Mismatch in Credit Markets

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Abstract: This paper studies a decentralized credit market where borrowers and lenders engage in costly search to establish credit relationships. We endogenize the market participation decision by borrowers to capture the entry and exit of small and medium-sized enterprises who depend on access to credit for survival. We also allow incentive frictions in the form of moral hazard to interact with search frictions in setting up incentive compatible loan contracts. We show that free entry can in the limit eliminate information-based credit rationing in equilibrium, with search frictions determining whether or not credit markets break down. More generally, we find that entry and incentive frictions are important in determining the extent of credit rationing, while entry and search frictions are important for determining the likelihood of credit market breakdown.

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Time runs a close second to cash on every entrepreneur’s list of scarce resources. (W. E. Wetzel, Jr., The Portable MBA in Entrepreneurship, p. 185)

Credit markets are capricious and susceptible to occasional crises and even breakdowns. Various explanations for the turbulence in credit markets have been proposed largely based on theories of informational imperfections. But these theories do not fully explain the phenomenon of mismatches. Mismatches occur when idle funds coexist with profitable but unexploited investment opportunities and are implicit in many accounts of credit market crises. The phenomenon occurs partly because markets for information-intensive loans made to entrepreneurs tend to be localized and satisfy many of the characteristics of a search market as originally proposed by Stigler (1961). Understanding what determines the extent of mismatch helps explain unfulfilled demand for credit and the tightness of credit markets. And because access to credit is an important factor for the survival of small and medium-sized enterprises, mismatch ultimately helps determine credit market participation and the likelihood of breakdown.

Information and matching frictions coexist even when credit markets are functioning normally and financial intermediaries exist to overcome these frictions. Small and medium-sized enterprises that exist in the financial twilight zone between bankruptcy and survival

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1In credit markets, imperfections arising from moral hazard and adverse selection problems have been identified as an important factor leading to the rationing of credit and possible bank failures. There has been an extensive literature following Diamond and Dybvig (1983) on bank runs as well as a broader literature on financial fragility (e.g., see Allen and Gale (2004) and papers cited therein). Still, Gorton and Winton (2003) conclude their survey of the recent theoretical literature on financial intermediation and the empirical literature on credit crises with the following assessment: “Overall, it seems that the question of whether banking is inherently unstable or not remains unresolved.”

2Many theoretical studies of credit market crises focus on the fact of rising currency to deposit ratios that was first put forth by Friedman and Schwartz (1960). In this case, idle funds in the hands of individuals are not channeled to profitable investment opportunities. Instead we focus on the fact that reserves rise, loan quality deteriorates, and banks behave conservatively during credit crises (e.g., see Bernanke (1983)). In this case, idle funds in the hands of banks are not channeled to profitable investment opportunities. In short, we focus on the asset side of financial intermediary balance sheets and thus our work can be considered complementary to approaches that emphasize the liability side. We note that with information frictions as with search frictions, there is the possibility that not all investment opportunities are fully funded and that resources remain idle. The difference between the two frictions is that one emphasizes the limits on knowledge and the other emphasizes the limits on time.

3Also, see Spulber (1996) for a discussion on the search and matching dimensions of financial markets. As Blanchflower and Oswald (1998) observe, where to find funding is a paramount problem for existing and would-be entrepreneurs.
are racing against time on the lookout for funds. Sometimes they qualify for funding and sometimes they are rejected, but the search goes on until they find funds or else run out. On the other side of the market, lenders spend costly time and resources looking to turn their idle funds into active investments. Part of the cost comes from search to identify potentially viable credit relationships. Once viable credit partners have been identified, resources are used to negotiate contracts that are mutually advantageous and incentive compatible. While the difference between normal and abnormal credit market behavior might be considered fluid, at some point breakdown occurs and the market ceases to exist at a local level or even at a national level. Because such breakdowns have real effects with potentially large social costs, it is important to understand this phenomenon and to what extent informational and matching frictions reinforce one another.

In this paper we combine information-based incentive frictions with matching-based search and entry frictions in a simple general equilibrium model of decentralized trade. Our analysis determines the optimal loan contracts and characterizes equilibria that emerge in such an environment. This enables us to address two important questions – when credit rationing arises and when credit markets break down.

We develop a framework with several features that are of interest. Borrowers and lenders choose whether or not to participate in a decentralized loan market where search for bilateral credit relationships is costly. Some participating borrowers and lenders form credit relationships that enable borrowers to finance investment projects which yield a productive rate of return. These returns are divided up between the lender in the form of an interest payment and the borrower in the form of residual profits. Because of incentive frictions that arise from asymmetric information, borrowers may, at a default cost, abscond with the borrowed funds. When loan contracts are negotiated they must be incentive compatible to overcome the moral hazard problem. Thus, credit rationing, in the sense of borrowers

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4Search in credit markets can be loosely thought of as encompassing the time element of ex ante screening in the spirit of Boyd and Prescott (1986) though we do not consider the adverse selection problem explicitly.

5While the number of credit crises worldwide has escalated in the last decades, their causes are imperfectly understood according to Gorton and Winton (2003). Suggestions as to what triggered these episodes are plentiful, but in the main models of crises can be divided into those that emphasize external shocks and those that emphasize sunspots (see Allen and Gale (2004) for a model that combines both approaches).

receiving fewer funds than desired, may emerge endogenously. In equilibrium, optimal loan contracts and the extent of credit rationing are determined jointly with market liquidity (or aggregate lending), borrower market participation (or endogenous entry), and the tightness of the credit market (or the excess demand for loans as measured by the ratio of unmatched borrowers to unmatched lenders).

We find that the nature of the optimal incentive-compatible contract in equilibrium varies with macroeconomic fundamentals as represented by borrower productivity. If productivity falls below a threshold that is determined by default costs, credit markets break down and cease to exist. If productivity exceeds this threshold, firms will be rationed unless lenders are sufficiently patient, in which case there is no rationing. Even very productive firms may be rationed if the rate of time preference is sufficiently low or the length of the loan contract period is sufficiently short. Intuitively, a low rate of time preference raises the present discounted value of absconding with borrowed funds and a short contract duration lowers the value of the match for a borrower who must search again in the credit market. Also, we find that credit market tightness and the extent of credit rationing are positively related when entry is exogenous. But when entry is endogenous, there generally does not exist a monotonic relationship between the two. Intuitively, credit may continue to be rationed in a market where it is relatively easy for borrowers to locate lenders because high market interest rates drive productive firms out of the loanable funds market. Finally, we show that entry and incentive frictions are important in determining the extent of credit rationing, while entry and search frictions are important for determining the likelihood of credit market breakdown. We show that in the limit free entry can eliminate information-based credit rationing in equilibrium, with search frictions determining whether or not credit markets break down.

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7 Credit rationing here follows the notion of Jaffee and Russell (1976) rather than the notion of Stiglitz and Weiss (1981), where observationally identical agents may or may not receive funds.

8 In this sense, our work is related to Bernanke and Gertler (1989) who emphasize that macroeconomic shocks to borrowers balance sheets can have real affects through the banking sector.

9 Free entry of borrowers causes self-selection that reduces the extent of credit rationing. This result suggests its opposite: competition by banks weakens banks’ bargaining position and increases the extent of credit rationing by reducing individual banks ability to dictate loan contract terms. The literature on bank market structure has focused instead on other issues related to the competitiveness of the banking sector. For instance, Gehrig (1995) shows that endogenous entry by banks in a spatial model reduces individual bank size and increases the fragility of the banking sector. Similarly, Broecker (1990) shows that average loan quality decreases and riskiness increases when the number of banks increases.
Related Literature

Our perspective differs in some significant ways from previous work on bilateral search in credit markets. Diamond (1992) is the original paper in the area and most closely related to our work.\textsuperscript{10} We diverge along several dimensions and consider endogenous matching probabilities and endogenous market participation. However, the most important innovation is that we integrate information frictions into a search framework and thus allow for the possibility of endogenous rationing. Den Haan, Ramey, and Watson (2003) and Wasmer and Weil (2004) also analyze search and credit but with an entirely different purpose. While Den Haan, Ramey, and Watson are mainly interested in how liquidity shocks are propagated and amplified in credit markets when there is a hold-up problem for lenders, Wasmer and Weil are interested in how credit market imperfections interact with labor market frictions. Becsi, Li, and Wang (2005) show how the entry of heterogeneous borrowers can affect the aggregate loan composition under perfect information. Here, we focus instead on how entry can affect credit rationing and loan contracts in equilibrium and the potential for market breakdown.

1 The Basic Environment

Time is continuous. There are two types of economic agents, those endowed with resources ("lenders") and those endowed with an “investment” technology which uses those resources to generate a positive return ("borrowers"). Our model focuses on the loanable funds market where available funds provided by a continuum of lenders are channeled to a continuum of potential borrowers through a decentralized credit market. Let \( N^L \) and \( N^B \) represent the mass of lenders and borrowers in this environment. For convenience, we normalize the measure of lenders to unity. Lenders and borrowers (bilaterally) meet with each other for the purpose of establishing a credit relationship and the matching technology that brings borrowers and lenders together is given by:

\[
m = m_0 M(N^L_u, N^B_u)
\] (1)

\textsuperscript{10}Using a search theoretic framework to model credit markets builds on Stiglitz and Weiss (1981) notion that labor and credit markets exhibit many similarities. Dell’Arriccia and Garibaldi (2004) provide empirical evidence that this notion is justified from a search perspective. For search and matching frameworks in the context of labor economics, the reader is referred to classic work by Mortensen (1982) and Pissarides (1984).
where $m$ measures flow matches, $m_0 > 0$ indicates the efficacy of credit market matching, and $M$ is strictly increasing and concave, satisfying the constant-returns-to-scale property, the standard Inada conditions, and the boundary conditions $M(0, \cdot) = M(\cdot, 0) = 0$. We define $N_u^i$ to be the number of unmatched agents and $N_m^i$ as the number of matched agents of type $i$ where $i = L, B$. By normalizing the mass of lenders to unity, we have: $N_u^L + N_m^L = 1$. A central feature of the loan market highlighted by our model is that market liquidity is determined by credit market tightness. Given the populations of lenders and borrowers, a measure of credit market tightness in our set-up is given by the ratio of unmatched lenders to unmatched borrowers: $\tau \equiv \frac{N_u^B}{N_u^L}$. Intuitively, if $\tau$ is high, then there are many potential borrowers relative to lenders with idle funds. Because it is more difficult for borrowers to locate potential lenders under these circumstances, we say that the credit market is “tight” from the borrowers’ viewpoints.

Utility generated from consumption is assumed to be linear for both types of agents. Since the focus of this paper is on how credit market frictions and market liquidity affect credit arrangements between borrowers and lenders rather than the intertemporal consumption and saving decisions of households, this simplifying assumption is adopted without loss of generality for our purpose. An unmatched lender consumes his flow endowment $\omega$ as he searches for borrowers with whom to trade this endowment for the promise of a future payment. A borrower begins the search period with only his investment technology and searches for potential lenders to finance their project. Lenders contact borrowers at a rate of $\mu$, while borrowers contact lenders at a rate of $\eta$. Due to asymmetric information about the borrower’s behavior, the lender is unsure about whether the borrower will invest in a productive project or take the money and run. Thus, once a borrower and a lender meet, the lender will set an incentive compatible loan contract to prevent the borrower from absconding with the funds. The contract specifies a gross interest payment, $R$, and the fraction of available funds actually lent out, $q \leq 1$. When this loan contract is established, the lender gives up a portion of the endowment, $q\omega$, to the borrower and consumes the residual portion $(1 - q)\omega$ while waiting for the end of the contract period. The contract period ends when borrowers and lenders are separated at which time the lender pays the borrower an amount $Rq\omega$ (i.e., loan repayment). The exogenous separation rate is given by $\delta$, and hence the length of the contract period is given by $1/\delta$ and the corresponding gross interest rate is
given by $\delta R$. After both members of the match become separated, they re-enter the pool of unmatched borrowers and lenders and again search for credit opportunities.

If borrowers in this model know with certainty that the loan will be repaid, our preferences imply that it will be optimal for the lender to set $q = 1$ and lend all of the endowment to the borrower in exchange for the future payment. However, due to hidden action, borrowers in our model may choose to default on the loan and abscond without repayment. When this occurs, the defaulter bears two costs. First, we assume that the defaulter is excluded from any future credit transactions. Second, we assume that the borrower must forfeit a real resource cost that is measured as a fraction $\theta$ of total loanable funds. This cost is meant to capture the outside penalty of default and may represent legal or institutional features. The better the legal or the monitoring system is, the higher is the default cost. This moral hazard feature is what may cause loanable funds to be rationed (i.e., $q < 1$). That is, lenders will use this quantity rationing feature of the loan contract so as to insure incentive compatibility and repayment.

We can now characterize the dynamic problem facing borrowers and lenders in our economy. Let $J_u$ and $J_m$ denote the lender's value associated with being in the unmatched ($u$) and matched ($m$) states. These asset values can be expressed as:

$$rJ_u = \omega + \mu (J_m - J_u)$$  \hspace{1cm} (2)

$$rJ_m = (1 - q)\omega + \delta[Rq\omega + (J_u - J_m)]$$  \hspace{1cm} (3)

where $r > 0$ is the rate of time preference. Equation (2) says that the flow value associated with an unmatched lender is the flow of consumption from his endowment and arrival rate of borrowers times the net value gained when a loan contract is implemented and the match is formed. Equation (3) says that the flow value associated with a matched lender is the flow of consumption of the residual endowment and the rate at which the contract expires times the interest payment and net value of returning to the unmatched pool.

Similarly, let $\Pi_u$ and $\Pi_m$ denote the borrowers’s value associated with being in the unmatched and matched states, respectively. Their asset values in the two states are:

$$r\Pi_u = \eta (\Pi_m - \Pi_u)$$  \hspace{1cm} (4)

$$r\Pi_m = Aq\omega + \delta[-Rq\omega + (\Pi_u - \Pi_m)]$$  \hspace{1cm} (5)
Equation (4) simply states that the flow value associated with an unmatched borrower is the rate at which they contact lenders times the net value gained when becoming matched with a lender. Equation (5) says that the flow value associated with a matched borrower is the stream of returns the borrower obtains from implementing the investment project and the value associated with separation which occurs at rate $\delta$. When this occurs, the borrower makes the interest payment $Rq\omega$, gains the state of returning to the unmatched borrowers’ pool, and loses the state of being a matched borrower.$^{11}$

Subtracting (2) from (3) gives us the lenders’ value of being matched relative to being unmatched as:

$$J_m - J_u = \frac{(\delta R - 1) q\omega}{r + \delta + \mu}$$

Notice that equation (6) implies that a necessary condition for an active loan market requires $J_m - J_u > 0$ or $\delta R > 1$. Otherwise, the economy will degenerate into an autarchic state where no credit activity occurs. We will assume that this condition holds.

Similarly, subtracting (4) from (5) gives us the borrowers’ value of being matched relative to being unmatched as:

$$\Pi_m - \Pi_u = \frac{(A - \delta R) q\omega}{r + \delta + \eta}$$

We note that The relative values given by (5) and (6) are aggregate expressions. However, each individual borrower is atomistic and take their unmatched value $\Pi_u$ as given when evaluating their matched value. We take this into account and rewrite (5) gives,

$$\Pi_m = \frac{(A - \delta R) q\omega + \delta \Pi_u}{r + \delta}$$

In the presence of the moral hazard problem, a loan contract must be incentive compatible to eliminate borrowers’ default in equilibrium. In our framework, this means that the value associated with being a matched firm must be at least as great as the value associated with taking the funds and absconding:

$$(1 - \theta) \frac{q\omega}{r} \leq \Pi_m$$

$^{11}$For simplicity we have assumed that borrowers do not have any assets. More generally, if borrowers also have an asset $\omega^B$, equation (3) must be modified to $r\Pi_u = \omega^B + \eta(\Pi_m - \Pi_u)$. If these assets are jointly productive with the endowment of the lender then equation (4) changes to $r\Pi_m = A(q\omega + \omega^B) + \delta[-Rq\omega + (\Pi_u - \Pi_m)]$. Another possibility (that is also beyond the scope of this paper) is that borrower’s assets could be used as collateral for loans. For a discussion of these and related issues see, for instance, Hart and Moore (1994).
where the left hand side of (9) gives the present discounted value of absconding as the discounted value of the funds borrowed net of the cost expressed as a fraction $\theta$ of the loan.\textsuperscript{12} Substitution of (7) into this above inequality (9) gives,

$$q \omega \left[ (1 - \theta) \left( \frac{r + \delta}{r} \right) - (A - \delta R) \right] \leq \delta \Pi_u \tag{10}$$

From (10) we see that an increase in the loan interest rate $\delta R$ or an increase in the total quantity of the loan $q \omega$ increases the likelihood of absconding. An incentive compatible loan contract is defined as a pair $(q, R)$ such that (10) is satisfied.

When borrowers and lenders meet, they bargain over the terms of the contract. We will assume that the outcome of this bargaining game is consistent with the Nash bargaining solution where the contract $(q, R)$ is designed to maximize the joint surplus of the funds suppliers and demanders $S = (J_m - J_u)^{1/2} (\Pi_m - \Pi_u)^{1/2}$, subject to the incentive compatibility constraint (10), $q \in [0, 1]$ and $R \geq 0$ Using (5) and (7), maximization of the joint surplus implies $\frac{dS}{dq} > 0$ and, from $\frac{dS}{dR} = 0$, one obtains:

$$\delta R q \omega = (A q \omega - r \Pi_u) \Gamma + (1 - \Gamma) \tag{11}$$

where $\Gamma \equiv \frac{r + \delta + \mu}{2(r + \delta) + \mu}$ is monotone increasing function of $\mu$. Notice that if the pair $(q, R)$ is not incentive constrained (that is, the incentive compatibility constraint is not binding), then it is always optimal for the lender to loan out his entire endowment: $q = 1$.\textsuperscript{13}

\section{Characterization of the Loan Contract}

We proceed by describing the optimal loan contract in the presence of incentive frictions that cause a moral hazard problem. The optimal contract must be incentive compatible and

\textsuperscript{12}We may instead allow borrowers to divert assets and returns rather than only the productive assets, as in Hart and Moore (1998). In this case, borrowers can use the loanable funds to produce before absconding, implying that the incentive compatibility condition changes from (9) to $(1 - \theta) \frac{A \omega}{r} \leq \Pi_m$. Moreover, we assume, like Diamond (1990), that the penalty for defaulting is exclusion from the credit market forever, which may be viewed too harsh in light of modern bankruptcy laws. If instead credit market participation is allowed after payment of the default penalty, the incentive compatibility constraint becomes $(1 - \theta) \frac{A \omega}{r} + \Pi_u \leq \Pi_m$. One can easily verify that most of our main results in the following sections remain valid under these two alternative setups (see Section 2 below).

\textsuperscript{13}Suppose instead we assume that production is finalized after separation (not before as assumed in (4)). In this case (5) changes to $r \Pi_m = \delta [(A - R) q \omega + (\Pi_u - \Pi_m)]$. Ultimately, this means that (11) simplifies to $\delta R q \omega = \frac{(\delta A - 1) q \omega - r \Pi_u}{2}$. While we do not pursue this simplification here, it should be noted that the Nash bargaining condition becomes independent of $\mu$ under this new timing assumption. In turn, the loan contract becomes independent of loan market tightness.
satisfy the Nash bargaining condition. Both conditions can be represented in an intuitive graphical fashion.

Formally, an optimal loan contract is a pair \((R,q)\) such that (i) \(q = 1\) and \(R\) solves (11) if this pair satisfies (10), or otherwise (ii) \(q < 1\) and \(R\) solves (11) and (10) with equality. We define this latter case as a situation where the optimal loan contract is characterized by credit rationing. Defining \(Q \equiv q\omega\), we can then express the optimal loan contract in terms of the amount of rationed funds and the gross interest rate, \((Q,\delta R)\), by rewriting equations (11) and (10) as:

\[
\delta R = \left( A - \frac{r\Pi_u}{Q} \right) \Gamma + (1 - \Gamma) \tag{12}
\]

\[
\delta R \leq B + \frac{\delta\Pi_u}{Q} \tag{13}
\]

where \(B \equiv A - (1 - \theta)\frac{\mu}{r}\).

The determination of the optimal incentive compatible loan contract can be accomplished graphically in \((Q,\delta R)\) space with the origin defined as \(Q = 0\) and \(\delta R = 1\). We construct the graph in three steps.

First, we plot the surplus maximization condition (12) and call it the SM locus. This locus is upward-sloping and concave with a horizontal intercept \(Q_{SM} = \frac{r\Pi_u}{A - \Gamma}\) and slope \(\frac{r\Pi_u}{Q^2}\). An optimal loan contract must be along this SM locus. The slope of the SM locus can be interpreted in terms of the bargaining power of borrowers versus lenders. For example, a steep SM curve, resulting from a faster arrival rate of borrowers, \(\mu\), or long loan contract period, low \(\delta\), increases the bargaining power of lenders. Hence they can demand a higher interest rate \((\delta R)\) for a given increase in the loan quantity \(Q\) and this implies a steeper SM locus.

Second, we plot the incentive compatibility condition (13) with equality in \((Q,\delta R)\) space and call it the IC locus. This locus is downward-sloping and convex to the origin such that \(\lim_{Q\to 0} \delta R = \infty\) and \(\lim_{Q\to\infty} \delta R = B\) and has slope \(-\frac{\delta\Pi_u}{Q^2}\). Also, for \(B < 1\), IC has a horizontal intercept at \(Q_{IC} = \frac{\delta\Pi_u}{\delta}\). Any \((Q,\delta R)\) in the area below the IC locus satisfies the incentive compatibility constraint.

Third is the property that an optimal loan contract must always have \(q\) as large as possible within the feasible range \([0, 1]\). Formally, we can plot \(Q = \omega\) as the upper bound for \(q\). This means that if an optimal loan contract exists, it must be on the part of the SM
locus that is to the right of the IC locus and to the left of the \( Q = \omega \) locus with the highest \( q \). The existence of an active loanable funds market for all \( \delta R > 1 \) requires the condition that \( A > 1 \) and this is satisfied for a sufficiently productive economy.

Figure 1 segments our characterization of the optimal loan contract into three cases, depending on the relative position of the SM and IC loci. Case I indicates a situation when the SM locus is everywhere below IC. Hence all combinations of \((Q, \delta R)\) along the SM locus are incentive compatible and the optimal loan contract is the one with the highest value of \( q \), implying \( Q^* = \omega \) and the absence of credit rationing (see point E).

When the IC locus crosses the SM locus as in Case II, the amount of funds available in the economy is central for characterizing the optimal incentive compatible loan contract. When the amount of funds available is low (say, \( \omega = \omega^L \)), the optimal loan contract represented by \((Q^L, \delta R^L)\) is at point \( E^L \) where credit rationing is absent (i.e., \( q = 1 \)). However, when the amount of funds available is high with \( \omega = \omega^H \), the incentive compatibility constraint is now binding. As a consequence, there will be a unique optimal loan contract represented by \((Q^H, \delta R^H)\). In this case credit rationing emerges because the highest \( q \) attainable is strictly less than one (see point \( E^H \) where \( Q^H < \omega^H \)). Intuitively, loaning out a higher quantity of available funds increases the incentives for the borrower to abscond and leads to a binding incentive compatibility constraint. Since credit rationing only occurs conditionally, depending on a sufficiently large endowment, we will refer to this as a case of “conditional” credit rationing.\(^{14}\)

Finally, Case III shows that if the IC locus crosses the horizontal axis at a point lower than SM, there exists no incentive compatible combination of \((Q, \delta R)\) and no underwriting of a loan contract. This is the “non-active” credit market outcome and is comparable to a credit market breakdown. After completely characterizing the steady-state equilibrium, we will return to a more detailed analysis of the conditions consistent with each of these possible equilibrium outcomes.\(^{15}\)

\(^{14}\)Other assumptions may change the size of the credit rationing region. For example, allowing the borrower to use the loanable funds to produce before absconding yields an incentive compatibility constraint as \((1 - \theta) \frac{\Delta q^C}{r} \leq \Pi_m\). In this case, equation (13) is the same except for \( B^* = A[1 - (1 - \theta) \frac{\Delta \Pi_r}{r}] < B \). This implies that the IC locus is everywhere below the original IC curve derived above. Hence, the set of incentive compatible contracts shrinks and credit rationing becomes more likely.

\(^{15}\)When credit market participation is allowed after payment of the default penalty, the incentive compatibility constraint becomes \((1 - \theta) \frac{\Delta q^C}{r} + \Pi_u \leq \Pi_m\) and, in this case, the IC locus is upward-sloping, \( \delta R \leq B - \frac{\Pi_u}{Q^*}\).
3 Steady-State Equilibrium

The previous section discussed the properties of incentive compatible optimal loan contracts given the rates by which borrowers and lenders are matched. We now close the model by characterizing the steady state process by which lenders and borrowers meet. This will in turn pin down the equilibrium contact rates by which agents are matched, the steady state population of matched and unmatched borrowers and lenders, and hence equilibrium credit market tightness.

The flow of lenders into the state of being matched is given by $\mu N_u^L$ and the flow of borrowers into the matched state is given by $\eta N_u^B$. In equilibrium, the flow of funds supplied must be equal to the flow of funds demanded:

$$\mu N_u^L = \eta N_u^B = m$$

(14)

Recalling the credit market tightness measure $\tau \equiv \frac{N_u^B}{N_u}$, we can use (1) and the constant returns property of the matching technology to rewrite the steady state condition in terms of our tightness measure:

$$\mu = \eta \tau = m_0 M(1, \tau)$$

(15)

It is straightforward to show that (15) implies $\mu$ is increasing in $\tau$ whereas,

$$\eta \equiv \eta(\tau) = m_0 M\left(\frac{1}{\tau}, 1\right)$$

(16)

is decreasing in $\tau$; moreover, both $\mu$ and $\eta$ are increasing in $m_0$, $\frac{\mu}{\eta} = \tau$ (independent of $m_0$), $\lim_{\tau \rightarrow 0} \mu(\tau) = 0$ and $\lim_{\tau \rightarrow \infty} \eta(\tau) = 0$. This relationship is often referred to as the Beveridge curve in the search equilibrium literature. For labor markets the curve relates the unemployment rate to the vacancy rate (or establishes a relationship between the associated flow contact rates), while for credit markets we relate the capital unemployment rate to a measure of how much idle funds there are in the system.

From (6) and (7), we can eliminate $\Pi_m$ to obtain the unmatched value facing each potential borrower:

$$\Pi_u = \frac{\eta}{r + \delta + \eta} \frac{(A - \delta R)Q}{r}$$

Thus, it is possible to have coexistence of a credit rationing equilibrium and an inactive equilibrium.
There is a large mass of potential borrowers. Borrower entry into the loan market is determined by assuming each borrower faces a fixed cost \( v \) for setting up the investment technology. By equilibrium entry, borrowers enter into the unmatched pool of borrowers until their unmatched value is driven down to the entry cost, or,

\[
\Pi_u = v
\]  

Using the Beveridge curve relationship, we can substitute (17) into (18) to yield:

\[
\frac{\eta(\tau)}{r + \delta + \eta(\tau)} (AQ - \delta RQ) = rv
\]  

or, after substituting in (12) and \( \frac{\mu}{\eta} = \tau \),

\[
Q = \frac{rv}{A - 1} \left( 1 + \tau + \frac{2(r + \delta)}{\eta(\tau)} \right)
\]  

This is referred to as the ex ante zero profit (ZP) locus, which is strictly increasing and strictly concave in \((Q, \tau)\) space with a horizontal intercept \( \frac{rv}{A - 1} \).

A steady-state loanable funds equilibrium with unrestricted entry of borrowers is therefore a triplet \((Q^*, \delta R^*, \tau^*)\) that satisfies (i) the optimal incentive compatible loan contract (12) and (13) as specified in the previous section, and (ii) the ex ante zero profit condition given by (20). Figure 2 provides an illustration of this steady-state equilibrium by combining Case II from Figure 1 in the top panel with the ZP locus in the bottom panel (whereby we note that this locus has the same horizontal intercept as the SM locus). Thus, a loanable funds equilibrium is determined in a recursive manner. The optimal incentive compatible contract pins down the equilibrium \((Q^*, \delta R^*)\). Then the ZP locus determines equilibrium entry and hence the market tightness measure \( \tau^* \) that is consistent with the optimal contract.

Once this triplet is determined, it is straightforward to derive the steady state populations of matched and unmatched borrowers and lenders. From (16) and (15), we have \( \eta^* \) and \( \mu^* \), respectively. Because the inflow of unmatched lenders being matched must equal the outflow of matched lenders being separated: \( \mu^* N_u^L = \delta N_m^L \). Substituting the population identity, \( N_u^L + N_m^L = 1 \), into this equilibrium flow condition gives:

\[
N_m^L = \frac{\mu^*}{\delta + \mu^*} = N_m^{B*} \quad \text{and} \quad N_u^L = \frac{\delta}{\delta + \mu^*}
\]  

\[\text{A sufficient condition for the concavity of the ZP locus is given by the envelope condition } \frac{(1/\tau)MmMn}{Mm} > -2. \]  

For example, this is trivially satisfied for the Cobb-Douglas case.
From this it is easy to see that the equilibrium number of unmatched borrowers is \( N_{u}^{B*} = \tau^{*}N_{u}^{L*} \). It is straightforward to verify that when a loanable funds equilibrium exists, it is unique. Hence, in general, not only does the optimal loan contract determine market tightness and liquidity, but market tightness in turn also affects the optimal loan contract.

4 Equilibrium Credit Rationing and Market Breakdown

We now analyze the properties of the steady-state loanable funds equilibrium. We identify the conditions that are consistent with existence of the steady state and show that we can differentiate between three possible regimes. Specifically, based on the equations underlying Figure 2, we have:

Proposition 1. (*Steady-state Loanable Funds Equilibrium*) Let \( r = \frac{(1-\theta)(2\delta+\mu)}{(A-1)-2(1-\theta)} \). A steady-state loanable funds equilibrium features one of the following three outcomes:

(i) If \((A-1) < (1-\theta)\), then there does not exist an incentive compatible loan contract and the loan market is non-active (Case III).

(ii) If \((1-\theta) \leq (A-1) < 2(1-\theta)\) and \(\omega\) is sufficiently high, then there exists a credit rationing equilibrium with \(q < 1\) (Case II).

(iii) If \(2(1-\theta) \leq (A-1)\) and \(\omega\) is sufficiently high, then

a. for \(r < \overline{r}\), there exists a credit rationing equilibrium with \(q < 1\) (Case II);

b. for \(r \geq \overline{r}\), the incentive compatibility constraint never binds and the equilibrium loan contract is not rationed (Case I).

**Proof:** See Appendix. \(\square\)

Proposition 1 outlines the region of the parameter space consistent with the various possible equilibrium outcomes discussed in the previous section. This Proposition has a very intuitive interpretation. Loosely, if the productivity of the investment project is sufficiently low relative to the incentive friction, then there will always the incentive incentive to abscond.
for any loan contract along the SM locus. In this case (Case III), there is no active loanable funds equilibrium and the loan market breaks down. Once productivity begins to exceed a threshold level (Case II), lenders begin channeling loanable funds to borrowers, but the quantity is rationed. Finally, if productivity is sufficiently high, then whether or not there is rationing can be expressed in terms of the rate of time preference. In particular, if the rate of time preference is sufficiently small, then the present discounted value of consumption generated from absconding for the borrower becomes greater than the value of being matched in the loanable funds market. Lenders must continue to ration loans so that the incentive compatibility binds (Case II). If, on the other hand, the rate of time preference is very large, there will never be an incentive for the borrower to abscond and all loan contracts are incentive compatible (Case I).

Proposition 1 establishes that credit rationing of productive firms depends on a threshold $r$. Next, we consider the underlying changes to this threshold that make credit rationing of productive firms more likely.

**Proposition 2. (Credit Rationing).** Suppose that the investment project is sufficiently productive such that $(A - 1) > 2(1 - \theta)$. Then a steady-state loanable funds equilibrium possesses the following properties.

(i) An exogenous increase in market tightness ($\tau$) increases the likelihood of credit rationing. For all $r > 0$, there exists $\tau < \infty$ sufficiently large such that equilibrium credit rationing will occur.

(ii) An increase in the duration of the loan contract (lower $\delta$) leads to an increase in the set of incentive compatible contracts and an increase in the loan market equilibrium interest rate ($\delta R$). If the latter effect dominates the former, credit rationing is likely to occur.

**Proof:** Part (i) follows directly from Proposition 1. From (15) an (exogenous) increase in $\tau$ increases the frequency at which lenders meet borrowers, $\mu$. Since $\frac{\partial \tau}{\partial \mu} > 0$ it follows that an increase in market tightness expands the set of feasible rates of time preference consistent with the credit rationing equilibrium. As $\tau$ becomes arbitrarily large, $\tau \rightarrow \infty$. To prove part (ii), we observe that $\lim_{Q \rightarrow \infty} \delta R_{SM}$ and $\lim_{Q \rightarrow \infty} \delta R_{IC}$ are decreasing in $\delta$. $\square$
To see the intuition behind this result, suppose that the initial steady state equilibrium is given by Case I. In this case, every loan contract that maximizes the joint match surplus of borrowers and lenders is incentive compatible. The Beveridge curve relationship given by (15) implies that an increase in market tightness increases the rate that lenders contact borrowers. This increases the threat point and bargaining power of lenders when negotiating the loan contract. As a result, the SM locus shifts upwards and this shrinks the set of $Q$ and $\delta R$ combinations consistent with incentive compatibility. If the increase in market tightness is sufficiently large, IC will eventually intersect with the SM locus and credit will begin to be rationed at a higher equilibrium interest rate. Thus, we are more likely to see credit rationed in an illiquid credit market where it is difficult for borrowers to find loan opportunities.

A longer contract duration (low $\delta$) increases the set of incentive compatible contracts since the borrower can enjoy the productive benefits supported by the loanable funds for a greater period of time. This is captured by an upward shift of the IC locus. However, a longer contract duration also makes the match more valuable to the borrower and biases the bargaining power towards lenders. Consequently, the SM locus shifts upwards as well. In Case II, the market equilibrium loan rate increases in both the case where there is no rationing ($\omega = \omega^L$) and when there is rationing ($\omega = \omega^H$). Whether or not credit rationing is more likely depends upon whether the bargaining effect dominates the incentive compatibility effect. If the bargaining effect dominates, then the a longer contract period both increases rationing and the equilibrium loan rate.

Proposition 2 showed that an exogenous increase in the tightness of the credit market will eventually lead to credit rationing. However, the link between tightness and credit rationing is severed once the entry decision of borrowers is allowed to be endogenous. In particular, we find

**Proposition 3.** *(Market Tightness versus Credit Rationing)* There is no necessary positive relationship between credit market tightness and the extent of credit rationing in general equilibrium.

To illustrate this proposition, consider the following comparative steady state analysis of an increase in funds matching efficacy ($m_0$) that improves matching for both lenders and borrowers in Case II. This is illustrated in Figure 3. Under our funds matching framework,
a rise of $m_0$ raises the effective contact rate of funds suppliers $\Gamma$ and strengthens their bargaining power. As a consequence, joint surplus maximization grants relatively higher returns to the suppliers, implying an increase in $R$ for each given value $Q$. That is, the SM locus rotates upwards. From the ZP relationship, an increase in $m_0$ will raise the matching rate $\eta(\tau)$ of borrowers. For a given $Q$ determined by the optimal loan contract, more potential borrowers enter and hence the loan market becomes tighter. That is, the ZP locus twists toward the vertical axis.

For the case of $\omega = \omega^L$ where the incentive compatibility constraint is not binding, the equilibrium loan rate rises as market participation (or tightness) increases. For the case of $\omega = \omega^H$, rationing increases in response to the increased entry of potential borrowers and a higher equilibrium loan rate is required to satisfy incentive compatibility. However, the increased severity of credit rationing reduces potential borrowers’ expected profit, thereby decreasing their entry. Due to this latter opposing effect, the net change in the tightness of the loan market is ambiguous. Hence, an observed increase in credit rationing need not imply increased tightness in the credit market.

5 The Role of Market Frictions

We next explicitly consider the impact of search frictions ($f_0$), market entry frictions ($\nu$), and incentive frictions ($\theta$) on the structure of optimal lending arrangements and steady state equilibrium in the loanable funds market. Each of these frictions is considered separately so that their relative contributions to explaining credit rationing and market breakdown can be isolated and analyzed.

5.1 Search Frictions

Without search frictions, credit market participants do not have to wait to set up a credit arrangement. When matches are instantaneous, we have

**Proposition 4. (Search Frictions).** In the absence of search frictions, the equilibrium loan contract and the existence of equilibrium credit rationing are independent of market tightness.
The absence of search frictions occurs in the limiting case where \( m_0 \to \infty \). The Beveridge curve relation implies that while \( \mu \) and \( \eta \to \infty \), \( \tau \equiv \mu/\eta \) will remain bounded by the constant returns to scale property of the matching technology. Using these and \( \lim_{m_0 \to \infty} \Gamma = 1 \), the steady state equilibrium conditions (12), (13), and (20) are now given by \( \delta R = A - \frac{\delta v}{Q} \), \( \delta R \leq B + \frac{\delta \eta}{Q} \), and \( Q = \frac{\delta v}{A-1}(1 + \tau) \), respectively. Since \( \tau \) no longer appears in the SM and IC loci, the optimal loan contract is independent of market thickness. \( \square \)

This result says that search frictions are crucial for a linkage between market tightness, the optimal loan contract, and credit rationing. If borrowers and lenders can meet and enter into a lending agreement instantaneously, their relative bargaining position will not be affected by the tightness of the market. In this situation the only equilibrium outcomes are the conditional (Case II) and non-active (Case III) steady states. Because of the entry costs on the borrower’s side, a reduction in search frictions increases the relative ease with which lenders locate borrowers. As in Proposition 2, this causes an upward shift in the SM locus. As the loan equilibrium interest rate rises, lenders must ration in order to keep the contract incentive compatible. The absence of search frictions in general equilibrium can be seen as a limiting case of this sequence of partial equilibrium events.

5.2 Entry Frictions

Here we consider costless entry of borrowers into the credit market. Under these circumstances, the demand for funds is perfectly elastic and we establish

**Proposition 5. (Entry Frictions).** In the absence of entry frictions, there does not exist a credit rationing equilibrium. Equilibrium in the loan market is characterized by either an active no rationing loanable funds market or a non-active loan market.

**Proof:** The absence of firm entry frictions occurs in the limiting case where \( v \to 0 \). Since there is now unrestricted borrower entry, the steady state conditions described (12), (13) are now given by \( \delta R = \Gamma(A - 1) + 1 \), \( \delta R \leq B \). Existence of an active loanable funds market requires \( \Gamma(A - 1) + 1 < B = A - (1 - \theta)(\frac{\delta \mu}{\tau}) \). Satisfaction of this condition implies that all combinations of \((Q, \delta R)\) along the (horizontal) SM locus is incentive compatible and there is no credit rationing, \( q = 1 \). \( \square \)
To obtain intuition behind this result, suppose that the initial steady state equilibrium is given by the active no rationing equilibrium of Case I. In this situation, $B > \Gamma(A - 1) + 1$. Lower entry frictions encourages the entry of borrowers and this drives down their unmatched value. This rotates the SM locus clockwise as lenders take advantage of their increased bargaining power. At the same time, a lower unmatched value for the borrower must be compensated by a decrease in the loan rate or loan quantity to maintain incentive compatibility. In the limiting case as these costs vanish, no contract that offers an interest rate above $B$ will be incentive compatible and no interest rate above $\Gamma(A - 1) + 1$ will be consistent with the optimal loan contract. Hence, there will only be an active no rationing equilibrium.

To show that credit rationing dissappears with free entry, suppose that the initial steady state equilibrium is given by the conditional equilibrium of Case II. In this situation, $B < \Gamma(A - 1) + 1$. Lower entry frictions encourages the entry of borrowers and this drives down their unmatched value. As lenders take advantage of their increased bargaining power, the interest rate consistent with the optimal loan contract rises and the SM locus rotates clockwise. In the limiting case as entry costs vanish, the interest rate approaches its maximum value given by $\Gamma(A - 1) + 1$. At the same time, a lower unmatched value for the borrower must be compensated by a decrease in the loan rate or loan quantity to maintain incentive compatibility and this is captured by a counterclockwise rotation of the IC locus. In the limiting case as these entry costs vanish, no contract that offers an interest rate above $B$ will be incentive compatible. Hence, because no optimal loan contract will be incentive compatible, the credit market fails to function and there will only be a non-active equilibrium.

Furthermore, recall that Case I captures an active loan market equilibrium with no credit rationing only if the potential supply of loanable funds ($\omega$) is sufficiently large. If $\omega$ is small then there may not be any positive rate of interest consistent with the optimal loan contract. However, as entry costs for borrowers are driven to zero, there will emerge an active non-rationing credit market equilibrium. If, on the other hand, the economy was initially characterized by the conditional Case II, possibly with credit rationing, then a removal of entry barriers will lead to a breakdown of the credit market and non-existence, which may be referred to as a problem of "over-crowding" in entry of borrowers with limited loanable funds supply.
5.3 Incentive Frictions

Finally, we investigate the role of moral hazard for credit arrangements in the decentralized market. We detail how the equilibrium is affected if the cost of absconding is driven down to zero.

**Proposition 6.** (Incentive Frictions). In the absence of incentive frictions, there only exists an active, no rationing loanable funds market equilibrium.

**Proof:** The absence of incentive frictions corresponds to the limiting case where the costs of absconding as a fraction of total funds, \( \theta \to 1 \). It is immediate from Proposition 1 that for any given \( A > 1 \), we can rule out (i) the non-active equilibrium and (ii) the credit rationing equilibrium with \( (A - 1) \in (1 - \theta, 2(1 - \theta)) = \emptyset \). In the case where \( (A - 1) > 2(1 - \theta) \),

\[
\tau = \frac{(1-\theta)[2\delta+\mu]}{(A-1)-2(1-\theta)} = 0.
\]

Hence, all \( r > 0 \) satisfies \( r > \tau \) and in this case there exists the active, no rationing equilibrium. \( \square \)

Proposition 5 says that the moral hazard problem arising from incentive frictions is crucial in explaining the existence of both credit rationing and credit market failure. In the presence of these incentive frictions, search frictions provide a link between market liquidity and credit market tightness and credit rationing. Finally, borrower entry frictions also play a crucial role in explaining credit rationing. While the absence of such frictions does not preclude the non-active equilibrium, it does rule out credit rationing as an equilibrium outcome.

6 Duration of Loan Contracts

An innovative aspect of this our search model of the credit market is that it incorporates a parameter \( \frac{1}{\delta} \) (that corresponds to the duration of the loan period). For example, in the previous section we discussed how an increase in the length of the loan contract can affect the incentive compatibility, the interest rate offer, and the possibility of credit rationing. An interesting application of this framework would be to study how the duration of the loan contract affects the interest rate offer and vice versa.

A simple illustration of how the contract’s duration affects the loan rate would be to consider the active equilibrium without credit rationing (Case I). Here, the optimal contract
interest rate is just given by substituting $Q^* = \omega$ into (12) to get:

$$\delta R^* = \left[A - \frac{rv}{\omega}\right]\Gamma + (1 - \Gamma)$$

(22)

Notice that $R^* > 0$ as long as $(A - 1) > \frac{rv}{\omega}$, which is satisfied if the supply of loanable funds $\omega$ or the productivity of the investment project $A$ is sufficiently large. In this case a decrease in $\delta$, or increase in the length of the loan contract, increases $\Gamma$ and hence the optimal loan interest rate $\delta R^*$. One could interpret result as an upward sloping yield curve in $(\frac{1}{\delta}, \delta R)$ space. While beyond the focus of this paper, it would be of interest in future work to more completely analyze the term structure properties of a search model of credit.

A related issue would be to extend our model to address the endogenous joint determination of the quantity, loan rate, and loan contracting period. One way to approach pinning down $(Q, \delta R, \delta)$ is to have all three objects be the outcome of decentralized bilateral bargaining between lenders and borrowers. In other words, have them be a solution to a Nash bargaining problem (P) which maximizes the joint surplus of the lender and borrower:

$$\max_{q, \omega, \delta R, \delta} (J_m - J_u)^{1/2}(\Pi_m - \Pi_u)^{1/2}$$

(23)

s.t. $J_m - J_u = \frac{(1-q)\omega + \delta Rq\omega - rJ_u}{r + \delta}$ and $\Pi_m - \Pi_u = \frac{(A - \delta R)q\omega - r\Pi_u}{r + \delta}$

In addition to the bargaining condition for $\delta R$ given in (12), we now have an additional first order condition associated with $\delta$ that, after simplification, is given by,

$$Rq\omega\left(\frac{1}{J_m - J_u} - \frac{1}{\Pi_m - \Pi_u}\right) = \frac{1}{r + \delta}$$

(23)

Because the expression in brackets is equal to zero $(J_m - J_u = \Pi_m - \Pi_u)$, this implies an optimal choice of $\delta^* \to \infty$. That is, instantaneous credit transactions and separations maximize the joint surplus of a borrower-lender pair when production is instantaneous at the time of a match. This result delivers an important message. In our admittedly highly stylized framework, where long-term relationships only lower search costs but do not alter incentive frictions, there exists no reason to continue a “long-term” credit relationship. This is because the marginal gain in matched values from continuing a relationship is dominated by the marginal loss in unmatched values (via the bargaining threat points). Undoubtedly, adding more realistic features to the model will provide additional incentive to prolonging credit
relationships. For instance, allowing learning and diminishing incentive frictions over the
time agents are matched may yield outcomes that favor long-term relationships. Although
it is beyond the scope of the current paper to go further along this path, we would like to
point out that there is an active literature exploring long-term credit relationships.\footnote{For instance, Hart and Moore (1998) investigate mechanisms and incentives to renegotiate debt contracts in the face of credit market imperfections and incomplete contracting.}

7 Concluding Remarks

This paper has presented a simple search-theoretic model of the credit market. The model
features endogenous entry and moral hazard because they are particularly important factors
in determining the fortunes of entrepreneurs. Our analysis describes the optimal incentive
compatible loan contracts and equilibria that emerge in such an environment. While we
tie the extent of mismatch to the tightness of credit markets, we find that mismatch and
tightness are somewhat of a “red-herring” for understanding the extent of credit rationing.
This is because endogenous entry severs any relationship between credit market tightness
and credit rationing. We also show that entry and incentive frictions are important factors
for determining when credit markets breakdown. In other words, the ease of entry into
the credit market and the ease of exit from the credit market (via default) are important
for market breakdown. This suggests that understanding the boundaries of the market
is fundamental for understanding credit market breakdown. Finally, we show that search
and incentive frictions are important for determining the extent of credit rationing. In other
words, it is the ease of finding a credit partner and the ease of undoing a partnership that are
important for rationing. This suggests that understanding the boundaries of a relationship
is fundamental for understanding rationing. We believe that our simple framework will be a
useful vehicle for further investigation into these boundary issues.

There are several other possibilities for future work. First, it would be interesting to re-
examine the optimal loan contract and the likelihood of market breakdown under alternative
versions of the moral hazard problem. For instance, we have identified in passing several
possibilities that might be of general interest. In particular, one may allow the borrower to
use the loanable funds to produce before absconding or permit the absconder to participate
in the loan market after paying for the default penalty. Moreover, following Hart and Moore (1994 and 1998), one could contrast the situation where the lender can divert project returns only to the situation where part of the underlying assets can be diverted. In any case, the incentive compatibility constraint must be modified, which may lead to different equilibrium outcomes. Second, one might argue there is too much randomness in our matching model and that this randomness exaggerates the moral hazard problem and diminishes the benefit of long-term relationships. One way to address this issue is to adopt the directed-search price-posting game developed by Peters (1991). Specifically, there are two segregated sub-markets: one similar to the environment in the present paper and one with lenders requiring full credit documentation (that presumably minimizes the incentive frictions). Since all borrowers are identical *ex ante*, each lender in each segregated submarket posts for all borrowers the flow interest rate and the duration of the loan contract to maximize the expected value subject to a no-arbitrage condition that ensures all borrowers receive equal value *ex ante*. As a consequence, the loan contracts are generally different between the two submarkets, and free mobility of borrowers results in different matching probabilities and hence different measures of tightness within the two credit markets.
Appendix

Proof to Proposition 1

(i) Consider the case where \((A-1) < (1-\theta)\). This implies that \(B \equiv A - (1-\theta)\left(\frac{r+\delta}{r}\right) < 1\) so that the IC locus has a horizontal intercept at \(Q_{IC} = \frac{\delta\Pi}{1-B}\). Suppose there exists an active loanable funds equilibrium. This implies: \(\frac{\delta\Pi}{1-B} = Q_{IC} < Q_{SM} = \frac{r\Pi}{A-1}\), or, \(r(1-B) < \delta(A-1)\), or by manipulating,

\[
r[(1-\theta) - (A-1)] < \delta[(A-1) - (1-\theta)]
\]

Since the right hand side of this expression is negative while the left-hand side is positive, we have a contradiction. Hence, no active loanable funds equilibrium exists.

(Case III)

(ii) Consider the case where \((1-\theta) \leq (A-1) \leq 2(1-\theta)\). From the SM locus given by (12) notice that \(\lim_{Q \to \infty} \delta R = \Gamma(A-1) + 1\). Similarly, from the IC locus given by (13), \(\lim_{Q \to \infty} \delta R_{IC} = B\). If \(B < 1\), then from (24) we know \(Q_{IC} \leq Q_{SM} = \frac{r\Pi}{A-1}\), or, \(r(1-B) < \delta(A-1)\), or by manipulating,

\[
r[(A-1) - 2(1-\theta)] < \delta(A-1) - (1-\theta)]
\]

Since the right-hand side of (24) is non-positive and the right-hand side is strictly positive, this condition holds. Thus, there is a unique credit rationing equilibrium where the SM locus intersects the IC locus (Case II).

(iii) Consider now \((A-1) > 2(1-\theta)\). Solving for \(r\) in (24) gives, \(r < \frac{(1-\theta)(2\delta+\mu)}{(A-1) - 2(1-\theta)} \equiv \tau > 0\). This condition is sufficient to guarantee that \(\lim_{Q \to \infty} \delta R_{SM} > \lim_{Q \to \infty} \delta R_{IC}\), or, \(\Gamma(A-1) + 1 > B\), which can be rewritten as:

\[
r[(A-1) - 2(1-\theta)] < (1-\theta)[2\delta + \mu]
\]

Measures of Credit Market Tightness

An alternative measure of credit market tightness is given by the ratio of unmatched borrowers to the total pool of borrowers or the “capital unemployment rate”: \(\kappa = \frac{N^B_N}{N^N_m+N^B_m}\). Since \(N^B_u = \tau N^L_u\) and \(N^B_m = N^L_m = \frac{\mu}{\delta} N^L_u\), we have:

\[
\kappa = \frac{\tau}{\tau + \mu/\delta} = \frac{\tau}{\tau + m_0 M(1,\tau)/\delta} = \frac{1}{1 + \frac{m_0 M(1,\tau)}{\delta}} = \kappa(\tau)
\]

From (24), it is easily verified that \(\kappa\) is monotonically increasing in \(\tau\).
References


Chart 1: The Structure of the Economy

Lenders (N_u^L)

flow endowment
\( \omega \)

unmatched value
\( J_u \)

Credit Market

matching
\( \mu N_u^L = \eta N_u^B = m(N_u^L, N_u^B) \)

production
\( \Lambda q \omega \)

surplus sharing

\( (1-q)\omega + \delta Rq \omega \) to L

\( (A-\delta R)q \omega \) to B

Borrowers (N_u^B)

entry cost
\( v \)

unmatched value
\( \Pi_u \)
Figure 1: Optimal Incentive Compatible Loan Contract \( Q = \omega q \) and \( \delta R \)

Case I: \( (A-1)>2(1-\theta) \) and \( r \geq \bar{r} \)

Case II: \( (A-1)>2(1-\theta) \) and \( r<\bar{r} \)
or \( (1-\theta) \leq (A-1) \leq 2(1-\theta) \)

Case III: \( (A-1)<(1-\theta) \)

Notes: There are three cases, depending on the relative position of the SM and IC loci:
1. Case I incentive compatibility constraint never binds;
2. Case II incentive compatibility constraint binds only when funds available are high and in that case, the amount of loan is rationed;
3. Case III optimal incentive compatible loan contract does not exist.
Figure 2: Steady-State Loanable Funds Equilibrium

Figure 3: Equilibrium Responses to a Reduction in Search Frictions (Higher $m_0$)