A THEORY OF POLITICAL CYCLES*

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Abstract

Empirical studies on political cycles show that the proximity of elections affects the real economy and policy choices. Previous theoretical studies assume that the policymaker can only affect his re-election probability with his last action before the election. This assumption directly produces a cycle without presenting a theory of why the policymaker’s behavior is different closer to the election. We shall explain how, without this assumption, existing political-agency models can still produce cycles. Important insights into the empirical implications of these models are presented. In contrast with previous (theoretical and empirical) studies, this paper considers how the policymaker’s optimal decision indeed depends on his reputation. Since the policymaker’s reputation most likely changes over time, in general, one cannot characterize the dynamics of his decisions without considering how his reputation is changing and how his decisions depend on his reputation. More generally, our findings deepen the understanding of the behavior of agents in positions in which they face infrequent firing decisions (or other discontinuous compensation schemes).

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1 Introduction

Empirical evidence indicates that both the real economy and policy choices may be affected by the proximity of elections (the so-called political business and budget cycles). What explains these cycles? In order to answer this question, the recent theoretical literature considers two characteristics of elections. First, as suggested by the empirical evidence (for a review, see Lewis-Beck and Stegmaier [2000]), good economic conditions help an incumbent to get reelected. The incumbent’s good performance indicates to the voters that the incumbent has the ability necessary to deal with the challenges he is facing (voters are learning the incumbent’s ability and this explains why the support for a policymaker changes over time). Second, opportunistic incumbents want to increase their reelection probability and may manipulate economic policy for improving economic outcomes. This approach seems reasonable and it is adopted in this paper. On the other hand, the policymaker would want to increase his reelection probability with his actions in every period. What makes the last period before the election special? Previous studies simply assume that only the last policymaker’s action before an election can affect the reelection probability. Therefore, no theory of why the policymakers’ behavior is different closer to the election is presented. Is it possible to explain political cycles without this assumption? Would the empirical implications of these models change if an incumbent policymaker can affect election outcomes with his actions in every period? The purpose of this paper is to provide answers to these questions.

This paper shows that, even without assuming that only the last policymaker’s action before an election can affect the reelection probability, existing political-agency models can still produce cycles. Following

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2 Previous theoretical studies assume that only the last incumbent’s performance before the election is useful as an indication of his future performance and/or only the last incumbent’s action before the election is not observed by the voters (as it is
the current consensus in the literature (see Brender and Drazen [2004] and Shi and Svensson [2003]), a political-agency model of career concerns is presented. As explained by Brender and Drazen [2004], “an incumbent might be rewarded at the polls only if he can hide the manipulation and make the public believe that the good economic conditions reflect the success of his policy or his high ability.” Thus, the incumbent’s action can be interpreted as an unobservable variable (like effort as in most agency models or deviating resources to less socially beneficial uses as in Persson and Tabellini [2000]) or as an observable variable in a model with uninformed voters (as in the model presented by Shi and Svensson [2002] where the incumbent manipulates fiscal policy producing political budget cycles). For concreteness, the paper refers to unobservable effort. The incumbent’s ability is unknown and may be learned through the amount of public good he delivers each period. Every two periods (i.e., after two output observations), the voters replace the incumbent if their expected lifetime utility with the incumbent in office (that depends on his expected productivity) is lower than their expected lifetime utility with a challenger. Each period, the incumbent decides on his action by in part trying to improve his reputation (the beliefs about his ability).

Why are there political cycles if a policymaker can affect election results with his effort in every period? Two periods before the elections, the incumbent policymaker knows that he can affect his future reputation explained below, the incumbent can only affect the election result if his action is not observed by the voters). These studies generate the cycle through these assumptions. It may be reasonable to assume that the more recent incumbent’s performance is more useful as an indication of his future performance (similarly, it could be assumed that the incumbent’s actions further from the election are more easily observed). This is assumed in this paper but it is not the key element for explaining the cycles. In fact, it is explained that, even in this case, the incumbent’s action further from the election could be more effective in affecting the election results. Most empirical studies on economic voting do not discuss explicitly the time horizon considered by voters. However, some studies reject the hypothesis of voters considering only the incumbent’s performance close to the election (see, for example, Abuelaifa and Meloni [2000], Brender [2003], Fair [1996], Meloni [1997], Panzer and Paredes [1991], and Peltzman [1990, 1992]). An exception is presented by Eisenberg and Ketcham [2004] who find that “only the most recent year of economics performance significantly determines the incumbent’s party’s vote share.” However, they consider four economic measures simultaneously and only 17 observations so their analysis has limited power. Moreover, when they consider county-level performance (21,368 observations) they find that “voters appear to consider each of the three most recent years about equally.”

(and the election results) with his efforts in the current period or in the next period. Therefore, he could exert more effort in the current period and less effort in the next period (or vice versa) and have the same reelection probability. In this paper, assumptions on the discount rate and the information structure play a role but are not crucial for understanding this tradeoff. Two channels are instead in focus.

First, the last effort before the election only affects the election result through the learning process in that period. Two periods before the election, effort affects the election result through the learning process in the current period and it also affects the result through the next-period learning process. If this indirect effect is negative, the last effort before the election could be higher. The voters infer the incumbent’s competence from the amount of public good he produces. This inference depends on the effort voters believe the incumbent exerted. For example, for a given amount of public good, if the voters believe the incumbent produced it exerting a higher effort, they would infer that it was produced with a lower competence. In particular, suppose that the incumbent can exert effort at periods 0 and 1 in order to change the reelection probability at period 2. Suppose that a better reputation implies that the voters believe that the incumbent exerts a higher effort. A higher period-0 effort implies that the incumbent period-1 reputation is better and, therefore, he is expected to exert more effort. Consequently, for a given period-1 output, the period-0 effort decreases the period-1 inference of the incumbent’s competence and it is less effective in affecting the election results than the period-1 effort.

Second, two periods before the election, the incumbent compares the marginal disutility of exerting effort in the current period with the expected marginal disutility of exerting effort next period (the incumbent is uncertain about his future reputation and he knows that the effort he will choose next period depends on his next-period reputation). This expected marginal disutility may be high or low depending on how reputation maps into effort next period. If the marginal cost of the next-period effort is a concave (convex) function of the reputation, Jensen’s inequality implies that it could be better for the policymaker to exert more effort when there is less (more) reputation uncertainty, i.e., when the election is close (far away).

Empirical studies on political cycles ignore that the incumbent’s reputation affects his decisions. In
models of career concerns, in general, the incumbent’s optimal decision depends on his reputation (see Martinez [2005]). This is not the case in previous theoretical studies on political cycles only because of carefully chosen assumptions adopted for tractability. In contrast, the present paper shows that, for understanding why there are political cycles, it is crucial to consider the relationship between the policymaker’s reputation and the strength of his incentives. Given that the incumbent’s reputation is most likely changing over time, for explaining how his decisions evolve over time and why a political cycle may be observed, it is necessary to consider the changes in his reputation. Consequently, this paper suggests to revisit previous empirical findings. For empirical analysis, past performance, the percentage of votes obtained in the previous election, or approval ratings could be used as an indication of the incumbent’s reputation.

Results from comparative-statics exercises (which represent comparisons across different economic and/or institutional environments) are also different if the incumbent can influence the reelection probability with his actions in every period. For example, in contrast with the result by Shi and Svensson [2002], a change in the per-period value a policymaker assigns to being in office has almost no effect on the importance of the cycles.

The incentives a policymaker has because he wants to win elections are not fundamentally different from the incentives any worker has if he wants to keep his job. The model presented here could be reinterpreted to study the way in which the importance of moral hazard problems depends on the proximity of the next firing decision in any agency relationship (if the agent is concerned about being fired). Let us consider, for example, the end of a contract that commits the principal to work with a certain agent. If the contract ends, the principal can choose to replace this agent with a new one. Stiroh [2003] and Wilczynski [2004] present empirical evidence of a renegotiation cycle: performance improves in the year before signing a multi-year contract, but declines after the contract is signed. For concreteness, the rest of the paper refers to the relationship between voters and policymakers.

The rest of this paper is structured as follows. Section 2 describes the model and defines equilibrium.

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5 As discussed by Martinez [2005], the incentives generated by firing are similar to the ones generated by other compensation schemes that imply a discontinuous increase in the compensation when the agent’s reputation is good enough (this is the case, for example, with promotions).
Section 3 characterizes the results. Section 4 concludes and suggests possible extensions.

2 The model

The paper presents a standard political-agency model of career concerns in which the information structure is not assumed to be such that the incumbent can only influence the next election result with his last action before this election. For expositional simplicity, this paper studies a political-agency version of Holmstrom’s [1999] model and it focuses on the stationary case of his model. The principal’s (voters’) decision is whether to replace the incumbent and he makes this decision every two periods. The rest of the assumptions in Holmstrom [1999] are respected.

A dynamic game played by the voters and the incumbents is presented. The existence of conflicts among voters is not considered. The incumbent policymaker wants to produce more public good for the voters because his ability is learned through the amount of public good he delivers and the voters’ reelection decision depends on the incumbent’s expected productivity that in turn depends on his reputation (the beliefs about his ability).

2.1 The environment

Time is discrete and indexed by $t$. Elections occur every two periods. At the beginning of an election period, voters reelect the incumbent if and only if their expected utility is higher with the incumbent than with a challenger. After an election, or at the beginning of a period without election, the incumbent decides on his effort $a_t \geq 0$. At period $t$, the amount of public good available $y_t$ is a stochastic function of the incumbent’s ability $\bar{\eta}_t$ and his effort $a_t$. In particular,

$$y_t = a_t + \bar{\eta}_t + \varepsilon_t,$$

where $\varepsilon_t$ is a normally distributed random variable with expected value 0 and precision $h_\varepsilon$ (the variance is $\frac{1}{h_\varepsilon}$). After the incumbent chooses his effort, the noise in the production process $\varepsilon_t$ is realized.

6This paper may be interpreted as considering situations where the decisive voter cares about future performance and not about ideology. Moreover, the model could be extended to include probabilistic voting as done by Shi and Svensson [2002].
The incumbent’s ability evolves as a random walk. In particular, $\eta_{t+1} = \bar{\eta}_t + \beta_t$ where $\beta_t$ is assumed to be normally distributed with mean 0 and precision $h_\beta$.\textsuperscript{7}

The players (the voters and the incumbent) are ignorant of the incumbent’s ability.\textsuperscript{8} They have the same belief about the ability of a new incumbent. This belief is normally distributed with mean $b_0$ and precision $h_\eta$ (and this is the belief about the period-0 incumbent’s ability).

Each period, the incumbent is said to be more competent if he obtains more with a given effort. Thus, the incumbent’s competence $\eta_t$ is defined by

$$\eta_t \equiv y_t - a_t = \bar{\eta}_t + \varepsilon_t.$$

The voters’ per-period utility is given by $y_t$. For expositional simplicity, voters are restricted to replacing the incumbent only with policymakers that were not in office before.\textsuperscript{9} A policymaker’s per-period utility is normalized to zero if he is not in office. The incumbent receives $R$ after winning an

\textsuperscript{7}As it is explained later, assuming that ability evolves over time allows us to focus on the case in which the precision of the beliefs about the incumbent’s ability does not depend on the number of periods the incumbent was in office. This assumption may represent situations in which the incumbent’s tasks are changing over time and his ability depend on the tasks he is focusing on (let us consider, for example, the president of a country that becomes involved in a war). Martinez [2004] presents a firing model of career concerns in which an agent’s ability does not change over time and the main results presented here are not affected.

\textsuperscript{8}Having the incumbent not know his ability allows us to consider situations where a policymaker in a new position may be ignorant of his ability when met with new tasks. This assumption also helps to understand situations where a policymaker’s success does not only depend on his individual ability but also on the ability of others working with him. Moreover, in adverse-selection models, a high ability incumbent may increase his reelection probability by degrading future possibilities because doing this is a signal of high ability and voters, knowing that the incumbent ruined the future, decide to reelect him. This may not be the most appealing framework for explaining political cycles. Furthermore, empirical evidence indicates that political cycles are more important if the share of informed voters in the electorate is smaller (see, for example, Shi and Svensson [2001]). This is consistent with models of career concerns and it is not consistent with signaling models.

\textsuperscript{9}This is an interesting starting point and it simplifies the analysis. The main results do not change much if this assumption is removed. Previous studies analyze models of optimal firing of agents where the optimal action for the principal does not involve hiring a previously tried agent other than the incumbent (see, for example, Banks and Sundaram [1990]). Martinez [2004] shows that, in models of career concerns, it may be optimal to hire someone who was fired before. On the other hand, this is not the case in the examples presented in this paper.
election and in a period without elections.\footnote{Martinez [2005] explains that the compensation scheme considered by the incumbent would be similar if the per-period compensation is allowed to depend on the incumbent’s reputation. Diermeier, Keane, and Merlo [2005] find that the mean of the monetized value of a House seat and a Senate seat in 1995 dollars are equal to $616,228 and $1,673,763 respectively.} The incumbent may choose his effort level. There is a cost to exerting effort, given by $c (a)$, with $c' (a) > 0$, $c'' (a) > 0$, and $c' (0) = 0$.

Players observe $y_t$ while $\eta_t$ is unobservable. The voters do not observe the incumbent’s effort $a_t$ (which is known by the incumbent).\footnote{Political-budget-cycles models of career concerns (see, for example, Shi and Svensson [2002]) assume that the incumbent’s action (for example, government expenditure) is observable but some voters are uninformed.}

### 2.2 The learning process

From this point forward, \textit{belief} refers to \textit{belief about the incumbent’s ability} unless stated otherwise (as when referring to the voters’ beliefs about the effort the incumbent exerted).

Players learn about the incumbent’s ability using Bayesian learning. Let $b_{vt}$ and $b_{it}$ denote the mean of the voters’ and the incumbent’s beliefs at the beginning of period $t$ (from here on, at period $t$). For simplicity, the precision of the noise in the random walk ability process is chosen to make the mean of the distribution sufficient for characterizing beliefs. Thus, we assume that

$$h_{t} = \frac{h_{\bar{\eta}}^2 + h_{\bar{\eta}} h_{\varepsilon}}{h_{\varepsilon}}.$$  

With this assumption, the period-$t+1$ precision in the beliefs about the incumbent’s competence is always equal to the period-$t$ precision and does not depend on the number of observations of the output produced by the incumbent. This precision is given by

$$H \equiv \frac{h_{\bar{\eta}} h_{\varepsilon}}{h_{\varepsilon} + h_{\bar{\eta}}}.$$ 

Consequently, there is no tenure effect in the determination of players’ decisions.\footnote{The tenure effect presented by Holmstrom [1999] is clear. With more output observations, the belief about the incumbent’s ability becomes more precise and new observations have less weight in the future beliefs. Given that the policymaker’s effort affects only these new observations, the incentives to exert effort are weaker when a policymaker has been in office longer (and the beliefs about his ability are more precise). Thus, the equilibrium effort level declines with tenure.} We shall refer to \textit{belief with mean $b$} as \textit{belief $b$}.
2.3 Equilibrium strategies

We shall assume that the incumbent plays a pure strategy and we shall use Perfect Bayesian Equilibrium as the equilibrium concept. Following Martinez [2005], it can be shown that, for all histories of the game that imply the same beliefs, the players’ equilibrium actions are the same.

The incumbent knows that the voters believe that he exerted the equilibrium effort in every period (according to the voters’ beliefs) and, therefore, the incumbent is able to infer the voters’ beliefs. The incumbent’s optimal strategy is a mapping from both $b_i$ and $b_v$ to $a$, and depends on the proximity of elections and on $t$. This strategy is denoted by $\hat{\alpha}_t(b_i, b_v)$ two periods before the next election and $\alpha_t(b_i, b_v)$ if there is an election next period. Let $\hat{\alpha}_t(x) \equiv \hat{\alpha}_t(x, x)$ and $\alpha_t(x) \equiv \alpha_t(x, x)$ for all $x$ denote the incumbent’s optimal strategies if the voters’ and the incumbent’s beliefs are coincidental (for example, on the equilibrium path).

In a Perfect Bayesian Equilibrium, the voters always believe they are on the equilibrium path (i.e., the voters believe that the incumbent exerted the equilibrium effort in every period). Consequently, the voters believe that the incumbent’s belief coincide with their beliefs and it is characterized by $b_i = b_v$. Thus, the voters’ beliefs are sufficient for characterizing their optimal reelection decisions. Let the voters’ reelection strategy be denoted by $\iota_t(b_v)$, where $\iota_t(b_v)$ equals one if the incumbent is reelected, and zero if otherwise (and $t$ is an election period).

2.4 Equilibrium learning

Observing $y_t$ allows the players to infer $\eta_t$ (and to update their beliefs). The incumbent knows $a_t$ and he is always able to infer $\eta_t$ correctly ($\eta_t = y_t - a_t$). Voters do not know $a_t$ and use their beliefs about $a_t$ to infer competence. In a Perfect Bayesian Equilibrium, voters consider that the incumbent chooses the equilibrium effort effort in every period.
equilibrium effort level in every period. Thus, voters’ inferred competence is given by

\[ \eta_{vt}(b_v, \eta, a) \equiv y - \alpha_t(b_v) = \eta + a - \alpha_t(b_v) \tag{1} \]

one period before the next election, and by

\[ \hat{\eta}_{vt}(b_v, \eta, a) \equiv y - \hat{\alpha}_t(b_v) = \eta + a - \hat{\alpha}_t(b_v) \]

two periods before the next election.

On the equilibrium path, the incumbent exerts the equilibrium effort in every period and the competence inferred by voters is equal to the competence inferred by the incumbent. Voters’ inference may be wrong, however, when deviations from equilibrium behavior are considered.

According to Bayes’ rule, the mean of the beliefs at \( t \) is a weighted sum of the mean at \( t - 1 \) and the inferred competence where the weight of the mean at \( t \) is given by:

\[ \mu = \frac{h\hat{\eta}}{h\eta + h\varepsilon}. \tag{2} \]

Thus, the incumbent’s belief at \( t \) is characterized by

\[ b_{it} = B\left(b_{it-1}, \eta_{t-1}\right) \equiv \mu b_{it-1} + (1 - \mu) \eta_{t-1}. \]

At any period \( t \) such that there is an election at \( t \), voters’ beliefs are represented by

\[ b_{vt} = B_{vt-1}\left(b_{vt-1}, \eta, a\right) \equiv B\left(b_{vt-1}, \eta_{vt-1}(b_{vt-1}, \eta, a)\right) = \mu b_{vt-1} + (1 - \mu) (\eta + a - \alpha_{t-1}(b_{vt-1})) \tag{3} \]

Similarly, one period before the next election,

\[ b_{vt} = \hat{B}_{vt-1}\left(b_{vt-1}, \eta, a\right) \equiv B\left(b_{vt-1}, \hat{\eta}_{vt-1}(b_{vt-1}, \eta, a)\right) = \mu b_{vt-1} + (1 - \mu) (\eta + a - \hat{\alpha}_{t-1}(b_{vt-1})) \tag{4} \]

On the equilibrium path, given that the competence inferred by voters is equal to the competence inferred by the incumbent, the voters’ and the incumbent’s beliefs are coincidental (at the beginning of the game, their beliefs are assumed to be coincidental). Their beliefs may be different, however, when deviations from the equilibrium behavior are considered.
2.5 Equilibrium definition

Let $\delta \in (0, 1)$ denote the discount factor and let $f_b$ denote the density function for a normally distributed random variable with mean $b$ and precision $H$. At the beginning of an election period, a voter’s expected lifetime utility is given by

$$
\tilde{W}_t(b_v) = \max_{I \in \{0, 1\}} \left\{ I \left[ \int_{-\infty}^{\infty} [\tilde{\alpha}_t(b_v) + \eta + \delta V_{t+1}(B(b_v, \eta))] f_{b_v}(\eta) \, d\eta \right] + \ldots \right\}
$$

where

$$
V_t(b_v) \equiv \int_{-\infty}^{\infty} \left[ \alpha_t(b_v) + \eta + \delta \tilde{W}_{t+1}(B(b_v, \eta)) \right] f_{b_v}(\eta) \, d\eta
$$

denotes his expected lifetime utility at the beginning of a period without elections. The voters’ equilibrium strategy $\mu_t(b_v)$ is given by the solution of problem 5.

Two periods before the next election, at the time the incumbent chooses his effort (right after the current-period election), $\tilde{W}_t(b_i, b_v)$ denotes his expected lifetime utility. Similarly, one period before the next election, $W_t(b_i, b_v)$ denotes the incumbent’s expected lifetime utility. Let $W_t(x) \equiv W_t(x, x)$ and $\tilde{W}_t(x) \equiv \tilde{W}_t(x, x)$ for all $x$ represent the incumbent’s expected lifetime utility when the beliefs are coincidental and represented by $x$. One period before an election, the incumbent’s problem reads

$$
W_t(b_i, b_v) = \max_a \left\{ R - c(a) + \delta \int_{-\infty}^{\infty} \tilde{W}_{t+1}(B(b_i, \eta), B_v(b_v, \eta, a)) \, \eta_{t+1}(B_v(b_v, \eta, a)) f_{b_v}(\eta) \, d\eta \right\}
$$

where

$$
\tilde{W}_t(b_i, b_v) = \max_a \left\{ R - c(a) + \delta \int_{-\infty}^{\infty} W_{t+1}(B(b_i, \eta), \tilde{B}_v(b_v, \eta, a)) \, f_{b_v}(\eta) \, d\eta \right\}.
$$

One period before the next election, the incumbent’s optimal effort $\alpha_t(b_i, b_v)$ is given by the solution of problem 7. Two periods before the next election, the incumbent’s optimal effort $\tilde{\alpha}_t(b_i, b_v)$ is given by the solution of problem 8.

**Definition 1** An equilibrium consists of the functions $V_t(b_v)$, $W_t(b_i, b_v)$, and $\tilde{W}_t(b_i, b_v)$, and strategies $\mu_t(b_v)$, $\alpha_t(b_i, b_v)$, and $\tilde{\alpha}_t(b_i, b_v)$ such that, for each period in the game ($t$):

1. $W_t(b_i, b_v)$ satisfy the incumbent’s problem if $t + 1$ is an election period.

2. $V_t(b_v)$ and $\tilde{W}_t(b_i, b_v)$ satisfy the voters’ and the incumbent’s problems if $t + 2$ is an election period.
3. \( \iota_t(b_v) \) solves the voters’ problem if \( t \) is an election period.

4. \( \alpha_t(b_i, b_v) \) solves the incumbent’s problem if \( t + 1 \) is an election period.

5. \( \hat{\alpha}_t(b_i, b_v) \) solves the incumbent’s problem if \( t + 2 \) is an election period.

6. The beliefs are obtained using the equilibrium strategies and Bayes’ rule.

3 Results

It is shown how the incumbent’s effort depend on the proximity of the next election. Differences in the incumbent’s effort imply differences in the amount of public good \( y \) and might be used for explaining political cycles. The model could be modified to study political budget cycles if the incumbent’s action represents a fiscal policy choice (as in the model presented by Shi and Svensson [2002]).

It will be shown that policymakers make effort-smoothing decisions. It is explained that, in order to understand why policymakers may decide to exert a lower effort two periods before an election (for a given reputation), one has to consider the relative effectiveness in increasing reelection probabilities of the effort exerted each period and the incumbent’s uncertainty about his future actions (that depend on his future reputation). Important insights into the empirical implications of models of political cycles are obtained.

Results from comparative-statics exercises are also discussed. It will be shown that these results are different from findings in previous studies (where only the last action before an election can affect this election outcome).

3.1 One election

For expositional simplicity, a one-election version of the model is discussed first. This allows us to show that, without assuming that only the last action before the election can affect the reelection probability, existing political-agency models can still produce cycles. In order to have a better understanding of the way in which the results change if more periods are considered, the limit of the finite-horizon solution in an infinite-horizon version of the model is discussed later.
3.1.1 The political cycle

Let us consider a three-period model. There is a new incumbent in office at period 0 who exerts effort in periods 0 and 1 in order to affect the probability of reelection at period 2.

The model is solved using backward induction. At period 2, the incumbent has no incentives to exert effort and $\alpha_2(b) = 0$ for all $b$. Therefore, the voters want to reelect the incumbent if and only if $b_v2 > b_0$, i.e., $\nu_2(b_v2) = 1$ if and only if $b_v2 > b_0$.

On the equilibrium path, if the players’ beliefs are denoted by $b$, the incumbent’s period-1 problem reads:

$$W_1(b) = \max_a \{ R - c(a) + \delta R P_b [B_{v1}(b, \eta, a) > b_0] \}$$

where $P_b [x]$ denotes the probability of $x$ when $\eta$ is distributed according to $b$. This problem is a particular case of problem 7. It can be written making explicit that, with his effort, the incumbent modifies the minimum realization of competence $\eta$ required for reelection:

$$W_1(b) = \max_a \left\{ R - c(a) + \delta R P_b \left[ \eta > \frac{b_0 - \mu b}{1 - \mu} + \alpha_1(b) - a \right] \right\}.$$  \hspace{1cm} (10)

The incumbent’s equilibrium strategies are characterized through the first-order conditions of the incumbent’s problems.\footnote{The last term in (10) is not globally concave. The incumbent’s optimal effort as a function of the effort the voters expect him to exert may be discontinuous if there is a strong enough convexity in the objective function. Consequently, a fixed point for the incumbent’s equilibrium strategy may not exist (for examples of this situation in a similar framework see Martinez [2004]). However, there are many ways of assuring global concavity. One way is to assume that the marginal cost function increases rapidly enough. For example, one could find an upper bound for the slope of the marginal benefit curve and assume that the slope of the marginal cost curve is always higher (this is particularly easy for problem 10). Another alternative is to assume that $c(a) = a^n$ and $n$ is high enough. Consequently, the marginal cost is very low for a low $a$ and, for a high enough $a$, it starts increasing very rapidly assuring that the marginal cost curve crosses the marginal benefit curve only once (from below) and the problem is globally concave. In particular, this makes the incumbent’s problem globally concave in the examples discussed in this paper.}

Thus, at period 1, the incumbent’s equilibrium strategy $\alpha_1(b)$ is given by:

$$c'(\alpha_1(b)) = \delta R f_b \left( \frac{b_0 - \mu b}{1 - \mu} \right).$$

Martinez [2005] shows that the maximum period-1 effort is obtained from an incumbent whose reputation is represented by $b_0$ and a lower (higher) effort level is obtained from an incumbent with better reputation.
if and only if the incumbent’s reputation is better (worse) than the average (as illustrated in figure 1). Moreover, $\alpha_1$ is a concave function around $b_0$ and a convex function for extreme values of $b$.

At period 0, the beliefs are coincidental an denoted by $b_0$. The incumbent’s problem is as described in equation 8. The next equation presents the Euler equation for this problem evaluated in equilibrium (for the derivation of a similar Euler equation see Martinez [2005]).

$$c_0'(\alpha_0(b_0)) = \delta \int_{-\infty}^{\infty} r_0(B(b_0, \eta)) c_0' (\alpha_1(B(b_0, \eta))) f_{b_0}(\eta) d\eta$$

(12)

where

$$r_0(b) \equiv \frac{\partial B_{\epsilon_1}}{\partial b_v} \bigg|_{b_v=b} = \mu - (1 - \mu) \alpha_1'(b).$$

(13)

The period-0 equilibrium effort $\alpha_0(b_0)$ can easily be obtained from equation 12 given the period-1 equilibrium strategy $\alpha_1(b)$ defined by equation 11.

Equation 12 represents the typical intertemporal tradeoff in dynamic models: having less utility today allows the incumbent to have more utility next period. In this case, the marginal cost from a higher effort level today is compensated with an expected lower effort level next period. Exerting a higher effort at period 0 may allow the incumbent to exert a lower effort at period 1 and still have the same reelection probability at period 2.

The next proposition shows that, without assuming that only the last action before the election can affect the reelection probability, existing political-agency models can still produce cycles, i.e., the equilibrium effort level may be higher at period 1.

**Proposition 1** For small enough changes in the incumbent’s reputation, the period-0 equilibrium effort is lower than the period-1 effort. For large enough changes in the incumbent’s reputation, the period-0 equilibrium effort is higher than the period-1 effort.

**Proof.** Recall that $\alpha_1(B(b_0, \eta))$ is a symmetric function with maximum at $\eta = b_0$ ($B(b_0, b_0) = b_0$) and, therefore, $c' (\alpha_1(B(b_0, \eta)))$ is a symmetric function with maximum at $\eta = b_0$. Moreover, $f_{b_0}(\eta)$ is a symmetric function with maximum at $\eta = b_0$, $r_0(b_0) = \mu$, and, for any $A \in \mathbb{R}$, $r_0(b_0 + A) - r_0(b_0) = r_0(b_0) - r_0(b_0 - A)$. Consequently,

$$c' (\alpha_0(b_0)) = \delta \mu \int_{-\infty}^{\infty} c' (\alpha_1(B(b_0, \eta))) f_{b_0}(\eta) d\eta.$$
Given that $\delta \mu < 1$,

$$c'(\alpha_0(b_0)) < \int_{-\infty}^{\infty} c'(\alpha_1(B(b_0, \eta))) f_{b_0}(\eta) \, d\eta.$$ 

Given that $\alpha_1(b_0) > \alpha_1(b)$ for all $b \neq b_0$ (see Martinez [2005]), $c'(\alpha_1(b_0)) > c'(\alpha_1(b))$ for all $b \neq b_0$. Therefore,

$$c'(\alpha_1(b_0)) > \int_{-\infty}^{\infty} c'(\alpha_1(B(b_0, \eta))) f_{b_0}(\eta) \, d\eta.$$ 

Consequently, $c'(\alpha_1(b_0)) > c'(\alpha_0(b_0))$, and $\alpha_1(b_0) > \alpha_0(b_0)$ (by $c'' > 0$). The proposition follows from the properties of $\alpha_1(b)$ (illustrated in figure 1 and discussed by Martinez [2005]).

Would the incumbent exert more effort at period 1? Proposition 1 shows that in order to answer this question it is necessary to consider that the incumbent’s reputation may change over time. This is the case in standard models of political cycles if the extreme assumption of the incumbent only being able to modify election results with his last action before the election is abandoned. Empirical studies on political cycles ignore that the incumbent’s reputation affects his decisions. Consequently, this paper suggests to revisit previous empirical findings. In order to illustrate this, let us consider the following example: $c(a) = a^n$, $n = 5$, $\delta = 0.9$, $R = 20$, $b_0 = 0$, $h_\eta = 0.75$, and $h_\epsilon = 0.75$. Figure 1 shows the equilibrium effort levels for this example. As illustrated in figure 1, if after one period in office the incumbent’s reputation becomes...
very good or very bad his period-1 effort is lower than his initial effort at period 0. Thus, an inverted political cycle would be observed. On the other hand, if the incumbent reputation does not change much, proposition 1 shows that the incumbent’s period-1 effort is higher than his initial effort at period 0. Thus, the framework developed here can explain both positive political cycles and inverted political cycles (the so-called post electoral cycles discussed, for example, by Alesina [1987]).

Why the incumbent does not exert the same effort in every period? Under the assumptions of previous studies, the incumbent’s period-0 actions would not be effective in affecting the period-2 reelection probability and, consequently, the incumbent would only exert effort at period 1 (the cycle is a direct result of the assumptions). Here, it is assumed that the incumbent’s more recent performance is more useful as an indication of his future performance.\(^\text{15}\) However, the relative effectiveness of the incumbent’s actions is endogenous here and the period-0 action could be more effective than the period-1 action in changing the reelection probability. This is the case if with his period-0 action the incumbent can make the voters believe that he will exert a lower period-1 effort and, therefore, he can make the voters infer a higher period-1 competence. For example, suppose that the period-1 effort expected by the voters \(\alpha_1 (b_{v1})\) is decreasing with respect to the reputation \(b_{v1}\) (as illustrated in figure 1 for high reputation levels). Then, at period 1, if the incumbent’s reputation is better \((b_{v1} \text{ is higher})\) and the voters believe the incumbent exerted a lower effort \(\alpha_1 (b_{v1})\), for any output \(y_1\), the voters infer a higher competence \(\eta_{v1} (b_{v1}) \equiv y_1 - \alpha_1 (b_{v1})\). The voters think that the output \(y_1\) is the result of a low effort and a high competence. Consequently, the incumbent’s period-0 effort (that increases \(b_{v1}\)) has a positive effect on the voters’ learning at periods 0 and 1 and it may be more effective than the period-1 effort in affecting the period-2 reelection probability (even if the period-1 incumbent’s performance is more useful than his period-0 performance as an indication of his period-2 performance).

In equation 12, \(r_0\) represents the relative effectiveness in changing the voters’ period-2 beliefs \(b_{v2}\) and, therefore, the probability of receiving \(R\) at period 2 of the incumbent’s period-0 effort (compared with his period-1 effort). As described above, the incumbent’s period-0 effort affects \(b_{v1}\) directly and it affects \(b_{v2}\) through \(b_{v1}\) (as indicated in equation 3). His period-1 effort affects \(b_{v2}\) directly. Thus, the relative

\(^{15}\)In the voters’ period-2 belief \(b_{v2} = \mu^2 b_0 + \mu (1 - \mu) \eta_{v0} + (1 - \mu) \eta_{v1}\), the period-0 competence they inferred \(\eta_{v0}\) is weighted less than the period-1 competence they inferred \(\eta_{v1}\).
Effectiveness $r_0$ is the derivative of the voters period-2 beliefs $b_{v2} = B(b_{v1}, \eta_{v1}(b_{v1}))$ with respect to their period-1 belief $b_{v1}$. If the relative effectiveness $r_0$ is higher (lower) than one, it implies that the period-0 effort was relatively more (less) effective than the period-1 effort in changing $b_{v2}$.

Considering the expected relative effectiveness is not enough for understanding the differences between the effort exerted by the incumbent at periods 0 and 1. Equation 12 indicates that the incumbent’s uncertainty about his future actions has to be considered. For example, let us compare the incumbent’s period-0 effort with the period-1 effort he exerts if the reputation does not change (and it is equal to $b_0$). Equivalently, let us compare the marginal cost of these equilibrium effort levels $c'(\alpha_0(b_0))$ and $c'(\alpha_1(b_0))$. Equation 12 shows that the period-0 marginal cost of exerting effort $c'(\alpha_0(b_0))$ is equal to the expected marginal cost of exerting effort at period-1 (discounted by $\delta$ and the expected relative effectiveness). The period-1 marginal cost $c'(\alpha_1(b_0))$ is the marginal cost of the next-period effort $\alpha_1(B(b, \eta))$ evaluated at the expected next-period reputation $b_0$. Is the expected marginal cost higher or lower than the marginal cost of the expected effort? In this case the comparison is simple because the marginal cost of the expected effort is the highest possible marginal cost. Consequently (if the result is not reverted by the relative effectiveness), the period-0 effort would be lower than the period-1 effort (as it is shown to be true in proposition 1).

3.1.2 Comparative statics

Comparative-statics exercises have been used to identify under what circumstances political cycles would be of higher magnitude. This section shows how considering that the incumbent can influence the election result with his actions in every period affects this analysis.

Differences in the per-period office value $R$ are analyzed. In a model where only the last action before an election can affect this election outcome, Shi and Svensson [2002] show that if the per-period office value (denoted here by $R$) is higher, political budget cycles are amplified. They find empirical evidence that supports this prediction.\footnote{They also analyze the effect of differences in the proportion of uninformed voters. The same could be done here if the model is reformulated.} The intuition behind this result is simple. A higher $R$ implies that there are stronger incentives to increase reelection probabilities. In their model, given that reelection probabilities...
can only be increased with the last action before the election, an increase in $R$ increases the importance of the cycles.

What can be learned about this relationship from the model presented here? If the incumbent can affect election results with his actions in every period, a higher office value $R$ implies a higher effort level in every period. Let us compare the effort levels at periods 0 and 1. Equation 11 shows that a higher $R$ implies a higher period-1 effort level for any reputation $b$. Equation 12 shows that if the incumbent expects a higher period-1 effort, he exerts a higher period-0 effort. In particular, if $c'$ is a homogeneous function, the next proposition shows that the office value $R$ only has a scale effect on the political cycles and the difference between the effort levels observed at periods 0 and 1 as a percentage of the period-0 effort level is independent of $R$.

**Proposition 2** Assume that $c'$ is a homogeneous function of order $j$. Then, for any period-1 reputation $b$, \( \frac{\alpha_1(b) - \alpha_0(b_0)}{\alpha_0(b_0)} \) is independent of $R$.

**Proof.** Let us consider any office value $R = R_0$. Let us suppose that there is a change in the office value from $R_0$ to $R_1 = \lambda R_0$ with $\lambda \in \mathbb{R}$. Let $\alpha_t(b; R)$ denote the equilibrium effort level if the beliefs are represented by $b$ and the per-period office value is $R$. The period-1 equilibrium effort level if the beliefs are represented by $b$ and $R = R_0$, $\alpha_1(b; R_0)$, satisfies

\[
c' (\alpha_1(b; R_0)) = \delta R_0 f_b \left( \frac{b_0 - \mu b}{1 - \mu} \right).
\]

If $R = R_1$, $\alpha_1(b; R_1)$ satisfies

\[
c' (\alpha_1(b; R_1)) = \delta R_1 f_b \left( \frac{b_0 - \mu b}{1 - \mu} \right).
\]

Therefore, $c' (\alpha_1(b; R_1)) = \lambda c' (\alpha_1(b; R_0))$. Given that $c'$ is homogenous of order $j$,

\[
\lambda c' (\alpha_1(b; R_0)) = c' \left( \lambda^j \alpha_1(b; R_0) \right)
\]

and $\alpha_1(b; R_1) = \lambda^j \alpha_1(b; R_0)$. For $R_0$, the period-0 equilibrium effort is given by

\[
c' (\alpha_0(b; R_0)) = \delta \mu \int_{-\infty}^{\infty} c' (\alpha_1(B(b_0, \eta); R_0)) f_{b_0}(\eta) \, d\eta
\]

and, for $R_1$, the period-0 equilibrium effort is given by

\[
c' (\alpha_0(b; R_1)) = \delta \mu \int_{-\infty}^{\infty} c' (\alpha_1(B(b_0, \eta); R_1)) f_{b_0}(\eta) \, d\eta.
\]

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Therefore, \( c'(\alpha_0(b_0; R_1)) = \lambda c'(\alpha_0(b_0; R_0)) \) and \( \alpha_0(b_0; R_1) = \frac{1}{\lambda^2} \alpha_0(b_0; R_0). \) Thus,

\[
\frac{\alpha_1(b; R_0) - \alpha_0(b; R_0)}{\alpha_0(b_0; R_0)} = \frac{\alpha_1(b; R_1) - \alpha_0(b; R_1)}{\alpha_0(b_0; R_1)}
\]

and \( \frac{\alpha_1(b) - \alpha_0(b_0)}{\alpha_0(b_0)} \) is independent of \( R. \)

This result implies empirically testable implications that allow us to distinguish the model in this paper from the one presented by Shi and Svensson [2002]. On the other hand, this result does not contradict the empirical findings in Shi and Svensson [2002] because it does not refer to differences between the effort levels observed in periods 0 and 1 but to these differences as a percentage of the period-0 effort.

### 3.2 Multiple elections

The limit of the finite-horizon solutions if players are far enough from the termination of the game is discussed here. The infinite-horizon case has particular interest because of its stationarity: the incumbent’s incentives (the number of elections he may face in the future, the value of winning these elections, and his future actions) do not depend on time. Analyzing a model with more than one election allows us to consider that, two periods before the next election, the incumbent’s reputation may be different from \( b_0. \) Moreover, one period before the next election, the incumbent make an effort smoothing decision similar to the period-0 decision discussed above.

Given the complexity of the problem studied here, a numerical approach is necessary. It is assumed that the optimal reelection rule is to reelect a policymaker if and only if his expected ability is higher than the one for an unknown challenger and, after finding the incumbent’s equilibrium strategies, this is checked to be true (for all the parameter values considered, this voting strategy is an equilibrium strategy). The Euler equations described in equations 15 and 14 below, and the expected lifetime utility for an incumbent two periods before the next election evaluated in equilibrium (when the incumbent’s and the voters’ beliefs are coincidental) constitute a system of three functional equations with three unknowns (the functions \( \hat{W}(b), \hat{\alpha}(b), \) and \( \alpha(b) \)). Numerical methods allow us to find these functions. In order to perform this calculation, the parameter values discussed in the example presented before are used.

Two periods before the next election, the incumbent’s problem is as described in (8) (and, for the infinite-horizon case, the time subscripts can be omitted). Incentives are as described for period-0 in the
one-election version of the model. These incentives are represented in the following Euler equation:

\[ c' (\hat{\alpha}(b)) = \delta \int_{-\infty}^{\infty} \hat{r}(B(b, \eta)) c' (\hat{\alpha}(B(b, \eta))) f_b(\eta) d\eta \]  

(14)

where

\[ \hat{r}(b) \equiv \frac{\partial B_v}{\partial b} \bigg|_{b_v=b} = \mu - (1 - \mu) \hat{\alpha}'(b) . \]

denotes the relative effectiveness of the incumbent’s action two periods before the next election. Equation 14 gives the incumbent’s equilibrium effort for any reputation \( b \). However, two periods before the next election (and after the current period election), given the equilibrium voting rule, only agents with a good enough reputation can be in office.

One period before the next election, the incumbent’s problem is as described in (7). The following Euler equation describes the incumbent’s incentives when the beliefs are coincidental and represented by \( b \).

\[ c' (\alpha(b)) = \delta \hat{W}(b_0) f_b \left( \frac{b_0 - \mu b}{1 - \mu} \right) + \delta \int_{\frac{b_0 - \mu b}{1 - \mu}}^{\infty} r(B(b, \eta)) c' (\hat{\alpha}(B(b, \eta))) f_b(\eta) d\eta \]  

(15)

where

\[ r(b) \equiv \frac{\partial B_v}{\partial b} \bigg|_{b_v=b} = \mu - (1 - \mu) \hat{\alpha}'(b) \]

denotes the relative effectiveness of the incumbent’s action one period before the next election.

The first term in the right-hand side of equation 15 represents the gain from increasing the next-election reelection probability. The value of winning the election is endogenous and it is given by \( \hat{W}(b_0) \) (in the one-election version of the model, in the period-1 marginal benefit of exerting effort, the value of winning the election is equal to \( W_T(b_0) = R \)).

The second term in the right-hand side of equation 15 represents the gain from increasing the reelection probability in the future elections. These incentives are similar to the ones an incumbent faces two periods before the next election: in order to increase future reelection probabilities, the incumbent may decide to exert effort now or in the future. The difference is that, one period before the next election, the incumbent may not enjoy the future benefit because he may lose the next-period election (in equation 15, this is represented in the lower bound in the integral).

Figure 2 illustrates how the proximity of the next election affects the incumbent’s decisions (for all reputations \( b \)). It shows that the numerical approach used for computing the solutions in the infinite-
election version of the model produces results that are consistent with the ones obtained from closed-form solutions in the one-election version of the model.

Figure 2 shows that, for a given reputation \( b \), the incumbent may decide to exert a higher effort further from the next election. For understanding this, for a given \( b \), let us compare the marginal cost of exerting effort one and two periods before the next election, \( c'(\alpha(b)) \) and \( c'(\hat{\alpha}(b)) \) respectively. Equation 14 shows that the marginal cost of exerting effort two periods before the next election \( c'(\hat{\alpha}(b)) \) is equal to the expected marginal cost of exerting effort one period before the next election \( c'(\alpha(B(b,\eta))) \) (weighted by the relative effectiveness and discounted by \( \delta \)). In order to understand the way in which this expected marginal cost (and, therefore, \( c'(\hat{\alpha}(b)) \)) compares with \( c'(\alpha(b)) \) (the marginal cost of \( \alpha(B(b,\eta)) \) evaluated at the expected \( B(b,\eta) \)) Jensen’s inequality has to be considered. If \( c'(\alpha(B(b,\eta))) \) is a convex (concave) function, Jensen’s inequality implies an inverted (positive) political cycle, i.e., it predicts that the incumbent’s effort level is higher (lower) further from the election. One period before the next election, the equilibrium strategy is convex for extreme reputations \( b \) and it is concave for reputations \( b \) close to \( b_0 \). However, for good reputations (high \( b \)) the effort-smoothing incentives are also important one period before the next election (the incumbent is very likely to win the election and, therefore, the lower bound in the integral in equation 15 is low) and the differences between the equilibrium efforts one and two periods before the next election are small.
4 Conclusions and extensions

The paper presents a model of political cycles that contrast with previous studies because, in the framework discussed here, a policymaker can alter election outcomes with his actions in every period (and not only with his last action before an election). It is shown that, in the standard political-agency model, for producing political cycles, it is not necessary to assume that only the last policymaker’s action before the election can affect the election result. Cycles are explained here by the endogenous relative effectiveness of the policymakers’ actions in altering future reelections probabilities, and the incumbent’s uncertainty about his future actions (that depend on his future reputation).

The model in this paper is a truly dynamic framework where policymakers’ optimal decisions depend on his reputation and these decisions influence his future reputation. In the existing literature on political cycles, dynamics are sidestepped. Thus, in contrast with previous studies, this paper produces empirical implications on the amplitude and the timing of cycles that are conditional on the incumbent’s reputation. Empirical studies on political cycles ignore that the incumbent’s reputation affects his decisions. Consequently, this paper suggests to revisit previous empirical findings.

The framework developed here can explain both positive political cycles and inverted political cycles (the so-called post electoral cycles) something that is not possible in previous studies. Moreover, it is shown that if policymakers can affect reelection probabilities with their actions in every period, the results from comparative-statics exercises are different from the findings in previous work.

This paper is useful for understanding the implicit incentives generated by firing decisions in any agency relationship in which the principal cannot fire an agent in every period and, therefore, the agents’ behavior depends on the proximity of the period in which the agent can be fired. This is the case, for example, of an assistant professor in a tenure track position. Gibbons and Murphy [1992] show that considering incentives from career concerns is necessary for designing optimal contracts that complement these incentives. They show that contract incentives are stronger when career-concern incentives are weaker and present empirical evidence supporting their finding. Consequently, analyzing the way in which career-concern incentives depend on the proximity of the firing period (for each reputation) could be important for understanding the way in which contracts should depend on the proximity of the firing period.17

17 In a model without learning about ability where the principal uses long-term contracts for providing incentives to the
The next step in this research project is to test the empirical implications of the model. This necessitates a measure of the incumbent policymakers’ reputation. A direct measure is possible if approval ratings are available. Alternatively, as the model suggests, a reputation measure can be constructed using observable variables related with past performance.

Analyzing the way in which the framework developed here could help explaining differences in the frequency of elections (or the length of contracts) is also an interesting extension. In this framework, if the incumbent’s action is effort or stealing, an incumbent policymaker prefers to postpone elections while voters prefer to increase the frequency of elections (assuming that the election cost is not too large). This suggests that the frequency of elections may be decided in a bargaining process. The intensity of the players’ preferences about the frequency of elections depends on the incumbent’s reputation and parameter values. Differences in these variables could help explaining differences in the frequency of elections. Moreover, the dynamic model presented here may help us explain changes in the frequency of elections. On the other hand, if the incumbent’s action represents fiscal policy, elections may create incentives to choose some suboptimal policy and, therefore, voters may dislike elections.

Additional natural extensions are analyzing cases with asymmetries in the learning processes, term limits and/or retirement for the policymakers, and a finite number of policymakers (political parties) participating in elections. Situations in which the incumbent’s action affects the voters’ capacity to learn (as in Le Borgne and Lockwood [2004]) could also be considered.

References


incumbent, Spear and Wang [2005] present an alternative reason for which the principal may want to replace the incumbent: it may be more costly to induce the incumbent to exert effort than to induce a new agent to exert effort. If the career-concern incentives discussed in this paper were complemented with incentives contracts, the firing motives considered by Spear and Wang [2005] could appear.


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