Limited Participation, Income Distribution and Capital Account Liberalization*

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Abstract

This paper examines theoretically, using a two-country real-business-cycle model, the effects of capital-market liberalization when there is limited participation in national financial markets. It is assumed that workers cannot smooth consumption as well as do stockholders, and therefore, liberalization may hurt workers. This dynamic model evaluates some claims—made particularly by the "anti-globalization" movement—that capital movements hurt workers, while benefitting stockholders. Quantitatively, liberalization makes workers better off in the long run, since the new capital allocation and increased insurance foster capital accumulation, raising wages that offset the output fluctuations due to capital flows. However, transitional effects may overturn these long-run benefits.

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1 Introduction

In the late 1980s and the early 1990s, many nations began implementing a wave of economic reforms. Among those reforms was capital account liberalization. As a result, today international financial markets are substantially more active than twenty years ago, though there are still many countries with various sorts of capital controls. Economic theory predicts that a complete opening of all financial markets should be welfare-improving: intertemporal consumption smoothing, consumption insurance, and better allocation of investment across countries would presumably all contribute their share to this. However, it is not clear what effect partial liberalization, or liberalization in the presence of other frictions that cannot be eliminated, would have. In this case Pareto improvements are not guaranteed, and at the very least some groups may lose from liberalization while others gain. A complete account of the actual experience from past liberalizations is yet to be made; see, for example, the survey of empirical research by Das and Mohapatra (2003). The purpose of this paper, however, is theoretical, studying the effects of capital account liberalization in conjunction with a particular friction: limited participation in the stock market. In other words, in this paper it is studied whether the fact that some nationals do not have access to stock markets could limit the benefits of international liberalization, whether in size or in the differential impact across groups in society.

The group that does not participate in national stock markets (prior to and after liberalization) is a particularly interesting one to study. The presumption here is that consumers with little wealth, referred to as “workers” here, are very inactive in intertemporal as well as in insurance markets. Thus, would those consumers lose from liberalization? In particular, would they suffer from increased movement of capital across borders, which might lead to a large increase in wage volatility and perhaps job insecurity? Some observers stress this as a real possibility. In the debate, there has been an increasing and significant movement against “globalization.” Part of the target of this movement is the liberalization of capital markets,
and one of the arguments put forth (though not the only one) is based on a concern that inequality will rise. Thus, the “anti-globalization” movement claims that the presumable benefits of the efficient allocation of capital throughout the world that comes with stock market liberalization favor only stockholders and not workers. Capital liberalization may lead to higher capital supply elasticities, affecting the welfare of those workers with limited mobility. The approach taken in this paper allows us to study those possibly asymmetric effects theoretically using standard macroeconomic theory extended to allow inequality. In particular, inequality is a function of the institutional scheme adopted in this model, since wages and returns to capital are endogenous and have an important influence on the distribution of wealth and welfare across individuals within a country (in addition to their possibly asymmetric effects on individuals in different countries).

There is plenty of microeconomic evidence of heterogeneity among households in terms of how their financial wealth is allocated. In particular, a very striking pattern in household survey data is that only a small fraction of households own stocks. Among workers, moreover, a significant fraction holds only liquid assets. Thus, an argument can be made that—at least as a rough approximation—those households can be described as “hand-to-mouth” consumers: they consume their labor income as it arrives and keep bank deposits only to facilitate the spending of that income.

There are also arguments from the theoretical literature on macroeconomics and inequality that suggest that one should in fact expect this kind of characterization of financial inequality across households. In particular, several one-country models of inequality lead to endogenous splits of the population into two groups: the high-wealth consumers, who are active stockholders and whose savings behavior is key in determining interest rates, and low-wealth consumers who are essentially passive financially. The model of Krusell and Smith (1998) derives those results based on assumptions of heterogeneity in time discount rates, and Guvenen (2003) studies a two-group model where the difference among agents
has its roots in risk attitudes and the intertemporal substitution elasticity.\textsuperscript{1} The model here departs from those papers: it assumes that there are two groups of agents—“workers” and “stockholders”—and it assumes limited participation of an extreme form: workers are simply hand-to-mouth consumers. The presumption is that a model based on incomplete insurance against idiosyncratic income risks along with preference heterogeneity, like a combination of the frameworks studied by Krusell and Smith and by Guvenen, would deliver a setup with an “almost-reduced” form like the model studied here.

In order to focus on the redistribution effects of international portfolio diversification, the focus on two groups greatly simplifies the analysis both numerically and analytically. In particular, it is possible to find closed-form solutions under the parameterizations assumed. Thus, the model is considerably more tractable than the frameworks studied in the incomplete-markets literature, while still capturing the essential features of those much more complex models. Thus, there are two countries and two types of agents in each country. Countries face country-specific technological shocks as in a standard two-country stochastic neoclassical growth model. Workers (low-wealth agents) do not have access to the stock market, while stockholders (high-wealth agents) do. Adjustments in the labor market occur only through a price dimension, e.g., movements in wages, while the adjustments through quantities, either in the intensive or in the extensive margin are not possible since labor supply is inelastic. Even though in principle adjustments through quantities may be important, Albuquerque and Rebelo (2000) provide an ample survey of literature that suggests a small impact in labor markets associated with many trade reforms.

The model can be solved for two institutional arrangements. In the first arrangement stockholders can buy contingent claims for every possible state of the world from an investment fund, and it is this firm that decides how to allocate the collected funds between the

\textsuperscript{1}Guvenen assumes limited participation but discusses how it would arise endogenously under fixed costs of participation. Krusell and Smith do not assume or derive limited participation but show how the portfolios of the wealthy will be more geared toward stock.
two technologies available in the world so as to maximize profits. In this setup, we would discuss stock prices and stock-market liberalizations. In the second arrangement, stockholders in each country accumulate capital as wealth, thus directly deciding how much capital to invest in each country. Here, we could use the cost of capital in the two countries as the relevant prices and the label foreign direct investment (FDI) would become appropriate. As expected, the equilibria of both models coincide, although the shape of the decision rules differs. The second arrangement is presented in this paper.

Leaving aside preference heterogeneity, countries in this model can be different in three ways: first, their average productivity can differ; second, even if their average productivity is the same, their volatilities do not need to be identical; and third, there can be differences in the abundance of inputs. One of the goals of the paper is to examine how those asymmetric distributions of primitives influence the outcomes for observables and for the economic welfare of different consumers. Moreover, the transition to steady state needs to be taken into account, so initial conditions—the initial relative capital stocks and productivity levels—can be important in determining outcomes.

The most striking result I have found is that of steady-state comparisons: it turns out that for a wide range of parameters, world workers actually benefit from capital account liberalization policies. This occurs because world wealth increases in the open economy and so does the capital invested in every country. The two key ingredients for this result are that the intertemporal elasticity of substitution (IES) of the stockholders’ utility function is equal to 1 and the concavity of the production functions. Empirical studies suggest that the IES for stockholders is close to 1, while the IES for workers is lower (around 0.1)\(^2\). Thus, the increase in the overall capital stock outweighs the potential losses from increased fluctuations in wages.

Another interesting result is that transition effects indeed can be quite important quantitatively. In the welfare comparisons it is thus possible that the positive long-run effects of

liberalization can be reversed: workers in one of the countries involved in the liberalization process may, for some initial conditions, be worse off. In those cases, thus, the gains in the long run are not sufficient to outweigh the losses in the near future due to the adjustment in capital flows.

I also calibrate the economy and compare the results obtained with those in related representative-agent two-country models. For the logarithmic-utility case, and given the closed-form nature of the results, it is straightforward to compare both steady states and transition paths with and without capital account liberalization.

The previous literature has studied the welfare gains from international risk sharing under a representative-agent assumption with output uncertainty. Cole and Obstfeld (1991) calibrate a model using U.S. and Japanese data and find very small gains from asset trade because relative price movements across different consumption goods induce insurance indirectly. Moreover, their results are in line with the findings in Lucas (1987), who estimates the cost of postwar United States consumption variability to be quite small. Obstfeld (1994) obtains much larger welfare gains than Lucas does using a model with endogenous growth. However, the empirical evidence suggests that stock-market liberalizations lead to important transitory growth effect without much influencing long-run growth outcomes (Fuchs-Schundeln and Funke (2001)). Finally, a model with heterogeneous agents of the type considered here is studied by van Wincoop (1996), but the purpose of his analysis is unrelated to the questions asked in this paper.

The paper is organized as follows. First a simple finite-horizon model with uncertainty is analyzed, and for illustration the focus is on perfect negative correlation between the technology shocks of the two countries. Subsequently, the infinite-horizon problem under incomplete markets arrangements is described, together with some comparative-static experiments. Finally, to measure the importance of the different effects that come into play during the capital liberalization process the US and Korean economy are calibrated. The last section concludes and suggests some extensions.
2 Finite-horizon models

2.1 Uncertainty with negatively and perfectly correlated shocks

The simplest but still interesting economy to study is described by a model with two countries that only lasts for two periods where shocks are negatively and perfectly correlated. The technological shocks today are indicated by $A$ and $A^*$ in each country while tomorrow there will be $A'$ and $A'^*$ (asterisks denote the foreign country variables and primes denote tomorrow’s variables).

For the sake of simplicity, I borrow the underlying one-country model from Krusell (2002).

Each country is populated by a large number of identical stockholders with measure $\mu$ ($\mu^*$) for the home (foreign) country, normalized to 1 for convenience, and a large number of identical workers with measures $n$ and $n^*$ respectively. Stockholders from the same country have identical wealth, not necessarily equal to that of stockholders in the other country. All agents in both economies have identical tastes.

In this two-period model, countries do not know if tomorrow their productivity shock will be high or low, but they do know that the correlation between the shocks of the two countries is minus one. Since I want to focus on the effects of uncertainty and risk sharing between the different groups of agents in the two countries, it will be assumed that the countries have the same initial wealth, the same participation rates (or factor abundance) and equal levels of the productivity shocks (i.e. $A_h = A^*_h$, and $A_l = A^*_l$).

It is worth noticing that in this case with uncertainty, the effects that opening stock markets internationally have on the groups non participating in them, this is, the workers, are different depending on the specific value of the IES coefficient of the stockholders’ utility function.

Again, the problem is solved for the simplest case with IES=1 (logarithmic utility).

The problem in autarky for the domestic stockholder is as follows:
\[ \text{Max}_{k_d'} \log(Rk_d - k_d') + \beta E \log(Rk_d'), \]

taking \( R' \) as given. \( R' \) now can take two values, \( R'_h \) and \( R'_l \).

\( k_d \) is today’s stock of capital of the representative domestic stockholder invested in the home country, and \( R \) is the rate of return of capital in the home country.

In equilibrium, the F.O.C. requires that

\[ k_d' = \frac{\beta}{1 + \beta} Rk_d, \]

and prices clear markets so that

\[ R'_h = \alpha A'_h k_d'^{\alpha - 1} n^{1 - \alpha}, \quad \text{and} \]
\[ R'_l = \alpha A'_l k_d'^{\alpha - 1} n^{1 - \alpha}. \]

Workers supply one unit of labor every period and consume their labor income. Therefore domestic workers consume

\[ c_d^w = (1 - \alpha) A k_d n^{1 - \alpha}, \quad \text{and} \]
\[ c_d'^w = (1 - \alpha) A' k_d'^{\alpha} n^{1 - \alpha}, \]

with \( A' \) taking values \( A_h \) or \( A_l \).

\( c_d^w \) is today’s consumption of the domestic worker which equates the return of labor in the home country.

The problem is symmetric for the foreign country.

When capital mobility is allowed, stockholders from any of the two countries can invest in two assets so their set of portfolio choices is larger. For the domestic stockholders the problem is (and analogously for the foreign stockholders):
\[
\max_{k_d',k_d''} \log(Rk - k_d' - k_d'') + \beta E \log(R'k_d' + R''k_d''),
\]
with prices \(R'\) and \(R''\) taken as given.

In equilibrium, domestic stockholders allocate half of their savings in every country because the production technologies are completely symmetric, so
\[
k_d' = \frac{\beta}{2(1 + \beta)} Rk, \quad \text{and}
\]
\[
k_d'' = \frac{\beta}{2(1 + \beta)} Rk,
\]
and so do the foreign stockholders with
\[
k_f' = \frac{\beta}{2(1 + \beta)} R^* k^*, \quad \text{and}
\]
\[
k_f'' = \frac{\beta}{2(1 + \beta)} R^* k^*,
\]
but their total savings do not change. Therefore consumption of stockholders in the first period does not change relative to the autarky case. However consumption in the second period does. Since shocks are perfectly and negatively correlated, stockholders are able to fully insure. Thus they are clearly better off with the liberalization of capital markets.

Workers’ consumption in the first period is given by initial conditions, and those do not change either in autarky or in the capital liberalization case. Workers’ consumption in the second period is given by their country’s specific shock (as before) and by the capital invested in the country in which they live. Even though stockholders diversify their portfolio investing half of their savings in each country, what matters for workers is the evolution of total savings, specifically how much of the total savings is invested in their country. Since the problem is completely symmetric, looking at total savings is sufficient to know how much capital is invested in each country.

How total savings evolve is given by the intertemporal elasticity of substitution of stockholders. As mention before empirical evidence suggests that stockholders have an IES around
1, so that the logarithmic utility for stockholders that we analyze would be consistent with the data.

For the logarithmic-utility case, total savings do not change since stockholders save a proportion of their wealth that does not depend on the expected return. In this case neither will consumption of the workers in the second period, making them indifferent between capital autarky or capital mobility. The fact that workers’ welfare is unaffected by the liberalization of capital markets in this dynamic two-period model may be misleading if one stopped here, since the inclusion of extra periods will break this result. Here is the intuition: when including an extra period, the benefits from smoothing that stockholders enjoy will start to leak to workers. Even if they do not participate in the stock market, workers dislike volatility too (concave utility). Their labor income volatility depends on two factors, one is their country specific shock, and the other is the capital invested in their countries. The former does not change since they cannot save or migrate, but the latter can change through stockholders’ optimal saving decisions. In the three period model with capital mobility, the expected income received by stockholders does not depend on the states of the technology shocks in the second period (thanks to diversification), which means that neither will their future savings. Moreover, and this is specific to the logarithmic utility, since stockholders total savings today do not depend on tomorrow’s expected return, stockholders’ asset income tomorrow will increase (they save the same amount of capital, but the availability of the new asset increases the expected return). In the two-period model, only stockholders enjoy this increase in their asset income, consuming all the increase in the second and last period. With three periods, and given that they save a constant proportion of their income, workers will also benefit from the increase in stockholders’ asset income in the second period, since this means a higher level of capital in the third period. Workers’ wages are increasing in capital. Therefore workers’ consumption grows in the third period, making expected utility of workers higher with capital mobility than without it.

When the IES for stockholders is larger than 1, the liberalization of the capital account
induces stockholders to save more than before, so that workers’ consumption increases in the second period too. In this case, workers’ welfare increases both in the two-period and in the three-period model compared to the capital autarky environment.

When the IES for stockholders is smaller than 1, the liberalization of the capital account induces stockholders to save less, so workers’ consumption decreases in the second period, making workers’ welfare fall in the two-period model. The inclusion of a third period will compensate workers for the loss of consumption in the second period, at least partially, through the decrease in the volatility of the capital investment as explained above, and therefore the decrease in the volatility of workers wages.

**Proposition 1** When countries are totally symmetric and productivity shocks are perfectly and negatively correlated, an elasticity of intertemporal substitution for stockholders larger than or equal to one increases world savings under capital market liberalization, and makes all agents better off.

3 The infinite-horizon model

3.1 Incomplete markets for stockholders

As before, stockholders maximize their lifetime utility. In order to do so, they can accumulate physical capital. They do not work, so that they only receive capital income and leisure is not valued. In autarky, a stockholder can only invest in his own country’s capital. The autarky problem for the domestic stockholder is standard and can be written as follows:

\[
\text{Max}_{\{c_{cap,t}\}} \mathbb{E} \sum_{t} U(c_{cap,t})
\]

\[
s.t
\]

\[
c_{cap,t} + k_{t+1} = R_t k_t \quad \forall t.
\]
As in the previous subsection, each worker supplies one unit of labor inelastically and consumes his labor income every period.

Both prices, $R_t$ and $w_t$, equal the marginal productivities of their corresponding factors.

The only source of heterogeneity within a country is between those two types of agents. In the home country, there is a measure $\mu = 1$ of identical stockholders, and a measure $n$ of identical workers.

### 3.1.1 Recursive formulation for a stockholder in the open economy

To keep the model manageable, only two types of stockholders are allowed, the ones from the home country and the ones from the foreign country. Within each country, all stockholders have the same wealth. However, the wealth between countries does not have to be the same.

The problem of a stockholder with wealth $\omega_\kappa$, in a world with wealth $\omega_t$, and home and foreign shocks $(A, A^*)$ is:

$$V(A, A^*, \omega_\kappa, \omega_t) = \max_{c_\kappa} \log(c_\kappa) + EV(A', A'^*, \omega'_\kappa, \omega'_t)$$

s.t

$$c_\kappa + k'_\kappa + k'^*_\kappa = \omega_\kappa,$$

where

$$\omega'_\kappa = \alpha A^T \bar{k}^{1-\alpha} n^{1-\alpha} + \alpha A'^* \bar{k}^{1-\alpha} n'^{1-\alpha} k'^*_\kappa,$$

and

$$\omega'_t = \alpha A^T \bar{k}^{1-\alpha} n^{1-\alpha} + \alpha A'^* \bar{k}^{1-\alpha} n'^{1-\alpha},$$

given the laws of motion for aggregates

$$\bar{k}' = f(A, A^*, \omega_t),$$

and

$$\bar{k}'^* = f^*(A, A^*, \omega_t).$$

In words,
\( \omega_t^\kappa \) : wealth of stockholder \( \kappa \) tomorrow.

\( \omega_t^T \) : total or aggregate tomorrow’s wealth, this is, wealth of the domestic and foreign stockholders tomorrow.

\( k_\kappa \) : today’s stock of capital invested in the home country that belongs to stockholder \( \kappa \).

\( k^*_\kappa \) : today’s stock of capital invested in the foreign country that belongs to stockholder \( \kappa \).

\( \bar{k} \) and \( \bar{n} \) are aggregate capital and labor in the home country.

\( \bar{k}^* \) and \( \bar{n}^* \) are aggregate capital and labor in the foreign country.

### 3.1.2 Closed-form solutions

In this setup, every-period stockholders’ total saving is a percentage \( \beta \) of their wealth. The way they allocate their savings among the two technologies matters not only for them but also for the workers in both economies since the labor income they receive depends on the aggregate capital invested only in the country in which they live and work.

Let’s call \( \lambda_{ij} \) the proportion of wealth invested in capital in the home country when shocks today are \((A_i, A_j^*)\); then \((\beta - \lambda_{ij})\) is the proportion of wealth invested in capital in the foreign country for the same case. The optimal values for \( \lambda_{ij} \) depend on the primitives of the economy such as the values of the technological shocks as well as the transition probabilities of those processes, the factor shares between capital and labor, the discount factor of the agents and the labor supplies in each country.

The policy functions for a stockholder of wealth \( \omega \) are:

\[
\begin{align*}
k' &= g_{ij}(\omega) = \lambda_{ij}\omega \\
k^* &= g^*_{ij}(\omega) = (\beta - \lambda_{ij})\omega.
\end{align*}
\]

Therefore tomorrow aggregates in each country are \( \bar{k}' = \lambda_{ij}\omega_t \) and \( \bar{k}^* = (\beta - \lambda_{ij})\omega_t \) respectively.
The value function for a stockholder with wealth $\omega$, in a world with a total wealth of $\omega_t$ when the shocks today are $(A_i, A_j^*)$ can be written as:

$$V(A_i, A_j^*, \omega, \omega_t) = a_{ij} + b_{ij} \log(\omega) + c_{ij} \log(\omega_t),$$

where $a_{ij}, b_{ij}$ and $c_{ij}$ are constants that depend on the parameters of the two economies.

The value function for a worker living in country $d$ with country wealth being $\omega_d$, in a world with a total wealth of $\omega_t$ when the shocks today are $(A_i, A_j^*)$ is:

$$V(A_i, A_j^*, \omega_d, \omega_t) = \tilde{a}_{ij} + \tilde{b}_{ij} \log(\omega_d) + \tilde{c}_{ij} \log(\omega_t),$$

where $\tilde{a}_{ij}, \tilde{b}_{ij}$ and $\tilde{c}_{ij}$ are constants that depend on the parameters of the two economies.

Notice that for a worker the relevant variables are the worldwide wealth and the wealth of the country in which he or she lives. The workers do not care who is the owner of the capital invested in their country, but only how much capital is invested.

4 The i.i.d. case

Before doing any serious calibration, it is worth exploring the simple case in which shocks are identically and independently distributed (i.i.d.) across time and across countries, where persistence is not an issue and the equations of the problem are greatly simplified.

Because the two country technological processes are i.i.d., it is proved in the appendix that the proportion of wealth that stockholders invest in each stock does not depend on the current world shocks $(A_i, A_j^*)$. Therefore $\lambda_{ij} = \lambda$ and $(\beta - \lambda_{ij}) = \lambda^* \forall ij$.

To heighten understanding the model, I analyze separately for the four types of agents in the global economy the effects on quantities and prices of three possible sources of heterogeneity between two countries that allow for international asset trade.
In this section, a steady state analysis is described, that is, a description of the changes in prices and quantities in the ergodic set moving from a world composed of two closed economies to an integrated (costless asset trade for stockholders) economy is provided. The results shown are computed for the case where the intertemporal substitution for the workers is 1. However, the conclusions are the same when the intertemporal substitution of workers is 0.1\textsuperscript{3}. Notice that in order to measure the steady state welfare of the agents in the closed economies, initial conditions of wealth do not matter. However, the same is not true for the asset-integrated world economy; actually the changes in stockholders’ welfare in the world with capital mobility depend greatly on the relative initial wealth between the two stockholders.

4.1 Comparative statics in the steady states

4.1.1 Differences in volatility

Imagine that Home and Foreign countries are about to open their stock markets between their national stockholders, and assume that the volatility of the productivity shocks differs between the Home and Foreign countries. How is redistribution among the agents in the world economy affected by this volatility? When should we expect larger movements in prices or capital flows? Those are the questions this section answers.

As earlier emphasized, even to compute welfare in the steady state, initial conditions matters. What matters exactly is the starting relative wealth between the representative stockholders of the two countries. Start by considering the most symmetric case where this relative wealth is 1.

All the following graphs should be read vertically. Points in a vertical line reflect the different levels of the variable indicated in the closed economy steady state and in the free capital mobility steady state for the same set of parameters.

\textsuperscript{3}Details available upon request.
The standard deviation of shocks in Home country is kept equal to 10% during the experiment, while the standard deviation of the foreign shocks varies as indicated by the graphs.

The first noticeable pattern that can be observed in Figure 2 is that workers in both countries are better off with the international opening of the stock markets. Even though the volatility of their labor income may increase or decrease compared to the closed economy level (Figure 4), global wealth always increases in the open world. Moreover the capital invested in each country also rises, slightly increasing wages (or workers’ consumption) in both countries, although more pronouncedly in the domestic (relatively low-risk) country, and making all workers better off. For domestic workers, only the capital income obtained for the use of the domestic technology is important, and when the economy is in asset autarky, aggregate capital income coincides with aggregate income of the domestic stockholders. However, when international portfolio diversification is allowed, this two aggregates do not need to coincide since then, stockholders can invest in another country as well; this is why the movements in consumption for stockholders and workers of the same country do not need to go in the same direction when the economy is open.

To understand Figure 1 remember that home and foreign stockholders’ welfare are different in autarky because the volatility of their countries-productive shocks are different (always 10% for the home country, and a range of levels for the foreign country), but once that capital markets open internationally and given that they start with the same wealth, their level of welfare in the new open steady state will be the same for both. For the stockholders, there is an area in Figure 1, where volatilities of the two countries are relatively close, in which both are better off in the open economy. However, when the volatilities of technology shocks are very different between the two countries, there is redistribution from stockholders of the low volatility country to those in the large volatility country. The former are worse off and the latter are better off in the open economy. The intuition for this result is simple; when the two assets are relatively similar, the gains from diversifying for both
stockholders compensate for the effects that opening the market for capital have on prices; however when the volatility of the two assets is quite different, the effect on prices (return to capital) for the stockholder of the relatively low volatility country is such that his or her wealth decreases and so does consumption in the ergodic set.

Another prediction of the model is that the larger the global volatility of the technology processes, the larger the fall in the cost of capital will be in both countries in the aftermath of the capital liberalization (graph not shown).

In this symmetric case, even though for some of the parameterizations of the model some stockholders may lose welfare in the open economy ergodic set, there are global welfare gains from opening the stock markets internationally.

To study the effects of the initial conditions in the steady state welfare, the model has been simulated for the same value of the parameters as above, but with a starting relative wealth between the home and the foreign stockholder equal to two. That is, the initial wealth of the home stockholder is twice that of the foreign stockholder. This could be thought as if the home country is ahead in its transition path towards the steady state when the stock market opens internationally.

Figure 1: Welfare of stockholders
Figure 2: Welfare of workers

Figure 3: Volatility of stockholders’ consumption
In this case, the predictions of prices and workers’ welfare of both countries are similar to the ones obtained in the symmetric case above. However, the relative initial wealth proves powerful in determining the stockholders’ welfare changes from opening the capital markets. For the case studied, the wealthy stockholder always gains from having access to the foreign technology no matter how volatile it is. Meanwhile the opposite happens for the poor stockholder.

In this non symmetric case, the loss in the poor stockholders’ welfare in the open economy is large enough that there are global welfare losses from opening the stock markets internationally. This is due to the concavity of the utility functions.

### 4.1.2 Differences in factor abundance

In the previous experiment, for all the cases analyzed, the measure of stockholders in both countries was the same (it was 1 in each country), as was the measure of workers (four in each country). This section examines what happens when the measure of foreign workers changes holding everything else constant. Notice that this changes both the population size as well as the participation rate in the stock market of the foreign country.
As before, initial conditions matter, so again the most symmetric case in which the starting relative wealth is 1 is considered.

For the range of parameters analyzed, Figure 6 illustrates that workers in both countries are better off with the international opening of the stock markets irrespective of the changes in the volatility of their labor income relative to their closed economy level (Figures 7 and 8). Global wealth remains unchanged or slightly increases in the open world, and so does the wealth invested in every country, increasing wages in both countries to a small extent and making all workers better off. Notice however that if the difference between the measure of workers between the two countries was large enough, some workers (the ones in the more populated country) could be worse off with the liberalization.

For the stockholders, and taking into account that now the risky technological shocks are identical in both countries, there does not exist an area as before, where both stockholders are better off in the open economy (Figure 5). Here there is always redistribution from stockholders of the labor-abundant country to those in the labor-scarce country such that the former are worse off and the latter are better off in the open economy.

The intuition for this result is simple; since return of capital is increasing on labor, capital liberalization makes stockholders of the labor-abundant country share their abundant resources with the new comers without the same advantages in return.

The cost of capital falls in both countries, due to the increase in the world’s wealth, as well as the amount of capital invested in each country. Besides, the cost of capital falls more in the smaller country.

In this symmetric case, and spite of the fact that some stockholders may lose welfare in the open economy while others win, there are global welfare gains from opening the stock markets internationally.

As before, initial conditions are very important at determining the stockholders’ welfare changes from opening the capital markets, but they are not for workers in either country. The graphs above show the steady state comparisons for the case in which both stockholders
Figure 5: Welfare of stockholders

Figure 6: Welfare of workers
Figure 7: Volatility of stockholders' consumption

Figure 8: Volatility of workers' consumption
start with the same wealth.

4.1.3 Differences in productivity

This section studies what happens when a country opens trade in assets with another country with a different level of productivity. That is, here the problem is solved for different levels of foreign productivity (assuming that foreign volatility is zero), and leaving the rest of the parameters of the model unchanged.

As in the previous two cases, initial conditions matter, so again consider the case where the starting relative wealth between the two stockholders is 1.

Figure 10 shows that workers in both countries are basically unaffected by the international opening of the stock markets irrespective of the changes in the volatility of their labor income compared to their closed economy level (Figures 11 and 12). Global wealth remains practically unchanged in the open world, and so does the wealth invested in every country.

For the stockholders, and taking into account that the standard deviation of the home process is 10% and that of the foreign process is always zero, there does not exist an area
Figure 10: Welfare of workers

where both stockholders are better off in the open economy (Figure 9). Whenever the foreign productivity is larger than the average home productivity and asset markets open internationally, there is redistribution from foreign stockholders to home stockholders. The intuition is that when opening the market for capital, the stockholders in the country with higher productivity have to share the benefits of the previously exclusive technology with the rest of the world’s stockholders. This reduces their wealth and therefore their consumption; and the opposite occurs for the stockholders in the relatively low productive country.

As in the other cases studied, the cost of capital falls in both countries, decreasing relatively more in the less productive country.

In this symmetric case, and in spite of the fact that some stockholders may lose welfare in the open economy while others win, there are global welfare gains (although very small) from opening the stock markets internationally.

Summarizing what can be learned about long-run outcomes from studying the different sources of heterogeneity, one striking result is clear. Workers tend to benefit from liberalizing capital markets, at least in the long run, and this occurs even though they do not participate
Figure 11: Volatility of stockholders’ consumption

Figure 12: Volatility of workers’ consumption
directly in capital markets. This result is robust to different starting conditions between the national stockholders of the two countries in the model.

In the simulations done so far, spillovers between the technologies of the two countries have not been considered, and the inclusion of them might change the results discussed above. Also, this is a model of rational agents, so I am excluding contagion episodes, usually related with irrational behavior, which some may argue are an important source of changes in the volatility of capital flows and therefore of welfare changes. In defense of the model though, we have seen that opening capital accounts does bring changes in the volatility workers in different countries face, but those changes come from rational decisions and do not contradict the fact the workers are still better off with them. Thus the general idea proclaimed by the anti-globalization movement that workers must lose with capital account liberalizations does not hold up, at least under a long-run perspective.

In all the results shown above, a logarithmic utility function has been assumed for both workers and stockholders, and although this is consistent with the empirical findings that say the IES for stockholders is around 1, the IES for workers is much lower than 1, around 0.1. (For a deeper discussion about the different IES between stockholders and workers see Guvenen (2003)). Even though workers do not save in this model, a low EIS means that workers would like to smooth consumption even more than stockholders, so the increase in wealth caused by the capital liberalization will be even more appreciated, making the steady state results robust to this change in workers’ utility.

4.2 Comparative statics including the transition path

Looking only at steady state welfare comparisons is useful because it allows us to think in the long run. However it excludes the adjustments and mechanisms that led us there in the first place. Moreover, the welfare variations during the transition may be such that they can reverse the sign of the welfare changes when only considering the steady states.
Throughout this paper, only unanticipated capital account liberalizations have been con-
sidered. If agents were able to anticipate capital liberalizations, they might change their
saving behavior before the actual liberalization takes place, so some effects would be missing
in the present analysis.

For all the cases studied above, it is useful to think in the following terms: in the long
run world wealth rises even when the production functions of the two countries differ and
world wealth is not allocated equally between them. The amount of capital invested in every
country is larger than it would have been, had the capital markets remained closed. In the
short run however, differences in the production functions between the two countries matter
more, as in the two-period model without uncertainty described in section 2.1. If the return
to capital differs between countries in autarky, capital will flow from one country to the
other to bring together the expected returns to capital invested in each country. Thus the
workers living in the country whose return was low in autarky will suffer since capital will
flow out in the short run and their wage and consumption will fall. If there are costs to
adjusting capital, the flow of capital from the low-return country to the high-return country
will be gradual, and so will the adjustmnet in workers’wages and consumption. But losses
to workers’ welfare could still be significant. For sufficient differences between the returns
to capital in the two countries in autarky, the decrease in the short-run consumption for
the workers in the low-return country will be such that the short-run loss in welfare could
outweigh the long-run benefits from opening stock markets.

As shown below, even small differences in the technological processes between the two
countries may cause sufficient losses during the transition path for the workers in the low-
return country such that the total change in welfare for them of liberalizing stock markets
could be negative. This result has two clear implications. If we think of the autarkic low-
return country as a rich or developed country, and the autarkic high-return country as a
developing country, capital liberalization could lead to an increase in income inequality in
the rich country and a decrease in income inequality in the developing country. In the long-
run, both economies grow so welfare of workers in both countries improves. However, the changes in the distribution of income from capital liberalization could persist if participation rates in the stock markets do not change over time.

Solving the model for a lower, more realistic intertemporal elasticity of substitution for workers will make the transition even more painful for the workers in the initially low-return country.

5 Calibration

5.1 Productivity parameters

To calibrate the process for productivity, annual measures of per worker GDP ($y_t^i$) and capital stock ($k_t^i$) in constant international prices are available from Heston, Summers and Aten (2002) database. The Solow residuals for country $i = Korea, US$ come from

$$\log(A_i^t) = \log(y_i^t) - \alpha_i \log(k_i^t)$$

Data on capital output shares come from Barro and Sala-i-Martin (1995, Table 10.8 Panel B).

After detrending the productivity time series using the Hodrick-Prescott filter, the following independent AR(1) specifications are reached:

$$\log(A_{t}^{Korea}) = 0.573 \log(A_{t-1}^{Korea}) + 0.016 \varepsilon_{t}^{Korea},$$

and

$$\log(A_{t}^{US}) = 0.442 \log(A_{t-1}^{US}) + 0.0099 \varepsilon_{t}^{US}.$$
those AR(1) processes can be approximated by a two-state Markov chain in the following way. If the transition matrix is

\[
P = \begin{pmatrix}
    p_{hh} & 1 - p_{hh} \\
    1 - p_{ll} & p_{ll}
\end{pmatrix}
\]

then the stationary probabilities are given by

\[
\pi_h = \frac{p_{lh}}{p_{hl} + p_{lh}}, \pi_l = 1 - \pi_h.
\]

Using this information \((\log A_h, \log A_l, p_{hh}, p_{ll})\) are picked to match the following moments:

\[
\sum_{i=h,l} \pi_i \log A_i = 0
\]

\[
\sum_{i=h,l} \pi_i (\log A_i)^2 = \frac{\sigma^2_\varepsilon}{1 - \rho^2}
\]

\[
\rho_{\log A_t, \log A_{t-1}} = \rho
\]

\[
\pi_h = \pi.
\]

Given a value for \(\pi\), this is a system of 4 equations in 4 unknowns that can be uniquely solved to obtain the values of \((\log A_h, \log A_l, p_{hh}, p_{ll})\) for each country. For \(\pi = 0.737\), \(^4\)

Korea:

\[
\log A_h = 0.0117, \log A_l = -0.0327, p_{hh} = 0.8877, p_{ll} = 0.685
\]

US:

\(^4\) Following Hamilton (89)
\[ \log A_h = 0.0134, \log A_l = -0.0376, p_{hh} = 0.9513, p_{ll} = 0.864 \]

Since full depreciation is assumed, the length of one period in the model is ten years. Consequently all the parameters shown above are transformed into their corresponding ten-year analogous ones.

The annual rate of preference \( \beta \) is 0.94 for all agents in the model.

The relative average productivity between the two countries is obtained as the ratio between \( \hat{A}_{US} \) and \( \hat{A}_{Korea} \), where

\[
\hat{A}_{Korea} = \sum_t \frac{TPF_{Korea}}{T}
\]
\[
\hat{A}_{US} = \sum_t \frac{TPF_{US}}{T}
\]

and \( TPF_t = \log y_t - \alpha \log k_t \) is total factor productivity.

For a sample from 1965 until 1990, \( \frac{\hat{A}_{Korea}}{\hat{A}_{US}} = 0.972 \).

The participation rate in the stock market in US is assumed to be 20\%, roughly consistent with data from the Panel Study of Income Dynamics. Although participation has increased, particularly in the 90s, the Korean stock market was liberalized in 1987, and the stock wealth is still very concentrated. Similar data for the Korean economy is not available, so the model is calibrated for values of 10\% (Case 1) and 20\% (Case 2); 10\% is a more likely value since the US stock market is the most developed in the world, and those of other countries will follow with a considerable lag.

The measure of stockholders in US is normalized to 1. Therefore, using population data for both countries, we can find the measure of the rest of the agents in the model for the two cases studied here.

Case 1:

\[ \mu_{Stock}^{US} = 1, \mu_{Wor}^{US} = 4, \mu_{Stock}^{Korea} = 0.085, \text{ and } \mu_{Wor}^{Korea} = 0.77. \]

Case 2:

\[ \mu_{Stock}^{US} = 1, \mu_{Wor}^{US} = 4, \mu_{Stock}^{Korea} = 0.17, \text{ and } \mu_{Wor}^{Korea} = 0.69. \]
Participation rates, together with data on stock of capital per worker in the data pin down the relative levels of stock of capital per worker in the model for the year of the liberalization of the Korean capital market. If the participation rate in the Korean stock market is assumed to be 10%, then

\[ \frac{k_{US}^{1987}}{k_{Korea}^{1987}} = 2.7. \]

If instead it is 20%,

\[ \frac{k_{US}^{1987}}{k_{Korea}^{1987}} = 2.3. \]

### 5.2 Numerical results

Steady state results for Case 1 are reported in the first line of Table 1. The first thing to notice is that redistribution occurs mainly among stockholders.

**Table 1. Effects of liberalizing the capital account on expected lifetime consumption (measured as a percentage change in annual consumption)**

<table>
<thead>
<tr>
<th>% of stockholders in Korea</th>
<th>Initial</th>
<th>Stockholders US</th>
<th>Stockholders Korean</th>
<th>Workers US</th>
<th>Workers Korean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady state only</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>10%</td>
<td>2.7</td>
<td>2.36%</td>
<td>-7.42%</td>
<td>-0.00%</td>
</tr>
<tr>
<td>Case 1*</td>
<td>10%</td>
<td>1</td>
<td>1.36%</td>
<td>-2.15%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Case 2</td>
<td>20%</td>
<td>2.3</td>
<td>1.27%</td>
<td>-4.55%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Plus transition costs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>10%</td>
<td>2.7</td>
<td>0.89%</td>
<td>-2.46%</td>
<td>-0.25%</td>
</tr>
</tbody>
</table>

For all cases the % of stockholders in US is 20%

The changes in stockholders’ welfare are of a much larger magnitude than the corresponding changes experienced by workers. This was expected since national markets are segmented.
and only stockholders participate actively in the capital liberalization process. To understand why US stockholders will be better off in the long run, while Korean stockholders will not, we have to look at the steady state welfare that each representative stockholder would enjoy under portfolio autarky, together with their relative initial wealth. It turns out that even though average productivity in US is slightly higher than in Korea, the smaller volatility of the Korean productivity shocks together with the smaller participation rates in the stock market (or relatively smaller stockholder-worker ratio) would make Korean stockholders better off than US stockholders under the closed-economy case, this is,

\[
\text{LifetimeUtility}^{\text{Closed ss}}_{\text{Stock Korea}} > \text{LifetimeUtility}^{\text{Closed ss}}_{\text{Stock US}}.
\]

Therefore, when capital is freed up to move between countries, US stockholders will have the chance to participate in the advantageous foreign technology, so they will be better off than otherwise. On the contrary and in spite of the diversification benefits, Korean stockholders will be worse off since the new technology they have access to, is relatively worse than the one they have to offer.

The exact changes in stockholders’ welfare are quite sensitive to their relative wealth at the moment of the capital account liberalization since this ratio will also dictate their relative wealth at the new steady state. In the second line of the table, the representative stockholders of both countries are assumed to have the same wealth at the date of the liberalization (this might be the case if free movement of capital is also accompanied by more availability of foreign borrowing in the Korean markets), so the redistribution among stockholders is less pronounced than before, though still important.

The other interesting feature shown in the first two rows of the table refers to the differences between the changes in welfare experienced by workers of every country. In other words, why do workers in the country where labor is scarce in absolute terms (Korea) improve more from the capital liberalization than do workers in the other country? As explained before, setting the intertemporal substitution of stockholders at 1, together with the concavity
of the production functions, increase world wealth in the long run. But how much wealth is invested in each country is particularly important in determining the welfare of workers. Although differences in the relative levels of initial capital help determine the changes in stockholders’ welfare in steady state, what matters most for workers’ changes in the long-run welfare are the differences between the technology and the size of the two countries. By looking at the calibrated parameters of the two economies, we observe that the features of the two production functions, this is, their average productivity and the volatilities of their technology shocks are not very different, their participation rate in the stock market could be (at least for the case where the participation rate in Korea is 10%), but the size of the two countries is very different. If two countries with identical technologies but different labor supply were left in portfolio autarky, the representative worker in every country would attain the same lifetime welfare in steady state since stockholders of each country would accumulate enough capital such that the capital-labor ratio in both countries would be the same, although stockholders of the country where participation in the stock market is more concentrated (Korea in Case 1) will have accumulated more capital so they will consume more and bear a larger volatility (just because their individual capital will be larger). However, when capital is allowed to move freely between the two countries, the differences in risks due to the different productive structure in each country, together with the proportion of the world production provided by each country will support different returns to capital in equilibrium. In general capital will increase in the new steady state in both countries, although it will increase relatively less or it may slightly decrease (as in the calibration) in the country that produces the larger proportion of the world production.

Results for Case 2 are shown in the third row of Table 1. The line of the arguments provided before still applies. It is worth noticing that the gains for US stockholders are smaller than in Case 1 though still very significant. This occur because Korean technology provides a good investment opportunity for US stockholders, but not as good as in Case 1 (now the amount of labor available in Korea is 0.69 compare to the previous 0.77).
When transition paths are included in the computation of welfare changes (see last row of Table 1), two main trends can be noticed: First of all, redistribution between stockholders is not as strong as before, and second, redistribution between workers grows.

US stockholders can take full advantage of the Korean technology only when the economy is already in the new steady state. This is also the time when Korean stockholders lose the maximum because of the sharing of their technology. Therefore when transition is taken into account, the gains of US stockholders fall compared to the steady state computation, while the loses of Korean stockholders partially recover.

The flood of capital invested in the Korean economy coming from US stockholders accelerate the accumulation of capital in Korea favoring their workers relatively more in the short run. By the same token, US workers lose relatively more, and also in absolute terms, when including the transition tours the new steady state. During the transition, either in the closed or open capital account case, the volatility of wages are higher than their corresponding steady state values. Moreover, volatilities in the closed-economy transition differ from those in the freed-capital one (as also do in the different steady states), but even in the transition, changes in the average dominate changes in the volatility.

6 Conclusions and extensions

This paper has examined the redistribution that takes place not only among countries but also among different groups within countries from opening the capital markets internationally. In order to provide some insights into the benefits of liberalization, and in contrast to earlier studies, we have emphasized as a key feature the presence of heterogeneity among nationals in their access to capital markets.

The main conclusion of the paper is that in the long run, for most of the parametrizations workers are better off, while if one takes into account the transition (or the short-run effects) following liberalization, workers of some of the countries involved may be worse off. Moreover,
this paper shows that, at least from a long-run perspective, there does not need to be a conflict in interest between stockholders and workers.

Liberalizing a country’s capital markets per se has redistribution effects between agents within that country, and also between countries. Furthermore, the free mobility of capital is likely to affect tax revenues. The model presented here is rich enough to provide some insights about the effects on redistribution under capital account liberalizations; at the same time, it is sufficiently simple to allow us to include taxation in the problem. Recent work by Mendoza and Tesar (2003), Quadrini (2003), and Klein, Quadrini, and Rios-Rull (2003) studies the changes in taxation over time associated with changes in the degree of capital mobility among countries. The aim of those studies is to assess the welfare consequences of tax competition under free capital mobility. A version of the present model with taxes would enrich the representative-agent setups employed in the above-mentioned papers by allowing us to investigate the interplay between inequality between agents in a country and tax competition. Tax competition may have very different effects on different groups, and it is also possible that the presence of incomplete (participation in) national financial markets can influence the aggregate welfare comparisons between systems with tax competition and systems with tax coordination.

7 Appendix

7.1 Intercepts of the stockholder’s value function in a closed economy

c_h \ c_l solve the system of two equations:

\[ c_i = \log(1 - \beta) + \beta(b \log(\alpha \beta^{\alpha-1} n^{1-\alpha}) + d \log(\alpha \beta^{\alpha} n^{1-\alpha}))) + \beta \sum_j \pi_{ij}(c_j + (b + d) \log A_j), \]

35
for $i = h, l$ and $j = h, l$.

If shocks are i.i.d. $c_h = c_l = c$ where

$$
c = \frac{\log(1 - \beta) + \beta(b \log(\alpha \beta^{\alpha - 1} n^{1 - \alpha}) + d \log(\alpha \beta^{\alpha} n^{1 - \alpha})) + \frac{\beta}{2}(b + d)(\log A_h + \log A_l)}{1 - \beta}.
$$

### 7.2 Derivation of the optimal policy rules for the stockholder under incomplete markets

The problem of stockholder with wealth $\omega$ when world wealth is $\omega_{to}$ and domestic and foreign current shocks are $(A_i, A_j^*)$ is:

$$
V(A_i, A_j^*, \omega, \omega_{to}) = \max_{k', k^{*'}} U(\omega - k' - k^{*'}) + \beta EV(A_i', A_j^{*'}, \omega', \omega'_{to})
$$

s.t.

$$
\begin{align*}
\omega &= R_i k + R_j k^* \\
\omega_t &= R_i k' + R_j k^{*'}.
\end{align*}
$$

The FOCs are:

$$
-U_c + \beta V_k(A_i', A_j^{*'}, \omega', \omega'_{to}) = 0 \\
-U_c + \beta V_{k^*}(A_i', A_j^{*'}, \omega', \omega'_{to}) = 0.
$$

Using the Envelope condition we obtain:

$$
\begin{align*}
V_k(A_i, A_j^*, \omega, \omega_{to}) &= R_i U_{cij} \\
V_{k^*}(A_i, A_j^*, \omega, \omega_{to}) &= R_i^* U_{cij}.
\end{align*}
$$
For the logarithmic utility and the Cobb-Douglas production function with full depreciation, FOCs read as follow:

\[
\frac{1}{c_{ij}} = \beta \left[ \pi_{ii} R_i^t \left( \frac{\pi_{ji}^*}{c_{ii}} + \frac{\pi_{ij}^*}{c_{ij}} \right) + \pi_{ij} R_j^t \left( \frac{\pi_{ji}^*}{c_{ji}} + \frac{\pi_{ij}^*}{c_{jj}} \right) \right]
\]

\[
\frac{1}{c_{ij}} = \beta \left[ \pi_{ji}^* R_i^t \left( \frac{\pi_{ii}^*}{c_{ii}} + \frac{\pi_{ij}^*}{c_{ji}} \right) + \pi_{jj}^* R_j^t \left( \frac{\pi_{ji}^*}{c_{ij}} + \frac{\pi_{ij}^*}{c_{jj}} \right) \right];
\]

since prices are competitive

\[
R_i = A_i \alpha k^{1-\alpha} \text{ and } R_j^* = A_j^* \alpha k^{1-\alpha},
\]

guessing the next functional form for the policy rules:

\[
k' = \lambda_{ij} \omega
\]

\[
k^{*'} = \lambda_{ij}^* \omega,
\]

and plugging those in the FOCs above, we get

\[
\frac{1}{\omega(1 - \lambda_{ij} - \lambda_{ij}^*)} = \beta \sum_z \pi_{iz} A_z \alpha (\lambda_{ij} \omega_{t0})^{\alpha-1} l^{1-\alpha} \left( \sum_y \omega_{zy}(1 - \lambda_{zy} - \lambda_{zy}^*) \right) \text{ and}
\]

\[
\frac{1}{\omega(1 - \lambda_{ij} - \lambda_{ij}^*)} = \beta \sum_y \pi_{jy}^* A_y^* \alpha (\lambda_{ij}^* \omega_{t0})^{\alpha-1} l^{1-\alpha} \left( \sum_z \omega_{zy}^*(1 - \lambda_{zy} - \lambda_{zy}^*) \right),
\]

but again

\[
\omega_{zy}' = R_z^t (\lambda_{ij} \omega) + R_y^t (\lambda_{ij}^* \omega)
\]

\[
= A_z \alpha (\lambda_{ij} \omega_{t0})^{\alpha-1} l^{1-\alpha} (\lambda_{ij} \omega) + A_y \alpha (\lambda_{ij}^* \omega_{t0})^{\alpha-1} l^{1-\alpha} (\lambda_{ij}^* \omega),
\]

so the FOCs simplify to:
\[ \frac{1}{(1 - \lambda_{ij} - \lambda_{ij}^*)} = \beta \sum_z \pi_{iz} A_z \lambda_{ij}^{\alpha-1} l^{1-\alpha} \left( \sum_y \left( A_z \lambda_{ij}^{\alpha} l^{1-\alpha} + A_z^* (\beta - \lambda_{ij})^\alpha l^{1-\alpha} (1 - \lambda_{zy} - \lambda_{zy}^*) \right) \right) \]

\[ \frac{1}{1 - \lambda_{ij} - \lambda_{ij}^*} = \beta \sum_y \pi_{jy}^* A_y^{\lambda_{ij}^*} l^{1-\alpha} \left( \sum_z \left( A_z \lambda_{ij}^{\alpha} l^{1-\alpha} + A_z^* (\beta - \lambda_{ij})^\alpha l^{1-\alpha} (1 - \lambda_{zy} - \lambda_{zy}^*) \right) \right). \]

(1)

(2)

those were the FOC’s for the stockholder when the two-country shocks were \((A_i, A_j^*)\), so we would have another six equations for pairs \((A_i, A_j^*), (A_j, A_i^*), (A_j, A_j^*)\), so in total we are left with eight unknowns and eight equations that do not depend on wealth.

But we can go even further and prove that \(\lambda_{ij} + \lambda_{ij}^* = \lambda_{ii} + \lambda_{ii}^* = \lambda_{jj} + \lambda_{jj}^* = \beta\). In order to prove this claim, we just have to show that if \(\lambda_{ij} = \lambda\) and \(\lambda_{ij}^* = \beta - \lambda\) solve Equation (2), then Equation (3) also holds. So assume that the next equality is true:

\[ 1 = \beta \sum_z \pi_{iz} A_z \lambda^{\alpha-1} l^{1-\alpha} \left( \sum_y \left( A_z \lambda^{\alpha} l^{1-\alpha} + A_z^* (\beta - \lambda)^\alpha l^{1-\alpha} \right) \right), \]

rename the terms as follows so it will be easy to see that the two mentioned equations are not independent:

\[ z_1 = A_i \lambda^{\alpha} l^{1-\alpha} \]
\[ z_2 = A_j \lambda^{\alpha} l^{1-\alpha} \]
\[ z_3 = A_i^* (\beta - \lambda)^\alpha l^{1-\alpha} \]
\[ z_4 = A_j^* (\beta - \lambda)^\alpha l^{1-\alpha}, \]

and
\[ a = \pi_{ii}\pi^*_ji \]
\[ b = \pi_{ii}\pi^*_jj \]
\[ c = \pi_{ij}\pi^*_ji \]
\[ d = \pi_{ij}\pi^*_jj, \]

so now Equation (2) and Equation (3) read:

\[
\frac{\lambda}{\beta} = \left[ z_1\left( \frac{a}{z_1 + z_3} + \frac{b}{z_1 + z_4} \right) + z_2\left( \frac{c}{z_2 + z_3} + \frac{d}{z_2 + z_4} \right) \right]
\]
\[
\frac{\beta - \lambda}{\beta} = \left[ z_3\left( \frac{a}{z_1 + z_3} + \frac{c}{z_2 + z_3} \right) + z_4\left( \frac{b}{z_1 + z_4} + \frac{d}{z_2 + z_4} \right) \right],
\]

and plugging Equation (2) in Equation (3) and using the fact that \(a + b + c + d = 1\), Equation (2) is also satisfied so the claim is true.

7.3 The i.i.d. case

As we just saw, when the country-specific shocks follow general transition probability matrices, we can say that, given the current realization of the shocks \((A_i, A^*_j)\), if the optimal propensity to invest in the domestic asset is \(\lambda_{ij}\), then the optimal propensity to invest in the foreign asset is \((\beta - \lambda_{ij})\). Apart from that, we cannot obtain a closed-form solution for the relationship among the optimal propensities to invest in the domestic asset for the different states, this is, \(\lambda_{ij}, \lambda_{ii}, \lambda_{ji}, \) and \(\lambda_{jj}\).

When both, home and foreign country shocks are i.i.d., current shocks do not give us any information about the future, apart from the one already included in today’s wealth, and therefore the propensity to invest in an asset from current wealth should be equal for any realizations of the shocks. In this case, the four equations that determine \(\lambda_{ij}, \lambda_{ii}, \lambda_{ji}, \) and \(\lambda_{jj}\) collapse into only one that solve for \(\lambda\).
\[
\lambda = \beta \sum_{zy} \pi_{iz} \pi_{jy}^* \left[ A_z \lambda^\alpha l^{1-\alpha} \frac{1}{A_z \lambda^\alpha l^{1-\alpha} + A_y^*(\beta - \lambda) l^{1-\alpha}} \right],
\]

or more intuitively

\[
\lambda = \beta E\left( \frac{y}{y + y^*} \right),
\]

where \( y \) is production in the domestic country and \( y^* \) is production in the foreign country.

### 7.4 Derivation of the coefficients of the value functions under incomplete markets

Proposed guess for the value function of a stockholder with wealth \( \omega \) when world wealth is \( \omega_{to} \) and domestic and foreign current shocks are \((A_i, A_j^*)\):

\[
V(A_i, A_j^*, \omega, \omega_{to}) = a_{ij} + b_{ij} \log(\omega) + c_{ij} \log(\omega_{to});
\]

substituting the optimal rule:

\[
V(A_i, A_j^*, \omega, \omega_{to}) = \log[(1 - \beta)\omega] + \beta \sum_{zy} \pi_{iz} \pi_{jy}^* (a_{zy} + b_{zy} \log(\omega_{zy}) + c_{ij} \log(\omega'_{zyto})).
\]

Remember that

\[
\omega'_{zy} = R_{z}^'(\lambda_{ij}\omega) + R_y^{*'}(\lambda_{ij}^\alpha \omega)
\]

\[
= \alpha [A_z(\lambda_{ij}\omega_{to})^{\alpha-1}l^{1-\alpha}(\lambda_{ij}\omega) + A_y((\beta - \lambda_{ij})\omega_{to})^{\alpha-1}l^{1-\alpha}((\beta - \lambda_{ij})\omega)],
\]

\[
\omega'_{zyto} = R_{z}^'(\lambda_{ij}\omega_{to}) + R_y^{*'}(\lambda_{ij}^\alpha \omega_{to})
\]

\[
= \alpha [A_z(\lambda_{ij}\omega_{to})^{\alpha-1}l^{1-\alpha} + A_y((\beta - \lambda_{ij})\omega_{to})^{\alpha}l^{1-\alpha}],
\]

and so on for \( zy = hh, hl, lh, \) and \( ll \). Substituting those in the above value function and renaming some terms as follows:
\[ h_{zy} = \alpha (A_z \lambda_{ij}^{\alpha} l^{1-\alpha}) + A_y^\alpha (\beta - \lambda_{ij})^{\alpha} l^{(1-\alpha)}, \]

we obtain:

\[
V(A_i, A_j^*, \omega, \omega_{to}) = \log((1 - \beta)\omega) + \\
\beta \sum_{zy} \pi_{iz} \pi_{jy}^* [a_{zy} + b_{zy} \log \omega + (\alpha(c_{zy} + b_{zy}) - b_{zy}) \log \omega_{to} + (b_{zy} + c_{zy}) \log(h_{zy})],
\]

for \( ij = hh, hl, lh, \) and \( ll \). With this system of equations in the unknown coefficients, we obtain that

\[
b_{ij} = b_{cap} = \frac{1}{1 - \beta} > 0, \forall i \forall j \quad \text{and} \quad c_{ij} = c_{cap} = \frac{\beta(\alpha - 1)}{(1 - \beta)(1 - \alpha \beta)} < 0, \forall i \forall j \quad \text{and} \quad a_{ii}, a_{ij}, a_{ji}, \text{ and } a_{jj}
\]

solve system of four independent equations (available upon request), that comes from equating the terms that do not multiply either \( \log \omega \) or \( \log \omega_{to} \) in the above expressions of the value functions. 

Now we proceed to derive the value function for the workers of the domestic country (the derivation for the workers of the foreign country would be analogous).

Proposed guess for the value function of a domestic worker whose country’s wealth is \( \omega_d \), when world wealth is \( \omega_{to} \), and domestic and foreign current shocks are \((A_i, A_j^*)\):

\[
V(A_i, A_j^*, \omega_d, \omega_{to}) = \bar{a}_{ij} + \bar{b}_{ij} \log(\omega_d) + \bar{c}_{ij} \log(\omega_{to}),
\]

\[
V(A_i, A_j^*, \omega_d, \omega_{to}) = \log\left[ \frac{(1 - \alpha)\omega_d}{\alpha l} \right] + \beta \sum_{zy} \pi_{iz} \pi_{jy}^* (\tilde{a}_{zy} + \tilde{b}_{zy} \log(\omega'_{zyd}) + \tilde{c}_{ij} \log(\omega'_{zyto})).
\]

Remember that

\[
\omega'_{zyd} = R'_z(\lambda_{ij}\omega_{to}) = \alpha [A_z(\lambda_{ij}\omega_{to})^{\alpha} l^{1-\alpha}], \text{for pairs } zy = hh, hl, lh, \text{ and } ll.
\]

Substituting those expressions in the workers’ value functions together with the expressions for tomorrow’s total wealth for the different states, we get:

\[
V(A_i, A_j^*, \omega_d, \omega_{to}) = \log\left[ \frac{(1 - \alpha)\omega_d}{\alpha l} \right] + \beta \sum_{zy} \pi_{iz} \pi_{jy}^* (\tilde{a}_{zy} + \alpha (\tilde{b}_{zy} + \tilde{c}_{zy}) \log \omega_{to} + (\tilde{b}_{zy} + \tilde{c}_{zy}) \log(h_{zy})].
\]
And proceeding analogously for $V(A_i, A_i^*, \omega_d, \omega_{to})$, $V(A_j, A_j^*, \omega_d, \omega_{to})$, and $V(A_j, A_j^*, \omega_d, \omega_{to})$ we can solve for the unknown coefficients such that:

$$\tilde{b}_{ii} = \tilde{b}_{ij} = \tilde{b}_{ji} = \tilde{b}_{jj} = b_{wor} = 1 > 0,$$

and

$$\tilde{c}_{ii} = \tilde{c}_{ij} = \tilde{c}_{ji} = \tilde{c}_{jj} = c_{wor} = \frac{\beta \alpha}{(1 - \beta \alpha)} > 0.$$ 

And $\tilde{a}_{ii}$, $\tilde{a}_{ij}$, $\tilde{a}_{ji}$, and $\tilde{a}_{jj}$ solve system of four independent equations (available upon request), that comes from equating the terms that do not multiply either $\log \omega_d$ or $\log \omega_{to}$ in the above expressions of the workers’ value function.

References


