Commitment, Banks and Markets

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March 14, 2005

Preliminary Draft

Abstract

This paper examines how banks and financial markets interact with one another to provide liquidity to investors. The critical assumption in our paper is that financial markets are characterized by limited enforcement of contracts. Hence in the event of default only a fraction of the borrowers assets can be seized. Limited enforcement reduces market participation since individuals face debt constraints, allows for equilibria where banks and markets coexist together and allows banks to provide more liquidity than markets in equilibrium. Furthermore, in equilibrium, all long term assets are held by banks because pooling resources allows a bank to increase its collateral base and therefore avoid debt constraints. Our assumption of limited enforcement therefore explains the maturity mismatch between the assets and liabilities of a bank and allows us to interpret banks as commitment mechanisms.

1 Introduction

In their paper "Bank Runs, Deposit Insurance, and Liquidity", Diamond and Dybvig show how deposits issued by banks provide liquidity\textsuperscript{1} to investors. Their model, however, has an important limitation that was pointed out by Jacklin (1987). Allowing investors to trade simultaneously in financial markets limits the amount of liquidity that a bank can provide. The reason for this is that the existence of a market provides more alternatives for a depositor to withdraw early. Jacklin’s critique has led to two fundamental questions. First, can banks improve on the liquidity provided by a market

\textsuperscript{1}An asset is liquid if it allows investors to consume as and when they want to.
when they coexist together? Second, under what conditions do we have equilibria where both banks and markets coexist? Papers that have examined banks and markets in an integrated framework address these questions by assuming restricted market participation (Diamond (1997), Allen and Gale (2004)). That is a certain fraction of agents do not have access to markets but have access to banks.

Restricted market participation allows banks to increase liquidity and gives rise to equilibria with both banks and markets. We, however, wish to dig one level deeper and specify explicit market frictions that cause restricted market participation. So in addition to asking the two questions above we can also examine other reasons as to why banks exist. In particular, we wish to focus on the role of banks as a commitment mechanisms.

In this paper, financial markets are characterized by an imperfect enforcement mechanism (a court or a law enforcement agency) that can only recover a fraction of the loan in the event of default. We are interested in three main questions. First, how does imperfect enforcement affect market participation? Second, how does imperfect enforcement affect the ability of banks to provide liquidity when they coexist with markets? Third, what roles do banks and markets play in equilibrium when markets are characterized by imperfect enforcement?

The basic model that we build on is Diamond (1997) which in turn builds on Diamond and Dybvig (1983). In his model, investors may have a privately observable liquidity shock and therefore need to consume early. They have two choices available to them. They can invest in real assets directly and trade in financial markets or they can deposit their funds in a bank in exchange for a stream of payments. Diamond assumes that a certain fraction of investors without shocks (lenders or demanders of claims) do not have access to markets. He then derives two main results from this assumption. First, liquidity is endogenous in this framework and depends on the fraction of lenders who participate in the market. With a smaller number of lenders the demand for claims falls and hence reduces the total claims traded in a market. This reduces the consumption of those investors who have shocks. Second, investors who cannot participate in markets have a lower incentive to withdraw early and can therefore cross subsidize those with liquidity shocks. Hence ex-post heterogeneity of investors has a very important role to play in his framework.

We depart from Diamond's framework by introducing an explicit friction in financial markets. If a borrower defaults in a financial market then only a fraction of the loan can be recovered. This fraction is known with certainty by all agents in the economy. This assumption is supposed to capture the
inefficiencies and delays involved with decisions made by a court of law. Examples of papers that use variants of this assumption are Kehoe and Levine (2001) and Azariadis and Lambertini (2003). We assume that banks can commit to pay back their depositors. This is based on the argument of Diamond and Rajan (2000) where banks are vulnerable to runs if they go back on their commitments to depositors.

We split our analysis into two possible cases. First consider the case where banks cannot trade in markets. Because of limited enforcement, borrowers in a financial market can only use a fraction of their long term returns as collateral and hence face endogenous debt constraints. Restricted supply of claims drives the prices of claims up. This increase in the price of claims allows banks to provide more insurance to those who need to consume early. This is because patient depositors have less incentive to withdraw early and lend in a financial market. Since banks provide better insurance, investors deposit all of their funds with the bank. Hence in this scenario banks can increase liquidity but we do not have an equilibrium with a mixed system.

Now consider the case where banks can also participate in markets. Arbitrage restrictions ensure that the price of claims is equal to the marginal rate of transformation. This places constraints on the amount of liquidity that a bank can provide even though banks do better than markets on this front. The key result here is that all long term assets are held by banks in equilibrium. The reason for this is as follows. When individuals have a liquidity shock they use all of their long term returns as collateral and hence face debt constraints. A bank by pooling resources can increase its collateral base and avoid debt constraints. Pooling resources therefore allows the bank to insure individuals against commitment risk.

Following Jacklin (1987), there have been several studies that examine the viability of banks as liquidity providers when investors have access to markets on the side. von Thadden (1998b) reviews the literature on liquidity provision by banks. He emphasizes two factors which constrain liquidity provision. First, if real assets are reversible then liquidity constraints are more severe. This reasoning holds good even in a dynamic continuous time framework (von Thadden (1998a)). The second reason is based on the co-existence of markets and is more closely related to our work. Other papers that examine banks and markets in the Diamond and Dybvig framework are Allen and Gale (2004) and Fecht, Huang, and Martin (????). Allen and Gale (2004) construct a general equilibrium model with banks and markets. In their framework, banks have two roles to play. First they provide liquidity to investors and second they trade Arrow securities in markets on behalf of individual investors to insure them against real shocks. One crucial differ-
ence in their framework is that individuals cannot participate in markets. Also we do not have any real shocks in our paper. Fecht, Huang, and Martin (????) focus on the tradeoff between liquidity provision by banks and growth that results from direct investment in assets.

The rest of the paper is organized as follows. Section 2 considers a situation with financial markets alone and shows how the provision of liquidity increases with the effectiveness of enforcement. Section 3 shows that the optimal quantity of liquidity desired by an individual is in excess of that provided by financial markets. Hence banks have a potential role to play as providers of liquidity. Section 4 considers a situation where banks and financial markets co-exist. The relative costs of banks and financial markets mentioned above are traded off to determine the sizes and roles of financial markets and banks. We conclude our paper with Section 5.

2 Basic Model with Financial Markets

2.1 Technology and Preferences

There are three dates $t = 0, 1, 2$. There is a single good in the economy which is used for consumption and investment purposes. There are a continuum of agents on the interval $[0,1]$ who are identical at date 0 and each agent is endowed with one unit of the good. There are two assets in the economy.

A short term asset yields $R_1 > 0$ in date 1 and 0 in date 2. $\epsilon > 0$ in date 1 and $R_2 > R_1$ in date 2 where $\epsilon$ is very small. Both of the assets satisfy constant returns to scale and all investment takes place at date 0. The following table summarizes the asset structure where one unit of the consumption good is invested at date 0.

<table>
<thead>
<tr>
<th>Date</th>
<th>$t = 0$</th>
<th>$t = 1$</th>
<th>$t = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short Term Asset</td>
<td>$-1$</td>
<td>$R_1$</td>
<td>0</td>
</tr>
<tr>
<td>Long Term Asset</td>
<td>$-1$</td>
<td>$\epsilon &gt; 0$</td>
<td>$R_2$</td>
</tr>
</tbody>
</table>

At date 1, with probability $q$ an agent has a liquidity shock that is privately observable which makes him want to consume at both dates and with probability $(1 - q)$ he does not have a liquidity shock and only wants to consume at date 2. We refer to individuals who have a shock as impatient or Type 1 individuals. Likewise individuals who do not receive a shock

2 Unlike Diamond and Dybvig (1983) all investment takes place at date 0. Hence the short term asset cannot be reinvested at date 1. This assumption simplifies the banks problem later in the analysis.
are referred to as patient or Type 2 individuals. An individual’s ex-ante preferences at date 0 are, therefore, given by the utility function

\[ U(c_{11}, c_{12}, c_{22}) = \begin{cases} 
  u(c_{11}) + \phi c_{12} & \text{with prob } q \\
  u(c_{22}) & \text{with prob } (1 - q)
\end{cases} \]

where \( u \) is strictly increasing, twice continuously differentiable, strictly concave, and satisfies the Inada conditions and \( \phi > 0 \). The Law of Large Numbers implies that a fraction \( q \in (0, 1) \) have liquidity shocks at date 1 and the remaining fraction \((1 - q)\) derive utility from date 2 consumption only. The preferences differ from Diamond and Dybvig in that individuals with liquidity shocks also get utility out of consuming at date 2. In addition we also make the following assumption.

Assumption M: \( \phi > 0 \) is sufficiently small so that \( \frac{\phi}{u'(\frac{R_1}{R_2}q)} \leq \frac{R_1}{R_2} \).

Assumption M states that the marginal rate of substitution of date 2 consumption for date 1 consumption for the impatient type is less than the marginal rate of transformation for all technologically feasible consumption bundles. In other words, an impatient type has preferences biased towards date 1 consumption.

### 2.2 Financial Markets

At date 0, individuals invest a fraction of their endowment \( \alpha \) in the short term asset and the remaining fraction \((1 - \alpha)\) in the long term asset to try and insure themselves against possible liquidity shocks. At date 1, after the shock is realized, they can buy or sell claims that pay out one unit of the good at date 2. The price of each claim is \( p \) and is measured in terms of the good at date 1. At date 1, impatient types consume their returns from the short term asset and supply claims (borrow) against a fraction \( \beta \) of their returns from the long term asset, which is used as collateral. The remaining fraction of date 2 returns is consumed at date 2. Patient types, on the other hand, demand claims at date 1 (lend) and consume their returns from date 2. Hence consumption \( c_{it} \) where \( i \) indicates the type of an individual and \( t \) indicates the date of consumption is given by

\[ c_{11} = \alpha R_1 + \beta (1 - \alpha) R_2 p \quad (2.1) \]
\[ c_{12} = (1 - \beta)(1 - \alpha) R_2 \quad (2.2) \]
\[ c_{22} = \frac{\alpha R_1}{p} + (1 - \alpha)R_2 \]  

(2.3)

We assume that there is an external enforcement agency that can seize a fraction \( \theta \) of the borrowers assets in case he chooses to default. In order to prevent borrowers from defaulting strategically, \( \alpha \) and \( \beta \) must satisfy a debt incentive constraint given by

\[
(1 - \theta)(1 - \alpha)R_2 \leq (1 - \beta)(1 - \alpha)R_2
\]

The left hand side of the constraint is what the borrower gets when he defaults on his loan at date 2 and the right hand side is what he gets if he decides to pay back his creditors. The constraint above reduces to

\[
\beta \leq \theta
\]

(2.4)

This debt incentive constraint has a simple interpretation. The constant cost of default per unit of the good is \( \theta \) and the constant benefit of default per unit of the good is \( \beta \). In order for default to not take place in equilibrium the benefit per unit of the good must be less than or equal to the cost per unit of the good. An alternative interpretation is that only a fraction \( \theta \) of long term returns can be used as collateral and this therefore restricts the total number of claims that a borrower can supply. In other words, limited enforcement leads to overcollateralization in financial markets.

2.3 Equilibrium

An equilibrium in a financial market is defined as \((p, \alpha, \beta)\) such that the following conditions hold

1. \( \alpha, \beta \in \arg\max qu(c_{11}) + q\phi c_{12} + (1 - q)u(c_{22}) \) subject to (2.1)-(2.4)

2. \( (1 - q)\alpha R_1 = qp\beta(1 - \alpha)R_2 \)

The above conditions state that taking prices as given, individuals maximize expected utility at date 0 and that markets for claims clear.

Hence an equilibrium is characterized as a solution to the following problem.

\[
\max_{\alpha, \beta} qu(c_{11}) + q\phi c_{12} + (1 - q)u(c_{22})
\]

subject to (2.1)-(2.4) and the market clearing condition. The First Order Conditions to the problem are as follows:
\[ qu'(c_{11}) (\frac{R_1}{R_2} - \beta p) - q \phi(1 - \beta) + (1 - q) u'(c_{22}) \left( \frac{R_1}{R_2 p} - 1 \right) = 0 \tag{2.5} \]

\[ qu'(c_{11}) (1 - \alpha) R_2 p - q \phi(1 - \alpha) R_2 = \lambda \tag{2.6} \]

\[ \beta \leq \theta \quad \text{and} \quad \lambda \geq 0 \quad \text{w.c.s} \tag{2.7} \]

The following lemma and proposition examine the effect of limited enforcement on the liquidity provided by financial markets. The proofs are in the appendix.

**Lemma 1.** In equilibrium we have \( \frac{R_1}{R_2} \leq p \leq \frac{R_1}{R_2 \theta} \) and \( \beta = \theta \). The inequalities are strict if \( \theta < 1 \).

The intuition behind the lemma is as follows. If prices of claims are too high then both borrowers and lenders would want to only invest in the long term asset and markets do not clear. If prices are too low then both borrowers and lenders will invest in the short term asset and markets do not clear once again. To understand why the debt incentive constraint binds let us combine the inequality \( p \geq \frac{R_1}{R_2} \) with Assumption M. This implies that the quantity of date 1 consumption that an impatient agent wants in exchange for a unit of date 2 consumption is less than what the market gives him. Hence impatient agents will not want to consume at all at date 2. Limited enforcement of contracts, however, allows only a fraction \( \theta \) of long term returns to be used as collateral. The rest has to be consumed at date 2.

**Proposition 1.** There exists a unique equilibrium with \( \alpha \leq q \) and \( c_{11} \leq R_1 \) and these values are strictly increasing in \( \theta \).

Proposition 1 illustrates how consumption levels of those with shocks are positively related to the limited enforcement parameter \( \theta \). Therefore, financial markets may not be able to provide adequate insurance against these shocks. The intuition for the result is as follows. When \( \theta \) falls, it affects both the first order conditions (and hence the demand function) of the investor and the market clearing condition. First consider the effect on the demand function. Suppose we start out with an initial \( \theta_0 \) and and \( p_0 \) and \( \alpha_0 \) are the associated equilibrium price and investment in short term asset. Then the first order conditions imply that at \( (p_0, \alpha_0) \) the investor is indifferent between both short term and long term assets. If \( \theta \) falls then at
prices \( p_0 \) the utility derived from investing in the short term asset exceeds that of the long term asset. This reduces the supply of claims to 0. In order for markets to clear prices have to rise. However, since liquidity shocks are random and since those with shocks derive some utility from date 2, the rise in prices does not have to exactly offset the decline in \( \theta \). Now let us examine the market clearing condition. Since the rise in \( p \) does not offset the decline in \( \theta \), \( \alpha \) must fall and must be less than \( \frac{1}{2} \). Lemma 1 and Proposition 1 allow us to endogenize liquidity provided by a financial market. Consumption levels of both types now depend on the limited enforcement parameter \( \theta \). The above results also indicate a potential role for banks as a commitment mechanism.

### 3 Optimal Liquidity

The results from the previous section demonstrate the effect of the limited enforcement parameter \( \theta \) on the liquidity provided by a financial market. In this section we find the efficient levels of consumption in an environment with perfect information and compare this with consumption levels in a financial market. We will show that financial markets cannot adequately insure those with shocks and that limited enforcement compounds the problem.

Suppose there are no informational asymmetries at date 1. Then the optimal provision of liquidity is the solution to the following maximization problem.

\[
\max_{c_{11}, c_{12}, c_{22}} q u(c_{11}) + q \phi c_{12} + (1 - q) u(c_{22})
\]

subject to the resource constraint

\[
\frac{qc_{11}}{R_1} + \frac{qc_{12}}{R_2} + (1 - q)c_{22} = 1 \tag{3.1}
\]

and subject to the non-negativity constraint

\[ c_{12} \geq 0 \]

The First order conditions to the problem are

\[
qu'(c_{11}) = \frac{\lambda q}{R_1} \tag{3.2}
\]

\[
q\phi - \frac{\lambda q}{R_2} = -\mu \tag{3.3}
\]
(1 - q)u'(c_{22}) = \frac{\lambda(1 - q)}{R_2} \tag{3.4}

where \( \lambda \) is the multiplier associated with the resource constraint and \( \mu \) is the multiplier associated with the non-negativity constraint.

**Lemma 2.** If the coefficient of relative risk aversion is strictly greater than 1 then the first best consumption levels must satisfy \( c_{11} > R_1 \), \( c_{22} < R_2 \) and \( c_{12} = 0 \). If this is the case financial markets provide too little liquidity.

**Proof.** Substituting \( \lambda \) from (3.2) into (3.3) and using Assumption M we have \( \mu > 0 \) which implies \( c_{12} = 0 \) at the optimum. Combining (3.2) and (3.4) we have,

\[
\frac{u'(c_{22})}{u'(c_{11})} = \frac{R_1}{R_2}
\]

If the coefficient of relative risk aversion is greater than 1 then the resource constraint implies that \( c_{11} > R_1 \) and \( c_{22} < R_2 \).

In the following section we try to examine whether banks can improve liquidity by insuring those with liquidity shocks and providing higher levels of consumption.

## 4 Banks and Financial Markets

Banks are modelled as depository institutions as in Diamond and Dybvig (1983). They take individuals decisions as given and choose a deposit contract that pays \( d_1 \) at date 1 to depositors who claim to be impatient and \( d_2 \) at date 2 to depositors who claim to be patient\(^3\). Since banks cannot observe depositor types these contracts have to be incentive compatible. Competition between banks ensures that they maximize the expected utility of an individual at date 0. Investors take the deposit contract as given and decide how to divide their endowment between real assets and a bank. So at date 0 an investor has three choices. He can invest a fraction of his endowment \( \alpha_1 \) in the short term asset, another fraction \( \alpha_2 \) in the long term asset and the remaining fraction \( \alpha_3 \) with the bank.

We assume that banks can commit to pay back depositors. This assumption is based on the reasoning provided by Diamond and Rajan (2000). Suppose banks earn rents on long term assets. Then if a bank does not pay

\(^3\)In general banks can offer contracts to each type that are date contingent. We will, however show that the optimal contract involves banks offering each type consumption at a particular date only.
back depositors at date 1 then its long term assets are seized and liquidated at a very high cost. The same is true if a bank defaults at date 2. We have not, however, explicitly accounted for banks earning rents in this model. We could partially relax the assumption that banks can commit to pay back depositors but it would make the model more complicated without adding any new insights.

We split our analysis into two possible cases. In the first case banks do not participate in markets. We relax this assumption and allow banks to participate in markets in the next subsection.

4.1 Banks do not participate in markets

Consider the case where banks cannot participate in markets. Let us examine the banks problem in closer detail. Consumption of an individual now depends on the fraction of his endowment deposited with a bank and his investment in long and short term assets. Consumption for type $i$ at date $t$ is given by

$$c_{11} = \alpha_1 R_1 + \beta(1 - \alpha_1 - \alpha_3) R_2 p + \alpha_3 d_1$$  \hspace{1cm} (4.1)

$$c_{22} = \frac{\alpha_1 R_1}{p} + (1 - \alpha_1 - \alpha_3) R_2 + \alpha_3 d_2$$  \hspace{1cm} (4.2)

$$c_{12} = (1 - \beta)(1 - \alpha_1 - \alpha_3) R_2$$  \hspace{1cm} (4.3)

In addition, the deposit contract has to satisfy a resource constraint and incentive compatibility conditions given by

$$\frac{qd_1}{R_1} + \frac{(1 - q)d_2}{R_2} = 1$$  \hspace{1cm} (4.4)

$$d_2 \geq \frac{d_1}{p}$$  \hspace{1cm} (4.5)

$$u(\alpha_1 R_1 + \beta p\alpha_2 R_2 + \alpha_3 d_1) \geq u(\alpha_1 R_1 + \beta p\alpha_2 R_2 + \beta p\alpha_3 d_2) + \phi(1 - \beta)\alpha_3 d_2$$  \hspace{1cm} (4.6)

4.2 Equilibrium

An equilibrium with banks and financial markets consists of a price $p$, an individuals portfolio ($\alpha_1, \alpha_2, \alpha_3$) and a deposit contract $(d_1, d_2)$ such that individuals maximize expected utility subject to (2.4) and (4.1)-(4.6) and
such that markets clear. Markets open up after individuals have withdrawn from a bank and the market clearing conditions is given by

\[ q(1 - \alpha_1 - \alpha_3)R_2\beta p = (1 - q)\alpha_1 R_1 \]  

(4.7)

Hence, an equilibrium is characterized as a solution to the following problem

\[ \max \ (qu(c_{11} + q\phi c_{12}) + (1 - q)u(c_{22})) \]

subject to all the constraints specified above. The First Order Conditions to the individuals problem are

\[ qu'(c_{11})(R_1 - \beta pR_2) + (1 - q)u'(c_{22})(\frac{R_1}{p} - R_2) - q\phi(1 - \beta)R_2 \leq 0 \]  

(4.8)

\[ qu'(c_{11})(d_1 - \beta pR_2) + (1 - q)u'(c_{22})(d_2 - R_2) - q\phi(1 - \beta)R_2 \leq 0 \]  

(4.9)

\[ q\alpha_2 R_2(u'(c_{11})p - \phi) = \lambda \]  

(4.10)

where \( \lambda \) is the multiplier associated with the debt incentive constraint.

The First Order Conditions for the banks problem can be summarized as

\[ q\alpha_3(u'(c_{11}) - \frac{R_2}{R_1}u'(c_{22}) = \mu(\frac{qR_2}{R_1(1 - q)} + \frac{1}{p}) \]  

(4.11)

where \( \mu \) is the multiplier associated with the incentive compatibility condition of type 2.

We want to find necessary and sufficient conditions under which banks can provide higher liquidity when individuals have the option of investing in both institutions. It turns out that \( \theta < 1 \) is both necessary and sufficient for banks to provide more liquidity than markets. \( \theta \) can be interpreted as cost of markets. Our results will therefore indicate that markets must have a cost associated with them in order for banks to provide better liquidity.

First let us examine the necessary conditions. The following proposition demonstrates that \( \theta < 1 \) is necessary for banks to increase liquidity.

**Proposition 2.** Let \( \theta = 1 \) then in equilibrium we must have \( p = \frac{R_1}{R_2} \) and \( c_{11} = R_1, \ c_{12} = 0 \) and \( c_{22} = R_2 \). In this case individuals are indifferent between markets and banks because they provide the same liquidity.
The proof of Proposition 2 is in the appendix. Proposition 2 gives us necessary conditions for banks to improve on the liquidity provided by markets. The intuition behind the proposition is as follows. If enforcement is perfect then individuals with a liquidity shock will consume nothing at date 2. Hence the only price at which both types of individuals are indifferent between the long and short asset is \( p = \frac{R_1}{R_2} \). If \( p > \frac{R_1}{R_2} \) then both types will only demand long term assets and markets never clear. Likewise if \( p < \frac{R_1}{R_2} \) then both types will only demand the short term asset and hence markets never clear. Suppose banks did try to increase liquidity by setting \( d_1 > R_1 \) then patient depositors can withdraw early and lend in a financial market. But this violates incentive compatibility. Hence conditions in a financial market limit a bank’s ability to increase liquidity. Proposition 2 yields the same result as Jacklin (1987). Banks and markets are perfect substitutes because they provide the same amount of liquidity.

The following lemma allows us to characterize prices in equilibrium. It also tells us that the debt incentive constraint always binds.

**Lemma 3.** In equilibrium we must have \( p \geq \frac{R_1}{R_2} \) and hence \( \beta = \theta \). If \( \theta < 1 \) then the inequality is strict.

The proof is in the appendix. The intuition for the lemma is exactly the same as Lemma 1. If prices of claims are too low then both types will want to invest in the short term asset and markets never clear. Since prices of claims are high enough, individuals will want to use all of their long term assets as collateral and hence the debt constraint binds.

The following lemma allows us to ignore the nonlinear incentive compatibility condition for impatient types whenever the incentive compatibility condition for patient types is satisfied.

**Lemma 4.** If \( p \geq \frac{R_1}{R_2} \) and if the incentive compatibility condition for Type 2 binds then the incentive compatibility condition for Type 1 holds.

The proof is in the appendix. The results from Lemma’s 3 and 4 are used to prove the following proposition which states that banks can increase liquidity if markets have costs associated with them.

**Proposition 3.** If \( \theta < 1 \) then there exist a continuum of equilibrium prices \([p, \bar{p}]\) with \( p > \frac{R_1}{R_2} \) and \( p \) is strictly decreasing in \( \theta \). Furthermore all investors deposit their entire endowment with the bank.
The proof of the proposition is in the appendix. This proposition demonstrates how banks use imperfections in a market to increase liquidity. If $\theta < 1$ then the bank can increase liquidity in the following way. If $\theta$ falls then because of the debt incentive constraint the supply of claims falls. This increases the prices of claims which in turn reduces the incentive for a patient depositor to withdraw early and lend in a financial market. Since banks can increase liquidity and since there are no costs associated with banks individuals decide to deposit all of their endowment in the bank.

4.3 Banks participating in markets

The previous subsection placed restrictions on banks participating in markets. We now examine a case where banks have complete access to markets. We are again interested in the same questions as the earlier sections. Can banks increase liquidity relative to markets and do we have equilibria where both institutions coexist? However, we would like to go a step further and also examine the role that a bank plays in equilibrium. In particular we are interested in the role of a bank as a commitment mechanism.

We can start by writing out the budget constraints that a bank faces. These are given by

\[ qd_1 = \gamma_1 \delta R_1 + p\gamma_2(1 - \delta)R_2 \]
\[ (1 - q)d_2 = (1 - \gamma_1)\frac{\delta R_1}{p} + (1 - \gamma_2)(1 - \delta)R_2 \]

where $\delta$ is the fraction of the per capita endowment invested in the short term asset, $\gamma_1$ is the fraction of short term returns paid to early withdrawers and $(1 - \gamma_2)$ is the fraction of long term returns paid directly to late withdrawers.

We are also interested in examining the debt incentive constraint of a bank. Suppose $\theta_B$ is the fraction of long term assets that can be seized by the bank in the event of default\(^4\). Let $(1 - \delta)R_2 = R'_2$. Then the banks debt incentive constraint can be written as $(1 - \gamma_2)R'_2 \geq (1 - \theta_B)R'_2$. This reduces to $\gamma_2 \leq \theta_B$. Notice that if the bank only invests in long term assets then this constraint reduces to $q \leq \theta_B$. The intuition behind this is that the bank has to use a fraction $q$ of its long term assets to pay those with liquidity shocks. As long as $q < \theta_B$ the bank has enough collateral to borrow against.

The following lemma places restrictions on prices in equilibrium. The lemma still has to be proved formally.

**Lemma 5.** In equilibrium $p = \frac{R_1}{R_2}$

\(^4\theta_B\) can be different from $\theta$
The intuition behind lemma 5 is as follows. First suppose $p < \frac{R_1}{R_2}$. Then both types of individuals will only want to invest in the short term asset. This is because $R_1 \geq pR_2$. This is also true for banks. By shifting all of their resources to the short term asset they can offer higher levels of consumption for both types without violating incentive compatibility conditions. A similar line of reasoning works if $p > \frac{R_1}{R_2}$. First notice that banks can provide higher liquidity because the incentive constraint for patient types is relaxed. This results in all individuals depositing their endowment with the bank. If we can show that banks do not hold any short term assets then we are done since markets will never clear. Now suppose banks choose $\delta > 0$. Then by shifting resources to the long term asset the bank can yield higher returns because of higher prices.

The following proposition characterizes the equilibrium when both banks and markets coexist and when banks trade in markets. The proposition has to be proved formally.

**Proposition 4.** Let $\theta < 1$. Then in equilibrium all long term assets are held by the bank.

The intuition behind Proposition 4 is as follows. If $\theta < 1$ and $p = \frac{R_1}{R_2}$ then from Lemma 1 all individuals will only want to invest in the short term asset. Hence in equilibrium it must be the case that banks hold all the long term assets in the economy. But why do banks have an incentive to hold long term assets in the economy? This can be understood by examining the debt incentive constraint for the bank. The debt incentive constraint for banks is given by $(1 - \theta_B)(1 - \delta)R_2 \leq (1 - \gamma_2)(1 - \delta)R_2$ where $\theta_B$ is the fraction of banks assets that can be seized in the event of default. This simplifies to $\gamma_2 \leq \theta_B$. If the bank only holds long term assets this condition reduces to $q \leq \theta_B$. Hence the debt incentive constraint holds for a sufficiently high $\theta_B$. This is the key result that arises from this proposition. Banks by pooling resources do not face debt constraints that individuals in markets face. Hence in equilibrium it is possible for them to hold long term assets. The short term assets can be held by individuals in a market or by banks. If banks have small variable costs associated with them then all of the short term assets will be held by individuals in the market. This is similar to the assumption Diamond (1997) makes to get a mixed system.
5 Conclusion

This paper studies interactions between banks and markets when the main function of the financial system is to provide liquidity. Financial markets are characterized by limited enforcement and we study how this friction affects coexistence and the provision of liquidity by both banks and markets. We find that when banks have restricted access to markets then they face fewer constraints to provide liquidity but we do not have a mixed system. Allowing banks to participate in markets places more restrictions on liquidity provision but allows for mixed systems. Furthermore banks hold all long term assets in the economy and play a role as a commitment mechanism. This result can be interpreted within the context of securitization. Several studies have questioned the role of banks with the development of markets for securitized assets. This study, however shows that banks do not face the same collateral constraints as individuals do on a market and hence play an important role in securitized markets.

One important limitation of the analysis is that we cannot relate our limited enforcement parameter $\theta$ to financial structure in the economy. This could be overcome by introducing costs to the banking sector. The threat of bank runs is an example.

APPENDIX

Proof of Lemma 1: Suppose $p = (\frac{R_1}{R_2} - \epsilon)$ in equilibrium where $\epsilon > 0$. Define the marginal utility of investing in the short term asset as $MU(\alpha)$. Then

$$MU(\alpha) = qu'(c_{11})(\frac{R_1}{R_2} - \beta(\frac{R_1}{R_2} - \epsilon)) - q\phi(1 - \beta) + (1 - q)u'(c_{22})(\frac{R_1}{R_2p} - 1)$$

which can be rewritten as

$$MU(\alpha) = qu'(c_{11})\frac{R_1}{R_2}(1 - \beta) - q\phi(1 - \beta) + K$$

where

$$K = qu'(c_{11})\beta\epsilon + (1 - q)u'(c_{22})(\frac{R_1}{R_2p} - 1) > 0$$

since $u'(.) > 0$

From Assumption M we have that $MU(\alpha)$ is strictly positive and markets never clear. Hence in equilibrium $p \geq \frac{R_1}{R_2}$
Now suppose \( p = \left( \frac{R_1}{R_2 \theta} + \epsilon \right) \) in equilibrium where \( \epsilon > 0 \). Then the marginal utility of investing in the short term asset is strictly negative and markets do not clear again. Hence in equilibrium \( p \) has to lie in the interval specified above. Similar arguments show that the inequalities are strict if \( \theta < 1 \)

Since \( p \geq \frac{R_1}{R_2} \) in equilibrium, the marginal utility of \( \beta \) is strictly positive implying \( \lambda > 0 \). The complementary slackness conditions imply \( \beta = \theta \).

**Proof of Proposition 1:** In order to show existence substitute the market clearing condition into (2.5). Define

\[
g(\alpha) = q \frac{R_1(q - \alpha)}{R_2 q(1 - \alpha)} u'(\frac{\alpha R_1}{q}) - q\phi(1 - \theta) \\
+ (1 - q)(\frac{q\theta(1 - \alpha)}{(1 - q)\alpha} - 1)u'(1 - \alpha) R_2(\frac{q\theta}{(1 - q)} + 1)) \tag{2.10}
\]

From the inada conditions on \( u \) we have \( \lim_{\alpha \to 0} g(\alpha) = \infty \) and \( \lim_{\alpha \to 1} g(\alpha) = -\infty \). Since \( u \) is twice continuously differentiable it follows that (2.10) is continuous in \( \alpha \) over [0, 1]. Hence from the intermediate value theorem we know that there exists \( \alpha \in (0, 1) \) such that the above \( g(\alpha) = 0 \). Since \( p \leq \frac{R_1}{R_2 \theta} \) in equilibrium it follows that \( \alpha \leq q \).

In order to show uniqueness notice that

\[
g'(\alpha) = \frac{R_1}{R_2}(u'(c_{11}) (q - 1) (1 - \alpha)^2 + \frac{(q - \alpha)}{(1 - \alpha)} u''(c_{11}) \frac{R_1}{q} \\
+ (1 - q)(u'(c_{22}) \frac{-q\theta}{(1 - q)^2\alpha^2} - \frac{q\theta(1 - \alpha)}{(1 - q)\alpha} - 1)(u''(c_{22}) \frac{q\theta}{(1 - q)} + 1) R_2)\]

Since \( u \) is concave and since \( p \geq \frac{R_1}{R_2} \) we have \( g'(\alpha) < 0 \). Hence in equilibrium \( \alpha \) is unique.

Now substitute \( p \) from the market clearing condition into equation (2.1). We then have

\[
c_{11} = \alpha R_1 + \theta(1 - \alpha) R_2 \left( \frac{1 - q}{q\theta(1 - \alpha)} \frac{\alpha R_1}{R_2} \right)
\]

which can be reduced to

\[
c_{11} = \frac{\alpha R_1}{q}
\]
Since \( \alpha \leq q \), it follows that \( c_{11} \leq R_1 \). If \( \theta < 1 \) then for the same reasons the inequalities are strict. Since \( g(\alpha, \theta) \) is strictly increasing in \( \theta \) and since \( g \) is strictly decreasing in \( \alpha \) it follows that \( \alpha \) is strictly increasing in \( \theta \). ■

**Proof of Lemma 3:** Let \( p \) be the price of claims in equilibrium. Define \( MU(\alpha_1) \) as the marginal utility of investing in the short term asset. Suppose \( p < \frac{R_1}{R_2} \). Then \( MU(\alpha_1) \) is strictly positive which implies \( \alpha_1 = 1 \). If this is the case then markets never clear which contradicts the fact that \( p \) is an equilibrium price. Using Assumption M and the fact that \( p \geq \frac{R_1}{R_2} \), we know that the debt incentive constraint (2.4) binds in equilibrium. ■

**Proof of Proposition 2:** The incentive compatibility conditions, (4.5) and (4.6) when put together give us \( d_2 = \frac{d_1}{p} \). Define \( MU(\alpha_1) \) and \( (MU(\alpha_3) \) as the marginal utilities associated with the short term asset and deposit. Both \( MU(\alpha_1) \) and \( (MU(\alpha_3) \) are equal to 0 only if \( p = \frac{R_1}{R_2} \). If \( p > \frac{R_1}{R_2} \) then from the resource constraint we have \( d_1 > R_1 \) and \( d_2 < R_2 \) and both \( MU(\alpha_1) \) and \( (MU(\alpha_3) \) are negative which contradicts the first order conditions. A similar argument rules out \( p < \frac{R_1}{R_2} \). ■

**Proof of Lemma 4:** Suppose \( d_1 = d_2p \). Define \( c_{11} = \alpha_1 R_1 + \beta p^2 \alpha_2 R_2 + \alpha_3 d_1 \) and \( c'_{11} = \alpha_1 R_1 + \beta p \alpha_2 R_2 + \beta p \alpha_3 d_2 \). The inequality (4.6) holds iff
\[
 u(c_{11}) - u(c'_{11}) \geq \phi(1 - \beta) \alpha_3 d_2
\]
Dividing both sides by \( \alpha_3 d_2 p (1 - \beta) \) we get
\[
\frac{u(c_{11}) - u(c'_{11})}{\alpha_3 d_2 p (1 - \beta)} \geq \frac{\phi(1 - \beta) \alpha_3 d_2}{\alpha_3 d_2 p (1 - \beta)}
\]
which reduces to
\[
\frac{\phi(c_{11} - c'_{11})}{u(c_{11}) - u(c'_{11})} \leq p
\]
Using Assumption M and the result from Lemma 3 we have
\[
\frac{\phi(c_{11} - c'_{11})}{u(c_{11}) - u(c'_{11})} < \frac{R_1}{R_2} \leq p
\]
Hence the incentive compatibility condition for Type 1 holds.

Proof of Proposition 3: Suppose the incentive constraint for type 2 binds. From Lemma 3 we know that \( p > \frac{R_1}{R_2} \). The resource constraint (4.4) and the incentive constraint for Type 2 then imply \( d_1 > R_1 \). Hence whenever (4.8) is equal to 0 then (4.9) is strictly positive. Therefore \( \alpha_1 = \alpha_2 = 0 \) and \( \alpha_3 = 1 \). Combining equations (4.4) and (4.5) we have \( c_{11} = d_1 = \frac{1}{S} \) and \( c_{22} = d_2 = \frac{1}{S_p} \) where \( S = \left( \frac{q}{R_1} + \frac{(1-q)}{R_2p} \right) \). If \( p = \frac{R_1}{R_2} \) then the left hand side of (4.8) is strictly positive and if \( p = \frac{R_1}{R_2\theta} \) then the left hand side of (4.8) is strictly negative. Using the intermediate value theorem, there exists a \( p \) such that the marginal utility of \( \alpha_1 = 0 \). Hence at \( p \), \( \alpha_3 = 1 \) and \( \alpha_1 = \alpha_2 = 0 \) satisfy both the first order conditions and the market clearing condition. The upper limit on the equilibrium price \( \overline{p} \) is determined by the price where (4.9) is equal to 0. Since \( u \) is concave we can show that \( \overline{p} \) is unique.

References


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