Information Flows and Aggregate Persistence*

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Abstract

This paper studies the effect of imperfect information on aggregate output and price dynamics. I argue that imperfect information can lead monetary shocks to have persistent real effects. In the environment with unobserved aggregate (monetary) and real demand shocks, price-setting agents face constant probability of updating to full information. Between full updating, agents use market prices and quantities to infer the state of the economy.

The general equilibrium model allows for an explicit aggregation. The economy is more informative if (a) the fraction of fully updating agents is high; (b) shocks to the money supply are more volatile than the sector-specific shocks; and (c) the degree of real rigidity is small. I find that the effect of monetary shocks on output and inflation is bigger in economy that is less informative. Dynamics in uninformative economies can be well approximated by the equilibrium where signals convey no information (as in Mankiw and Reis (2002)).

*Preliminary and incomplete. All remaining errors are mine. The views expressed in this paper are those of the author. No responsibility for them should be attributed to the Bank of Canada.

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1 Introduction

What is the nature of monetary nonneutrality and how important is it for business cycles? These questions have been guiding the agendas of generations of business cycle researchers for decades. In his defining work Lucas (1972) laid out the foundations of the nonneutrality of money under rational expectations and imperfect information about fundamental disturbances. In Lucas’ model the information becomes public knowledge soon after the monetary shock rendering real effects counterfactually short lived\(^1\).

In the absence of convincing theory of persistent information, alternative sources of monetary nonneutrality attracted more attention. In particular, sticky price models introduced by Calvo (1983) and Taylor (1980) have become a dominant workhorse of business cycle modellers. In the last several years, however, there has been a growing literature reevaluating theoretical appeal and empirical relevance of models with nominal rigidities. Chari, Kehoe, and McGrattan (2000) show that models with sticky prices alone cannot generate the long lasting responses of real economy to monetary shocks. Adding real rigidity in the sense of Ball and Romer (1990) on one hand, and adjustment costs on control variables on the other hand, helped propagating the nonneutrality\(^2\). However, to achieve plausible degree of persistence, the number of such frictions needs to be large and adjustment costs substantial. In addition, Sims (1998) argues that without assuming "pervasive adjustment costs" standard theories cannot replicate the evidence of sluggish cross-variable interactions found in empirical macroeconomic models. Finally, Bils and Klenow (2004) and Klenow and Kryvtsov (2004) present the new empirical evidence and find small degree of nominal price stickiness. As a result, yet another reevaluation of the driving force behind real persistence is warranted.

The new approach proposed by Sims (2003) assumes that individuals have limited information-processing capacity. Such individuals behave as if they face a signal-extraction problem with the noise due to the constrained processing of information. Sims’ theory provides microfoundations for persisting wedge between observed and processed information.

Recently many business cycle researchers incorporated Sims’ idea into the models of business cycles. Woodford (2001) considers an aggregated model in which imperfect information is due to agents facing \textit{exogenous} idiosyncratic noisy signals. In symmetric equilibrium real effects are persistent because higher order expectations adjust slowly\(^3\). To generate plausible persistence the noise have to be assumed large\(^4\).

An alternative way of generating lasting imperfect information is to assume cost of information acquisition or to impose information delays. Moscarini (2004) derives optimal time-dependent rules under limited information capacity constraint. There is a trade-off between the benefit of fresh information and cost of higher rate of information acquisition. As a result, optimal inertia is unresponsive and nonmonotonic in the predictability of the

\(^1\)The literature on identification and effects of monetary shocks is voluminous. See, for example, Christiano et al. (1996, 2000), Romer and Romer (1989, 2004), Bernanke and Mihov (1998).

\(^2\)See, for example, Christiano, Eichenbaum, and Evans (2005), Bouakez, Cardia, and Ruge-Murcia (2000), Dotsey and King (2001).

\(^3\)Importance of higher order expectations was emphasized by Phelps (1970) and recently by Morris and Shin (2002)

\(^4\)Other research along this line includes Hellwig (2002) who studies information interactions under public and private signals, Lorenzoni (2004) investigates dynamic responses in economy with technology shocks.
environment. Mankiw and Reis (2002) write a model in which agents update their information sets infrequently. In between updating, information sets are unchanged. The model is able to replicate some attractive features of the monetary transmission mechanism. The assumption that agents do not receive or are not able to process any information for long periods of time allows for tractability at the cost of potentially missing some important aspects of agents’ decisions.

The distinct feature of the above models of adjustment under imperfect information is the exogenously imposed nature of information acquisition. Equilibrium outcomes have no effect on the rate of information flows. The contribution of this paper is to study optimal adjustment in economy where agents draw information from their market place. In this case, the endogenous signals are directly determined by equilibrium outcomes.

More specifically, this paper studies the effect of imperfect information on aggregate output and price dynamics. I argue that imperfect information can lead monetary shocks to have persistent real effects. In the environment with unobserved aggregate (monetary) and real demand shocks, price-setting agents face constant probability of updating to full information. Between full updating, agents use market prices and quantities to infer the state of the economy. There are two channels through which new information affects aggregate output and inflation. The "passive" channel is due to full updating of information by a fraction of agents in each period. "Active" channel is determined by agents’ ability to infer the state of the economy from the endogenous signals in between periods of full updating.

The general equilibrium model allows for an explicit aggregation. The active learning is strong if (a) the fraction of fully updating agents is high; (b) the sector-specific shocks are more volatile than shocks to the money supply; and (c) the degree of real rigidity is small. I find that the effect of monetary shocks on output and inflation is bigger in economies with weaker active channel of information transmission. Equilibrium dynamics in such economies can be well approximated by the equilibrium where signals convey no information (as in Mankiw and Reis (2002)). I also estimate the wedge between most and least informed agents in a stationary equilibrium. For plausible parameter values the wedge is around 8% of quarterly consumption.

The paper proceeds as follows. Section 2 introduces the model and defines equilibrium. Section 3 explains how the model is aggregated and solved. Section 4 presents and discusses the results of model simulations. Section 5 concludes.

2 Model

The economy consists of a finite number of structurally identical island economies. Each island is populated by a unit measure of final good producers, a unit measure of households producing intermediate goods, and a local government. There is no trade or communication across islands. Let time be indexed by $t = 1, 2, \ldots$. There are two sources of uncertainty on each island. The first source is due to fluctuating rate of money supply growth on the island, characterized by a random variable $\mu_t$. The other source of uncertainty comes from stochastic demand for intermediate goods, denoted by a random variable $\phi_t$. It is assumed that innovations to the money growth process are equal and perfectly synchronized across islands, whereas innovations to the demand are independent across islands. In addition, the (across island) sum of demand disturbances is restricted to be constant in all periods. Hence, money growth goods demand are interpreted as aggregate and island specific shocks to the aggregate economy respectively.
Let $z_t = \{\mu_t, \phi_t\}$ denote the state of uncertainty on an island in period $t$, and $z^t = \{z_0, z_1, ..., z_t\}$ denote the history of states through period $t$. Each period a fraction of households directly observes the history of states. For all remaining households information is transmitted indirectly via prices and quantities in the markets where they trade.

The households will be indexed by a pair $(i, \tau)$, where $i \in [0,1]$ and $\tau$ denotes the number of periods since the last time the state history was directly observed. The informed households have an index of zero, so $\tau = 0, 1, 2,...$

In the rest of this section I describe the island economy’s setup and equilibrium.

### 2.1 Final goods producers

Final good is produced by a continuum of competitive producers using intermediate goods as inputs. The production function is Dixit-Stiglitz (CES) over variety of differentiated input goods. Let $Y_t$ denote the quantity of the final good produced, $y_t(i)$ be the quantity of input goods purchased from household $(i, \tau)$, $P_t$ and $P^*_t(i)$ - prices of final and intermediate goods respectively. Final goods producers solve the following problem:

$$\max_{\{y_t(i)\}} P_tY_t - \int P^*_t(i)y_t(i)di$$

subject to

$$Y_t = e^{\phi_t} \left[ \int y^*_t(i) \left( \frac{1}{\sigma} \right) d\bar{i} \right]^{\frac{\theta}{\sigma - 1}}$$

Here $e^{\phi_t}$ is a random shift of the demand for intermediate inputs. We assume that $\phi_t$ follows an AR(1) process $\phi_{t+1} = \rho \phi_t + \varepsilon_{\phi t+1}$, with mean zero and i.i.d. errors $\varepsilon_{\phi t+1} \sim N(0, \sigma^2_{\phi})$.

First-order conditions yield the demand function for intermediate goods:

$$y^*_t(i) = Y_t \left( e^{\phi_t} \right)^{\theta - 1} \left( \frac{P_t}{P^*_t(i)} \right)^{\theta}, \quad \tau = 0, 1, ...$$

Zero-profit condition implies that the island price index is

$$P_t = e^{-\phi_t} \left[ \int P^*_t(i)^{1-\theta} d\bar{i} \right]^{\frac{1}{1-\sigma}}$$

### 2.2 Intermediate goods producers

There is a unit measure of self-employed households on each island. Every period a household trades money and nominal bonds, purchases final good for consumption, and produces a differentiated good to trade to the final good producers.

The timing of events within a period is the following. The household $(i, \tau)$ starts period $t$ with nominal wealth $W^*_t(i)$. At the beginning of period $t$ current state $z_t$ is realized. Each household with probability $\lambda$ directly observes the history of states through current period, $z^t$. Hence in any period a fraction $\lambda$ of households fully updates their information. Next the prices of differentiated goods are set. The asset and goods markets are then open. The trader of the household buys money $M^*_t(i)$ and nominal debt $B^*_t(i)$ with gross nominal return $R_t$ payable in period $t + 1$. The worker of the household $(i, \tau)$ produces $y^*_t(i)$ units of differentiated good to satisfy all the demand at the preset price. The shopper of the
household purchases $c_t^i(i)$ units of the final good for consumption by the household. By the end of period $t$ household’s nominal wealth is $W_t^r(i)$.

Let $s_t^r(i)$ denote prices and quantities observed by the household $(i, \tau)$ in period $t$. These prices and quantities contribute the signal $s_t^r(i)$. Let $s^{\tau, t}(i)$ denote the history through period $t$ of signals since last full updating, $s^{\tau, t}(i) = \{s_{t-\tau}^r(i), s_{t-\tau+1}^r(i), \ldots, s_t^r(i)\}$. The information set of agent $(i, \tau)$ at the beginning of the period is $I_t^r(i) = \{s_{t-\tau}, s^{\tau-1, t-1}(i)\}$. Before asset and goods markets close household $(i, \tau)$ updates their information sets after observing new market signals $s^{\tau, t}(i)$, so that the information set at the end of the period is $\hat{I}_t^r(i) = \{s_{t-\tau}, s^{\tau, t}(i)\}$. Hence all decisions in period $t$ except for the preset prices, are made with respect to the updated information.

Each period utility is derived from consumption, holding of real balances at the end of asset markets and leasure. Let $V_t^r(i)$ denote the value function of household $(i, \tau)$ at the beginning of period $t$. This household solves the following optimization problem:

$$V_t^r(i, W_t^r) = \max_{P_t^r} \left[ \max_{c_t^r(i), y_t^r(i), M_t^r(i), B_t^r(i)} U(c_t^r(i), M_t^r(i)/P_t, y_t^r(i)) \right]$$

$$+ \beta(1 - \lambda)E \left[ V_{t+1}^{r+1}(i, W_{t+1}^r) | \hat{I}_t^r(i) \right]$$

$$+ \beta \lambda E \left[ V_0^r(i, W_0^r) | \hat{I}_t^r(i) \right]$$

(4)

subject to (2), period $t$ budget constraint

$$M_t^r(i) + B_t^r(i) \leq W_t^r(i) \pi_t$$

(5)

the law of motion for households nominal wealth

$$W_{t+1}^{r+1}(i) = M_t^r(i) + R_t B_t^r(i) + P_t^r(i) y_t^r(i) - P_t c_t^r(i) + \Pi_t$$

(6)

nonnegativity constraints on consumption, output and money holdings, and no-Ponzi schemes constraints on the level of nominal debt holdings. Initial conditions consist of the distribution of nominal wealth holdings at the beginning of period 0, $\{W_0^r(i)\}$.

In the problem above, $\beta$ denotes a discount factor, $\beta \in (0, 1)$. The period utility function is continuous, strictly increasing and strictly concave in consumption and real money balances, and decreasing in labor. Claims on profits of final good producers are denoted by $\Pi_t$.

As reflected in the budget constraint (5), agents receive a subsidy proportional to their nominal wealth at the beginning of each period. This assumption is made to abstract from the effects of the fiscal policy on the price level, so that the price level is determined only by the quantity of money in the economy.

Furthermore, I assume that all fully updating firms pool their wealth at the beginning before entering the asset market. The beginning of period $t$ wealth of fully updating households is then given by

$$W_t^0(i) = \sum_{t=1}^{\infty} \nu^t W_{t-1}^{r-1}(i)$$

(7)

Let $U_{ct}(i) \equiv \partial U(c_t^r(i), M_t^r(i)/P_t, y_t^r(i))/\partial c_t$ denote the first derivative of the utility with respect to its argument $x$, where $x = \{c, M/P, y\}$. First-order conditions yield

$$P_t^r(i) = \frac{\theta}{\theta - 1} \frac{E \left[ (\sum_{t}^{r} U_{ct}(i) \cdot Y_t \cdot (e^{\phi_t})^{\theta - 1} P_t^0 | I_t^r(i) \right] }{E \left[ U_{ct}(i)/P_t \cdot Y_t \cdot (e^{\phi_t})^{\theta - 1} P_t^0 | I_t^r(i) \right] }$$

(8)
\[
\frac{U_{mt}^i(i)}{U_{ct}^i(i)} = R_t - 1
\]
\[
\frac{U_{ct}^i(i)}{P_t} = \beta(1 - \lambda)E \left[ R_{t+1} \pi_{t+1} \frac{U_{ct+1}^i(i)}{P_{t+1}} | \tilde{Z}_t \right]
\]
\[
W^i_t = M^i_t (R_t - 1) + R_t \pi_t W^i_t(i) - P_t e^\phi_t(i) + P_t^\tau(i)(e^\phi_t)^{\theta-1} \left( \frac{P_t}{P^\tau_t(i)} \right)^{\theta} + \Pi_t
\]

According to the pricing equation (8), each household sets its price to a markup over the ratio of expected marginal disutility of output to the expected marginal utility of consumption. In equation (9) the marginal rate of substitution between consumption and real money balances is equal to the nominal interest rate. The Euler equation (10) says that the disutility of giving up one unit of consumption in period \( t \) is equal to the increase in utility from an extra one unit of consumption in period \( t + 1 \) augmented by the nominal interest rate. Finally, (11) is the budget constraint written in terms of household’s nominal wealth.

### 2.3 Information processing

It follows from the first-order conditions (8)-(11) that the set of observed prices and quantities in period \( t \) can be represented by a triplet \( \tilde{s}_t(i) = \{P_t, Y_t(e^\phi_t)^{\theta-1}, R_t\} \). I assume that the signal \( s^\tau_t(i) \) is a geometric mean over the vector \( \tilde{s}_t(i) \):

\[
s^\tau_t(i) = P_t^{1-\omega_Y-\omega_R} \left( Y_t(e^\phi_t)^{\theta-1} \right)^{\omega_Y} R_t^{\omega_R}
\]

where weights \( \omega_Y, \omega_R \in [0,1] \).

Equation (12) represents the information processing constraint on households: the signal is an imprecise representation of the observed variables. The benefit of having the processing constraint is that there is no need to introduce more than two sources of uncertainty. Without the constraint agents would be able to precisely infer the state unless the number of non-collinear sources of uncertainty is bigger than the number of observed variables, in our case three. Introducing additional sources of uncertainty not only decreases tractability, but also greatly convolutes the informational structure of the model.

The interpretation of the processing constraint is similar in spirit to many post-Sims (2003) models. Agents cannot process all the information they receive each period, and instead update their probability distributions based on imperfect or noisy signals of new information. Notice, however, that specification (12) contains no noise, so that the signal \( s^\tau_t(i) \) is only the function of the state history.

Since the observed variables, \( P_t, Y_t(e^\phi_t)^{\theta-1} \) and \( R_t \) are island-wide variables, all signals are common knowledge among households on the same island. This together with the assumptions of identical preferences and wealth pooling implies that households who fully update in the same period set the same prices and quantities. From here on I omit index \( i \).

The evolution of households’ probability distributions are given by Bayes laws. In the linearized model in Section 3 the Bayes updating implies that the laws of motion for conditional expectations of the state are given by the Kalman filter equations.

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\[^5\] \( P_t \) and \( R_t \) are observed directly, \( Y_t(e^\phi_t)^{\theta-1} \) is derived from the demand function (2) and direct observation of demand, \( y_t^c \), and relative price, \( P_t^\tau/P_t \).
2.4 Local government

Let $\mu_t$ denote the rate of growth of the money supply, $e^{\mu t} = \frac{M_t}{M_{t-1}}$. Local government supplies money according to the stochastic process $\{\mu_t\}$, where $\mu_t$ is assumed to follow an AR(1) process $\mu_{t+1} = (1-\rho_\mu)\mu + \rho_\mu \mu_t + \varepsilon_{\mu t+1}$, with mean $\mu$ and i.i.d. errors $\varepsilon_{\mu t} \sim N(0, \sigma_\mu)$.

On the fiscal side, the local government chooses the sequence of nominal debt, $\{B_t\}$ such that its budget is balanced in all periods:

$$(M_t + B_t) \pi_t^{-1} = M_{t-1} + R_{t-1} B_{t-1}$$

(13)

2.5 Market clearing conditions

There are four markets on each island: final and intermediate good markets, money and debt markets. Intermediate goods market clearing is already implied by using the same notation for the demand and supply of intermediate inputs. The remaining market clearing conditions are

$$\sum_{\tau=0}^{\infty} \nu^\tau c_t^\tau = Y_t$$

(14)

$$\sum_{\tau=0}^{\infty} \nu^\tau M_t^\tau = M_t$$

(15)

$$\sum_{\tau=0}^{\infty} \nu^\tau B_t^\tau = B_t$$

(16)

2.6 Equilibrium

Island equilibrium consists of sequences of prices $\{P_t, P_t^r, R_t\}_{t=0}^{\infty}$, allocations $\{Y_t, c_t^*, M_t^*, B_t^*, W_t^*, W_t, y_t^*\}_{t=0}^{\infty}$, value functions $\{V_t^*(\cdot)\}_{t=0}^{\infty}$ such that, given aggregate prices $\{P_t\}_{t=0}^{\infty}$, $\{R_t\}_{t=0}^{\infty}$ and initial nominal wealth distribution $\{W_t^*(i)\}$, for all state histories,

(1) allocations and individual prices $\{P_t^r\}_{t=0}^{\infty}$ solve the optimization problem of the final good producers and households;

(2) value functions $\{V_t^*(\cdot)\}_{t=0}^{\infty}$ satisfy Bellman equations (4)

(3) households’ probability distributions are updated according to the Bayes law

(4) final good producers collect zero profits

(5) all markets clear.

The local government’s budget constraint holds by Walras law.

It will be useful for solving the model to rewrite the household’s budget contraint in terms of discounted sum of future surpluses. Denote $E_t^\tau \equiv E \cdot [\mathcal{I}_t^\tau (i)], \hat{E}_t^\tau \equiv E \cdot [\hat{\mathcal{I}}_t^\tau (i)]$.

Multiplying period $t+k$ budget constraint by $\hat{E}_t^\tau \left( \prod_{h=0}^{k} R_t^{-1} \right) / P_{t+k}$ and summing forward we get:

$$\sum_{k=0}^{T-1} \hat{E}_t^\tau \left( \prod_{h=0}^{k} R_t^{-1} \right) \left[ \frac{M_{t+k+1}^* + B_{t+k+1}^*}{P_{t+k}} - \frac{y_{t+k}^* \theta_{t+k-1} \left( \frac{P_{t+k}}{P_{t+k}^*} \right) \theta_{t+k} - M_{t+k}^* + R_{t+k} B_{t+k}^*}{P_{t+k}} \right] = 0$$
Rearranging terms we obtain:
\[
\frac{W_t^\tau}{P_t} = \hat{E}_t^\tau \sum_{k=1}^{T-1} \prod_{h=1}^{k} R_{t+1+h}^{-1} \left[ (R_{t+k} - 1) \frac{M_{t+k}^\tau + Y_{t+k} \phi_{t+k}^{\theta-1} \left( \frac{P_{t+k}}{P_{t+k}^\tau} \right)^{\theta-1}}{P_{t+k}^\tau} \right] + \hat{E}_t^\tau \left( \prod_{h=1}^{T} R_{t+1+h}^{-1} \right) \frac{M_{t+T}^\tau + B_{t+T}^\tau}{P_{t+T}}
\]

Assume that the following transversality conditions hold:
\[
\lim_{T \to \infty} \hat{E}_t^\tau \left( \prod_{h=1}^{T} R_{t+1+h}^{-1} \right) \frac{M_{t+T}^\tau + B_{t+T}^\tau}{P_{t+T}}, \quad \forall \tau
\]

Then taking \( \lim_{T \to \infty} \), using transversality conditions and cash in advance constraint yields the expression for household \( \tau \)'s real wealth at the end of period \( t \):
\[
\frac{W_t^\tau}{P_t} = \hat{E}_t^\tau \sum_{k=1}^{T-1} \prod_{h=1}^{k} R_{t+1+h}^{-1} \left[ \frac{U_{ct+k}^\tau}{U_{ct+k}^\tau} \frac{M_{t+k}^\tau + Y_{t+k} \phi_{t+k}^{\theta-1} \left( \frac{P_{t+k}}{P_{t+k}^\tau} \right)^{\theta-1}}{P_{t+k}^\tau} \right] + \hat{E}_t^\tau \left( \prod_{h=1}^{T} R_{t+1+h}^{-1} \right) \frac{M_{t+T}^\tau + B_{t+T}^\tau}{P_{t+T}}
\]

The island equilibrium is a solution of the system of equations (1)-(3), (7)-(18), and the Bayesian law of motion for conditional probability distributions.

### 3 Aggregation and Solution

For concreteness, I choose the following form for the period utility function:
\[
U(c, M/P, y) = \frac{1}{1 - \sigma} \left[ (1 - \omega) \left( \frac{M}{P} \right)^{\frac{1}{\omega}} + \omega \right]^{\frac{1}{1-\sigma}} - \psi \frac{y^{1+\xi}}{1+\xi}
\]

Here \( \sigma \) is a coefficient of relative risk aversion, \( \eta \) - elasticity of substitution between consumption and real money balances, \( \omega \) - consumption share, \( \psi \) - weight on labor, \( \xi \) - coefficient of aversion to working.

To have a stationary system, I divide all nominal variables in period \( t \) by island price index in period \( t-1 \), except for the end of period wealth, which is divided by \( P_t \).

In equilibrium with complete information, when all households observe the history of states through current period, all nominal variables grow with the rate of money supply growth and all real variables are constant:
\[
\begin{align*}
p^\tau &= \pi = \mu \\
c^\tau &= y^\tau = Y = 1 \\
R &= 1 + r = \beta^{-1} \mu \\
m &= \mu \left( \frac{\omega}{1-\omega} \left( \mu \beta^{-1} - 1 \right) \right)^{-\eta} \\
b &= 0 \\
w &= m \mu^{-1}
\end{align*}
\]

I log-linearize equilibrium equations around complete information steady state. Appendix A contains the details of log-linearization.
Next, I aggregate equilibrium equations to obtain a closed system in terms of island variables and their (average) expectations. Average expectations are defined as:

$$\hat{E}_t (\cdot) \equiv \sum_{\tau=0}^{\infty} \nu^\tau \hat{E}_t^\tau (\cdot) \quad (21)$$

Here are the main steps of aggregation (see details in Appendix B):

1) write households’ wealth in terms of their consumption and island variables; sum across cohorts to get island wealth;
2) write households’ consumption in terms of their current and past wealth, current and past island variables and their expectations, and past consumption; substitute for wealth using results of step 1;
3) sum equations, obtained in step 2 to get the first aggregate equation;
4) to get the second aggregate equations, sum pricing equations substituting for consumption using results of step 2.

The resulting system has two equations (45) and (46) in terms of current, past and (average expected) future output and interest rate.

Let

$$z_t = [\mu_t, \phi_t]'$$

To solve the equilibrium system I am going to approximate the state history by a truncated vector

$$Z_t = \left[ z_t', z_{t-1}', ..., z_{t-T}' \right]'$$

This method was used for example by Hellwig (2004) and Lorenzoni (2004).

The expanded state vector evolves according to

$$Z_t = AZ_{t-1} + B \varepsilon_t$$

where

$$\varepsilon_t = [\varepsilon_{\mu t}, \varepsilon_{\phi t}]', \quad A = \begin{bmatrix} \rho & 0_{2,2(T-1)} \\ I_{2(T-1)} & 0_{2(T-1),2} \end{bmatrix}, \quad B = \begin{bmatrix} I_2 \\ 0_{2(T-1),2} \end{bmatrix}, \quad \rho = \begin{bmatrix} \rho_{\mu} & 0 \\ 0 & \rho_{\phi} \end{bmatrix}.$$

Each household on the same island observes:

$$\bar{s}_t^i = \left[ 0 \quad \hat{Y}_t \\ \hat{\phi}_t \right] + \left[ \mu_t + (\theta - 1) (\phi_t - \phi_{t-1}) \\ (\theta - 1) \phi_t \quad 0 \right]$$

where hats denote log-linear (for output) and linear (for interest rate) deviations from the steady state.

Assume that output and interest rate are functions of the truncated state history:

$$\bar{s}_t^i = H \bar{s}_t Z_t$$

where

$$H \bar{s} = \begin{bmatrix} 0 \\ H_Y \quad H_r \end{bmatrix} + \left[ \begin{bmatrix} 1, \theta - 1, 0, 1 - \theta, 0_{1,2(T-2)} \\ 0, \theta - 1, 0_{1,2(T-1)} \end{bmatrix} \right]$$

and $H_Y, H_r$ are unknown $(1 \times 2T)$ matrices.
To update their prior households use the weighted signal given by the processing constraint (12)

$$s_t^\tau = \Omega^\tau s_t = H_s Z_t$$

(22)

where \( \Omega = [1 - \omega_Y - \omega_R, \omega_Y, \omega_R]' \), \( H_s = \Omega^\tau H_s \).

After observing the signal, agents update their expectations according to the Kalman filter equation\(^6\):

$$\hat{E}_t^\tau Z_t = A \hat{E}_t^{\tau-1} Z_{t-1} + K_\tau H_s [Z_t - A \hat{E}_t^{\tau-1} Z_{t-1}], \quad \tau \geq 1$$

(23)

$$\hat{E}_t^0 Z_t = Z_t$$

(24)

where

$$K_\tau = \Sigma^\tau_{\tau-1} H_s (H_s \Sigma^\tau_{\tau-1} H_s')^{-1}$$

$$\Sigma^\tau_{\tau+1} = A [\Sigma^\tau_{\tau-1} - \Sigma^\tau_{\tau-1} H_s (H_s \Sigma^\tau_{\tau-1} H_s')^{-1} H_s \Sigma^\tau_{\tau-1}] A' + Q$$

$$vec(\Sigma_{1|0}) = [I - (A \otimes A)]^{-1} \cdot vec(Q)$$

and \( Q = B \begin{bmatrix} \sigma_\mu^2 & 0 \\ 0 & \sigma_\phi^2 \end{bmatrix} B' \).

Summing (23) and (24) over all cohorts yields:

$$\hat{E}_t Z_t = \lambda Z_t + (1 - \lambda) A \hat{E}_{t-1} Z_{t-1} + (1 - \lambda) K H_s [Z_t - A \hat{E}_{t-1} Z_{t-1}] + R_t$$

(25)

where \( K = \sum_\tau \nu^\tau K_\tau \), \( R_t = (1 - \lambda) \sum_\tau \nu^\tau K_\tau H_s [\chi E_t Z_t - E_t^\tau Z_t] \).

In numeric simulations \( \chi \) is chosen such that \( E [R_t] = 0 \).

Aggregated Kalman filter equation (25) closes our equilibrium system. The remaining details about numeric solution are given in Appendix C.

4 Simulation of equilibrium dynamics

In this section I present the results of model solution and simulation. First, I describe and discuss the main features of the monetary transmissions mechanism. Second, I separate two channels of information flows and analyze their importance for the size of nonneutrality. Finally, I propose a measure of the implicit costs needed to keep the uninformed households from updating their information.

Assumed benchmark parameter values are given in Table 1. The unknown parameters are the rate of full information updating \( \lambda \), and signal weights \( [\omega_\pi, \omega_Y, \omega_R] \). Their values are set arbitrarily for now. The rate of updating is 0.25 corresponding to the expected time between full updating of 1 year. Signal weights are all 1/3, implying that all three observed variables contribute equally to the signal. The discount factor \( \beta \) is 0.97\(^{1/4} \) so that a period is one quarter. The elasticity of goods substitution \( \theta \) is 7 implying the steady state markup of 17\%, which is within the range of used in the literature. The aversion to labor \( \xi \) is 1 implying unit labor supply elasticity. Coefficient of risk aversion \( \sigma \) is 1, and money demand is interest inelastic, \( \eta = 0 \). Benchmark monetary and real demand shocks are equally volatile, \( \sigma_\mu = \sigma_\phi = 0.01 \). Monetary shocks are persistent with serial correlatoin \( \rho_\mu \) equal to 0.6 (e.g. Chari, Kehoe, and McGrattan (2000) use 0.57). Real demand shocks are short-lived with \( \rho_\phi = 0 \).

4.1 Monetary transmission mechanism

Figure 1A shows impulse responses of households’ expectations of the concurrent (detrended) quantity of money in the benchmark economy. The fully informed cohort ($\tau = 0$) knows precisely the amount of money. Cohort 1 is uninformed in the first period after the shock, but fully updates in the second period as its expectations jump to the actual level. The behavior of the higher cohorts is similar: expectations are adjusted based on the endogenous signals until full updating is allowed and expectations jump to the actual quantity of money at around 2.5%. The average expectation adjusts sluggishly, getting within 0.1% from the actual level of money only after 12 quarters. The slow adjustment of average expectations is due to two reasons. First, the rate of full updating is relatively low at 0.25 so that expectations of older cohorts receive bigger weight in the average expectations. Second, expectations of older cohorts grow very slowly, leveling at about 0.3%. The speed of the endogenous learning by households who are not allowed to update to full information is determined by how informative the endogenous signal is about the state of the economy. To illustrate this point, I repeat the previous experiment in economy where monetary shocks are more volatile (see Figure 1B). The ratio of standard deviations, $\sigma/\sigma_\phi$ is now 3.3 (as opposed to 1 for Figure 1A). Since the economy is more predictable, not updating households endogenously learn faster about the shock. Expectations about monetary shock now level at around 1.6%.

Hence Figures 1A, 1B demonstrate that in this economy information disseminates via two distinct channels. The "passive channel", is due to the exogenous regular renewal of information sets by fully updating households. The second, "active channel", is driven by households inferring the unobserved state of the economy from processing endogenous signals. The importance of the active channel depends on how informative the endogenous signal is. In the extreme case, when signals convey no information, households who do not update would completely miss the monetary shock, expecting the quantity of money to stay at the pre-shock level.

Figure 2 depicts the equilibrium aggregate output as a function of history of monetary shocks. For the economy with high fraction of informed households ($\lambda = 0.5$) aggregate output is a function of a few recent monetary shocks. As the fraction of uninformed agent increases, the weight of older monetary shocks becomes more important. Since output is part of the endogenous signal, lower rate of full updating implies that the signal is more dispersed over the state history. With Kalman filter putting more weight on the most recent shocks, smoothing out the signal over the state history delays the adjustment of expectations. Hence, increases (decreases) in the frequency of full information updating make both passive and active learning channels stronger (weaker).

4.2 The size of nonneutrality of money

To see the effects of the monetary transmission mechanism on the model economy, I look at impulse responses of output and inflation to a 1% increase in the rate of money growth. Figure 3 compares such responses for economies with different fraction of fully updating households. The size of the responses, both in magnitude and persistence, varies dramatically with across the economies. Magnitude is defined by the maximal value of the response. Persistence of a response is measured by its half-life, the time it takes the response to decrease in half after reaching its maximal value. Tables 2 documents, for an economy with equally volatile aggregate and island-specific shocks, the increase in the fraction of
fully informed households from 0.05 to 0.99, decreases the half-life of output from 19.9 to 1.5 quarters and the magnitude from 1.9% to 0.08%. Consistent with output, inflation responses are small in magnitude and persistent for economies with small fraction of fully updating households, and large and transient for economies with high rate of full updating.

Next I investigate the effect of the relative size of aggregate and island specific shocks on equilibrium dynamics. In my experiments, I change the size of the money growth shocks, keeping the size of island demand shocks fixed. Figure 4A provides output and inflation impulse responses for economies with different relative size of shocks, measure by the ratio of their standard deviations, $\sigma_\mu / \sigma_\phi$. As monetary shocks become more important, responses become more transitory. Table 3 shows that when the ratio of standard deviations decreases increases from 0.1 to 10, half-life of output responses goes down from 7.6 to 2.4 quarters. This decline in persistence is due to the fact that when money growth shocks are relatively bigger, households’ probability distributions shift away from real demand shocks onto monetary shocks. Since Bayesian updating assigns bigger weight on recent signals, household will learn about larger monetary shocks faster.

Finally, I analyze the importance of real rigidity for generating real persistence. As one can see from the households’ pricing equations (30), (31), the elasticity of the (pre)set price with respect to output is equal to $\xi / (1 + \xi)$). The fact that this elasticity is smaller than 1 implies that households pricing decisions are complementary: in response to increase in demand changes in individual prices are slow to rise. This rigidity is an additional source of persistence of real effects in the economy. As it was the case with change in the rate of updating, increase in the real persistence implies that endogenous signals are dispersed over longer state history, and therefore they become less informative about the current state.

The role of real rigidities in generating persistence has been widely emphasized in the monetary business cycle literature\(^7\). Figure 4B demonstrates the importance of real rigidity in the present model. As the elasticity of goods substitution goes down from 11 to 3 (so that for $\xi$ fixed at 1, the elasticity $\xi / (1 + \xi)$ increases from 1/12 to 1/4), the output response half-life decreases from 8.7 to 4.3 (see Table 4). Hence, real rigidities augment the conventional source of persistence with the additional propagation due to their effect on the information content of the endogenous signals.

4.3 Active vs passive learning

The preceding discussion showed that there are two distinct channels through which information about changing fundamentals disseminates into agents’ information sets and aggregate economy. The passive channel is due to the fraction of fully updating households. When this fraction is sufficiently small, the effects of monetary shocks are long lived. The active channel is driven by households inferring the unobserved state of the economy from processing endogenous signals. The importance of this channel depends on how informative is the signal, captured by the rate of updating to full information $\lambda$, relative size of aggregate and island specific shocks $\sigma_\mu / \sigma_\phi$, and the degree of real rigidity $\frac{\xi}{1 + \xi}$. When information flows rapidly via two channels, the effects of monetary shifts are small and short-lived. When the flow of information is slow, there is sizeable real persistence.

To document the importance of each of these two channels, I compare the degree of persistence in the present model to the one in the model where signals are uninformative. To simulate the economy in which signals convey no information, I restrict the vector of

\(^7\)See for example, Christiano, Eichenbaum, and Evans (2005), Bouakez, Cardia, and Ruge-Murcia (2000), Dotsey and King (2001).
signal weights, $H_s$ in (22), to contain all zeros. The economy with uninformative signals mimics the "sticky information" economy of Mankiw and Reis (2002). Figures 5A, 5B and 5C compare half-life of output responses to monetary shocks for two economies.

Figure 5A compares persistence across economies with different fraction of fully updated households. In economy with equally volatile monetary and real demand shocks the active learning is weak, and so persistence is determined by the passive channel. As a result, for any rate of information acquisition, nonneutrality in the economy with endogenous signals lasts almost as long as in the economy with uninformative signals. As the relative size of monetary shocks grows, active learning becomes stronger rendering nonneutrality less persistent. For $\sigma_\mu/\sigma_\phi = 3.3$ and $\lambda = 0.1$ half-life of output is 7.1 quarters (as opposed to 13.8 quarters in economy with $\sigma_\mu/\sigma_\phi = 1$, and 16.5 in economy with uninformative signals), and for $\lambda = 0.05$ it is 8 quarters (vs 19.9 and 25.2 quarters, see Table 2). Hence, real persistence in economy with endogenous learning depends on how strong is the active channel of information flows relative to the passive channel.

Figure 5B compares half-life of output across the range of (log of) ratio of standard deviations of two shocks. Impulse responses in the economy with uninformative signals are insensitive to the change in relative size of the shocks. On the contrary, in the economy with endogenous signals, as monetary shocks become more volatile than island demand shocks, signals become more informative, reenforcing active learning and bringing down real persistence. For $\sigma_\mu/\sigma_\phi = 5$, the response of output lasts 4 quarters, twice less than in the uninformative signal economy, 8.1 quarters (see Table 3).

Finally, Figure 5C looks at real persistence as a function of elasticity of goods substitution. When the elasticity is low (and hence the degree of real rigidity is small), prices respond quicker and hence convey the information faster. As a result persistence between informed and uninformed economies' output responses differs significantly: for $\theta = 3$, output half-lives differ by 2.3 quarters.

I summarize by saying that the duration of real effects of nominal disturbance depend on the agents' ability to infer the state of the economy from the endogenous signals. This ability is stronger when the rate of full updating is sufficiently high, monetary shocks are smaller than demand shocks, and the degree of real rigidity is not significant. When the endogenous learning is weak, nonneutrality can be very persistent. In this case, the economy with information delays, similar to Mankiw and Reis (2002), approximates well the dynamics in the economy with informative signals.

4.4 Estimating the wedge between the most and the least informed households

In this paper the rate with which households update to full information is exogenously fixed. Future research will relax this assumption. One way to do that is to assume that price setters face fixed costs of updating their information sets. In this case, for the marginal agent, the fixed costs is equal to the difference in values between updating to full information and not updating. Although the restrictions made in this paper do not allow for this analysis, I estimate the average upper bound of such fixed costs of updating, which I will call the wedge between the most and the least informed households.

The unconditional expectation of the value of the firm in the zero cohort:

$$E [V_t^0] = E [U_t^0] + \beta(1-\lambda)E [V_t^1] + \beta\lambda [E [V_t^0] - \Delta^1]$$

(26)
where $E\left[U_{t}^{0}\right]$ is the unconditional mean of the period utility at the period of full adjustment, $E\left[V_{t}^{1}\right]$ is the unconditional mean of the value of the firm in the first cohort, $\Delta^{1}$ is the deterministic wedge between cohorts zero and one.

Similarly the unconditional expectation of the value of the least informed firm:

$$E\left[V_{t}^{\infty}\right] = E\left[U_{t}^{\infty}\right] + \beta(1 - \lambda)E\left[V_{t}^{\infty}\right] + \beta \lambda E\left[V_{t}^{0}\right] - \Delta^{\infty}$$

where $E\left[U_{t}^{\infty}\right]$ is the unconditional expectation of the period utility of the least informed cohort, $\Delta^{\infty}$ is the deterministic wedge between cohorts $\infty$ and zero.

Use (26) to define $E\left[V_{t}^{0}\right]$ to be the upper bound on $E\left[V_{t}^{0}\right]$:

$$E\left[V_{t}^{0}\right] = E\left[U_{t}^{0}\right] + \beta(1 - \lambda)E\left[V_{t}^{0}\right] + \beta \lambda E\left[V_{t}^{0}\right] = E\left[U_{t}^{0}\right] \frac{1}{1 - \beta}$$ (27)

where we took into account that

$$E\left[V_{t}^{0}\right] \geq E\left[V_{t}^{1}\right]$$

and

$$\Delta^{1} \geq 0$$

Analogously,

$$E\left[V_{t}^{\infty}\right] = E\left[U_{t}^{\infty}\right] + \beta(1 - \lambda)E\left[V_{t}^{\infty}\right] + \beta \lambda E\left[V_{t}^{0}\right] - \Delta^{\infty}$$ (28)

Define the wedge $\Delta^{\infty}$ as:

$$\Delta^{\infty} = E\left[V_{t}^{0}\right] - E\left[V_{t}^{\infty}\right]$$ (29)

Solving the system (27)-(29) yields the expression for the estimate of the wedge:

$$\Delta^{\infty} = \frac{E\left[U_{t}^{0}\right]}{1 - \beta} - \frac{E\left[U_{t}^{\infty}\right]}{1 - \beta \lambda}$$

Since we use linear method of computing equilibrium, we use the second-order approximation to compute $E\left[U_{t}^{0}\right]$ and $E\left[U_{t}^{\infty}\right]\text{s}$. For tractability, I report the wedge in units consumption rather than utils.

For the benchmark parameter values the estimated wedge is 8.4%. Zbaracki, Ritson, Levy, Dutta, and Bergen (2004) provide the empirical estimate of the cost of price change related to gathering, processing and communicating information. Their estimate of these "managerial" is around 1.1% of firm’s quarterly revenue. Provided that Zbaracki et al estimate the average cost of information processing, my estimate of the maximal cost of information updating is within a reasonable range.

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*I simulate equilibrium dynamics for a string of shocks over 200 quarters. First 100 quarters are then discarded to ensure stationary equilibrium. Across time second moments are means of second moments over 100 such simulations.*
5 Conclusion

This paper is a contribution to the theory of monetary nonneutrality in environments with imperfect information about the source of observed disturbances. Uninformed agents face a signal-extraction problem subject to the exogenous probability of full updating and the restriction on information processing. Individual optimality conditions are aggregated to yield a closed system for equilibrium aggregate output and inflation. The size of real effects of monetary shocks depends on parameters determining the predictability of environment: the rate of full information updating, the relative size of aggregate and sector-specific shocks, and the degree of strategic complementarity of pricing decisions. The more informative is the economy, the smaller is the nonneutrality of money. When the economy is not informative the benefit from fully updating information measured by the difference in values of the most and least informed agents is relatively small. For reasonable parameter values, this extra benefit is around 8% of quarterly consumption.

The paper abstracts from the endogenous timing of information updating decisions, or endogenous timing of pricing decisions with infrequent information. It is an interesting topic for future research to investigate the importance of endogenous changes in the fraction of adjusting agents in the aggregated general equilibrium framework. A standard way of introducing the endogeneity of extensive margin is to assume fixed costs of adjustment. It is very often, that movements in the fraction of adjusting agents play a crucial role in such models. For example, Bonomo and Garcia (2001) study the optimal $Ss$-type rules in a model with infrequent information and fixed costs of price adjustment. In their model, the number of adjusting agents increases with aggregate uncertainty. In the $Ss$-type price adjustment model Golosov and Lucas (2003) show that the extremely volatile fraction of adjusting agents renders the real effects of monetary shocks very transient.
Appendix A. Log-linearized equilibrium equations

All indexed variables denote log-linear deviations from the deterministic steady state. The log-linearized system consists of:

- pricing equations:

\[
\begin{align*}
    p_t^0 &= \frac{\xi}{1 + \theta \xi} (Y_t + (\theta - 1) \phi_t) + \frac{\sigma}{1 + \theta \xi} c_t^0 + \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r) \frac{r_t}{1 + \theta \xi} + \pi_t, \\
    p_t^\tau &= \bar{E}_{t-1}^{-1} \left[ \frac{\xi}{1 + \theta \xi} (Y_t + (\theta - 1) \phi_t) + \frac{\sigma}{1 + \theta \xi} c_t^\tau + \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r) \frac{r_t}{1 + \theta \xi} \right], \quad \tau \geq 0
\end{align*}
\] (30)

- money demand equation:

\[
-\pi_t + m_t^\tau = c_t^\tau - \frac{r_t}{\tau}, \quad \tau \geq 0
\] (32)

- Euler equations:

\[
\sigma c_t^\tau = \bar{E}_{t}^{-1} \left[ -\frac{r_{t+1}}{1 + r} + \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r) (r_{t+1} - r_t) + \sigma c_{t+1}^\tau \right], \quad \tau \geq 0
\] (33)

- households’ flow budget constraints:

\[
\begin{align*}
    \dot{\hat{w}}_t^0 &= \mu \beta^{-1} \hat{w} \left( \frac{r_t}{R} + \dot{w}_{t-1} \right) - \omega^{-1} f'(r) r_t - \omega^{-1} f(r) c_t^0 \\
    &\quad + (Y_t + (\theta - 1) \phi_t) + (\theta - 1) (\pi_t - p_t^0)
\end{align*}
\] (34)

\[
\begin{align*}
    \dot{\hat{w}}_t^\tau &= \mu \beta^{-1} \hat{w} \left( \frac{r_t}{R} + \dot{w}_{t-1}^\tau \right) - \omega^{-1} f'(r) r_t - \omega^{-1} f(r) c_t^\tau \\
    &\quad + (Y_t + (\theta - 1) \phi_t) + (\theta - 1) (\pi_t - p_t^\tau) \quad \tau \geq 1
\end{align*}
\] (35)

- households’ wealth constraint:

\[
\begin{align*}
    \dot{\hat{w}}_t^\tau &= \bar{E}_{t} \sum_{k=1}^{\infty} (\beta \mu)^{-1} \left\{ \left[ \omega^{-1} f'(r) r_{t+k} + \omega^{-1} f(r) c_{t+k}^\tau \right] - (Y_{t+k} + (\theta - 1) \phi_{t+k}) \\
    &\quad - (\theta - 1) \left( \pi_{t+k} - p_{t+k}^\tau \right) \right\} - \frac{\omega^{-1} f(r)}{R} \frac{1}{\tau} \sum_{h=1}^{k} \tau_{t+h}
\end{align*}
\] (36)

- resource constraint:

\[
\sum_{\tau} \nu^\tau c_t^\tau = Y_t
\] (37)

- inflation index:

\[
-\phi_t + \sum_{\tau} \nu^\tau p_t^\tau = \pi_t
\] (38)

and the law of motion for aggregate real money balances:

\[
\mu_t = m_t - m_{t-1} + \pi_{t-1}
\] (39)
Appendix B. Aggregation

Let’s simplify households’ wealth equation (36). By the law of iterated expectations, pricing equations (31) imply:

\[
\hat{E}_t^r \left[ \frac{\xi}{1 + \theta \xi} (Y_{t+k} + \theta - 1) \phi_{t+k} + \frac{\sigma}{1 + \theta \xi} \sigma c_{t+k} + \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r) r_{t+k} + \pi_{t+k} - p_{t+k}^* \right] = 0, \quad k = 1, 2, ...
\]

Then

\[
\hat{w} \hat{w}_{t}^r = \hat{E}_t^r \sum_{k=1}^{\infty} (\beta \mu)^{-k} \left[ \left( \omega^{-1} f'(r) + \frac{(\theta - 1) \sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r) \right) r_{t+k} + \left( \frac{\omega^{-1} f(r)}{\sigma} + \frac{\theta - 1}{1 + \theta \xi} \right) \sigma c_{t+k} \right] - \frac{1 + \xi}{1 + \theta \xi} (Y_{t+k} + (\theta - 1) \phi_{t+k}) - \frac{\omega^{-1} f(r) - 1}{(1 - \beta \mu)^{-1} R} r_{t+k}
\]

(40)

To obtain \( \hat{E}_t^r \sum_{k=1}^{\infty} (\beta \mu)^{-k} c_{t+k}^* \) we sum over Euler equations:

\[
\sigma \hat{E}_t^r c_{t+1}^* = \sigma c_t^* - \hat{E}_t^r \left[ -\frac{r_{t+1}}{1 + r} + \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r) (r_{t+1} - r_t) \right]
\]

\[
\sigma \hat{E}_t^r c_{t+k}^* = \sigma c_t^* + \frac{\beta \mu^{-1}}{1 - \beta \mu^{-1}} c_t^* + \frac{\beta \mu^{-1}}{1 - \beta \mu^{-1}} \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r) r_t
\]

\[
\sigma \sum_{k=1}^{\infty} (\beta \mu)^{-k} \hat{E}_t^r c_{t+k}^* = \sigma \frac{\beta \mu^{-1}}{1 - \beta \mu^{-1}} c_t^* + \frac{\beta \mu^{-1}}{1 - \beta \mu^{-1}} \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r) r_t
\]

\[
- \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r) \sum_{k=1}^{\infty} (\beta \mu)^{-k} \hat{E}_t^r r_{t+k} + \frac{1}{1 - \beta \mu^{-1}} \hat{E}_t^r \sum_{k=1}^{\infty} (\beta \mu)^{-k} r_{t+k}
\]

After plugging this back into (40) we obtain:

\[
\hat{w} \hat{w}_{t}^r = \beta \mu^{-1} \eta_1 \sigma c_t^* + \eta_2 r_t + \hat{E}_t^r \sum_{k=1}^{\infty} (\beta \mu)^{-k} \left[ \eta_3 r_{t+k} + \eta_4 (Y_{t+k} + (\theta - 1) \phi_{t+k}) \right]
\]

(41)

where

\[
\eta_1 = (1 - \beta \mu^{-1})^{-1} \left( \frac{\omega^{-1} f(r)}{\sigma} + \frac{\theta - 1}{1 + \theta \xi} \right)
\]

\[
\eta_2 = \left( \frac{\omega^{-1} f(r)}{\sigma} + \frac{\theta - 1}{1 + \theta \xi} \right) \frac{\beta \mu^{-1}}{1 - \beta \mu^{-1}} \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r)
\]

\[
\eta_3 = \left( \frac{\omega^{-1} f'(r)}{\sigma} + \frac{(\theta - 1) \sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r) \right) - \frac{\omega^{-1} f(r) - 1}{R (1 - \beta \mu^{-1})}
\]

\[
+ \left( \frac{\omega^{-1} f(r)}{\sigma} + \frac{\theta - 1}{1 + \theta \xi} \right) \left( \frac{1}{R (1 - \beta \mu^{-1})} - \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r) \right)
\]

\[
\eta_4 = \frac{1 + \xi}{1 + \theta \xi}
\]
Aggregating (41) over cohorts yields:

\[
\hat{w}_t = \beta \mu^{-1} \eta_1 Y_t + \eta_2 r_t + \hat{E}_t \sum_{k=1}^\infty (\beta \mu^{-1})^k \left[ \eta_3 r_{t+k} + \eta_4 (Y_{t+k} + (\theta - 1) \phi_{t+k}) \right]
\]

(42)

To get the expressions for \( c_t^\tau \) we will use pricing equations

\[
0 = \frac{\xi}{1 + \theta \xi} (Y_t + (\theta - 1) \phi_t) + \frac{\sigma}{1 + \theta \xi} c_t^0 + \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r) r_t + \pi_t - p_t^0
\]

\[
0 = \hat{E}_{t-1} \left[ \frac{\xi}{1 + \theta \xi} (Y_t + (\theta - 1) \phi_t) + \frac{\sigma}{1 + \theta \xi} c_t^{\tau - 1} + \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r) r_t + \pi_t - p_t^\tau \right]
\]

and budget constraints:

\[
\hat{w}_t^0 = \mu \beta^{-1} \hat{w} \left( \frac{r_t}{R} + \hat{w}_{t-1} \right) - \omega^{-1} f'(r) r_t - \omega^{-1} f(r) c_t^0 + (Y_t + (\theta - 1) \phi_t) + (\theta - 1) \pi_t - p_t^0
\]

\[
\hat{w}_t^\tau = \mu \beta^{-1} \hat{w} \left( \frac{r_t}{R} + \hat{w}_{t-1}^{\tau - 1} \right) - \omega^{-1} f'(r) r_t - \omega^{-1} f(r) c_t^{\tau - 1} + (Y_t + (\theta - 1) \phi_t) + (\theta - 1) (\pi_t - p_t^\tau)
\]

\[
= \mu \beta^{-1} \hat{w} \left( \frac{r_t}{R} + \hat{w}_{t-1}^{\tau - 1} \right) - \omega^{-1} f'(r) r_t - \omega^{-1} f(r) c_t^{\tau - 1} + (Y_t + (\theta - 1) \phi_t) - (\theta - 1) \hat{E}_{t-1} \left[ \frac{\xi}{1 + \theta \xi} (Y_t + (\theta - 1) \phi_t) + \frac{\sigma}{1 + \theta \xi} c_t^{\tau - 1} + \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r) r_{t-1} \right]
\]

Then

\[
\eta_6 c_t^0 = -\hat{w}_t^0 + \mu \beta^{-1} \hat{w} \pi_{t-1} + \eta_8 (Y_t + (\theta - 1) \phi_t) + \eta_9 r_t
\]

(43)

After taking into account (41) and (42) we obtain

\[
\eta_{10} c_t^0 = \eta_8 (Y_t + (\theta - 1) \phi_t) + \eta_{11} r_t - \hat{E}_t \sum_{k=1}^\infty (\beta \mu^{-1})^k \left[ \eta_3 r_{t+k} + \eta_4 (Y_{t+k} + (\theta - 1) \phi_{t+k}) \right]
\]

\[
+ \eta_1 \sigma Y_{t-1} + \mu \beta^{-1} \eta_2 r_{t-1} + \mu \beta^{-1} \hat{E}_{t-1} \sum_{k=1}^\infty (\beta \mu^{-1})^k \left[ \eta_3 r_{t-1+k} + \eta_4 (Y_{t-1+k} + (\theta - 1) \phi_{t-1+k}) \right]
\]

where

\[
\eta_{10} = \eta_6 + \mu \beta^{-1} \eta_1 \sigma
\]

\[
\eta_{11} = \eta_9 - \eta_2
\]

Consumption in cohort \( \tau \geq 1 \) is

\[
\omega^{-1} f(r) c_t^\tau = -\hat{w}_t^\tau + \mu \beta^{-1} \hat{w} \pi_{t-1}^{\tau - 1} + \left[ \frac{\mu \beta^{-1} \hat{w}}{R} - \omega^{-1} f'(r) \right] r_t + (Y_t + (\theta - 1) \phi_t)
\]

\[
- (\theta - 1) \hat{E}_{t-1} \left[ \frac{\xi}{1 + \theta \xi} (Y_t + (\theta - 1) \phi_t) + \frac{1}{R} r_t \right]
\]

\[
- \frac{\theta - 1}{1 + \theta \xi} \left[ \sigma c_{t-1}^{\tau - 1} + \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r) r_{t-1} \right]
\]

(44)
Summing across (43) and (44) across \( \tau \) gives

\[
\omega^{-1} f(r) Y_t = -\lambda \frac{\sigma(\theta - 1)}{1 + \theta^\xi} \eta_{10}^{-1} \left[ \eta_8 (Y_t + (\theta - 1) \phi_t) - \hat{E}_t^{\beta} \sum_{k=1}^{\infty} (\beta \mu^{-1})^k [\eta_3 R_{t+k} + \eta_4 (Y_{t+k} + (\theta - 1) \phi_{t+k})] \right] \\
+ \eta_{11} r_t + \eta_{12} Y_{t-1} + \mu \beta^{-1} \eta_{12} r_{t-1} + \mu \beta^{-1} \hat{E}_{t-1} \sum_{k=1}^{\infty} (\beta \mu^{-1})^k [\eta_3 R_{t-1+k} + \eta_4 (Y_{t-1+k} + (\theta - 1) \phi_{t-1+k})] \\
- \beta \mu^{-1} \eta_1 \sigma Y_t - \eta_2 r_t - \hat{E}_t \sum_{k=1}^{\infty} (\beta \mu^{-1})^k [\eta_3 R_{t+k} + \eta_4 (Y_{t+k} + (\theta - 1) \phi_{t+k})] \\
+ \eta_1 \sigma Y_{t-1} + \mu \beta^{-1} \eta_{2} r_{t-1} + \mu \beta^{-1} \hat{E}_{t-1} \sum_{k=1}^{\infty} (\beta \mu^{-1})^k [\eta_3 R_{t-1+k} + \eta_4 (Y_{t-1+k} + (\theta - 1) \phi_{t-1+k})] \\
+ \left[ \lambda \eta_8 + (1 - \lambda) \left( \frac{\mu \beta^{-1} \hat{\omega}}{R} - \omega^{-1} f'(r) \right) \right] r_t + \left[ \lambda \eta_8 + (1 - \lambda) \right] (Y_t + (\theta - 1) \phi_t) \\
- \frac{(\theta - 1)(1 - \lambda)}{1 + \theta^\xi} \left\{ \hat{E}_{t-1} \left[ \xi (Y_t + (\theta - 1) \phi_t) + \frac{r_t}{R} \right] + \sigma Y_{t-1} + \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r) r_{t-1} \right\}
\]

where

\[
\eta_6 = \omega^{-1} f(r) + \frac{\sigma(\theta - 1)}{1 + \theta^\xi} \\
\eta_8 = \frac{1 + \xi}{1 + \theta^\xi} \\
\eta_9 = \frac{\mu \beta^{-1} \hat{\omega}}{R} - \omega^{-1} f'(r) - \frac{\theta - 1}{1 + \theta^\xi} \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r)
\]

or

\[
\eta_{12} \phi_t + \eta_{13} Y_t + \eta_{14} r_t + \eta_{15} Y_{t-1} + \eta_{16} r_{t-1} + \eta_{17} \hat{E}_{t-1} \left[ \xi (Y_t + (\theta - 1) \phi_t) + \frac{r_t}{R} \right] \\
+ \eta_{18} \hat{E}_t \sum_{k=1}^{\infty} (\beta \mu^{-1})^k [\eta_3 R_{t+k} + \eta_4 (Y_{t+k} + (\theta - 1) \phi_{t+k})] \\
- \hat{E}_t \sum_{k=1}^{\infty} (\beta \mu^{-1})^k [\eta_3 R_{t+k} + \eta_4 (Y_{t+k} + (\theta - 1) \phi_{t+k})] \\
+ \mu \beta^{-1} (1 - \eta_{18}) \hat{E}_{t-1} \sum_{k=1}^{\infty} (\beta \mu^{-1})^k [\eta_3 R_{t-1+k} + \eta_4 (Y_{t-1+k} + (\theta - 1) \phi_{t-1+k})] = 0
\]

where
\[ \eta_{12} = \left( \lambda \eta_8 + (1 - \lambda) - \lambda \frac{\sigma(\theta - 1)}{1 + \theta \xi} \eta_{10}^{-1} \right) (\theta - 1) \]

\[ \eta_{13} = \lambda \eta_8 + (1 - \lambda) - \omega^{-1} f(r) - \lambda \frac{\sigma(\theta - 1)}{1 + \theta \xi} \eta_{10}^{-1} \eta_8 - \beta \mu^{-1} \eta_1 \sigma \]

\[ \eta_{14} = \lambda \eta_9 + (1 - \lambda) \left( \frac{\mu \beta^{-1} \hat{w}}{R} - \omega^{-1} f'(r) \right) - \eta_2 - \lambda \frac{\sigma(\theta - 1)}{1 + \theta \xi} \eta_{10}^{-1} \eta_{11} \]

\[ \eta_{15} = \left[ \eta_1 \left( 1 - \lambda \frac{\sigma(\theta - 1)}{1 + \theta \xi} \eta_{10}^{-1} \right) - (1 - \lambda) \frac{(\theta - 1)}{1 + \theta \xi} \right] \sigma \]

\[ \eta_{16} = \mu \beta^{-1} \eta_2 \left( 1 - \lambda \frac{\sigma(\theta - 1)}{1 + \theta \xi} \eta_{10}^{-1} \right) - (1 - \lambda) \frac{(\theta - 1)}{1 + \theta \xi} \frac{\sigma \eta_1}{\eta - 1} f(r)^{-1} f'(r) \]

\[ \eta_{17} = \frac{(1 - \lambda) (\theta - 1)}{1 + \theta \xi}, \quad \eta_{18} = \lambda \frac{\sigma(\theta - 1)}{1 + \theta \xi} \eta_{10}^{-1} \]

The aggregated pricing equation is

\[ -(1 - \lambda) \left[ \mu_t - Y_t + Y_{t-1} + \frac{\eta}{r} (r_t - r_{t-1}) - \frac{1}{1 + \theta \xi} \left( \sigma Y_{t-1} + \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r) r_{t-1} \right) \right] \]

\[ - \phi_t + (1 - \lambda) \hat{E}_{t-1} \left[ \mu_t - Y_t + Y_{t-1} + \frac{\eta}{r} (r_t - r_{t-1}) \right] \]

\[ + \lambda \left[ \frac{\xi}{1 + \theta \xi} (Y_t + (\theta - 1) \phi_t) + \frac{\sigma \eta^{-1} f(r)^{-1} f'(r)}{1 + \theta \xi} r_t \right] + (1 - \lambda) \hat{E}_{t-1} \left[ \frac{\xi}{1 + \theta \xi} (Y_t + (\theta - 1) \phi_t) + \frac{1/R}{1 + \theta \xi} r_t \right] \]

\[ + \frac{\lambda \sigma}{1 + \theta \xi} \eta_{10}^{-1} \left[ \eta_8 (Y_t + (\theta - 1) \phi_t) + \eta_{11} r_t - \hat{E}_{t}^{\theta} \sum_{k=1}^{\infty} (\beta \mu^{-1})^k \left[ \eta_3 r_{t+k} + \eta_4 (Y_{t+k} + (\theta - 1) \phi_{t+k}) \right] \right] \]

\[ + \frac{\lambda \sigma}{1 + \theta \xi} \eta_{10}^{-1} \left[ \eta_1 \sigma Y_{t-1} + \mu \beta^{-1} \eta_2 r_{t-1} + \mu \beta^{-1} \hat{E}_{t-1} \left( \beta \mu^{-1} \right)^k \left[ \eta_3 r_{t-1+k} + \eta_4 (Y_{t-1+k} + (\theta - 1) \phi_{t-1+k}) \right] \right] \]

\[ = 0 \]

or

\[ -(1 - \lambda) \mu_t - \eta_{19} \phi_t + \eta_{20} Y_t + \eta_{21} r_t + \eta_{22} Y_{t-1} + \eta_{23} r_{t-1} \]

\[ + (1 - \lambda) \hat{E}_{t-1} \left[ \mu_t + \frac{\xi(\theta - 1)}{1 + \theta \xi} \phi_t + \left( \frac{\xi}{1 + \theta \xi} - 1 \right) Y_t + \left( \frac{1/R}{1 + \theta \xi} + \frac{\eta}{r} \right) r_t + Y_{t-1} - \frac{\eta}{r} r_{t-1} \right] \]

\[ - \frac{\lambda \sigma}{1 + \theta \xi} \eta_{10}^{-1} \hat{E}_{t}^{\theta} \sum_{k=1}^{\infty} (\beta \mu^{-1})^k \left[ \eta_3 r_{t+k} + \eta_4 (Y_{t+k} + (\theta - 1) \phi_{t+k}) \right] \]

\[ + \frac{\lambda \sigma}{1 + \theta \xi} \eta_{10}^{-1} \mu \beta^{-1} \hat{E}_{t-1} \left( \beta \mu^{-1} \right)^k \left[ \eta_3 r_{t-1+k} + \eta_4 (Y_{t-1+k} + (\theta - 1) \phi_{t-1+k}) \right] \]

\[ = 0 \]
where

\[
\eta_{19} = 1 - \frac{\lambda \xi (\theta - 1)}{1 + \theta \xi} - \frac{\lambda \sigma}{1 + \theta \eta_{10} \eta_{8}} (\theta - 1)
\]

\[
\eta_{20} = 1 - \lambda + \frac{\lambda \xi}{1 + \theta \xi} + \frac{\lambda \sigma}{1 + \theta \eta_{10} \eta_{8}}
\]

\[
\eta_{21} = \frac{\lambda}{1 + \theta \xi} \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r) - (1 - \lambda) \frac{\eta}{r} + \frac{\lambda \sigma}{1 + \theta \eta_{10} \eta_{11}}
\]

\[
\eta_{22} = \frac{\lambda \sigma}{1 + \theta \xi} \eta_{10} \eta_{1} - (1 - \lambda) \left(1 - \frac{\sigma}{1 + \theta \xi}\right)
\]

\[
\eta_{23} = \frac{\lambda \sigma}{1 + \theta \xi} \eta_{10} \mu \beta^{-1} \eta_{2} + (1 - \lambda) \left(\frac{\eta}{r} + \frac{1}{1 + \theta \xi} \frac{\sigma \eta - 1}{\eta - 1} f(r)^{-1} f'(r)\right)
\]

**Appendix C. Numeric solution**

Express first-order expectations in terms of the current state as

\[
\hat{E}_t Z_t = \Xi Z_t
\]

Since the state variable is truncated, we use approximation:

\[
Z_{t-1} = U Z_t
\]

where

\[
U = \begin{bmatrix}
0_{2T-2,2} & I_{2T-2} \\
0_{2,2} & 0_{2,2T-2}
\end{bmatrix}
\]

The Kalman equation implies the equation for \(\Xi\):

\[
\Xi = \lambda I + (1 - \lambda) A \Xi U + (1 - \lambda) KH_s (I - \chi A \Xi U)
\]

or

\[
\Xi = \lambda I + (1 - \lambda) (I - \chi KH_s) A \Xi U + (1 - \lambda) KH_s
\]

Which can be solved for \(\Xi\) after taking into account the fact that \(vec(ABC) = [C' \otimes A] vec(B)\):

\[
vec(\Xi) = W^{-1} \cdot vec(\lambda I + (1 - \lambda) KH_s)
\]

where \(W = I - (1 - \lambda) (U' \otimes (I - \chi KH_s) A)\).

We will use the following approximation

\[
\hat{E}_t \sum_{k=1}^{\infty} (\beta \mu^{-1})^k Z_{t+k} = \hat{E}_t \left[ \sum_{k=1}^{T} (\beta \mu^{-1})^k Z_{t+k} + \sum_{k=T+1}^{\infty} (\beta \mu^{-1})^k Z_{t+T} \right]
\]

\[
= \hat{E}_t \left[ \sum_{k=1}^{T} (\beta \mu^{-1})^k A^k Z_t + \sum_{k=T+1}^{\infty} (\beta \mu^{-1})^k A^T Z_t \right]
\]

\[
= \beta \mu^{-1} A \hat{E}_t \left[ (I - (\beta \mu^{-1})^{T-1} A^{T-1}) (I - \beta \mu^{-1} A)^{-1} + \frac{(\beta \mu^{-1})^{T-1} A^{T-1}}{1 - \beta \mu^{-1}} \right] Z_t
\]

\[
= \beta \mu^{-1} A T \hat{E}_t Z_t
\]

\[
\hat{E}_t \sum_{k=1}^{\infty} (\beta \mu^{-1})^k Z_{t+k} = \beta \mu^{-1} A T Z_t
\]
Then rewrite our system of equilibrium equations in matrix form:

\[
\begin{align*}
\gamma_1 + \gamma_2 H + \gamma_3 \Delta U + 
\gamma_4 A \Xi U + \gamma_5 H A \Xi U + \gamma_6 H \Xi U - \frac{\lambda \sigma}{1 + \theta \xi} \eta_{10}^{-1} ( \gamma_0 H + \gamma_00 ) \mathcal{A} \mathcal{T} ( \beta \mu^{-1} I - \Xi U ) & = 0 \\
\gamma_7 + \gamma_8 H + \gamma_9 \Delta U + \gamma_10 A \Xi U + \gamma_{11} H A \Xi U + ( \gamma_0 H + \gamma_00 ) \mathcal{A} \mathcal{T} [ \beta \mu^{-1} ( \eta_{18} I - \Xi ) + (1 - \eta_{18}) \Xi U ] & = 0
\end{align*}
\]

where

\[
\begin{align*}
\gamma_0 & = [ \eta_4, \eta_3], \ \gamma_00 = [0, \eta_4(\theta - 1), 0_{n_z(T - 1)}] \\
\gamma_1 & = [-1 + \lambda, -\eta_{19}, 0_{n_z(T - 1)}], \ \gamma_2 = [\eta_20, \eta_21], \ \gamma_3 = [\eta_{22}, \eta_{23}] \\
\gamma_4 & = (1 - \lambda) \left[ 1, \frac{\xi(\theta - 1)}{1 + \theta \xi}, 0_{n_z(T - 1)} \right], \ \gamma_5 = (1 - \lambda) \left[ \frac{\xi}{1 + \theta \xi} - 1, \frac{1/R}{1 + \theta \xi} + \eta \right] \\
\gamma_6 & = (1 - \lambda) \left[ 1, -\frac{\eta}{r} \right], \ \gamma_7 = [0, \eta_{12}, 0_{n_z(T - 1)}], \ \gamma_8 = [\eta_{13}, \eta_{14}], \ \gamma_9 = [\eta_{15}, \eta_{16}] \\
\gamma_{10} & = [0, \eta_{17}(\theta - 1), 0_{n_z(T - 1)}], \ \gamma_{11} = \eta_{17} \left[ \xi, 1/R \right]
\end{align*}
\]

or, in matrix form,

\[
M_0 + M_1 H + M_2 H \Delta U + M_3 H A \Xi U + M_4 H \Xi U + M_5 H \mathcal{A} \mathcal{T} ( \beta \mu^{-1} I - \Xi U ) + M_6 H \mathcal{A} \mathcal{T} [ \beta \mu^{-1} ( \eta_{18} I - \Xi ) + (1 - \eta_{18}) \Xi U ] = 0
\]

where

\[
\begin{align*}
M_0 & = \left[ \gamma_1 + \gamma_4 A \Xi U - \frac{\lambda \sigma}{1 + \theta \xi} \eta_{10}^{-1} \gamma_00 \mathcal{A} \mathcal{T} ( \beta \mu^{-1} I - \Xi U ) \right] \\
M_1 & = [\gamma_2; \gamma_8], \ M_2 = [\gamma_3; \gamma_9], \ M_3 = [\gamma_5; \gamma_{11}], \\
M_4 & = \epsilon_{01} \gamma_{10}, \ M_5 = -\epsilon_{01} \frac{\lambda \sigma}{1 + \theta \xi} \eta_{10}^{-1} \gamma_0, \ M_6 = \epsilon_{02} \gamma_0
\end{align*}
\]

where

\[
\epsilon_{01} = [1, 0]', \ \epsilon_{02} = [0, 1]'
\]

Again using vectorization equation,

\[
W_H \text{vec}(H) = -\text{vec}(M_0)
\]

where

\[
W_H = [I \otimes M_1] + [U' \otimes M_2] + [(A \Xi U)' \otimes M_3] + [(\Xi U)' \otimes M_4] \\
+ [(A \mathcal{T} (\beta \mu^{-1} I - \Xi U))' \otimes M_5] + [(A \mathcal{T} [\beta \mu^{-1} (\eta_{18} I - \Xi ) + (1 - \eta_{18}) \Xi U])' \otimes M_6] \]
References


Table 1. Parametrization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Benchmark value</th>
<th>Range</th>
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</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.25</td>
<td>(0, 1)</td>
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<tr>
<td>$[\omega_\pi, \omega_Y, \omega_R]$</td>
<td>$[1/3, 1/3, 1/3]$</td>
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<td>$\beta$</td>
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<td>$\theta$</td>
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<td>[3, 11]</td>
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<td>$\xi$</td>
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<td>[1, 10]</td>
</tr>
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<td>$\sigma_\mu$</td>
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<td>[0.001, 0.1]</td>
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<tr>
<td>$\sigma_\varphi$</td>
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<td>$\rho_\varphi$</td>
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Table 2. Persistence of output responses to +1% monetary impulse
(for different fraction of fully updating households)

<table>
<thead>
<tr>
<th>Fraction of fully informed households $\lambda$</th>
<th>Expected duration between full updating $1/\lambda$</th>
<th>Half-life of output responses, quarters</th>
</tr>
</thead>
<tbody>
<tr>
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<td>informative signals $\sigma_\mu/\sigma_\varphi = 1$</td>
<td>uninformative signals $\sigma_\mu/\sigma_\varphi = 3.3$</td>
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<tr>
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<td>1.5</td>
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</table>

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Table 3. Persistence of output responses to +1% impulse to money growth rate (for different size of money growth shocks)

<table>
<thead>
<tr>
<th>Relative size of monetary and demand shocks $\sigma_m/\sigma_q$</th>
<th>Half-life of output responses, quarters</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1</td>
<td>7.4</td>
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Table 4. Persistence of output responses to +1% to money growth rate (for different size of real rigidity)

<table>
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<tr>
<th>Demand elasticity of individual price wrt output $\theta$</th>
<th>Elasticity of individual price wrt output $\xi/(1 + \theta\xi)$</th>
<th>Half-life of output responses, quarters</th>
</tr>
</thead>
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<tr>
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<td></td>
<td>Informative signals</td>
</tr>
<tr>
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<td>1/12</td>
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</table>
Figure 1A. Households' expectations of the current quantity of money, responses to +1% impulse to money supply growth, $\frac{\sigma}{\mu} = 1$

$\frac{M_t}{\mu_t}$, % deviations

quarters

aggregate
cohort 0
cohort 1
cohort 5
cohort 10
cohort 15
Figure 1B. Households' expectations of the current quantity of money, responses to +1% impulse to money supply growth, $\frac{\sigma_\mu}{\sigma_\phi} = 3.3$
Figure 2. Aggregate output as a function of history of monetary shocks.
Figure 3. Responses to +1% impulse to money supply growth. Varying the fraction of updating households, $\lambda$.
Figure 4A. Responses to +1% impulse to money supply growth.
Varying the size of monetary shocks, $\sigma_\mu$ ($\sigma_\phi=0.01$)
Figure 4B. Responses to +1% impulse to money supply growth. Varying the demand elasticity, $\theta$.
Figure 5A. Half-life of output.
Responses to +1% impulse to money supply growth.
Varying the fraction of updating households, $\lambda$

Informative signals, $\sigma_\mu / \sigma_\phi = 1$
Informative signals, $\sigma_\mu / \sigma_\phi = 3.3$
Uninformative signals
Figure 5B. Half-life of output.
Responses to +1% impulse to money supply growth. Varying the size of monetary shocks, $\sigma_\mu$ ($\sigma_\phi = 0.01$)
Figure 5C. Half-life of output.
Responses to +1% impulse to money supply growth.
Varying the demand elasticity, $\theta$