Endogenous Growth and the Emergence of Equity Finance

Niloy Bose$^{a,b}$

Abstract:
This paper characterises the development of equity markets as a dynamic process that both influences and is influenced by the development of the real sector of the economy. In an overlapping generations economy borrowers seek funds to run risky investment projects by drawing up contracts which may take the form of either equity or debt issue. In the presence of information asymmetry between borrowers and lenders, the optimal contract is determined by trading off information dilution costs against bankruptcy costs. Significantly, the equilibrium choice of contract depends on the state of the economy which, in turn, depends on the contracting regime. Based on this analysis, the paper provides a theory of the joint determination of real and financial development with the ability to explain the emergence of a stock market along the path of real development.

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a. Department of Economics, University of Wisconsin-Milwaukee, WI 53201, USA.
b. Centre for Growth and Business Cycle Research, School of Economic Studies, University of Manchester, Manchester, M13 9PL, England.

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1. Introduction

Over the last decade, a substantial volume of research has been devoted towards understanding the relation between financial development and real economic activity. At the empirical level, evidence has been found of a strong positive correlation between the level of financial development and long-run growth (King and Levine, 1993a,b; Demetriades and Hussein, 1996). At the theoretical level, various models have been proposed to account for this correlation. In some models, the structure of the financial market is imposed exogenously, and attention is focussed on how financial development reinforces economic growth by increasing the social marginal productivity of investment, and/or by increasing the fraction of savings channelled to investment (e.g. Bencivenga and Smith, 1991; Cooley and Smith, 1998). In other models, financial development is an endogenous outcome of the growth process with consideration given to the co-evolution of real and financial activity (e.g., Greenwood and Jovanovic, 1990; Bose and Cothren (1997).

Most of the above research considers economies in which all investment is financed either by entirely debt or by equity. Yet it has been widely recognised for some time that financial development is a multi-faceted process that takes place through various distinct stages – from the emergence and expansion of bank-intermediated debt finance to the materialisation of stock markets and the increasing use of equity as an additional instrument by which firms raise funds (e.g., Gurley and Shaw, 1955, 1960; and Goldsmith, 1969). Thus, during the later stages of financial development, an economy typically undergoes transition from a financial system based wholly or predominantly on the issue of debt to one involving a much greater reliance on the issue of equity. Empirical studies have recently sought to identify the events that lead to this transition (Levine and Zervos, 1996, 1998; Antje and Jovanovic, 1993; Demirguc-Kunt and Levine, 1996a, 1996b). Almost unanimously these studies suggest that, even after allowing room for some significant exceptions, measures of equity market activity are positively correlated with measures of real development across many different countries (and that the association is particularly strong for developing economies).

Despite strong suggestions from the empirical literature, modelling the transition from one stage of financial development to another has so far eluded the attention of most researchers and there remains little by way of formal analysis of the relationship between equity market activity and real development within a fully-specified dynamic general
equilibrium context. This paper models the co-evolution of the real and financial sectors in a way that explains why the emergence of stock markets occurs relatively late in the process of economic development.

Over the last two and a half decades, a sizeable amount of research identifies the costs and benefits associated with a firm’s decision to go public. On the one hand, factors such as administrative expenses and fees (Ritter, 1987), loss of confidentiality (Yosha, 1992), and adverse selection problem (Leland and Pyle, 1977) have shown to contribute toward the costs of going public. On the other hand, it has been suggested that, by going public, firms generally overcome borrowing constraints, improve their bargaining position with existing lenders (Rajan, 1992), and obtain better liquidity positions. In addition, benefits such as recognition of investors (Merton, 1987), opportunities for diversification (Pagano, 1993), and opportunities to exploit mispricing of equity (Ritter, 1991) have been identified as further incentives to issue equity. More recently, Bolton and Freixas (2000) have proposed a framework in which a firm’s decision to issue equity depends on a trade off between bankruptcy costs and informational dilution costs – a mechanism on which the analysis in the present paper draws heavily. Bankruptcy cost is the loss that a borrower incurs in his current and/or future profit when he is unable to honour a mutually agreed fixed payment that is associated with debt issue. By contrast, an informational dilution cost arises in the presence of informational asymmetries between firms and investors (e.g., Myers and Majluf, 1984). This cost is incurred by good quality firms that are pooled together with inferior quality firms with the result that the offered contract falls short of their first best contract. While, in the absence of any pre-committed payment arrangements, there is no bankruptcy cost associated with equity financing, there may be a higher dilution cost for a good firm offering equity since a firm sells claims on cash flows which depend entirely on the borrower’s type. By contrast, under debt financing, while the dilution costs could be lower, a firm may be forced into bankruptcy and liquidation when the firm’s debt is high. This trade off between the two types of costs plays an important role in determining the optimal financing choice of a firm.

While the literature has made significant contributions in understanding factors that are relevant to a firm’s decision to go public, the analysis has been confined to static, partial equilibrium environments. To draw a connection between the level of economic development and the financing choices of firms, it is imperative to investigate how the costs and benefits of issuing equity could be influenced by the state of the economy along the path of real development. With this purpose in mind, we examine the trade-off between bankruptcy costs and dilution costs (e.g. Bolton and Freixas, 2000) in a dynamic
general equilibrium setting to establish a connection between the financing choice of firms and real development. In particular, we consider an economy in which an informational asymmetry is present between borrowers and lenders. Lenders fund risky investment projects by drawing up financial contracts involving either debt or equity financing. Each period, a representative borrower evaluates the trade-off between bankruptcy costs and dilution costs to determine his preferred mode of financing. Significantly, the loss (in future income) that a borrower incurs in the event of bankruptcy depends on the level of capital accumulation in the economy. As a result, a borrower’s choice between debt financing and equity financing depends crucially on the state of the economy. In turn, the economy’s rate of return to capital depends on the prevailing mode of financing. Given this mutual dependency, we jointly determine the equilibrium mode of financing along the path of development. Our analysis produces results consistent with the stylised facts. In particular, we show that, at low levels of capital accumulation, borrowers rely primarily on debt finance as the main instrument of raising funds. As capital accumulates, however, more and more borrowers use equity as their preferred mode of financing. Consequently, the amount of equity market activity increases along the path of real development.

The paper is organised as follows. In Section 2 we present a description of the economic environment. In Section 3 we study firms’ optimal choice of financing in a partial equilibrium setting. In Section 4 we jointly determine the equilibrium financing choice and the path of capital accumulation within a dynamic general equilibrium setting. Section 5 concludes with some remarks.

2. The Economy

Time is discrete and indexed by \( t = 1, 2, \ldots, \infty \). There is an infinite sequence of two-period-lived agents with a set of initial old agent present at \( t = 1 \). All generations are identical in size and composition. Each generation is divided at birth into two groups of market participants – lenders and borrowers. For convenience, we normalise the size of each group to one. We assume that all agents are risk neutral and wish to consume only in the second period of their lives.

Each borrower begins life with zero resources, except for a risky investment project that converts time \( t \) output into time \( t + 1 \) capital. In order to run such a project, a borrower must acquire external finance from lenders (of the same generation). We assume that borrowers are heterogeneous. Specifically, a newly born borrower can be either of two types: a type 1 or a type 2. A given fraction, \( 0 < v < 1 \), of young borrowers are
assumed to be of type 1. In this economy, output of an investment project at time $t$ is jointly determined by the realisation of a project specific shock, $\Omega_t$, and the type of a borrower who is operating the project. A borrower does not have any control over the realisation of $\Omega_t$. In addition, unless the project is taken on, a borrower is unable to observe the realisation of $\Omega_t$ and knows only its probability distribution which we assume to be identical and independent across projects, and to be given by $\Omega_t = \Omega_1$ (indicating a good state) with prior probability $p$ and $\Omega_t = \Omega_2$ (indicating a bad state) with prior probability $(1-p)$. When $\Omega_t = \Omega_1$, a Type 1 borrower operating an investment project is able to convert 1 unit of time output into $Q_1 > 1$ units of time capital. By contrast, under the same circumstance, a Type 2 borrower is able to produce only $q < 1 < Q$ units of capital. When $\Omega_t = \Omega_2$, an investment project fails and yields nothing irrespective of a borrower’s type.

We assume that by being an operator of an investment project, a young borrower acquires valuable experience irrespective of his type and the project outcome. Through such experience, a borrower is able to develop entrepreneurial skills that can be productively employed during his adulthood. In particular, we assume that each adult borrower is endowed with one unit of labour which, when combined with acquired entrepreneurial skill, produces one unit of entrepreneurial input that has productive use in output production.

Each young lender is endowed with one unit of labour that is supplied inelastically to producers of output at the competitively-determined wage rate, $w_t$. A lender is able to convert his time wage, $w_t$, into $w_t$ units of capital in time $t+1$, and can rent this amount to output producing firms. Alternatively, a lender may wish to lend his wage to a capital-producing borrower in return for capital in $t+1$. To ensure that loan transactions take place between borrowers and lenders, we assume $Q$ to be sufficiently large.

In the second period of life, each old borrower manufactures final output using a common, non-stochastic technology. The inputs to manufacturing are labour (hired from the young of the next generation), physical capital and entrepreneurial input (acquired

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1 This assumption may require some clarification. Transferring skills acquired in one activity to another are common in practice and are often cited in literature. For example, in the 'Stepping Stone' model of Jovanovic and Nyarko (1996), agents are able to transfer skills acquired in one task to other occupations. The positive spillover effect from one activity to another can also be justified on the basis of empirical evidence. For example, it has been documented that firms are often able to increase the level of skills and productivity in some of their branches by transferring skills from others (e.g. Blomstrom, M., et.al 1994)
from investment projects undertaken previously). An adult borrower employing \( l_{t+1} \) units of labour, \( k_{t+1} \) units of capital, and \( s_{t+1} \) units of entrepreneurial input is able to produce \( y_{t+1} \) units of output according to

\[
y_{t+1} = \bar{k}_{t+1}^{\theta} k_{t+1}^{\alpha} l_{t+1}^{1-\alpha} + \bar{k}_{t+1}^{\sigma} k_{t+1}^{\alpha} s_{t+1}^{\sigma-\alpha}, \theta, \alpha, \sigma \in (0,1).
\]

Where, \( \bar{k}_{t+1} \) is the ‘average per firm capital stock’, implying an externality in production of the type considered by Shell (1966), Romer (1986), or Prescott and Boyd (1987a,b). For simplicity, it is assumed that \( \sigma = 1 - \alpha \). In addition, for the economy to have non-trivial transitional dynamics, we assume \( \alpha + \theta = \gamma < 1 \).

Complete factor mobility ensures that producers of output face the same competitively-determined wage rate, \( w_{t+1} \), rental rate of capital, \( \rho_{t+1} \), and rate of return on the entrepreneurial input, \( m_{t+1} \). In equilibrium, all output producers must employ equal amounts of \( l_{t+1} \), \( k_{t+1} \), and \( s_{t+1} \). Since there are equal number of borrowers and lenders, we obtain the amount of labour per firm as \( l_{t+1} = 1 \). Similarly, \( k_{t+1} \) (the quantity of capital employed by each output producer) must equal \( \bar{k}_{t+1} \) in equilibrium. Notice that, in equilibrium, for a given value of \( s_{t+1} \), the production function, takes a form that is similar to that considered by Jones and Manuelli (1990). Thus, the output technology that we study is a generalisation of the idea that the marginal product of capital remains bounded. Given the above, the equilibrium values of \( w_{t+1} \), \( \rho_{t+1} \), and \( m_{t+1} \) are then given by the marginal productivity relations

\[
w_{t+1} = (1-\alpha)k_{t+1}^{\gamma} \tag{2}
\]

\[
\rho_{t+1} = \alpha k_{t+1}^{\gamma-1} + \alpha s_{t+1}^{\sigma-\alpha} \tag{3}
\]

\[
m_{t+1} = (1-\alpha)k_{t+1}s_{t+1}^{-\alpha} \tag{4}
\]

Finally, we assume that, while the distribution of borrower types and the distribution of the project specific random shock are common knowledge, a lender cannot distinguish ex-ante between a type 1 and a type 2 borrower because a borrower’s type is private information. This informational asymmetry plays an important role in shaping the financial contract between borrowers and lenders. The design of the financial contract is discussed in the following section.
3. The Credit market and the Optimal Choice of Financial Contract

To keep our exposition transparent, we first analyse agents’ choice of financial contract for a given set of values of the state variables. In particular, we assume that, at the beginning of period \( t \), each borrower offers a financial contract while taking the capital stock, \( k_t \), wage rate, \( w_t \), and the rental rate of capital for period \( t+1 \), \( \rho_{t+1} \), as given. Subsequently, we demonstrate how the optimal financial contract is influenced by the evolution of these state variables along the growth path.

We assume that a financial contract offered by a borrower can take one of two possible forms: a bond issue or an equity issue. A bond issue specifies a fixed repayment (\( R \)) to lenders at a specified date. By contrast, equity issue entitles lenders (as outside shareholders) to a share \( \delta \in [0,1] \) of produced capital. Also, we assume that the terms of contracts offered in the market are public knowledge and that competition drives lenders’ economic profits to zero.

In the credit market, informational frictions between borrowers and lenders play a pivotal role in determining the nature of the financial contract. On the one hand, as indicated earlier, a lender is unable to distinguish \textit{ex-ante} between a type-1 and a type-2 borrower. On the other hand, we assume that, once the project output is realised, each borrower faces an opportunity to sever ties with lenders by moving to another location\(^2\). By undertaking such fraudulent action, a borrower is able to evade his payment commitments. Such action is costly, however, since it results in a loss of a borrower’s future earnings. Recall that each adult borrower is endowed with one unit of labour that enables him to earn income from acquired entrepreneurial skills. Relocation uses up a fraction, \( \tau \), of a borrower’s second period labour endowment, thus lowering a borrower’s prospect of future earnings. At the same time, relocation means that a borrower loses all his claims on the produced capital. In what follows, we identify the state of events governing the decision of a borrower whether or not to sever ties with lenders. Knowledge of such events is essential in pinning down bankruptcy costs in our models.

Let \( B (\tilde{B}) \) denote the return from the entrepreneurial skills that a borrower receives during his adulthood if he does not (does) re-locate. Given that each adult borrower is endowed with one unit of labour, equation (4) implies,

\[
B_{t+1} = (1 - \alpha) k_{t+1} \rho_{t+1}^{-\alpha} \tag{5}
\]

\(^2\) The story we have in mind is similar to that proposed by Galor and Zeira (1993) and Banerjee and Newman (1993).
\[ \hat{B}_{t+1} = (1-\alpha)(1-\tau)k_{t+1}S_t^{\alpha} \]  

**Proposition 1:** If \( \hat{B}_{t+1} > B_{t+1} - (R_t - qw_t)\rho_{t+1} \), then

(i) A type-2 borrower offering a debt contract would always prefer to sever ties with lenders.

(ii) A type-1 borrower offering a debt contract would sever ties with lenders only if the bad state (\( \Omega_u = \Omega_2 \)) has occurred.

**Proof:**

(i) If \( \Omega_u = \Omega_1 \) and the borrower is a type-2 borrower then the project is able to convert \( w_t \) amount of time \( t \) output into \( qw_t < w_t \) amount of time \( t+1 \) capital. Under such circumstances, if a type-2 borrower decides not to sever ties with lenders then the lenders are able to extract the residual claim, \((R_t - qw_t)\rho_{t+1}\), from his adult income. Accordingly, the net lifetime income of a type-2 borrower is \( B_{t+1} - (R_t - qw_t)\rho_{t+1} \). By re-locating and thereby relinquishing all claims on the produced capital, a type-2 borrower is able to obtain \( \hat{B}_{t+1} \) amount of lifetime income. Thus, when \( \hat{B}_{t+1} > B_{t+1} - (R_t - qw_t)\rho_{t+1} \), it pays a type-2 borrower to sever ties with lenders and re-locate. When \( \Omega_u = \Omega_2 \), the project produces nothing and the equivalent condition is \( \hat{B}_{t+1} > B_{t+1} - R_t\rho_{t+1} \), which is always true given that \( \hat{B}_{t+1} > B_{t+1} - (R_t - qw_t)\rho_{t+1} \).

(ii) When \( \Omega_u = \Omega_1 \), it is possible for a type-1 borrower to honour his payment commitments. Under such a circumstance, given that \( B_{t+1} > \hat{B}_{t+1} \), it is optimal for a type-1 borrower not to sever ties with lenders. In contrast, when \( \Omega_u = \Omega_2 \), the project produces nothing and the condition \( \hat{B}_{t+1} > B_{t+1} - (R_t - qw_t)\rho_{t+1} \) is sufficient to induce a type-1 borrower to sever ties and re-locate.

**Lemma 1:** Borrowers do not sever ties with lenders when they issue equity contracts.

**Proof:** An equity issue specifies a share \( \delta \in [0,1] \) of the produced capital that outside shareholders are entitled to. Thus, a borrower is not committed to any fixed re-payments when an equity contract is issued. Since he is always able to honour his payment commitments, and since \( B_{t+1} > \hat{B}_{t+1} \), a borrower has no reason to sever ties with lenders.
For the remainder of this section, we assume that the inequality \( \hat{B}_{t+1} > B_{t+1} - (R_t - qw_t)\rho_{t+1} \) holds. In Section 4 we explicitly discuss the conditions required to satisfy this inequality.

The above results are useful in identifying the bankruptcy cost which a borrower would incur if he is unable to honour the fixed payments associated with the issue of the debt contract. Following convention (e.g. Bolton and Freixas, 1988, 2000) we view the bankruptcy cost as a firm’s loss in future earning/profit opportunities. The above analysis suggests that a borrower who is unable to honour the debt contract would sever ties with lenders. The loss in future earnings incurred by a borrower in such a case is the difference between \( B_{t+1} \) and \( \hat{B}_{t+1} \). Accordingly, we view \( B_{t+1} - \hat{B}_{t+1} \) as the bankruptcy cost in our model.

Notice that a borrower issuing an equity contract is not committed to any fixed payment and is able to avoid bankruptcy and the associated cost. This does not, however, imply that, from a borrower’s perspective, equity financing is always the preferred mode of raising funds. Under an equity contract, the payments that a lender receives depend entirely on a borrower’s type. As a result, good quality firms incur a higher dilution cost when they are pooled together with inferior quality firms. To illustrate this, consider the scenario in which a debt contract has been offered by a borrower. If \( \Omega \neq \Omega_i \) and the borrower is a Type 2 borrower, then the borrower would re-locate, leaving the opportunity for the lender to appropriate any capital \( (qw_t) \) produced. However, under an equity contract, a lender is only able to obtain a fraction \( (\delta) \) of the output \( (qw_t) \) produced by the Type-2 borrower. Thus, when a lender offers a contract by pooling the two types of borrowers together, an equity contract gets more diluted (i.e., falls short of the first best contract) than a debt contract. In what follows, we illustrate explicitly the effects of this dilution cost on a type-1 borrower’s project payoff. Let \( W_E \) and \( W_D \) denote the expected amount of the capital that a Type 1 borrower retains from the project under an equity and a debt contract, respectively.

Proposition 2: \( W_D > W_E \).

Proof: An equity contract specifies a share \( \delta \in [0,1] \) that the lenders are entitled to. Since competition drives lenders’ profits to zero, the value of \( \delta \) must be consistent with the zero profit constraint of the lender. For a given loanable fund, \( w_t \), the zero profit constraint of the lender is given by \( w_t = \delta p[vQ + (1-v)q]w_t \). This, in turn, implies
\[ \delta = \frac{1}{p[vQ + (1-v)q]). \] When offering a debt contract, a borrower promises a fixed repayment, \( R \), that must also satisfy the zero profit constraint of the lender, i.e.,

\[ vpR + (1-v)pqw_t = w_t. \]

Accordingly, \( R_t = \frac{w_t - (1-v)pqw_t}{vp} \). Given these observations, we obtain

\[ W_E = pQw_t(1-\delta) = pQw_t \times \frac{p[vQ + (1-v)q] - 1}{p[vQ + (1-v)q]} \equiv X_1w_t \] (7)

\[ W_D = p[Qw_t - R_t] \equiv \left[ \frac{Qvp - 1 + (1-v)pq}{v} \right]w_t \equiv X_2w_t \] (8)

It is easily verifiable that \( X_2 > X_1 \Leftrightarrow Qvp + (1-v)pq > 1 \). The latter condition must hold for the project to be feasible. Hence, \( W_D > W_E \)

The results obtained so far suggest that borrowers face the prospect of incurring bankruptcy costs under a debt contract. At the same time, the presence of a higher dilution cost may discourage a borrower to raise funds by issuing an equity contract. As in the case of Bolton and Freixas (2000), a borrower’s optimal choice of contract then rests on the trade-off between the levels of bankruptcy costs and dilution costs. In what follows, we analyse this trade-off and determine the condition governing the optimal contracting form.

Two issues are worth emphasizing at this point. First, lenders operate in a competitive framework which drives their economic profit to zero. Any contract that makes extra-economic profit for the lenders cannot prevail in the market as the lenders would compete against each other and would try to win borrowers by offering all extra-economic profits to the borrowers. This amounts to saying that the competition drives the lenders to maximize the utility of the borrowers subject to their zero profit constraint. Hence, the borrowers’ preference determines the optimal contracting form. Second, a type-2 would always mimic any preference revealed by a type-1 borrower. These two facts together imply that the preference of a type-1 borrower governs the optimal financing choice. Accordingly, we derive the optimal contract by comparing the utilities of a type-1 borrower.

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3 This approach is common in the existing literature. Examples include, Bencivenga and Smith (1991, 1993), Azariadis and Smith (1993), among others.

4 As in Bolton and Freixas (2000), we do not consider the possibility where Type-1 borrowers could attempt to partially reveal themselves by offering a menu of contracts. Such a separating equilibrium can only be supported by ad-hoc beliefs.
borrower under a debt and an equity contract (each of which makes zero profit to the lenders).

Recall that a type-1 borrower derives his life-time earnings from two sources. First, a borrower is able to rent out his earnings (capital) from the project during his adulthood at the competitively determined rental rate, $\rho_{r+1}$. Second, a borrower obtains income from his acquired entrepreneurial skills. Let $U_E$ and $U_D$ represent a type-1 borrower’s life-time utility under an equity contract and a debt contract, respectively.

Making use of equations (7) and (8), and the fact that when a borrower issues an equity contract he does not re-locate, we write the expressions for the life-time utility of a borrower under an equity contract as

$$U_E = \rho_{r+1}W_E + B_{r+1} = \rho_{r+1}X_i w_i + B_{r+1}$$

Here, the first term represents a Type-1 borrower's project earnings (expressed in terms of output) and the second term represents his earnings from the acquired entrepreneurial skills. However, if a debt contract is issued, a type-1 borrower prefers to re-locate when $\Omega = \omega_2$ (which occurs with a probability $(1 - p)$). Accordingly, the expression for the life-time utility of a type-1 borrower offering a debt contract is given as

$$U_D = \rho_{r+1}W_D + pB_{r+1} + (1 - p)\hat{B} = \rho_{r+1}X_2 w_i + pB_{r+1} + (1 - p)\hat{B}_{r+1}$$

After substituting the expressions for $B_{r+1}$ and $\hat{B}_{r+1}$ from equation (5) and (6) into equation (9) and (10), a straightforward comparison reveals that the issue of equity is the preferred mode of financing from the point of view of a type-1 borrower when

$$\tau(1 - p)(1 - \alpha)k_{r+1}^{1 - \alpha} > \rho_{r+1}w_i$$

Thus, the optimal mode of financing essentially depends on the variables, $k_{r+1}, w_i$ and $\rho_{r+1}$, which, in turn, are determined by the economy’s capital dynamics. It is this feature that accounts for the endogeneity of the contracting regime along the economy’s path of development. We address this issue in detail in the following section.

4. The Capital Accumulation Path

The economy’s capital stock at time period $t + 1$ originates from the projects run by type-1 and type-2 borrowers who are successful in converting $w_i$ into capital. Since the population size of borrowers and lenders has each been normalised to one, the time $t + 1$ capital stock per firm is given by $k_{r+1} = p[vQ + (1 - v)q]w_i$. After substituting the
expressions for $w_t$ from equation (2), we obtain the dynamic capital accumulation path of the economy as

$$k_{t+1} = p(1-\alpha)[vQ + (1-v)q]k_t^\gamma$$

(12)

Evidently, both capital dynamics and the time $t$ wage rate are independent of the prevailing contracting regime.$^5$ By virtue of equation (3), however, the time $t+1$ marginal product of capital, $\rho_{t+1}$, is influenced by the contracting regime that prevails at time $t$. To see this, consider the case in which all borrowers are offering equity contracts at time $t$. In the absence of any need to re-locate, all borrowers supply one unit of entrepreneurial input during their adulthood. With equal numbers of borrowers and lenders, the entrepreneurial input per output-producing firm equals 1. Accordingly, by virtue of equation (3), the marginal product of capital at time $t+1$ is given by

$$\rho_{t+1}^E = \alpha k_{t+1}^{\gamma-1} + \alpha$$

(13)

By contrast, if debt financing is in place at time $t$, a fraction, $(1-pv)$, of borrowers will re-locate, each of whom is able to supply $(1-\tau)$ entrepreneurial input during their adulthood. As a result, the amount of entrepreneurial skill available per firm at time $t+1$ is $pv + (1-pv)(1-\tau)$, and the time $t+1$ marginal product of capital is given by

$$\rho_{t+1}^D = \alpha k_{t+1}^{\gamma-1} + \alpha[pv + (1-pv)(1-\tau)]^{(1-\alpha)} = \alpha k_{t+1}^{\gamma-1} + \alpha s^{\gamma-1}(1-\alpha)$$

(14)

Thus, the time $t+1$ return to capital depends on the mode of financing that prevails at time $t$. A straightforward comparison of equations (13) and (14) yields $\rho_{t+1}^E > \rho_{t+1}^D$.

Before proceeding, it is appropriate to comment on the inequality, $\hat{B}_{t+1} > B_{t+1} - (R_t - qw_t)\rho_{t+1}$, which we have used to establish the results in the previous section. Substituting the expressions for $\rho_{t+1}, B_{t+1}, \hat{B}_{t+1}$, and $R_t$, from equation (3), (5), (6), and (8) respectively, and after making use of the relation, $k_{t+1} = p[vQ + (1-v)q]w_t$, it is straightforward to see that

$$\hat{B}_{t+1} > B_{t+1} - (R_t - qw_t)\rho_{t+1} \iff k_{t+1}^{\gamma-1} > s_{t+1}^{\gamma-1} \left[ \frac{vp^2\tau(1-\alpha)}{(1-pq)\alpha} \{vQ + (1-v)q\} - s_{t+1} \right] \equiv s_{t+1}^{\gamma-1} \Theta(s_{t+1})$$

(15)

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$^5$ This, however, is not true for the output growth. An increase in the use of equity financing would result in higher per capita output growth for the economy due to an increase in the level of entrepreneurial skill per firm. This result is in tune with the findings by Levine and Zervos (1998) which suggest that the value of stock trading relative to the size of the market and the value of trading relative to the size of the economy positively and significantly predict the current and future rates of output growth.
As long as $Q$ is not too large, the expression $\Theta(s_{t+1})$ remains negative even when $s_{t+1}$ takes the minimum value, $\hat{s}$ (as implicitly defined by equation (14)). Thus, for A1 to hold at all times, we impose an upper bound on the values of the parameter $Q$.\(^6\)

After making use of the fact that $k_{t+1} = p[vQ + (1 - v)q]w_t$, we re-write equation (11) as

$$\frac{\tau p(1 - p)(1 - \alpha)[vQ + (1 - v)q]^{\tau'\alpha}}{X_2 - X_1} \equiv \Psi(s_{t+1}) > \rho_{t+1} \quad (16)$$

According to the above equation, a borrower’s financing choice at time $t$ depends on the date $t + 1$ rate of return to capital, $\rho_{t+1}$, and the entrepreneurial input level per firm, $s_{t+1}$. As indicated by equations (13) and (14), $\rho_{t+1}$, in turn, depends upon the financing choice of borrowers at time $t$. Further, $s_{t+1}$ is also influenced by borrowers’ choice of financing at time $t$. In what follows, we take into account these mutual dependencies in establishing the link between the level of capital accumulation and the equilibrium financing choice of borrowers.

To begin, we consider a scenario where, at time $t$, $\mu_t \in (0,1)$ fraction of borrowers offer equity financing and the rest $(1 - \mu_t)$ offer debt financing. As a result, the level of entrepreneurial input available per firm at time $t + 1$ is

$$s_{t+1}^m = \mu_t + (1 - \mu_t)\hat{s}, \quad (17)$$

and the resulting time $t + 1$ marginal product of capital is given by

$$\rho_{t+1}^m = \alpha k_{t+1}^{\mu_t} + \alpha [s_{t+1}^m]^{1-\alpha} = \alpha k_{t+1}^{\mu_t} + \alpha [\mu_t + (1 - \mu_t)\hat{s}]^{1-\alpha} \quad (18)$$

**Proposition 3**: Any $\mu_t = \mu_t^* \in (0,1)$ for which $\rho_{t+1}^m = \Psi(s_{t+1}^m)$, supports an equilibrium in the credit market at time $t$ where $\mu_t^*$ fraction of borrowers offer equity finance and the rest $(1 - \mu_t^*)$ offer debt financing.

**Proof**: For any given $\mu_t \in (0,1)$, if $\Psi(s_{t+1}^m) > \rho_{t+1}^m$, then (by virtue of equation (16)) borrowers who currently offer debt contracts would deviate and offer equity contracts, instead. Accordingly, such a $\mu_t$ cannot support an equilibrium. Similarly, any $\mu_t \in (0,1)$}

\(^6\) For $\Theta(\hat{s}) < 0$ requires $\alpha(1 - p\hat{q})[p\hat{v} + (1 - p\hat{v})(1 - \tau)] > \tau p v' (1 - \alpha)[vQ + (1 - v)q]$. The parameter value $\nu = 0.5, \hat{q} = 0.5, \hat{p} = 0.5, \tau = 0.11$, and even an exceptionally large value of $Q = 10$, satisfies the above condition with a sufficient margin.
for which \( \Psi(s_{t+1}^m) < \rho_{t+1}^m \) holds cannot support an equilibrium since the borrowers who currently offer equity contracts would deviate and offer debt contract. Thus, any \( \mu_t = \mu_t^* \in (0,1) \) for which \( \rho_{t+1}^m = \Psi(s_{t+1}^m) \) holds can support a credit market equilibrium at time \( t \). □

**Lemma 2:** \( \mu_t^* \) is unique and is increasing in \( k_t \).

*Proof:* Define \( T(\mu_t) = \Psi[\mu_t + (1 - \mu_t)\hat{s}] - \alpha[\mu_t + (1 - \mu_t)\hat{s}]^{(1-\alpha)} \). Making use of equation (17) and (18), it is easy to see that \( \rho_{t+1}^m = \Psi(s_{t+1}^m) \Leftrightarrow ak_{t+1}^y = T(\mu_t) \). Further, the definition of \( \Psi(,) \), together with the fact that \( \hat{s} = pv + (1 - pv)(1 - \tau) < 1 \), implies that the function \( T(\mu_t) \) is monotonically decreasing in \( \mu_t \) over the interval \( \mu_t \in (0,1) \). Therefore, any \( \mu_t = \mu_t^* \in (0,1) \) that solves \( ak_{t+1}^y = T(\mu_t) \) is unique. In addition, since \( \gamma < 1 \), and since \( k_{t+1} \) is an increasing function of \( k_t \) (through equation 12), \( ak_{t+1}^y \) is decreasing in \( k_t \). Thus, to maintain the equality \( ak_{t+1}^y = T(\mu_t^*) \), \( \mu_t^* \) must increase with \( k_t \). □

One may note, however, the possibility that \( \mu_t^* \) may not lie in the interior of its own domain. Notice that the function \( T(\mu_t) \) attains its highest and lowest values at \( \mu_t = 0 \) and at \( \mu_t = 1 \), respectively. For a small enough \( k_t \), it is possible that \( ak_{t+1}^y \geq T(0) \) may hold. Similarly, a large enough \( k_t \) may give rise to a scenario where \( ak_{t+1}^y \leq T(1) \). Both situations imply an absence of \( \mu_t^* \in (0,1) \). We characterize the outcomes under such circumstances, as follows.

**Proposition 4:**

(i) If \( ak_{t+1}^y \geq T(0) \), then the credit market at time \( t \) is characterized by a unique equilibrium where all borrowers offer debt contracts.

(ii) If \( ak_{t+1}^y \leq T(1) \), then the credit market at time \( t \) is characterized by a unique equilibrium where all borrowers offer equity contracts.

*Proof:* Suppose that borrowers offer debt contracts at time \( t \), so that \( \Psi(s_{t+1}) = \Psi(\hat{s}) \) and the time \( t + 1 \) marginal product of capital is \( \rho_{t+1}^D = \alpha[k_{t+1}^y + \alpha \hat{s}^{1-\alpha}] \) as given by equation (14). The definition of \( T(\mu_t) \) implies

14
\[ \alpha k_{t+1}^{-1} \geq T(0) \Leftrightarrow \alpha k_{t+1}^{-1} \geq \Psi(\hat{s}) - \alpha s^{1-a} \Leftrightarrow \rho_{t+1}^D \geq \Psi(\hat{s}). \] Since \( \rho_{t+1} = \rho_{t+1}^E \geq \Psi(\hat{s}) \), equation (16) suggests that no borrower has an incentive to deviate and offer an equity contract when all others offer debt contracts. Debt financing is therefore the equilibrium choice of the borrowers. To see that this is the unique equilibrium, suppose that all borrowers offer equity contracts. Accordingly, \( \rho_{t+1} = \rho_{t+1}^E \), as given by equation (13), and \( \Psi(s_{t+1}) = \Psi(1) \).

Recall that \( \Psi(\hat{s}) > \Psi(1) \) and that \( \rho_{t+1}^E > \rho_{t+1}^D \). Therefore, \( \rho_{t+1}^D \geq \Psi(\hat{s}) \) implies \( \rho_{t+1}^E > \Psi(1) \).

According to equation (16), an individual borrower in such a situation would deviate and offer a debt contract when other borrowers offer equity contracts. Therefore, equity financing cannot qualify as an equilibrium financing choice. By employing similar arguments, it is easy to see that equity finance is the unique equilibrium choice of borrowers when \( \alpha k_{t+1}^{-1} \leq T(1) \).

Based on the above observations, we now analyse the evolution of the financial regimes along the path of economic development. Let \( k^D \) and \( k^E \) represent the values of \( k_i \) for which \( \alpha k_{t+1}^{-1} = T(0) \) and \( \alpha k_{t+1}^{-1} = T(1) \), respectively. It is then possible to characterize the equilibrium in the financial market in terms of the relation of the current capital stock, \( k_i \), with \( k^D \), \( k^E \), and the initial capital stock per firm, \( k_0 \). For example, when \( k^D \geq k_i \geq k_0 \), we have \( \alpha k_{t+1}^{-1} \geq T(0) \) and debt financing is the market equilibrium choice of financing. When \( k_i \) exceeds \( k^D \) and is such that \( k^E > k_i > k^D \), the market equilibrium at time \( t \) is characterized by \( \mu_t^* \in (0,1) \) fraction of borrowers offering equity financing and the rest choosing debt financing. As illustrated by Lemma 2, \( \mu_t^* \) increases monotonically with capital accumulation in this interval implying that the market increasingly relies on equity as a means of raising funds along the growth path. Finally, when \( k_i > k^E \), we have \( \alpha k_{t+1}^{-1} \leq T(1) \), which represents the extreme scenario where at a very high level of capital accumulation, borrowers rely exclusively on equity financing.

We illustrate the above results using dynamic simulations of a numerical version of the model. As well as giving an idea of the orders of magnitude involved, these simulations allow us to observe the changes in the optimal financing choice along the transition path of capital. The transition process is generated by equation (16), together with equation (12) which describes the dynamics of capital accumulation. We begin by fixing the initial capital stock per firm, \( k_0 \), which determines \( k_i \) according to equation (12). Denoting \( \mu_0^* \) as the equilibrium fraction of borrowers offering equity finance at
We next evaluate \( s^m_t(\mu^*_0) \), \( \rho^m_t(\mu^*_0) \), and \( \Psi(s^m_t(\mu^*_0)) \) according to equations (17), (18) and (16), respectively. For \( \mu^*_0 \) to support an equilibrium, the relation 
\[
\rho^m_t(\mu^*_0) = \Psi(s^m_t(\mu^*_0))
\]
must hold. This equality yields the value of \( \mu^*_0 \). In cases, where the value of \( \mu^*_0 \) is less than zero or greater than one, we return values \( \mu^*_0 = 0 \) and \( \mu^*_0 = 1 \), respectively. We repeat the above process taking \( k_1 \) as the starting value of the capital stock per firm, and so on and so forth. This process of iteration yields a sequence \( \{(k_0, \mu^*_0), (k_1, \mu^*_1), (k_2, \mu^*_2), \ldots\} \), which, in turn, demonstrates the joint evolution of the capital stock and the financing choice in the credit market.

Our benchmark parameter values are given by
\[
\{v = 0.50, q = 0.50, p = 0.50, \tau = 0.111, Q = 6, \alpha = 0.36, \theta = 0.33, \gamma = 0.69, \sigma = 0.64\}
\]
These values satisfy the parameter restrictions assumed up to now and are sufficient for illustrative purposes. We begin the iteration process with an initial capital stock per firm, \( k_0 = 0.5 \). In accordance with equation (12), the capital stock per firm of this economy converges to a value, \( k_{ss} = 1.375 \). The diagrammatic presentation of 25 iterations is presented in Figure 1. The results strongly support the predictions of our theoretical analysis. For example, there is a complete absence of equity finance (i.e., \( \mu^*_t = 0 \)) until the economy reaches a certain level of maturity (i.e., \( k_{t+5} = 1.09 \)). From then on, the value of \( \mu^*_t \) increases monotonically with the capital stock per firm and converges to \( \mu^*_t = 0.783 \) as the economy reaches its steady state. Thus, the above chain of events describes a process of transition from low to high economic development in which the stock market becomes an increasingly important source of funding for the borrowers.

5. Concluding Remarks

It is well documented (e.g., Michie 1987; Gurley and Shaw 1967) that equity market development is strongly associated with real development in the history of the United States and United Kingdom. More recently, empirical studies have gone beyond country-specific studies. Using cross-country analysis, these studies suggest that strong equity market activity is typically associated with high levels of economic development. Against this backdrop, our paper seeks to explain the positive correlation between stock market activity and economic growth, which has received relatively little theoretical attention.
In drawing a connection between equity market development and real development, one may wish to consider routes (although, to our best knowledge have not been pursued) that are different from those presented in this paper. For example, one may appeal to a more conventional micro-approach (e.g., Cass and Stiglitz, 1972) which highlights the wealth effect on portfolio choices of individuals. Alternatively, one may wish to view equity market development as an outcome of risk-taking behaviour of individuals in the face of a declining rate of return on capital along the path of development. Recently, Boyd and Smith (1996, 1998) have proposed a framework in which producers of capital choose between two different technologies that are financed in two different ways. The first type of technology is one that yields a relatively low expected return, is publicly observable and is financed by means of equity at no expense. The second type of technology is one that yields a relatively high expected return, is not directly observable by lenders and is financed by means of debt subject to a standard costly verification problem (e.g. Townsend 1979; Diamond (1984). Assuming plausible parameter values, it is shown that there is a critical level of per capita income below which only a debt market exists. However, as capital accumulation takes place, the cost of state verification increases due to a fall in the relative price of capital. Eventually (i.e., once the critical level of income is reached), a stock market emerges as firms begin to make more use of the observable technology and less use of the unobservable technology, implying an increase in the amount of equity finance relative to debt finance.

Rather than appealing to the above conventional approaches or rather than restricting the mode of financing to a particular type of technology (as in Boyd and Smith, 1996, 1998), our paper focuses on the information structure in the capital market to provide an explanation for why economies increasingly rely on equity market along the path of economic development. We build this explanation in terms of a trade-off between information dilution costs and bankruptcy cost along the path of development. On the one hand, this approach enables us to offer a relatively simple framework capable of connecting equity market activity and real development. On the other hand, by embedding a mechanism frequently used in the micro-finance literature for evaluating firms' optimal financing choice, this paper bridges the gap that commonly exists between the mainstream finance literature and the literature on economic development.

Before concluding, it is worth emphasising that the objective of this paper has been to shed light on the observed trend that the level of equity market activity increases along the path of real development. In reality, however, one may occasionally come across notable exceptions to such a trend as it is sometime the case that economies whose level of
economic development is comparable differ in terms of the volume of equity market activity. For example, in the U.K. the ratio of stock market value to GDP is five times larger than in Germany, France, Denmark and Finland, and six times larger than Italy and Norway (Pagano, 1993). Such aberrations are most commonly explained on the basis of differences in institutional and regulatory arrangements in financial markets across these countries, the discussion and analysis of which is beyond the scope of the present paper.

References


Figure 1: Capital Accumulation and Financing Choice