Enclosure

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Abstract

We explore the relationship between capital accumulation and the development of property rights. In our analysis, the development of property rights is an endogenous process, driven by capital accumulation. Property rights are defined as institutions that internalize the portion of the return to capital that is otherwise treated as common property. This enclosure further encourages capital accumulation and sustains economic growth.

We model such institutions as multilateral agreements among agents, which can only be implemented and sustained when agents are sufficiently patient, so that the long-run benefits of sustaining agreements outweigh the short-run incentive to defect. In this model, patience is determined by the marginal product of capital; the marginal product of capital shrinks, and consequently patience increases, as capital grows. Hence property rights can be established only when capital is sufficiently abundant.

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1 Introduction

De Soto (1989) has highlighted how tenuous property rights are in poor countries. To buy a house or to start a business in Peru is a major undertaking in no small part because the ownership of a piece of land might be ill-defined, or because the myriad licensing requirements of a business effectively block its legitimate formation. In rich countries the notion of the transfer of property is an elementary process because the delineation of property rights is at the very core of the state’s purpose. De Soto’s thesis is that the definition of property rights increases when the need exists. His primary example is the development of squatter camps in Peru, which as time passes, develop internal, unofficial systems of property rights that eventually are ratified by the state. But this process proceeds with glacial slowness in poor countries, to the obvious detriment of their economies.

If the benefits from property rights are so great, countries wishing to emerge from poverty would support them without outside prodding, and with far greater alacrity than they do in practice. Why don’t they? We provide an answer.

Our theory treats property rights as the expression of a multilateral contract that can be implemented and sustained only when agents are sufficiently patient. The degree of patience is endogenous: it is determined by the market interest rate, which in equilibrium is identical to the marginal product of capital. As growth progresses, the marginal product of capital, and therefore the interest rate, shrinks, and patience increases. Increased patience leads firms to adhere to the contract, rather than opt for the short-run benefits of free riding. Property rights therefore develop sequentially just as in the situations De Soto describes.

In our theory property rights are therefore the consequence of growth as well as its cause. Because property rights formation results in a more efficient use of capital, growth can actually accelerate when property rights form. Our theory is therefore consistent with the empirical reality that wealthy countries can grow faster than poor countries in some stages of development.

The phenomenon identified by De Soto is far from unique to present-day Latin America.
It is strikingly similar to the enclosure of public fields in England that has been famously studied in the economic history literature:

...5,000-odd acts of Parliament had swept [the system of open fields] away, transforming numerous and vague rights of use to open fields, commons, and waste into unambiguous rights of ownership to enclosed plots, free of village direction. (McCloskey, 1972)

We therefore term the transition to well-defined property rights “enclosure”.

In this paper we expand the concept of enclosure to encompass capital in general. For example, we view intellectual output such as inventions as a kind of commons, and the institution of patent protection as the equivalent of enclosure.

We do not differentiate capital into physical capital and human capital. We do, however, model ownership of capital as diffuse: capital nominally owned by a firm is used directly by that firm, but is also an element of the production functions of other firms. This representation of production technology can be viewed as the expression of technology spillovers as documented by Griliches (1958), and Bernstein (1988). We interpret these spillovers as the consequence of poorly defined property rights, rendering part of the return to investment, which would efficiently be private, as common property. In order to emphasize the fungibility of the extent of these spillovers, we will refer to the spillover component of firms’ production functions as complementary capital.

When firms pay each other the marginal value of the complementary capital that they use, firms are encouraged to produce additional capital; we equate the extent of these payments with the extent of property rights, and interpret the unwillingness of firms to make the payments as the absence of property rights. We then examine whether positive prices can be supported in a non-cooperative setting.

Our non-cooperative solution concept stems from the theory of repeated games. We ask whether individual firms would wish to refrain from defection even if they have the opportunity to free ride temporarily when they defect. The free-riding period, during which
a defecting firm is paid by other firms, but does not pay others, is followed by permanent exclusion from the property rights agreement, in which access to its complementary capital is limited. Reverting to permanent exclusion may be an extreme punishment for a defector. However, if we can show that such grim punishments fail to constrain defection, we have demonstrated that cooperation cannot be an equilibrium.

Our main finding is that property rights—firms appropriately compensating each other for the use of complementary capital—cannot be supported in poor economies, but may eventually become supportable as capital accumulates.

The driving force behind the result is the folk theorem. The folk theorem establishes that cooperation can be an outcome in a repeated game if the players have sufficient patience. Patience plays a role because the gains from defection are short-lived, since punishments are initiated once a defection is perceived, and the subsequent losses are long-lived. When agents are patient, the long-run punishment is discounted less severely so that it can dominate the short-term gains from defection and therefore deter it.

In standard repeated games the discount rate or degree of patience is fixed. But during economic growth, the discount rate, which is identical to the marginal product of capital, is not fixed: it declines as capital accumulates. In our model, poor economies with low capital have high marginal products of capital, and therefore high discount rates.\(^1\) They therefore cannot sustain the cooperative outcome. As they accumulate capital their discount rate shrinks, and when they cross a threshold level of capital the discount rate reaches the point at which cooperation is sustainable. Property rights then form.\(^2\)

\(^1\)Our theory predicts that interest rates and development will be negatively correlated in the long run, just as the standard neoclassical model does. Viewing the long run in terms of centuries, we should expect that interest rates have broadly declined over time. There is some limited evidence for this: McCloskey and Nash (1984) showed that the interest rate, measured by the dynamics of the price of grain in medieval England, was high relative to post-industrial revolution interest rates.

\(^2\)The linkage between property rights and growth is an active subject of investigation. A leading investigation by Ginarte and Park (1997) showed that there is a positive correlation between patent rights and
Clearly, capital accumulation would be accelerated if the overall marginal return on capital increases due to enclosure. Conversely, capital accumulation supports enclosure. Because of this positive feedback between property rights and capital accumulation, our model predicts that rich countries, which can sustain property rights, can sometimes grow faster than poor countries.\(^3\)

Our study supplements the existing literature on intellectual property rights, such as Grossman and Helpman 1991a, 1991b, and 1994, Helpman 1993, and Grossman and Lai 2002. In these models, the ability to garner monopoly profits is the only incentive for a firm to invest in knowledge. The externality of knowledge takes the form of costless imitation, which, if not discouraged by property rights, drives the profits of the initial investor to zero. In this set-up, there is an optimal level of property rights protection that a benevolent government could establish at any time by balancing the tradeoff between the loss of consumer surplus due to monopoly power and the loss of investment incentives due to the absence of property rights protections. We go beyond this literature in two senses. First, in our model the extent of property rights is endogenous, and cannot be consciously imposed by a government. Second, our model is dynamic, and so we can say when property rights form and characterize their extent.

The theory of property rights is also central in the new institutional economics literature. income per capita. In a more recent empirical study by Djankov, La Porta, Lopez-de-Silanes and Shleifer (2004) which examines the efficiency of legal systems, a positive correlation between income and property rights also emerges.

We provide additional details in section 6.

\(^3\)A corollary is that capital does not necessarily flow from rich countries to poor countries. Tornell and Velasco (1992), in a model of growth and property rights, posit two investment opportunities available to agents in a developing economy: high-return domestic capital and low-return foreign capital. Investors shun investment in high-return domestic projects because their returns are subject to expropriation and redistribution due to poorly developed property rights in the domestic economy—the entire capital stock is shared by all agents in the economy, just as in a fishery or commons. Capital therefore flows from poor countries to rich countries, rather than the reverse.
This literature springs in part from a thesis of Demsetz (1967), that property rights develop when a resource becomes economically valuable. The increase of property value could be due to discoveries of resources, such as the discovery of the Comstock lode (Libecap, 1978), government deregulation in the case of airport landing slots (Riker and Sened, 1991), or technological change, such as improving salmon fishing productivity and precipitating the institutionalization of fishing rights (Higgs, 1982). Our mechanism similarly triggers enclosure when a resource becomes sufficiently valuable: lower discounting increases the firm values with property rights. But more central to our model is that the increased value simultaneously initiates the commitment that is the foundation of property rights.

In this paper we set out a simple demonstration of our concept. In the next section we set out the basic structure of our model. We then describe the efficient growth path as a benchmark. Using shooting methods to numerically simulate growth paths, we then demonstrate our central result, that enclosure occurs after a period of autarky. Aside from establishing that enclosure does occur, we also establish that investment can accelerate in pre-enclosure economies and continues at a high rate once enclosure occurs. In our concluding sections we provide some limited empirical support for our model.

2 The model

Our model is an extension of the standard deterministic neoclassical growth model. We model households and firms separately. Households behave in an entirely conventional way, holding equity in the firms. The households do not make physical investment decisions, and perceive growth only through changes in the value of their equity holdings. The endogenous discounting that drives the model occurs on the firm side. Firms make investment decisions, and the discounting effects of the model appear through the interest rate, which is driven by the marginal product of capital.

Time is continuous. There is a continuum of identical households with unit measure. The instantaneous discount rate of households is $\rho$. Households obtain utility from consuming
the single good with instantaneous felicity function $u(c)$.

There is a continuum of firms, also with unit measure. Production requires capital but no labor. We do not differentiate capital into physical capital and human capital. We view knowledge simply as a variety of capital, the outcome of investment, no different from the outcome of investment in a railroad. We do however make a distinction between types of capital by their ownership. It is possible that capital owned by one firm can benefit the production of another firm. Capital, which in our model encompasses knowledge, typically does have externalities, but we label them as complementarities.\footnote{Another strand of literature links growth with the positive externalities generated by technology. The empirical evidence for the existence of positive externalities in technology is compelling: calculations of the rate of return to investment in research and development show it to be extraordinarily high, far higher than market rates (see for example Griliches, 1958, and Bernstein, 1988). Technology spillovers documented by Bernstein and Yan, 1997 and Coe, Helpman and Hoffmaister, 1997—the ability of poorer countries to incorporate technology from developed economies through imitation—are apparently an engine of growth. The impetus for government policy would seem to be to subsidize R & D and to encourage technology transfer through imitation and learning. The theoretical models that support this perspective, such as those of Grossman and Helpman (1991 and 1994), and Grossman and Lai (2002), rest on the assumption that innovators garner \textit{monopoly} profits, whether temporarily or permanently: this is because in the absence of property rights, innovators lose all potential rents due to imitation by competitors in the absence of monopoly rights attaching to new technology. By contrast, our model views these spillovers as symptoms of incompletely developed property rights.}

There are thus two types of capital that enter the production function: firm-specific and complementary. The firm-specific capital corresponds to the usual expression of capital in a production function. Complementary capital is a function of the average of all other firms’ capital, and enters the production function separately. Because there is a continuum of firms, each individual firm has no direct impact on the complementary capital of other firms.

We denote the capital stock for firm $i$ as $k^i$, and firm $i$’s complementary capital as $k^{-i}$. The production function for firm $i$ is $f^i(k^i, k^{-i})$, net of depreciation. We require
that the production function display decreasing returns to scale in all capital. Each firm’s investment $x^i$ is subject to convex adjustment costs, $\phi(x^i)$, where $\phi(0) = 0$, $x^i\phi'(x^i) > 0$, and $\phi''(x^i) > 0$.

The price of output is normalized to 1. The market value of firm $i$ is $q^i_t$. The instantaneous interest rate is $r_t$. Both firms and households can borrow and lend at this rate. To simplify notation, use $R^i_s = \int_s^t r_\tau d\tau$ to denote the discounting between time $s$ and $t \geq s$.

2.1 The household’s problem

Because all households are identical, we can aggregate them into a single representative household that holds shares in all the firms, and consumes the aggregated output of all firms.

The share of firm $i$’s stock held by the representative household is $\alpha^i_t$. The representative household maximizes discounted utility in continuous time for one good:

$$\max_{c_t, \{\alpha^i_t\}} \int_0^\infty e^{-\rho t} u(c_t) dt$$

subject to

$$c_t + \int_0^1 \alpha^i_t q^i_t di = \int_0^1 \alpha^i_t (\pi^i_t + \dot{q}^i_t) di$$

The budget constraint states that the flow of consumption and portfolio adjustments must equal income from profit and capital gains.

Because the model is deterministic, the household’s portfolio-choice decision problem can be simplified. Let $r^i_t$ denote the instantaneous holding return of stock for firm $i$, that is,

$$r^i_t = \frac{\pi^i_t + \dot{q}^i_t}{q^i_t}$$

The household’s optimal portfolio holding is the following

$$\alpha^i_t = -\infty \text{ if } r^i_t \neq \arg \max_i r^i_t$$

$$\alpha^i_t \in [0, \infty) \text{ if } r^i_t = \arg \max_i r^i_t$$
The household’s problem is well defined if and only if the holding returns are equalized and are equal to the interest rate. That is

\[ r^i_t = r_t \quad \text{for all} \quad i. \]

Therefore from the household’s perspective the portfolio can be aggregated into a single asset \( a_t \) with rate of return \( r_t \). The representative household’s problem can then be restated as follows.

\[
\max_c \int_0^\infty e^{-\rho t} u(c_t) dt
\]

subject to

\[
\dot{a}_t = r_t a_t - c_t
\]

The solution to the household problem is then characterized by the law of motion (2) and the Euler equation (3):

\[
\frac{\dot{c}_t}{c_t} = -\frac{u'(c_t)}{u''(c_t)c_t} (r_t - \rho)
\]

which we express as the growth rate of consumption as the product of the intertemporal elasticity of substitution and the interest rate net of time preference.

### 2.2 Property rights and the firm’s problem

Since the capital of firm \( i \) affects the productivity of other firms and vice versa, firms can implement a multi-lateral agreement to ensure compensation for this productivity—property rights. We model this agreement as specifying a per-unit fee \( \omega^i_t \) that is paid to firm \( i \) by other firms for the use of firm \( i \)’s capital as complementary capital, and a per-unit fee \( \omega^{-i}_t \) that is paid to other firms by firm \( i \) at time \( t \). If \( \omega^i_t = \omega^{-i}_t = 0 \), then no such agreement exists at time \( t \). A property-rights agreement is regular if \( \omega^i_t \) and \( \omega^{-i}_t \) are piecewise continuous in \( t \) and both converge to a constant over time. A property-rights agreement is symmetric if \( \omega^i_t = \omega^j_t \) and \( \omega^{-i}_t = \omega^{-j}_t \) for all \( i, j \) and \( \omega^i_t = \omega^{-i}_t \). In our analysis, we consider only regular and symmetric agreements.
If no property rights agreement exists at time $t$, firms may engage in activities to deter free-riding by other firms. We represent the effect of such activity as a reduction in the available complementary capital: $k^{-i}_t = \theta \int_{j \neq i} k^j_t \, dj$ where $0 < \theta < 1$. Once mutual property rights are established, $k^{-i}_t = \theta_i / \sum_{j \neq i} k^j_t \, dj$.\footnote{Activity to limit the uncompensated use of complementary capital is costly, and we are suppressing the explicit representation of this cost, and the means by which free riding is impeded, in this version of the model.}

Let $K_s$ denote the capital of all firms at time $s$, i.e., $K_s \equiv \{k^i_s\}_i$. Let $\Omega_s \equiv \{\omega^i_t, \omega^{-i}_t\}_{t=s}^{\infty}$ denote a property-rights agreement from time $s$ on. Given the property-rights agreement $\Omega_s$, the objective of the firm $i$ is to choose an investment plan $x^i_t$ to maximize its market value.

$$q^i_s(K_s, \Omega_s) = \max_{x^i_t} \int_s^\infty e^{-rt} [f^i(k^i_t, k^{-i}_t) - x^i_t - \phi(x^i_t) + \omega^i_t k^i_t - \omega^{-i}_t k^{-i}_t] \, dt$$

subject to

$$\dot{k}^i_t = x^i_t$$

$$k^{-i}_t = \theta \int_{j \neq i} \kappa^j_t \, dj$$

$$\theta_t = \begin{cases} 1 & \text{if } \omega^i_t, \omega^{-i}_t > 0 \\ \theta & \text{otherwise} \end{cases}$$

That is, the representative firm maximizes the discounted value of output net of investment and adjustment costs of investment, along with net payments from complementary capital.

Let $f^i_1$ denote the marginal product of firm $i$’s own capital, and $f^i_2$ the marginal contribution of its complementary capital. Then the solution of the firm’s problem satisfies the following differential equation.

$$\phi''(x^i_t) \dot{x}^i_t = (1 + \phi'(x^i_t)) r_t - f^i_1(k^i_t, k^{-i}_t) - \omega^i_t$$ \hspace{1cm} (4)

Observe that if adjustment costs were zero we would have the standard equation of the marginal product of the firm’s own capital and its receipts from other firms with the equi-
librium interest rate. The higher agreed payment induces firms to attain a higher level of capital in the long run, an incentive that is absent without property-rights.

2.3 Symmetric equilibrium

In equilibrium the goods market and the capital market must clear, and prices must be consistent with the optimization of firms and consumers.

The goods market-clearing condition is that consumption, investment, and the adjustment cost of investment must be financed by output:

$$c_t + \int_0^1 [x^i_t + \phi(x^i_t)]di = \int_0^1 f^i(k^i_t, k^{i-1}_t)di$$  \hspace{1cm} (5)

The assets held by the representative household must equal the value of the firms:

$$a_t = \int_0^1 q^i_t di$$

Recalling the Euler conditions of consumers in equation (3) and firms in equation (4), we have the requirement that the interest rate satisfy the condition

$$r_t = \rho + \left(-\frac{w'(c_t)c_t}{u'(c_t)}\right)\frac{\dot{c}_t}{c_t} = \frac{f^i_1(k^i_t, k^{i-1}_t) + \omega^i_t}{1 + \phi'(x^i_t)} + \frac{\phi''(x^i_t)}{1 + \phi'(x^i_t)} \dot{x}^i_t$$ \hspace{1cm} (6)

Assume that all firms have the same production function and initial capital. Under a symmetric property-rights agreement, all firms choose the same investment plan, that is

$$c_t + x^i_t + \phi(x^i_t) = f^i(k^i_t, k^{i-1}_t).$$ \hspace{1cm} (7)

The profit, $\pi^i_t$, and market value, $q^i_t$, of a typical firm are

$$\pi^i_t = f^i(k^i_t, k^{i-1}_t) - x^i_t - \phi(x^i_t) = c_t$$

$$q^i_t = a_t$$

Combining these equations we obtain the equilibrium condition

$$\ddot{q}^i_t = \dot{a}_t = r_t a_t - c_t = r_t q^i_t - \pi^i_t$$
which implies that the interest rate facing firm \( i \), \( r^i_t \), is identical to \( r_t \) for all \( i \), ratifying the simplification of the household’s problem in the previous section.

Totally differentiating the goods market-clearing condition (5) yields

\[
\dot{c}_t + \int_0^1 (1 + \phi'(x^i_t)) \dot{x}^i_t di = \int_0^1 \left[ f^1_t(k^i_t, k^{-i}_t) \dot{k}^i_t + f^2_t(k^i_t, k^{-i}_t) \theta_t \int_{j \neq i} \dot{k}^j_t dj \right] di
\]

(8)

Solving this equation for \( \dot{c} \) and substituting into (6) yields

\[
\left[ \frac{\phi''(x^i_t)}{1 + \phi'(x^i_t)} - \frac{u''(c_t)}{u'(c_t)}(1 + \phi'(x^i_t)) \right] \dot{x}^i_t = \rho - \frac{u''(c_t)}{u'(c_t)} \left[ f^1_t(k^i_t, k^{-i}_t) + \theta_t f^2_t(k^i_t, k^{-i}_t) \right] x^i_t - \frac{f^1_t(k^i_t, k^{-i}_t) + \omega^i_t}{1 + \phi'(x^i_t)}
\]

(9)

The symmetric equilibrium is thus characterized by (9) and by

\[
\dot{k}^i_t = x^i_t
\]

(10)

where \( c_t \) is determined by the market clearing condition (5).

If \( \omega^i_t \) converges to a constant \( \omega \) over time, the steady state of the economy is given by the following equations:

\[
f^1_t(k^i, k^{-i}) + \omega = \rho
\]

(11)

\[
c = f^i(k^i, k^{-i})
\]

(12)

where \( k^{-i} = k^i \) if \( \omega > 0 \); otherwise \( k^{-i} = \theta k^i \).

Imposing our symmetry assumption so that \( \omega^i_t = \omega^-i = \omega \) for all \( t \), we can use phase diagram methods to study the qualitative features of the model. The global phase diagram of this problem is complicated due to the presence of the adjustment cost. One can show that the \( \dot{x}^i = 0 \) locus intersects with the \( \dot{k}^i = 0 \) locus only once at the steady state, but the locus is not necessarily monotonically decreasing.\(^6\) As long as the complementarity between

\(^6\)The slope of \( \dot{x} = 0 \) locus is \( A/B \), where

\[
A = \sum_{j=1}^{2} f^j x^j - (1 + \phi') \left( \frac{u''}{u'} \right)^2 \left( f^2 \right)^2 + (1 + \phi') \left( -\frac{u''}{u'} \right) \sum_{j=1}^{2} \sum_{k=1}^{2} f^j_k x^j
\]

\[
B = \frac{\phi''}{1 + \phi'} f^1 x^1 + (1 + \phi') \left( -\frac{u''}{u'} \right) \sum_{j=1}^{2} f^j x^j - (1 + \phi') \left( -\frac{u''}{u'} \right) \left( 2 \left( u''^2 \right)^2 - \frac{u'' u'}{u'^2} \right) \left( f^j x^j \right)^2
\]
the firm’s own capital and the complementary capital is not too high however, the slope of $\dot{x}^i = 0$ locus is locally negative around the intersection of the two loci, as shown in Figure 1. Therefore the growth path has standard behavior in the neighborhood of the steady state; in our computed examples this holds globally as well.

3 The social optimum and autarky

We next analyze the social planner’s problem and autarky as a benchmark. A social planner would force firms to internalize the externalities associated with complementary capital and attain the efficient neoclassical growth path.

A social planner solves the following problem: maximize the discounted utility of the representative consumer through the choice of consumption and investment:

$$\max_{c_t, \{x_t^i\}} \int_0^\infty e^{-\rho t} u(c_t)dt$$

subject to

$$c_t + \int_0^1 [x_t^i + \phi(x_t^i)]di = \int_0^1 f^i(k_t^i, k_t^{i-1})di$$
Figure 2: Comparison of Autarky and Social Optimum

The symmetric solution satisfies the differential equation

\[
\dot{k}_t^i = \int_{j \neq i} k_t^j d_j
\]

\[
\dot{k}_t^i = x_t^i
\]

as well as the market clearing condition (7) and the law of motion (10).

The socially optimal capital at the steady state, \( k_{op} \), satisfies

\[
f_1(k_{op}, k_{op}) + f_2(k_{op}, k_{op}) = \rho.
\]

Under autarky, in which firms free ride on each other so that \( \omega_t^i = \omega_t^{-i} = 0 \), the steady state capital under autarky, \( k_{au} \), satisfies \( f_1(k_{au}, \theta k_{au}) = \rho \). The socially optimal capital level at the steady state is therefore higher than the competitive equilibrium under autarky, as shown by figure 2).

The social optimum can be achieved with a property-rights agreement that pays the marginal contribution of complementary capital every period, that is, \( \omega_t^i = \omega_t^{-i} = f_2(k_t^i, k_t^j) \).

We refer to such an agreement as a complete property-rights agreement. Agreements that
lie between autarky and the social optimum are partial property-rights agreements. We denote the completeness of different property-rights agreements by $\nu$. More specifically, the property-rights agreement we study takes the following form

$$\omega_i^t = \nu f^i_2(k^i_t, k^{-i}_t),$$

i.e., the agreed payment is $100 \times \nu$ percent of the marginal product of complementary capital.

4 The transition to property rights

Not all property-rights agreements can be enforced. Firms have incentives to compromise the property rights of other firms by reneging on their payments for complementary capital to other firms. This reneging constitutes defection in the game sense.

We assume that the detection of defection takes time. Until a defecting firm is detected, it receives payments from other firms, but does not pay other firms for the complementary capital that they provide. After a lag of $\tau$, the defection is detected and the defecting firm no longer receives payments from other firms, and at that time the firm is permanently excluded from the agreement. Thenceforth the value of its complementary capital is reduced as well, because other firms take measures to prevent its use—only the fraction $\theta$ of complementary capital can then be used.

The gain from defection is twofold: the first is the immediate but short-run retention of payments that were made to other firms under the contract, and the second is the possible increase of profit obtained by either by low investment or sale of the excess capital previously accumulated. This latter gain accrues because the firm’s autarkic steady-state capital is lower than the property-rights contract steady state capital. The cost of defection is the loss of full utilization of complementary capital as $\theta$ shrinks when defection is detected, and lower long-run profits entailed by moving toward a lower steady state capital.

As with our abstraction of the effort to limit the uncompensated use of complementary capital, we are abstracting away from explicitly modeling the effort needed to monitor and respond to defection.
Because the set of firms is a continuum, the effect of one firm’s defection is negligible. The consequence of this assumption is that the interest rate used by the defecting firm to calculate its discounted payoffs from defection is still the equilibrium rate under full cooperation, and the defecting firm therefore does not affect the investment decision of non-defecting firms.

With the game structured in this way we can analyze the off-equilibrium-path payoff of a defecting firm. For each level of capital we consider an equilibrium path in which a property-rights agreement is in force and examine the payoff of a firm which defects. If a representative firm prefers such a deviation then because of symmetry so will all firms, and the proposed equilibrium path is not sustainable at that level of capital. However, if there is a threshold level of capital such that no firm wishes to defect once that level is attained, the equilibrium path is sustainable.

4.1 The problem of a defecting firm

Given a regular and symmetric property-rights agreement, where \( \omega_t > 0 \) for all \( t \), the time \( s \) defecting firm solves the following problem.

\[
\hat{q}_t^i(K_s, \Omega_s) = \max_{\hat{x}_t^i} \int_s^{s+\tau} e^{-R_t^i} \left[ f^i(\hat{k}_t^i, k_t^{-i}) - \hat{x}_t^i - \phi(\hat{x}_t^i) + \omega_t^i \hat{k}_t^i \right] dt \\
+ \int_{s+\tau}^{\infty} e^{-R_t^i} \left[ f^i(\hat{k}_t^i, k_t^{-i}) - \hat{x}_t^i - \phi(\hat{x}_t^i) \right] dt
\]

subject to

\[
\hat{\dot{k}}_t^i = \hat{x}_t^i \\
k_t^{-i} = \begin{cases} 
\int_{j \neq i} k_t^j dj & t \leq s + \tau \\
\theta \int_{j \neq i} k_t^j dj & t > s + \tau 
\end{cases}
\]

where the notation \( \hat{x} \) and \( \hat{k} \) denotes the defection path, distinct from the non-defection path followed by other firms.
The optimal investment plan of the defecting firm is

\[
\dot{x}_i^t = \begin{cases} 
\frac{1+\phi'(\hat{x}_i^t)}{\phi''(\hat{x}_i^t)} r_t - \frac{f_i'(\hat{k}_i^t, k_{t-1}^i) + \omega_i^t}{\phi''(\hat{x}_i^t)} & t \leq s + \tau \\
\frac{1+\phi'(\hat{x}_i^t)}{\phi''(\hat{x}_i^t)} r_t - \frac{f_i'(\hat{k}_i^t, k_{t-1}^i)}{\phi''(\hat{x}_i^t)} & t > s + \tau 
\end{cases}
\]

When we calculate the value of defection, the investment path during the initial stage of defection \((t < \tau)\) is pasted to the post-detection path \((t \geq \tau)\).

4.2 The transition path

A property-rights agreement \(\Omega_s\) is enforceable at time \(s\) with capital \(K_s\) if \(q_i^s(K_s, \Omega_s) \geq \hat{q}_i^s(K_s, \Omega_s)\). A property-rights agreement is enforceable starting from time \(s\) if it is enforceable for all \(t \geq s\).

The enforceability of the contract depends on the harshness of the punishment, which is parameterized by time lag of detection, \(\tau\), and ability to block free-riding, \(\theta\). The smaller are \(\tau\) and \(\theta\), the less incentive there is for a firm to defect.

The enforceability of the contract also depends on the average current capital stock: the lower the level of capital, the higher the equilibrium interest rate. Future losses accruing from the loss of complementarity during the punishment phase are therefore discounted at a higher rate when the capital stock is low, reducing the severity of the punishment. The contractual arrangement is therefore not supportable at low capital levels, but becomes supportable as capital reaches a threshold level.

The resulting equilibrium path is illustrated in Figure 3. The figure shows two growth paths. The path terminating at \(A\) corresponds to a pure autarky equilibrium in which enclosure never occurs. The second path, terminating at \(B\), illustrates the enclosure process. The path has two legs. The second leg, terminating at \(B\), corresponds to the equilibrium path under a property-rights agreement. At low levels of capital this path is not enforceable; the economy starts on an autarky leg, with a property rights agreement commencing only when capital attains \(k^*\), the enclosure threshold. Observe that even though there is a period of autarky, the rate of investment on this pre-enclosure leg is higher than it would be under
a permanent autarky regime because enclosure is anticipated.

5 Numerical Experiments

In this section we use numerical examples to demonstrate the enclosure process. We will show that the degree of completeness of property rights arrangements that is enforceable increases with the level of capital. In order to simplify our numerical analysis we restrict $\nu$ to be a constant; we will comment on the wider consequences of this assumption at the end of this section.

We consider a specification of the model with power utility, quadratic adjustment cost, and Cobb-Douglas production function.

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

$$\phi(x) = \delta x^2$$

$$f^i(k^i, k^{-i}) = (k^i)^\alpha (k^{-i})^\gamma$$

Notice that there is no explicit total factor productivity (TFP) term in the production function; the existence of complementary capital in the production function implies a TFP
term of $A \equiv (k^{-i})^\gamma$.

The parameter values for our experiments are listed in Table 1. The risk aversion coefficient $\alpha$ is 0.5. The subjective discount factor $\rho = .04$ implies a 4% interest rate at the steady state. The share of own capital $\alpha$ is 0.3, which is roughly the capital share for the US economy. In the baseline example, the share of complementary capital $\gamma$ is low. Later on we will discuss robustness of our results with respect to this parameter. The parameters $\tau$ and $\theta$ are chosen so that the transition path is non-trivial, that is, so that enclosure actually occurs.

### 5.1 Algorithm

Both the competitive equilibrium under a property-rights agreement and the optimal control problem of a defecting firm display the saddle path property. The transversality conditions force the equilibrium path and the defection path to follow saddle trajectories and converge to their corresponding steady states.

The shooting algorithm we use finds these saddle paths. Given initial capital $k_0$, the shooting algorithm begins with a conjecture of an initial value for $x_0$, then follows the path
generated by the differential equation system for $(\dot{k}, \dot{x})$, (8-9). If capital exceeds the steady state level (which is known analytically) along this conjectured path for large values of $t$, we conclude that the conjecture was incorrect, the initial guess of $x_0$ is adjusted, and the process repeated. The initial guess of $x_0$ is iteratively adjusted in this fashion to get the terminal capital as close to the steady state as possible.

Once the saddle path is reasonably approximated, we can compute the interest rate $r_t$ and profit $\pi^i_t$ of a firm that abides by the property-rights agreement, as well as the profit $\tilde{\pi}^i_t$ of a defecting firm. When computing the saddle path of a defecting firm, the interest rate and capital of all the other firms remain the equilibrium values under cooperation, because each firm is negligible due to the continuum assumption.

5.2 An upper bound on $\nu$

The highest possible degree of completeness of property rights $\nu$ that is enforceable must be the one that is sustainable at the steady state. The steady-state capital $k^{ss}$ is such that with property rights agreements with completeness $\nu$,

$$f^i_1(k^{ss}, k^{ss}) + \nu f^i_2(k^{ss}, k^{ss}) = \rho$$

A cooperative firm has value $f^i(k^{ss}, k^{ss})/\rho$ at the steady state. The value of a defecting firm is computed numerically using method described in the previous section. Figure 4 plots the value difference between a cooperative firm and a defecting firm for different values of $\nu$ at the steady state. The figure illustrates that the marginal value of cooperation decreases as the degree of completeness increases. In particular, it is possible to sustain only a partial property rights agreement with a value of $\nu$ of only 60%, even at the steady state.

A property-rights agreement with degree of completeness $\nu$ is sustainable at the steady state only if the value of a cooperative firm is higher than the value of a defecting firm. The value of defecting increases as $\nu$ increases, due to the short-run gain from avoiding the payment $\omega^{-i}$. For larger $\nu$, property rights agreements are therefore no longer sustainable, even at the steady state. In our baseline experiments, the maximum sustainable $\nu^{max}$
is about 60%. If we interpret the contractual agreement as patent protection, when the interest rate is 4%, $\nu = 60\%$ roughly corresponds to a patent with lifespan of 23 years.\(^8\)

5.3 Sustainability of partial property rights along the saddle path

For a fixed degree of completeness $\nu < \nu^{\text{max}}$, we can numerically determine when enclosure will occur by comparing the value of a cooperative firm against the value of a defecting firm. Figure 5 plots the value differences for various levels of capital with $\nu = 0.55$. Because of the patience effect, the incentive for cooperation is monotonically increasing in capital.

In the early stages of economic development, capital is low, with the interest rate consequently high. Since the gain from cooperation is in the future while the gain from defection is immediate, the initial high interest rate leads firms to prefer defection. Only when capital reaches a threshold point $k^*(\nu)$, and the associated interest rate is low enough, will defection be dominated by cooperation. From that point on, the partial property rights agreement becomes enforceable.

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\(^8\)That is, the present value of 23 year of a flow at an interest rate of 4% is about 60% of the value of an infinite term for that same flow.
5.4 The transition path and enclosure

If an economy with low initial capital anticipates enclosure, firms will not stay on the pure autarky-equilibrium saddle path even in the early pre-enclosure stage of development. As we pointed out in Figure 3, the equilibrium path still satisfies the system (8-9) under autarky, but smoothly pastes to the saddle path of an equilibrium under a partial property-rights agreement at the point where capital reaches the threshold value $k^*(\nu)$.

Figure 6 plots this transition path for three different value of $\nu$. One can see that the critical value $k^*(\nu)$ increases when the degree of completeness increases. In other words, as an economy accumulates more capital, it also could sustain a more complete property rights agreement. The more complete property rights agreement leads to higher steady state capital. This gradual enclosure of the marginal contribution of complementary capital counters the force of the diminishing marginal return on capital, rendering the growth rate and investment rate of a more developed economy high.

In order to illustrate the effect of enclosure on growth and investment, we compare growth rates and investment rates in a economy that has a partial property-rights agreement to one without such an agreement (Figure 7). The dashed lines correspond to autarkic paths;

---

9This increase is not very significant in the baseline model but will be when we increase the value of $\gamma$. 

the solid lines correspond to paths in which autarky is followed by enclosure and the property rights path. It is worth emphasizing that if autarky prevailed permanently, a standard growth path would emerge: investment would steadily decrease as the autarkic steady-state were approached. In the economy with a property rights agreement, the pasting of the pre-property-rights path to the property-rights path results in an increase of the growth rate and investment. There is a pre-enclosure investment “frenzy”; investment takes off in Rostow’s sense before enclosure and continues at a high rate thereafter.¹⁰

A more complete model would entail continuous and gradual enclosure, with ever-increasing degrees of internalization of complementarities. This more complete model would have the degree of internalization determined by firms having a zero marginal incentive to defect at all times. Thus, \( \nu \) will in general also be a function of the state, increasing as the

¹⁰Our model is thus reminiscent of the “take-off” literature of the 1950’s. Rostow’s 1956 article is an example. Rostow’s thesis is that a specialized industry, such as the silk industry in Japan, gets organized in a modern way: they start using industrial machinery for spinning and weaving, they develop organized markets, especially abroad, and they develop relationships with suppliers of capital that enable them to fund large projects such as investment in machinery and also to ride out the vagaries of demand shocks. This results in that one key industry serving as a model which other industries follow: take-off then ensues. Rostow argues that this event occurs over a period of about 20-30 years, after which the country is transformed into a modern economy and state. The hallmark of the event is that economic growth rates become radically high during this period, followed by a relative slowdown.
steady state is approached. Our simulations, which show a sharp transition between the absence of a property rights agreement and its institution, approximate a model in which the completeness of property rights, characterized by $\nu$, increased steadily over time. In a model with a continuously increasing $\nu$, the sharp transition would disappear: the interest rate in particular would follow a smoother path, and would decline in the long run at a much slower rate than it would under pure autarky or under a pure neoclassical regime; this slower decline would far better match the empirical long run path of interest rates.
5.5 Sensitivity analysis of $\gamma$

Prescott (1998) observes that the neoclassical model requires total factor productivity to differ across countries:

(A)dding intangible capital does not make the neoclassical growth model a theory of international income differences. Only if investment in intangible capital is as large as GDP do savings rate differences have important consequences for output per worker. A model with a human capital producing sector fails for similar reasons. The fraction of time allocated to enhancing human capital must be larger than time allocated to market for differences in human capital investment rates to be important. The neoclassical growth model accounts for differences across countries only if total factor productivity differs across countries.

Our model provides an explanation of cross-country differences in total factor productivity. The magnitude of TFP in our model is partially explained by the quantity of capital: TFP is comprised of the contribution of complementary capital, $k^\gamma$.

We examine this effect across countries. Let $A_{it}$ and $k_{it}$ be the measured Solow residual and capital per worker for country $i$ at time $t$ respectively. We fit a power curve in the form of $A_{it} = A_t k_{it}^{\gamma_t}$, allowing a common component $A_t$, not contained in our model, as well, with $\gamma_t$ as the parameter of interest.

We use data from the Penn World Table (Mark5.6) for the years 1965 to 1990 for 64 countries that have data on the physical capital stock. If we assume that the parameters are fixed over time and pool all the data together, the estimated value of $\gamma$ is 0.30. Allowing for evolution of the parameters over time and estimating $\gamma$ year by year, the value of $\gamma$ falls between 0.24 to 0.34. Thus, our model provides a framework that can explain cross-country TFP patterns in a consistent way.

The value of 0.1 for $\gamma$ in our baseline numerical model is smaller than the estimated value. However, all the results we presented in the previous subsections continue to hold
qualitatively for larger values of $\gamma$.\footnote{The computed example become numerically unstable for $\gamma$ values that are higher than $\alpha$, the share of own capital in production.} One regularity is that as $\gamma$ increases the upper bound of the degree of completeness of property rights, $\nu$, first decreases slightly and then increases. Holding all other parameters fixed, the increase of the power of the externality through the increased share of complementary capital increases the unit payment under the property-rights agreement, which increases the value of defection and makes it harder to enforce property rights agreements with higher completeness $\nu$. On the other hand, the increased share of complementary capital dramatically increases the level of the steady state capital under the more complete property rights agreement, which works to sustain cooperation. Initially the first effect is strong but eventually the second effect dominates. (Table 2)

6 Empirical Discussion

Our model predicts that rich countries are able to sustain more complete property-rights agreements, and more complete property rights increase subsequent growth rates. Empirically, it is extremely difficult to compare the strength of property rights on a consistent basis across countries. There are few attempts to develop qualitative rankings based on legal documents, especially those about intellectual property rights. Rapp and Rozek, 1990 and Ginarte and Park, 1997 are among them. Both studies focus exclusively on patents. Ginarte and Park extended the methodology of Rapp and Rozek significantly and developed a panel of indexes by rating the patent laws of most countries every five years from 1960 to

\begin{table}
\centering
\caption{Sensitivity analysis of $\gamma$}
\begin{tabular}{|c|c|c|c|}
\hline
$\gamma$ & 0.1 & 0.2 & 0.3 & 0.4 \\
\hline
$\nu$ & 0.59 & 0.54 & 0.60 & 1.00 \\
\hline
\end{tabular}
\end{table}
Table 3: Regression analysis by Ginarte and Park (Table 6)

<table>
<thead>
<tr>
<th></th>
<th>OLS(1)(^a)</th>
<th>GLS(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-1.060 (0.203)</td>
<td>-0.120 (0.310)</td>
</tr>
<tr>
<td>GDP per capita</td>
<td>0.232 (0.025)</td>
<td>0.060 (0.038)</td>
</tr>
<tr>
<td>R&amp;D/GDP</td>
<td></td>
<td>0.035 (0.019)</td>
</tr>
<tr>
<td>Secondary enrollment rate</td>
<td>-0.029 (0.022)</td>
<td></td>
</tr>
<tr>
<td>Political freedom</td>
<td>-0.012 (0.033)</td>
<td></td>
</tr>
<tr>
<td>Openness</td>
<td></td>
<td>0.114 (0.030)</td>
</tr>
<tr>
<td>Market freedom</td>
<td></td>
<td>0.550 (0.130)</td>
</tr>
<tr>
<td>Adjusted R-square</td>
<td>0.31</td>
<td>0.31</td>
</tr>
</tbody>
</table>

\(^a\)Panel regression. The dependent variable is the Ginarte-Park patent index. Standard errors are in parentheses. All variables are in logs except for the openness measure. GDP is the 1960-1990 average.

They also showed that a significant positive correlation between patent rights and GDP per capita exists (Table 3 first column). However it is not the case that every country strengthens their patent protection as its wealth increases. Patent protection in most Latin American countries actually worsened between 1960-1990. The more elaborated regression analysis by Ginarte and Park (Table 3 column 2) shows that the effect of the level of development significantly weakens once other variables are added. The more significant determinants of the strength of patent protection are the size of the R & D investment, openness and market freedom.


\(^1^2\)The Ginarte-Park index is based on five components: duration of protection, extent of coverage, membership in international patent agreements, provisions for loss of protection, and enforcement measure. Some components are further broken down into characteristics that they think closely describe their effective strength. All scores are aggregated with equal weights. The final index has a minimum possible score of 0.0 and a maximum of 5.0.
Their paper compares the efficacy of legal systems in providing clear and practical property rights to parties in ordinary commercial activities: eviction of tenants for non-payment and collection of nsf checks. They carefully surveyed law firms in more than 100 countries in order to measure costs and durations in carrying out these court transactions. Although their focus is on the effects of English and French legal systems on the outcomes, their regressions also establish that per-capita GDP is negatively related to property rights in the sense of their definition.

7 Conclusion

We view our model as meeting a challenge posed by North (1996):

There is no mystery why the field of development has failed to develop during the five decades since the end of the Second World War. Neoclassical theory is simply an inappropriate tool to analyze and prescribe policies that will induce development. It is concerned with the operation of markets, not with how markets develop. How can one prescribe policies when one doesn’t understand how economies develop? The very methods employed by neoclassical economists have dictated the subject matter and militated against such a development. That theory, in the pristine form that gave it mathematical precision and elegance, modeled a frictionless and static world. When applied to economic history and development, it focused on technological development and more recently human capital investment but ignored the incentive structure embodied in institutions that determined the extent of societal investment in those factors. In the analysis of economic performance through time it contained two erroneous assumptions: first, that institutions do not matter and, second, that time does not matter.
Our model is indeed a neoclassical model that does in fact endogenize the formation of institutions over time.

An extended model would entail continuous and gradual enclosure, with ever-increasing degrees of internalization of complementarities, and would also encompass heterogeneous initial capital across firms or countries. This more complete model would have the degree of internalization determined by firms having a zero marginal incentive to defect at all times, and would result in a smoother, more slowly decreasing path of interest rates. Our model is symmetric, but actual capital is distributed highly asymmetrically, and the institution of property rights is highly asynchronous. Our model also is deterministic. An extended model with random technological changes would allow the marginal product of capital to increase at random times; this could have the effect of sundering rather than strengthening property rights, with the consequence that growth might be slowed, or at least that it would fail to accelerate, to the degree predicted by elementary theory.

We end with an observation about policy. Our model is a completely positive one, and the only comment it can make about policy is to suggest a sense of patience. England could easily loan enough capital to Ukraine to double Ukraine’s output, and if that could be done both England and Ukraine would benefit hugely in discounted welfare terms. Repeating this experiment around the globe would usher in an era of unprecedented growth and prosperity. Such an event must be preceded by the willingness of the Ukrainians and of the British to undertake it, and to be committed to the subsequent payments by Ukrainians to the British—to be committed to property rights. Our model shows that this commitment cannot simply be assumed or externally imposed, but that it develops naturally over time. We are therefore optimistic about the long run, but to reach it we must wait.
References


