Development Accounting with Endogenous TFP

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Abstract

Influential works by Klenow & Rodriguez-Clare (1997), Hall and Jones (1999), and Parente & Prescott (2000), among others, have argued that most of the cross country differences in output per worker is explained by differences in total factor productivity (TFP). This conclusion, however, is obtained in a framework that explicitly or implicitly assumes TFP to be exogenous. We study whether this conclusion holds when TFP is endogenous. We device a general model that can accommodate diverse endogenous and exogenous TFP models in the literature such as Romer (1990), Jones (1995), and Klenow & Rodriguez-Clare (2004). The law of motion of TFP involves two components. The first reflects that TFP is costly to accumulate as it requires to divert resources out of production. The second component reflects the speed at which free knowledge is diffused in the world. We show analytically that allowing for TFP endogeneity always increases the role of factors in explaining cross-country income differences. We also find that unless the speed of diffusion is un-plausible large, the main conclusion of the studies above is overturned. Most of the cross-country differences in output per worker are explained by differences in savings rates and human capital.

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1 Introduction

Consider the almost 36-fold difference in output per worker between the United States and Niger in 1988. According to Hall and Jones (1999), physical capital per worker was around 80 times larger in the US than in Niger, and human capital was around 3 times larger in the US. For standard physical and human capital shares, of 1/3 and 2/3 respectively, differences in these two factors alone can explain a 9-fold gap in output per worker. The remaining 4-fold of the gap must be attributed to differences in the efficiency in the use of those factors, or total factor productivity (TFP.). These simple calculations indicate that the main reason why the US produces much more output per worker than Niger is because of its larger endowments of physical and human capital.

Hall and Jones (1999), however, reached a completely different conclusion. They argued that part of 80-fold difference in capital per worker is actually due to the 4-fold different in TFPs. This is because a country with lower TFP produce less output and can accumulates less capital if savings rates are about the same. Consequently, Hall and Jones impute part of the capital differences to TFP differences, following a methodology proposed by Klenow & Rodriguez-Clare (1997). Taking into account the direct effect of TFP on output, and the indirect effect of TFP on capital, Hall and Jones find that the 4-fold TFP gap accounts for an almost 8-fold difference in output per worker1. The remaining 4.5-fold gap is explained by factors intensity. As a result, they conclude that the main reason for Niger relative poverty is its low TFP. More generally, they conclude that most of the differences in the productivity of labor across countries is explained by TFP differences. A similar conclusion is obtained by Klenow & Rodriguez-Clare, and Parente & Prescott (2000), among others, using a similar methodology. These works call for theories of TFP, and have being instrumental in changing the focus of researchers and policymakers toward TFP and away from factors.

A shortcoming of the studies mentioned above is that they explicitly or implicitly assume TFP to be exogenous. In particular, they do not allow for any feedback from factor endowments, and their determinants, to explain TFP differences. This feedback clearly occurs in models of endogenous growth, such as Romer (1990), models in which the amount of factors affects the extent of research and development activities and, consequently, the TFP level of the economy.

The objective of this paper is to study what are the sources of cross-country income differences

1To be precise, Hall and Jones find a 7.7 difference.
within a framework that allows TFP to be endogenous. For this purpose, we generalize the textbook Solow growth model to allow for endogenous TFP. The model is sufficiently flexible so that for certain parameter values TFP is actually exogenous. More importantly, the model can also accommodate the basic mechanisms of other endogenous growth models such as Romer (1990), Jones (1995), Eaton and Kortum (1996), or Klenow & Rodriguez-Clare (2004).

In our model, a country can accumulate TFP from two sources. One source is costless adoption of technologies available elsewhere. These technologies include better production methods, but it also may include better institutions that increase productivity. The speed at which technological diffusion occurs turns out to be the critical parameter.

The second source of TFP accumulation are costly TFP investments. They are governed by a production function that transform inputs into TFP. One can interpret this function directly as an R&D sector, as in the endogenous growth literature. More generally, however, it captures the idea that efficiency improvements are costly because they require to divert resources such as capital, and labor from the production of output. As an example, consider the case of Niger. Its 4-fold TFP gap with the U.S. may be related to poor institutions, poor enforcement, corruption, extreme inequality, or more generally, to low levels of social capital as described by Hall and Jones. The TFP production function postulates that social capital and TFP can improve but at a cost. This cost is determined by the degree of returns to scale of the TFP production function. If returns to scale are zero, then the cost of increasing TFP is infinite. We show that this is the implicit assumption in current accounting exercises. We relax this assumption by increasing the returns to scale toward a value more in line with the endogenous growth literature, which is typically one (e.g., Romer 1990, Jones 1995.)

We find that unless the speed of technological diffusion is implausibly large, the main conclusion of the studies above is overturned. Most of the cross country differences in output per worker is explained by differences in savings rates and human capital. For example, assuming constant returns to scale in the TFP production function, and under the seemingly extreme assumptions that capital is not an input in the production of TFP and that there are no externalities in the production of TFP, we find that more than 60% of the cross country dispersion of output per worker is explained by differences in savings rates and human capitals. This number is only 40% when TFP is assumed to be exogenous. The explanatory power increases with the share of capital in the
production of TFP, and with the degree of externalities in the production of TFP. If the share of capital in the production of TFP is increased to 1/3, just as it is in the production of output, then differences in saving rates and human capital can explain 100% of the cross country dispersion of income per worker. Our preferred value for the role of factor is 75%, a value that is supported on regression analysis.

The paper is organized as follows. Section 2 provides a review of the existing accounting methods and results. Section 3 is the core of the paper. It presents the model of endogenous TFP. Section 3.1 focuses on the model without scale effects and presents the results of this version. Section 3.2 focuses on the model with scale effects and presents the corresponding results. Section 4 concludes.

2 Existing Literature

This section provides a critical review of the existing literature on the topic of development accounting. We organize the discussion using a unified framework, and report results using a common database, the one provided by Hall and Jones. Some of the results in this section are new, and intended to provide a complete picture about development accounting results with exogenous TFP. The next section contrasts these results with the ones obtained in an endogenous TFP framework.

Existing accounting methodologies differ in two key aspects: in the representation of the production function, and in the precise statistic constructed to assess the contributions of factors and TFP.

2.1 Formulations of the production function

There are two representations of the production function used in the literature. One is the technological production function that describes how aggregate physical capital $K$, and aggregate human capital $H$, are transformed into aggregate output $Y$. The production function is typically assumed to have the Cobb-Douglas form:

$$Y = AK^\alpha H^{1-\alpha}$$

where $A$ is the total factor productivity. Output per-worker is then given by

$$y = Ak^\alpha h^{1-\alpha} \equiv A_1 \cdot X_1$$  \hspace{1cm} (1)
where $k \equiv \frac{k}{L}$ and $h = \frac{H}{L}$ are the amount of physical and human capital per worker, $X_1 \equiv k^{\alpha} h^{1-\alpha}$ denotes “factors of production,” and $A_1 = A$. This representation of the production function is similar to the one used by Denison (1967), King and Levine (1994), and Caselli (2003). Given information about $y$, $k$, $h$, and $\alpha$, the residual $A_1$ can be computed as $A_1 = y/X_1$. The residual in this representation is just the TFP.

An alternative representation of the production function has been popularized by Klenow and Rodríguez-Clare (1997), and Hall and Jones (1999). They argue that part of the cross-country differences of capital stocks result from TFP differences. As a result, part of the capital differences must be imputed to the TFP differences. For this purpose, they rewrite (1) as\footnote{Klenow and Rodríguez-Clare (1997) use a different production function than the one used here. Theirs is $Y = K^{\alpha} H^{\alpha_H} (AL)^{1-\alpha K - \alpha_H}$, where $H$ is human capital, and $L$ is labor force. This is Hall and Jones formulation.}

$$y = A_1^{1-\alpha} \kappa^{1-\alpha} h = A_2 \cdot X_2$$

where $\kappa \equiv \frac{k}{y}$, $A_2 = A_1^{1-\alpha}$ and $X_2 = \kappa^{1-\alpha} h$. The residual under this formulation is obtained as $A_2 = y/X_2$. This representation was used by Mankiw, Romer, and Weil (1990) in their study of the Solow model. In the Solow model the steady state capital-output ratio is completely determined by the saving rate independently of TFP levels. Thus, an exogenous permanent increase of TFP affects output but it does not affect the long term capital-output ratio. As a result, capital has to increase in response to a higher TFP. For the Solow model, and more generally for models of exogenous TFP, $A_2$ and $X_2$ truly provide a decomposition in terms of fundamentals. Notice that the residual in the second representation is not the TFP but a modified TFP. It is instructive to write $A_2$ in the following way:

$$A_2 = \underbrace{A}_{\text{Direct TFP Effect}} \cdot \underbrace{A_1^{1-\alpha}}_{\text{Indirect TFP Effect}}$$

This representation makes clear that the residual in the second formulation provides both a direct and an indirect role for TFP. Below we normalize TFPs and factors by the richest country in the sample. In that case, $A$ is typically below 1 for a poor country, and therefore $A_2 < A$. This means that more of the country’s relative poverty is explained by the residual in the second representation of the production function than in the first representation. Hence, less of the country’s relative poverty is explained by the factors term $X_2$ relative to the factors term $X_1$.\footnote{Klenow and Rodríguez-Clare (1997) use a different production function than the one used here. Theirs is $Y = K^{\alpha} H^{\alpha_H} (AL)^{1-\alpha K - \alpha_H}$, where $H$ is human capital, and $L$ is labor force. This is Hall and Jones formulation.}
The main difference between the two representations of the production function is that one describes a short term relation between inputs and outputs, while the other describes a long term relationship. The first representation is useful to decompose output in terms of the actual factor endowments and TFPs available at certain point in time. The second formulation adopts a theory of capital and TFP accumulation that allows to impute observed differences in factors as arising from current differences in TFPs. Such imputation only makes sense if capital has had enough time to adjust to TFP differences and those differences have prevail for some time.

2.2 Measures of contribution

Once factors (either $X_1$ or $X_2$) and residuals (either $A_1$ or $A_2$) are computed, accounting methodologies also differ in how to assess their independent role in explaining cross-country difference in output per worker. Following closely the growth accounting methodology, Denison (1967), and King and Levine (1994) measure the contribution of factors and TFP in the following way. First, the ratio of output per worker in countries $i$ and $j$ satisfies $\frac{y_i}{y_j} = \frac{A_i}{A_j} \frac{X_i}{X_j}$, where $A$ refers to either $A_1$ or $A_2$, and $X$ refers to the corresponding $X_1$ or $X_2$. Taking logs in both sides gives

$$\ln \frac{y_i}{y_j} = \ln \frac{A_i}{A_j} + \ln \frac{X_i}{X_j}. \tag{4}$$

Taking country $j$ to be the richest country in the sample (in terms of $y$), this expression suggests that the contribution of factors and TFP in accounting for country’s $i$ relative poverty be measured as

$$\Phi_{Xi}^1 = \frac{\ln X_i/X_j}{\ln Y_i/Y_j} \text{ and } \Phi_{Ai}^1 = \frac{\ln A_i/A_j}{\ln Y_i/Y_j}.$$ 

A natural way to assess the average contribution of factors and TFP for a group of countries is to take the average $\Phi’s$ in the sample:\footnote{Neither Denison (1967) nor King and Levine (1994) use this average contribution. Denison did not provide an average, and King and Levine use a more complicated measure that requires to classify countries by deciles of income.}

$$\Phi_X^1 = \frac{1}{I} \sum_{i=1}^{I} \Phi_{Xi}^1 \text{ and } \Phi_A^1 = \frac{1}{I} \sum_{i=1}^{I} \Phi_{Ai}^1 \label{5}$$

Note that by construction $\Phi_{Xi}^1 + \Phi_{Ai}^1 = 1$, and therefore $\Phi_X^1 + \Phi_A^1 = 1$.

A different measure is used by Klenow and Rodríguez-Clare (1997)(KRC henceforth). Using a
more statistical approach based on variance decomposition, they measure the contribution of factors as the fraction of the cross-country income dispersion that can be attributed to cross-country factors dispersion. More specifically, equation (4) implies that

$$\text{var} \left( \ln \frac{y_i}{y_j} \right) = \text{var} \left( \ln \frac{A_i}{A_j} \right) + \text{var} \left( \ln \frac{X_i}{X_j} \right) + 2 \text{cov} \left( \ln \frac{A_i}{A_j}, \ln \frac{X_i}{X_j} \right).$$

KRC propose to measure the contribution of factors and TFP as follows

$$\Phi^2_X = \frac{\text{var} \left( \ln \frac{X_i}{X_j} \right) + \text{cov} \left( \ln \frac{A_i}{A_j}, \ln \frac{X_i}{X_j} \right)}{\text{var} \left( \ln \frac{y_i}{y_j} \right)}$$

$$\Phi^2_A = \frac{\text{var} \left( \ln \frac{A_i}{A_j} \right) + \text{cov} \left( \ln \frac{A_i}{A_j}, \ln \frac{X_i}{X_j} \right)}{\text{var} \left( \ln \frac{y_i}{y_j} \right)}$$

A useful property of these measures is that they add up to one\(^4\). An important drawback, however, is the arbitrary assignment of the covariance. Under the hypothesis that TFP is exogenous, the covariance term should be zero. However, it turns out to be large in the data. KRC acknowledge that this implies that TFP is actually endogenous. In order to give some role to factors in affecting TFP accumulation, KRC assign half of the covariance to the factors term and half to the TFP term. However, they still use (2) to compute the residuals, a formulation that assumes an exogenous TFP and therefore bias the variance terms toward TFP. We are able to show below how the KRC methodology bias the results towards TFP. In particular, we argue, all the covariance term should be assigned to factors.

Finally, Caselli (2004) advocates the use of counterfactuals to assess the role of factors. In particular, he asks: “what would be the world dispersion of output per worker if only factors differ across countries but not TFPs?” If factors are important, then this dispersion must be large relative

\(^4\)To understand the differences between (5) and (6) measures, consider a case in which there is one rich country and a large number of poor countries. Assume that for all poor countries \(X_i/X_j = 0.5\) while for the other half \(A_i/A_j = 1\). Since all countries except one have the same amount of factors, \(\text{var} \left( \ln \frac{X_i}{X_j} \right) = 0\), \(\text{cov} \left( \ln \frac{A_i}{A_j}, \ln \frac{X_i}{X_j} \right) = 0\), and therefore \(\Phi^2_X \simeq 0\). In words, (6) would state that none of the cross-country income dispersion is explained by the dispersion of factors. All is explained by TFP dispersion. On the other hand, notice that for all countries except one, \(y_i/y_j \simeq 0.5\) because poor countries have the same amount of factors and almost the same amount of TFP. Thus, \(\ln X_i/X_j \simeq 1\) and therefore \(\Phi^2_X \simeq 1\). In words, (5) states that all the gap between the poor countries and the rich country is fully explained by factors, and none by TFP. Thus, (5) provides an assessment about underdevelopment even in the absence of dispersion.
to observed dispersion. He measures the contribution of factors by

$$\Phi^3_X = \frac{\text{var}(\ln X_i/X_j)}{\text{var}(\ln Y_i/Y_j)}$$

Since Caselli analyzes the counterfactual in which only factors differ across countries but not TFPs, he does not evaluate the contribution of TFP. Evidently, the latter is not $1 - \Phi^3_X$, but

$$\Phi^3_A = \frac{\text{var}(\ln A_i/A_j)}{\text{var}(\ln Y_i/Y_j)}$$

which corresponds the question of what fraction of the observed dispersion of output per worker can be accounted for by TFP dispersion only. A limitation of $\Phi^3_X$ and $\Phi^3_A$ is that they do not add up to one. However, one may assess the relative contributions by comparing $\Phi^3_X$ and $\Phi^3_A$.

We also study an alternative counterfactual to the one proposed by Caselli. It answers the question of how much the world income inequality would be reduced if factors of production were equalized across countries but TFPs were kept at current levels? The following statistic answers this question

$$\Phi^4_X = 1 - \frac{\text{var}(\ln A_i/A_j)}{\text{var}(\ln Y_i/Y_j)}.$$

$\Phi^4_X$ points to a policy question by providing an assessment of the potential benefits of reducing factors dispersion. Notice that $\Phi^4_X = 1 - \Phi^3_A$. Finally, the following statistic gives an ideas about the benefits of reducing TFP dispersion:

$$\Phi^4_A = 1 - \frac{\text{var}(\ln X_i/X_j)}{\text{var}(\ln Y_i/Y_j)}.$$

$\Phi^4_A$ responds the question of how much would the world income inequality would be reduced if TFPs were equalized across countries but factors of production were kept at current levels.

### 2.3 Development Accounting with Exogenous TFP

We now report the statistics described in the previous section for the two representations of the production function. For this purpose, we use the Hall and Jones (1998) data set. It contains information about output, physical capital, and human capital for 126 countries for the year 1988.

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5 The data is available at http://emlab.berkeley.edu/users/chad/HallJones400.asc
Using information about population size from the World Development Indicators (2001), we construct weighted version of the statistics, where the weights were the relative population sizes. It turns out that weighted statistics provide similar results as their unweighted counterparts particularly when Luxemburg is excluded from the sample. We decided to exclude Luxemburg, and report only the more widely use unweighted statistics. Table 1 reports the statistics. Probably the most well-known statistics are ϕ^2_{A^2} and ϕ^2_{X^2}, the ones used by Klenow & Rodriguez-Clare, which produces a 60% contribution for TFP and 40% for factors with this data set. The counterfactual used by Caselli (2004), ϕ^3_{X^1} produces a similar result: 41% of the dispersion of percapita output per worker is explained by the factors only model.

Table 1 serves to put these well-known results into context. First, the representation of the production function matters significantly. The role of TFP is substantially larger under the second representation for all statistics. If the first representation is used, the message of Table 1 is that factors matter substantially more than TFP to explain development. For example, the version of the King and Levine (1994) statistics, ϕ^1_{A^1} and ϕ^1_{X^1}, provides a 30% contribution of TFP and 70% contribution of factors. Table 1 also shows a limitation of the counterfactual employed by Caselli. Although the factors only model can reproduces 41% of the observed dispersion, the TFP only model reproduces even less, 20%. The remaining 39% is covariance, which Caselli seems to

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6 Luxemburg is the second richest country in the sample, with a similar percapita output to the U.S. but very different levels of factors and TFP. As a result, ϕ^1_{X^1} and ϕ^1_{A^1} were particularly large for Luxemburgh, which distorted the some unweighted statistics significantly.

7 King and Levine found a much lower contribution of factors. However, they did not include human capital into their computations as we do.
implicitly assign to TFP.

We now relax the assumption of an exogenous TFP implicit in Table 1, and assess how the results of Table 1 changes, in particular the second half of the Table.

3 A Model of Endogenous TFP and Capital

3.1 The model

Consider the following Solow model extended to incorporate TFP accumulation along the lines of Jones (1995) and Klenow and Rodriguez-Clare (2004):

\[ Y_t = K_t^\alpha (Z_t hL_{Yt})^{1-\alpha} \]  
\[ K_{t+1} = (1 - \delta) K_t + sY_t \]  
\[ L_{t+1} = \gamma L_t \]  
\[ Z_{t+1} - Z_t = B_t \left( K_t^\theta (hL_{Zt})^{1-\theta} \right) ^\psi \]  
\[ B_t = \lambda \left( \frac{Z_t}{Z_t} \right)^\eta Z_t^\sigma \]  
\[ L_{Zt} = \mu_L L_t, \quad L_{Yt} = (1 - \mu_L)L_t, \quad 0 < \mu_L < 1 \]  
\[ K_{Zt} = \mu_K K_t, \quad K_{Yt} = (1 - \mu_K)K_t, \quad 0 < \mu_K < 1. \]

The first three equations correspond to the core of the Solow model. Equation (7) describes the production of aggregate output, \( Y_t \), which requires capital, \( K_t \), TFP, \( A_t \equiv Z_t^{1-\alpha} \), and quality adjusted labor, \( hL_{Yt} \), where \( h \) is the average level of human capital in the economy. Equation (8) states that capital can be accumulated and that investment is a constant fraction of aggregate output. Equation (9) states that labor grows exogenously at the constant gross rate \( \gamma L \).

Equations (10) and (11) describes how TFP is accumulated\(^9\). Equation (10) states that TFP investments require capital, \( K_{Zt} \), and quality adjusted labor, \( hL_{Zt} \). The parameter \( \psi \) is degree of returns to scale in the production of TFP, with \( \psi = 1 \) signifying constant returns to scale. In line

\(^8\)The first half of the table is identical because it is a short term decomposition. TFP is mainly exogenous in the short term in our model.

\(^9\)More precisely, how \( Z \) is accumulated, which also implies that TFP, \( A \), is accumulated. In what follows we use loosely the term TFP to refer both to \( A \) and \( Z \).
with the endogenous growth literature, (11) allows for the existence of externalities in the production of TFP which occurs when \( \phi \neq 0 \). Moreover, the term \( \left( \frac{Z_t^*}{Z_t} \right)^\eta \) captures international spillovers. Here, \( Z_t^* \) is the technological frontier (which can be country specific, but assumed to be exogenous to the country). For \( \eta > 0 \), the term \( \left( \frac{Z_t^*}{Z_t} \right)^\eta \) captures the idea that lagging behind the technology frontier facilitates technological progress via adoption. Equation (11) can be written as:

\[
B_t = \lambda Z_t^\eta Z_t^\phi
\]

where \( \phi \equiv \sigma - \eta \).

Finally, equations (12) and (13) state that constant fractions of total labor and total capital are allocated to the production of goods and to the production of TFP.

Equations (7) to (13) describe a very general model of capital and TFP accumulation that incorporates other models in the literature. For example, the standard Solow model with exogenous technological change is obtained making \( \psi = \mu_L = \mu_K = 0, \phi = 1 \) and \( \eta = 0 \); A barebone version of Romer’s model of endogenous growth is obtained when \( \theta = \mu_K = 0, \phi = \gamma_L = 1 \) and \( \psi = 1 \) and \( \eta = 0 \); Jones (1995) model of endogenous growth is obtained making \( \theta = \mu_K = 0, \psi = 1 \) and \( \phi < 1, \eta = 1 \). Our model thus allows us to reproduce Hall and Jones (1998) results, but more importantly, allows to check the robustness of those results. Microfoundations for this type of reduced form model have been provided by Romer (1990) and Jones (1995.)

We now proceed to characterize the dynamics of this model economy. Denote by lower case letters the per capita variables of the model. Thus, for example, \( y_t \equiv Y_t / L_t \). As before, denote \( \kappa_t \equiv \frac{K_t}{Y_t} \) the capital-output ratio. Using (7) and (12), \( y_t \) can be written as

\[
y_t = (1 - \mu_L) Z_t (\kappa_t)^{1-\alpha} h.
\]

This is a version of the equation employed by Klenow & Rodriguez-Clare and Hall & Jones to perform their decomposition (see Equation 2 and notice that \( Z_t = A^{1-\alpha} \)). Since \( Z_t \) is endogenous in our model, we need to go beyond this equation. At this point, we restrict the parameters of the model to preclude scale effects, in the sense of Jones (1995), from arising in our model. This is probably the most relevant case because scale effects are hard to defend empirically. Klenow and Rodriguez-Clare (2004) also employ this assumption.
3.2 Characterization of the Equilibrium

Consider the case in which the scale economy, either in terms of population, capital, or output, does not affect the long term growth rate of the economy. It turns out that the following parameter restriction eliminates this type of scale effect from the model.

Assumption 1: \( \phi + \psi \theta < 1 \).

We now characterize the dynamics of the model economy under Assumption 1. Denote \( \gamma_{V,t} = \frac{V_{t+1}}{V_t} \) the gross growth rate of an arbitrary variable \( V \), and \( g_{V,t} = \gamma_{V,t} - 1 \) its net growth rate. The following Proposition states the main result of the paper, a generalized version of Equation (2) that allows for endogenous TFP.

**Proposition 1** Let Assumption 1 hold. Then \( y_t \) satisfies

\[
y_t = A_{3t} \cdot X_{3t} \tag{16}
\]

where \( X_{3t} \equiv h^{1+\frac{1}{1-\phi-\psi}} \frac{\alpha}{1-\phi-\psi} (1+\frac{\psi}{1-\phi-\psi}) \) and \( A_{3t} \equiv (1-\mu_L) \left( \frac{\lambda}{g_{Zt}} \mu^\phi (1-\mu_L) \psi^\theta \frac{\mu}{\mu_L} (1-\theta)^{L_t^\psi} \right) \frac{1}{1-\phi-\psi} \)

**Proof.** According to equations (10) and (11)

\[
g_{Zt} = \lambda Z_t^{*\eta} Z_t^{\phi-1} (\mu_K K_t)^{\psi} (\mu_L h L_t)^{\psi(1-\theta)}, \tag{17}
\]

and solving for \( Z_t \),

\[
Z_t = \left( \frac{\lambda Z_t^{*\eta}}{g_{Zt}} (\mu_K K_t)^{\psi} (\mu_L h L_t)^{\psi(1-\theta)} \right)^{\frac{1}{1-\sigma}} \tag{18}
\]

On the other hand, (15) implies that

\[
K_t = \kappa_t Y_t = \kappa_t (1-\mu_L) (\kappa_t)^{\frac{1}{1-\sigma}} \hspace{1pt} h Z_t L_t
\]

\[
= (1-\mu_L) (\kappa_t)^{\frac{1}{1-\sigma}} h Z_t L_t
\]

Substituting this result into (18), and solving for \( Z_t \) we obtain

\[
Z_t = \Omega_t h^{\frac{\psi}{1-\phi-\psi}} (\kappa_t)^{\frac{1}{1-\sigma}} h Z_t L_t
\]

where \( \Omega_t = \left( \frac{\lambda Z_t^{*\eta}}{g_{Zt}} \mu^\phi (1-\mu_L) \psi^\theta \frac{\mu}{\mu_L} (1-\theta)^{L_t^\psi} \right) \frac{1}{1-\phi-\psi} \). Equation (19) makes clear that TFP lev-
els depend on the factor endowments of the economy. Substituting (19) into (15) one obtains the desired result.

In order to understand Equation (16), consider first the case of $\psi = 0$. In that case, according to Equations (10) and (11), TFP evolves exogenously, just as in the Solow model, and Equation (16) collapses into equation (2). More precisely, if $\psi = 0$, $X_{3t}$ equals $X_{2t}$, and the residual $A_{3t}$ equals the residual $A_{2t}$. Thus, standard accounting results can be replicated in this framework and interpreted as the result of extremely high (infinite) TFP investments costs. We can gradually reduce this cost within our framework by gradually increasing the returns to scale parameter, $\psi$. The effect of increasing $\psi$ is to increase the exponents of $h$ and $\kappa$ in Equation (16), which implies that cross country differences in human capital and capital-output ratios are amplified, i.e. produce larger output differences. In other words, the role of factors increase if the endogeneity of TFP is taken into account.

Alternatively, consider the case $\theta = \alpha$. In that case, $X_{3t} = (X_{2t})^{1+\frac{\psi}{1-\phi-\psi}}$. Again, the exponent of this expression is larger than 1 which means that a given difference in factors can produce a larger differences in output. Thus, factors alone can explain more of the cross-country income differences when TFP is endogenous. This result arises because the production function for output becomes in fact increasing returns to scale in factors when TFP is endogenous, even though it is constant returns to scale for a given TFP level.

### 3.2.1 Balanced Growth Path

Before performing the accounting exercise using equation (16), it is important to understand why (16) provides a meaningful decomposition in our model. In the Solow model, (2) provides a fundamental decomposition because the underlying parameters determining $\kappa$ are different from the parameters determining $A$. In particular, $A$ evolves exogenously in the Solow model, and $\kappa$ is determined by the exogenous savings rate. We now show that (16) provides a fundamental decomposition also as $A$ and are determined by different parameters in the model.

Consider the evolution of this economy along a balanced growth path. Along that path, $g_{Zt}$ is constant so that, according to equation (17)

$$1 = \frac{g_{Zt+1}}{g_{Zt}} = \gamma_Z^n \gamma_Z^{\phi - 1} (\gamma_K)^{\psi \theta} (\gamma_L)^{\psi (1-\theta)}.$$
It is straightforward to check that along a balanced growth path $\gamma_K = \gamma_Z \gamma_L$. Using this result into the previous equation, it follows that the long term growth rate of the economy is given by

$$\gamma_Z = (\gamma^*_Z, \gamma_L)^{\frac{s}{\psi + \phi}}.$$ 

This is analogous to the result in Jones (1995) but for a model with capital in the R&D sector and technological diffusion. The long term growth rate of the economy is thus scale free. Moreover, according to equation (8) $\gamma_{K,t} = 1 - \delta + \frac{sY_t}{K_t}$ so that $\kappa_t = \frac{s}{\gamma_{K,t} + \delta - \psi}$. Thus, in the long term,

$$\kappa = \frac{s}{(\gamma^*_Z)^{\frac{\psi}{\psi + \phi}} \gamma_L^{\frac{\psi(1 - \theta) + 1 - \phi}{\psi + \phi}} + \delta - 1}. $$

We can now state in what sense (16) provides a fundamental decomposition. In one hand, changes in saving rates only affect $\kappa$ but not $\hat{A}$. On the other hand, changes in $\lambda$, $\mu_K$, $\mu_L$, and once and for all changes of $L_t$ affect $\hat{A}$ but not $\kappa$. Finally, changes in $\gamma_Z$ (meaning changes in $\gamma_L$ and $\gamma^*_Z$) affect both $\hat{A}$ and $\kappa$.

### 3.2.2 Development Accounting

We know reproduce the results of Table 1 for the case of endogenous TFP, based on equation (16). Following standard endogenous growth literature we assume constant returns to scale in the production of ideas, $\psi = 1$. Moreover, we choose $\theta = \alpha$ so that the production of ideas and output are equally capital intensive. The results are not very sensitive to this choice. We can write (16) as

$$y_t = A_{3t} \cdot (X_{2t})^{1 + \frac{1}{1 - \phi - \sigma}}$$

This equation makes clear that the critical parameter is $\phi \equiv \sigma - \eta$, and in particular $\eta$. The results of Table 1 hold if $\eta$ is very large so that $1 + \frac{1}{1 - \phi - \sigma}$ is close to 1. As an example, consider first the case $\sigma = \eta$ so that $\phi = 0$. This a case in which the positive and negative externalities derived from the production of ideas cancel each other out. The first part of Table 2 shows the results for this case. It that case, factors explain all of the income differences or even more. This example seems to imply that $\eta > \sigma$ is needed if TFP components are to play any role. This implies, that the negative externalities associated to the adoption must substantially overweight its positive
Table 2
Development Accounting with Endogenous TFP

<table>
<thead>
<tr>
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<th>$X$</th>
<th>$A$</th>
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</thead>
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<tr>
<td>$\Phi^1$</td>
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</tr>
<tr>
<td>$\Phi^2$</td>
<td>0.99</td>
<td>0.00</td>
</tr>
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<td>$\Phi^3$</td>
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</tr>
<tr>
<td>$\Phi^4$</td>
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<td>-0.60</td>
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<td>$\phi = -1$</td>
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<td>0.13</td>
</tr>
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<td>0.63</td>
<td>0.34</td>
</tr>
<tr>
<td>$\phi = -1.33$</td>
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<td></td>
</tr>
<tr>
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<tr>
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<td>0.37</td>
</tr>
<tr>
<td>$\Phi^4$</td>
<td>0.63</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Figure 2:

externalities.

What is a sensible choice for $\eta$? We can estimate $\eta$ using the following stochastic version of (20).

$$\ln y_t = a + b \ln X_{2t} + e_t$$

Mankiw, Romer, and Weil (1992) used a similar procedure to estimate the human capital share in a model with exogenous TFP. The main criticism to such approach is that the error term $e_t$ may not be orthogonal to the explanatory variable $\ln X_{2t}$. This criticism, however, is not valid here. Our theory that states that the error term is independent of the explanatory variable. Our theory also implies that

$$b = 1 + \frac{1}{1 - \phi - \alpha}$$

A simple regression gives $b \approx 1.5$ so that $\phi = -1.33$. The third Part of Table 2 report the result for this case. In particular, the Klenow and Rodriguez-Clare result is reversed. The main cause of cross-country income differences are differences in saving rates and human capital.
4 Concluding Comments

[To be written]
References


