Increasing Returns to Savings and Wealth Inequality∗

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Abstract

In this paper I present an explanation to the fact that in the data wealth is substantially more concentrated than income. Starting from the observation that the composition of households' portfolios changes towards a larger share of high-yield assets as the level of net worth increases, I first use data on historical asset returns and portfolio composition by wealth level to construct an empirical return function. I then augment the standard neoclassical growth model with idiosyncratic labor income risk and missing insurance markets to allow for returns to savings to be increasing in the level of accumulated assets. The quantitative properties of the model are examined and show that an empirically plausible difference between the return faced by poor and wealthy agents is able to generate a substantial increase in wealth inequality compared to the basic model, enough to match the Gini index of the U.S. Distribution of wealth.

Keywords: Wealth inequality, self-insurance, portfolio composition, increasing returns.

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1 Introduction

Empirical studies like Hurst, Luoh and Stafford (1998), Díaz-Giménez, Quadrini and Ríos-Rull (1997), and Budría-Rodríguez et al. (2002) have shown that earnings, income and wealth are very concentrated, with distributions that are skewed to the right. Of the three variables wealth is by far the most concentrated with a Gini coefficient of 0.78, while the same index for earnings and income is 0.63 and 0.57.\(^1\)

The latter fact is a regularity that is observed over time and across countries as well and has drawn considerable attention in the quantitative macroeconomic literature. The basic framework used to explain this fact is the one in Aiyagari (1994) and Huggett (1996) and is based on heterogeneous labor earnings shocks, missing insurance markets and borrowing constraints. In Aiyagari’s setting agents are ex-ante identical, but at any point in time each of them has experienced a different history of realized labor shocks. Inequality is generated because agents with a history of good shocks have accumulated savings to insure themselves against the possibility of bad shocks, while agents with a history of bad shocks have depleted their accumulated assets. Huggett adds saving to finance consumption during retirement as a further ingredient that enhances wealth inequality by introducing heterogeneity across agents of different ages. Both models are successful at reproducing qualitatively the empirical evidence. However they are incapable of matching the data quantitatively so that various features, like heterogeneous subjective discount factors, bequest motives and entrepreneurship have been used in later work to improve the performance of the basic model.

Both the basic model and the extensions that followed share one key assumption about the assets available to the agents to carry out their saving plans. This assumption is that there is a single asset in the economy. A consequence is that all agents, no matter what their income or wealth is, face the same return on their investments. This assumption is clearly at odds with reality, since real world households may choose to hold assets as diverse in terms of return, risk and liquidity as for example housing and stocks or life insurance policies and checking accounts. To the extent that portfolio composition and returns vary systematically among households, these will have different incentives to save adding a further potential source of wealth inequality.

The goal of this research is to incorporate this basic feature of households’ investment decision in an otherwise standard precautionary savings model and test whether the existence of increasing returns to savings is a quantitative relevant source of wealth inequality. It turns out that it is: a modest difference in the return faced by the poorest

\(^1\)The values reported here are taken from Díaz-Giménez, Quadrini and Ríos-Rull (1997) and are based on the 1992 Survey of Consumer Finances.
and wealthiest households in the economy is sufficient to match the Gini index and the share of the different quintiles of the U.S. distribution of wealth. This suggests that so far the most relevant explanation for the massive concentration of wealth of real economies has been overlooked, explaining the difficulties that past models have faced in their effort. A caveat is that this paper models an infinite-horizon economy. In a finite-horizon setting it is unlikely that small differences in returns could generate huge differences in wealth during the course of a lifetime. However an extension of the life-cycle model that adds intergenerational links like in Castañeda, Díaz-Giménez and Rios-Rull (2002) and De Nardi (2001) would reproduce the results of the dynastic model considered here.

The model assumes exogenously that returns to saving are increasing in the level of wealth; however, this feature of investment opportunities has strong support in the data. Empirical research by Bertaut and Starr-Mcluer (2000), Kennickell et al. (2000) and Samwick (2000) clearly shows that the composition of households’ portfolios shifts towards larger fractions of high-yield assets, like stocks and business equity, as the household’s net worth increases. In the paper I first construct an empirical return function using data on portfolio composition by wealth levels reported in Samwick (2000) and data on returns of broad categories of assets from a variety of sources. This exercise reveals that while the poorest 60 percent of the population faces an average return to its wealth which is close to 1 percent, the top 1 percent invests its wealth at an average return in excess of 4.5 percent. Then, using standard values for the preference and technology parameters, I show that a difference in the return between rich and poor agents that is consistent with the empirical evidence generates a substantial amount of extra inequality in the basic Aiyagari model, enough to match the Gini index observed in the U.S. economy.

The reason why a model with increasing returns to savings can match the observed level of wealth inequality is the following. In the standard model, as discussed in Carroll (2000), the incentive to save declines to negligible levels before the large amount of asset holdings observed in the data is reached. In the present framework instead, agents who receive a sufficiently long string of good labor income realizations to accumulate a large amount of wealth face higher returns that keep up their propensity to save, leading to further increases of their net worth. At the same time this behavior of wealthy people will lead to a rate of return to small amounts of assets that is further away from the subjective discount factor than it would be in the standard case of constant returns, leading unlucky agents to accumulate even less. The two forces jointly spread out both ends of the wealth distribution making it more consistent with the empirical evidence.

Before moving to the remaining sections of the paper it is important to spend a few words about the interpretation of the positive relation between net worth and portfolio
returns that is observed in the data and, based on that evidence, is assumed here. There are two alternative but not mutually exclusive stories that can be told to explain this relation. The first is a market imperfection one: it may be necessary to pay information costs to gain knowledge of the functioning of some asset markets and even then other trading costs are required to actually participate in those markets. As a consequence only households that have accumulated enough wealth may find it attractive to spend the time and money needed to participate in those higher return markets. In support of this view come a number of studies like, for example, Paiella (2001) and Vissing-Jørgensen (2001) about the costs of participating in the stock market and by Hong, Kubik and Stein (2001) that find participation to the stock market being positively related to sociability as a result of the effect that communication with peers has in lowering information costs.

The second story is a behavioral one: according to this interpretation some agents dislike some assets and decide not to participate even if this would be optimal based on their risk preferences and on the asset return. In support of this possibility the paper cited above by Hong et al.(2001) reports that participation in the stock market in the U.S. is substantially higher for white than for non-white even after controlling for wealth, income, education and survey measures of risk tolerance; Guiso, Sapienza and Zingales (2004) reach a similar conclusion with Italian data when comparing participation rates between southern and northern Italians.2

The model presented in this paper is more consistent with the first interpretation since it implies a positive feedback from wealth accumulation to higher returns and again to further accumulation; moreover it does not assume heterogeneity in tastes. However it is not in contrast with the behavioral story: by showing that small differences in the return to assets can generate a realistic concentration of wealth it says that small ex-ante differences in behavior concerning portfolio composition may lead to the large observed wealth inequality. Moreover the same positive relation between net worth and the return on savings assumed in this paper would still be obtained as an ex-post result in a behavioral one.

The rest of the paper is organized as follows. In Section 2 I first review the quantitative literature on wealth inequality. In Section 3 I present a short account of the empirical evidence on wealth inequality and on portfolio composition at different wealth levels; I then construct an empirical schedule that maps net worth into average portfolio returns. In Section 4 I describe the model, the choice of parameters and the results. Finally

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2The three authors suggest that financial contracts and stocks in particular are trust-intensive contracts. They then exploit variation within Italy of measures of social capital (of which trust is an important element) and show that these are positively related to the use of checks, participation in the stock market and the availability of credit.
2 Literature Review

A large number of papers present quantitative models that attempt to explain the observed wealth distribution\(^3\). These models share a set of basic assumptions. First they are populated by agents who receive an exogenous stochastic flow of income. Second, markets are assumed to be incomplete so that it is not possible to fully insure consumption risk. Finally there is some form of borrowing constraints. A notable example of this kind of models is Aiyagari (1994). Agents use accumulated savings in order to buffer negative shocks to income and therefore to smooth consumption. While agents are ex-ante homogeneous, ex-post each of them will have experienced a different history of past realized incomes leading to a different level of accumulated wealth. This model, as presented in the above mentioned paper by Aiyagari, generates a distribution of wealth which is more concentrated than the distribution of income, a feature that is qualitatively consistent with the data. However at a quantitative level it grossly underpredicts the observed concentration of wealth. In particular it fails to explain the two tails of the distribution, that is, the very low level of wealth accumulation by poor agents and the accumulation of huge estates at the very top of the wealth distribution.

The model in Aiyagari (1994) considers an economy populated by infinitely-lived dynasties in which saving occurs for precautionary reasons. In a related paper Huggett (1996) uses similar assumptions about market structure, but casts the model in a finite-horizon framework where agents face a realistic lifetime profile of earnings and go through the working and retirement stages of life. In this framework saving also occurs to finance retirement consumption. The model fares well in terms of matching the Gini index but it obtains this result by having a large fraction of households with no or even negative wealth while still underestimating the large accumulation of assets of the very rich. Moreover those with no wealth are entirely concentrated among very young households that face an upward sloping earnings profile and would like to borrow.

Following these two papers various mechanisms have been proposed to improve the ability of quantitative models to match the observed concentration of wealth. These attempts may be broadly classified based on whether their main focus is on the left or the right tail of the distribution.

A prototypical example of the first group is the paper by Hubbard, Skinner and Zeldes (1995). Their model is cast in a finite-horizon framework. It features different lifetime

\(^3\)Quadrini and Ríos-Rull (1997) present an excellent review of the attempts that have been made to explain the quantitative properties of the US wealth distribution.
profiles and risk of earnings for different educational groups and adds health risk. The crucial element is the presence of means-tested government programs that provide a safety net in the form of a floor on consumption in case of very bad luck. This induces very poor agents not to accumulate assets at all and rely on public social insurance instead. While not directly focused on measuring wealth inequality this model is able to generate a substantial number of agents with very low or no savings at all without having them entirely concentrated among younger agents.

Another institutional feature that has the potential to reconcile the data on the uneven wealth distribution with the output of quantitative models is the progressivity of the social security system. This has been used by Domeij and Klein (2002) to account for the large portion of Swedish households with very little wealth and, coupled with lifetime differences in earning abilities, has also been proposed by Huggett and Ventura (2000) to explain why low income households as a group save a lower fraction of their income than high income households do.

As far as the right tail of the wealth distribution is concerned two mechanisms have been proposed so far. Based on the empirical observation, reported in Gentry and Hubbard (2000), that entrepreneurs both make a significant share of very wealthy households and tend to have higher wealth-income-ratios, Quadrini (2000) constructs a model where entrepreneurship is recognized as the critical element to add to a quantitative model to generate realistic wealth concentration. In his model there are three features that drive savings of business households up. First, imperfections in financial markets limit the amount that can be borrowed, forcing agents who have entrepreneurial ideas to accumulate wealth to overcome that constraint. Second, intermediation costs drive a wedge between borrowing and lending rates so that the marginal return to saving and investing in the private firm is higher than market returns. Finally, consistent with the empirical evidence, Quadrini assumes that the income flow generated by a business is more risky than the income of paid employees, increasing precautionary saving. While the model is able to generate a more realistic wealth concentration and to account for the higher wealth-income ratio and upward mobility of entrepreneurial households, it basically neglects the fact that a substantial share of the top of the wealth distribution is made by non-entrepreneurial households; moreover it is silent about the left tail of the distribution.

The second mechanism exploits intergenerational links in the form of altruism and correlation between the earning abilities of successive members of a family. Castañeda et al. (2002) is an example along these lines: the authors use intergenerational links and a stylized representation of the U.S. security system and estate taxation to check if it is possible to find a process for earnings that allows the model to match the distribution of earnings and wealth simultaneously. While they claim their effort is successful, the
resulting labor earnings process is highly unrealistic. In a slightly different fashion De Nardi (2001) also constructs a model populated by finitely lived agents in which parents and children are linked by voluntary bequests and persistence within families in earnings abilities. Her model is calibrated on U.S. and Swedish data and shows how the two intergenerational links are important to explain the emergence of the large estates we observe at the top of the wealth distribution.4

Finally a completely different approach that looks jointly at the two ends of the wealth distribution has been followed by Krusell and Smith (1998). The key feature of their model is the assumption of heterogeneous subjective discount factors. While many economists would look with suspicion at a model based on an unobservable variable like the discount factor, there is some experimental evidence in favor of preference heterogeneity.5 The economy in Krusell and Smith is populated by infinitely lived agents but retains some features of a finite-horizon model by assuming that discount factors, an individual specific characteristic, change stochastically over time with an average frequency close to the average length of life. As a result of this assumption some agents will be patient, accumulate wealth and therefore fix the equilibrium interest rate. The rest will have a discount factor well below the interest rate and therefore act as hand-to-mouth consumers. As a result the model is able to generate both a large number of agents with low or no assets at all and an empirically plausible concentration of wealth at the top of the distribution.

The present work is most closely related to Quadrini (2000) and Krusell and Smith (1998). It shares with Quadrini’s paper the idea that an important contribution to the explanation of the high concentration of wealth that we observe in the data comes from the fact that different agents face different investment opportunities and that those who are wealthier face higher returns which in turn induces them to accumulate further. The theory presented here is more general in many respects though. It has the merit that it acknowledges that high returns may come from stocks as well as ownership of a private enterprise and that empirically high returns to investment are a common feature of the portfolio of all wealthy agents and not only of entrepreneurs. It also recognizes the role played by the low returns faced by poor households in determining their very limited

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4 The two approaches are brought together by Cagetti and De Nardi (2002). First, they explicitly model a market friction that limits entrepreneurial borrowing generating higher returns to investment in own firms than on the market. Second, their economy is populated by stochastically aging agents who go through the stages of working life, retirement and death, therefore allowing for voluntary bequests. The joint operation of higher marginal returns to business investment and the bequest motive enable their model to reproduce the high concentration of wealth at the top of the distribution, although it is still true that the very wealthy are active or retired entrepreneurs, which leaves the empirically observed share of wealthy non entrepreneurs unaccounted for.

5 See for example Barsky et al. (1997).
accumulation of assets

The relation with Krusell and Smith is that in both models the source of the high concentration of wealth comes from the fact that different agents face returns that are at different distance from their subjective discount factors: in Krusell and Smith there is only one rate of return and discount factors are heterogeneous, while here there is only one discount factor and returns are heterogeneous. The advantage of the framework here is that heterogeneity in discount factors is not observable while the fact that returns to portfolios are increasing in their size is an empirically well documented observation, as it will be shown in the next section.

3 The Empirical Evidence

The purpose of this section is twofold. First, it reports some basic data about inequality and in particular it shows the well known fact that the concentration of wealth is greater than the concentration of income and earnings. Second, it briefly describes some facts about the observed changes in portfolio composition as we move along the wealth distribution. Here the main finding is that richer households tend to have more complex portfolio structures with a larger fraction of their net worth held in high yielding assets than poorer households. Using data on portfolio composition by level of net worth, together with data on returns to different assets I then compute an empirical return schedule. It will be shown that the range of this schedule in not large but still not negligible.

3.1 Some Facts about Inequality

There exist very good and extensive surveys on different dimensions of inequality like Díaz-Giménez et al. (1997), Hurst et al. (1998), Wolff (2000) and Budría-Rodríguez et al. (2002). In this subsection therefore I briefly report on some aspects of inequality in the U.S. data focussing in particular on the relative features of earnings compared to wealth inequality.

Table 1 reports the Gini coefficient and the share of net worth held by different quantiles of the distribution in 1992.6 The most striking feature that emerges from a look at Table 1 is the extreme concentration of wealth at the top of the distribution. The

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6The table is based on Wolff (2000) who in turn used the Survey of Consumer Finances as the source of data. The SCF is a survey sponsored by the Federal Reserve and the Department of the Treasury and is conducted every three years on a randomly selected sample of households. Its main feature is that it oversamples very wealthy households and therefore it is the most reliable source of data to study the financial choices of the very rich.
Table 1: Wealth Inequality (data)

<table>
<thead>
<tr>
<th>Year</th>
<th>Gini coefficient</th>
<th>Top 1%</th>
<th>Top 5%</th>
<th>Top 20%</th>
<th>Bottom 40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1983</td>
<td>0.779</td>
<td>32.6</td>
<td>54.4</td>
<td>79.6</td>
<td>1.7</td>
</tr>
<tr>
<td>1992</td>
<td>0.802</td>
<td>35.9</td>
<td>58.3</td>
<td>81.9</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Table 2: Income Inequality (data)

<table>
<thead>
<tr>
<th>Year</th>
<th>Gini coefficient</th>
<th>Top 1%</th>
<th>Top 5%</th>
<th>Top 20%</th>
<th>Bottom 40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1982</td>
<td>0.480</td>
<td>12.8</td>
<td>26.1</td>
<td>52.0</td>
<td>12.3</td>
</tr>
<tr>
<td>1991</td>
<td>0.528</td>
<td>15.7</td>
<td>30.5</td>
<td>56.3</td>
<td>10.5</td>
</tr>
</tbody>
</table>

one percent richest households owned, in 1992, 35.9 percent of total national wealth of the U.S. economy and if we look at a slightly broader set of families, that is, the top 5 percent, the share in total net worth increases to 58.3 percent. The second important fact is the negligible amount of wealth held by a large fraction of the population; the 40 percent poorest families owned in 1992 only 1.2 percent of total net worth. Looking at the two rows of the table it is clear that the large degree of wealth concentration is a quite stable feature of the U.S. economy, as the share of total net worth at different quantiles of the distribution did not move much between 1983 and 1992.

Table 2 reports the same statistics as Table 1, but refers to the distribution of income. While income is still very concentrated it is considerably less concentrated than wealth. The Gini index is now only 0.480 in 1982 and 0.528 in 1991. The share of the top 1 percent in 1991 was 15.7 percent and that of the top 5 percent was 30.5 percent in the same year. Finally the 40 percent poorest in the population earned a little more than 10% of total income, a small but not completely negligible share.

3.2 Wealth and Portfolio Composition

In this subsection I report some statistics that illustrate how the composition of household portfolios change with the level of net worth. There are some good surveys on the topic like Bertaut and Starr-Mcluer (2000), Kennickell et al. (2000) and Samwick (2000). The main messages that consistently emerge from all of them are that the structure of family portfolios increases in complexity as their wealth increases and that wealthier households

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7Díaz-Giménez et al. (1997) and Budría Rodríguez et al. (2002) report also statistics about the concentration of earnings. These are similar to the ones for income and therefore are not reported here.
invest larger shares of their net worth in higher-return and higher-risk assets. As far as the first point is concerned, Bertaut and Starr-Mccluer (2000) classify households based on the number of different types of assets they hold in their portfolios. Then they compute the median value of financial wealth for the groups obtained in this way and find that there is a monotonically increasing relationship between the two variables.

More relevant to the purpose of the present paper is the second point which I will illustrate by means of Table 3 and Table 4. In Table 3 I report the percentage of households who own the particular asset indicated in the first column of the table.8 The figures are reported by wealth quartiles and separately for the wealthiest 5 percent families. It is clear that liquid accounts, a group of assets that pay a very low return, are very widespread even among the bottom quartile of the distribution and their use is basically universal from the next quartile up. On the contrary high yielding assets like stocks, mutual funds and business assets, are held by about 2 to 4 percent of the families in the bottom quartile of the wealth distribution, a very tiny minority. The fraction of households owning those assets increases monotonically as we move to upper quartiles of the distribution so that each of them is owned by more than half of the 5 percent richest families.

While Table 3 reports data on participation to different asset markets, Table 4 takes an alternative perspective and looks at the share that assets with different returns represent in the average portfolios of families belonging to different quantiles of the wealth distribution.9 The reported shares are relative to wealth net of the value of owner oc-

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8The table is an excerpt from Table 4, pp 29, in Bertaut and Starr-Mccluer (2000).
9This table is based on Table 9.3 and Table 9.4 in Samwick, (2000). The tables report data for 1983,
ocupied house. Liquid accounts make 13.7 percent of the average portfolio of households in the bottom 40 percent of the wealth distribution. This percentage goes up to 16.2 at the second decile and then declines monotonically to a low 6.6 for the top percentile. The share of business equity in total assets shows a monotonic increase across the whole distribution, starting from a low 3.9 percent for the bottom 40 percent and increasing to a high 40.2 percent for the top percentile. The fraction of directly and indirectly held stocks shows a non monotonic pattern. Stocks represent 12.6 percent of the portfolio of the average household in the bottom 40 percent of the wealth distribution, 26.2 percent when we consider households in the 95th to 99th percentile and then a slightly lower 19.4 for the top percentile. However both stocks and business equity are high yielding assets and if the two are summed the fraction of this sum in total non housing wealth shows a clear pattern of monotonic increase across all the wealth distribution.

Summarizing, Table 4 clearly shows that as wealth increases the share of high-yielding assets increases and that of low-yielding assets decreases, so that households face a return schedule that is increasing in their asset holdings. This statement will be made more precise in the next section where an empirical return schedule is constructed.

### 3.3 Portfolio Returns

In this section I take the available evidence on portfolio composition by percentiles of the wealth distribution described in the previous section and using data about the return to different categories of assets I construct an empirical function that maps wealth holdings into returns on that wealth.

Samwick (2000) reports the composition of the portfolio of the average household belonging to different percentiles of the wealth distribution, classifying assets into the following groups: financial, own home, other properties, private equity and a miscellaneous entry that includes all other assets that do not fall in any of the previous categories. In turn he breaks down financial assets into interest bearing accounts, taxable bonds and equity, reporting a separate figure for the share held through retirement accounts, tax-exempt bonds and a residual group which includes, among else, cash value of life insurance policies and trust accounts. He also reports, for the same percentiles of the wealth distribution, the net worth of the average household in that percentile. The data are constructed based on the 1983, 1989, 1992 and 1995 issues of the Survey of Consumer Finances.

In order to construct an empirical return function I proceed as follows. First I elim-
inate from the computation the assets in the residual categories, since it is difficult to impute a measure of their return. Overall they represent about 10 percent of total assets in the four surveys; when we look through the wealth distribution this share is slightly below 10 percent at the top and about 20 percent at the bottom of the wealth distribution.\footnote{Most of this share and its variation across wealth groups comes from the category miscellaneous non-financial assets. This could be a consequence of the inclusion of cars and other motor vehicles in the category. These assets are durable goods that have a consumption value but - as it is clear in the case of cars - they pay a negative return. Including them in the computation of portfolio returns would further increase the difference in the returns faced by poor and wealthy agents.} Next I collect the share of stocks held directly or in retirement accounts and do the same for bonds adding in this case also tax-exempt bonds. Finally I consider own house and other property as a single asset, even though the latter may include land and other non-residential properties. By doing this regrouping I end up splitting net worth into five classes: interest-bearing accounts, stocks, bonds, property and business assets.

Interest-bearing accounts are a heterogeneous group of assets that includes checking and saving accounts, which pay a negative real interest, and certificates of deposit and money market accounts, which pay a small positive interest; therefore, I conventionally set the return to this category to 0.

I set the return to stocks to 8 percent, the value reported in Jagannathan et al. (2000) for the return to the S&P 500 index for the period 1926 to 1999. Based on Moskowitz and Vissing-Jørgensen’s (2002) claim that the return to private equity is no different than the return to the public equity index I also set the return to business assets at 8 percent.

As far as bonds are concerned, these are again a heterogeneous category of assets including government, corporate and foreign bonds as well as municipal and state bonds that have a preferential tax treatment. Not having a finer subdivision of the category I attribute to it the return to 20-year U.S. Treasury securities of 1.9 percent reported in Jagannathan et al. (2000) and referring again to the period 1926-1999.

Finally, Goetzmann and Spiegel (2000) report that according to the Office of Housing Enterprise Oversight the real price of housing has increased at a 0.5 percent annual compounded rate over the period 1980 to 2000. While the return to residential property includes the housing services that it provides, this class of assets has special costs like property taxes and maintenance. Moreover high costs and risks of transaction may have a strong negative impact on the return, especially when the holding period is short. The two authors then suggest that the return to this asset may be even lower than the 0.5 percent per year that their price appreciation suggests. Given all these considerations I take the value of 0.5 percent as the return to property.

With this in mind I can construct a return to the average portfolio of different per-
Table 5: Return by Wealth Percentiles

<table>
<thead>
<tr>
<th></th>
<th>0-60</th>
<th>60-80</th>
<th>80-90</th>
<th>90-95</th>
<th>95-99</th>
<th>99-00</th>
<th>all</th>
</tr>
</thead>
<tbody>
<tr>
<td>1983</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>0.13</td>
<td>0.67</td>
<td>1.30</td>
<td>2.41</td>
<td>5.70</td>
<td>30.71</td>
<td>1</td>
</tr>
<tr>
<td>R(a)</td>
<td>0.95</td>
<td>1.14</td>
<td>1.67</td>
<td>2.61</td>
<td>3.54</td>
<td>4.87</td>
<td>3.02</td>
</tr>
<tr>
<td>1989</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>0.11</td>
<td>0.69</td>
<td>1.33</td>
<td>2.34</td>
<td>5.3</td>
<td>33.4</td>
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</tr>
<tr>
<td>R(a)</td>
<td>0.87</td>
<td>1.22</td>
<td>1.56</td>
<td>2.07</td>
<td>2.94</td>
<td>4.05</td>
<td>2.63</td>
</tr>
<tr>
<td>1992</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>a</td>
<td>0.12</td>
<td>0.68</td>
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<td>5.86</td>
<td>29.29</td>
<td>1</td>
</tr>
<tr>
<td>R(a)</td>
<td>0.93</td>
<td>1.39</td>
<td>1.88</td>
<td>2.29</td>
<td>3.22</td>
<td>4.41</td>
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</tr>
<tr>
<td>a</td>
<td>0.11</td>
<td>0.63</td>
<td>1.24</td>
<td>2.36</td>
<td>5.31</td>
<td>34.83</td>
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</tr>
<tr>
<td>R(a)</td>
<td>1.13</td>
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<td>1.83</td>
<td>2.50</td>
<td>3.42</td>
<td>5.09</td>
<td>3.05</td>
</tr>
</tbody>
</table>

Note: a: Average percentile wealth normalized by average wealth
R(a): Percentage return to average portfolio by percentile

centiles of the wealth distribution for the four years considered by Samwick; the results are reported in Table 5. The level of wealth in the table is normalized by the value of average net worth for that particular year. There are two points that are worth noting in the table. First, in all the years considered, the average return that households face on their portfolio is monotonically increasing in the level of net worth. The second point concerns the magnitude of the difference between the return faced at the top and at the bottom of the distribution. In 1983 the average household in the top 1 percent of the population held 30.7 times the average net worth and faced a return on its assets of 4.87 percent, while the average return faced by a household in the bottom 60 percent of the distribution, which held only 0.13 times the average wealth in the population, was 0.95 percent, corresponding to a difference of 3.92 percentage points. Looking at the other years the difference between the return faced by poorest and wealthiest households was 3.18 percentage points in 1989, 3.48 percentage points in 1995 and 3.96 percentage points in 1995. Overall it seems that the difference is not big but still significant and can be bounded between 3 and 4 percentage points.

It will be the focus of the modelling section to see to what extent a difference in returns of this magnitude can be held responsible for the extreme concentration of wealth, compared to the one of earnings, that we observe in the data.
4 The Model

The model in this paper builds on the standard stochastic growth model. Agents are ex-ante identical; however, they receive a stochastic endowment of efficiency units of labor which can be only partially insured by saving. This introduces ex-post heterogeneity like in Aiyagari (1994). The model features a second source of ex-post heterogeneity in the form of differences in the return to assets that are related to their level.

4.1 The Economic Environment

The economy is populated by a large (measure one) number of infinitely lived agents whose preferences are defined over streams of consumption and are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

where $\beta$ is the subjective discount factor and the period utility function $U$ is of the standard isoelastic form, that is,

$$U(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

where $c$ is period consumption and $\gamma$ is the coefficient of relative risk aversion.

In each period agents receive a stochastic endowment of efficiency units of labor which I denote with $l$. The logarithm of the endowment evolves over time following a first-order autoregressive process

$$\log l_t = \rho \log l_{t-1} + \varepsilon_t$$

where $\varepsilon_t$ is $iid$ over time and normally distributed.

Agents have a single asset to carry out their saving plans; the amount of the asset held at time $t$ is denoted by $a_t$ and a tight borrowing constraint, $a_t \geq 0$ is assumed. The crucial and distinctive assumption in the model is that the return to this asset is increasing in the amount held by the agent. We can pictorially visualize this assumption by thinking that production takes place in a single factory while consumption is carried out at decentralized locations where agents live. Agents receive the compensation for labor services at the factory and carry it with them at no cost to their living quarters. They are also entitled to capital income in an amount which is the product of their asset holdings and the marginal product of capital, but this is shipped to their location and during transportation part of this output is lost, with the amount lost growing less than proportionally with the amount to be delivered.
The assumption of a return function which is increasing in the level of assets is a reduced form to capture the evidence, presented in the previous section, that households invest larger proportions of their portfolios in higher-yield assets as they become wealthier.\footnote{An alternative way of modelling that is more directly linked to the actual portfolio choice of households, but that is equivalent in practical terms to the one chosen here, would have been to assume that there are two assets with different returns and an exogenous rule that assigns weights to the two assets with the share of the higher yield one being increasing in wealth.}

I will denote with $R$ the price of the services of capital that in a competitive equilibrium will be equal to the marginal product of capital; I will use the notation $r(a)$ to refer to the function that describes the actual compensation received by agents who own capital. This function can then be written as:

$$r(a) = 1 + \Phi(a)$$

and

$$\Phi(a) = R - I(a).$$

The chosen functional form for $I$ is a logistic defined by:

$$I(a) = \nu \frac{1}{1 + e^{\lambda(a - \overline{a})}}.$$

Here $\nu$, $\overline{a}$, and $\lambda$ are three constants which control the difference in returns faced by the poorest and wealthiest households, the level of wealth at which the returns start to pick up and the speed of this increase respectively.

At the aggregate level capital and labor are turned into output of the single good $Y$ according to the usual Cobb-Douglass technology: $Y = K^\alpha L^{1-\alpha}$ where $K$ and $L$ denote aggregate capital and labor respectively. Finally the single output good is transformed into current consumption and future capital according to:

$$C + K' = Y + (1 - \delta) K - TW$$

where $\delta$ is the depreciation rate, $K'$ is next period capital and $TW$ is the total waste of output that occurs when shipping capital income to the households entitled to it. If we denote with $\mu(a,l)$ the measure of agents with asset holdings $a$ and labor efficiency units $l$, the variable $TW$ is defined as:

$$TW = \frac{Z}{l} \int_a I(a) \mu(a,l) \, da \, dl.$$
With the description of the economic environment in mind and omitting the time index we can formulate the optimal decision problem in recursive form as follows:

\[ V(a, l) = \max_{c, a_0} \{ U(c) + \beta E[V(a', l') | l] \} \]

subject to the following set of constraints:

\[ c + a' = ar(a) + wl \]
\[ a' \geq 0, \ c \geq 0 \]

where \( w \) is the wage rate and a prime denotes next period variables.

The solution to the Bellman equation described above gives rise to an optimal value function \( V^*(a, l) \) and associated decision rule \( g(a, l) \).

A stationary competitive equilibrium for this economy is a value function \( V^*(a, l) \) and decision rule \( g(a, l) \), a probability distribution \( \mu^*(a, l) \) and positive real numbers \((R, w)\) such that:

a) Given prices \( R \) and \( w \), \( g(a, l) \) solves the household’s optimization problem with value function \( V^*(a, l) \).

b) Prices \( R \) and \( w \) are determined competitively, that is, \( w = (1 - \alpha)(K/L)^{\alpha} \) and \( R = \alpha(K/L)^{\alpha - 1} - \delta \).

c) The probability distribution \( \mu^*(a, l) \) is a stationary distribution associated with \([g(a, l), P]\) where \( P \) is the transition probability over labor efficiency units. The stationarity condition can be formally expressed as:

\[ \sum_{l'} \sum_{a'} \mu^*(a', l') = \sum_{l_0, l' \in g(a, l)} \mu^*(a, l) P(l, l') d\lambda. \]

d) Markets clear, that is, \( C + K' = K^{\alpha}L^{1-\alpha} + (1 - \delta)K - TW \).

The model has no closed-form solutions and therefore it is solved numerically.

4.2 Calibration

In this section the values chosen for the parameters of the model will be described. Given the choice of the basic parameters defining preferences, technology and the stochastic labor income process, the model is first solved for a constant rate of return to assets, that is, for the case in which \( r(a) = 1 + R \). Then a comparative analysis with different triples \((\nu, \lambda, \pi)\) is made in order to explore whether the assumption of increasing returns to savings can give a quantitatively relevant contribution to the explanation of the empirically observed wealth concentration.
The values of the calibration exercise are taken from Aiyagari (1994). Preferences are defined by two parameters, that is, the subjective discount factor $\beta$ and the coefficient of relative risk aversion $\gamma$. Consistent with a model period of one year $\beta$ is set at 0.96, a value used in most macroeconomic studies. The coefficient of relative risk aversion $\gamma$ is fixed at a value of 3, in line with most studies as well.

Production technology is also defined by two parameters, the share of capital in the production function $\alpha$ and the depreciation rate of physical capital $\delta$, which are assigned the values of 0.36 and 0.08 respectively.

At the aggregate level the total amount of efficiency units of labor is normalized to one. At the individual level the labor endowment follows an $AR(1)$ process in logarithms, with autocorrelation coefficient $\rho$ equal to 0.9 and variance of log-labor earnings $\sigma^2_e$ equal to 0.2.\textsuperscript{12}

4.3 Results

In this section I will present the main results of the model. The section is divided in two parts: the first one is devoted to illustrate individual behavior as suggested by an examination of the decision rules from the consumer’s optimization problem, the second one considers the general equilibrium results with a focus on the wealth distribution.

4.3.1 Value and Policy Functions

Both the value function and the decision rules associated with this problem differ qualitatively from the corresponding solutions to a standard problem with constant rate of return.

A look at the individual optimization problem reveals that with a non-concave return function, like the logistic one that has been assumed, the problem itself ceases to be concave. This can, at least potentially, lead to non monotonicities in the optimal value function. Whether this will be the case depends on the amount of non concavity in the return function and is an issue that can be solved through the numerical analysis of the problem. The numerical methods used to solve the optimization problem however do not require concavity.\textsuperscript{13}

Figure 1 reports the value function computed for the following choice of parameters of the return function: $\nu$ is set at 0.04, $\overline{\pi}$ equals 4 and $\lambda$ equals 1. The three lines

\textsuperscript{12}The two values imply a standard of the innovation $\sigma_\varepsilon$ equal to 0.195.

\textsuperscript{13}At each state-space point the static maximization problem defined by the right-hand side of the Bellman Equation is solved with a direct search method. This method is slow, but finds the global maximum independently of the shape of the maximand.
correspond to three different levels of the individual endowment of labor efficiency units and are plotted at the equilibrium of that particular economy. As can be seen from the graph, the value function is still increasing but it shows a slight non-concavity around the value of 5, that is, slightly to the right of the point where the return function with the given parameters exhibits an inflection point. The non concavity however is minor and the value functions still retain quasi concavity. The analogous plots for different choices of the parameters that characterize the return functions are omitted since they show the same qualitative pattern.

As far as the decision rules are concerned it is useful to first write down the first-order conditions for the optimal choice of assets. At an interior point this will read:

$$u'(c) = E \left\{ \beta [r(a') + a'r'(a')]u'(c') \right\}$$

where for notational simplicity we can define the marginal return to savings as:

$$mrs(a) = r(a) + ar'(a).$$

If we consider a given value of the marginal product of capital $R$, given the specification of $r(a)$ used here, it is always true that the first term in $mrs(a)$ is smaller than $1 + R$ itself. However a logistic return function also implies that $r'(.)$ is positive, so that it cannot be said a priori whether $mrs(.)$ is greater or smaller than $1 + R$.

Figure 2 shows rates of returns to savings at the equilibrium of the model with the same choice of parameters considered above. Two curves are plotted. The dashed line is the return function $r(a)$ while the continuous line is the marginal return to savings function $mrs(a)$. The most striking feature of the two functions is that while $r(a)$ converges to $1 + R$, but is always below it, the marginal return to savings function (almost) coincides with $r(a)$ for low or high values of the level of assets, but it is above it in an intermediate range where the return function shows a marked increase: this is the effect of the term $ar'(a)$. What emerges from the figure is that $mrs(.)$ can exceed the marginal product of capital for the corresponding parametrization of the model and in a narrower range of wealth it can also exceed the subjective rate of intertemporal preference.

From a qualitative point of view this is very important, since as we can see from the first order condition, the marginal return to savings is crucial in determining the rate of growth of consumption and therefore the saving rate. When $mrs(a) > 1 + R$, the Euler equation implies a lower $u'(c')/u'(c)$ ratio and therefore, with concave utility function, a higher rate of growth of consumption, that is, a higher saving rate; the opposite happens when $mrs(a) < 1 + R$. 

17
This can be seen in Figure 3. The graph reports the policy function at equilibrium for the same parametrization of the economy previously used in this section. It does so for the median value of the labor efficiency unit shock. It also reports the corresponding policy function for a model that uses the same preference and technology parameters, but adopts the conventional assumption of constant return to saving.

The decision rule for the model with non-constant returns, represented by the dashed line, is clearly non-linear and as suggested by the joint examination of the first-order conditions and the marginal return to savings function, it is below the decision rule for the constant return case for low levels of wealth, where the marginal return to savings function lies below the marginal product of capital. Around the point where marginal returns overtake the marginal product of capital the decision rule for the non-constant case becomes higher than the one for the constant case and then converges to it for even higher levels of wealth.

This pattern suggests that the model with increasing returns to savings has the potential to explain the observed high level of wealth concentration. This is because at low levels of wealth agents want to dis-save at a faster pace than in the constant return case, possibly clustering around zero asset holdings. Having reached a certain critical level of wealth however they want to save more than in the constant return case, moving upwards more quickly. The overall effect is to magnify the difference in asset holdings of agents with different histories of labor shocks. How important is this effect is a quantitative issue and will be examined for a number of different parametrizations of the return function in the next section.

4.3.2 General Equilibrium Results: The Preferred Parametrization

In this section I give a detailed account of the general equilibrium results for a particular choice of the parameters of the return function and I compare them with the constant interest rate case and with the actual data from the U.S. economy. For ease of reference I will label the case detailed in this section the preferred parametrization.

In the case considered here \( \nu \) is given the value 0.03, \( \lambda \) is set at 0.125 and \( \bar{\pi} \) is fixed at 12. The corresponding return and marginal return functions are plotted in figure 4 and they show the same qualitative pattern as the ones described in the previous section. Notice that the difference in the return faced by the very wealthy and the poorest agents in the economy is only about 2.3 percentage points, a value that is consistent with the findings reported in section 3.3.

Figure 5 plots the Lorenz curve for the preferred parametrization and for the baseline case of constant interest rate, while Table 6 reports statistics about the wealth distri-
Figure 3: Decision Rules

The graph shows the relationship between current assets and next period assets, with decision rules indicated by lines $r$ and $r(a)$. The axes are labeled as follows:

- **Current assets**
- **Next period assets**

The graph is a straight line indicating a direct relationship between the two variables.
Figure 4: Return Functions (Preferred Parametrization)
Table 6: Wealth Inequality (Model and Data)

<table>
<thead>
<tr>
<th>Year</th>
<th>Gini</th>
<th>Top 1%</th>
<th>Top 5%</th>
<th>Top 10%</th>
<th>Top 20%</th>
<th>Bottom 40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1983</td>
<td>0.779</td>
<td>32.6</td>
<td>54.4</td>
<td>66.4</td>
<td>79.5</td>
<td>1.7</td>
</tr>
<tr>
<td>1992</td>
<td>0.802</td>
<td>35.9</td>
<td>58.3</td>
<td>69.9</td>
<td>82</td>
<td>1.2</td>
</tr>
<tr>
<td>R</td>
<td>0.474</td>
<td>4.4</td>
<td>17.1</td>
<td>29.5</td>
<td>48.7</td>
<td>9.6</td>
</tr>
<tr>
<td>R(a)</td>
<td>0.801</td>
<td>9.7</td>
<td>41.2</td>
<td>72.2</td>
<td>87.6</td>
<td>0.8</td>
</tr>
</tbody>
</table>

It is apparent from a look at Figure 5 that the small difference in returns faced by agents with different levels of assets is enough to generate a substantial amount of extra wealth inequality compared to the one implied by precautionary saving alone.

Table 6 makes this point clearer. The Gini index for wealth in the model with constant interest rate is 0.474 while the one of the model with increasing returns is 0.801. This value exactly matches the one computed for the U.S. economy based on the 1992 Survey of Consumer Finances and overpredicts the one for the year 1983. A closer look at the wealth distribution reveals that the share held by the 1 percent wealthiest portion of the population is about 35 percent in the data, it is a puny 4.4 percent in the baseline model and it is still only 9.7 percent in the preferred parametrization. Despite this failure at the very top of the wealth distribution, the preferred parametrization fares very well in the next percentiles. The share of the top 5 percent is 41.2 percent and the top 10 and 20 percent hold respectively 72.2 and 87.6 percent of assets in the preferred parametrization. The figures for the U.S. economy at the same percentiles are about 55, 69 and 80 percent respectively. The model’s output is then very close to real world data and actually overpredicts the share of the top 10 and 20 percent of the population. Finally it does a very good job at matching the extremely low share of the 40 percent poorest agents in the economy, with a model share of 0.8 percent, the same order of magnitude of the one observed for the U.S. economy. To assess the performance of the preferred parametrization it is also useful to point out that the baseline model with constant interest rate grossly underpredicts the share of all the top percentiles of the distribution while it generates a 9.6 percent figure for the bottom 40 percent, largely above the 1.2 or 1.7 percent observed in the data.

A look at the distribution of wealth, reported in Figure 6 helps understand the mechanics of the model. The figure reports the distribution for the baseline case of constant returns and for the current choice of parameters. Clearly the preferred model is better able at approximating the Pareto shape of the wealth distribution that we observe in
Figure 5: Lorenz Curves (Fixed Return and Preferred Case)
the data\textsuperscript{14}, with a large number of agents piled up at or very close to the borrowing constraint and a very elongated right tail.

A peculiar feature of the distribution generated by the model with increasing returns and not observed in the data, is that there are very few agents in an intermediate range of wealth, between 10 and 30, and then a larger group at the right of this interval. This second feature of the distribution can be interpreted by looking at the plot of the marginal return function and observing that it exhibits a spike at an asset level slightly greater than 20, where the return is above the subjective discount rate, and is substantially below that at low levels of wealth. Agents who get to accumulate enough wealth for precautionary reasons will then start to accumulate assets very fast and pile up at a higher wealth level, where the marginal return to savings converges to the lower marginal product of capital. As a consequence few agents at any point in time will hold an amount of assets in the region where rapid accumulation takes place. This also contributes to explaining why the perfect match of the Gini index is obtained by underestimating the share of the top 1 and 5 percent of the population and overestimating the one of the top 10 and 20 percent: within the group of lucky agents a little more inequality would be needed for a perfect match.\textsuperscript{15} For the parameters used convergence occurs around a level of assets of 50, that is about 9 times the average stock of capital, resulting in a wealth-income ratio for the top 1 percent of 7.3, still below the one in the data which is around 12, although much larger than the 4.06 of the constant return model.

The large number of agents with very little or no wealth at all is explained instead by the fact that those who get bad draws of the labor efficiency shock and therefore are forced to decumulate, will at some point do this much faster than in the constant return case because the rate of return that they face is well below their subjective discount rate, much more than it would be in the constant return case. This latter fact can be better appreciated by also looking at aggregate results. The equilibrium interest rate for the model with constant returns is 3.36 percent and by definition is the return faced by every agent in the economy; the return faced by poor agents in the preferred parametrization is instead only 1.12 percent.

Summarizing, the preferred parametrization shows that the hypothesis of increasing returns to savings is a powerful mechanism for matching the observed concentration of wealth quantitatively. It is worth stressing that this is a consequence not of the impact it

\textsuperscript{14}See Quadrini and Ríos-Rull (1997) for a plot of the wealth distribution in the US data.

\textsuperscript{15}This feature is a consequence of the functional form of the return function and not of the particular parameters chosen, as it will become more clear in the next section when a sensitivity analysis will be performed. A possible way out is to assume that different groups of the population face different return schedules so that the asset level where rapid accumulation occurs is different for different agents.
Figure 6: Wealth Distribution (Fixed Return and Preferred Case)
has on the wealth distribution per se, but of the fact that this impact is obtained with a small difference in the returns faced by wealth-poor and wealth-rich agents, a difference which in the model is about 2.5 percentage points, perfectly consistent with the empirical evidence. As a further support to the virtues of this model, if one interprets the amount of output lost in the model economy as cost of intermediation and compares the figure to the value added of the financial sector, which performs this intermediation role in actual economies, it turns out that the share for the model is 1.9 percent, the same order of magnitude of the figure for the U.S. economy of 4.02, reported in Díaz-Giménez et al. (1992).16

Once again it should be stressed that the goal of this paper is to show how a simple and empirically well documented fact, so far largely overlooked, is responsible for the largest part of the observed wealth concentration. This should be kept in mind when assessing the fact that the model underestimates the share of wealth held by the top 1 percent of the population. One should take into account both that the existence of increasing returns to savings is the only mechanism to generate wealth inequality in the model and above all that the earnings process used here is very simplified and very far from even approaching the observed earnings inequality. The associated Gini index is 0.049 and the share of the top 5 percent in total earnings is 5.97 percent compared to 0.46 and 19 percent in the U.S. data.

4.3.3 Sensitivity Analysis

In this section results for other choices of the parameters that define the return function are presented. The analysis is less detailed than the one in the previous section for the preferred parametrization and is confined to an examination of the wealth concentration statistics. The choice of the parameters is the following. I consider two different values of $a$, the inflection point of the return function, namely $a = 8$ and $a = 12$. Then, based on the fact reported in section 3.3, that the range of the empirical return schedule is between a little more than 3 percent and a little less than 4 percent I choose to run the simulations for values of the parameter $\nu$ of 0.03 and 0.04.17 Finally I consider two values

16Díaz-Giménez et al. (1992) report data on the fraction of the financial sector in total output for selected years in the US. That sector accounted for 2.7 percent of output in 1950 and has steadily grown to reach 5.6 percent at the end of the 80s. The figure reported in the main text is the average of those reported in the cited paper. The figures reported above and the ones in the model are of the same order of magnitude.

17This choice implies that the range of the return function $r(a)$ over the entire real line is an interval of length 0.04 or 0.03. In practice however, given the choice of the other parameters, the function gets very close to the lower asymptote only for negative values of assets. Given the tight borrowing constraint of 0 this implies that the actual observed range of the return function in the model is somewhat below the one in the data.
The results are reported in Table 7 and Table 8. The first fact that emerges from the tables is that the Gini coefficient ranges from a lowest 0.714 to a highest 0.835. Looking back at Table 1, the index for the U.S. economy was 0.779 in 1983 and 0.802 in 1992. Clearly the ability of the increasing return to savings function to generate a level of wealth concentration that is in line with the empirical evidence is a robust feature of the model considered here.

Looking at the share of different percentiles of the distribution it appears that the top 1 percent, oscillating between 6 percent and 9.9 percent is always below the corresponding figure of about 35 percent for the U.S. economy and also the share of the top 5 percent in the model, ranging between 24.9 percent and 42.3 percent is somewhat below the 55 percent figure found in the data. However the share held by the top 20 percent of the model population varies between 76 percent and 93.1 percent, values that are situated around the 81.9 percent share observed in the U.S. economy in 1992. The share of the bottom 40 percent in the model is once again very close to the one of data, ranging from 0.3 to 1.2 percent to be compared to the 1.2 percent observed in the U.S. economy.

Another important feature of the results of the sensitivity analysis is that if we fix the value of \( \nu \) and \( \bar{\pi} \) then results both in terms of the Gini coefficient and in term of the share of wealth held by the top 5 percent of the population improve substantially when \( \lambda \) is reduced from 0.25 to 0.125. The interpretation of these results is straightforward. The marginal return displays a spike at some level of assets slightly to the right of \( \bar{\pi} \). This spike marks a watershed in that those agents holding an amount of wealth below this level will face very low returns and accumulate almost no wealth at all. Once the critical level of wealth is reached agents will tend to accumulate very quickly until the marginal return to savings settles down to its asymptotic value. If \( \lambda \) is relatively high
Table 8: Wealth Distribution, a=8

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>Top 1</th>
<th>Top 5</th>
<th>Top 10</th>
<th>Top 20</th>
<th>Bottom 40</th>
<th>Gini index</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>4.4</td>
<td>17.1</td>
<td>29.5</td>
<td>48.7</td>
<td>9.6</td>
<td>0.474</td>
</tr>
<tr>
<td>$R(a)$:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda = 0.125$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu = 0.03$</td>
<td>8.2</td>
<td>34.3</td>
<td>60.0</td>
<td>82.9</td>
<td>1.2</td>
<td>0.757</td>
</tr>
<tr>
<td>$\nu = 0.04$</td>
<td>8.5</td>
<td>36.3</td>
<td>65.1</td>
<td>89.7</td>
<td>0.5</td>
<td>0.801</td>
</tr>
<tr>
<td>$\lambda = 0.25$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu = 0.03$</td>
<td>6.0</td>
<td>24.9</td>
<td>44.7</td>
<td>76.0</td>
<td>0.9</td>
<td>0.714</td>
</tr>
<tr>
<td>$\nu = 0.04$</td>
<td>6.3</td>
<td>26.7</td>
<td>48.3</td>
<td>82.8</td>
<td>0.3</td>
<td>0.753</td>
</tr>
</tbody>
</table>

then this convergence occurs very quickly so that a large number of agents will cluster in a relatively small interval of wealth holdings: the share of wealth of the lucky agents who reached the critical level of assets will be high compared to the rest of the population, but within this group the distribution will be fairly egalitarian. When $\lambda$ is reduced the marginal return function will converge more slowly maintaining the incentive to save at even higher level of assets; this creates more inequality also within the lucky part of the population and increase the share of a tiny minority even more.

Finally, an increase in the difference between the lowest and highest returns, keeping the other parameters fixed leads, as expected, to an increase in wealth inequality.

5 Conclusions

The economic fortunes of the households in real economies are very unequal with wealth being substantially more concentrated than other measures like income and earnings. This fact has attracted a lot of attention among macroeconomists. The basic framework of precautionary saving outlined in Aiyagari has proven capable of reproducing the fact reported above qualitatively, but not quantitatively leading to successive extensions that include different features, like social security, intergenerational links, entrepreneurship or heterogeneous preferences but retain the basic assumption that a single asset is available in the economy. This paper explicitly acknowledges the fact that in reality there is a menu of assets with different returns that households may use to carry out their saving plans and that there is a systematic positive relationship between asset holdings and the return to these holdings.

To accomplish this task I have considered a variant of Aiyagari’s (1994) model where I assume that agents face a return to their savings that is increasing in the level of assets they hold. This feature is able to increase substantially the level of wealth inequality
compared to the standard case of constant returns. As a matter of fact the model is able to match the concentration of wealth observed in the U.S. data as measured by the Gini index and by most quintiles of the distribution. The model still fails to match the huge fortunes that are accumulated by a few wealthy households at the very top (1 percent) of the distribution. This is hardly surprising though, given the fact that the stochastic process for the efficiency units of labor used here is very simplified and generates an earnings concentration that falls short of the one in the data by a large amount.
References


