commitment. They find that, compared to full information, partial information harms the credit market. Debt levels and default rates both decline under partial information. In this sense better consumer information is welfare improving. Narajabad (2008) finds that improved information leads to more borrowing and increased default rates for high-risk consumers but less borrowing and lower default rates for low-risk consumers.

\textsuperscript{10}This is not strictly valid. For instance, a home or auto loan implies some assets are observable even if partially owned. This is irrelevant for the present paper and Chatterjee, Corbae and Rios-Rull (2008a) since both papers only consider unsecured debt.

\textsuperscript{11}Related empirical work includes Ausubel (1999), Berger (2003), and Chiappori and Salanie (2008).
Livshits, MacGee and Tertilt (2007) also study a life cycle model and find that the increase in debt levels and default rates are largely explained by reduced lending costs as well as reduced costs of default.

In contrast to the preceding literature full information here leads to complete credit market failure. This is because once actuarial risk is known, there is no incentive to repay loans. Credit markets can only survive under imperfect information about types in the insurance market and perfect information leads to welfare losses. Welfare results derived solely from the trade off between payoffs in current and future credit markets do not hold when the trade off is changed to payoffs in current credit and a future insurance market! This is one of the core messages of this paper. Increased use of credit history in non-credit applications makes welfare implications based solely on credit market outcomes less meaningful if not suspect. Further, most quantitative papers\textsuperscript{12} on unsecured credit use arbitrary off-equilibrium beliefs. These arbitrary beliefs are useful in a computational sense and help derive unique predictions and stationary distributions for calibration. However, they run the risk of disregarding alternative outcomes which can arise from less stringent restrictions on off-equilibrium beliefs. The quantitative literature also does not consider mixed strategy equilibria which are particularly relevant for the subprime market\textsuperscript{13}. The result on learning requires a high degree of sophistication on part of the consumer and is driven by experiencing surprises some of which are costly. This is related to recent work by Agarwal, Driscoll, Gabaix and Laibson (2008) who document evidence of learning in the credit card market.

2 Model

Agents

There are two time periods\textsuperscript{14} and a unit measure of consumers with constant income $e$ in each period. Consumers differ on two dimensions: (Actuarial) Risk Type and Preference Type. Type is private information. The number of types in each dimension is two, which corresponds to high and low values. The risk type of an agent refers to a probability of an income loss $\Delta$ after the end of the second period. I denote this probability by $\pi$. The probability of loss $\Delta$ can be either high or low. I refer to different risk types as high risk or low risk consumers. I denote actuarial risk types by $B$ (big, high risk) and $S$ (small, low risk) with $\pi^S < \pi^B$. A fraction $\mu_1$ of consumers are of the low-risk type.

The other dimension of variation in consumer types refers to random preference shocks. In each period the preference shock can be either high or low and these shocks are independent and identically distributed across periods. The probability of a high preference shock is $\lambda$ and is independent of risk type. I use $\theta$ to represent preference shocks.

\textsuperscript{12}Chatterjee, Corbae and Rios-Rull (2008a) and Athreya, Tam and Young (2008)

\textsuperscript{13}See Adams, Einav and Levin (2007) for a study of a market for subprime auto loans.

\textsuperscript{14}There is a third period in which an insurance market operates but no strategic decisions are made.
Preferences

Let $c_t$ denote consumption in period $t$ and $θ_t$ denote the corresponding preference shock. The utility function for a consumer is given by

$$θ_1 \ln c_1 + E[θ_2 \ln c_2 + V(I)]$$

(1)

The expectation in the utility function is taken over the probability distribution of preference shocks for a given risk type as well as the loss probability $π^i, i ∈ \{B, S\}$. $V(I)$ refers to the payoff to a consumer from accepting insurance contract $I$. Insurance contracts are described later in this section. A key assumption is that the support for preference shocks depends on risk type. Using $H$ and $L$ to denote high and low preference shocks, the assumption can be formally represented as

ASSUMPTION 1

$$θ^S_L < θ^B_L < θ^S_H < θ^B_H$$

(2)

Assumption 1 simply requires that the high preference shock for a low-risk type is larger than the low preference shock for a high-risk type agent. This implies that the preference shocks for the high-risk consumers first order stochastically dominate those for the low risk consumers. I place a further restriction on the preference shocks for the two types. Specifically, I assume that the low (high) preference shock for high-risk consumers is $α > 1$ times the low (high) preference shock for the low-risk consumers.

ASSUMPTION 2

$$θ^B_L = αθ^S_L \text{ and } θ^B_H = αθ^S_H$$

Assumption 2 seems restrictive and merits an explanation. The impact of assumption 2 is that the borrowing and saving needs are independent of risk type. This is because, the need to borrow or save (from any consumer’s first order condition) simply depends on the ratio of the first-period preference shock and the expected second-period preference shock. By assumption 2, this ratio is independent of risk type. As will be seen later, in equilibrium, even without assumption 2, all borrowers borrow identical amounts. Assumption 2 simplifies the analysis by making the amount of saving identical across types as well.

To keep the focus of the analysis on the interplay between borrowing, default and the insurance market, I assume that consumers with a low first-period preference shock behave non-strategically and necessarily save. The impact of relaxing this assumption is presented later when checking for robustness. The assumption is motivated by the observation that consumers with
persistent, high levels of unsecured (usually credit card) debt typically have small saving balances. Similarly, consumers with persistent, high levels of savings do not have large amounts of unsecured debt. A different way of motivating the assumption is that most consumers finance car and house loans at the same institutions at which they have checking or saving accounts. This introduces a degree of securitization on loans. It seems unrealistic to allow consumers to default on their loans when they have accumulated savings at the same institution.

**Firms**

There are two types of profit-maximizing firms: banks and insurance companies. Banks accept saving deposits and offer loans. Insurance companies offer contracts to insure against the income loss $\Delta$. The two types are separate and cannot contract on each other’s actions. Both industries, banking as well as insurance, are assumed to be perfectly competitive with free entry.

**Information Intermediaries**

Information intermediaries have no objective function and therefore no strategic role. They maintain historical records of consumer borrowing and saving behavior. They also keep track of loan repayment and default. I assume that information intermediaries do not report levels of saving, only the decision to save. This is motivated by the fact that consumer credit reporting agencies do not report asset holdings. The levels of borrowing however are reported and hence observable by insurance companies.

**Timing**

All consumers know their risk type from the start. At the beginning of period 1, each consumer draws a preference shock. Based on this draw and their expectation for second period preference shock, agents borrow or save to smooth their consumption over the two periods. At the start of period 2, consumers who have borrowed have the choice to default on their debt. There are no direct costs associated with default. Based on their asset market behavior, as relayed by information intermediaries, consumers are offered insurance contracts. Insurance contracts and the associated payoffs are explained below.

**Strategies**

- Consumers: A strategy for a consumer of risk-type $i$ is $(l, d_{ijk}(l))$ where $l$ is the amount loan (negative for saving) and $d_{ijk}(l)$ is the probability of default for consumer of risk-type $i$ following a preference shock $j$ in the first period and a preference shock $k$ in the second period. The default probability also depends on the amount borrowed $l$. Note that I am restricting
attention to pure strategies over borrowing amounts. Further note that by assumption con-
sumers with low first-period preference shock save. This implies they do not have a default
decision and therefore $d^i_{Lk} = 0$.

- **Banks:** A strategy for a bank is simply a gross interest rate $R(l)$ on loans. It is assumed that
  savings earn a gross interest rate identical to the risk free rate $(1 + r)$. The interest rate on
  loans depends on the amount loaned out which is $l$ from above.

- **Insurance Companies:** Offer insurance contracts based on their beliefs about a consumer’s
type. The contracts are described below (see equation 3). The beliefs are based on prior
distribution of types and on observed actions such as amount borrowed and default decision.
  Beliefs following repayment and default are represented by $\mu_0(l)$ and $\mu_1(l)$ respectively. These
  represent the conditional probability that a consumer is of the low-risk type. Note that for
  consumers who save, beliefs are the same as the unconditional prior probability of low-risk
  type, which is represented by $\mu_1$.

**Insurance Contracts**

An insurance contract takes the form $(X, m)$ where $X$ is indemnity (payout if the loss is realized)
and $m$ is premium per unit of indemnity purchased. Insurance contracts depend on the fraction of
low-risk type consumers. Let $\mu$ represent the fraction of low-risk consumers. Insurance contracts
take the following form

$$
(X, m) = \begin{cases}
  (\Delta, m^P), & \text{if } \mu \geq \mu^* \text{ (Pooling Contract)} \\
  \{(\Delta, \pi^B), (X^*, \pi^S)\}, & \text{if } \mu < \mu^* \text{ (Separating Contract)}
\end{cases}
$$

(3)

The type of insurance contracts offered depends on a threshold, $\mu^*$, for the fraction of low-risk
consumers. When the fraction of low-risk consumers exceeds this threshold, then all consumers
are offered full-insurance, pooling contracts with premium $m^P$. When the fraction of low-risk
consumers is below the threshold insurance companies offer separating contracts. In the separating
case, consumers self select from a menu of contracts which consists of: a full insurance contract at
high premium $(\Delta, \pi^B)$ or a partial insurance contract at low premium $(X^*, \pi^S)$. These contracts
are identical to those presented in Wilson (1977) and Chatterjee, Corbae and Rios-Rull (2008a).

To keep the insurance market separate from the asset market, I assume that the loss in income
$\Delta$ happens after all asset market outcomes and that the loss is relative to an endowment $e$ which
is the same as the income in the first two periods. Let $V^i(X, m)$ represent the value of insurance
contract $(X, m)$ to type $i$. 

9
When $\mu < \mu^*$ (Separating Contract)

\[ V^S(X^*, \pi^S) = (1 - \pi^S) \ln(e - \pi^S X^*) + \pi^S \ln(e - \Delta + X^* - \pi^S X^*) \]  \tag{4}

\[ V^B(\Delta, \pi^B) = \ln(e - \pi^B \Delta) \]  \tag{5}

When $\mu \geq \mu^*$ (Pooling Contract)

\[ V^S(\Delta, m) = V^B(\Delta, m) = \ln(e - m^P \Delta) \]  \tag{6}

where $m^P = \pi^B - \mu(\pi^B - \pi^S)$

The payoffs to each risk-type from the different insurance contracts are presented in figure 1.

**Determination of the threshold $\mu^*$**

- The values for the threshold $\mu^*$ and the level of partial insurance $X^*$ can be obtained using either the Wilson (1977) or the Rothschild-Stiglitz (1976) equilibrium concept. It involves the following steps

  - $X^*$ solves

    \[ V^B(\Delta, \pi^B) = V^B(X^*, \pi^S) \]  \tag{7}

    This keeps the high-risk type indifferent between full and partial insurance.

  - $\mu^*$ solves

    \[ V^S(\Delta, \pi^B - \mu^*(\pi^B - \pi^S)) = V^S(X^*, \pi^S) \]  \tag{8}

The threshold on the fraction of low-risk consumers $\mu^*$ discussed above leads us to a restriction on parameter values.

**ASSUMPTION 3:**

\[ \frac{\alpha \theta_H}{\theta_L} < \frac{\ln(e - \pi^S \Delta) - \ln(e - \pi^B \Delta)}{\ln(e - \pi^S \Delta) - \ln(e - \pi^B \Delta + \mu^*(\pi^B - \pi^S) \Delta)} \]  \tag{9}

The above restriction ensures that there is a large enough separation between the high preference shock for a high-risk agent and the low preference shock for a low-risk agent. Without this restriction, the preferences of the two risk-types are not different enough to obtain separation in the insurance market through any signaling via loan repayment decisions.
3 Equilibrium

The main feature of the two-dimensional type structure developed here is that for any given level of borrowing, the high-risk consumers are more likely to find themselves in situations that trigger default. The equilibrium concept I use is Perfect Bayesian Equilibrium (PBE).

Let \( l^i_j \) represent the loan undertaken by a consumer of risk type \( i \) following a first-period preference shock \( j \). For instance, \( l^B_H \) denotes the amount borrowed by a high-risk consumer. Note that \( l^i_L \) is negative which implies that the such consumers save. In what follows I restrict attention to pure strategies in levels of borrowing and saving, I permit mixed strategies in default decisions.

Recall that the default probabilities \( d_{jk}^i(l) \) and beliefs \( \{\mu_0^i(l), \mu_1^i(l)\} \) depend on the amount borrowed \( l \). However, to keep the notation uncluttered I drop the argument \( l \) from these terms.

**Proposition 1:** There is an equilibrium in which insurance companies disregard asset market behavior and no loans are extended. \( \mu_2^0 = \mu_2^1 = \mu_1, R = \infty, l^i_H = 0 \) and \( -l^i_L = a^* \).

\[
a^* = \left[ \frac{[\lambda \theta_H + (1 - \lambda \theta_L)](1 + r) - \theta_L}{[\lambda \theta_H + (1 - \lambda \theta_L)] - \theta_L} \right] e \frac{1}{1 + r}
\]

Proposition 1 states that if insurance firms do not update beliefs based on asset market behavior, then there is no incentive to repay loans. This implies that banks will refuse to extend any loans. Consumers with low first-period preference shocks however do save and this optimal level of saving is given by \( a^* \). By assumption 2, \( a^* \) is independent of risk-type and is derived purely from the consumption smoothing motive for such consumers.

The second result is that in any equilibrium with positive amounts of lending, all agents who borrow must borrow identical amounts. If different risk-types borrow different amounts, then types are identified by their choice of loan amount. If types are identified, then there is no ambiguity in insurance market outcomes. Each risk type is offered an actuarially fair, full insurance contract. This in turn implies there is no incentive to repay the loans. Anticipating this, banks refuse to extend loans to the any consumer.

**Lemma 1:** In any equilibrium with positive amount of lending, both risk types borrow identical amounts \( l^i_H = l^B_H \).

Now consider the second period where borrowers have the choice to default on their loans. This decision is made after the second-period preference shock is realized. Let \( d^i_{ik} \) represent the probability of default by a consumer of risk type \( i \) following a history of preference shock \( k \in \{H, L\} \). For instance, \( d^B_{HL} \) denotes the default probability for a high-risk consumer with a high first-period preference shock and a low second-period preference shock.

Define a *continuation equilibrium* as the equilibrium of a game starting in period 2 with some arbitrary, positive level of debt for either type of consumer.
Lemma 2: In any continuation equilibrium, if \( d_{HL}^i > 0 \) then \( d_{HH}^i = 1 \) for \( \forall i \in \{B, S\} \).

Lemma 2 states that irrespective of risk-type, if an agent with low second-period preference shock defaults with positive probability, then an agent of the same risk-type with high second-period preference shock must default with certainty.

The intuition for lemma 2 is that for a given type, the gain in the insurance market from repayment is independent of preference shock. The gain from default, however, is higher for the agent with higher second-period shock.

Lemma 3: In any continuation equilibrium if \( d_{HL}^B > 0 \) then \( d_{HH}^S = 1 \).

Lemma 3 states that if a high-risk agent with low second-period preference shock defaults with positive probability, then a low-risk agent with high second-period preference shock must default with certainty. To see this, note that the gain from default is strictly higher for the low-risk agent since \( \theta_H > \alpha \theta_L \). However, the gain in the insurance market for the low-risk type is always less than that for the high-risk type. Therefore, if the high-risk agent with low shock is willing to default, the low-risk agent with high preference shock must default with certainty.

Before proceeding any further, I place the following restriction on off-equilibrium beliefs.

Condition 1: \( \mu_0^B \geq \mu_1 \) and \( \mu_2^B \leq \mu_1 \).

Condition 1 requires that following repayment the posterior belief of a low-risk type cannot be less than the prior. It also requires that following default the posterior belief of a low-risk type cannot exceed the prior. Condition 1 reflects the central theme of the current study that agents resist opportunistic behavior in credit market to obtain better outcomes in the insurance market.

Lemma 4: In any continuation equilibrium that satisfies condition 1, if \( d_{HL}^S > 0 \) then \( d_{HL}^B \geq d_{HL}^S \) and \( d_{HH}^S = d_{HH}^B = 1 \).

Lemma 4 states that if a low-risk borrower with low second-period preference shock defaults with positive probability, then all agents with high second period preference shock default with certainty and high-risk agents with low second period preference shock are at least as likely to default as the low-risk agents with low preference shock.

Lemma 5: In any continuation equilibrium that satisfies condition 1, if \( d_{HH}^B < 1 \) then \( d_{HL}^S = d_{HH}^S = d_{HH}^B = 0 \).

Lemma 5 states that if a high-risk borrower with high second-period preference shock repays with positive probability, then all other types of agents repay with certainty.

Lemma 4 and lemma 5 help us rule out some uninteresting cases where either all agents default or all agents repay with certainty. I only consider cases where the low-risk agent with low second-period preference shock repays the loan with certainty and that the high-risk agent with high second period shock defaults with certainty. That is, from now on I set \( d_{HL}^S = 0 \) and \( d_{HH}^B = 1 \).
I simplify notation by using $d^S$ for $d^S_{HH}$ and $d^B$ for $d^B_{HL}$.

- Posterior belief of low-risk type following repayment

$$
\mu^0_2 = \frac{\mu_1(1-\lambda) + \mu_1 \lambda (1-d^S)}{\mu_1(1-\lambda) + \mu_1 \lambda (1-d^S) + (1-\mu_1)(1-\lambda)(1-d^B)} \tag{10}
$$

- Posterior belief of low-risk type following default

$$
\mu^1_2 = \frac{\mu_1 \lambda d^S}{\mu_1 \lambda d^S + (1-\mu_1)(1-\lambda)d^B + (1-\mu_1)\lambda} \tag{11}
$$

**Lemma 6:** There exists a threshold $\mu^S \in [0,1]$ such that when $\mu_1 \leq \mu^S$, $d^S = 1$ for all $l \geq 0$. For $\mu_1 > \mu^S$, $d^S = 0$ for all $l \in (l, l]$ and the interest rate is

$$
R = 1 + \frac{r + \lambda(1-\mu_1)}{\mu_1 + (1-\lambda)(1-\mu_1)} \tag{12}
$$

$$
\bar{l} = \frac{e}{R} \left[ 1 - \left( \frac{e - \pi^B \Delta + \mu^* (\pi^B - \pi^S) \Delta}{e - \pi^B \Delta + \mu^0_2 (\pi^B - \pi^S) \Delta} \right)^\frac{1}{\mu^*_L} \right] \tag{13}
$$

$$
\underline{l} = \frac{e}{R} \left[ 1 - \left( \frac{e - \pi^B \Delta}{e - \pi^B \Delta + \mu^1_2 (\pi^B - \pi^S) \Delta} \right)^\frac{1}{\mu^*_H} \right] \tag{14}
$$

For loans in the support of $l$ the posterior beliefs are given by

$$
\mu^0_2 = \frac{\mu_1}{\mu_1 + (1-\mu_1)(1-\lambda)}, \mu^1_2 = 0.
$$

The threshold $\mu^S$ is given by

$$
\mu^S = \frac{(1-\lambda)\mu^*}{1-\lambda \mu^*} \leq \mu^*, \quad \lim_{\lambda \to 0} \mu^S = \mu^*
$$

Lemma 6 states that there is a threshold for the prior fraction of low-risk type such that when the prior is less than the threshold, low-risk agents with a high second period shock default with certainty. For prior values above the threshold, low-risk agents repay with certainty provided the amount of the loan is within a certain range. This range is derived from two conditions. The upper bound of the range makes the low-risk consumers with low second-period shock just indifferent
between default and repayment. The lower bound ensures that high-risk agents with high second-period shock default with certainty as assumed earlier. Lemma 6 also pins down the equilibrium interest rate for a region of the parameter space above the threshold.

The intuition behind the lemma is as follows. For a given level of debt \( l \) and interest rate \( R \), the gain from default for a low-risk agent with high preference shock is fixed but independent of the prior. The benefits from repaying the debt however accrue from the resulting pooling equilibrium which increases monotonically (see figure 1) with the prior fraction of low-risk type \( \mu_1 \). When the prior is very small, there are no gains in the insurance market. However, when the prior is large, there are significant gains in the insurance market which outweigh any gains from default. From figure 1 there will be a unique value for the prior at which the low-risk consumer is indifferent between repayment and default; \( \mu^S \) denotes this threshold.

Since the gain from default is increasing in the amount owed \( (l) \), we can derive the bounds on the level of debt that makes low-risk agents repay with certainty and high-risk agents default with certainty. To see why the interest rate above the threshold is independent of the amount borrowed, note that the fractions of agents repaying and defaulting is independent of the amount borrowed. From the zero expected profit condition for banks we can derive the interest rate which only depends on fractions (or measures) of agents repaying and defaulting, and hence independent of the level of debt.

**Lemma 7:** When \( 0 < \mu_1 < \mu^* \), in any continuation equilibrium with debt \( l \in [0, l^B] \) \( \exists \, d^{B*}(l) \in (0,1) \) such that, \( d^B = d^{B*}(l) \). Further, the interest rate is given by

\[
R(d^{B*}(l)) = 1 + \frac{r + \lambda + (1 - \mu_1)(1 - \lambda)d^{B*}(l)}{(1 - \lambda)(\mu_1 + (1 - \mu_1)(1 - d^{B*}(l)))}
\]

\( l^B \) solves

\[
\alpha \theta_L \ln(e - R^B l^B) + \ln(e - (\pi^B - \mu^* (\pi^B - \pi^S)) \Delta) = \alpha \theta_L \ln(e) + \ln(e - \pi^B \Delta)
\]

where \( R^B = 1 + \frac{r + \lambda}{1 - \lambda} \) (15)

The posterior probability following repayment is

\[
\mu_2^0(d^{B*}) = \frac{\mu_1(1 - \lambda)}{\mu_1(1 - \lambda) + (1 - \mu_1)(1 - \lambda)(1 - d^{B*})} = \frac{\mu_1}{\mu_1 + (1 - \mu_1)(1 - d^{B*})}.
\]

Lemma 7 states that for prior probabilities less than the threshold \( \mu^* \) and loan amounts in the range \([0, l^B]\), an agent of the high-risk type with low second-period preference shock randomizes between default and repayment. This randomization probability depends on the level of outstanding debt. Moreover, the interest rate also depends on the level of debt indirectly through the default probability. The randomization probability is implicitly obtained as a function of debt from the
agent’s indifference condition which is presented in the proof.

The intuition for the result is the following. From lemma 6, a low-risk agent with high preference shock defaults with certainty. From the discussion following lemma 5, a low-risk agent with low shock repays with certainty whereas a high-risk agent with high preference shock defaults with certainty. Now note that a high-risk agent with low preference shock cannot default with probability 1. If he does, then repayment implies a posterior probability of 0, resulting in an offer of full insurance with the lowest premium \((\Delta, \pi^S)\). This provides an incentive to deviate and repay. On the other hand such an agent cannot repay with certainty either. Repayment with certainty results in a posterior that is less than \(\mu^*\) resulting in a separating insurance contract. Therefore the high-risk agent with low preference shock must randomize. The interest rate is obtained from the zero expected profit condition and depends on the variable probability of default. The resulting interest rate can then be substituted into the high-risk agent’s indifference condition to implicitly solve for the default probability as a function of debt.

The support for loans is obtained in steps. First note that the interest rate is increasing in the rate of default. The rate of default however is declining with the prior probability of low-risk type. At the limit, when the prior probability is equal to the threshold \(\mu^*\), the default rate drops to zero. This corresponds to the lowest interest rate and provides the upper bound on loans. The lower bound on loans is zero and this happens when the prior is close to zero leading to a default rate of close to one.

**Equilibrium Characterization**

**Proposition 2:** If there is an equilibrium with positive amounts of borrowing or lending, it must belong to one of two following types.

(A) **CCR Equilibrium:** When \(\mu_1 > \mu^*\), the support for loans is given by \([l, \bar{l}]\) and the interest rate is \(R\) where \(l, \bar{l}\) and \(R\) are defined in lemma 6. Only the high-risk consumers with high second-period preference shock default \((d^S = d^B = 0)\). Default leads to separating contracts, repayment leads to full-insurance pooling contracts with a premium based on the posterior and given by \(\pi^B - \mu_2^0(\pi^B - \pi^S)\). Savers (consumers with low first period preference shock) get full-insurance pooling contracts at a premium based on the prior \(\pi^B - \mu_1^1(\pi^B - \pi^S)\). Beliefs off the equilibrium path are undefined. This implies the equilibrium level of borrowing is not unique.

(B) **Mixed Strategy Equilibrium:** When \(\mu_1 < \mu^S\), the support for loans is given by \([0, l^B]\) and the interest rate is \(R(d^{B^*})(l)\). Consumers of either risk type default following a high second period preference shock. Low-risk consumers with low second period preference shock repay with certainty while high-risk agents with low second period preference shock default with probability \(d^{B^*}(l)\). Repayment leads to pooling contracts whereas default leads to separating contracts. Savers are offered separating contracts. As in the CCR case above, off equilibrium
beliefs are undefined and therefore the equilibrium level of borrowing is not unique.

(C) Either Type: When $\mu^S < \mu_1 < \mu^*$, either type A or type B equilibrium described above can arise.

**Participation Constraints**

For type A (CCR) equilibrium, the following ex-ante participation constraints must be satisfied.

Participation constraint for a low-risk consumer $\forall l \in (\underline{l}, \bar{l}]$

\[
\theta_H \ln(e + l) + \lambda \theta_H \ln(e - Rl) + (1 - \lambda) \theta_L \ln(e - Rl) + \ln(e - \pi^B \Delta + \mu_2^0(\pi^B - \pi^S)\Delta) \\
\geq \theta_H \ln(e) + (\lambda \theta_H + (1 - \lambda) \theta_L) \ln(e) + \ln(e - \pi^B \Delta + \mu_1(\pi^B - \pi^S)\Delta)
\]

Participation constraint for a high-risk consumer $\forall l \in (\underline{l}, \bar{l}]$

\[
\alpha \theta_H \ln(e + l) + \alpha \lambda \theta_H \ln(e) + \alpha(1 - \lambda) \theta_L \ln(e - Rl) + \lambda \ln(e - \pi^B \Delta) \\
+ (1 - \lambda) \ln(e - \pi^B \Delta + \mu_2^0(\pi^B - \pi^S)\Delta) \\
\geq \alpha \theta_H \ln(e) + \alpha(\lambda \theta_H + (1 - \lambda) \theta_L) \ln(e) + \ln(e - \pi^B \Delta + \mu_1(\pi^B - \pi^S)\Delta)
\]

For type B (Mixed strategy) equilibrium, the following ex-ante participation constraints must be satisfied.

Participation constraint for a low-risk consumer $\forall l \in [0, l^B]$ and $R = R(d^B)$

\[
\theta_H \ln(e + l) + \lambda \theta_H \ln(e) + (1 - \lambda) \theta_L \ln(e - Rl) + \lambda \ln(e - \pi^B \Delta + \mu^*(\pi^B - \pi^S)\Delta) \\
+ (1 - \lambda) \ln(e - \pi^B \Delta + \mu_2^0(\pi^B - \pi^S)\Delta) \\
\geq \theta_H \ln(e) + (\lambda \theta_H + (1 - \lambda) \theta_L) \ln(e) + \lambda \ln(e - \pi^B \Delta + \mu^*(\pi^B - \pi^S)\Delta) \\
+ (1 - \lambda) \ln(e - \pi^B \Delta + \mu_2^0(\pi^B - \pi^S)\Delta)
\]
Participation constraint for a high-risk consumer $\forall l \in [0, L^B]$

\[
\begin{align*}
\alpha \theta_H \ln(e + l) &+ \alpha(\lambda \theta_H + d^{B^*}(1 - \lambda) \theta_L) \ln(e) \\
&+ \alpha(1 - \lambda)(1 - d^{B^*}) \theta_L \ln(e - RL) + (\lambda + (1 - \lambda)(1 - d^{B^*})) \ln(e - \pi^B \Delta) \\
&+ d^{B^*}(1 - \lambda) \ln(e - \pi^B \Delta + \mu_2^0(\pi^B - \pi^S) \Delta) \\
\geq \alpha \theta_H \ln(e) &+ \alpha(\lambda \theta_H + (1 - \lambda) \theta_L) \ln(e) + \ln(e - \pi^B \Delta)
\end{align*}
\]

The two types of equilibrium presented in proposition 2 only arise conditional on positive amounts of borrowing and lending. If the utility to either type of consumer from not participating in credit markets at all is higher than the utility prescribed in either equilibrium above, then the result would be a no-borrowing-no lending equilibrium (proposition 1). The first type of equilibrium in proposition 2 replicates that presented in Chatterjee, Corbae and Rios-Rull (2008). Low-risk type agents with high preference shocks choose to repay the loan to signal their low risk. However, the signal does not result in unique identification. Note that savers receive pooling insurance contracts with higher premiums than agents who borrow and repay. This is because repayment always shifts the posterior to the right of the prior. Since no information is revealed about savers, the posterior and prior coincide.

Note that the equilibrium is not unique. Although the interest rate is pinned down, we can only identify the support of borrowing, not a unique level. This is because beliefs are indeterminate off the equilibrium path. Standard techniques of equilibrium refinement are not useful in this environment. Suppose there is a belief refinement that leads to identification off the equilibrium path, then the agent receives full insurance at the complete information premium. However, such identification implies there is no longer any incentive to repay loans. This is a remarkable result because in most signaling games, it is quite easy to obtain separation in types. Separation is not possible in the current environment because of the interplay between credit and insurance markets. Any separating behavior leads to certain default and is therefore undone by creditors who refuse to extend loans.

The second type of equilibrium, in which the high-risk consumers randomize between repayment and default is new to the current work. This equilibrium did not arise in Chatterjee, Corbae and Rios-Rull (2008)’s analysis because they only consider a subset of the parameter space, specifically, they only study the case where the prior probability of low-risk type is higher than $\mu^*$. As with the CCR case, only the support of loans can be determined. Since beliefs are undefined off the equilibrium path, we cannot obtain uniqueness.
Existence of Equilibrium

When are equilibria with positive levels of borrowing and lending likely to arise? To answer this question we need to look at the participation constraints more carefully. First note that some of the participation constraints are trivially satisfied. For instance under the CCR equilibrium, that is when $\mu_1 > \mu^S$, a low-risk consumer’s participation constraint always holds. For the mixed strategy case, that is when $\mu_1 < \mu^*$, it is again simple to check that a high-risk consumer’s participation constraint always holds.

Rewrite the participation constraint for a high-risk agent under the CCR equilibrium as follows

$$
\alpha \theta_H \ln(e + l) + \lambda \ln(e - \pi^B \Delta) + \alpha(1 - \lambda) \theta_L \ln(e - Rl) \\
+ (1 - \lambda) \ln(e - \pi^B \Delta + \mu^0_2(\pi^B - \pi^S)\Delta) \\
\geq \alpha \theta_H \ln(e) + \alpha(1 - \lambda) \theta_L \ln(e) + \lambda \ln(e - \pi^B \Delta + \mu_1(\pi^B - \pi^S)\Delta) \\
+ (1 - \lambda) \ln(e - \pi^B \Delta + \mu_1(\pi^B - \pi^S)\Delta)
$$

The above is trivially satisfied when the fraction of high preference shock consumers is low, that is when $\lambda \to 0$. However, when the fraction of high preference shock agents is very large, then the above condition may fail to hold. To see note that when $\lambda \to 1$ and $\mu_1 \to 1$, the above participation constraint simplifies to

$$
\alpha \theta_H \ln(e + l) + \ln(e - \pi^B \Delta) \geq \alpha \theta_H \ln(e) + \ln(e - \pi^S \Delta)
$$

For the participation constraint to hold, the loan amount $l$ that satisfies the constraint must also belong to the set $[L, \bar{L}]$ derived in lemma 6 from ex-post participation constraints. It is easy to show that the lower bound is easily met. Satisfying the upper bound on borrowing in turn implies that the following condition hold.

$$
\left[ \frac{e - \pi^S \Delta}{e - \pi^B \Delta} \right]^{\frac{1}{\alpha \theta_H}} - 1 < \frac{1}{(1 + r)} \left[ 1 - \left( \frac{e - \pi^B \Delta + \mu^*(\pi^B - \pi^S)\Delta}{e - \pi^S \Delta} \right)^{\frac{1}{\theta_L}} \right]
$$

This condition is not satisfied when the threshold on the fraction of low-risk consumers $\mu^*$ is too low.

Now consider the mixed strategy case. The participation constraint of a low-risk consumer can be stated as

$$
\theta_H \ln(e + l) + (1 - \lambda) \theta_L \ln(e - Rl) + (1 - \lambda) \ln(e - \pi^B \Delta + \mu^0_2(\pi^B - \pi^S)\Delta) \\
\geq \theta_H \ln(e) + (1 - \lambda) \theta_L \ln(e) + (1 - \lambda) \ln(e - \pi^B \Delta + \mu^*(\pi^B - \pi^S)\Delta)
$$

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The above condition is violated when the fraction of low-risk consumers is too low. It is easiest to see this for the case when $\mu_1 \to 0$. In this case the condition is violated for all values of $\lambda$. This is due to the interest rate begin exceedingly high, which makes it not worth borrowing irrespective of the potential gains in the insurance market. The interest rate for this case is derived in lemma 7 and goes to infinity as $\mu_1 \to 0$. Therefore when the fraction of low-risk consumers is extremely low, there is not borrowing or lending in equilibrium.

The bounds on loan amounts presented in lemma 6 and 7 are derived from second period incentive constraints, the ex-ante participation constraints cannot expand those sets. Note that the participation constraints do not help restrict loan amounts further than what appear in lemma 6 and lemma 7. Figure 2 presents an illustration of when the different types of equilibria can arise in the parameter space. In figure 2, NBNL refers to the no borrowing no lending case where participation constraint for some type of consumer is not satisfied. When the fraction of low-risk consumers is very small, then the participation constraint for the low-risk consumer is violated. This happens for all distributions of preference shocks and appears on the left hand margin of figure 2. On the other hand, when the fraction of low-risk consumers is extremely high and the fraction of high preference shock consumers is also very high, then the participation constraint of high-risk consumers is violated. This appears in the north-east corner of of figure 2. For other values of the parameter space, equilibria with positive amounts of borrowing and lending can arise.

**Corollary to Proposition 2:** $R < R(d_Bs(l))$ and $l > l_B$.

The corollary states that interest rates are higher in the mixed strategy case than the CCR equilibrium. This is obvious given the higher default rate on loans. Further, the upper bound on loans in the mixed strategy case is less than the lower bound on loans in the CCR equilibrium which implies the support for loans is disjoint in the two cases. The primary reason for the supports being disjoint is the fact that there is a strictly positive lower bound in the CCR equilibrium. In the CCR equilibrium the loan amount has to be high enough to ensure default by the high-risk agent with high second period preference shock.

The proprietary nature of data sets involving credit scores and insurance premiums makes it difficult (at present) to test the predictions of the model. It is interesting to note that at least in one important dimension the model’s predictions are confirmed in some commonly observed markets. Figure 3 and 4 are generated from information available at http://www.myfico.com. These figures illustrate how interest rates for home and auto loans vary with FICO credit scores. Figure 5 which is from Furletti (2003) shows the relationship between finance charge differentials from the highest risk consumer groups. All three figures share a common feature, interest rates are relatively flat at the higher end of credit scores but rapidly rise as the score falls. The model’s prediction, which is consistent with this observation is presented (for a parameterized example) in figure 6. In this case, the prior probability of low-risk type is analogous to a credit score. The sharp rise in interest rates occurs at the point when equilibrium outcomes involve mixed strategies. This is when default
rates become significantly higher. It must be stressed that the comparison is qualitative as can be noted from the extremely high interest rate for low credit scores.

4 Robustness of Equilibrium

4.1 Strategic Borrowing

A critical assumption made so far is that consumers are not allowed to save as well as borrow at the same time. Specifically, consumers with low first-period preference shocks are assumed to behave in a non-strategic manner. This assumption was made to keep the focus of the analysis on the interplay between borrowing, default and the insurance market. The assumption is motivated by the observation that consumers with persistent, high levels of unsecured (typically credit card) debt have small saving balances. Similarly, consumers with persistent, high levels of savings do not have large amounts of unsecured debt. In the current section, I relax this assumption. Consumers are free to both borrow as well as save. The main question of interest is: Is the equilibrium derived in proposition 2 robust to this variation? If not, what will the new equilibrium look like?

Claim 1: The equilibrium presented in proposition 2 is not an equilibrium when consumers are free to borrow and save at the same time.

The result is easy to see for the mixed strategy case. In this case, consumers with low first period preference shock (savers) are offered the same set of insurance choices as borrowers who default. A saver can profitably gain by taking out a loan and defaulting on it. This is because default leads to no loss in terms of insurance choices and the loan strictly increases consumption. Of course, they may choose to repay the loan, but the main point is that there is a profitable deviation from the equilibrium described in proposition 2. Therefore the mixed strategy case described in proposition 2 cannot be an equilibrium when consumers are free to both save as well as borrow at the same time.

Things are somewhat different under the CCR case. In this scenario, savers get insurance on better terms than borrowers who default. Borrowing followed by default implies a loss for savers in terms of insurance choices. However, note that savers can potentially get better terms on insurance by borrowing and repaying. This behavior is called strategic borrowing. It is ‘strategic’ because there is no need for these consumers to borrow. They actually want to save. The point of borrowing and repaying the loan is to get better terms on insurance. To formally show that the first part of proposition 2 (CCR case) cannot be an equilibrium when consumers are free to both save as well as borrow at the same time.

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Lemma 8: In any equilibrium with positive amounts of borrowing and lending ($l > 0$), if $d_{Hk}^i < 1$ then $d_{Lk}^i = 0$ for $i \in \{B, S\}$ and $k \in \{H, L\}$.

Call consumers with high first period preference shock genuine borrowers. Consumers with low first period preference shock are called savers. Lemma 8 ties the repayment behavior of genuine borrowers and savers of a given risk type for a given loan amount. Lemma 8 states that for a given

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risk type, if a genuine borrower repays a loan with positive probability, then a saver of the same risk type with the same second period preference shock must repay a similar loan with certainty. The intuition behind the result is that for a given risk type and second period preference shock, savers enter period 2 with more assets than borrowers. Since the risk type is the same, the gain in the insurance market from repayment is fixed for savers and borrowers. However, due to the higher asset level, the gain from default is less for savers than genuine borrowers. Therefore, if the genuine borrowers are repaying with positive probability, the savers must repay with certainty.

**Corollary to lemma 8:** In any equilibrium with positive amounts of borrowing and lending \((l > 0)\), if \(d_{Lk}^i > 0\) then \(d_{Hk}^i = 1\) for \(i \in \{B, S\}\) and \(k \in \{H, L\}\).

This implies that if a saver defaults with positive probability, then the corresponding genuine borrower with the same loan and the same second period preference shock must default with certainty. This is driven by the same intuition that lies behind lemma 8. For a given risk type, the gain in insurance market is fixed. But due to their higher asset holdings, the gain from default is less for savers than for genuine borrowers.

Lemma 8 implies the following repayment profile for the CCR case \((\mu_1 > \mu^*)\). If any saver engages in strategic borrowing with the same support of loans as genuine borrowers, then except a high-risk saver with a high second period preference shock, all other types repay the loans with certainty. If a low-risk saver engages in strategic borrowing, he repays the loan with certainty irrespective of second period preference shock. The expected payoff to a low-risk saver from *not engaging* in strategic borrowing is given by (from proposition 2)

\[
\theta_L \ln(e - a^*) + [\lambda \theta_H + (1 - \lambda) \theta_L] \ln(e + (1 + r)a^*) + \ln \left( e - \pi^B \Delta + \mu_1 (\pi^B - \pi^S) \Delta \right)
\]

Here, \(a^*\) refers to the optimal level of saving by consumers with low first-period preference shock when they do not engage in strategic borrowing. This optimal level of saving for either risk-type is given by

\[
a^* = \left[ \frac{[\lambda \theta_H + (1 - \lambda) \theta_L] (1 + r) - \theta_L}{[\lambda \theta_H + (1 - \lambda) \theta_L] - \theta_L} \right] \frac{e}{1 + r}
\]

The expected payoff to a low-risk saver from engaging in strategic borrowing *and repaying* is given by

\[
\theta_L \ln(e - a^* + \epsilon) + [\lambda \theta_H + (1 - \lambda) \theta_L] \ln \left( e + (1 + r)(a^* - \epsilon) - (R - 1 - r)l \right) + \ln \left( e - \pi^B \Delta + \mu_2^0 (\pi^B - \pi^S) \Delta \right)
\]

Here \(\epsilon\) refers to some adjustment in the level of savings to account for the loss in net wealth implied by interest payments on the loan.
Lemma 9: In any equilibrium with positive amounts of borrowing and lending \((l > 0)\), there exists \(\epsilon > 0\) such that expected utilities in equations 16 and 17 are equal.

Engaging in strategic borrowing and then repaying with certainty is costly because of interest payments. However, if this results in better terms on insurance then strategic borrowing provides a profitable deviation from proposition 2. For a given \(l\) in the support of loans, there exists an \(\epsilon\) that makes the payoff in 17 equal to that in 16. This implies that in the neighborhood of \(\epsilon\), strategic borrowing provides a profitable deviation for the low-risk saver from the equilibrium described in proposition 2.

Equilibrium with Strategic Borrowing

Claim 1 clearly states a negative result; the equilibrium in proposition 2 cannot be an equilibrium in the current framework when consumers are free to engage in strategic borrowing. The natural question to ask is what equilibrium outcomes arise with strategic borrowing? Unfortunately the answer to this question is less clear. I cannot obtain clear predictions without putting additional restrictions on the environment. Several outcomes are possible. For instance, if there is any lending in equilibrium, it will necessarily be associated with some level of strategic borrowing. What further complicates the analysis is that strategic borrowing may also involve default.

Table 1 presents the repayment strategies that can arise in equilibrium with strategic borrowing when the support of loans is common for both genuine as well as strategic borrowers. The values in table 1 follow from lemma 8 and its corollary. For any of these strategy profiles to constitute an equilibrium, they must also satisfy participation constraints for all types of consumers. I present a numerical example below to illustrate one such equilibrium. Specifically I present an equilibrium in which savers of both risk-types engage in strategic borrowing and both repay their loans with certainty. Compared to the original equilibrium when no strategic borrowing occurs, two things change: the posterior probability following repayment and the interest rate.

Parameterized Example: This example illustrates that the original equilibrium derived in proposition 2 is not robust to the variation in which strategic borrowing is allowed. I only illustrate the case where \(\mu_1 > \mu^*\) and equilibrium is of the CCR type. This is because as mentioned previously, there is always a profitable strategic-borrowing deviation in the mixed strategy case since defaulters are treated the same way as savers. The example is then extended to derive one equilibrium for the case when strategic borrowing is allowed.

Let \(\theta_L = 0.8, \theta_H = 1.5, \mu_1 = 0.7, \lambda = 0.5, \alpha = 1.2, r = 4\%, e = 10, \Delta = 8, \pi^S = 15\%, \pi^B = 22\%.\) For this case, the threshold \(\mu^* = 0.6621\), the equilibrium amount of savings by either risk-type is \(a^* = 1.9527\) and the interest rate on loans is \(R = 22\%.\) It is easy to show that this equilibrium satisfies all participation constraints. To illustrate that this equilibrium is not robust to allowing savers to engage in strategic borrowing, we simply have to find a profitable deviation. The equilibrium level of borrowing is not uniquely determined in equilibrium. For this example, I assume
Table 1: Default probabilities with common support for loans

<table>
<thead>
<tr>
<th>Prior</th>
<th>Type $S_{jH}$</th>
<th>Type $S_{jL}$</th>
<th>Type $B_{jH}$</th>
<th>Type $S_{jL}$</th>
<th>Borrowing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\mu_1 &gt; \mu^*)$</td>
<td>$d^S_{HH} = 0$</td>
<td>$d^S_{HL} = 0$</td>
<td>$d^B_{HH} = 1$</td>
<td>$d^B_{HL} = 0$</td>
<td>genuine</td>
</tr>
<tr>
<td></td>
<td>$d^S_{LL} = 0$</td>
<td></td>
<td></td>
<td>$d^B_{LL} = 0$</td>
<td>strategic</td>
</tr>
<tr>
<td>$(\mu_1 &lt; \mu^*)$</td>
<td>$d^S_{HH} = 1$</td>
<td>$d^S_{HL} \in {0,1}$</td>
<td>$d^B_{HH} = 1$</td>
<td>$d^B_{HL} \in {0,1}$</td>
<td>genuine</td>
</tr>
<tr>
<td></td>
<td>$d^S_{LL} = 0$</td>
<td></td>
<td></td>
<td>$d^B_{LL} = 0$</td>
<td>strategic</td>
</tr>
</tbody>
</table>

that is it is the same as the highest level of borrowing that can be supported in equilibrium. This makes the existence of strategic borrowing particularly stark because it forces strategic-borrowers to borrow the highest possible amount borrowed by genuine borrowers. It is again easy to check that one profitable deviation for a low-risk saver with a low first-period preference shock is to reduce saving by $\epsilon = 0.1055$ and borrow (and repay) the identical amount as genuine borrowers which in this case is $l = 0.2914$.

In contrast, the equilibrium with strategic borrowing where savers of both risk-types borrow and repay involves an interest rate of $R = 12\%$, borrowing amount $l = 0.2914$ which is unchanged from the original, net savings of $a_S = 1.7801$ by a saver of low-risk type and net savings $a_B = 1.7449$ for the high-risk type. Only a genuine borrower of the high-risk type defaults following a high second-period preference shock. Note that although the amount borrowed by both savers is identical, their adjusted levels of savings are different with strategic borrowing because the consumption-smoothing cost or utility cost of engaging in strategic borrowing is different for the two groups. Further note that the interest rate with strategic borrowing is less than the interest rate where strategic borrowing is not permitted. This is because in this particular equilibrium with strategic borrowing, all strategic borrowers repay their loans with certainty. Therefore in this particular case, strategic borrowing reduces the borrowing costs for all agents. However this comes at a price, the posterior probability following repayment is lower than the case without strategic borrowing. This implies the gain in the insurance market from repayment is lower with strategic borrowing.

This result is related to a puzzle: Why do consumers borrow at the high interest rates often charged by credit cards when they have cheaper options or when they don’t actually need to borrow. This result suggests that one reason people borrow at high interest rates is to create a favorable credit score which helps guarantee lower interest rates or insurance premiums in the future.

Thus far, we have only considered the case that savers borrow from the same support as genuine borrowers. This is an acceptable route as long as the objective is to check for and derive profitable deviations from a prescribed equilibrium. However, in general, savers may choose to borrow from a different support. Equilibrium in this case will have different features than those described in lemma 8 and lemma 9 and is presented later.

The case in which savers borrow from a different support from genuine borrowers is structurally different from the case where they borrow from the same support. When the support is
common for savers and genuine borrowers, savers of just a single risk-type can engage in strategic borrowing by themselves. This is possible because they can pass themselves off as genuine borrowers. In contrast, note that when savers borrow from a different support, savers of both risk types must engage in strategic borrowing. Otherwise risk-type is identified from the different support and such loans will not be extended (since they will not be repaid).

**Lemma 10:** Suppose there is an equilibrium with positive amounts of strategic borrowing from a support that is disjoint from the support of loans for genuine borrowers. Then

\[(1 - \lambda)(d^S_{LH} - d^B_{LH}) < \lambda(d^B_{LL} - d^S_{LL}).\]

Lemma 10 states that if savers borrow from a support that is different from genuine borrowers, then there is no equilibrium in which all strategic borrowers repay or default with certainty. In any equilibrium with strategic borrowing from a disjoint support, some strategic borrowers must repay and some must default. Further, the posterior probability following repayment must be higher than the prior and the posterior following default must be lower than the prior. The condition in lemma 10 ensures that this is indeed the case. The next result relates the supports for strategic borrowing and genuine borrowing.

**Lemma 11:** Suppose there is an equilibrium in which the supports for strategic and genuine borrowing are disjoint. Let the support for strategic borrowing be \((l^F, \bar{l}^F)\) and that for genuine borrowing be \((l, \bar{l})\). Then, \(l^F > \bar{l}\).

Lemma 11 presents a surprising result. It states that if the supports for genuine and strategic borrowing are disjoint, then the levels of strategic borrowing must be strictly higher than the levels of genuine borrowing. The intuition behind the result is the same as that for lemma 8. Strategic borrowers (savers) enter the second period with positive amounts of assets. From lemma 10, they cannot repay or default with certainty. For a saver to default, the gain from default must exceed the gain in the insurance market following repayment. For a given loan amount, the gains in the insurance market simply depend on posterior beliefs regarding risk-type. However, for a given loan amount, the gain from default is decreasing in the level of assets. Therefore, the higher the accumulated savings, the higher the strategic loan amount needs to be to make default feasible.

Table 2 lists the possible default strategies when the support for strategic and genuine borrowing are disjoint. Tables 1 and 2 suggest that there are several possible equilibrium outcomes. For instance, from table 1 when \(\mu_1 > \mu^*\) there are two possible outcomes, one in which high-risk savers default following a high second period preference shock and in which they repay with certainty. Without placing additional restrictions on the environment I cannot specify which outcome will arise. In fact, without making further assumptions we cannot even state when strategic borrowing occurs and which, if any, risk-type choose to engage in it. A full characterization of equilibrium is therefore not presented. This is not because a full characterization is outside the
Table 2: Default probabilities with disjoint support for loans

<table>
<thead>
<tr>
<th>Prior</th>
<th>Type $S_{jH}$</th>
<th>Type $S_{jL}$</th>
<th>Type $B_{jH}$</th>
<th>Type $S_{jL}$</th>
<th>Borrowing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\mu_1 &gt; \mu^*)$</td>
<td>$d^S_{HH} = 0$</td>
<td>$d^S_{HL} = 0$</td>
<td>$d^B_{HH} = 1$</td>
<td>$d^B_{HL} = 0$</td>
<td>genuine</td>
</tr>
<tr>
<td>$(\mu_1 &lt; \mu^S)$</td>
<td>$d^S_{HH} = 1$</td>
<td>$d^S_{HL} \in {0, 1}$</td>
<td>$d^B_{HH} = 1$</td>
<td>$d^B_{HL} \in (0, 1]$</td>
<td>strategic</td>
</tr>
</tbody>
</table>

scope of the current project, but simply because the main message here is to show the impact of strategic borrowing on borrowing and repayment strategies.

4.2 Deletion of Credit Histories

In this subsection I analyze the possibility that agents have the means to erase their credit history without any cost. The decision to erase records, is made before the insurance market opens. The question again is whether the original equilibrium presented in proposition 2 is robust to this variation. To break ties I assume that when consumers are indifferent between deleting their record and not, they choose not to.

Claim 2: Suppose consumers have the option to erase their credit market history with no direct cost. The equilibrium in proposition 2 is robust to this change if insurance firms treat consumers with no credit history as the high-risk type. The equilibrium is not robust to deletion of credit histories when insurance firms treat consumers of unknown type as if they were of average risk.

Since we are checking for robustness, I do not derive equilibrium strategies from scratch. Instead, given the change in the environment I simply check for profitable deviations from the prescribed equilibrium. In this environment, savers have no decision to make beyond the first period. For borrowers there are two cases: (i) $0 < \mu_1 < \mu^S$ where only mixed strategy equilibria can arise with positive amounts of lending and (ii) $\mu_1 > \mu^*$ where only the CCR equilibrium can arise with positive levels of lending. There is of course a third case for intermediate values of $\mu_1$. The arguments from the first two cases will remain valid since only one of either type of equilibrium can arise for this intermediate case.

Recall that although savings are unobservable, consumers who save still have a credit history, one associated with zero level of borrowing. The result of erasing one’s history therefore is to be treated as an unknown type. The robustness of the original equilibrium critically depends on how insurance firms treat consumers with unknown type. Suppose insurers treat consumers of unknown type to be of the high-risk type. From proposition 2, when $0 < \mu_1 < \mu^S$, agents with high second period preference shock default with certainty. Low-risk agents with low second period preference shock repay with certainty whereas high-risk agents with low second period preference shock randomize. Default results in the same menu of insurance contracts that are offered to savers.
High-risk agents who default are indifferent between having the records erased or kept. Low-risk consumers who default however strictly prefer not to erase their records. This is because erasure results in full insurance at the highest possible premium. If they choose to leave the record intact, they are offered a menu of contracts which includes partial insurance $X^*$ at a low premium $\pi^S$ which they prefer to full insurance at the highest premium $(L, \pi^B)$. Repayment however leads to attractive pooling contracts. Those who repaid strictly prefer their records to be kept intact since these lead to better terms on insurance. Therefore there is no profitable deviation by erasing the records for any type of consumer. Note that for $0 < \mu_1 < \mu^S$ the above argument continues to hold even if insurance firms treat consumers of unknown type to be of average risk. Average risk refers to the prior fraction of low-risk consumers which is $\mu_1$ and since this results in a choice between separating contracts, there is no profitable deviation from erasing credit history.

Now consider the case when $\mu_1 > \mu^*$. Savers are offered pooling contracts. All agents who repayed their debt again strictly prefer the records to be kept since this leads to pooling contracts on terms better than what is offered to savers. Suppose insurance firms treat consumers of unknown type as the high-risk type. Again no agent finds it profitable to erase their borrowing record. Doing so results in insurance at the highest possible premium. However, if insurance firms treat consumers of unknown type to be of average-risk, then erasing credit histories is profitable. In this case, agents who default and erase their records are treated the same way as savers. Therefore the original equilibrium is not robust to the option of deleting records when insurance companies treat unknown consumers to be of average-risk.

This result leads to a puzzle. In the United States, consumers without credit histories are often treated as if they belong to the worst possible risk type. This is evident in cell phone and used car markets. In the cell phone market consumers without credit history are often required to make large deposits in order to get service. Typically, these deposits are in the $600-$800 range and are returned after a year of service. In the used car market, consumers without credit histories are charge interest rates of close to 30% per year. This is an extraordinarily large figure considering that a car loan is a (partially) secured loan. In the United States it is very difficult to erase one’s credit history. Credit histories are identified through and associated with a consumer’s social security number and the records are permanent. Only some forms of bankruptcies are erased from credit reports, although after a significant amount of time, typically 7 years or longer. The puzzle is that if credit histories are so hard to erase and the fraction of new consumers small compared to existing ones, why are consumers without credit histories treated so badly? Perhaps this is suggestive of ways consumers are able to manipulate credit histories. One mechanism through which credit histories can be manipulated are through joint loans. For instance a husband and wife may jointly finance a house. Even if one of them has a damaged credit history, the joint loan tends to favorable offers than an independent loan. This issue is also related to the optimal length of credit records. One particular issue of interest to policymakers is how long should negative reports,
such as bankruptcies, remain on a consumer’s credit file? If the length is too short, insurance companies and creditors will continue to treat consumers of unknown type as if they were the worst possible type.

4.3 Learning about Information Linkage

In this section I explore the possibility that agents may be unaware of the link between their credit market behavior and insurance premiums. I address the following question; Is there any mechanism through which unaware agents can endogenously learn about the link between credit and insurance markets?

Suppose agents are unaware of the informational link between credit and insurance markets. If default is costless, then there cannot be any learning.

Claim 3: Suppose there is a cost \( \phi > 0 \) for default. Then there are four thresholds \( \phi_1, \phi_2, \phi_3 \) and \( \phi_4 \) such that

- if \( \phi \in [\phi_1, \phi_2) \) or \( \phi \in [\phi_3, \phi_4) \), then agents will learn about the informational link.
- for all other values of \( \phi \) no learning occurs.

The four thresholds correspond to the gain from default for different agents as explained below. When agents are unaware of the informational link, they default with certainty. This in turn implies the unsecured credit market is inactive. With no active credit market, insurers gain no information about types. There can be no learning in this environment. Therefore, for any learning to occur, I must assume that there is an exogenous cost to default. This cost is represented by \( \phi \). This cost is public information and the same for all agents.

To illustrate how learning can occur, consider the case where the fraction of high types is such that insurance companies are offering pooling contracts, that is the case \( \mu_1 > \mu^* \). Recall that the gain from default for any type is given by

\[ \theta_2[u(e) - u(e - Rl)] \]

Define \( V^{Def} \equiv [u(e) - u(e - Rl)] \). From assumptions 1 and 2

\[ \theta_L V^{Def} = \phi_1 < \alpha \theta_L V^{Def} = \phi_2 < \theta_H V^{Def} = \phi_3 < \alpha \theta_H V^{Def} = \phi_4 \]

These four (in order) represent the gain from default for low-risk type with low preference shock, high-risk type with low preference shock, low-risk type with high preference shock and finally the high-risk type with high preference shock. Consider utility cost from default \( \phi \in [\phi_1, \phi_2) \). With this level of cost, low-risk agents with low preference shock choose not to default. Insurance companies can now make inferences about type from repayment. This results in an offer of the
insurance contract \((\Delta, \pi^S)\) for the low-risk agent who repayed the loan. This surprises the agent and the agent learns about the link between repayment and insurance premiums. Note however that the posterior probability following default changes too. Specifically, the posterior probability of low-risk type following default is less than the prior. This implies the agents that defaulted on their loans also get a surprise when their premiums are unexpectedly raised. The change in insurance may be small, as in an increase in premium but still a pooling contract, or the change could be drastic when the posterior falls below \(\mu^*\). In the latter case agents who defaulted are offered a menu of separating contracts.

This mechanism assumes a high degree of sophistication on the part of agents. I believe that when viewed in a repeated setting, the mechanism is certainly plausible. Agents are obviously more likely to draw the correct inference from changes in insurance premiums when such ‘surprises’ occur over a period of time. For instance, every time a low-risk agent draws a high second period preference shock, he defaults and is faced with a high insurance premium. At other times the agent has a low shock, repays the loan and consequently is offered a low premium.

The case when \(\mu_1 < \mu^*\) or \(\phi \in [\phi_3, \phi_4)\) can be similarly explained. Note however, that in the current environment, when \(\phi \in [\phi_2, \phi_3)\), then repayment and default both lead to the same posterior as the prior and therefore no learning occurs.

One way of implementing such learning mechanisms may be to tie initial offers for insurance with suggestions that offers may be contingent on past credit market behavior. One example when this happens in reality is when consumers are denied credit or insurance based on information contained in credit reports.

5 Conclusion

I study the interaction between credit and insurance markets. It is assumed that consumers are different on two dimensions: actuarial risk and preference type. Preference type refers to either high or low preference shocks in each of two time periods. Agents have the right to default on their debt without any exogenous cost. The support for preference shocks depends on an agent’s risk type. Specifically I assume that high-risk agents are more likely to find themselves in situations that can trigger default. Repayment of debt therefore acts as a signal of low risk.

Conditional on positive amounts of borrowing and lending, two types of equilibrium can arise. When the fraction of low-risk consumers is above a certain threshold, then all default decisions are in pure strategies. Specifically, above the threshold, high-risk agents with high second period preference shocks default with certainty. All other agents repay their debt with certainty. When the fraction of low-risk consumer is below the threshold, then all consumers with a high second period preference shock default with certainty. Low-risk consumers with low second period preference shock repay with certainty. However, high-risk consumers have to randomize between default and repayment following a low second period preference shock. In certain regions of the parameter
space, either type of equilibrium can arise.

Most of the quantitative papers in the credit literature only ‘allow for’ unique predictions. The multiplicity of equilibria derived here is therefore important for empirical work. Most of the literature finds that better information improves welfare. In the current environment, full information reduces welfare through credit market failure. If the costs of default, utility or otherwise, are indeed decreasing as in Athreya (2004) and Livshits, MacGee and Tertilt (2007), then perhaps we should start paying more attention to the other costs which are increasing. Credit-based auto, home and mortgage insurance premiums are all sources of indirect costs of poor credit market decisions.

The only way to create a credit history is to borrow. Given the importance of credit history in consumer finance, there are incentives for agents to engage in borrowing purely to create a favorable credit record. I call this strategic borrowing. Strategic borrowing results in a rich set of equilibrium outcomes. I present one such equilibrium and contrast it with the outcome when strategic borrowing is not permitted. I find that when strategic borrowing is allowed, it results in a trade off. On the one hand strategic borrowing reduces the borrowing costs for all agents. However, strategic borrowing limits the gains in the insurance market from signaling through repayment. As an experiment, I consider the possibility that agents can erase their credit history. This is motivated by two facts: (i) Bankruptcies are often deleted from credit reports after a certain amount of time, and (ii) Consumers can manipulate the use of credit records by applying for a loan using their spouse’s credit record. I find that such behavior is unlikely to arise in equilibrium when insurers treat consumers with unknown type as if they were the worst possible type. This however leads to a puzzle. In the United States credit histories are linked to social security numbers and are quite difficult to manipulate. If records are hard to manipulate or erase, then there is no incentive for insurance companies to treat unknown consumers too harshly. This issue is obviously related to the policy question of just how long should negative reports, such as bankruptcies, be included in credit reports. Finally I address the possibility that consumers are unaware of the information link between credit market behavior and insurance premiums. I find that when there are exogenous costs to default and these costs are within some broad ranges, consumers can endogenously learn about the information link. In particular learning occurs through unexpected changes in the terms of coverage of insurance contracts. Learning in this manner requires a high degree of sophistication. Recent work in this area however suggests that the process is certainly plausible. Agarwal, Driscoll, Gabaix and Laibson (2008) document evidence of learning in the credit card market. In their study learning is driven by payment of avoidable fees such as late payment fines.

Future Research: One extension of the current framework is to analyze the impact of integration in credit and insurance firms. The merger of Citigroup and Travelers Insurance is one instance. Consolidation in the financial services sector has increased at a rapid pace. The current paper provides a useful apparatus to study the impact of such consolidations. Recent work by Cutler,
Finkelstein and McGarry (2008) presents a model of preference heterogeneity in insurance markets to explain an empirical puzzle. They conjecture that in some insurance markets consumers who represent low risk are also more risk averse and in such cases, low-risk individuals purchase more insurance than high-risk individuals. This structure may also be useful for the joint analysis of credit and insurance markets.

Proofs

Proof of Lemma 1

The proof is by contradiction. Suppose \( l^B_H \neq l^S_H \). Then irrespective of the default decision, types are identified, \( \text{Prob}\{i = S|l = l^S_H\} = 1 \) and \( \text{Prob}\{i = S|l = l^B_H\} = 0 \). With no cost of default, all borrowers default on their loans. This strategy would be undone by banks in the first period. Banks will refuse to extend any loans and therefore borrowing different amounts cannot be part of any equilibrium.

Proof of Lemma 2

Suppose \( d^i_{HL} > 0 \). Assume \( d^i_{HH} < 1 \). Let the gain from repayment in the insurance market for risk-type \( i \) be represented by \( \Delta V^i \). From lemma 1 all borrowers will borrow identical amounts. Let this amount be represented by \( l \). \( d^i_{HL} > 0 \) implies the gain from default must be at least as large as the gain from repayment through the insurance market. This implies

\[
\theta^i_L[\ln(e) - \ln(e - l)] \geq \Delta V^i
\]  

(20)

On the other hand, \( d^i_{HH} < 1 \) implies,

\[
\theta^i_H[\ln(e) - \ln(e - l)] \leq \Delta V^i
\]  

(21)

Since \( \theta^i_H > \theta^i_L \), (20) and (21) contradict each other. Therefore, \( d^i_{HH} = 1 \).

Proof of Lemma 3

Suppose \( d^i_{HL} > 0 \). Assume \( d^i_{HH} < 1 \). From figure (1) the potential gain in the insurance market following either default or repayment is always higher for the agent of the high-risk type. Denote this by the inequality

\[
\Delta V^B \geq \Delta V^S
\]  

(22)

\( d^i_{HL} > 0 \) implies,

\[
\alpha \theta^i_L[\ln(e) - \ln(e - l)] \geq \Delta V^B
\]  

(23)
and \( d_{HH}^i < 1 \) implies
\[
\theta_H \left[ \ln(e) - \ln(e - l) \right] \leq \Delta V^S
\]  \hspace{1cm} (24)

From (22) and using \( \alpha \theta_L < \theta_H \) (from assumption 2), (23) and (24) contradict each other. Therefore \( d_{HH}^i = 1 \).

**Proof of Lemma 4**

Suppose \( d_{HL}^S > 0 \) and let \( d_{HL}^B = 0 \). From lemma 2, \( d_{HH}^S = 1 \). Now the posterior following a repayment is given by
\[
\mu_2^0 = \frac{\mu_1(1 - \lambda)(1 - d_{HL}^S)}{\mu_1(1 - \lambda)(1 - d_{HL}^S) + (1 - \mu_1)(1 - \lambda) + (1 - \mu_1)\lambda(1 - d_{HL}^B)}
\]

It is easy to show that there are no values of \( d_{HL}^B \) for which this posterior would satisfy condition 1. Therefore \( d_{HL}^B > 0 \). By lemma 2 this implies \( d_{HH}^B = 1 \). Now the posterior following repayment is given by
\[
\mu_2^0 = \frac{\mu_1(1 - \lambda)(1 - d_{HL}^S)}{\mu_1(1 - \lambda)(1 - d_{HL}^S) + (1 - \mu_1)(1 - \lambda) + (1 - \mu_1)\lambda(1 - d_{HL}^B)}
\]

The only way the above relationship can satisfy condition 1 is if \( d_{HL}^B > d_{HL}^S \).

**Proof of Lemma 5**

Suppose \( d_{HH}^B < 1 \). From lemma 2, this implies \( d_{HL}^B = 0 \). From lemma 4, \( d_{HL}^S = 0 \) implies \( d_{HH}^S = 0 \). Suppose \( d_{HH}^S > 0 \). The posterior probability following a default is given by
\[
\mu_2^1 = \frac{\mu_1 d_{HL}^S}{\mu_1 d_{HL}^S + (1 - \mu_1)\lambda d_{HL}^B}
\]

It is easy to show that this posterior does not satisfy condition 1 for any choice of \( d_{HH}^B \). Therefore \( d_{HH}^S = 0 \).

**Proof of Lemma 6**

The gain from default for a low-risk agent with low second-period preference shock is
\[
\theta_L \left[ \ln(e) - \ln(e - Rl) \right]
\]

The gain from a pooling equilibrium resulting from repayment is given by
\[
\ln(e - \left( \pi^B - \mu_2^0 \left( \pi^B - \pi^S \right) \right) \Delta) - \ln(e - \left( \pi^B - \mu^* \left( \pi^B - \pi^S \right) \right) \Delta)
\]

From lemma 3, for a low-risk agent with high second-period shock to be repaying with any
probability, a high-risk agent with low preference shock must be repaying with certainty. Using the zero expected profit condition for the asset market, we can obtain the interest rate $R$ for the part of the parameter space where the low-risk agent with high shock is repaying as

$$R = 1 + \frac{r + \lambda(1 - \mu_1)}{\mu_1 + (1 - \lambda)(1 - \mu_1)} \quad (25)$$

The interest rate is independent of the amount of debt in the region where the low-risk agent with low preference shock is repaying. To obtain the threshold on the prior, note that repayment leads to pooling insurance contracts only if the posterior following repayment $\mu_2^B$ exceeds $\mu^*$. This places an implicit lower bound on the prior $\mu_1$ and the bound is represented by $\mu^S$ which is given below

$$\mu^S = \frac{(1 - \lambda)\mu^*}{1 - \lambda\mu^*}$$

Note that $\mu^S \leq \mu^*$. For a low-risk agent with low second-period preference shock to be indifferent between default and repayment, the loan amount $l$ must solve

$$\theta_L \ln(e - Rl) + \ln(e - (\pi^B - \mu_0^0(\pi^B - \pi^S)) \Delta) = \theta_L \ln e + \ln(e - (\pi^B - \mu^*(\pi^B - \pi^S)) \Delta).$$

$$l = \frac{e}{R} \left[ 1 - \left( \frac{e - \pi^B \Delta + \mu^*(\pi^B - \pi^S) \Delta}{e - \pi^B \Delta + \mu_0^0(\pi^B - \pi^S) \Delta} \right)^{\frac{1}{\theta_L}} \right] \quad (26)$$

Similarly, for a high-risk agent with high second-period preference shock to be indifferent between default and repayment, the loan amount $l$ must solve

$$\alpha \theta_H \ln(e - Rl) + \ln(e - (\pi^B - \mu_0^0(\pi^B - \pi^S)) \Delta) = \alpha \theta_H \ln e + \ln(e - \pi^B \Delta).$$

$$l = \frac{e}{R} \left[ 1 - \left( \frac{e - \pi^B \Delta}{e - \pi^B \Delta + \mu_0^0(\pi^B - \pi^S) \Delta} \right)^{\frac{1}{\alpha \theta_H}} \right] \quad (27)$$

The above indifference conditions for the two risk types provide the bounds on loan amounts presented in lemma 6. Assumption 3 guarantees that $l < \bar{l}$.

**Proof of Lemma 7**

From the discussion following lemma 5, $d_{jH}^B = 1$ and $d_{jL}^S = 0$. From lemma 6, $d_{jH}^S = 1$. 

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Suppose \( d^B_{jL} = 0 \). Then,

\[ \mu_2^1 = 1 \]

This implies repayment leads to full insurance at the lowest possible cost. The high-risk type with low shock can have a profitable deviation by repaying. Therefore \( d^B_{jL} > 0 \)

Now suppose \( d^B_{jL} = 1 \). Then,

\[ \mu_2^1 < \mu^* \]

This implies repayment leads to the same menu of insurance contracts as before. The agent can profitably deviate by defaulting. Therefore \( d^B_{jL} < 1 \). Hence the high-risk agent with low shock must randomize.

The posterior probability following repayment is given by

\[
\mu_2^0(d^{B*}) = \frac{\mu_1(1 - \lambda)}{\mu_1(1 - \lambda) + (1 - \mu_1)(1 - \lambda)(1 - d^{B*})} = \frac{\mu_1}{\mu_1 + (1 - \mu_1)(1 - d^{B*})}
\]  

(28)

For the high-risk agent to be willing to randomize he must be indifferent between defaulting and repayment the loan. This implies the expected utility from exercising either option must be the same

\[
\alpha \theta L \ln(e - Rl) + \ln(e - (\pi^B - \mu_2^0(d^{B*})(\pi^B - \pi^S)) \Delta) = \alpha \theta L \ln(e) + \ln(e - \pi^B \Delta)
\]

(29)

The zero expected profit condition for banks implies that the interest rate also depends on the probability of default. This is given by

\[
R(d^{B*}) = 1 + \frac{r + \lambda + (1 - \mu_1)(1 - \lambda)d^{B*}}{(1 - \lambda)(\mu_1 + (1 - \mu_1)(1 - d^{B*}))}
\]

(30)

Substituting equation 30 into equation 29 for \( R \), \( d^{B*}(l) \) can be obtained as an implicit function of the level of debt.

For the any agent to be repaying we must have \( \mu_2^0 \geq \mu^* \). From equation 28, note that \( \mu_2^0 \rightarrow \mu^* \) as \( d^{B*} \rightarrow 0 \). This implies the threshold \( \mu^S \) is equal to \( \mu^* \) when \( \lambda \rightarrow 0 \). To find the support for loans note that the interest rate is increasing in the rate of default. The lowest interest rate is associated with \( d^{B*} = 0 \). This value is represented by \( R^B \).

\[
R^B = 1 + \frac{r + \lambda}{1 - \lambda}
\]

The highest loan amount that can be supported is one that results from the lowest interest
rate which happens when $d^{B^*} = 0$ and $R = R^B$. This upper bound is denoted by $l^B$ and is obtained by plugging in $R^B$ and $\mu_2^0 = \mu^*$ into equation 29.

Proof of Proposition 2

All agents are offered a choice of two asset contracts: A loan contract with gross interest rate $R$ and a savings contract with gross interest rate $(1 + r)$. An agent can always choose not to borrow or save. Therefore both types of equilibrium described in proposition 2 will fail to be equilibrium if it is profitable for any agent not to participate in credit markets. Conditional on borrowing in the first period, and the discussion following lemma 5, $d^{B^*}_{jH} = 0$ and $d^{S}_{jL} = 1$ for all $\mu_1$.

From lemma 6 and lemma 7,

$$
\begin{align*}
    d^{S}_{jH} &= \begin{cases} 
        1, & \text{for } \mu_1 < \mu^* \\
        0, & \text{otherwise}
    \end{cases} \\
    d^{B}_{jL} &= \begin{cases} 
        d^{B^*}(l), & \text{for } \mu_1 < \mu^* \\
        0, & \text{otherwise}
    \end{cases}
\end{align*}
$$

The corresponding interest rates and support for loans are derived in lemma 6 and lemma 7. Given these strategies and insurance market outcomes, each agent decides whether to accept the loan/saving contract offered. The insurance market outcomes follow directly from above strategies and beliefs derived for any continuation equilibrium. All profitable deviations are ruled out by lemmas 1-8.

Except for the case where there is no borrowing and no loans are extended, equilibrium is not unique. This is because off equilibrium beliefs are undefined. To see this, consider the CCR equilibrium. Suppose there is a unique level of borrowing associated with this equilibrium. In such a case, what would beliefs be following a different level of debt is observed? If such off-equilibrium beliefs are associated with a particular risk-type, say by using a refinement concept such as Cho and Kreps’ (1987) intuitive criterion, then there is no incentive to repay the loan. This implies such loans will not be extended.

Proof of Corollary to Proposition 2

In the CCR equilibrium, the lower bound of the support is chosen such that it guarantees default by a high-risk consumer following a high second period preference shock. This lower bound can be obtained from the consumer’s indifference condition between repayment and default. The lower bound (in general the support) depends on the prior fraction of low-risk type $\mu_1$.

$$
I(\mu_1)R = e \left[ 1 - \left\{ \frac{e - \pi^B \Delta}{e - \pi^B \Delta + \mu_2^0(\pi^B - \pi^S)\Delta} \right\}^{\frac{1}{\alpha \theta_H}} \right] \tag{31}
$$

For the mixed strategy case, the loan amount is such that it makes a high-risk agent with a low second period preference shock indifferent between repayment and default. Representing this amount by $l^B$, it solves

34
\[ l^B(\mu_1) R(d^{B*}) = e \left[ 1 - \left\{ \frac{e - \pi^B \Delta}{e - \pi^B \Delta + \mu^0_2 (\pi^B - \pi^S) \Delta} \right\}^{\frac{1}{\pi L}} \right] \] (32)

Note that the posterior following repayment, \( \mu^0_2 \), is different in the two cases. It is straightforward to check that the right hand side of equation 31 is lowest and the right hand side of equation 32 is highest for \( \mu_1 \to \mu^* \). At this point, the interest rates are given by \( R \) for the CCR case in equation 12 and \( R^B \) for the mixed strategy case in equation 15. \( R^B \) is the lowest value for \( R(d^{B*}) \). Therefore, \( R < R(d^{B*}) \) in all cases. To compare the supports, note that as \( \mu_1 \to \mu^* \), equations 31 and 32 imply (using \( \theta_H > \theta_L \))

\[ l^B R > l^B R^B \Rightarrow l > l^B \text{ since } R < R^B \]

Therefore the supports are disjoint.

**Proof of lemma 8**

Only the case for a low-risk consumer is proved. The proof for a high-risk consumer is identical. For a low-risk borrower to be repaying with positive probability the following must hold in period 2

\[ \theta_k[\ln e - \ln(e - Rl)] \leq \ln \left( e - \pi^B \Delta + \mu^0_2 (\pi^B - \pi^S) \Delta \right) \]
\[ - \ln \left( e - \pi^B \Delta + \mu^* (\pi^B - \pi^S) \Delta \right) \text{ for } k \in \{L, H\} \] (33)

A low-risk saver enters the second period with higher assets. Let the higher assets be represented by \( e' > e \)

\[ e' > e \Rightarrow \ln(e') - \ln(e' - Rl) < \ln(e) - \ln(e - Rl) \] (34)

Comparing 33 and 34 for \( k \in \{L, H\} \)

\[ \Rightarrow \theta_k[\ln(e') - \ln(e' - Rl)] < \ln \left( e - \pi^B \Delta + \mu^0_2 (\pi^B - \pi^S) \Delta \right) \]
\[ - \ln \left( e - \pi^B \Delta + \mu^* (\pi^B - \pi^S) \Delta \right) \]
\[ \Rightarrow d^S_{Lk} = 0 \]

Therefore, if \( d^H_{Hk} < 1 \) then \( d^L_{Lk} = 0 \)

**Proof of lemma 9**
Setting 16 and 17 equal to each other we obtain

\[
\begin{bmatrix}
e - a^* + \epsilon \\
e - a^*
\end{bmatrix}^{\theta_L} \begin{bmatrix}(e - \pi^B \Delta + \mu_0^B(\pi^B - \pi^S)\Delta) \\
(e - \pi^B \Delta + \mu_1(\pi^B - \pi^S)\Delta)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
e + (1 + r)a^* \\
e + (1 + r)(a^* - \epsilon) - (R - r - 1)l
\end{bmatrix}^{(\lambda \theta_H + (1 - \lambda) \theta_L)}
\]

(35)

Given \(l\) and \(a^*\), we can find an \(\epsilon\) that solves the above equation. This is when a low-risk saver is indifferent between engaging in strategic borrowing and not. Therefore in some neighborhood of \(\epsilon\), the low-risk saver will strictly prefer to deviate from the equilibrium in proposition 2 and engage in strategic borrowing. The prescribed strategies in proposition 2 then do not constitute an equilibrium.

**Proof of claim 1**

As stated in the text, it is easy to derive a profitable deviation in the mixed strategy case. In the mixed strategy case, borrowing (by savers) any amount from the support of loans and defaulting with certainty is one profitable deviation from the equilibrium presented in proposition 2. This strategy strictly increases consumption and involves no loss in insurance outcomes.

For the CCR case, from lemma 8, if a low-risk saver borrows from the same support as genuine borrowers, then the saver must repay with certainty. From 35 we can find an \(\epsilon\) adjustment in savings that makes a low-risk saver just indifferent between engaging in strategic borrowing and not. Therefore for some value in the neighborhood of \(\epsilon\) the low-risk consumer can profitably deviate from the CCR equilibrium in proposition 2.

**Proof of lemma 10:**

Suppose all strategic borrowers borrow from a different support and default with certainty, then no loans will be extended with that support. Now suppose all strategic borrowers repay with certainty, then the posterior is the same as the prior. This leads to no change in insurance outcomes and provides an incentive to deviate through default.

In general, the posterior following repayment is given by

\[
(\mu_2^0)^\text{strategic} = \frac{\mu_1[\lambda(1 - d^S_{LH}) + (1 - \lambda)(1 - d^S_{LL})]}{\mu_1[\lambda(1 - d^S_{LH}) + (1 - \lambda)(1 - d^S_{LL})] + (1 - \mu_1)[\lambda(1 - d^B_{LH}) + (1 - \lambda)(1 - d^B_{LL})]}
\]

Setting \((\mu_2^0)^\text{strategic} = \mu_1\) gives the condition in lemma 10.

**Proof of lemma 11:**

Suppose \(\mu_1 > \mu^*\) and further suppose that strategic borrowers are replicating the a CCR (proposition 2) type repayment strategy. Consider a genuine borrower of the high-risk type and a strategic borrower also of the high risk-type. Further suppose the genuine borrower is borrowing
from the lower bound of his support $l$. Let the high-risk strategic borrower borrow an amount $l^{F} \neq l$.

The gain from default for the genuine borrower following a high second period preference shock is

$$\alpha \theta_H \{\ln e - \ln(e - Rl)\} \quad (36)$$

The gain from default for the strategic borrower following a high second period preference shock is

$$\alpha \theta_H \{\ln(e + (1 + r)(a + l^{F})) - \ln(e + (1 + r)a - (R - 1 - r)l^{F})\} \quad (37)$$

where $a$ is some given level of savings. The gain in the insurance market from repayment for either borrower is

$$\ln(e - \pi^B \Delta + \mu_2^0(\pi^B - \pi^S)\Delta) - \ln(e - \pi^B \Delta) \quad (38)$$

At $l$, the genuine borrower is just indifferent between default and repayment. Therefore the payoffs in 36 and 38 are identical. From lemma 10, there must be some level of default associated with strategic borrowing and a high-risk consumer with a high second period preference shock is the most likely to default. For default to be profitable for the strategic borrower, the value of $l^{F}$ must be such that the payoff in 37 exceeds that in 38. But from the concavity of the log function, this is possible only if $l^{F} > l$. This argument holds for any amount of borrowing by the genuine borrower from the entire support. Therefore strategic borrowers must borrow at higher levels than genuine borrowers.

References


$\mu = \mu^*$

Figure 1: Value of Insurance Vs Fraction of Low-Risk Type
$\mu^* = \frac{(1-\lambda)\mu^*}{1-\lambda\mu^*}$

Figure 2: Equilibrium
Figure 3: Home Loans with FICO Scores. Source: www.myfico.com
Figure 4: Auto Loans with FICO Scores. Source: www.myfico.com
Figure 5: Credit Card Discounts with FICO Scores. Source: Furletti (2003)
Figure 6: Model Generated Interest Rates