Asymmetric information and the lack of international portfolio diversification*

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Abstract

There is pervasive evidence that individuals invest primarily in domestic assets and thus hold poorly diversified portfolios. Empirical studies suggest that informational asymmetries may play a role in explaining the bias towards domestic assets. In contrast, theoretical studies based on asymmetric information fail to produce significant quantitative effects. The present paper develops a theoretical model in which the presence of informational asymmetries explains a significant fraction of the home equity bias observed in the data. The main departure from previous theoretical work is the assumption that local investors outperform foreign investors in identifying the correct ranking of local investment opportunities, instead of possessing superior information about the aggregate performance of the domestic stock market. The other key assumption is based on the evidence that short-selling is a costly activity. This is modelled by assuming that agents face a short-sales constraint. This paper studies the case of a two-country world. There are two assets in each country. Only local investors receive informative signals about local assets. Thus, domestic agents have an incentive to concentrate their investments in the local asset favored by the signal realization, and reduce the position held in the other local asset. When the signal is sufficiently informative, the short-sales constraint binds, i.e., local investors are not able to finance purchases of the perceived ‘good’ local asset by selling short the perceived ‘bad’ local asset. In that case, local agents decide to invest a lower fraction of their portfolio in foreign securities in order to invest in the ‘good’ local asset. Since this mechanism relies on a first order effect, it can generate significant levels of home equity bias.

Keywords: International portfolio diversification, home bias, asymmetric information.

JEL Classification: D82, F30, G11, G15

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1 Introduction

The last decades have witnessed a remarkable increase in international capital flows. Gross cross-border transactions in bond and equity of US residents represented 4% of the GDP in 1975, but increased to 320% by 2003. Given this evidence, we would expect to observe an internationally diversified portfolio composition. Yet this is not the case. The fraction of US portfolio invested overseas has been quite low. Figure 1 on page 2 uses two simple measures to illustrate the recent behavior of US investors in the bias towards domestic stocks. It shows that by the end of 2003 foreign stock holdings accounted for 13.5% of the overall equity holdings of US residents. The figure is larger than the mere 1% that was observed until the first half of the 80’s, but it is still significantly below what the theory of international portfolio diversification predicts. For instance, the international version of the capital asset pricing model predicts that the composition of every investor’s portfolio coincides with the world market portfolio composition. Thus, by the end of 2003 American investors should have allocated around 50% of their equity portfolios in foreign stocks.\(^1\) The low investment in foreign assets is known in the literature as the home equity bias puzzle and was initially documented by French and Poterba (1991), Cooper and Kaplanis (1994) and Tesar and Werner (1995).

This evidence has motivated a vast body of literature. There may be various reasons why domestic investors are reluctant to invest abroad. There are domestic regulations that limit the foreign exposure of institutional investors. Some foreign countries impose limits on the fraction of a firm that can be owned by non-nationals. Transaction costs may be higher for cross-border transactions. Exchange rate fluctuations increase the risk of investing in foreign assets if domestic investors care only about returns nominated in domestic currency. However, none of these factors has offered a satisfactory explanation. Lewis (1999) and Karolyi and Stulz (2003) offer a detailed survey of the literature.

An alternative explanation rests on the intuitive assumption that local investors can collect more precise information about domestic assets.\(^2\) The typical setup in this literature is given by a two-country world with a single asset in each country. Domestic investors receive informative signals about domestic and foreign assets, but the signal about the local asset is more precise. The reason why this generates home equity bias is that the informational disadvantage about the foreign asset turns it into a more risky investment option from the perspective of local investors. However, this explanation faces two

\(^1\)Baxter and Jermann (1997) argue that the amount invested overseas should be even larger than that, given that the returns on human capital are correlated with the returns on domestic assets.

\(^2\)See Gehrig (1993), Zhou (1998) and Coval (2000). A more thorough description of this work is provided below.
limitations. First, it relies on effects that are of second order importance. Once the parameters in the
model are replaced with realistic values, the bias generated is quite low. Second, there is a bias only in
ex ante terms, i.e., in expectation, before the parameters determining the distribution of dividends are
drawn, and before agents receive their signals. Ex post, it may happen that local agents invest more
heavily in the foreign asset. In other words, the model displays high volatility in the fraction invested
overseas. This is not observed in the data.

The distinctive feature of this paper is that the information asymmetry between domestic and foreign
investors relates to the performance of individual stocks rather than the market portfolio. The paper
assumes that domestic investors do not outperform foreign investors in predicting the performance of the
local stock market index, but they do enjoy an advantage in identifying the best individual investment
opportunities. The latter is motivated on the grounds that there is more scope for the existence of
disparities in information about individual firms than about aggregates like the stock market. The

Figure 1: Measures of foreign investment positions. Sources: Department of Commerce and Inter-
national Federation of Stock Exchanges
second assumption that separates this work from previous theoretical papers is based on the extensive evidence that shows that short-selling is a costly activity.\textsuperscript{3} For simplicity, this paper assumes that agents face short-sales constraints.

The paper maintains the two-country world setup but with two technologies in each country. All technologies are publicly listed, so every individual can invest in any technology. Each technology receives either a high productivity shock (high returns) or a low productivity shock (low returns). The probability distribution over next period’s productivity shock is not publicly observed. Instead, domestic investors receive an informative signal about local assets. A fraction $\phi$ of domestic investors receives a signal favorable to the local asset that it is more likely to pay high returns. The remaining local investors receive an incorrect signal, encouraging investment in the local asset that pays high returns with the lowest probability. Each investor does not know whether the signal received is correct or not. For simplicity, it is assumed that local agents do not receive information about foreign assets.

Agents diversify their portfolios in two dimensions: across countries and across assets within each country. The information structure described above has two implications regarding portfolio composition. First, agents want to concentrate the local component of their portfolios in the local technology favored by the signal realization (the ‘good’ asset). Second, since the information received is uncorrelated with any index of aggregate performance, local investors do not have an incentive to invest more heavily in the domestic or foreign country. This implies that when the degree of information is such that local agents still demand a positive amount of the local asset not favored by the signal realization, the home equity bias is nil. This result changes when the signal is very informative. In that case, agents are sufficiently optimistic about the perceived ‘good’ local asset that they are willing to finance the purchases of these stocks by selling short the perceived ‘bad’ local asset. Since it is assumed that short-selling activities are costly and never take place, local agents decide to finance the purchases of the ‘good’ local asset by investing less in foreign stocks. This mechanism is what generates home equity bias in the model. The result is driven by an effect of first order importance, i.e., differences in expected returns. This allows us to explain a high fraction of the bias observed in the data for realistic parameters values. In addition, the fraction of foreign investments generated by the model is stable over time.

The remainder of the paper is organized as follows. The next two subsections review related empirical and theoretical literature. Section 2 introduces the model and discusses the assumptions. Section 3

\textsuperscript{3}See section 2.1 on page 8.
extends the model to a case with positive cross-asset return correlation. Section 4 computes the shadow price of the short-sales constraint. Section 5 allows for endogenously determined stock prices in a context where prices are not fully revealing. Section 6 provides some analytical characterization of our main results using the CARA-Gaussian setup. Section 7 describes the testable implications of our model. Section 8 applies the baseline model to study the home consumption bias. Finally, Section 9 concludes.

1.1 Related empirical literature

This paper relies on the assumption that domestic investors have better information about domestic stocks than foreign investors. This can be justified on many grounds. Equity investment in foreign companies requires understanding different accounting practices and legal environments. Domestic investors are exposed to a wide array of sources of local news that can convey useful information about the performance of domestic companies. In addition, the geographic proximity allows for face-to-face contacts with local corporate executives, employees and other individuals that may have valuable private information.

The empirical literature in the area is not conclusive but suggests that in several cases foreign investment behavior is consistent with the presence of informational asymmetries. Kang and Stulz (1997) study foreign stock ownership in Japanese firms. They find that foreign investors concentrate their portfolios in large firms, firms with good accounting performance, and firms with high exports. These are the types of companies where the information asymmetries are presumed to be the lowest. Dahlquist and Robertsson (2001) find qualitatively similar results for Swedish firms. Choe et al. (2004) analyze foreign trades in Korea and find that foreign investors buy at higher prices than domestic investors and sell at lower prices. A similar result is found by Dvořák (2001) for Indonesia. Shukla and van Inwegen (1995) provide evidence that UK mutual funds obtain lower returns from their investments in the US compared to US funds. Ahearne et al. (2004) study the home bias of US investors against specific countries. They find that the home bias decreases with the fraction of the foreign country market value that is cross-listed in the US stock market. Frankel and Schmukler (1996) show that domestic investors were ‘front-runners’ in the Mexican crisis of 1994: they tried to sell their local investments before foreign investors did. Portes and Rey (1999) analyze the determinants of cross-border transaction flows. They find that distance has a significant negative impact, which they argue is a proxy for informational asymmetries. Hau (2001) finds that traders located in Frankfurt and in
German speaking cities in Europe show higher proprietary trading profits on German stocks. There is also evidence of higher profits for traders located near corporate headquarters.

On the contrary, other papers find evidence that foreign investors outperform local investors, which implies that either foreign agents do not have less information than local residents or that the informational disadvantage does not play a significant role. Karolyi (2002) shows that foreign investors obtain higher returns in Japan. Grinblatt and Keloharju (2000) reach the same conclusion for Finland. Seasholes (2004) provides evidence that foreign investors in Taiwan buy before price increases and sell before price decreases.

One reason behind the mixed results is that it is not easy to isolate the role played by differences in information. Even when the comparison is made across similar classes of agents, there may be other factors affecting the behavior of domestic and foreign agents. In this respect, the database used in Coval and Moskowitz (1999, 2001) allows for a more precise measure of the role played by informational asymmetries. The authors study the portfolio composition of more than 2000 mutual funds in the US. It is plausible to conjecture that their sample consists of a homogenous set of actors who are also subject to the same legal environment. They find evidence of home equity bias within US boundaries. Fund managers invest more in companies with headquarters located near the fund’s offices. Moreover, they earn substantial abnormal returns in nearby investments, while at the same time stocks held predominantly by local investors tend to show higher expected returns. The evidence strongly suggests that fund managers are exploiting an informational advantage in their selection of nearby stocks. Ivkovich and Weisbenner (2004) study a sample of 78,000 households and also find evidence of a strong preference for local stocks. In addition, the excess return of local investments is larger among companies not listed in the Standard and Poor 500 index. The latter are presumably firms with wider informational asymmetries between local and non-local investors.\footnote{Hubermann (2001) also documents that investors tilt their portfolio composition toward local companies. But he argues that this behavior is due to the fact that investors prefer to invest in firms that are familiar to them, independently of their prospects. If that were the case, we should not expect to observe abnormal returns on local investments. However, the evidence provided by Coval and Moskowitz (1999, 2001) and Ivkovich and Weisbenner (2004) suggests that a significant fraction of investors indeed behaves rationally.}

1.2 Related theoretical literature

All of the above is taken as evidence that not only does the assumption of local information advantage have an intuitive appeal, but it also receives some support from empirical evidence. From a theoretical
point of view, it has already been said that the present paper is not the first attempt to explain the home bias assuming asymmetric information between domestic and foreign investors. Gehrig (1993) uses the workhorse model of rational expectations equilibrium developed by Grossman (1976), Grossman and Stiglitz (1980) and Admati (1985). He assumes that every agent receives informative signals about the future performance of domestic and foreign assets. The domestic signal conveys more information than the foreign one. This leads to more imprecise assessments about future performance of the foreign asset. Domestic agents thus perceive the foreign stock as more risky than the local stock and reduce their holdings of foreign assets. However, Glassman and Riddick (2001) and Jeske (2001) argue that the implied risk aversion needed to generate quantitatively significant results is unreasonably high.

Zhou (1998) considers a two asset model with a more sophisticated learning process. Agents face the so-called ‘infinite regress’ problem: forecasting the forecasts of forecasts ... of others. But that feature does not help him to obtain any sizeable effect. Coval (2000) extends the framework in Zhou (1998) by introducing direct investment decisions and simplifies the learning process. He also obtains a small impact on the home bias.

In addition to the poor quantitative performance, Jeske (2001) argues that the previous modelling strategies do not seem suitable to address the home bias puzzle. Since domestic agents hold better information about domestic assets, sufficiently low expected local dividends induce residents to liquidate their local positions in favor of foreign assets. On the other hand, foreign investors unaware of the poor expected performance of local assets may find it convenient to purchase local stocks at a discount. As a consequence, these models predict unrealistic fluctuations of the home bias (which can turn into foreign bias for certain shock realizations). These limitations lead him to conclude that asymmetric information does not stand up as a compelling theoretical explanation for the home equity bias.

The reason behind the lack of success of previous attempts is that the burden of the explanation relies on effects that are of second order importance. In the setups considered, agents cannot be systematically pessimistic nor optimistic with respect to any asset. The explanation for why agents show a preference for domestic assets is that they are perceived to be less risky or that they provide better hedge against consumption risk (see Coval (2000)). But this plays a secondary role in the standard expected utility framework with the HARA utility functions commonly assumed for macroeconomic analysis.

The present paper differs from the previous literature in the sense that the main driving force of the bias towards domestic assets is of first order importance. Epstein and Miao (2003) and Alonso (2004)
assume preferences that allow for ambiguity aversion and are able to explain a significant fraction of the bias. The reason is that they also introduce a first order effect: domestic agents are systematically pessimistic about foreign stocks. The present paper shows that similar quantitative results can be obtained without a major departure from the mainstream model.

2 The model

Consider a two-country world. Each country is inhabited by a large number of infinitely-lived, identical agents. Agents have preferences defined over a stream of tradable consumption goods:

\[ E \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \mid I_0 \right] , \]

with

\[ u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma} \]

The probability distribution of future consumption flows depends on each agent’s initial information set, denoted by \( I_0 \).

Each country hosts two risky technologies. Each technology produces the same consumption and investment good. The output provided by each technology depends on the capital allocated in the previous period and the current productivity shock. The productivity shocks vary across technologies. For simplicity, it is assumed that productivity shocks may take either a high value (\( A_h \)) or a low value (\( A_l \)). Technologies do not require labor as an input, i.e., they are of the AK type. The probability that technology \( i \) is hit with a high productivity shock is denoted by \( \nu_i \), where \( i = 1, 2, 1^*, 2^* \). The superscript \( ^* \) is used to denote foreign variables. The probability values are drawn from a joint distribution with density \( f(\nu_1, \nu_2, \nu_{1^*}; S) \), where \( S \) represents an observable aggregate state.

Investors are not able to observe the probabilities that govern the distribution of productivity shocks. Instead, they receive informative signals about the relative expected performance of local technologies. A fraction \( \phi \) of domestic investors receive a signal favorable to the local technology that it is more likely to receive a high productivity shock, while a fraction \( 1 - \phi \) receive an incorrect signal, suggesting them to invest in the technology that is less likely to receive a high productivity shock. Each investor does not know whether the signal received is correct or not. The value of \( \phi \) is assumed to be larger than 0.5. Finally, it is assumed that agents are not allowed to hold short positions.
The timing in the model is fairly simple. Productivity shocks are realized at the beginning of each period. After that, Nature draws the vector \((\nu_1, \nu_2, \nu_{1^*}, \nu_{2^*})\), that determines the probability distribution over next period’s productivity shocks. This vector also conditions on the distribution of signals received by domestic and foreign agents. Finally, every agent chooses how much to invest in each asset given the information conveyed by the aggregate state realization and the signal received. In order to let the investor’s problem have a recursive structure, it is assumed that the density \(f\) defined before depends only on the current realization of productivity shocks.\(^5\)

From the perspective of an individual investor, the relevant state variables are his wealth level, the signal received and the current productivity shocks: \((\omega, s, S)\). The last two pieces of information are helpful in getting a better assessment of the probability distributions for next period’s shocks. The agent’s optimization problem can therefore be expressed as

\[
V(\omega, s; S) = \max_{k_1^*, k_2^*, k_{1^*}, k_{2^*}} \{u(c) + \beta E[V(\omega', s', S')]\}
\]

subject to

\[
c + k_1^* + k_2^* + k_{1^*} + k_{2^*} = \omega
\]

\[
k_i^*, k_{i^*} \geq 0 \quad \text{for } i = 1, 2
\]

2.1 Discussion of assumptions

The signal structure captures the hypothesis that local investors do not possess significantly better information about the future performance of the local stock market compared to what foreign investors know, but they do outperform foreign investors in spotting the best domestic investment opportunities. For instance, it is not clear why American investors should do systematically worse at predicting the performance of the German stock market when compared to German investors. In that case, the relevant information set consists mostly of public news and past performance of aggregate variables, which are readily available. It may be possible that some local investors get privileged access to information about policy decisions (like a proximate declaration of default), but this relates to rare events and should not play a significant role in developed countries, with good institutions. On the contrary, domestic investors are exposed to a wide array of sources of local news that can convey useful information about

\(^5\)If the density depended on past realizations of \(\nu_i\) and \(\nu_{i^*}\), agents would typically need to keep track of the entire history of shocks in order to update their beliefs.
the performance of domestic companies. In addition, the geographic proximity allows for face-to-face
contacts with local corporate executives, employees and other individuals that may have valuable private
information. Finally, several papers surveyed in the introduction seem to support that hypothesis that
local investors may hold privileged information about individual stocks.

It is clear that short-selling is more costly than buying a stock. The common method of shorting
an equity is to borrow the security and sell it. Later, the short-seller needs to buy it to return it to the
lender. The explicit cost of the transaction is the fee that the short-seller pays for the loan. But there
are other costs. The standard practice is that the equity lender can ask the loan to be repaid (‘recall’
the shares) at any time, which exposes the short-seller to risk. There are also various regulations that
increase the cost of short-selling: the proceeds from short-selling are taxed by the short-term capital
gain tax rate independently of how long the short position is open; sell orders that are short sales can
be executed only after the stock price has increased (on an ‘uptick’), etc. Some institutional investors
are even prohibited from taking short positions. Almazan et al. (2004) find that by 2000 69% of US
equity funds were not permitted to engage in short-selling practices. Among the ones that were not
constrained, only 10% held short positions. All of the above may help to explain why the market for
equity loans is so thin. D’Avolio (2002) reports that in June 2001 the total amount of stocks shorted
represented 1.7% of the market capitalization. Dechow et al. (2001) documents that short positions
represented only 0.2% of the market value in 1976 but increased to 1.4% in 1993. The previous evidence
shows that short-selling activities are quite limited in the US, the most developed financial market.
This suggests that although the assumption of no-short-sales constraints made is this paper may appear
extreme, it is in line with trading limitations observed in actual markets.

Other features of the model require further explanation. Notice that agents can only invest in four
assets while there are sixteen possible state realizations, depending on the realization of productivity
shocks. This means that markets are incomplete. Not only is this a realistic assumption, but it is also
necessary in order to have a well defined measure of home bias. Otherwise, it would not be clear how
to classify assets that pay contingent on joint domestic and foreign state realizations. Besides, under

\footnote{See D’Avolio (2002), Dechow et al. (2001) and Duffie et al. (2002) for a description of the institutional details of
equity lending markets and regulations applied to short-selling. An additional implicit cost of short-selling is described in
Lamont (2004). He argues that firms have incentives to impede short sales of their stock. He analyzes a sample of 266
firms who threatened, took action against, or accused short sellers of illegal activities. His findings suggest that those
firms succeeded in raising the costs of short selling.}

\footnote{Koski and Pontiff (1999) find that 79% of equity mutual funds make no use of derivatives suggesting that funds are
also not finding synthetic ways to take short positions.
complete markets there would be fifteen endogenous prices. That is more than enough to reveal all the information agents need to know.

The model laid down above does not require an equilibrium concept. The returns on capital are exogenously given by productivity shocks and share prices of the four technologies are equal to one: The sectors’ sizes fully adjust in every period in response to the aggregate demand of each stock. Assuming a more standard production process with decreasing marginal productivity on capital and labor would feature endogenous factor prices. But the market incompleteness implies that the model cannot be solved as if each economy was inhabited by a representative agent. In order to forecast future factor prices individuals may need to keep track of the wealth distribution in each country. Even though this is an interesting extension, it involves a high level of complexity and it is not necessary to illustrate the main result of the paper.

The constant-share-price result can be relaxed by imposing a sluggish adjustment in the supply of stocks. The standard model with asymmetric information used in the finance literature assumes a constant asset supply over time, except for shocks due to liquidity trading. We choose the opposite extreme in order to prevent investors from extracting valuable information from prices. If relative prices can differ, it would be necessary to allow for additional sources of uncertainty that can mask the expected relative performance of each asset. This is not a trivial extension. To the best of our knowledge, the only model structure with multiple assets and partially revealing prices corresponds to the one developed by Admati (1985). Her results relies on the Gaussian-CARA framework. However, it is not possible to accommodate that modelling strategy in the present work. The assumption of no-short-sales breaks down the Bayesian updating scheme over normally distributed variables, making the problem intractable. Section 5 develops a simple environment that allows for endogenously determined prices. It shows that the main conclusion of the paper is not affected as long as prices do not reveal too much information.

2.2 Implications for the home equity bias, a simple framework

This section makes two simplifications with respect to the model introduced above. First, the probability distribution from which the values of $\nu$ are drawn are assumed to be independent across technologies as well as from aggregate state realizations. For simplicity, they are assumed to be uniformly distributed
over the interval \([0, 1]\). The previous simplifications imply that there is no persistence in returns.\(^8\) The second implication induced by this simplified setup is that signals about domestic technologies do not convey information about foreign assets. It is straightforward to generalize the model and allow domestic agents to receive some information about foreign assets. The main results will not be affected as long as foreign signals are less informative than local ones.

The only change the new assumptions introduce to the optimization problem given by equation 1 is that the aggregate state is no longer a state variable. The expectation of the value function for the following period can then be expressed as

\[
E[V'(\omega_0, s_0) \mid s] = E[Pr(h, h, h, 1) \mid s] V'(\omega_{h,h,h,h}, 1) + \cdots + E[Pr(l, l, l, 1) \mid s] V'(\omega_{l,l,l,l}, 1) + \\
E[Pr(h, h, h, 2) \mid s] V'(\omega_{h,h,h,h}, 2) + \cdots + E[Pr(l, l, l, 2) \mid s] V'(\omega_{l,l,l,l}, 2)
\]  

(2)

where

\[
\omega'_{i,j,i',j'} = A_{i}k_1 + A_{j}k_2 + A_{i'}k_1' + A_{j'}k_2'.
\]

The term \(E[Pr(i, j, i', j', s') \mid s]\) denotes the conditional joint probability of receiving a signal \(s'\) in the following period and observing a future combination of productivity shocks \((i, j, i', j')\). The first two components refer to domestic productivity shocks to technologies 1 and 2 respectively, and the last two components denote shock realizations of foreign technologies 1 and 2 respectively. The conditional expectation is computed using Bayes’ rule. Namely,

\[
E[Pr(i', j', i^{*'}, j^{*'} , s') \mid s] = \frac{Pr(s') E[Pr(i', j', i^{*'}, j^{*'} , s)]}{Pr(s)} = \frac{Pr(s') E[Pr(A_{i^{*'}} = A_{i^{*'}})] E[Pr(A_{i} = A_{i})] E[Pr(A_{1} = A_{1}, A_{2} = A_{2}) \mid s]}{Pr(s)}
\]

where the second equation makes use of the assumptions stated at the beginning of this section. More explicitly, the previous expectation is computed as follows:

\(^8\)There is mixed evidence in this respect: some authors find that past prices do not convey useful information about future returns while other papers find some effect. But even the latter do not find a large effect. This suggests that eliminating serial correlation in returns is not a very restrictive assumption. See Malkiel (2003) for a discussion on the topic.
\[
E \left[ \Pr \left( i', j', i'^*, j'^*, s' \right) \mid s \right] = \left[ \frac{\Pr (\nu'_i > \nu'_2) \Pr (s' \mid \nu'_i > \nu'_2) + \Pr (\nu'_i < \nu'_2) \Pr (s' \mid \nu'_i < \nu'_2)}{\Pr (\nu_1 > \nu_2) \Pr (s \mid \nu_1 > \nu_2) + \Pr (\nu_1 < \nu_2) \Pr (s \mid \nu_1 < \nu_2)} \right] \times E \left[ \Pr \left( A'_{1^*} = A_{1^*} \right) \right]
\]

where \( \nu'_i \) denotes the next period draw of \( \nu \) for technology \( i \), with \( i = 1, 2 \), and \( \Pr^{i'}(\nu) \) denotes the probability that technology \( i \) receives productivity shock \( A_{i'} \) if the actual probability of being hit with a high productivity shock is \( \nu \). Formally,

\[
\Pr^{i'}(\nu) = \begin{cases} 
\nu & \text{if } i' = h, \\
1 - \nu & \text{if } i' = l.
\end{cases}
\]

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Figure 2: Change in local distribution of returns upon the arrival of the relative signal.

Figure 2 illustrates how an individual updates his beliefs about domestic technologies. For instance, after receiving a signal that favors technology 1, the investor assigns more weight to points below the
diagonal. The final conditional expectation of \((\nu_1, \nu_2)\) lies on a point like A. He becomes optimistic about asset 1 and pessimistic about asset 2. This explains the incentive to diversify the local component of his portfolio.

The dynamic optimization problem is solved by finding policy functions that satisfy the Euler Equations. It is easy to check that the assumptions of constant relative risk aversion utility function and linear technology lead to individual policy functions that are linear in wealth. However, it is not possible to find an explicit expression for the slope coefficients, so the problem is solved using numerical techniques.\(^9\) Appendix 6 considers a model with CARA utility function and normal productivity shocks. That framework allows us to fully characterize the optimal investment policy. We find qualitatively similar results to the ones described below.

The parameter values are chosen in such a way that the domestic return process resembles some key statistics of the US stock market. The assumption that foreign assets share the same characteristics with domestic assets is made for simplicity. A period in the model corresponds to one year. The value for the high productivity shock \((A_h)\) is set equal to 1.27. The value for the low productivity shock \((A_l)\) is set equal to 0.85. This yields an average return on each stock market of 6%, with a standard deviation of 14.7%.\(^{10}\) Finally, we choose standard preference parameters: a logarithmic utility function and a subjective discount factor \((\beta)\) of 0.96.

Figure 3 describes the main result of the paper. The graph shows how investors’ policy functions depend on the precision of the signal. A completely uninformative signal corresponds to the case where \(\phi = 0.5\). Half of domestic agents receives a signal favorable to technology 1 and the other half receives a signal favorable to technology 2 regardless of the actual probability distribution of domestic returns. In that case, agents fully diversify their portfolios and invest a quarter of their savings in each asset. As the signal becomes more informative, a majority of domestic investors are able to identify the domestic technology that yields higher returns. A typical local investor increases his position in the local technology favored by the signal realization and decreases his position in the other local technology. The overall proportion of local assets in his portfolio is barely affected: He still invests roughly half of

\(^9\) Equation 2 shows that each individual needs to allocate future consumption across 32 states. The assumption that technologies are identical means that the signal received does not affect the perceived discounted utility of future consumption streams. This reduces the future state space by half. In addition, investors do not receive any information than can help them differentiate between the two foreign assets so the latter are perceived to be equivalent. In practice, then, it is only necessary to solve for three policy functions: the investments in the two domestic technologies and the foreign one. Also, the state space can be reduced to 12 possible realizations. With logarithmic utility function, the solution for the policy functions consists of the root of a system of three polynomials of 12th order!

\(^{10}\) The statistics correspond to the case where both technologies receive the same weight.
his savings in foreign assets.

But the last conclusion changes if the signal is so informative that local investors want to short-sell the perceived low-returns asset to finance purchases of the perceived high-returns asset. Since short-selling is a costly activity, the strategy followed in order to finance purchases of the 'good' local asset is to lower foreign assets holdings. It is in this range where a significant home equity bias shows up. For instance, when the proportion of local investors that receive the ‘correct’ signal is around 70%, domestic investors always hold 80% of their portfolios in local assets.

3 A more general case with positive return correlation across assets

Given that the signals observed by local agents relate only to future relative performance of domestic assets, the role played by differences in information decreases with the correlation of local returns. For example, if the returns of both assets were perfectly correlated, there would be no scope for differences in performance. That case would resemble the framework analyzed in previous studies (one asset per country), which we already know does not help to explain the lack of international portfolio diversification.

In order to allow for cross-asset return correlation, it is assumed that the values of $\nu_1$ and $\nu_2$ are
drawn in two steps. First, Nature draws a value $\nu$ from a Uniform distribution over $[0, 1]$. The latter satisfies $\nu = \frac{\alpha + \nu_2}{2}$ and can be interpreted as an aggregate shock. Second, Nature draws a value $\eta$ from a Beta distribution. This determines the relative performance of domestic assets. Formally,

$$\nu_1 = \begin{cases} 2\nu\eta & \text{if } \nu < 0.5, \\ (2\nu - 1)(1 - \eta) + \eta & \text{if } \nu > 0.5. \end{cases}$$

When $\eta$ equals 0.5, the returns of domestic technologies share the same probability distribution. For values larger that 0.5, technology 1 yields higher expected returns. For values below 0.5 technology 2 yields higher expected returns. This section maintains the assumption that local assets have the same ex ante distribution of returns, which implies that $\eta$ has a mean of 0.5. This pins down one of the parameters of the Beta distribution. The remaining parameter is used to control for the volatility of $\eta$. The intuition is that if $\eta$ concentrates a high probability mass around its mean, local assets will tend to share a similar probability distribution and they will therefore display unconditional return correlation.\(^{11}\)

The random variables that determine aggregate and relative performance are assumed to be independent. Thus, the probability distribution over $\nu$ and $\eta$ can be written as

$$h(\nu, \eta) = \begin{cases} g^\beta(\eta; \sigma_\eta) \frac{2\alpha\nu + 1 - \alpha}{1 - \frac{1}{2}} & \text{if } \nu < 0.5, \\ g^\beta(\eta; \sigma_\eta) \frac{2\alpha(1 - \nu) + 1 - \alpha}{1 - \frac{1}{2}} & \text{if } \nu > 0.5. \end{cases}$$

The function $g^\beta(\eta; \sigma_\eta)$ denotes the density function of a random variable with Beta distribution and parameters $\frac{1}{8} \left[ \frac{1}{\sigma_\eta^2} - 4 \right]$. The second term allows us to determine how much probability mass is assigned near the extremes, i.e., when both assets pay high or low returns with certainty. The role of that term can be seen more clearly below. The density function over $\nu_1$ and $\nu_2$ is obtained after a change of variables.

$$f(\nu_1, \nu_2) = \begin{cases} g^\beta \left( \nu_1 + \nu_2; \sigma_\eta \right) \frac{2\alpha(\nu_1 + \nu_2) + 1 - \alpha}{1 - \frac{1}{2}} \frac{1}{2(\nu_1 + \nu_2)} & \text{if } \nu_1 + \nu_2 < 1, \\ g^\beta \left( 2 - \nu_1 - \nu_2; \sigma_\eta \right) \frac{2\alpha(2 - \nu_1 - \nu_2) + 1 - \alpha}{1 - \frac{1}{2}} \frac{1}{2(2 - \nu_1 - \nu_2)} & \text{if } \nu_1 + \nu_2 > 1. \end{cases}$$

The previous equation shows that the weight assigned to realizations of $\nu_1$ and $\nu_2$ close to the certainty points decreases with $\alpha$. Thus, the parameter $\alpha$ is useful to generate high return correlation. This section maintains the assumption that both countries are identical. This means that foreign assets are subject to the same return process as local assets. It is also assumed that returns from domestic assets are uncorrelated once we condition on the realizations of $\nu_1$ and $\nu_2$.\(^{11}\)
and foreign assets are uncorrelated. Before choosing the values of $\sigma_\eta$ and $\alpha$, it is necessary to determine what is a plausible level of correlation. The capital asset pricing model (CAPM) provides a simple framework in order to retrieve a sensible value. The CAPM states that:

$$R_i = R_f + \beta_i (R_m - R_f) + \epsilon_i,$$

where $R_i$ denotes return of asset $i$, $R_f$ denotes the risk free interest rate, $R_m$ denotes market return and $\epsilon_i$ denotes the idiosyncratic shock to asset $i$. This setup captures a simple mechanism that generates cross-asset correlation: The return of every asset depends on an aggregate variable, i.e., the excess return of the market portfolio. The next step is to provide an interpretation for the assets in our model. If the model were followed literally, each asset would correspond to a portfolio of local firms. This approach implies a high level of aggregation, so we would expect to obtain a strong cross-asset correlation. However, the same reason we utilized to abstract from informational asymmetries about aggregate variables could be applied to those portfolios. The way we would like to interpret local and foreign assets is as if they represent firms or specific industries. The fact that the paper considers only two stocks per country is just a simplification made for tractability purposes.

If each asset stands for a firm, the results in Fama and French (1992) imply a zero cross-asset correlation. They study a sample of 2267 stocks and conclude that the average estimation of $\beta$ is not significantly different than zero. Fama and French (1993) sort individual stocks into 25 portfolios according to firm sizes and book-to-market ratio. Their estimation of the single factor model produces a mean return correlation across portfolios of 78%. Fama and French (1997) sort stocks according to the industry they belong to. They construct 48 industry-portfolios and obtain a mean cross-asset correlation of 63%.

An alternative procedure to estimate the cross-asset return correlation is to use Equation (3) and assume that both assets have a $\beta$ of one, i.e., they are two representative assets. Then,

$$\text{Corr} (R_i, R_j) = \frac{\text{Cov} (R_i, R_j)}{\sigma (R_i) \sigma (R_j)} = \frac{\text{Var} (R_m - R_f)}{\text{Var} (R_m - R_f) + \text{Var} (\epsilon)} = \frac{1}{1 + \frac{\text{Var} (\epsilon)}{\text{Var} (R_m - R_f)}},$$

which shows that cross-asset correlation depends on the ratio of idiosyncratic risk to aggregate risk. Campbell et al. (2001) use the CAPM structure to estimate return volatility at the market, industry, and firm levels. Using their estimates, a correlation of 0.6 is obtained at the industry level, similar

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12 The implicit assumption is that idiosyncratic shocks (denoted by $\epsilon_i$) are independent across assets.

13 The mean $R^2$ is taken as the estimated cross-asset correlation.
to the value obtained in Fama and French (1997). As expected, the correlation at the firm level is significantly lower, it ranges from 0.19 to 0.25 depending on whether returns are computed on a daily or weekly basis.

The previous evidence illustrates that the correlation can take almost any positive value depending on how assets are defined. We choose a correlation of 0.25 as the benchmark value and report results for other values. The baseline value of \( \alpha \) is set to 0 and the baseline value of \( \sigma_\eta \) is set to 0.25. We also consider the case \( \alpha = -6 \) and \( \sigma_\eta = 0.1 \), which generates a correlation of 0.45. Figure 4 reports the results. As expected, the fraction invested overseas decreases as the correlation increases. But the bias can still be significant for reasonable levels of cross-asset return correlation. However, when the cross-asset correlation is relatively high, there is less room for disagreement about asset returns and agents tend to hold a more diversified portfolio.

![Figure 4: Sensitivity of home equity bias to cross-asset return correlation](image)

The figure shows the fraction invested in domestic assets as a function of the correlation between assets, for different values of the correlation parameter. The results indicate that the fraction invested overseas decreases as the correlation increases. But the bias can still be significant for reasonable levels of cross-asset return correlation. However, when the cross-asset correlation is relatively high, there is less room for disagreement about asset returns and agents tend to hold a more diversified portfolio.
The shadow price of the short-sales constraint

The restriction on short-sales captures in a simple way the fact that short-selling is a costly activity. A more general formulation can be developed assuming, for example, that agents are required to pay a fee in order to hold short positions. The fee should include not only the direct cost derived from the equity loan, but also the implicit cost due to legal restrictions and the extra risk incurred by short-selling (like an early recall). This section finds the implicit fee that prevents agents from selling short.

It is assumed that agents pay a fee $\tau$ whenever they short-sell. The fee is proportional to the amount sold short. The investor’s optimization problem is set out below. The only difference with respect to the optimization problem in the baseline model is in the individual’s budget constraint. Without loss of generality, we consider the problem of an agent with a signal realization that favors asset 1.

$$V(\omega) = \max_{k_1', k_2', k_1^{s*}, k_2^{s*}} \{ u(c) + \beta E[V(\omega' | s = 1)] \}$$

subject to

$$c + k_1' (1 + I(k_1') \tau) + k_2' (1 + I(k_2') \tau) + k_1^{s*} (1 + I(k_1^{s*}) \tau) + k_2^{s*} (1 + I(k_2^{s*}) \tau) = \omega,$$

where

$$I(x) = \begin{cases} 1 & \text{if } x < 0, \\ 0 & \text{if } x > 0. \end{cases}$$

The value of $\tau$ consistent with the observation that agents do not hold short positions can be retrieved from the first order derivatives in the problem with short-sales constraints. The results shown in Figure 5 are based on the benchmark parameterization used in Section 3, where a cross-asset return correlation of 0.25 is assumed. The graphs illustrate that the model is capable of generating significant levels of home bias without imposing high costs on short sales. For instance, the fraction invested locally is around 75% with a fee of 2.5%.

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14 As it was said before, policy functions are linear in wealth. Thus, the policy function of asset $i$ can be written as $k_i'(\omega) = \alpha_i \omega$ with $i \in \{1, 2, 1^*, 2^*\}$. There are 16 future state realization depending on the productivity shock faced by each local and foreign technology. Let $A_i^h$ denote the productivity shock received by technology $i$ in state $h$. Since we consider the case where the local signal favors technology 1, the short-sales constraint binds when the investor wants to short sale stocks of technology 2. The implicit value of $\tau$ consistent with no-short-sales is therefore obtained as follows:

$$\tau = 1 - \beta \sum_{h=1}^{16} \Pr(j | s = 1) \frac{A_2^h}{\sum_{i=1,2,1^*,2^*} \alpha_i A_i^h}.$$
5 Endogenous asset prices

The objective of this section is to illustrate that the main results described before do not depend on the assumption that asset prices are exogenously given. This section abandons the assumption of infinitely elastic asset supplies. An additional feature arises when prices are non-trivially determined in equilibrium: They typically reveal valuable information. In order to prevent prices from being fully revealing, it is necessary to allow for a richer model that incorporates additional sources of uncertainty. This is not an easy task, especially if we depart from the standard environment with a CARA utility function and Gaussian returns. In accordance with most of the previous theoretical literature, the model below assumes that prices are only partially revealing because of the existence of supply shocks (noise traders). The difference is that we restrict attention to an ad hoc structure of shocks that enables us to reduce the set of possible equilibrium prices. Instead of extracting information from a multidimensional

Figure 5: Home bias and shadow price of short-sales constraint in the baseline model with cross-asset return correlation of 0.25.
space, agents learn from a finite set of prices. We finally show that this simple economy can also display significant levels of home equity bias.

The main features of the model are the following ones: As before, the world is composed of two countries. There are two trees in each country. All trees are ex-ante identical. For simplicity, it is assumed that agents live for two periods. Agents are initially endowed with exogenous income and shares of trees. It is assumed that every agent is entitled with an equal amount of shares of local and foreign trees. There is a measure 1 of agents in each economy, so every agent is endowed with 0.5 shares of each tree. Consumption goods are perishable. That means that agents can only allocate consumption across time and states by trading shares of trees. As before, short-sales are not allowed.

Trees pay dividends in the second period and then die. Tree $i$ pays high dividends $d_h$ with probability $\nu_i$ and low dividends $d_l$ with probability $1 - \nu_i$. Dividend payoffs are independent across assets. Each $\nu_i$ is drawn from a uniform distribution with support $[0,1]$. Agents do not observe actual realizations of $\nu$’s but receive informative signals. The signal structure is the same as the one defined in Section 2.

The consumer’s optimization problem can be stated as follows:

$$\max_{a_1,a_2,a_1^*,a_2^*} \left\{ u(c_0) + \beta \sum_{i=l,h} \sum_{j=l,h} \sum_{i^*=l,h} \sum_{j^*=l,h} \Pr(i,j,i^*,j^* | I) u(c_{i,j,i^*,j^*}) \right\}$$

subject to

$$c_0 = y + p_1(0.5 - a_1) + p_2(0.5 - a_2) + p_1^*(0.5 - a_1^*) + p_2^*(0.5 - a_2^*),$$

$$c_{i,j,i^*,j^*} = d_i a_1 + d_j a_2 + d_i a_1^* + d_j a_2^* \quad \text{for} \quad i, j, i^*, j^* \in \{h, l\},$$

$$a_m \geq 0 \quad \text{for} \quad m \in \{1, 2, 1^*, 2^*\},$$

where $a_m$ denotes holdings of asset $m$, $p_m$ denotes the market price of asset $m$, $y$ denotes the exogenous income received in the first period, and $I$ denotes the agent’s information set.

It is useful to differentiate between two types of states in order to simplify the exposition below. The current state depends on the realization of $(\nu_1, \nu_2, \nu_1^*, \nu_2^*)$. If agents could pool all the information available they would only be able to differentiate four possible configurations of the world economy, depending on which is the best asset in each country. This means that there are four possible current states. They are described in Table 1. On the other hand, the future state realization is determined by

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15The approach taken in this paper is similar to Wallace (1992).

16This is due to the fact that the fraction $\phi$ does not depend on the absolute values of $(\nu_1, \nu_2)$ or $(\nu_1^*, \nu_2^*)$. In other words, the intensity of the signal is independent from the actual gap in expected relative performances.
the actual dividend shocks experienced by each of the four trees. As before, there are 16 possible future states.

<table>
<thead>
<tr>
<th>Current state</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>( \nu_1 &gt; \nu_2; \ \nu_1^* &gt; \nu_2^* )</td>
</tr>
<tr>
<td>II</td>
<td>( \nu_1 &gt; \nu_2; \ \nu_1^* &lt; \nu_2^* )</td>
</tr>
<tr>
<td>III</td>
<td>( \nu_1 &lt; \nu_2; \ \nu_1^* &gt; \nu_2^* )</td>
</tr>
<tr>
<td>IV</td>
<td>( \nu_1 &lt; \nu_2; \ \nu_1^* &lt; \nu_2^* )</td>
</tr>
</tbody>
</table>

Table 1: **Partitions of pooled information**

In the setup described so far, prices are fully revealing. For instance, suppose that current state I has taken place. Thus, a majority of domestic agents receive a signal that favors asset 1. The resulting higher demand of that asset translates into a higher relative price. In this case, domestic and foreign agents would be able to infer which one is the best domestic asset. If a similar result applies to foreign assets, there would be no heterogeneity across agents: On top of the egalitarian distribution of endowments, everyone would hold the same information. Therefore, prices would adjust in such a way that agents decide not to trade. They keep half of their wealth in foreign assets.

In order to make the problem more interesting, the current section features a non-trivial information structure. It assumes that the supply of trees is subject to shocks. These shocks can be thought of as asset demand that arises from unmodelled agents or due to non-informational reasons. The typical interpretation in the literature is that they reflect trades of investors faced with liquidity shocks. In this scenario, a high price of one of the assets does not necessarily signal high expected dividends. It is also possible that the asset has become very valuable because its demand was hit with a large inelastic component that left few shares available to the remaining agents.

Each supply shock consists of a fourth dimensional vector: one component per asset. There are four possible supply shocks for each current state realization. This means that unconditionally, there are sixteen possible supply shocks. One of the four possible shock vectors is the null vector, independently of the initial state that has been realized. When the null vector is realized, the asset supplies are unaffected, and the equilibrium prices reflect the current state realization. However, the remaining shocks are such that the resulting equilibrium prices can mimic prices observed in other states with zero shocks. Formally, denote by \( \{ \tilde{\mu}_{ij} \}_{j=1}^4 \) the set of possible supply shocks in current state \( i \). Notice that
\( \mu_{ij} \in \mathbb{R}^4 \) \( \forall i, j \) and \( \mu_{ii} = 0 \) \( \forall i \). Let us denote by \( \bar{\mu}_i \in \mathbb{R}^4 \) the equilibrium price vector in state \( i \) with zero shocks. It is assumed that when the vector of supply shocks takes a value \( \bar{\mu}_{ij} \), the equilibrium price vector equals \( \bar{p}_j \).

There is a probability \( q + \frac{1-q}{4} \) that the supply shock takes null values. All other shock realizations occur with probability \( \frac{1-q}{4} \). The degree of informativeness of market prices is summarized in the value taken by \( q \). If \( q = 0 \) prices are fully uninformative. If \( q = 1 \) prices are fully informative. Prices are partially revealing in all other cases.

The structure of shocks is such that prices are not fully revealing. This allows for heterogeneity across agents. Denote by \( \tilde{\lambda}_i \in \mathbb{R}^4 \) the vector of measures of agents in current state \( i \). The first two components of vector \( \tilde{\lambda}_i \) correspond to the fractions of domestic agents receiving signals 1 and 2 respectively. The last two components correspond to the fractions of foreign agents receiving signals 1 and 2 respectively.

These measures are summarized in Table 2.

<table>
<thead>
<tr>
<th>Current state</th>
<th>Local agents</th>
<th>Foreign agents</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Signal 1</td>
<td>Signal 2</td>
</tr>
<tr>
<td>I</td>
<td>0</td>
<td>1 - 0</td>
</tr>
<tr>
<td>II</td>
<td>0</td>
<td>1 - 0</td>
</tr>
<tr>
<td>III</td>
<td>1 - 0</td>
<td>0</td>
</tr>
<tr>
<td>IV</td>
<td>1 - 0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: Measure of agents depending on the current state realization

Let \( \bar{a} (\bar{p}, s) \) denote the vector of asset demands that solves optimization problem (5). Namely, \( \bar{a} (\bar{p}, s) = [a_1 (\bar{p}, s), a_2 (\bar{p}, s), a_1^* (\bar{p}, s), a_2^* (\bar{p}, s)] \). Let \( Z_{ij}^k (p) \) denote the excess demand of asset \( k \) in state \( i \) with supply shock \( ij \) and price vector \( \bar{p} \). Formally,

\[
Z_{ij}^k (p) = \sum_{l=1}^4 \bar{\lambda}_i (l) \bar{a}_k (\bar{p}, \bar{s}(l)) - (1 + \bar{\mu}_{ij} (l)) \quad \text{for} \ i, j \in \{I, II, III, IV\}, \quad k \in \{1, 2, 1^*, 2^*\},
\]

where \( \bar{x}(l) \) denotes the component \( l \) of vector \( \bar{x} \). The term \( \bar{s} \) denotes the vector of possible signal realizations. The current state, indexed by \( i \), determines the measure of agents receiving each signal as well as the values that the supply shocks may take. The shocks, indexed by \( j \), determine the available net supply of assets once the inelastic component has been incorporated.
Definition 1 A rational expectations equilibrium (REE) consists of a set of price vectors $\bar{p}_I$, $\bar{p}_{II}$, $\bar{p}_{III}$, $\bar{p}_{IV}$ and individual demands $\bar{a}(\bar{p}, s)$ such that:

(i) $\bar{a}(\bar{p}, s)$ solves each consumer’s optimization problem for market prices $\bar{p}$ and individual signal $s$.

(ii) Markets Clear: $Z^k_{ij}(\bar{p}) = 0 \forall i, j \in \{I, II, III, IV\}$ where $k \in \{1, 2, 1^*, 2^*\}$.

(iii) Agents update their beliefs using their private signal and market prices according to Bayes’ rule.17

Given the particular uncertainty structure assumed in this section, the equilibrium satisfies an extra condition:

- Prices are partially revealing: $\bar{p} = \bar{p}_j$ whenever $\bar{\mu} = \bar{\mu}_{ij} \ \forall i, j \in \{I, II, III, IV\}$.

The consumer’s problem defined in (5) does not allow for a closed-form solution. This implies that the equilibrium must be found using numerical techniques. Given that the trees are ex ante identical and that the optimization problems of domestic and foreign investors are entirely symmetric, it is sufficient to solve for equilibrium prices in two cases. First, when the price of domestic asset 1 is higher than the price of domestic asset 2 and $\nu_1 > \nu_2$. Second, when the same ranking of local prices is combined with $\nu_1 < \nu_2$. Local agents do not receive signals about foreign assets, so the results are invariant to the ranking of prices in foreign markets. We show the results for the first case. The second one features higher levels of home bias in the region we are interested, i.e., when prices do not reveal too much information. In the second case, a majority of domestic agents receives a signal favoring the cheapest local asset, which reinforces the desire to invest locally.

The model is solved assuming logarithmic utility function. The low dividend value is set to 0.8 and the high dividend value is set to 1.2. Finally, the model is solved for many values of $q$ and $\phi$. The first one controls for the degree of informativeness of market prices. The second one determines how informative individual signals are. The results are shown in Figure 6.

If prices are very informative or individual signals are not sufficiently informative, local agents invest roughly half of their portfolios in domestic assets. As prices become less informative or individual signals

17 Appendix contains a formal description of the beliefs’ updating scheme.
become more informative, investors start to bias their portfolio toward domestic securities. The graph shows that the home equity bias can be quite significant when prices are not very informative.

The graph also shows that the bias may decrease with the precision of the individual signal if the latter is already sufficiently informative. The reason is the following. Agents that display the strongest preference toward domestic assets are the ones receiving an incorrect signal. Their beliefs mirror the beliefs of agents receiving the correct signal, but the price of the asset for which they expect higher dividends is lower. As the signal becomes more precise ($\phi$ increases), the fraction of individuals with this strong preference for local assets decreases, driving down the overall home equity bias.

Table 3 illustrates the magnitude of supply shocks for a case where 65% of domestic portfolios are composed of local assets. It shows that it is not necessary to consider too extreme shocks in order to observe a significant level of home bias. Even though the model presented in this section relies on ad-hoc assumptions, we conjecture that extending the model to less arbitrary distributions of shocks or allowing for other sources of uncertainty that mask the current state realization, do not lead to different
qualitative conclusions. The difference is that in a more general case agents need to learn over a fourth dimensional space. This is due to the fact that there are four prices that convey useful information.

The mechanism leading to partially revealing prices would not be qualitatively different from the one assumed in this section, but the level of complexity would be significantly larger.

<table>
<thead>
<tr>
<th>Asset 1</th>
<th>Asset 2</th>
<th>Asset 1*</th>
<th>Asset 2*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>-0.080300</td>
<td>0.010677</td>
<td>-0.358955</td>
<td>0.437354</td>
</tr>
<tr>
<td>-0.358955</td>
<td>0.437354</td>
<td>-0.080300</td>
<td>0.010677</td>
</tr>
<tr>
<td>-0.439255</td>
<td>0.448031</td>
<td>-0.439255</td>
<td>0.448031</td>
</tr>
</tbody>
</table>

Table 3: Supply shocks to domestic and foreign assets when $q = 0.1$ and $\phi = 0.65$ Expressed as a fraction of the average supply of each asset.

6 Analytical characterization in the CARA-Gaussian framework

This section considers a model that shares the blueprints of the framework laid down in Section 2 and has the advantage of being able to solve analytically. However, it uses a more complex and nonstandard structure for macroeconomic analysis. In addition, it generates a volatile fraction of foreign investments. For these reasons it is not taken as our benchmark model.

For simplicity, it is assumed that agents live for two periods. They consume at the end of the second period. Only investment activities take place in the first period. As before, there are two technologies available in each country. Production technology is of the ‘AK’ type, but the productivity shock follows a different process. The shock to technology $i$ ($A_i$) consists of two parts,

$$A_i = \mu_i + \epsilon_i$$

where both $\mu_i$ and $\epsilon_i$ are normally distributed. More precisely,

$$\mu_i \sim N (\theta, \sigma^2_\mu)$$

$$\epsilon_i \sim N \left(0, \sigma^2_\epsilon\right) \quad \forall i = 1, 2, 1^*, 2^*$$

\(^{18}\)Since the only information agents receive is about $\mu_i$. Thus, the component $\epsilon_i$ determines how useful is that information.
This section maintains the assumption that the signal received by local agents reveals information about relative performance of local technologies, but not about the aggregate performance of the home country. Formally, each local agent observes a private signal $s$ that satisfies the following:

$$s = \mu_1 - \mu_2 + \xi.$$  

$$\xi \sim N \left(0, \sigma_{\xi}^2\right)$$

The realization of $\xi$ is idiosyncratic. It is assumed that agents cannot pool the signals. They only observe their own signal. Each foreign investor receives a private signal $s^*$, with

$$s^* = \mu_1^* - \mu_2^* + \xi.$$  

For simplicity, it is assumed that all normal variables introduced before are uncorrelated. The final modification with respect to our benchmark framework is that investors display preferences with constant coefficient of absolute risk aversion. The utility function of local and foreign agents has the following form:

$$u(c) = -e^{-\lambda c}$$

Consider now the problem of a local investor endowed with $\omega$ units of the good and signal $s$. His objective is to maximize his expected utility of consumption, i.e.

$$E \left[-e^{-\lambda \left[k_1 A_1 + k_2 A_2 + k_1^* A_1^* + (\omega - k_1 - k_2 - k_1^*) A_2^*\right] | s}\right],$$

subject to

$$k_i \geq 0 \quad \forall i = 1, 2, 1^*.$$  

The demand of the fourth asset (foreign technology 2) is obtained as a residual. With the utility function assumed in this section, the optimization problem resorts to maximizing the certainty equivalent of consumption. After rearranging terms, the objective function simplifies to the following expression:

$$\begin{bmatrix} k_1 & k_2 & k_1^* & \omega \end{bmatrix} \begin{bmatrix} E [A_1 - A_2^* | s] \\ E [A_2 - A_2^* | s] \\ E [A_1^* - A_2^* | s] \\ E [A_2^* | s] \end{bmatrix} - \frac{\lambda}{2} \begin{bmatrix} k_1 & k_2 & k_1^* & \omega \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_1^* \\ \omega \end{bmatrix}.$$
The matrix $\Sigma$ denotes the covariance matrix of $[A_1 - A_2, A_2 - A_2, A_1 - A_2]$ conditional on the information conveyed by the signal. The conditional expectations and covariance matrix can be found using the projection theorem.\footnote{Consider two normally distributed random vectors, say $X$ and $S$.}

$$\Sigma = \begin{bmatrix} 2\left(\sigma_\mu^2 + \sigma_\epsilon^2\right) - \frac{\sigma_\mu^4}{2\sigma_\mu^2 + \sigma_\epsilon^2} & \sigma_\mu^2 + \sigma_\epsilon^2 + \frac{\sigma_\mu^4}{2\sigma_\mu^2 + \sigma_\epsilon^2} & \sigma_\mu^2 + \sigma_\epsilon^2 - \left(\sigma_\mu^2 + \sigma_\epsilon^2\right) \\ \sigma_\mu^2 + \sigma_\epsilon^2 - \frac{\sigma_\mu^4}{2\sigma_\mu^2 + \sigma_\epsilon^2} & 2\left(\sigma_\mu^2 + \sigma_\epsilon^2\right) - \frac{\sigma_\mu^4}{2\sigma_\mu^2 + \sigma_\epsilon^2} & \sigma_\mu^2 + \sigma_\epsilon^2 - \left(\sigma_\mu^2 + \sigma_\epsilon^2\right) \\ -\left(\sigma_\mu^2 + \sigma_\epsilon^2\right) & -\left(\sigma_\mu^2 + \sigma_\epsilon^2\right) & \sigma_\mu^2 + \sigma_\epsilon^2 - \left(\sigma_\mu^2 + \sigma_\epsilon^2\right) \end{bmatrix}$$

and

$$\begin{bmatrix} E[A_1 - A_2 | s] \\ E[A_2 - A_2 | s] \\ E[A_1 - A_2 | s] \end{bmatrix} = \begin{bmatrix} \sigma_\mu^2 + \sigma_\epsilon^2 \\ \frac{-\sigma_\mu^4}{2\sigma_\mu^2 + \sigma_\epsilon^2} \\ 0 \end{bmatrix}. \quad (6)$$

The expressions for the conditional expectations show that the expected return of asset 1 is corrected upwards upon the arrival of a positive signal, while the expected return of asset 2 is corrected downwards. Also, the main diagonal of $\Sigma$ shows that the availability of some information about domestic assets drives down their perceived variance compared to that of foreign assets. The domestic signal does not carry any information about foreign assets, so its perceived probability distribution coincides with the unconditional distribution.

Equation (6) reports the optimal investment behavior in the unconstrained problem, i.e., when agents do not face short-sales constraints.

$$\begin{bmatrix} k_1 \\ k_2 \\ k_{1*} \end{bmatrix} = \frac{\omega}{4} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \frac{s}{\lambda} \begin{bmatrix} \frac{\sigma_\mu^2 + \sigma_\epsilon^2}{(\sigma_\mu^2 + \sigma_\epsilon^2)(2\sigma_\mu^2 + \sigma_\epsilon^2) - 2\sigma_\mu^4} \\ \frac{-\sigma_\mu^4}{(\sigma_\mu^2 + \sigma_\epsilon^2)(2\sigma_\mu^2 + \sigma_\epsilon^2) - 2\sigma_\mu^4} \\ 0 \end{bmatrix}. \quad (6)$$
If the signal is not very informative (\( \sigma_{\xi}^2 \) is high) asset holdings resemble the perfectly diversified portfolio, where an equal amount is invested in each asset. A similar result holds if investors are highly risk averse (high \( \lambda \)) or returns are volatile (high \( \sigma_{\mu}^2 + \sigma_{\xi}^2 \)). However, it is easy to verify that agents always allocate half of their portfolios in domestic assets, regardless of the signal realization.

In the constrained problem, the solution coincides with (6) whenever the signal does not take extreme values. Equation (7) describes the solution in the case where the short-sales constraint is binding for one of the local assets.

\[
\begin{aligned}
(k_i) &= \frac{1}{3 (\sigma_{\mu}^2 + \sigma_{\xi}^2) \left(2 \sigma_{\mu}^2 + \sigma_{\xi}^2\right) - 2 \sigma_{\mu}^4} \left[ \frac{\sigma_{\mu}^2}{\lambda} \left( \frac{2}{-1} \right) |s| + \omega \left(2 \sigma_{\mu}^2 + \sigma_{\xi}^2\right) \left(2 \sigma_{\mu}^2 + \sigma_{\xi}^2 - \frac{\sigma_{\mu}^2}{2 \sigma_{\mu}^2 + \sigma_{\xi}^2}\right) \right], \\
(k_1) &= \frac{1}{3 (\sigma_{\mu}^2 + \sigma_{\xi}^2) \left(2 \sigma_{\mu}^2 + \sigma_{\xi}^2\right) - 2 \sigma_{\mu}^4} \left[ \frac{\sigma_{\mu}^2}{\lambda} \left( \frac{2}{-1} \right) |s| + \omega \left(2 \sigma_{\mu}^2 + \sigma_{\xi}^2\right) \left(2 \sigma_{\mu}^2 + \sigma_{\xi}^2 - \frac{\sigma_{\mu}^2}{2 \sigma_{\mu}^2 + \sigma_{\xi}^2}\right) \right],
\end{aligned}
\]

where

\[
\begin{aligned}
i &= 1 \text{ if } s \in \left( \frac{\omega \lambda \left( (\sigma_{\mu}^2 + \sigma_{\xi}^2) \left(2 \sigma_{\mu}^2 + \sigma_{\xi}^2\right) - 2 \sigma_{\mu}^4 \right)}{\sigma_{\mu}^2}, \frac{\omega \lambda \left( (2 \sigma_{\mu}^2 + \sigma_{\xi}^2) \left(2 \sigma_{\mu}^2 + \sigma_{\xi}^2\right) - \sigma_{\mu}^4 \right)}{\sigma_{\mu}^2} \right), \\
i &= 2 \text{ if } s \in \left( \frac{-\omega \lambda \left( (\sigma_{\mu}^2 + \sigma_{\xi}^2) \left(2 \sigma_{\mu}^2 + \sigma_{\xi}^2\right) - 2 \sigma_{\mu}^4 \right)}{\sigma_{\mu}^2}, \frac{-\omega \lambda \left( (2 \sigma_{\mu}^2 + \sigma_{\xi}^2) \left(2 \sigma_{\mu}^2 + \sigma_{\xi}^2\right) - \sigma_{\mu}^4 \right)}{\sigma_{\mu}^2} \right).
\end{aligned}
\]

It is easy to check from the previous equation that the fraction invested in local assets grows as the signal increases in absolute value. Agents fully specialize in one of the domestic assets when the signal received is sufficiently large in absolute value.

The previous solution shows a result also observed in the benchmark model: The bias decreases with the degree of risk aversion (\( \lambda \)). From another perspective, we may also expect to observe more diversified portfolios as asset returns become more volatile. The following proposition shows that this is not always the case within the present model.

**Proposition 2** The home equity bias increases with \( \sigma_{\mu}^2 \) if

\[
2 \left( (\sigma_{\mu}^2 + \sigma_{\xi}^2) \left(2 \sigma_{\mu}^2 + \sigma_{\xi}^2\right) - \sigma_{\mu}^4 \right) \\
(3\sigma_{\xi}^2 \sigma_{\mu}^2 - 4\sigma_{\mu}^4) + 2\sigma_{\mu}^2 \left[ \sigma_{\mu}^2 \sigma_{\xi}^2 + 2 (\sigma_{\mu}^2 + \sigma_{\xi}^2) \sigma_{\xi} \right] > 0
\]

**Lemma 3** When \( \sigma_{\mu}^2 \) is sufficiently small, the home equity bias increases with \( \sigma_{\mu}^2 \) when \( \sigma_{\mu}^2 < \sqrt{\frac{3}{4}} \sigma_{\xi} \), and decreases with \( \sigma_{\mu}^2 \) when \( \sigma_{\mu}^2 > \sqrt{\frac{3}{4}} \sigma_{\xi} \).

The explanation is that changes in \( \sigma_{\mu}^2 \) induce a horse race between two effects: On the one hand, as the mean realization becomes more volatile, there is more room for a wider dispersion of beliefs at the individual level. This increases the bias. On the other hand, more volatile returns induce a stronger
desire to hold diversified portfolios. This discourages the concentration of investments in one of the local stocks and henceforth, it reduces the bias.

7 Testable implications

This section lays down the main empirical implications of our model. The fact of considering a framework with multiple assets distinguishes this paper from previous work and allows for a richer set of implications. We find that not only is the present model able to explain a significant fraction of the home equity bias observed in the data, but it is also consistent with several patterns of foreign investment behavior. Albuquerque et al. (2004) analyze the equity portfolios of a set of individuals who traded through a large investment broker between 1991 and 1996. They find evidence suggesting that the portfolio of foreign stocks is more diversified than the portfolio of local stocks. Local agents hold a larger fraction of their foreign equity investments through mutual funds, compared to the fraction of US stocks held through institutional investors. They also find that the share of foreign investments is negatively correlated with the degree of concentration of the portfolio of local stocks. US investors who invest heavily in a few domestic firms tend to allocate a lower fraction of their equity investments in foreign stocks.

For simplicity, the present paper restricts attention to the case where domestic agents do not receive information about foreign assets. In this scenario, local investors specialize in one domestic stocks but hold a perfectly diversified foreign portfolio: they invest the same amount in each foreign stock.\footnote{Allowing domestic agents to receive signals about foreign stocks induces a less diversified foreign portfolio. However, it does not affect the proportion invested in foreign equity as long as the foreign signal is not so informative that local investors fully specialize in one foreign asset.} This is in line with the first finding described above. With respect to the second finding, our model predicts that agents who receive precise information about the relative performance of local stocks display a bias towards local assets. In contrast, local investors who receive less informative signals hold a more diversified local portfolio and display no bias: they invest half of their wealth in foreign assets.

A general prediction of the current setup is that more concentrated portfolios show better performance. When there is heterogeneity in the quality of the information received, the model predicts that individuals with more precise information show a higher specialization and on average, enjoy higher returns. This is consistent with the evidence found by Ivkovich et al. (2004) for a sample of US households. They show that the stocks purchased by individuals with concentrated portfolios display higher returns.
than the stocks purchased by individuals with diversified portfolios. A similar finding is reported by Kacperczyk et al. (2004) for a sample of US equity mutual funds.

Tesar and Werner (1995) report that the turnover rate on foreign equity portfolios is significantly higher than the turnover rate on domestic equity portfolios. This finding has been used as evidence against theoretical explanations that rely on informational asymmetries. If domestic agents receive more precise information about domestic firms than foreign firms, they should trade more intensively on local stocks. However, Warnock (2002) argues that these findings are based on data published before reliable cross-border holdings data were available. He uses more accurate data and finds foreign turnover rates significantly lower than the ones reported in Tesar and Werner (1995), and roughly comparable to domestic turnover rates. Our model features lower foreign turnover rates but given the previous evidence, we do not interpret this fact as a severe limitation of the model.

8 Consumption and output correlations

Backus et al. (1992) illustrate that consumption and output fluctuations across countries are in strong disagreement with the prediction of the standard Real Business Cycle model. In an environment where agents can share risks across countries, we may expect co-movements of consumption across countries to be more pronounced than co-movements of output. Indeed, under complete markets, the former correlation is one regardless of the correlation between output levels. However, Backus et al. (1992) find that consumption is less correlated than output for a sample of OECD countries. The data used in Figures 7 and 8 correspond to a different time frame and frequency than the data used in Backus et al. (1992), but the conclusion remains intact. It can be conjectured that the relatively high consumption correlation is due to the fact that it is only in the last 15 years that we observe a significant amount of international capital flows. Figure 8 suggests that even though there has been an increase in international risk sharing during the last years, consumption growth correlation still tends to be slightly below output growth correlations. This feature has been called the consumption home bias puzzle and has motivated several papers in the field. Lewis (1999) surveys the literature and concludes that none of the existing papers provides a satisfactory answer to this puzzle.

Even though the baseline model analyzed in this paper does not feature complete markets, the class

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21 The exception is Brennan and Cao (1997). They develop a model with asymmetric information that generates higher turnover rates on foreign equity portfolios.
of market incompleteness considered does not induce a decrease in consumption correlation. It is worth noting that the portfolio composition of domestic and foreign investors coincide when there are no information asymmetries. This implies a consumption correlation of one. It is when the asymmetries in information bias the portfolios towards domestic assets that the correlation drops. But this does not provide a satisfactory answer if output correlations are nil, as is the case in the model specification introduced before.\footnote{This section takes a macro interpretation of the model laid down before.} This section extends the basic model in order to allow for positive output correlations. It also disentangles aggregate shocks from pure idiosyncratic shocks that only affect relative performance of technologies in each country.

There are three possible aggregate state realizations in each economy: 1) both technologies receive a high productivity shock 2) only one technology is hit with a high productivity shock 3) both technologies receive a low productivity shock. In fact, the intermediate state hosts two possible growth rates depending on whether the technology that receives the high productivity shock corresponds to the one that has received higher investments or not. We let that be governed by idiosyncratic shocks.

There are nine possible aggregate states in the world economy. The transition probabilities are state
independent, i.e., output shocks are transitory. The transition probabilities are also common knowledge. Agents only learn about idiosyncratic shocks. Thus, to fully specify the probability distribution over aggregate states it is necessary to determine the values of eight probabilities. Assuming symmetry across countries reduces the number to five, but this still implies too many degrees of freedom. The strategy chosen here is to select only two values. The extreme states (described by a square in Figure 9) share the same probability level, denoted by \( \pi^E \). The remaining states, with the exception of the ‘central’ state, share another value, denoted by \( \pi^N \).\(^{23}\)

\[
\begin{array}{ccc}
H & \circ & \square \\
M & \circ & \diamond & \circ \\
L & \square & \circ & \circ \\
\end{array}
\]

Figure 9: Aggregate shock combinations

The idiosyncratic shocks follow a process similar to the one specified in Section 2. When the domestic country aggregate state is non extreme, technology 1 is subject to a high productivity shock with probability \( \nu \), where \( \nu \) is uniformly distributed over the interval \([0, 1]\). In those cases, the shock to technology 2 is perfectly negatively correlated with the shock to technology 1. The information structure is the same as before: A fraction \( \phi \) of domestic investors receives a signal that favors technology 1 whenever \( \nu > 0.5 \), and a signal that favors technology 2 whenever \( \nu < 0.5 \). The remaining \( 1 - \phi \) fraction of local investors receive the ‘incorrect’ signal. The idiosyncratic shock and information structure in the foreign country mirror the ones in the domestic country.

The consumer dynamic optimization problem has the same formulation as equation 1 on page 8. The difference lies in the expectation of the value function for the following period. The assumptions made in this section lead to the following formulation:

\(^{23}\)The states with high growth in one country and low in the other country are weighted by half since they are not subject to idiosyncratic uncertainty
Wealth distribution tends to become very skewed in the simulations performed. For instance, after 100,000 periods, one agent in each country typically owns around 99% of the country’s wealth regardless of the initial wealth distribution. The reason is that tiny differences in the average rate of return produce a widespread wealth gap when they are accumulated over long periods. In theory, this feature does not affect the results. There are no prices to be determined and by the law of large numbers the distribution of wealth among individuals receiving a correct signal is the same as the distribution of wealth among individuals receiving an incorrect signal. However, numerical results are subject to large errors if there is only one agent owning almost all the country’s wealth.

For that reason, the results reported below correspond to a sample of 100 observations of simulation of the world economy. Both economies are simulated for 100 periods and 1000 agents. The value of $\pi^E$ is set equal to 0.15, while $\pi^N$ is set equal to 0.07. Assigning more weight on extreme realizations induce positive cross-country correlation of output growth rates. We choose a value of 1.09 for the high productivity shock and of 1.03 for the low productivity shock. We assume that a fraction of 54% of the population in each country receives a correct signal about the country specific idiosyncratic shock. Utility function is logarithmic in both countries. The previous parameterization induce a home bias of 92.6%, a mean output growth rate of 1.8% and a mean standard deviation of output growth of 2.1%. The results are illustrated in Figure 10.

The model predicts that cross-country consumption correlations are above output correlations. This result is not surprising, though. There is no mechanism in the model that can deliver the opposite result. Individual consumption is just a constant fraction of individual wealth, which in turn, is allocated mostly to one domestic asset while the rest is diversified overseas. This last component of wealth is what explains the higher consumption correlation. In the extreme case where agents invest only in domestic assets,
Consumption correlation would match output correlation.

Previous work has made the point that a large fraction of consumption goods are not internationally tradable, which may explain why aggregate consumption need not be correlated across countries even under efficient risk sharing arrangements. But then, a complete markets setup implies that consumption of tradable goods should display strong co-movements across countries, a fact rejected by the data. In that scenario, the only force than can reduce the correlation of tradable consumption is the presence of nonseparabilities in the utility function between traded and nontraded goods. Complete markets promote the equalization of marginal rates of substitution of traded goods across countries. If preferences are nonseparable, the marginal rates of substitution of traded goods also depend on consumption of nontraded goods. In that context, the domestic growth rate of consumption of traded goods acquires

25Stockman and Tesar (1995) find that cross-country correlation of internationally traded goods is below than that of aggregate consumption.
a more ‘independent life’ from its foreign counterparts.\textsuperscript{26} Lewis (1996) tries to quantify how much of the apparent low degree of risk sharing (suggested by the low consumption correlations of tradables) can be attributed to nonseparabilities between the consumption of traded goods, nontraded goods and leisure. For a sample of 72 countries, she finds that nonseparabilities can account for up to 13 percent of the cross-country variation in consumption of tradables.

Notice that the gap between output and consumption correlations displayed in Figure 10 is not much different from the role Lewis finds for nonseparabilities. This suggests that an extension of the baseline model that disaggregates between nontraded and traded goods may help to provide a theoretical explanation for the low cross-country consumption correlations.

\textsuperscript{26}See Lewis (1999) for a more detailed discussion of the topic.
9 Conclusions

There is pervasive evidence that individuals and institutional investors favor stocks of their own country. In addition, empirical studies show that there exists a home equity bias within US boundaries. Households and mutual funds prefer stocks of proximate companies. These studies also show that the returns agents enjoy on local stocks exceed the returns on non-local stocks. The evidence suggests that the lack of portfolio diversification is based on rational behavior. It also point towards the presence of informational asymmetries in financial markets.

This paper develops a theoretical model that can explain a significant fraction of the bias observed in the data. The model differs from the standard theory in three aspects. First, it considers the case of multiple stocks per country. Second, it assumes that local investors are able to collect more precise information about the ranking of local stocks than that of foreign stocks. Third, it assumes that short-sales are costly. In this environment, each domestic investor displays a strong preference for certain local stocks. When the information collected is sufficiently precise, local investors find it convenient to finance purchases of the perceived good local stocks by selling short the perceived bad local stocks. However, if the cost of short-selling is high, domestic investors decide to sacrifice the diversification services provided by their foreign investments in order to concentrate their equity portfolio on the (domestic) stocks that are thought to offer higher expected returns. Unlike previous papers in the literature, the underlying mechanism that explains the bias for local stocks is based on first order effects, i.e., differences in expected returns. This explains the ability of the model to generate significant quantitative results.

We show that the strong home equity bias implied by the model is robust to several changes in the baseline specification. However, there was one extension that was not pursued in the paper: the case of persistence in stock dividends. The introduction of permanent shocks has a double effect. On the one hand, it increases the power of private information. The latter can now be used to forecast the future stream of returns. Without persistence, it only helps to predict next period returns. This effect strengthens the mechanism that generates home bias. On the other hand, there is more public information available. If dividends have a persistent component, agents can learn from past realizations of dividends. This reduces the role of private information, and undermines the incentives to invest heavily in local stocks. However, this extension poses two challenges from a technical point of view.

\footnote{This conclusion depends on the fact that stock prices adjust to the information contained in past dividend realizations. If prices are constant over time, like in the ‘AK’ model, the adjustment is made through quantities. In this case, the fraction invested overseas is low on average, but can display a high volatility.}
First, it can only be solved under a recursive structure. The problem becomes intractable if agents need to keep track of all past dividend realizations and signals received in order to compute their beliefs. Second, it requires dealing with multiple state variables with continuous domain.

The data shows that cross-country consumption correlations tend to be lower than cross-country output correlations. This has been named the home consumption bias. Although the stylized model presented in the paper is not able to account for this fact, it can explain an important fraction of the discrepancy between the prediction of the standard theory and the data. In addition, the results obtained in previous studies suggest that allowing for nonseparabilities in the utility function can help to fully explain the aforementioned bias.
References


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A Beliefs updating scheme when agents learn from signals and prices

Agents need to infer the probability distribution over future states before solving their optimization problem. They receive two pieces of information: prices and individual signals. Both of them reveal information regarding the relative values of $\nu_1$, $\nu_2$, $\nu_1^*$, and $\nu_2^*$. Agents’ beliefs consist of the expected probability distribution over future states conditional on the information received. In order to simplify the exposition, it is assumed that the market price vector corresponds to the equilibrium prices agents would observe in current state $I$ without supply shocks, i.e., $\bar{p} = \bar{p}_I$. It is straightforward to generalize the formulas to other cases. The formal expression for the expected probability of future state $i, j, i^*, j^*$ given prices $\bar{p}$ and private signal $s$ is illustrated below.

$$E[\Pr (i, j, i^*, j^*) | \bar{p}, s] = \frac{E[\Pr (i, j, i^*, j^*, \bar{p}, s)]}{\Pr (\bar{p}, s)} =$$

<table>
<thead>
<tr>
<th>Expression</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pr (\nu_1 &gt; \nu_2; \nu_1^* &gt; \nu_2^*) \left( \frac{1-q}{4} + q \right) \Pr (s</td>
<td>\nu_1 &gt; \nu_2) E[\Pr (i, j, i^<em>, j^</em>)</td>
</tr>
<tr>
<td>$\Pr (\nu_1 &lt; \nu_2; \nu_1^* &gt; \nu_2^*) \frac{1-q}{4} \Pr (s</td>
<td>\nu_1 &lt; \nu_2) E[\Pr (i, j, i^<em>, j^</em>)</td>
</tr>
</tbody>
</table>

The second equation above uses the law of conditional probabilities and the third one uses Bayes’ rule. Every current state realization could lead to the observed market price $\bar{p}$. Thus, when agents compute their beliefs, they span over the four possible current states. The first element in each term on the numerator denotes the a priori probability of being in each current state. The second and third components capture the probability of observing prices $\bar{p}$ and signal $s$ for each current state. Finally, the fourth component computes the expected probability that the future dividend shocks take values $i, j, i^*, j^*$ for each current state realization.

---

28 It is sufficient to compute the expectation of these probabilities because the latter enter linearly in the individual’s first order conditions.
B Proof of proposition 2

Denote by $\Phi$ the fraction invested in local assets. There is a home bias when $\Phi > 0.5$. The aggregate fraction invested in local assets depends on the actual realizations of $\mu_1$ and $\mu_2$. The latter condition the distribution of information across agents. For instance, if the difference between these two variables is large, a high fraction of local investors will receive extreme signals. In order to allow for a general statement, we consider the ex ante expectation of $\Phi$. That is, the unconditional expected fraction invested in local assets. The latter is computed as follows:

$$
\begin{align*}
E(\Phi) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{2\pi \sigma_\mu^2} \exp \left[ - \frac{(\mu_1 - \theta)^2}{2\sigma_\mu^2} - \frac{(\mu_2 - \theta)^2}{2\sigma_\nu^2} \right] \\
& \times \left\{ 
\int_{-s}^{\infty} f(s | \mu_1, \mu_2) \, ds \, \mu_1 \, ds + \int_{-s}^{\infty} g(s) \, f(s | \mu_1, \mu_2) \, ds \, \mu_1 \, ds + \int_{-s}^{\infty} f(s | \mu_1, \mu_2) \, ds \, \mu_2 \, ds + \right\}
\end{align*}
$$

where

$$
\begin{align*}
\mathcal{g}(s) &= \frac{1}{3 (\sigma^2_\mu + \sigma^2_\nu)} \left\{ \sigma^2_\mu \omega s \left| s \right| + (2 \sigma^2_\mu + \sigma^2_\nu) \left( \sigma^2_\mu + \sigma^2_\nu \right) \right\} \\
\mathcal{g}(\bar{s}) &= \frac{\omega L \left\{ \sigma^2_\mu + \sigma^2_\nu \right\} \left( 2 \sigma^2_\mu + \sigma^2_\nu \right) - \sigma^4_\mu}{\sigma^4_\mu}
\end{align*}
$$

We are interested in the derivative

$$
\frac{\partial E(\Phi)}{\partial \sigma^2_\mu}
$$

It is easy to verify that the derivatives of the threshold values ($s$ and $\bar{s}$) with respect to $\sigma^2_\mu$ cancel out and therefore, do not play a role. This implies that the sign of Equation (B.1) depends on the sign of the derivatives of the values taken by $g(s)$. Without loss of generality, we consider now the case where $s > 0$.

$$
\frac{\partial g(s)}{\partial \sigma^2_\mu} = \frac{2s}{\sigma^2_\mu} \left( 3 \sigma^2_\mu \sigma^2_\nu - 4 \sigma^4_\mu \right) + 2 \sigma^2_\mu \left( \sigma^2_\nu + \sigma^2_\xi \right) + 2 \left( \sigma^2_\mu + \sigma^2_\nu \right) \sigma^2_\xi
$$

$$
\frac{\partial g(\bar{s})}{\partial \sigma^2_\mu} = \frac{2s \left( 3 \sigma^2_\mu \sigma^2_\nu \right) - 4 \sigma^4_\mu \left( \sigma^2_\mu + \sigma^2_\xi \right)}{3 \left( \sigma^2_\mu + \sigma^2_\nu \right) \left( 2 \sigma^2_\mu + \sigma^2_\xi \right) - 2 \sigma^4_\mu}
$$

43
The sign of the expression above is ambiguous. However, it is possible to find a sufficient condition that can help us to identify cases where the derivative takes positive values. If the sign is positive for $s = \bar{s}$, then it must be positive for all possible signals belonging to the range $[\underline{s}, \bar{s}]$. This substitution yields the sufficient condition stated in the text. From the previous equation, it is easy to see that when $\sigma^2_\mu$ is sufficiently small, the first term in the numerator dominates the entire expression and therefore, it determines the sign of the derivative. ■