Families as Roommates: Changes in U.S. Household Size from 1850 to 2000*

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Abstract

The average American today lives in a household of three and a half people, compared to seven in 1850. Household sizes are smaller for Americans of all ages, living in rural or urban areas. We investigate to what extent this decline can be explained by a rational response to changing economic conditions – namely, the growth of income per person. We develop a simple theory of household size choice. Living with others is beneficial because households share the costs of a local public good, but costly because it takes time to form and maintain the necessary relationships. We calibrate the model to fit facts about the relationship between household size, consumption patterns, and income in the cross-section at the end of the 20th century, and then project the model back in the time series to 1850. We find that changing income can account for 30 percent of the observed decline in household size through this channel. We then consider a model that explicitly incorporates children. Children themselves are a public good, and a calibrated model with children is able to account for most of the decline.

Keywords: household size, economies of scale, household equivalence scales, life cycle

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1 Introduction

While the average American in 1850 lived in a household of 6.7 members, the average American today lives in a household of only 3.5 members, a 48% drop. The decline affected people of all ages and birth cohorts. For example, a person born in 1830 lived with seven other people as a teenager and with four other people by the age of retirement. In contrast, a person born in 1950 lived with only four other people in his teens, and with just one other person by the age of retirement.[1] The fertility decline is a major contributor, with fewer young children in the average household. But the fall also encompasses a number of other trends, including a decline in the importance of marriage, an increased tendency for the elderly and young adults to live alone, and a decline in the importance of long-term boarding and lodging arrangements.

Traditionally, changes in household size have been associated with changes in institutions and societal frictions. In this paper, we propose a parsimonious theory of household formation that abstracts from institutional "details" such as marriage, divorce, and fertility. We solve a model with a single cost to household formation and no frictions, and show how such a model can explain a substantial portion of the changes in household size without appealing to institutional changes. Our point of departure is that institutions can explain some aspects of the decline in household size, but they cannot explain them all. For instance, the changes in marriage and divorce laws help to explain some of the shifts from married to single households, but cannot explain why the number of boarders or extended family members in a household have also fallen. Similarly, the drop in fertility cannot explain why the household size of never-married adults has also fallen. Our strategy consists of searching for a common driving force affecting all Americans and causing virtually every form of living together to become less prevalent.

We use a general equilibrium overlapping generations model with two goods, one private and one public within the household. Agents purchase and consume their own private good (clothes), but they can also purchase and share the public good (housing) with other members of their household. The two goods are imperfect substitutes, so agents always choose to consume positive amounts of both. Agents simultaneously determine how to allocate their income between private and public goods and how many other people to live with. Hence, the economies of scale to living together are endogenous to the household.

Finally, we allow income to increase exogenously over time. Given non-homothetic pref-

[1] Based on forecasts from the US Bureau of Census.

erences for the two consumption goods, an income effect leads agents of different cohorts to make different choices about their consumption basket and household size. Our hypothesis is that while both goods are normal goods, private goods are more income-elastic than household public goods. Thus, as people get richer, they choose to consume relatively more of the private good, which endogenously decreases the economies of scale to household formation and thereby makes smaller households more desirable. Data from the Consumer Expenditure Survey (CEX) support our channel. In a given year, richer households tend to have a higher budget share for private goods and fewer members. We calibrate our model to fit cross-sectional data, and then project it backwards to 1850 to explore the importance of this channel in explaining the decline in household sizes. We find that our model can account for about 30% of the decline.

Fertility fell substantially during the end of the 19th century and almost certainly played a large role in the fall in household size. We thus do not find it surprising that a model that makes no distinction between children and adults cannot account for the entire decline in household size. However, we find it plausible that the same mechanism may have also played a role in the fertility decline. As people earn higher incomes, they want to buy relatively more private goods for their children, such as cars, televisions, and restaurant dinners. This makes each additional child endogenously more costly and hence adults want fewer children. To capture this logic, we modify the model to include an explicit distinction between children and adults. The benefits of sharing a household with other adults is that they contribute additional income, while children contribute utility because adults are altruistic towards children in the household. Children themselves are public goods: both the burdens and the joys of parenthood are shared by all adult members of the household. This modification adds two new ideas. First, it adds an effect that is similar to, but different in detail from, the quantity-quality trade-off analyzed by Becker and Lewis (1973) and others. Second, in our calibrated model, the share of resources devoted to children falls over time. Since children themselves are public goods, this effect endogenously decreases the benefits from living with other adults, providing a second channel to explain why there are fewer adults per household.

Since we cannot separate out expenditures on children from expenditures on adults in the data, we use a different calibration strategy for the model with children treated asymmetrically. We target a set of moments about the relationship between average adults and children in the household, income, and age in the cross-section and the time series. We find that the quantitative model matches the timing of the decline in children and adults per household remarkably well. In the calibrated model, most of the fall in the number of children per household happens by 1940, while most of the fall in the adults per household happens after that date. Our model is overidentified, and the timing of the falls is asymmetric. We therefore interpret the ability of the model to match these facts with increasing GDP per capita as the sole driving force as a success.

In Section 2 we review the literature on household sizes and scale economies. In Section 3 we document the decline in household size empirically. In Section 4 we present the model. Section 5 contains the calibration and main results. In Section 6 we present an extension of the model that explicitly allows for children. Section 7 concludes.

2 Related Literature

2.1 Theories of Household Size and Decline

We begin with the literature on the decline of household sizes as a general phenomenon. Several authors have documented the decline in household sizes empirically. For instance, Kobrin (1976) shows that household sizes in the United States have been falling since the 1790 Census. Kuznets (1978) documents differences in the size of the average household between developed and developing countries, over time within the United States, and across various other divisions such as urban and rural households. He finds that more developed countries tend to have smaller average household sizes. Laslett (1969) provides some evidence using various small surveys of English households that household sizes may have been stable from roughly 1600 until as late as 1911, with the decline in household sizes beginning only in 1911.[2]

Several authors have also documented that there is a strong negative relationship between household size and income. Early work by Brady (1958) and Beresford and Rivlin (1966) uses data tabulations to support such a relationship. More detailed work was provided by Michael, Fuchs, and Scott (1980) and Pampel (1983), who use cross-state variation and individual microdata to study the relationship between income and household size. Ermisch (1981) provides evidence of the same relationship in the 1973 and 1976 General

[2]One problem that he acknowledges is that his data lacks a record of how a household was defined. It is unclear how arrangements such as boarders or lodgers may have been counted, although he describes some tests that can be used.

Household Surveys of Britain, where he estimates an income elasticity of household size of -0.20.

Researchers have suggested a number of different hypotheses for the decline of household sizes and the empirical regularity between development status or income and household size. Burch and Matthews (1987) provide an excellent summary of this research. They put hypotheses into six basic categories:

1. Rising real income, along with a notion of privacy as a normal or luxury good;
2. A decline in the availability of relatives to live with, and a strong preference for living with relatives over non-relatives;
3. Changes in tastes or preferences for privacy;
4. A decline in the specialization of different household members in different roles, resulting in less gains from household formation;
5. The entry of women into the workforce, a decline of household services produced at home, and the rise of market production of goods and services formerly produced at home, such as cooked meals;
6. Advances of technology that allow people to enjoy many of the benefits of companionship while living separately.

Most of this research has been undertaken by demographers or sociologists, with an emphasis on studying a number of possible explanations rather than formal models and quantitative estimates of the relative importance of the suggested channels. By contrast, the main focus of our paper is to explore the quantitative importance of our channel.

Most theoretical work on household formation focuses on a particular margin of family formation, such as marriage or divorce; we will cover these topics in the next section. Little work has been done on general household formation. An exception is Ermisch (1981), who models a household as a technology that turns the inputs of expenditures, non-market time, and household composition into a measure of per-person utility. The household size affects welfare in three ways: household members pool expenditures to buy a good which is locally public but congestible; household members pool their time to produce household services; and the number of household members enters utility directly, as a preference for privacy. His model is sufficiently general that it offers little theoretical guidance for the mechanism through which rising income may lead to falling household sizes. The importance of each of the mechanisms depends on substitution and income effects which he generally cannot sign. Instead, his focus is on empirically estimating the income-household size relationship.

Lam presents a model of households as clubs (Lam 1983, Lam 1984, Lam 1988), building on work done by Buchanan (1965). His papers have in common with ours that there are both local public and private goods. Otherwise they are quite different, as is suggested by their relationship to the club literature. He generally focuses more on the types and composition of households that arise when heterogeneous members jointly choose both household membership and the provision of goods; our work is more concerned with the strong relationship between household size and income.[3] Like the work of Ermisch, his papers abstract from the related questions of fertility and optimal number of children.

2.2 Theories of Decline in Specific Living Together Arrangements

A large body of research studies related demographic trends which can be viewed as components of the decline in household size. Among these demographic trends, five have received the most attention:

1. The decline in marriage and increase in age at first marriage;
2. The fertility decline;
3. The increase in the propensity for young adults to live alone;
4. The decline in extended family living together, particularly a much higher propensity for the elderly to live alone rather than with their adult children;
5. The removal of non-family members from the household, particularly boarders, lodgers, and employees.

All but the last item (5) have been extensively studied independently of the general trend towards smaller household sizes. For items (2)-(4), there exists substantial additional evidence that the changes are explicitly linked to income. Jones and Tertilt (2006) show that there is a strong negative relationship between fertility and income in the United States

[3]For instance, his 1988 paper focuses on how a view of households as clubs can generate a force towards positive assortative matching in the marriage market.

cross-section and time series. The income-fertility relationship that they find is remarkably stable over time, suggesting rising income may play a large role in falling fertility. Earnings ability and house prices are shown to affect the age at which young persons move out of their parents' home in the United States (Haurin, Hendershott, and Kim 1994) and in Britain (Ermisch and Di Salvo 1997). And several papers have connected various forms of old-age pensions to an increased tendency for the elderly persons and elderly widows to live alone (Costa 1997, Costa 1999, McGarry and Shoeni 2000).

A number of papers have explored mechanisms that might explain the decline in one of these arrangements for living together. So below we give a very selected survey with a special emphasis on papers that use macroeconomic tools and provide quantitative results.

Greenwood and Guner (2004) offer a model which helps explain the decline in marriage and the increase in divorce in the period since 1950 in the United States. They view home production and provision of nonmarket services as the essential reason to form a household. Their model features technological progress in the home production sector combined with strongly diminishing marginal utility in nonmarket goods compared to market goods, so that agents are increasingly more able to live in smaller households.[4]

An enormous literature studies the pronounced decline in fertility which has happened within the United States and around the world. These papers offer a variety of mechanisms through which fertility might fall. For some examples of recent research within the economics literature, see Greenwood, Seshadri, and Vandenbroucke (2005), Doepke (2004), Boldrin and Jones (2002), Greenwood and Seshadri (2002), Galor and Weil (2000), and Galor and Weil (1996).

Regalia and Rios-Rull (2001) explores the rise of single female and single mother households in the United States from the mid-1970's to the mid-1990's, which can be thought of as a mixture of items (1) and (3) in the above list. Their model explores the role of an increase in the relative wages of females to males in explaining these trends, and they find that it can explain almost 90% of the increase the propensity for females to live alone.

Bethencourt and Rios-Rull (2007) offer a model that explores a mechanism similar to ours to explain changes in the living arrangements of elderly widows. Their model allows widows to live alone or with children, with cohabitation providing some economies of scale

[4]The authors also use the framework to explain marriage and divorce patterns. Two of the authors use a similar story elsewhere to explain the rising female labor force participation rate; see Greenwood, Seshadri, and Yorukoglu (2005)

but also subject to some effort costs. Their model connects rising income to an increased tendency for widows to live alone in the time series between 1970 and 1990 as well as in the cross-section. Their analysis suggests that almost 80% of the variation in living arrangements is accounted for by the variation in income per capita.

Finally, it has been noted elsewhere that the tendency for children to move out of their parents' home earlier is not universal, with many European countries showing the opposite trend. Fogli (2000) offers as an explanation the differences in employment protection across countries. High European employment protection makes older workers more likely, and younger workers less likely, to find employment. In her model, employment protection leads young persons to live at home with their parents and enjoy the benefits of local public goods. The United States, with relatively weak employment protection, would not be prone to such forces.

2.3 Economies of Scale

Household sizes are of interest to researchers for several purposes. For instance, an understanding of household sizes allows researchers to create estimates of population from counts of dwellings (Chandler 1987). Likewise, it can be useful for projecting the demand for housing units (Ermisch and Overton 1985). Finally, the size and structure of households is closely related to the construction of equivalence scales.

Equivalence scales are born from a simple question: if a person living alone has expenditures x, how much does it cost for two people to live together and maintain the same standard of living? In general, the answer will be less than $2x$ because of shared household public goods and other economies of scale. Equivalence scales and economies of scale are of interest here because they provide the economic motive for household formation used in the paper.

Equivalence scales have typically been constructed in two very different ways. Some have been created using a notion of minimum expenditure, particularly those designed to adjust the poverty threshold based on household size and composition. For instance, Orshansky (1965) calculated the minimum additional food expenditure needed to support additional household members. With little more than CPI adjustment, her work serves as the basis for many poverty scales today.

Other equivalence scales or estimates of economies of scale come from estimates of how household expenditures on various categories of goods change as household size or composition changes; see for instance Lazear and Michael (1980) or Nelson (1988). Goods have large economies of scale (and lower x in the equivalence scale measure) if expenditures increase little in household size. Typically, housing is found to have large economies of scale, while goods such as clothing have smaller economies of scale. Counter-intuitively, food is often found to have large economies of scale (Deaton and Paxson 1998).

This literature raises several questions that we attempt to address in this paper. First, application of the large economies of scale estimated by these authors implies large costs to the nearly universal tendency to live in smaller households, particularly for those who live alone. Why are people foregoing these presumably large benefits? Second, much of the equivalence scale literature takes the question of household formation as being exogenous, with the resulting difficulty that estimates may be biased if the causality runs in the opposite direction: it may be that those with tastes for more or less of the household public goods are more or less likely to form large households. Finally, these studies ignore the possibility that the "right" estimates of equivalence scales or economies of scale may be time-varying, as was found in the recent work by Logan (2007), who examines changing consumption patterns using consumer expenditure data from 1888 to 1935. This paper seeks to address these questions by making household size decisions and economies of scale decisions both endogenous.

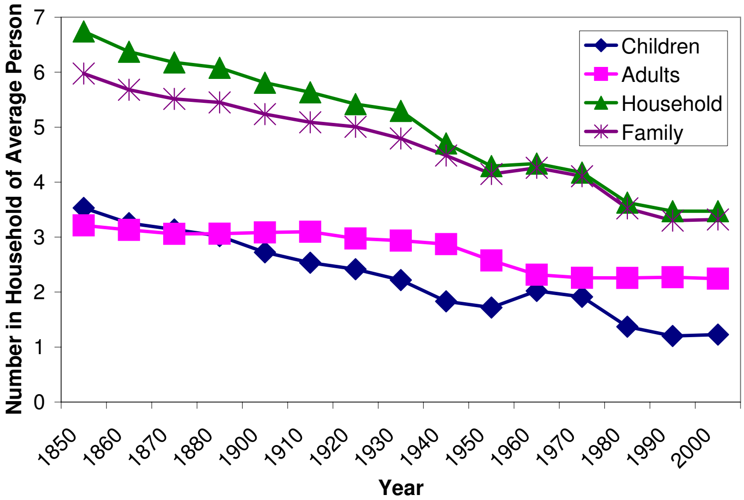
3 Changes in Household Size and Composition

Living arrangements of Americans have changed dramatically over the last 150 years. We use a 1% public use sample of the U.S. Census data to document these changes.[5] These data have been made available by the Minnesota Population Center at www.ipums.org (Ruggles et al 2004). We describe carefully our data analysis in Appendix A.

We choose to focus on the household size of the average American. We depart here from the standard measure of average household size for two reasons. First, people living in large households are under-represented in measures of average household size. Second, when documenting household sizes by age, we compute the average household size of all Americans of age a, rather than the average household size of household heads of age a. The second approach is more common in the literature, but we feel that it misses important

[5]Except for 1930 data, for which only a 0.5% sample is currently available.

Figure 1: Various Measures of Household Size (excluding group quarters)



aspects of the data.[6] The distinction is important because most people are not household heads most of the time, and those people who are tend to have different household sizes. For example, even the majority of 25 year old males are not household heads, and while the average 25 year old male born between 1900 and 1920 lived in a household of size of 4.4, the average household size for those who were household heads was only 3. Further, the fraction of individuals heading their own household at a given age has exhibited significant changes over time. For both of these reasons, we believe that a measure that tracks people rather than households is more meaningful. Using this definition of household size, the data reveal the following patterns:

1. Individuals today live in households that are about half the size of households 150 years ago. The average household today has fewer members of every type.

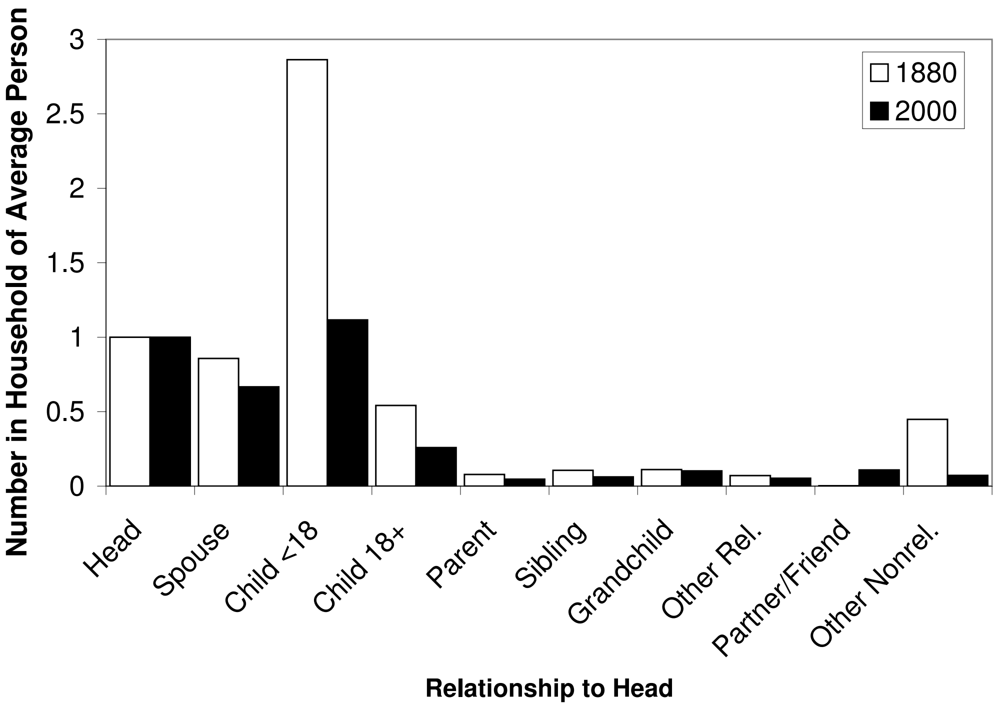
2. Most of the decline in adults per household occurred during the second half of the period, whereas children per household declined most rapidly during the first half of the period.

3. Household size varies systematically by age within a given cross-section. The overall profile displays peaks for young children and middle-aged adults, with lower values for young adults and the elderly.

[6]For example, Fernandez-Villaverde and Krueger (forthcoming) and Attanasio and Browning (1995).

4. There is a strong negative relationship between income and household size.

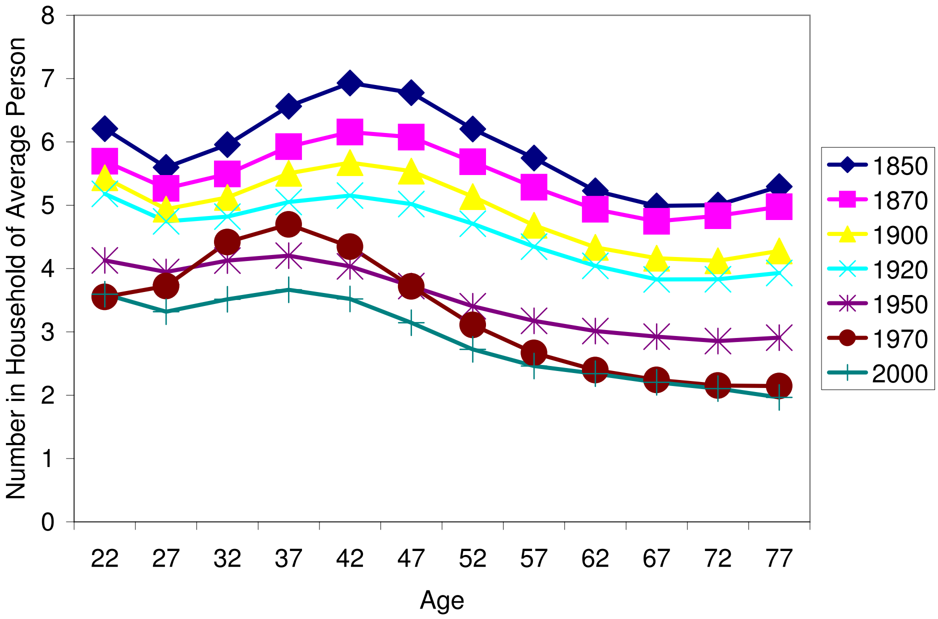
Figure 2: Household Members, 1880 and 2000



We now add more details on each of these facts. Figure 1 shows the decline in various measures of household size (see also Table 6 in the Appendix). Total household size fell from 6.7 people in 1850 to 3.5 people by 2000, a reduction of about 50%. Many people expect that large households were a primarily rural phenomenon, but similar patterns prevail for urban households: the average urban dweller experienced a household size decline from 6.5 to 3.5 members between 1850 and 1990. Splitting household members into adults (18 years and above) and children (17 years and younger), we find that the number of adults fell from 3.2 to 2.2, a decline of about one third, while the number of children fell from 3.5 to 1.2, a decline of about two thirds. The timing of the reduction in children and adults per household was asymmetric. 74% of the total observed decline for the number of children took place before 1940, while only 34% of the total observed decline for the number of adults took place by then.

The Census began collecting and tracking more detailed information of the relationship of members to the household head in 1880. Figure 2 compares the composition of households in 1880 to 2000. By definition the average person's household always has one head, but there are fewer members of virtually every other type. The only increase has been in the category "partners, friends, and visitors", which tracks a recent increase in nonmarried partners; this increase is more than offset by the decline in spouses. One large and sometimes unexpected decline is in "other non-family members", a group that primarily includes boarders, lodgers, and employees: in 1880, nearly every other person's household had such a member, but today only one in fourteen people have a household with a boarder. Again, patterns of decline for urban households are quite similar, although there is some evidence of level differences.

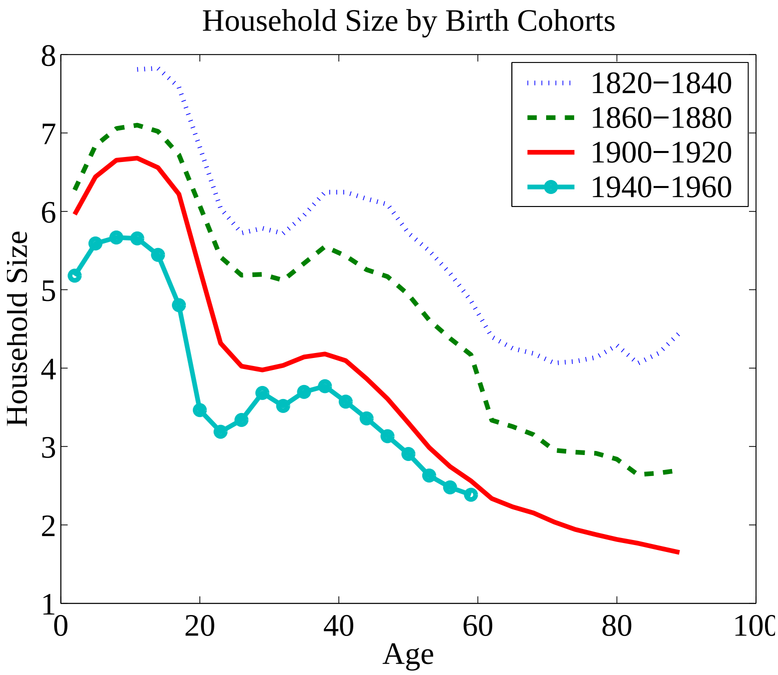
Figure 3: Average Household Size by Age, from Different Censuses



Living arrangements follow a clear pattern by age: the age profile looks almost double hump-shaped. Figure 3 plots the cross-section of household size by age constructed from a series of different Censuses. Very young adults (20-24 years) live in larger households than 25-35 year old adults. Persons in their mid-40s have the largest households, while very old people have the smallest households. Note also that the entire profile has shifted down almost uniformly. While the exact timing of the peak of the hump has appeared to change, this apparent shift is mostly an artifact of using cross-section data. Figure 4 plots life cycle household size constructed by following an age cohort across consecutive U.S. Censuses.

Here, the pattern is the same except that the timing of the peak of the hump remains unchanged. Also, the double-hump shape is more pronounced because Figure 4 includes the childhood years.

Figure 4: Average Household Size over the Life Cycle



Poorer people live in larger households and also have more children. We compute adults and children per household by age and income quintile. With the exception of people older than 60 years, we find a strong pattern: people in the bottom 20% of the income distribution live with more adults and more children than people in the top 20% of the income distribution. They are also more likely to live with unrelated individuals. Table 1 gives the average sizes by income quintiles for adults in the 25-29 and 30-34 age brackets from the 2000 Census.[7]

[7]The quintiles are computed based on total household income per adult in the household.

Table 1: Average Household Size by Income Quintiles, 2000

	25-29 year old persons				30-34 year old persons			
Quintile	children	adults	total	non-family	children	adults	total	non-family
1	1.42	2.63	4.03	0.42	1.69	2.44	4.13	0.32
2	1.24	2.55	3.79	0.34	1.55	2.31	3.86	0.23
3	0.99	2.35	3.34	0.25	1.40	2.14	3.54	0.16
4	0.77	2.22	2.99	0.22	1.17	1.99	3.17	0.11
5	0.53	2.13	2.66	0.20	0.96	1.92	2.88	0.09

4 The Model

4.1 Agents

We use an infinite horizon overlapping generations model. Agents live for a finite number of periods, $a \in \{0, 1, \ldots, \bar{a}\}$, where a stands for age. Let τ be the birth year of an agent; sometimes we call this cohort. In addition we assume there are different types of agents denoted by i. We can then identify an agent by his birth cohort and type (τ, i). We denote the productivity of agent (τ, i) at age a by $z(\tau, a, i)$. We also index all decision variables by (τ, a, i). For example, $s(\tau, a, i)$ is the household size of agent (τ, i) at age a. Let $f(\tau, i)$ denote the number of agents of birth cohort τ, and type i; we assume that all agents live until exactly age $\bar{a}$ so f is not a function of a. Note that cohort, age, and actual year are related by $t = \tau + a$.

4.2 Preferences

Agents have preferences over two types of consumption goods: private and household public goods. Private goods, such as restaurant dinners or health care, can only be consumed by one individual. We denote private goods consumption of agent (τ, i) at age a as $v(\tau, a, i)$. Household public goods, such as home security systems or heating, are consumed nonrivalrously by all agents living in a household. We denote household goods consumption

$h(\tau, a, i)$. Note that we define a good as public on the basis of whether or not it can be used by more than one agent, not whether or not it is. A public good in the home of a single person is still a public good for our purposes.

We assume that agent (τ, i) has preferences represented by the following utility function

$$W(\tau, i) = \sum_{a=0}^{\bar{a}} \beta^{\tau+a} U(\tau, a, i), \tag{1}$$

where the period utility is given by

$$U(\tau, a, i) = \left[\omega \frac{v(\tau, a, i)^{1-\phi}}{1-\phi} + \frac{h(\tau, a, i)^{1-\sigma}}{1-\sigma}\right]. \tag{2}$$

Note that the additively separable utility function allows us to produce wealth effects and generate nonlinear Engel curves if $\sigma \neq \phi$.

4.3 Households

An agent (τ, i) of age a lives in a household of size $s(\tau, a, i)$. For simplicity, we assume that agents live only with other agents of the same type and age.[8] Household size is allowed to take on any real number greater than one; the only restriction is that people live with at least themselves. The benefit of living in a larger household is splitting the cost of purchasing household public goods. There is a time cost associated with being in a household. We interpret this cost as the cost of forming and maintaining a household: for example, time spent finding roommates, resolving disputes, and so on. We assume that this cost varies systematically over the life cycle. To be concrete, we assume that there is no cost for living alone and that the cost is linear in additional household members: $B_a(s(\tau, a, i)-1)$. The age-specific term B_a captures life-cycle variation in the cost of finding other household members, which may vary for biological reasons or because a larger current household makes it less costly to find and maintain a larger household next year.

Agents in this economy are endowed with one unit of time every period and productivity $z(\tau, a, i)$. Given that they use a fraction $B_a(s(\tau, a, i) - 1)$ of their time in forming and maintaining their household, they spend the rest of their time producing goods using a

[8] Ermisch (1981) makes a similar assumption that agents live with others with the same tastes. Without this assumption, the decisions about what to purchase and who to allow becomes complicated as in the club literature. See the literature review for cites.

linear technology that uses efficiency units of time as the only input. Relative productivity in the private and public goods sectors is fixed over time and normalized to 1; we abstract from the relative technological progress captured in Greenwood, Seshadri, and Yorukoglu (2005). There is no leisure in the model and no capital.[9] Agents purchase two types of goods: their share of the household public good $h(\tau, a, i)$ as well as the private good, $v(\tau, a, i)$. Let $p(t)$ be the Arrow-Debreu price at time t. Then the agent's lifetime budget constraint can be written as:

$$\sum_{a=0}^{\bar{a}} p(\tau + a) \left[\frac{h(\tau, a, i)}{s(\tau, a, i)} + v(\tau, a, i) \right] \leq \sum_{a=0}^{\bar{a}} p(\tau + a) z(\tau, a, i) [1 - B_a(s(\tau, a, i) - 1)] \tag{3}$$

The constraint will be satisfied with equality given the nonsatiated preferences. As a matter of terminology, $p(\tau + a)z(\tau, a, i)[1 - B_a(s(\tau, a, i) - 1)]$ is income of agent (τ, i) at time $\tau + a$, $p(\tau + a)[\frac{h(\tau,a,i)}{s(\tau,a,i)} + v(\tau, a, i)]$ is (per capita) expenditures, and $p(\tau + a)[h(\tau, a, i) + s(\tau, a, i)v(\tau, a, i)]$ is total household expenditures; all are distinct from the agent's productivity $z(\tau, a, i)$.

4.4 Equilibrium

Definition 1 *An equilibrium for this economy is an allocation* $\{s(\tau, a, i), v(\tau, a, i), h(\tau, a, i)\}_{\tau,i}$ *and prices* $\{p(t)\}_{t\in T}$ *that:*

1. *Each agent* (τ, i) *maximizes (1) and* (2) *subject to the constraint (3) and:*

$$s(\tau, a, i) \geq 1 \qquad \forall a$$

$$v(\tau, a, i), h(\tau, a, i) > 0 \qquad \forall a$$

2. *The market clearing condition for production is satisfied at every point in time* t*:*

$$\sum_i \sum_{\{(\tau,a)|\tau+a=t\}} f(\tau, i) \left[\frac{h(\tau, a, i)}{s(\tau, a, i)} + v(\tau, a, i) \right] =$$
$$\sum_i \sum_{\{(\tau,a)|\tau+a=t\}} f(\tau, i) z(\tau, a, i) [1 - B_a(s(\tau, a, i) - 1)] \tag{4}$$

[9] The distinction between increases in GDP because of technological progress vs. increases in the capital stock is irrelevant for our analysis. In the empirical implementation, all changes in GDP p.c. are captured as improvements in labor productivity.

Let $\lambda(\tau, i)$ be the Lagrangian multiplier on the budget constraint for agent (τ, i). Then, assuming interiority, agent (τ, i)'s first order conditions can be written as:

$$v(\tau,a,i): \qquad \beta^{\tau+a}\omega v(\tau,a,i)^{-\phi} = \lambda(\tau,i)p(\tau+a) \tag{5}$$

$$h(\tau,a,i): \qquad \beta^{\tau+a}h(\tau,a,i)^{-\sigma} = \frac{\lambda(\tau,i)p(t+a)}{s(\tau,a,i)} \tag{6}$$

$$s(\tau,a,i): \qquad B_a z(\tau,a,i) = \frac{h(\tau,a,i)}{s(\tau,a,i)^2} \tag{7}$$

Given the strictly concave utility function, the only restriction that may bind is $s(\tau, a, i) \geq 1$. If it does, the deviation from the interior solutions is straightforward. For the propositions we will assume interiority, although the corner may bind for some agents in our numerical analysis. Given the restriction on $s(\tau, a, i)$, the agent's problem is convex and well-defined, so an equilibrium exists; see Appendix 2 for details.

Rearranging the first order conditions, we get the following relationships between variables.

$$h(\tau,a,i) = B_a z(\tau,a,i)s^2(\tau,a,i) \tag{8}$$

$$v(\tau,a,i) = \left(\frac{\omega h(\tau,a,i)^{\sigma}}{s(\tau,a,i)}\right)^{1/\phi} \tag{9}$$

and we can relate household sizes over the life-cycle as follows:

$$s(\tau,a,i) = \left(\frac{p(\tau)}{p(\tau+a)}\beta^a\right)^{\frac{1}{2\sigma-1}} \left(\frac{B_0 z(\tau,0,i)}{B_a z(\tau,a,i)}\right)^{\frac{\sigma}{2\sigma-1}} \tag{10}$$

By substituting these equations into the budget constraint (3), we can reduce the problem of agent (τ, i) to an equation with only one unknown, household size at age zero, $s(\tau, 0, i)$:

$$\begin{aligned}
&\sum_{a=0}^{\bar{a}} p(\tau+a)\left(\omega(B_a z(\tau,a,i))^{\sigma}\left[\left(\beta^a\frac{p(\tau)}{p(\tau+a)}\right)^{1/(2\sigma-1)}\left(\frac{B_0 z(\tau,0,i)}{B_a z(\tau,a,i)}\right)^{\frac{\sigma}{2\sigma-1}} s(\tau,0,i)\right]^{2\sigma-1}\right)^{1/\phi} \\
&+2\sum_{a=0}^{\bar{a}} p(\tau+a)B_a z(\tau,a,i)\left[\left(\beta^a\frac{p(\tau)}{p(\tau+a)}\right)^{1/(2\sigma-1)}\left(\frac{B_1 z(\tau,0,i)}{B_a z(\tau,a,i)}\right)^{\frac{\sigma}{2\sigma-1}} s(\tau,0,i)\right] \\
= &\sum_{a=1}^{\bar{a}} p(\tau+a)\left[1+B_a\right] z(\tau,a,i)
\end{aligned} \tag{11}$$

4.5 Properties of the Model

The model is general enough that it can generate a decrease or an increase in household size as income rises. Our goal is to show the conditions under which the model has the property that higher productivity leads to lower relative demand for private goods and lower household sizes.

Proposition 1 – ***Household Size and Productivity over the Life-Cycle***

Suppose $B_a = B$ for all a and that for all i, $z(\tau, a, i) = z(\tau, i)$ for all a.

a) If $\sigma > 0.5,\ \phi < \sigma$, then $z(\tau', i) > z(\tau, i)$ implies that $s(\tau', a, i) < s(\tau, a, i)$ for all (a, i).

b) If $\sigma > 0.5,\ \phi > \sigma$, then $z(\tau', i) > z(\tau, i)$ implies that $s(\tau', a, i) > s(\tau, a, i)$ for all (a, i).

Proof. This case can be solved analytically. From the first order conditions, we can express household size at age a as a function of household size at any other age. Guess that in equilibrium $p(t) = \beta^t$ and plug into (10). Then all terms cancel out and we have $s(\tau, a, i) = s(\tau, i)$. Since $h(\tau, a, i) = B_a z(\tau, a, i) s^2(\tau, a, i)$ and $v(\tau, a, i) = \left(\frac{\omega h^\sigma(\tau,a,i)}{s(\tau,a,i)}\right)^{1/\phi}$, all variables are constant over the life-cycle. Plugging this into the budget constraint (11) and also using the price guess, this equation reduces to $v(\tau, i) + \frac{h(\tau,i)}{s(\tau,i)} = [1 - B(s(\tau, i) - 1)]z(\tau, i)$. Hence, the equilibrium is autarky and market clearing is trivially satisfied. To see that $s(\tau, i)$ decreases if $z(\tau, i)$ increases, substitute in the solutions for $v(\tau, i)$ and $h(\tau, i)$ from above to derive

$$z(\tau, i)^{\frac{\sigma-\phi}{\phi}} (B^\sigma \omega)^{1/\phi} s(\tau, i)^{\frac{2\sigma-1}{\phi}} + 2Bs(\tau, i) = 1 + B$$

Now using the implicit function theorem, we have

$$\frac{ds}{dz} = \frac{-\frac{\sigma-\phi}{\phi}\left(\omega B^\sigma s^{2\sigma-1} z^{\sigma-2\phi}\right)^{1/\phi}}{\frac{2\sigma-1}{\phi}\left(\omega B^\sigma z^{\sigma-\phi} s^{2\sigma-1-\phi}\right)^{1/\phi} + 2B}$$

A sufficient condition for this expression to be negative is $\sigma > 0.5$ and $\sigma > \phi$, which completes the proof. *Q.E.D.*

In our numerical results we show that this proposition is true more generally, that is, letting B_a and income vary over the life cycle, we find that $\sigma > 0.5$ and $\phi < \sigma$ imply that average household size decreases in response to growing incomes. An analytical proof for the more general case is difficult as equilibrium prices will typically have no closed form solution.

Next, we derive some cross-sectional properties, that is, we compare equilibrium choices of two agents of the same birth cohort and age, but with different productivities. General results along these lines are easier, as the two agents face the same prices. We begin by showing that the same conditions on parameters guarantee a negative correlation between productivity and household size in the cross-section.

Proposition 2 – ***Household Size and Productivity in the Cross-Section***

Suppose that agents have a constant productivity over their life cycle, $z(\tau, a, i) = z(\tau, i)$ *for all* a*. Assume* $\sigma > 0.5, \sigma > \phi$*. Then* $z(\tau, i) > z(\tau, j)$ *implies that* $s(\tau, a, i) \leq s(\tau, a, j)$ *for all* a*, with strict equality if* $1 < s(\tau, a, j)$*.*

Proof. Divide equation (11) by $z(\tau, i)$ to derive

$$z(\tau, i))^{\frac{\sigma-\phi}{\phi}} \sum_{a=0}^{\bar{a}} p(\tau + a) \left(\omega(B_a \left[\left(\beta^a \frac{p(\tau)}{p(\tau + a)} \right)^{1/(2\sigma-1)} \left(\frac{B_0}{B_a} \right)^{\frac{\sigma}{2\sigma-1}} s(\tau, 0, i) \right]^{2\sigma-1} \right)^{1/\phi}$$
$$+2 \sum_{a=0}^{\bar{a}} p(\tau + a) B_a \left[\left(\beta^a \frac{p(\tau)}{p(\tau + a)} \right)^{1/(2\sigma-1)} \left(\frac{B_1}{B_a} \right)^{\frac{\sigma}{2\sigma-1}} s(\tau, 0, i) \right] = \sum_{a=1}^{\bar{a}} p(\tau + a) \left[1 + B_a \right]$$

Then the proof is essentially the same as the proof of Proposition 1. *Q.E.D.*

Additionally, using similar conditions to those in the previous two propositions, we can show that the consumption bundle is becoming increasingly dominated by private consumption.

Proposition 3 – ***Goods Composition***

Assume $\sigma > \phi, \sigma > 0.5$*. Then* $z(j) > z(i)$ *implies that*

i. $\frac{v(\tau,a,j)}{h(\tau,a,j)} > \frac{v(\tau,a,i)}{h(\tau,a,i)}$

ii. $\frac{v(\tau,a,j)}{s(\tau,a,j)v(\tau,a,j)+h(\tau,a,j)} > \frac{v(\tau,a,i)}{s(\tau,a,i)v(\tau,a,i)+h(\tau,a,i)}$

iii. If in addition $\sigma < 1$*, then* $\frac{h(\tau,a,j)}{s(\tau,a,j)v(\tau,a,j)+h(\tau,a,j)} < \frac{h(\tau,a,i)}{s(\tau,a,i)v(\tau,a,i)+h(\tau,a,i)}$

Proof. We will prove each part separately.

i. From (9) and (8) we can derive

$$\frac{h}{v} = \left(\frac{(B_a z(\tau, a, i))^{\phi-\sigma} s(\tau, a, i)^{2(\sigma-\phi)+1}}{\omega} \right)^{1/\phi}.$$

Then the result follows directly from the assumption on parameters together with Proposition 2.

ii. Similarly to i., we can derive here that $\frac{v(\tau,a,i)}{s(\tau,a,i)v(\tau,a,i)+h(\tau,a,i)} = [s(\tau, a, i) + s(\tau, a, i)^{1/\sigma}\omega^{-1/\sigma}v(\tau, a, i)^{(\phi-\sigma)/\sigma}]^{-1}$. Then using the same logic as in i., we find the suggested correlation.

iii. Again we can write $\frac{h(\tau,a,i)}{s(\tau,a,i)v(\tau,a,i)+h(\tau,a,i)} = \left[s(\tau, a, i)^{1-1/\sigma}v(\tau, a, i)^{(\sigma-\phi)/\phi}\omega^{1/\sigma} + 1\right]^{-1}$. Note the new condition on σ, but otherwise similar logic shows the relationship suggested in the proposition. *Q.E.D.*

The model makes strong predictions about cross-sectional relationships. If $\sigma > 0.5$ and $\psi < \sigma$, then more productive agents will live in smaller households and consume relatively more private goods. Proposition 4 shows that income varies systematically with productivity. Together, these propositions establish the testable conditions under which the model would behave according to our hypothesis: higher income should be associated with smaller household sizes and relatively more private consumption baskets.

Proposition 4 – ***Productivity and Income***

Assuming $z(\tau, a, i) = z(\tau, i)\ \ \forall a$ *and let* $\sigma > 0.5, \sigma > \phi$. *Then* $z(\tau, j) > z(\tau, i)$ *implies* $z(\tau, j)[1 - B_a(s(\tau, a, j) - 1)] \ > \ z(\tau, i)[1 - B_a(s(\tau, a, j) - 1)]$.

Proof. Income in our model is $z(\tau, i)(1 - B_a(s(\tau, a, i) - 1))$. Then the result follows immediately from Proposition 2 *Q.E.D.*

Having provided sufficient conditions for rising productivity to lead to falling household size and changing household consumption baskets, it is now useful to develop some intuition for the sufficient conditions. $\sigma > \phi$ guarantees that the private good is more income-elastic than the public good. As productivity rises, if the prices are held fixed, agents increase the budget share of the private good.

Choosing household sizes in this model is equivalent to paying to choose a price for the public good. Larger household sizes are costly, but cause lower prices for public goods; these effects are similar to standard income and substitution effects. Households that want to consume relatively more of the public good face a trade-off between increasing household size and lowering the price, and decreasing household size and spending more. $\sigma > 0.5$ ensures that increasing household size is a cost-effective method for increasing relative consumption of the public good.

5 Calibration and Results

We calibrate the model to match data on household sizes and relative consumption from the United States for the end of the 20th century. We then use the model to predict household sizes for all years between 1850 and 2000. This methodology allows us to infer what proportion of the actual decline in household size can be accounted for by the channels highlighted in this paper.

A crucial parameter in our model is the income elasticity of private and public goods. To identify this elasticity we use cross-sectional data on consumption for different income groups. We construct this data from the Consumer Expenditure Survey (CEX). In Section 5.1 we briefly introduce the CEX, discuss how we use the data, and provide some descriptive statistics. In Section 5.2 we describe how CEX data is used to calibrate the model parameters.

5.1 Consumption of private vs. household public goods

The microeconomic data are drawn from the 1980-2004 Consumer Expenditure Survey (CEX). The CEX provides a continuous and comprehensive flow of data on the expenditure habits of American consumers. The data are collected by the Bureau of Labor Statistics and used primarily for revising the CPI. Consumer units are defined as members of a household related by blood, marriage, adoption, or other legal arrangement, single persons living alone or sharing a household with others, or two or more persons living together who are financially dependent. The definition of the head of the household in the CEX is the person or one of the persons who owns or rents the unit.

The CEX is based on two components: the Diary, or record keeping survey, and the Interview survey. The Diary sample interviews households for two consecutive weeks, and is designed to obtain detailed expenditures data on small and frequently purchased items, such as food, personal care, and household supplies. The Interview sample is in the form of a rotating panel, and it follows survey households for a maximum of 5 quarters, although inventory and basic sample data are collected only in the first quarter. The data base covers about 95% of all expenditure, with the exclusion of expenditures for housekeeping supplies, personal care products, and non-prescription drugs. Following most previous research, our analysis below uses only the Interview sample.

The CEX collects information on a variety of socio-demographic variables, including characteristics of members, characteristics of housing unit, geographic information, inventory of household appliances, work experience and earnings of members, unearned income, taxes, and other receipts of consumer unit, credit balances, assets and liabilities, occupational expenses and cash contributions of consumer unit. Expenditure is reported in each interview (after the first) and refers to the three months of the previous quarter. Thus, a household interviewed in April 1980 reports expenditure for January, February, and March 1980. Income is reported in the second and fifth interview, and it refers to the previous twelve months. Holdings of financial assets are reported only in the last interview.

We refer the reader to the Appendix for step-to-step details on sample selection and consumption definition. Our sample selections are aimed at eliminating the most severe reporting errors in consumption. We end up discarding about 25% of observations through our selection procedure, mostly durables; our concern is that the difference between the timing of expenditure and the timing of consumption may bias our results. Our definition of expenditure on private goods includes expenditures on alcohol; apparel and footwear; records; food; vehicle gasoline, maintenance, and miscellaneous; public transportation; health; books; education; entertainment; tobacco; personal care; business services; and hotels and other lodgings. Our definition of expenditure on public goods includes utilities; home telephone and cable; textiles; house services, such as baby-sitting and gardening; rent, and imputed rent for homeowners; newspapers and magazines; and home insurance.

Table 2: Household Size and Consumption by Income Quintiles, 40-49 year old

quintile	income	HH size	h	v	h/v
1	2,845	4.33	1,600	534	3.00
2	6,210	3.90	2,046	741	2.76
3	9,158	3.56	2,360	929	2.54
4	13,301	3.07	2,658	1,166	2.28
5	25,640	2.32	3,200	1,757	1.82

We "deflate" consumption data by the chained CPI (all items) for urban Consumers (in

1982-84 dollars, as provided by the BLS). Given the overlapping panel nature of the CEX, each month a certain number of households enter the panel and an approximately equal number leave it. Monthly consumption data are aggregated to form quarterly consumption data for each household in the sample. Then, we aggregate across households to form moments of the quarterly consumption distribution. Note that households start their second interview (when consumption data are firstly collected) in different months. Thus, some households' second interview covers the months of January through March, some other households's second interview will have data for the months of February through April, and so forth. By the very design of the CEX, no households contributes multiple observations to adjacent overlapping quarters. In other words, a household that contributes data to January-March 1980 will not contribute data for February-April (or March-May). Its next contribution, if that exists, will be for April-June 1980. The income data is defined after-tax and it is an average of the records reported in the second and fifth interview (if available).

Table 2 summarizes our findings from the CEX data for middle-aged adults. We concentrate on 40-49 year olds in the data because we believe that actual income might be a relatively good proxy for productivity for this group.[10] We find that on average people consume 2.48 times as many public goods as private goods. The data also displays a strong negative relationship between the ratio of expenditures on public over private goods. People in the poorest income quintile consume about three times as much public goods as private goods, while people in the richest quintile consume not even twice as much of public goods relative to private ones. We calculate the income-elasticity of the public to private goods ratio based on the data reported in the table and find an elasticity of -0.24.

5.2 Calibrating the Model

We assume that a model period is equal to five years and that people live for 16 periods, from age 0 to 79. For simplicity, the distribution of people over types is assumed to be uniform, $f(\tau, i) = 1$.

We need to specify an exogenous productivity process for each birth cohort, age, and type of agent, $z(\tau, a, i)$. We use income per capita as a measure of productivity because the model predicts that they are correlated, and because income per capita is available in a long time series and in cross-sectional data sources. We use GDP per capita from

[10] As opposed to young workers who are still in school or older workers who may have retired.

Johnston and Williamson (2006) for 1850-2003. Outside of this horizon, we impute GDP per capita using an annual growth rate of 1.9% (the average annual growth rate between 1850-2000). We assume there are 5 different types i, corresponding to five income quintiles. We assume individuals stay in the same quintile over their life-cycle. Moreover, we assume that the relative income of the different quintiles is constant over time. We use the income reported in Table 2 to compute the fraction of average income that is earned by each quintile respectively: $(z_1, z_2, z_3, z_4, z_5) = (0.25, 0.54, 0.80, 1.16, 2.24)$. Then, we assume $z(\tau, a, i) = z_i Y_{\tau+a}$, where $Y_{\tau+a}$ is GDP per capita in year $\tau + a$ taken from Johnston and Williamson (2006).

We choose a standard annual discount factor $\beta = 0.98$. 19 parameters then need to be calibrated in the model: 16 household cost parameters $\{B_a\}_{a=1}^{16}$ and three utility parameters: σ, ϕ, ω. We pick the parameters to match 19 moments from cross-sectional U.S. data at the end of the 20th century. The moments are chosen to reflect the important mechanisms of the model. Since we are interested in how changes in income affect the optimal choice of public/private goods, we want our model to match the average h/v ratio as well as the income elasticity of this ratio. From the CEX data described above, we have an average h/v ratio of 2.48 for 40-49 year old persons. The income elasticity for the same age group is -0.24.[11] As is standard in the literature, we assume that the intertemporal elasticity of substitution is 0.5. Note that there is no closed form solution for the IES in this model. Instead, we solve for it computationally.[12] Finally, we compute average household sizes for 16 age groups from the U.S. Census data, i.e. for ages 0-4, 5-9, 10-14, ..., 75-79. Table 3 summarizes the 19 moments from the data. Table 4 provides the calibrated parameters. Appendix 3 gives additional details on the construction of the moments and the solution method.

[11]Obviously there are more moments in the data than the ones we currently match. In particular, one could compute the h/v ratio and the corresponding elasticity for different age groups. We plan to do more robustness analysis along these lines in the future. To incorporate more data, one would need to minimize a loss function rather than matching all moments exactly as we currently do.

[12]Concretely, we pick parameters such that the average I.E.S. across all people in 2000 is exactly 0.5. The computed IES varies by about 6.7% in the cross-section.

Table 3: Data Moments

Moment	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
Data	4.496	4.667	4.679	4.334	3.419	3.269	3.381	3.609	3.527	3.192
Moment	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}	s_{16}	$\frac{\bar{h}}{\bar{v}}$	$\varepsilon_{h/v,z}$	I.E.S.	
Data Value	2.775	2.486	2.342	2.218	2.105	1.957	2.48	-.24	0.5	

Table 4: Calibrated Parameters

Parameter	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
Value	0.0674	0.0633	0.0626	0.0695	0.0979	0.104	0.0979	0.0882	0.0905	0.104
Parameter	B_{11}	B_{12}	B_{13}	B_{14}	B_{15}	B_{16}	ω	σ	ϕ	
Value	0.127	0.148	0.160	0.172	0.184	0.204	0.057	1.91	1.66	

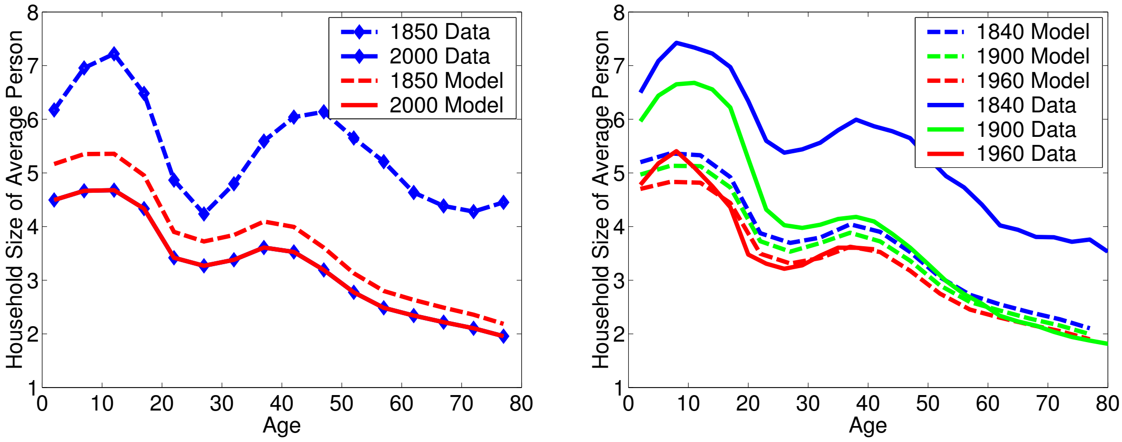
5.3 Results

Our model is calibrated to match cross-sectional data from the end of the 20th century. We assess how much of the overall decline in household size can be accounted for by the channel emphasized in this paper. Given the calibrated parameters, the model generates a decline in household size between 10-13 percent, depending on age. The fall in the data ranges from 27-56 percent. We conclude that the substitution from public to private goods in response to income growth accounts between 18 and 48 percent of the fall in household size, depending on age. Averaging across all age groups, the model accounts for about 30 percent of the observed fall.

Figure 5 plots two measures of the evolution of household size for the model and the data. Figure 5a shows the evolution of household sizes in the cross-section. The model does better at matching the decline in household sizes for young people, and less well at matching the decline for older people. Figure 5b shows the change in household size over the life-cycle for the three cohorts born in 1840, 1900, and 1960, respectively. Again, the model does better at explaining variation in earlier ages and worse in later ages.

One way of assessing the plausibility of the channel emphasized in the model is to

Figure 5: Model and Data Household Size



(a) Cross-Section Household Size

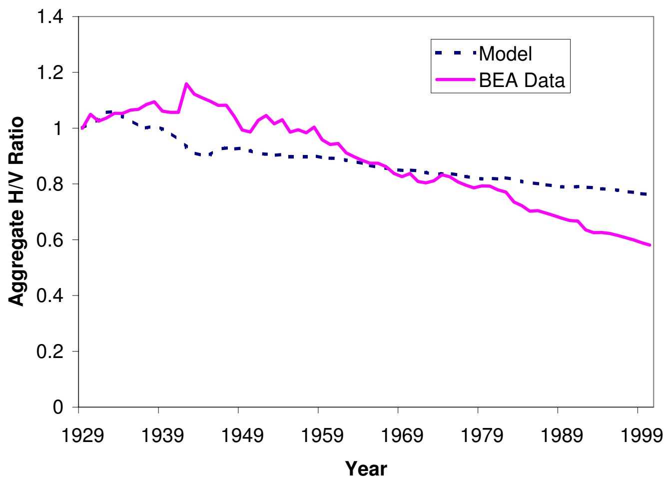
(b) Household Size Over Life Cycle, Selected Birth Cohorts

compare aggregate consumption implications generated by the model to the data. One such statistic for which data is available is the ratio of aggregate public expenditures relative to aggregate private expenditures. The Bureau of Economic Analysis has published a decomposition of personal consumption expenditures by product type since 1929. We break these product types into private or public. Private goods are primarily clothing, medical care, vehicles and transportation, and recreation. Public goods are primarily housing, household operation, furniture, utilities, and food. Although it is somewhat controversial to think of food as public, there are two reasons to do so. First, food itself is an input to the home production of cooked meals, which also involves cooking time and shopping time. The use of time in production makes the final meals consumed at home highly public; but since we cannot observe the value of these meals, we include the value of expenditures on food instead. Second, previous research has found that expenditure patterns on food are similar to expenditure patterns on other public goods (Deaton and Paxson 1998, Logan 2007).[13]

[13]By contrast, when data on food taken away from home is available separately, it shares all the characteristics of a private good. Since we cannot distinguish between the two, and since food taken at home is the dominant category of food over the time period, we put all food into public expenditures, which may underestimate our case.

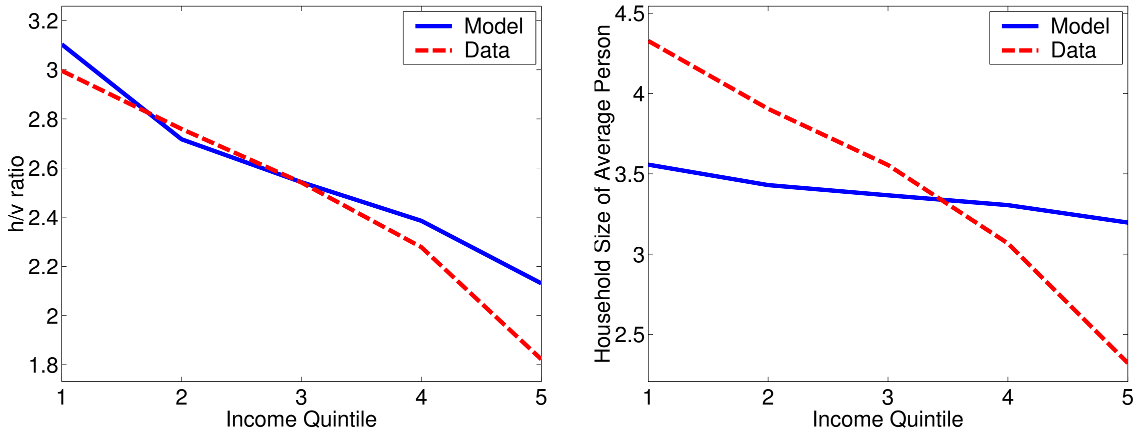
Figure 6 plots the ratio of aggregate public to private expenditures implied by the model and the data for the period 1929-2000, where the ratio for 1929 has been normalized to 1 for each series. If anything, the data show a larger decline than does the model, about 40% instead of 25%. Other than the changes driven by consumption restrictions in World War II, the primary contributors are a decline in the share spent on food and large increases in the shares spent on medical care and recreation.

Figure 6: Time Series of Aggregate Expenditures



We can also use the cross-sectional evidence that was not directly used for the calibration to assess the performance of the model. Figure 7a shows that the calibration captures the relationship between income and relative consumption fairly well. Since we calibrated the model to match the elasticity between these two variables, this should not be surprising. However, as Figure 7b shows, the model does a poor job in matching the relationship between income and household size: there is significantly less cross-sectional heterogeneity in the model than in the data. One interpretation would be that a declining expenditure share on household public goods is only one channel through which income and household size are related and that other factors are needed to fully capture this relationship. Our view is that the current model is missing the most important household public good: children. Relative demand for children may be an important extra channel which is missing from the current model. In the next section, we explicitly introduce children into the model to explore the potential of such an extended model to explain the history of household sizes in the U.S.

Figure 7: Model Elasticities, 2000 Cross-Section



(a) h/v ratio (b) Household Size

6 A Model with Children

6.1 Extending the Model

We now extend the framework to distinguish between adults and children in a household. The motivation for living with children is very different from the motivation for living with other adults. People share space and other public goods with other adults because each adult contributes either the income or time necessary to purchase or make the public goods. Children do not contribute to the public good. Instead, adults allow children into the household because they feel altruistically towards children, similar to Becker and Barro (1988). To keep the model tractable, we assume that the altruism extends only to those periods in which children are living in the household. That is, every period, adults derive utility from their own consumption and the consumption from all children living with them at the time.

Adults are the only decision makers in the economy. As before, they form together into households to share the cost of buying local public goods, but bear costs associated with living with other people. Adults also decide how many children to have in the household. They are altruistic towards children and care about the number and the welfare of all children in the household. Children are public in consumption and costs, in the sense that an adult derives enjoyment from all children in the home, helps pay for the consumption of all children in the home, and loses some time in care for all children in the home. Hence, adults maximize utility:

$$W(\tau, i) = \sum_{a=0}^{\bar{a}} \beta^{\tau+a} U(\tau, a, i) \tag{12}$$

$$U(\tau, a, i) = \omega \frac{v(\tau, a, i)^{1-\phi}}{1-\phi} + \frac{h(\tau, a, i)^{1-\sigma}}{1-\sigma} + \delta k(\tau, a, i)^{\alpha} \left\{ \Omega + \frac{h(\tau, a, i)^{1-\sigma}}{1-\sigma} + \omega \frac{(v^k(\tau, a, i))^{1-\phi}}{1-\phi} \right\} \tag{13}$$

subject to the budget constraint:

$$\sum_{a=0}^{\bar{a}} p(\tau+a) \left[\frac{h(\tau, a, i)}{s(\tau, a, i)} + v(\tau, a, i) + \frac{v^k(\tau, a, i) k(\tau, a, i)}{s(\tau, a, i)} \right]$$
$$\leq \sum_{a=0}^{\bar{a}} p(\tau+a) z(\tau, a, i) [1 - B_a(s(\tau, a, i) - 1) - B_a^k k(\tau, a, i)] \tag{14}$$

where $k(\tau, a, i)$ is the number of children in a household of adults of type (τ, i) at age a, $v^k(\tau, a, i)$ is the private consumption per child, and B_a^k is the fraction of time lost by an adult of age a in the care of each child. Adults can allocate private consumption asymmetrically between themselves and their children, but all members still share equally in the local public good. The parameter Ω, if sufficiently large, is a way to guarantee that the children's utility is positive even when $\sigma > 1$. This ensures that adults always want to have some children and that the corner $k^* = 0$ is never relevant. For other possible resolutions of a desire for positive fertility and high elasticity of substitution for consumption, see Jones and Schoonbroodt (2007).

6.2 Equilibrium

An equilibrium in this model is defined analogously to Section 4.4.

Definition 2 *An equilibrium for the economy with children is an allocation*

$\{s(\tau, a, i), v(\tau, a, i), h(\tau, a, i), v^k(\tau, a, i), k(\tau, a, i)\}_{\tau,i}$ *and prices* $\{p(t)\}_{t \in T}$ *that:*

1. *Each agent* (τ, i) *maximizes (12) and (13) subject to the constraint (14) and:*

$$s(\tau, a, i) \geq 1 \qquad \forall a$$

$$k(\tau, a, i) \geq 0 \qquad \forall a$$

$$v(\tau, a, i), h(\tau, a, i), v^k(\tau, a, i) > 0 \qquad \forall a$$

2. The market clearing condition for production is satisfied at every point in time t:

$$\sum_i \sum_{\{(\tau,a)|\tau+a=t\}} f(\tau, a, i) \left[\frac{h(\tau, a, i)}{s(\tau, a, i)} + v(\tau, a, i) + \frac{v^k(\tau, a, i)k(\tau, a, i)}{s(\tau, a, i)} \right]$$
$$= \sum_i \sum_{\{(\tau,a)|\tau+a=t\}} f(\tau, a, i) z(\tau, a, i)[1 - B_a(s(\tau, a, i) - 1) - B_a^k k(\tau, a, i)] \qquad (15)$$

In Appendix B we show that for certain conditions on parameters, this problem has a unique solution given by the first-order conditions. In the work that follows, we constrain ourselves to these portions of the parameter space.

6.3 Calibration

Introducing children into the model creates two difficulties. The first is that the model is sufficiently complicated that it is difficult to characterize the relationships between household size, consumption patterns, and income analytically. Our approach to assessing this model is thus purely computational. The second problem is that the CEX data do not distinguish between consumption of children and adults. We therefore use a different empirical strategy for this section. We calibrate the model to match a rich set of cross-section and time series moments on the patterns between income, age, and adults and children per household. We choose enough moments to guarantee that the model is overidentified, so that we can assess the fit of the model using its ability to match the moments. We are also particularly interested to see whether the model can match the asymmetric timing of the fall in adults and children per household (see Figure 1).

Much of the setup here is similar to that in Section 5. We assume that agents are adults and decision makers from 20-79. We set a period to represent 5 years, so that agents are adults for 12 periods in the model. We maintain that $f(\tau, i) = 1$, ignoring changes in demographics. The exogenous driving force is a smoothed version of GDP/capita which we input as productivity $z(\tau, a, i)$ in the model.

We calibrate 30 parameters: $\alpha, \delta, \omega, \sigma, \phi, \Omega, \{B_a\}_{a=1}^{12}, \{B_a^k\}_{a=1}^{12}$. We also impose some restrictions on the parameter space, to ensure that the problem is well-defined; these restrictions are explained in Appendix B. For completeness, they are $\delta > 0$, $\sigma > 1$, $0 < \phi < 1$,

$0 < \alpha - 1 + \phi < 1$. Ω also needs to be sufficiently large for the problem to be well-defined; we find that constraining it below by 0.000001 avoids any difficulties.

We use six time series moments: adults and children per household in 1850, the decline in each series between 1850 and 1940, and the decline in each series between 1940 and 2000. This set of moments is equivalent to choosing the levels in each year, but emphasizes that we are trying to match the asymmetry in the timing. We use ten moments on the cross-section between income and household size: the adults and children per household for each of the five income quintiles, calculated for agents age 25-29. Finally, we use 24 moments on the life cycle profile of adults and children per household: the average adults and children per household for agents of each of the 12 age groups. In total, we have 40 moments to match.

We calibrate all 30 parameters jointly to minimize a loss function over deviation from the 40 moments. The loss function is constructed so that the time series, income cross-section, and life cycle profile moments are weighted equally, and the loss for each moment is calculated as the square of percentage deviations between model and data.[14] The calibrated parameters are given in Figure 8 and Table 5.

Figure 8 plots the calibrated time costs of having adults and children for each age. The model suggests that children are much more costly than adults. Further, the costs of children at a given age seem to be closely related to expected fecundity of women at that or slightly younger ages. Finally, note that in a model with children, the costs of having adults vary little over the life cycle. Adults are less likely to live together when elderly because one of the main reasons to live together is to participate jointly in child-rearing, but elderly adults have few children and therefore lower benefits to living together. Table 5 gives the values for the other 5 parameters. Private goods display a much higher relative income elasticity in this model than in the model without children, as can be seen by the ratio σ/ϕ, which is 14.4 in this model versus 1.15 in the model without children.

6.4 Results

Since we cannot test up our model's predictions on expenditures, we use the overidentification of the model and its ability to match the asymmetry of the decline in adults and

[14]Precisely, the loss for each time series moment is weighted 1/6, the loss for each income cross-section moment is weighted 1/10, and the loss for each age cross-section moment is weighted 1/12.

Figure 8: Time Costs of Family Members

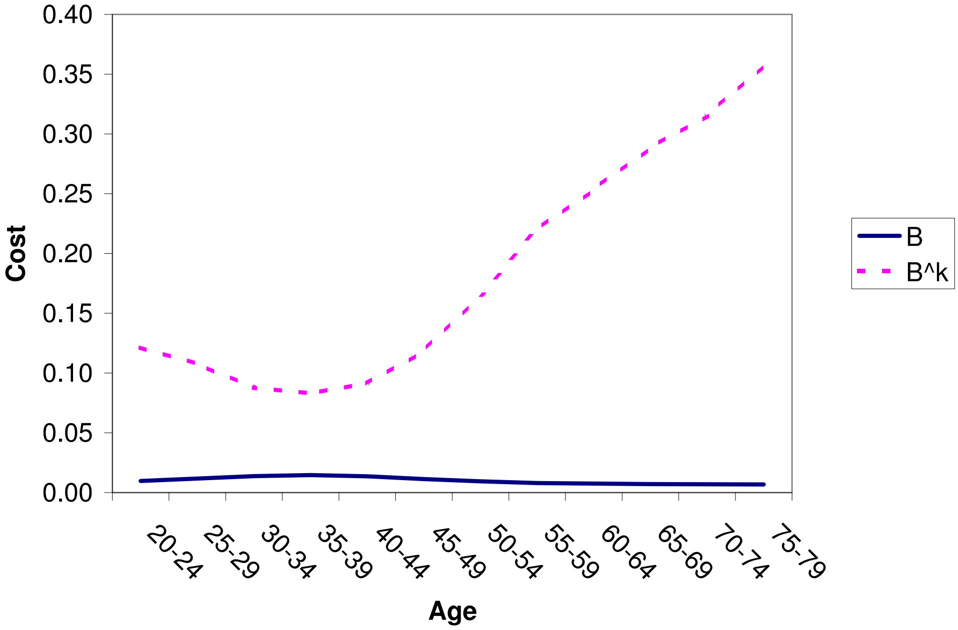


Table 5: Calibrated Parameters

Parameter	α	δ	ω	σ	ϕ	Ω
Value	0.5521	0.0795	1.68×10^{-5}	9.5176	0.6592	0.0011

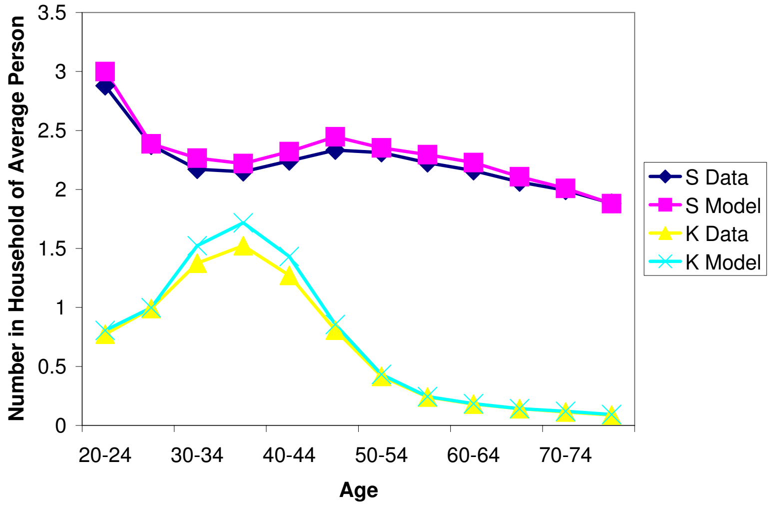
children per household to assess its fit. Figure 9 plots the model predicted and actual household sizes by age in 2000.[15] Since the model has free parameters B_a and B_a^k for each age, it is reassuring that the model matches these moments quite well.

The overidentification of the model binds elsewhere. Figure 10 plots the predicted relationship between income and household size in the cross-section. The model generally performs well, matching the elasticity of children in the household quite well and with a modest overestimation of the amount of elasticity of adults in the household.

Finally, Figure 11 plots the model's predicted time series paths for adults and children

[15] A table given all the moments for the model and the data is available in Appendix C.3.

Figure 9: Life Cycle Profile

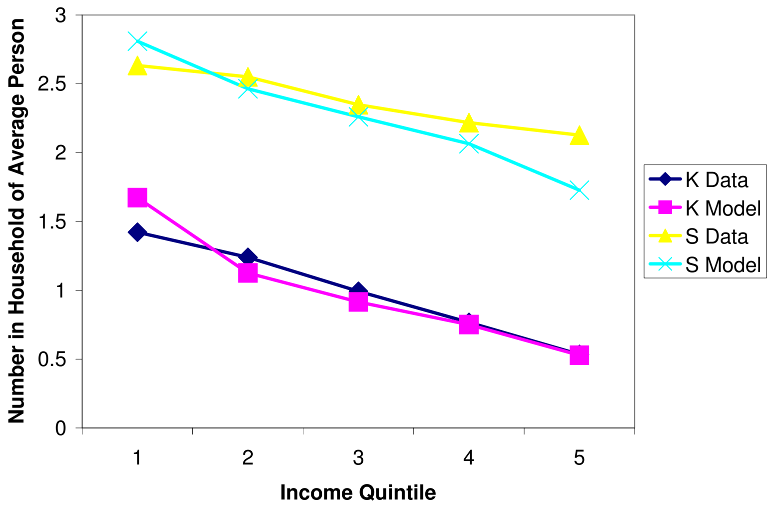


in the household. The model predicts substantially too few children in 1850, but otherwise does quite well. It predicts a sharper decline for children than for adults, and does a good job at matching the asymmetry in the decline of adults per household, although not as good of a job in matching the asymmetry in the decline of children per household. For adults, the model predicts that 31% of the total decline happens by 1940, as opposed to 24% in the data. For children, the model predicts that 49% of the total decline happens by 1940, as opposed to 60% in the data.

Interestingly, the model is able to replicate some of the key features of the data in the 1930-1960 period, including the decline and recovery of number of children in the household and the dramatic decline in adults in the household from 1935 to 1945. The only driving force in the model is changing income, so the model is translating the large income swing from the Great Depression through the post-World War II boom into a rapidly changing preference for quality of children over quantity.

Overall, the model with children predicts a 71% decline in children in the household, as opposed to 68% in the data. The key force in the model is the quantity-quality tradeoff. In the calibrated model, expenditures on private goods per child are a constant level relative to expenditures on private goods per adult. Hence, both adults and children substitute in a large way towards private goods, making children much more costly and causing substitution from quantity of children to quality. The model also predicts 26% decline in the

Figure 10: Cross-Section Relationship with Income



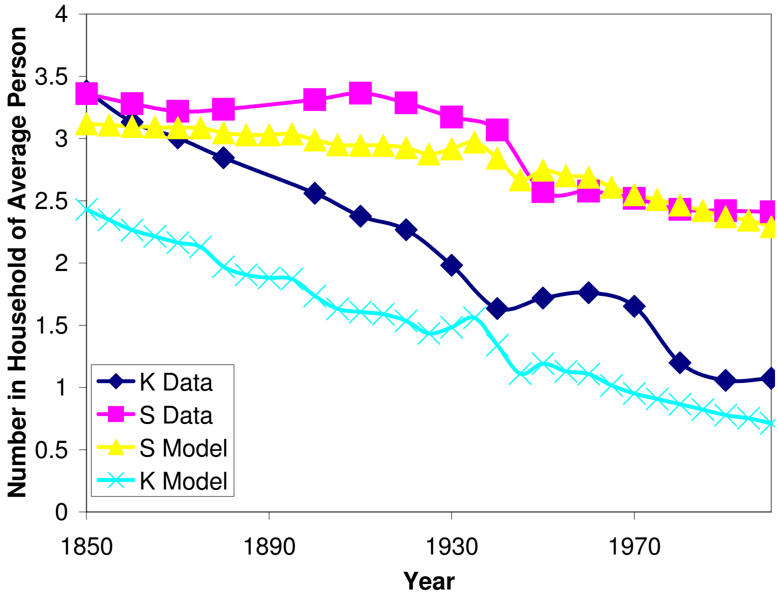
number of adults in the average person's household, as opposed to a 34% decline in the data. The key force here is that the number of children falls sufficiently rapidly that the budget share of children falls over time by about half. Since sharing expenditures on children becomes a less important motive for living together, adults live in smaller households as well.

7 Conclusion

There has been a systematic, robust decline in household size over the last 150 years, affecting virtually every type of agent in the economy and causing declines in nearly every form of living together. Motivated by this fact, we propose a model where agents choose their household size and their consumption of public and private goods. Agents form households to share the cost of public goods, but the downside is that they face a time cost to maintaining a household. In this model, we abstract from any institutions and frictions that surround household formation.

We combine productivity growth and non-homothetic utility as a mechanism for agents to prefer endogenously more private consumption over time. The mechanism underlying our explanation for the decline in household size is that, as income rises, agents exogenously prefer a relatively more private consumption basket, and thus they endogenously choose

Figure 11: Time Series



smaller household sizes. This mechanism is consistent with what we know about household sizes, relative consumption baskets, and income per capita from both micro and macro data. In particular, in the CEX more income per capita is associated with smaller households and a relatively more private consumption bundle.

Using the CEX data, we calibrate our model to fit the recent cross-section quite well. We then ask the question: how much of the fall in household size over the last 150 years can we explain through rising income and private goods being relative luxury goods? Our answer is that we can explain about 30 percent.

We then add children to the framework and find that the channel does also well in accounting for the decline in the average number of children per household. Children equally benefit from the public good, so that when a large fraction of household income is spent on household public goods, the marginal expenditures on children is low. As income goes up, and people prefer to spend more of their income on private goods, children become endogenously more costly. This effect is strengthened because altruistic parents also want to buy more child-specific private goods for their children, which makes children even more costly. The calibrated model is able to replicate an interesting asymmetry that is present in the data: the number of children per household declines very steeply between 1850 and 1940, and falls at a slower pace thereafter, whereas the number of adults per household is almost constant between 1850 and 1940 with most of the total fall occurring after 1940.

Our model proposes a mechanism broad enough to encompass many different facets of the decline in household sizes. Obviously we have not captured all the factors relevant for the observed decline in household size. Other factors that have been emphasized in the literature are the declining relative price of household appliances, the decrease in fertility, and social security which made living alone more affordable for retirees. Nevertheless, our work shows that income can play an important role in quantitatively generating a decline which affects many decisions and living together arrangements, and which helps unify a variety of different trends previously studied separately.

References

Attanasio, O., and M. Browning (1995): "Consumption over the Life Cycle and over the Business Cycle," *American Economic Review*, 85(5), 1118–1137.

Becker, G., and R. Barro (1988): "A Reformulation of the Theory of Fertility," *Quarterly Journal of Economics*, 103(1), 1–25.

Becker, G., and H. Lewis (1973): "On the Interaction between the Quantity and Quality of Children," *Journal of Political Economy*, 81(2, Part 2), S279–S288.

Beresford, J. C., and A. M. Rivlin (1966): "Privacy, Poverty, and Old Age," *Demography*, 3(1), 247–258.

Bethencourt, C., and J. Rios-Rull (2007): "On the Living Arrangements of Elderly Widows," *CAERP Discussion Paper # 21*.

Boldrin, M., and L. Jones (2002): "Mortality, Fertility and Saving Decisions," *Review of Economic Dynamics*, 5, 775–814.

Brady, D. S. (1958): "Individual Incomes and the Structure of Consumer Units," *The American Economic Review*, 48(2), 269–278.

Buchanan, J. M. (1965): "An Economic Theory of Clubs," *Economica*, 32(125), 1–14.

Burch, T. K., and B. J. Matthews (1987): "Household Formation in Developed Societies," *Population and Development Review*, 13(3), 495–511.

Chandler, T. (1987): *Four Thousand Years of Urban Growth: An Historical Census*. The Edwin Mellen Press, second edn.

Costa, D. L. (1997): "Displacing the Family: Union Army Pensions and Elderly Living Arrangements," *Journal of Political Economy*, 6, 1269–1292.

——— (1999): "A House of Her Own: Old Age Assistance and the Living Arrangements of Older Nonmarried Women," *Journal of Public Economics*, 72, 39–59.

Deaton, A., and C. Paxson (1998): "Economies of Scale, Household Size, and the Demand for Food," *Journal of Political Economy*, 106(5), 897–930.

Doepke, M. (2004): "Accounting for Fertility Decline During the Transition to Growth," *Journal of Economic Growth*, 9(3), 347–383.

Ermisch, J., and P. Di Salvo (1997): "The Economic Determinants of Young People's Household Formation," *Economica*, 64, 627–644.

Ermisch, J. F. (1981): "An Economic Theory of Household Formation," *Scottish Journal of Political Economy*, 28(1), 1–19.

Ermisch, J. F., and E. Overton (1985): "Minimal Housing Units: A New Approach to the Analysis of Household Formation," *Population Studies*, 39(1), 33–54.

Fernandez-Villaverde, J., and D. Krueger (forthcoming): "Consumption over the Life Cycle: Facts from the Consumer Expenditure Survey Data," *Review of Economics and Statistics*.

Fogli, A. (2000): "Endogenous Labor Market Rigidities and Family Ties," New York University, mimeo.

Galor, O., and D. N. Weil (1996): "The Gender Gap, Fertility and Growth," *The American Economic Review*, 86(3), 374–387.

——— (2000): "Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond," *American Economic Review*, 90(4), 806–828.

Greenwood, J., and N. Guner (2004): "Marriage and Divorce since World War II: Analyzing the Role of Technological Progress on the Formation of Households," *University of Rochester, Research Report No. 8*.

Greenwood, J., and A. Seshadri (2002): "The U.S. Demographic Transition," *American Economic Review Papers and Proceedings*, 92(2), 153–159.

Greenwood, J., A. Seshadri, and G. Vandenbroucke (2005): "The Baby Boom and Baby Bust," *American Economic Review*, 95(1), 183–207.

Greenwood, J., A. Seshadri, and M. Yorukoglu (2005): "Engines of Liberation," *Review of Economic Studies*, 72, 109–133.

Haurin, D. R., P. H. Hendershott, and D. Kim (1994): "Housing Decisions of American Youth," *Journal of Urban Economics*, 35, 28–45.

Johnston, L. D., and S. H. Williamson (2006): "The Annual Real and Nominal GDP for the United States, 1790 - Present," Economic History Services, http://eh.net/hmit/gdp/.

Jones, L., and M. Tertilt (2006): "An Economic History of the Relationship Between Occupation and Fertility - U.S. 1826-1960," *NBER Working Paper*, 12796.

Jones, L. E., and A. Schoonbroodt (2007): "Trends in Fertility and the Intertemporal Elasticity of Substitution in Dynastic Models," *University of Minnesota, mimeo.*

Kobrin, F. E. (1976): "The Fall in Household Size and the Rise of the Primary Individual in the United States," *Demography*, 13(1), 127–138.

Kuznets, S. (1978): "Size and Age Structure of Family Households: Exploratory Comparisons," *Population and Development Review*, 4(2), 187–223.

Lam, D. (1983): "The Economics of Household Composition in Developing Countries," Ph.D. thesis, University of California, Berkeley.

——— (1984): "Household Public Goods and the Shadow Price of Privacy: A Framework for Modeling the Interaction of Household Composition and Economic Behavior," Paper presented at the annual meeting of the Population Association of America, Minneapolis.

——— (1988): "Marriage Markets and Assortative Mating with Household Public Goods: Theoretical Results and Empirical Implications," *The Journal of Human Resources*, 23(4), 462–487.

Laslett, P. (1969): "Size and Structure of the Household in England Over Three Centuries," *Population Studies*, 23(2), 199–223.

Lazear, E., and R. Michael (1980): "Family Size and the Distribution of Real Per Capita Income," *American Economic Review*, 70(1), 91–107.

Logan, T. D. (2007): "Economies of Scale in the Household: Evidence and Implications from the American Past," Working Paper.

McGarry, K., and R. F. Shoeni (2000): "Social Security, Economic Growth,a nd the Rise in Independence of Elderly Widows in the 20th Century," *Demography*, 37, 221–236.

MICHAEL, R. T., V. R. FUCHS, AND S. R. SCOTT (1980): "Changes in the Propensity to Live Alone: 1950-1976," *Demography*, 17(1), 39–56.

NELSON, J. (1988): "Household Economies of Scale in Consumption: Theory and Evidence," *Econometrica*, 56(6), 1301–1314.

ORSHANSKY, M. (1965): "Counting the Poor: Another Look at the Poverty Profile," *Social Security Bulletin*, pp. 3–29.

PAMPEL, F. C. (1983): "Changes in the Propensity to Live Alone: Evidence from Consecutive Cross-Sectional Surveys, 1960-1976," *Demography*, 20(4), 433–447.

REGALIA, F., AND J.-V. RIOS-RULL (2001): "What Accounts for the Increase in the Number of Single Households?," University of Pennsylvania, mimeo.

RUGGLES, S., AND S. BROWER (2003): "Measurement of Household and Family Composition in the United States, 1850-2000," *Population and Development Review*, 29(1), 73–101.

A Appendix 1: Census Data Details

Measurement of household size using U.S. Census data is somewhat complicated by four issues. First, the Census definition of persons living together has changed somewhat over time. Second, persons living together may be categorized as households or as group quarters, with the distinction changing over time. Third, there exist some fragment records, or parts of households, as a result of data collection errors. Finally, the IPUMS system did not sample large households in their entirety in the early Census years. For completeness, we discuss our treatment of these issues here; for more detail on the first two issues, see also Ruggles and Brower (2003).

The Census definition of persons who live together is always based on sharing a house or a part of a house and sharing some resources. The measure of resources used has changed somewhat over time: early Censuses stressed shared income, but later Censuses used shared kitchens, shared dining tables, shared cooking equipment, or meals taken together as the measure of persons who live together. Although we suspect that the changing definition has had little impact on our measures, there is no room to evaluate that proposition analytically.

In 1930, Census enumerators split persons who live together into two categories: households, and group quarters. Group quarters are households which include a large number of unrelated persons; typical group quarters include hospitals, jails, asylums, retirement homes, barracks, college dormitories, and lodging houses. The Census also counted all households with a sufficient number of people unrelated by blood or marriage to the household head as being group quarters. Problematically, the exact number of persons changed somewhat over time. Residences with 5-9 persons unrelated to the household head were categorized as households in some years, and as group quarters in others. The IPUMS project has also coded residences as group quarters or households for the 1850-1920 period by using both definitions of group quarters (5 or more persons unrelated and 10 or more persons unrelated).

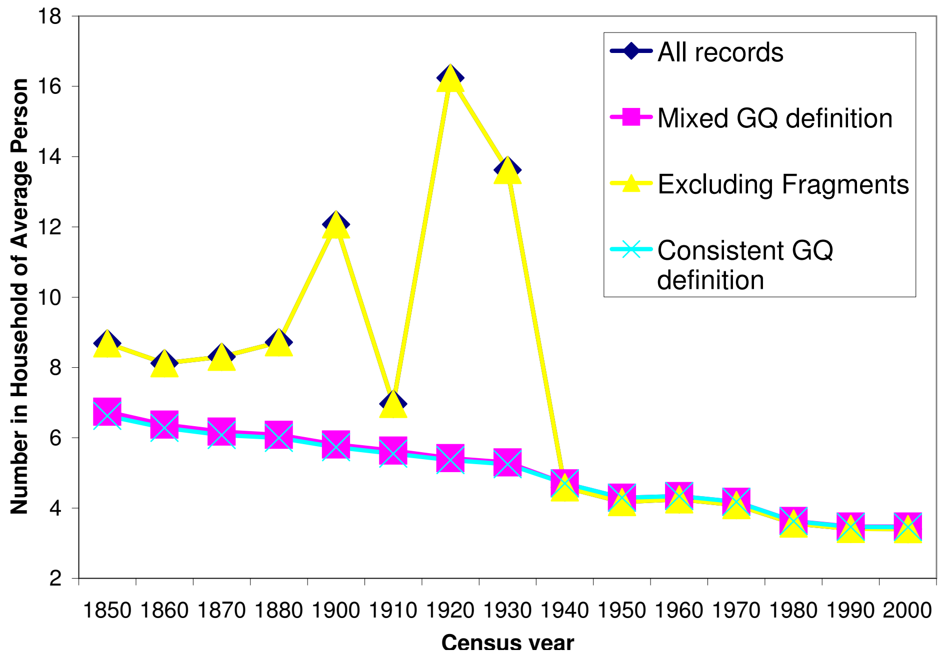
In our conceptual framework, the distinction between households and group quarters is irrelevant as long as the living arrangement is a choice. For example, students or boarders make a conscious decision to live together to share resources and lower costs. There are two problems with using persons living in group quarters in our analysis. First, for some people such as prison inmates living arrangements are not a choice, and the Census provides no consistent guidance on who was living in group quarters voluntarily. Indeed, for many years the Census only tells us that the persons are living in some sort of group quarters. Second, if a unit is classified as group quarters, there is no way to link members who live together, which prevents us from providing information on the composition and demographic characteristics of the household. Because of these issues, our basic principle is to exclude those living in group quarters. Finally, one component of the definition of group quarters changed over time: in some years, a small number of households with 5-9 persons unrelated to the household head were coded as group quarters, and in others they were coded as households. We include these persons when they are coded as households and exclude them when they are counted as group quarters. In principle this makes the definition inconsistent, but in practice so few persons are affected that the definitions are nearly identical.

A relatively small number of individuals from 1850-1930 are categorized as fragments. Typically, these are persons who were initially missed by Census enumerators, and were added at some later date. For these households it is impossible to verify the household or group quarters status. Further, there is no way to link these records to the other members of the household. For these reasons, we exclude all fragments.

The final issue is that the IPUMS system did not sample large households in their entirety for early Censuses. The data on household size is collected in two variables. Up until 1930, the Census included a question asking respondents to provide their own household size. Thus, we have an accurate count of the number of persons in each household until that date. However, the IPUMS system did not sample large households (30+ members) in their entirety for the period 1850-1930. Hence, we might know that there are 32 household members, but the Census data only include the first 10. We use appropriate weights to correct for this in computing the size and likely demographic characteristics of the missing persons. Most of these households are subsequently excluded because they qualify as group quarters: it is difficult to find households with 30 or more persons, 10 or fewer of whom are unrelated to the household head. From 1940 onward the IPUMS system sampled large households in their entirety, and they further provide a measure of the associated number of records, NUMPREC. For these years, we use NUMPREC is our measure of household size.

Figure 12 shows the time series of household size when excluding various records. Excluding fragments makes almost no difference: the averages for all records and records excluding only fragments are nearly identical. Similarly, there is little difference between the averages for consistent and time-varying definitions of group quarters. The primary benefits come from excluding the small fraction of the population consistently coded as

Figure 12: Household Size, Various Definitions (constructed from U.S. Census Data



group quarters (1.6 - 3.7 percent of the sample, depending on the year); the pre-1940 data is wildly irregular without this decision.

Table 6 gives the results of using the NUMPREC or NUMPERHH measures of household size. The two measures differ at most by 0.02 of a child, except for 1930 where the difference is a 0.15, but that is still less than a 3 percent difference. Excluding the group quarters we exclude makes the two measures nearly identical.

Table 6: Different Measures of Household Size

census yr	children	adults	numperhh	numprec	famsize	unrelated individuals*	# observations
1850	3.531	3.212	6.742	6.742	5.976	0.766	193,276
1860	3.254	3.130	6.372	6.384	5.680	0.692	269,628
1870	3.139	3.060	6.177	6.199	5.514	0.663	375,938
1880	3.018	3.063	6.081	6.081	5.450	0.631	491,663
1900	2.723	3.086	5.809	5.809	5.238	0.571	737,975
1910	2.533	3.098	5.634	5.631	5.089	0.545	354,570
1920	2.415	2.974	5.418	5.388	5.006	0.412	1,020,057
1930	2.218	2.936	5.297	5.154	4.794	0.503	235,260
1940	1.831	2.872	n/a	4.703	4.484	0.219	1,311,052
1950	1.718	2.575	n/a	4.293	4.148	0.145	1,904,786
1960	2.022	2.315	n/a	4.338	4.260	0.078	1,750,660
1970	1.913	2.261	n/a	4.174	4.107	0.067	1,972,150
1980	1.371	2.258	n/a	3.629	3.522	0.107	2,209,919
1990	1.201	2.270	n/a	3.471	3.300	0.171	2,447,860
2000	1.227	2.245	n/a	3.472	3.321	0.151	2,740,802

* Unrelated individuals are household members that are not related to the household head by blood, marriage or adoption. It equals the difference between household size and family size.

B Appendix 2: Technical Details

Here we present the proofs that the agents' problems have unique solutions given by the first-order conditions.

Proposition 5 *Solution to Agent Without Children's Problem*

For any $\{p(t)\}_{t=0}^{\infty}$ *such that* $p(t) > 0$ *for all* t*, there exists a solution to the agent's problem given by (1) - (3) for all* (τ, i)*.*

Proof. The non-degenerate constraint qualification holds trivially for this system of constraints, so the Kuhn-Tucker conditions describe the entire set of local maxima. As long as the budget set is compact and the utility is continuous, there exists a unique global maximum. Continuity of utility is obvious, so we verify compactness.

The inequality of the budget constraint guarantees that the set is closed from above. The set is not closed from below for $h(\tau, a, i)$ and $v(\tau, a, i)$, since we have to impose the restrictions $h(\tau, a, i) > 0$ and $v(\tau, a, i) > 0$ for the utility to be well-defined. But since the consumer's preferences satisfy the Inada conditions, then by standard arguments we can always create an equivalent budget set where we constraint $h(\tau, a, i) \geq \underline{\mathrm{h}}$ and $v(\tau, a, i) \geq \underline{\mathrm{v}}$ for some small $\underline{\mathrm{h}}$ and $\underline{\mathrm{v}}$, effectively making the budget set closed.

We have bounded the choice variables from below within the problem. To see that they are bounded from above, note that with positive prices, $v(\tau, a, i)$ and $s(\tau, a, i)$ are both directly limited by the total resources available. Given that $s(\tau, a, i)$ is bounded from above, it follows that $h(\tau, a, i)$ is bounded from above. Hence, the set is compact, there exists a unique global maximum, and it is described by the Kuhn-Tucker conditions. Since for the problem outlined above there is a unique point described by the Kuhn-Tucker conditions, the solution to the problem is that unique point. *Q.E.D.*

The problem for agents with children is somewhat more difficult and involves more technical assumptions. The conditions are standard for a model with Barro-Becker style altruism towards children (Becker and Barro 1988). Jones and Schoonbroodt (2007) shows that there are other conditions for which the problem is well-defined, but we focus our attention on this set for the time being. Otherwise, the proof is a combination of the intuition from a Barro-Becker style model with the proof just completed.

Proposition 6 *Solution to Agent With Children's Problem*

Suppose $\delta > 0$, $\sigma > 1$, $0 < \phi < 1$, $0 < \alpha - 1 + \phi < 1$. Then for any $\{p(t)\}_{t=0}^{\infty}$ such that $p(t) > 0$ for all t, there exists a $\underline{\Omega}$ such that for all $\Omega >\underline{\Omega}$, there exists a solution to the agent's problem given by (12) - (14) for all (τ, i).

Proof. The proof follows similar logic to the one above. To see the analogy to this problem with the usual fertility with altruism problem, define the agent's problem in terms of total private goods spending on children, $V^k(\tau, a, i) = k(\tau, a, i)v^k(\tau, a, i)$. The agent's utility is continuous, strictly increasing, and strictly concave in all arguments for $0 < \alpha - 1 + \phi < 1$; other than the addition of the public good, this utility function is identical to Becker and Barro (1988). Then it is necessary to verify that the budget set is compact.

The set is bounded below by assumption. $v(\tau, a, i)$, $s(\tau, a, i)$, and $k(\tau, a, i)$ are bounded above directly from the budget constraint. Since $s(\tau, a, i)$ is bounded below by 1, it follows that $h(\tau, a, i)$ and $V^k(\tau, a, i)$ are bounded above as well by the budget constraint. Hence, the problem is bounded.

The set is also closed from above by the inequality in the budget constraint. $s(\tau, a, i)$ and $k(\tau, a, i)$ are closed from below by assumption. As before, the other variables are open from below. However, if we can verify that the Inada conditions hold, then it is possible to define sufficiently small $\underline{\mathrm{v}}$, $\underline{V}^k$, and $\underline{\mathrm{h}}$ such that there is an equivalent budget constraint where $v(\tau, a, i)$, $V^k(\tau, a, i)$, and $h(\tau, a, i)$ are closed from below by the three lower bounds.

The Inada condition for $v(\tau, a, i)$ is straightforward. Since $\delta > 0$ and $k(\tau, a, i)$ is bounded from above, then the Inada condition also holds for $h(\tau, a, i)$. Define $\underline{\Omega} = \frac{\underline{h}^{1-\sigma}}{1-\sigma}$, so that children have positive utility and parents want to have children. Then the Inada condition also holds for $k(\tau, a, i)$, so that there exists a $\underline{\mathrm{k}}$ such that $k(\tau, a, i) \geq$ bk. Finally, as long as $k(\tau, a, i)$ is bounded below by a strictly positive number, it is straightforward to show that the Inada condition holds for $V^k(\tau, a, i)$ as well. Hence, the problem is equivalent to one which is closed from below, and the problem is equivalent to a compact one. Again, the set is compact, there exists a unique global maximum, and it is described by the Kuhn-Tucker conditions. Since for the problem outlined above there is a unique point described by the Kuhn-Tucker conditions, the solution to the problem is that unique point. *Q.E.D.*

C Appendix 3: Computational Methods and Calibration Details

C.1 Computational Details

In this appendix we address some additional details of the structure and solution of the model, as well as the construction of the data moments. We use an iterative two-step method to calibrate the model. In the first step, we guess a set of parameter values. In the second step, we solve for the prices and allocations of the economy governed by those parameters. Our economy with children treated symmetrically starts in 1750 with an allocation of optimizing initial old, and ends in 2100 with agents who optimally live alone and allocate income between private and public expenditures. The economy with children treated asymmetrically begins even earlier and ends later. We then extract information about the time period of interest, 1850-2000; we have verified that the extra years are a sufficiently long horizon that the 1850-2000 results are insensitive to the initial and terminal points. After solving the economy, we calculate the relevant moments; unless they are sufficiently close to the data moments, we return to Step 1 and begin again.

C.2 Construction of Moments for Model Without Kids

The construction of our data moments requires a bit of explanation. The cross-section of household sizes by age or by age and income are calculated from a 1% sample of the 2000 U.S. Census data. Historical household sizes were calculated similarly from older samples. The data on expenditure and income patterns are all taken from the Consumer Expenditure Survey. To create a sufficiently large sample of agents, we pool observations from 1995-2000 and treat them as a single cross-section. We break people down into five income quintiles, which correspond to five different types i in the model. For each type, we find the average household size, average income (ignoring issues of topcoding), average h, and average v.

To find an observation for the level of h/v, we use the total h of all observations from our constructed data set, divided by the total v. We prefer the ratio of averages because the average of the ratios is much more sensitive to errors and misreported values (a single reported $v = 0$ would greatly bias the average of the ratios, for instance).

Elasticities are a bit more difficult to compute because we do not have one fixed elasticity.

For the elasticity of relative consumption with respect to productivity, we have chosen to use the elasticity of response implied by the differences between the income quintiles in the CEX. Since we feel that the elasticities may not be well-estimated for young agents (who are not decision-makers to the extent that adults are) and old agents (for whom retirement may hamper income variation, providing a poor estimate of productivity), we choose to use only the elasticities of the agents between 40-49, whom we pool into one large sample. Finally, while elasticity is defined as an instantaneous derivative, we are approximating elasticities with very discrete data, so the elasticity is sensitive to the choice of the denominator; we choose to average across the two denominators. To be precise, the elasticity is:

$$\varepsilon_{\alpha z} = \frac{1}{4}\left\{\sum_{i\in\{2..5\}}\left(\frac{\bar{\alpha}(a,i)-\bar{\alpha}(a,i-1)}{\bar{\alpha}(a,i-1)+\bar{\alpha}(a,i)}\right)\left(\frac{\bar{z}(i-1)+\bar{z}(i)}{\bar{z}(i)-\bar{z}(i-1)}\right)\right\}$$

Where $\bar{\alpha} = \frac{\bar{h}}{\bar{v}}$. Here, we use i to denote the five quintiles constructed from the CEX data. a is held fixed at 40-49, while $\bar{z}(i)$ and $\bar{s}(a,i)$ denote the average observations of income per capita and household size for agents of age 40-49 and income quintile i. We calculate the model moments in the same manner. Finally, rather than empirically estimate the IES, we choose a value of 0.5, which is consistent with standard business cycle estimates.

C.3 Moments for Model With Kids

Table 7: Moments for Model with Children

Moment	Data Value	Model Value
1850 Adult Household Size	3.360	3.118
1850 Child Household Size	3.382	2.431
Fall in Adult HH Size, 1850-1940 (%)	-8.670	-8.859
Fall in Child HH Size, 1850-1940 (%)	-51.700	-44.912
Fall in Adult HH Size, 1940-2000 (%)	-27.300	-19.338
Fall in Kid HH Size, 1940-2000 (%)	-34.270	-46.794
Adult HH Size, Quintile 1, Age 25-29	2.633	2.810
Adult HH Size, Quintile 2, Age 25-29	2.551	2.464
Adult HH Size, Quintile 3, Age 25-29	2.348	2.260
Adult HH Size, Quintile 4, Age 25-29	2.218	2.064
Adult HH Size, Quintile 5, Age 25-29	2.127	1.727
Child HH Size, Quintile 1, Age 25-29	1.422	1.673
Child HH Size, Quintile 2, Age 25-29	1.240	1.126
Child HH Size, Quintile 3, Age 25-29	0.993	0.915
Child HH Size, Quintile 4, Age 25-29	0.767	0.751
Child HH Size, Quintile 5, Age 25-29	0.534	0.528
Adult HH Size, Age 20-24	2.880	2.999
Adult HH Size, Age 25-29	2.375	2.386
Adult HH Size, Age 30-34	2.171	2.265
Adult HH Size, Age 35-39	2.151	2.219
Adult HH Size, Age 40-44	2.242	2.321
Adult HH Size, Age 45-49	2.333	2.446
Adult HH Size, Age 50-54	2.312	2.352
Adult HH Size, Age 55-59	2.226	2.295
Adult HH Size, Age 60-64	2.160	2.228
Adult HH Size, Age 65-69	2.063	2.108
Adult HH Size, Age 70-74	1.992	2.009
Adult HH Size, Age 75-79	1.882	1.880
Child HH Size, Age 20-24	0.774	0.805
Child HH Size, Age 25-29	0.991	0.998
Child HH Size, Age 30-34	1.375	1.524
Child HH Size, Age 35-39	1.524	1.718
Child HH Size, Age 40-44	1.272	1.432
Child HH Size, Age 45-49	0.807	0.855
Child HH Size, Age 50-54	0.416	0.433
Child HH Size, Age 55-59	0.241	0.245
Child HH Size, Age 60-64	0.182	0.185
Child HH Size, Age 65-69	0.143	0.143
Child HH Size, Age 70-74	0.114	0.120
Child HH Size, Age 75-79	0.088	0.093