A Model of Occupational Choice, Offshoring and Immigration

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June 2017

Abstract
This paper develops a two-country model of offshoring and immigration with occupational choice and endogenous firm productivity. Individuals in Home choose to become entrepreneurs or workers, whereas those in Foreign can only be employed as workers. Entrepreneurs produce output using a fixed set of tasks that can be performed locally or abroad. The model is used to investigate the impact of offshoring and immigration policies on occupational choice, task allocation, productivity, income inequality, and welfare in Home. The model predicts that pro-immigration policies increase the number of entrepreneurs, raises productivity, and improves the aggregate welfare. It also predicts that lowering offshoring costs generates job polarization, increases inequality between workers and entrepreneurs, and improves the aggregate welfare.

JEL Classification: F1, J2, J3, J6, L1
Keywords: Entrepreneurship, Immigration, Offshoring, Occupational Choice
1 Introduction

Cheaper labor in developing economies combined with lower trade barriers and improved communication channels over the last three decades have intensified offshoring among companies in the developed countries. During the same period, immigration to advanced countries has increased substantially. For example, according to the US census data, immigrants’ share of the total U.S. population rose from about 6 percent in 1980 to more than 13 percent in 2015. The trends in offshoring and immigration have been hotly debated among policy makers and academics due to their economic and social implications.

This paper develops a two-country, general equilibrium model of task trade to investigate the impact of immigration and offshoring on occupational choice, firm productivity, income inequality, and welfare. The model has three important features. First, individuals choose to become entrepreneurs or workers depending on their abilities. Second, tasks are tradable as in Grossman and Rossi-Hansberg (2008), and each task can be performed by native workers, immigrant workers, or offshore workers. Third, entrepreneurs can improve their firm productivity by investing in managerial capital.

The model predicts that making immigrants more integrated to Home induces them to perform more complex tasks.¹ Since immigrants are more intensively used by non-offshoring firms, this policy will make non-offshoring firms relatively more profitable; as a result, the number of non-offshoring entrepreneurs increases and that of offshoring ones decreases. Under this policy, non-offshoring (offshoring) firms acquire more (less) managerial capital, and thus become more (less) productive. Finally, making immigrant workers more integrated to Home reduces the income inequality between workers and entrepreneurs, and improves the aggregate welfare. The model also predicts that increasing the number of immigrants has several desirable effects on the Home economy. It increases the mass of entrepreneurs, induces them to improve their firm productivity, lowers the inequality between workers and entrepreneurs, and improves the aggregate welfare.

¹Our analysis mainly focuses on the impact of immigration and offshoring policies on Home economy, where entrepreneurs can offshore their tasks. There is no entrepreneurial activity in Foreign, which mainly performs tasks for firms in Home.
Reducing variable offshoring costs increases the set of tasks performed by offshore workers by downgrading tasks performed by immigrant workers and upgrading tasks performed by native workers. This policy generates job polarization by increasing the mass of workers and offshoring entrepreneurs at the expense of the moderately skilled entrepreneurs. Under this policy, offshoring firms acquire more managerial capital (hence, have higher firm productivity), whereas non-offshoring firms acquire less managerial capital (hence, have lower firm productivity). Reducing variable offshoring costs increases the income inequality between entrepreneurs and workers, but improves aggregate welfare.

These results are generally consistent with recent empirical studies. The prediction that making immigrants more integrated to Home induces task upgrading of immigrant workers is consistent with Ottaviano et al. (2013). The finding that immigration increases the set of entrepreneurs enjoys support from Olney (2013) who finds that immigration has a positive impact on the number of establishments in US cities, and the effect is stronger among small establishments. The prediction that immigration has a positive impact on productivity is supported by Peri (2012) who shows that immigration does not crowd out employment of natives, while having a strong, positive effect on productivity.

Barba Navaretti et al. (2008) find that offshoring is more prevalent among larger and more productive firms in Italy. Similarly, using Chilean plant-level data, Kasahara and Lapham (2013) find that firms importing intermediate goods tend to be larger and more productive. The finding that lowering variable offshoring costs leads to task upgrading of native workers and task downgrading for immigrants in offshoring firms finds support from Ottaviano et al. (2013). The finding that reducing offshoring costs induces firms to import more intermediate goods is consistent with Goldberg et al. (2010) who find that lower input tariffs increased new input varieties in India. Using Hungarian firm-level data, Halpern et al. (2015) find that using more imported inputs increases firm productivity significantly.

This paper is related to recent literature that explores task trade introduced by Grossman and Rossi-Hansberg (2008). Baldwin and Robert-Nicoud (2014) develop a model with trade in goods and trade in tasks to analyze to what extent gains from trade and theorems in the Heckscher-Ohlin model change. Groizard et al. (2014) incorporate offshoring into Melitz’s
(2003) model to study the impact of offshoring on unemployment. Egger et al. (2015) present
a monopolistic-competition model of trade with occupational choice, and find that an exposure
to offshoring may lead to a welfare loss. Unel (2016) develops a small-open-economy model
of offshoring with unemployment to study the impact of credit constraints on offshoring and
unemployment.2

There are relatively few papers that have studied immigration and offshoring in a unified
framework. Barba Navaretti et al. (2008) develop a simple offshoring model to investigate the
impact of offshoring on native and immigrant workers using firm-level data from Italy.3 They
find that offshoring decreases the demand for native and immigrant workers. Olney (2012)
develops a partial equilibrium model to compare the impact of offshoring and immigration on
wages of U.S. native workers, and finds that offshoring has a more positive impact on low-skilled
wages than immigration. He assumes that immigrants are identical to natives in performing
tasks, and the supply of skilled and unskilled workers is exogenously fixed. Mandelman and
Zlate (2014) develop a stochastic growth model where offshoring and immigration can jointly
generate job polarization. Their model has two large symmetric countries and a small country
that is the source of unskilled immigrants. Furthermore, tasks are produced using only skilled
workers and there is a two-way offshoring between the two large economies.

Another related paper in this literature is Ottaviano et al. (2013) who, using data on the US
manufacturing industries over 2000–2007, investigate how declines in offshoring and immigration
costs affect the employment of native workers. They find that lowering offshoring costs leads to
task upgrading of natives and task downgrading of immigrants, whereas a reduction in immi-
gration costs leads to task upgrading of immigrants but has no effect on the task complexity of
natives. For their empirical analysis, they develop a partial equilibrium model where supply of
workers are fixed (i.e., no occupational choice), firms in each sector are identical (i.e., no firm
heterogeneity), and offshoring does not incur any fixed costs.

The rest of this paper is organized as follows. The next section introduces the model and

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2This paper also complements the literature that incorporates occupational choice into trade models. See, for
among many others.

3Although they empirically investigate the impact of offshoring on immigrant workers, their model does not
explicitly considers immigrants.
discusses its equilibrium properties. Section 3 presents a series of comparative static exercises to study the impact of offshoring and immigration policies on occupational choice, firm productivity, income inequality, and welfare. Section 4 concludes the paper.

2 Setup of the Model

There are two countries (Home and Foreign) that consume two homogeneous goods produced in perfectly competitive markets. Home is populated by natives and immigrants with total mass \(1 + M\), where \(M < 1\) is the size of immigrant workers. Natives differ with respect to their managerial ability, and choose to become entrepreneurs or workers; whereas immigrants always employ as workers.\(^4\) Home produces two homogeneous goods using labor as the only factor of production. Good 1 is produced locally using only workers, whereas good 2 is produced by a continuum of entrepreneurs each using her managerial ability and a set of tasks that can be performed locally or abroad.

Foreign is populated by identical workers with constant mass \(L^*\). Foreign produces good 1 and performs tasks for Home’s firms in sector 2 (and thus, good 2 is not produced there). In my subsequent analysis, all variables related to Foreign are denoted by an asterisk, and for simplicity, I will only show expressions for Home when this causes no confusion.

2.1 Preferences

Individuals’ preferences are described by the following Cobb-Douglas utility function

\[
 u = \left( \frac{q_1}{1 - \theta} \right)^{1-\theta} \left( \frac{q_2}{\theta} \right)^{\theta},
\]

where \(q_i\) is consumption of good \(i = 1, 2\), and \(\theta \in (0, 1)\) is an exogenous, constant parameter.

Let \(e\) denote individual expenditure. The demand for each good is given by

\[
 q_1 = (1 - \theta)e/p_1, \quad q_2 = \theta e/p_2,
\]

\(^4\)Since one of the main points of this paper is to explore how the existence of low-skill immigrants affects natives’ income distribution and welfare, I omit the possibility that immigrants can also be entrepreneurs. Extending the model by allowing heterogeneity among immigrants is left for a future study.
where $p_i$ is the price of good $i$. Good 1 is chosen as the numeraire by setting its price equal to one (i.e., $p_1 = 1$), and for notational simplicity, the price of good 2 is denoted by $p$.

Substituting $q_i$ from (2) into (1) yields the indirect utility

$$v(e, p) = p^{-\theta}e,$$

and aggregating (3) across all individuals yields $V(p) = p^{-\theta}E$, where $E$ denotes aggregate expenditure (income).

### 2.2 Production

Good 1 is competitively produced using only workers. Production of one unit of good 1 in Home requires one unit of native worker or $1/w_m$ units of immigrant workers, where $w_m < 1$ is an exogenous constant. Since markets are competitive, the wage rates of native and immigrant workers are $w = 1$ and $w_m$, respectively. In Foreign, production of one unit of good 1 requires $1/w^*$ units of worker, where $w^* < 1$ is an exogenous constant. It is assumed that goods are freely traded between two countries, which implies that the foreign wage rate equals $w^*$.

Good 2 is produced only in Home by a continuum of heterogeneous firms, each run by an entrepreneur under perfect competition. Entrepreneurs differ with respect to their managerial capital (or firm productivity), and an entrepreneur with managerial capital $z$ produces output according to

$$y_2(z) = \left(\frac{z}{1-\eta}\right)^{1-\eta} \left(\frac{L}{\eta}\right)^{\eta},$$

where $\eta \in (0, 1)$ is an exogenous parameter that measures the labor share in production and $L$ is a composite labor. The restriction $\eta \in (0, 1)$ ensures that firms have finite size, and thus $\eta$ also measures managers’ span of control (Lucas, 1978). The composite labor $L$ is produced by assembling a set of differentiated tasks as follows:

$$L = \exp \left[ \int_{0}^{1} \ln l(j) dj \right],$$

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5The conclusion $w_m < w$ is consistent with several empirical studies that have shown that, ceteris paribus, immigrants earn less than natives (Butcher and Dinardo 2002, Ottaviano et al. 2013, Beland and Unel 2016). Some studies argue that the wage gap between immigrants and natives reflects immigrants’ lack of linguistic skills, social capital, and unfamiliarity with the institutional rules (Lessem and Sanders 2017).
where $l(j)$ is the amount of task $j$ used in assembly.\footnote{For notational simplicity, it is assumed that tasks are aggregated through the Cobb-Douglas technology. Results qualitatively remain the same if one uses the CES function instead.}

Tasks are produced using only workers under constant returns to scale technology. As in Ottaviano et al. (2013), each task can be performed in one of the following three ways: production by native workers, production by immigrant workers, or production by workers in Foreign. Production of one unit of any task requires one unit of native workers, and thus the unit cost of production of any task by a native worker is one, i.e. $c(j) = 1$.

If task $j$ is performed by immigrant workers, production of one unit of the task requires $\beta_m t_m(j)/w_m$ units of immigrant workers, where division by $w_m$ is done for notational simplicity.\footnote{Ottaviano et al. (2013) assume production of one unit of task $j$ requires $t_m(j)$ units of immigrant, and an immigrant worker endowed with one unit of labor is able to provide only $1/\delta$ units of labor in Home. Clearly, $\delta$ and $\beta_m$ in the present model capture the same effects.} It is assumed that $\beta_m t_m(j)/w_m > 1$ for all $j \in [0, 1]$ so that tasks are performed more efficiently by native workers. The parameter $\beta_m$ captures immigrants' overall state of communication with native workers and managers as well as their familiarity with institutional rules in this sector. Function $t_m(j)$ measures heterogeneity across tasks, and it is assumed that tasks are indexed in increasing order of complexity, i.e. $dt_m(j)/dj > 0$. Since the wage paid to immigrants is $w_m$, the marginal cost of performing task $j$ by immigrant workers is $c(j) = \beta_m t_m(j)$.

Entrepreneurs wishing to offshore face both fixed and variable costs of offshoring. They first must pay an irreversible fixed costs $f_o$ measured in terms of good 1. The fixed cost $f_o$ covers foreign market entry costs as well as coordinating the performance of tasks to be produced abroad. Upon paying $f_o$, performing one unit of the task $j$ requires $\beta_o t_o(j)/w^*$ units of foreign workers. The parameter $\beta_o$ reflects the overall state of communication technology between the two countries as well as variable trade costs such as transport and tariffs, and $t_o(j)$ captures the heterogeneity in productivity across tasks. It is again assumed that $t_o(j)$ is continuously differentiable with $dt_o(j)/dj > 0$. It then follows that the marginal cost of performing task $j$ in Foreign is $c(j) = \beta_o t_o(j)$.

**Assumption 1.** The following conditions hold.

i. $\beta_m t_m(0) < \beta_o t_o(0) < 1$ and $1 < \beta_o t_o(1) < \beta_m t_m(1)$;
ii. $\beta_m t_m(j)$ and $\beta_o t_o(j)$ intersect only once at some $j_1 \in (0, 1)$ such that $\beta_m t_m(j) \leq \beta_o t_o(j) < 1$ for all $j \leq j_1$.

If a firm does not offshore, it divides its tasks between immigrants and natives. In this case, as shown in Figure 1, all tasks with $j < j_2$ will be performed by immigrants, and the rest by natives. The marginal cost of producing a task is given by

$$c_d(j) = \begin{cases} 
\beta_m t_m(j) & j \in [0, j_2), \\
1 & j \in [j_2, 1]. 
\end{cases}$$

(6)

Using the above cost function, the cutoff $j_2$ is given by

$$t_m(j_2) = \frac{1}{\beta_m},$$

(7)

and thus the task cutoff $j_2$ decreases as the parameter $\beta_m$ increases. In addition, the cutoff $j_2$ is independent of the managerial capital $z$, i.e. the set of tasks performed by immigrants is the same for all entrepreneurs producing domestically.

Assumption 1 implies that for an offshoring firm tasks are divided into three groups as shown in Figure 1. All tasks with $j < j_1$ are performed by imm immigrants, tasks with $j \in [j_1, j_3)$ by foreign
workers, and tasks with $j \geq j_3$ by natives. Thus, the marginal cost of producing a task is given by
\[
c_o(j) = \begin{cases} 
\beta_m t_m(j) & j \in [0, j_1), \\
\beta_o t_o(j) & j \in [j_1, j_3), \\
1 & j \in [j_3, 1]. 
\end{cases}
\tag{8}
\]
Using this cost distribution, the cutoffs $j_1$ and $j_3$ are given by
\[
t_o(j_1) = \frac{\beta_m}{\beta_o}, \quad t_o(j_3) = \frac{1}{\beta_o}. \tag{9}
\]
Given that $t'_o(j) > 0$, the cutoff $j_3$ decreases as the variable offshoring cost $\beta_o$ increases. Since $t'_m(j_1) > t'_o(j_1)$ as shown in Figure 1, the task cutoff $j_1$ decreases in $\beta_m$ and increases in $\beta_o$. Note that both cutoffs are independent of the fixed offshoring cost $f_o$ and the managerial capital $z$.

Lemma 1 summarizes these results (see Appendix A.1 for formal proofs).

**Lemma 1.** Consider Home just described.

i. $dj_1/d\beta_m < 0$, $dj_2/d\beta_m < 0$, and $dj_3/d\beta_m = 0$;

ii. $dj_1/d\beta_o > 0$, $dj_2/d\beta_o = 0$, and $dj_3/d\beta_o < 0$;

iii. $dj_i/dz = 0$ and $dj_i/df_o = 0$ for $i = 1, 2, 3$.

The division of tasks across worker types differs from Ottaviano et al. (2013) in one important aspect. Their model does not include firm heterogeneity or fixed offshoring costs (since they empirically investigate the effects of immigration and offshoring on the average task assignments across worker types). Consequently, all firms offshore and tasks are performed by three different worker groups as discussed above. In the present set-up, firm heterogeneity and fixed offshoring costs will induce only more productive firms to offshore; as a result, there will be non-offshoring firms that only use native and immigrant workers to perform their tasks.

I now turn to discuss formation of managerial capital $z$. If an individual chooses to become an entrepreneur, she can invest to improve the managerial capital (firm productivity). Following insights from the human capital theory (Becker 1994, Acemoglu 1996), I assume that an entrepreneur with managerial ability $a$ must spend $\lambda z^2/2a$ units of good 1 to acquire $z$ units of
managerial capital, where $\lambda > 0$ is an exogenous parameter. The costs of managerial capital decline with the level of ability and increase with the level of managerial capital. Ability levels are drawn from a common, exogenous cumulative distribution $G(a)$ with density $g(a)$ and support $[1, \infty)$. More specifically, following Helpman et al. (2010), I assume that ability levels follow the following Pareto distribution:

$$G(a) = 1 - a^{-k},$$

where $k$ is the shape parameter. To ensure variables have finite values I assume that $k > 1$.

An entrepreneur with managerial ability $a$ maximizes her earnings $e_2(a)$ by choosing the amount of task $l(j)$ and whether to offshore:

$$e_2(a) \equiv \max_{I_o, l(j), z} \left\{ p y_2(z) - \int_0^1 c(j)l(j) dj - I_o f_o - \frac{\lambda z^2}{2a} \right\},$$

where $y_2(z)$ is given by (4) and $I_o$ is an indicator function that equals one if the firm offshores, and zero otherwise.

Let $L_d$ and $L_o$ denote composite labor inputs used by non-offshoring and offshoring firms, respectively; and $W_d$ and $W_o$ denote aggregate wages associated with these labor inputs so that $W_v L_v = \int_0^1 c_v(j) l_v(j) dj$ (hence, $W_v$ denotes the unit composite-labor cost). Solving the cost minimization problem yields

$$W_d = \exp \left[ \int_{j_1}^{j_2} \ln[\beta_m t_m(j)] dj \right],$$

$$W_o = \exp \left[ \int_{j_1}^{j_2} \ln[\beta_m t_m(j)] dj + \int_{j_1}^{j_3} \ln[\beta_o t_o(j)] dj \right].$$

Note that $W_d$ is the same for all non-offshoring firms and $W_o$ is the same for offshoring ones. Further, offshoring firms face a lower aggregate wage than domestic firms, i.e. $W_o < W_d$, which creates an incentive for firms to offshore.  

Since markets are perfectly competitive, production technology (4) implies that $y_2v(z) = \frac{W_v L_v}{(\eta p)}$, and substituting this back into (4) yields

$$L_v = \frac{\eta z}{1 - \eta} \left( \frac{p}{W_v} \right)^{\frac{1}{1-\eta}}, \quad y_2v(z) = \frac{z}{1 - \eta} \left( \frac{p}{W_v} \right)^{\frac{\eta}{1-\eta}},$$

To see this, note that $W_o < W_d$ iff $\int_{j_1}^{j_2} \ln[\beta_v t_v(j)] dj + \int_{j_1}^{j_3} \ln[\beta_o t_o(j)] dj < \int_{j_1}^{j_2} \ln[\beta_m t_m(j)] dj$, which directly follows from $\beta_m t_m(j) > \beta_o t_o(j)$ for $j \in [j_1, j_2]$ and $\ln[\beta_o t_o(j)] < 0$. 

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where the second expression is obtained by substituting \( L_v \) back into (4). Substituting \( y_{2v}(z) \) from (13) and \( W_v L_v = \eta y_{2v}(z) \) into profit function (11), and then maximizing the resulting expression with respect to \( z \) yields

\[
\begin{align*}
z_v(a) &= \frac{a}{\lambda} \left( \frac{p}{W_v^\eta} \right)^{\frac{1}{1-\eta}}, \quad v = d, o.
\end{align*}
\]

(14)

Finally, substituting \( z \) back into profit function (11) yields

\[
\begin{align*}
e_{2v}(a) &= \frac{a}{2\lambda} \left( \frac{p}{W_v^\eta} \right)^{\frac{2}{1-\eta}} - \mathbb{I}_o f_o.
\end{align*}
\]

(15)

### 2.3 Occupational Choice

I begin my analysis by considering allocation of ability. A native individual chooses to become an entrepreneur if her entrepreneurial income is greater than the wage that she earns as a worker, i.e. \( e_2(a) \geq w = 1 \). The ability cutoff (denoted by \( a_d \)) at which an individual is indifferent between being an entrepreneur and a worker is given by \( e_{2d}(a_d) = 1 \). Using (15) with \( \mathbb{I}_o = 0 \) and \( v = d \) yields

\[
\begin{align*}
a_d &= 2\lambda \left( \frac{W_d^\eta}{p} \right)^{\frac{2}{1-\eta}}.
\end{align*}
\]

(16)

An entrepreneur chooses to offshore if \( e_{2o}(a) \{ \mathbb{I}_o = 1 \} \geq e_{2d}(a) \{ \mathbb{I}_o = 0 \} \), where \( e_{2v}(a) \) is given by (15). The ability cutoff level (denoted by \( a_o \)) necessary for offshoring is determined when the inequality holds with equality, which yields

\[
\begin{align*}
a_o &= \frac{2\lambda f_o}{(W_d/W_o)^{2\eta/(1-\eta)} - 1} \left( \frac{W_d^\eta}{p} \right)^{\frac{2}{1-\eta}},
\end{align*}
\]

(17)

where \( W_d \) and \( W_o \) are given by (12a) and (12b), respectively. Combining equations (16) and (17) yields

\[
\begin{align*}
a_o &= A a_d, \quad A = \frac{f_o}{(W_d/W_o)^{2\eta/(1-\eta)} - 1}.
\end{align*}
\]

(18)

It is assumed that \( f_o \) is sufficiently high so that \( A > 1 \); as a result, only more able entrepreneurs offshore.

With these ability cutoffs, native workers in Home are divided into three groups: those with \( a < a_d \) become workers; those with \( a \in [a_d, a_o) \) become entrepreneurs and produce domestically;
and finally, those with \( a \geq a_o \) become entrepreneurs and offshore the same set of tasks to Foreign. All immigrants are workers. Using these cutoffs, the managerial capital, firm revenue, and entrepreneurial income can be expressed as follows:

\[
\begin{align*}
    z_v(a) &= a \left( \frac{2}{\lambda a_d} \right)^{\frac{1}{\eta}} \left( \frac{W_d}{W_v} \right)^{\frac{a}{1-\eta}}, \\
    r_{2v}(a) &= \frac{2a}{(1-\eta)a_d} \left( \frac{W_d}{W_v} \right)^{\frac{2\eta}{1-\eta}}, \\
    e_{2v}(a) &= \frac{a}{a_d} \left( \frac{W_d}{W_v} \right)^{\frac{2\eta}{1-\eta}} - I_o f_o,
\end{align*}
\]

where \( W_v \) is the aggregate price index given by (12).

\[9\]

2.4 Income Distribution and Welfare

Income of a native worker is \( w = 1 \) and the entrepreneurial income is given by (19c). It then follows that the income distribution across natives in Home is given by

\[
e(a) = \begin{cases} 
1 & \text{if } a < a_d, \\
\frac{a}{a_d} & \text{if } a \in [a_d, a_o), \\
\frac{aW_d^{2\eta/(1-\eta)}}{a_dW_o^{2\eta/(1-\eta)}} - f_o & \text{if } a \geq a_o.
\end{cases}
\]

Using \( v(a, p) = p^{-\theta}c(a) \) together with (20) yields the welfare distribution in Home. The income of an immigrant or offshore worker is \( w_m \), and thus her welfare is given by \( v_m = w_m p^{-\theta} \).

With this distributional assumption, one can calculate the average entrepreneurial income to make between-group comparisons. Using (20) and (10), it is easy to show that the average entrepreneurial income is given by

\[
\bar{e}_2 = \frac{k + f_o A^{-k}}{k-1}, \quad \bar{e}_{2o} = \frac{f_o + kA}{k-1},
\]

where \( A \) is given by (18) and \( \bar{e}_{2o} \) is the average income of offshoring entrepreneurs. In addition, aggregating \( e(a) \) across all individuals yields the following aggregate income

\[
E = 1 + w_m M + \frac{1 + f_o A^{-k}}{(k-1)a_d^k},
\]

\[9\] For example, combining equations (12) and (16) yields \( z(a) \); and combining (15) and (16) yields \( e_2(a) \).
where $M$ is the number of immigrants, and $A$ is given by (18). Aggregate income in Foreign is given by $E^* = w^*L^*$ because all individuals employ as workers with wage rate $w^*$.

Home’s aggregate welfare is given by $V = p - \theta E$, and Foreign’s welfare is given by $V^* = p - \theta E^*$. However, from a policy perspective it is more interesting to investigate how immigration and offshoring policies affect Home’s national welfare, defined as the aggregate welfare of natives in Home:

$$V_n = p - \theta + (1 + f_oA - k)p - \theta(k - 1)a_kd,$$

(23) where $p$ and $a_d$ are related with each other through equation (16).

2.5 The Open-Economy Equilibrium

In equilibrium, the total (net) supply of each good must be equal to the total demand for the good. Let $Q_2$ and $Q_2^*$ denote the total quantity demanded for good 2 in Home and Foreign, respectively; it then follows that $Q_2 + Q_2^* = Y_2$, since Foreign does not produce good 2.\(^{10}\) The total amount of good produced in sector 2 is given by $Y_2 = \int y_2(a)g(a)da$, and using $y_2(a)$ from (19b) yields

$$Y_2 = \frac{2k(1 + f_oA^{-k})}{(k - 1)(1 - \eta)a^d_d},$$

(24) where $A$ is given by (18).

Demand function (2) implies that $pQ_2 = \theta E$, where $E$ is the aggregate income in Home given by (22). As a result,

$$p(Q_2 + Q_2^*) = \theta(E + E^*),$$

(25) where $Q_2^*$ and $E^*$ are the corresponding aggregate variables for Foreign. Substituting $Q_2 + Q_2^* = Y_2$ into this equation, and using (22) and (24) yields

$$a_d = \left\{ \frac{[2k - \theta(1 - \eta)](1 + f_oA^{-k})}{\theta(k - 1)(1 - \eta)(1 + Mw_m + w^*L^*)} \right\}^{\frac{1}{k}}.$$ 

(26)

Lemma 2. The ability cutoff $a_d$ is uniquely determined and given by equation (26).

\(^{10}\)For good 1, the corresponding equation is $Q_1 + Q_1^* = Y_1 - C_1 + Y_1^*$, where $C_1$ is the amount of good 1 used in formation of managerial capital and foreign-market entry (see Appendix A.2 for calculation of $C_1$).
Once the ability cutoff \( a_d \) is determined, one can easily determine other endogenous variables. For example, substituting \( a_d \) into (16) yields

\[
p = \left\{ \frac{\theta(k - 1)(1 - \eta)(1 + w_m M + w^* L^*)(2\lambda)^k W_d^{2k\eta/(1 - \eta)}}{[2k - \theta(1 - \eta)](1 + f_o A^{-k})} \right\}^{\frac{1 - \eta}{2k}},
\]

where \( W_d \) and \( A \) are given by (12a) and (18), respectively. Substituting \( a_d \) into (17) yields the ability cutoff \( a_o \). Finally, substituting \( a_d \) into (22) yields

\[
E = \frac{2k(1 + w_m M) + \theta(1 - \eta)w^* L^*}{2k - \theta(1 - \eta)}.
\]

Thus, the aggregate income \( E \) is independent of \( \beta_m \) and offshoring costs \( \beta_o \) and \( f_o \), which imply that the impact of a change in these variables on aggregate welfare \( V = Ep^{-\theta} \) only works through their impact on the relative price \( p \).

### 3 Immigration and Offshoring Policies

This section presents a series of comparative static exercises to investigate implications of immigration and offshoring policies for occupational choice, firm productivity, income inequality, and welfare in Home.

#### 3.1 Immigration Policies

There are two types of offshoring policies that I will investigate: a reduction in \( \beta_m \) and an increase in the number of immigrants \( M \). As discussed earlier, a reduction in \( \beta_m \) can be interpreted as improving immigrants’ communication channels and skills with managers as well as making them more familiar with rules in Home. According to Lemma 1, a reduction in \( \beta_m \) increases the cutoffs \( j_1 \) and \( j_2 \), but has no impact on \( j_3 \). Thus, the set of tasks performed by immigrants increases, but that performed by foreign workers decreases.

A reduction in \( \beta_m \) intuitively decreases the unit-labor costs measured by \( W_d \) and \( W_o \). Since markets are perfectly competitive, the relative price \( p \) falls as well. As formally shown in Appendix A.3, reduction in the aggregate wage index \( W_d \) is more substantial than that in the

\(^{11}\)Appendix A.2 also shows the amount of native, immigrant, and foreign labor used by sector 2.
relative price $p$, and thus the ability cutoff $a_d$ decreases. Further, reduction in $W_d$ is greater than that in $W_o$ (i.e., the unit-cost of producing domestically becomes cheaper relative to offshoring), as a result, some offshoring firms choose to produce locally, i.e. $a_o$ increases.

**Lemma 3.** In the open-economy equilibrium, $da_d/d\beta_m > 0$ and $da_o/d\beta_m < 0$.

Using the result in Lemma 3, equations (19a) and (19c) imply that reducing $\beta_m$ increases managerial capital $z$ and entrepreneurial income $e(a)$ of non-offshoring entrepreneurs. As the relative price of good 1 raises ($p \downarrow$), acquiring managerial capital becomes more expensive; consequently, these entrepreneurs have less incentive to acquire managerial capital as shown in equation (14). However, reducing $\beta_m$ reduces the unit-labor cost $W_d$ as well, and this effect dominates the former one; as a result, non-offshoring entrepreneurs acquire more managerial capital. In the case of offshoring entrepreneurs, the reduction in the unit-labor cost $W_o$ is not as substantial as the fall in $p$; consequently, they acquire less managerial capital and generate lower entrepreneurial income as shown in Figure 2.a.\textsuperscript{12}

Reducing $\beta_m$ increases $A$; as a result, equations in (21) imply that this policy reduces the inequality between entrepreneurs and workers, while increasing that between offshoring en-

\textsuperscript{12}Differentiating $\tilde{e} = (a/a_d)(W_d/W_o)^{2\eta/(1-\eta)}$ with respect to $\beta_m$ and using $da_d/d\beta_m$ and $d(W_d/W_o)/d\beta_m$ from Appendix A.2 yields

\[
\frac{d\tilde{e}}{d\beta_m} = \frac{2\eta(j_2 - j_1)(1 - A^{1-k})\tilde{e}}{(1 - \eta)(1 + f_e A^{-k}) \beta_m} > 0,
\]

since $A^{1-k} < 1$. 

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**Figure 2:** Impact of Reducing $\beta_m$ on Income and Welfare Distributions
entrepreneurs and workers. The inequality between all entrepreneurs and workers decreases because of a substantial increase in the mass of entrepreneurs \((a_d \downarrow)\), and the inequality between offshoring entrepreneurs and workers increases because the number of offshoring entrepreneurs decreases \((a_o \uparrow)\).

A reduction in \(\beta_m\) makes all workers better off because of a decline in the relative price \(p\). The welfare impact of this policy on entrepreneurs is ambiguous as shown in Figure 2.b: the welfare of more able entrepreneurs (i.e., \(a > \bar{a}\)) decreases, while that of others increases. However, the national welfare \(V^n\) increases, because the aggregate income remains unchanged and the relative price \(p\) decreases.

**Proposition 1.** Consider the two trading economies as described. A reduction in \(\beta_m\) has the following effects on Home. It

- **a.** increases (decreases) the set of tasks performed by immigrants (foreign workers);
- **b.** increases (decreases) the mass of non-offshoring (offshoring) entrepreneurs;
- **c.** increases (decreases) the productivity of non-offshoring (offshoring) firms;
- **d.** decreases (increases) the inequality between workers and all (offshoring) entrepreneurs;
- **e.** improves the aggregate welfare.

The finding that a reduction in \(\beta_m\) increases the set of tasks that immigrants perform is consistent with Ottaviano et al. (2013) who, using employment data on immigrants, natives, and offshore workers from the US over 2000–2007, find that easier immigration leads to task upgrading of immigrants.\(^{13}\) They also find that lower immigration costs have no impact on the average task complexity of native workers. In the present model, this policy has no impact on native tasks in offshoring firms, while pushing natives toward more complex tasks in non-offshoring firms. The analysis then suggests that when the share of offshoring firms in an industry

\(^{13}\)In their a small open economy model, Ottaviano et al. assume that an immigrant worker endowed with one unit of labor is able to provide only \(1/\delta\) units of labor in Home. Parameter \(\delta\) captures the cost of immigration, and reducing \(\delta\) means immigration becomes easier. The no-arbitrage condition implies that an immigrant’s wage rate in Home equals \(w_m = w^* \delta\), and the unit cost of production is \(w^* \delta t_m(j)\). Clearly, \(\delta\) and \(\beta_m\) capture the same effects.
rises, the impact of easier immigration will have a limited effect on the average complexity of native tasks. Ottaviano et al. consider only manufacturing industries where offshoring has increased dramatically.

I now turn to investigate the impact of immigration \((M \uparrow)\) on the Home economy. According to Lemma 1, an increase in the mass of immigrants \(M\) has no impact on any of task cutoffs; as a result, aggregate wage indexes \(W_d\) and \(W_o\) do not change. However, an increase in \(M\) puts a downward pressure on the real wage, and thus the relative price \(p\) increases, which in turn makes entrepreneurship more profitable.\(^{14}\) In this case, more able individuals choose to become entrepreneurs \((a_d \downarrow)\). Since the relative price of good 1 becomes cheaper, offshoring also becomes more profitable, and thus the ability cutoff \(a_o\) decreases as well.

**Lemma 4.** *In the open-economy equilibrium, \(da_d/dM < 0\) and \(da_o/dM < 0\).*

An increase in \(M\) increases the mass of entrepreneurs in both countries and the mass of offshoring entrepreneurs in Home. Since good 1 becomes relatively cheaper, entrepreneurs acquire more managerial capital (i.e., higher firm productivity) and generate higher income as shown in Figure 3.a. Although income of each existing entrepreneur increases, this policy has no effect on the income inequality between entrepreneurs and workers in either country (see equations in (21)). The increase in the supply of entrepreneurs with lower income completely cancels out the increase in the average income of the existing entrepreneurs.

Since an increase in \(M\) increases the relative price \(p\), it makes all workers and some entrepreneurs worse off as shown in Figure 3.b. However, the positive effect on more able entrepreneurs is strong enough to overcome the negative impact on others; as a result, Home’s national welfare increases.\(^{15}\) Clearly, the increased price of good 2 lowers Foreign welfare.

**Proposition 2.** *Consider the two trading economies as described. An increase in the mass of immigrants \(M\) has the following effects on Home. It*

\(^{14}\)Since \(W_d\) and \(W_o\) remain unchanged, an increase in \(M\) does not have any effect on \(A\); as a result, it will increase the relative price \(p\) as indicated by eq. (27).

\(^{15}\)Substituting \(a_d\) and \(p\) from (26) and (27), respectively, into (23), and differentiating the resulting expression with respect to \(M\) easily yields \(dV^n/dM > 0\).
Figure 3: The Impact of Increasing $M$ on Income and Welfare Distributions

- $a$. has no effects on task-allocation across different workers;
- $b$. increases the mass of all types of entrepreneurs;
- $c$. increases the productivity of all firms;
- $d$. does not affect the inequality between workers and entrepreneurs;
- $e$. improves the national welfare.

The results generally enjoy support from several empirical studies. Ottaviano et al. (2013) show that more immigration does not have a significant impact on the task complexity of native workers. However, they also find that more immigration induces immigrants to upgrade their tasks as they replace the tasks of offshore workers. Olney (2013), using data on immigration and the establishments in US cities, finds that immigration has a positive impact on the number of establishments, and the effect is stronger among small establishments. The prediction that immigration has a positive impact on productivity is supported by Peri (2012) who analyzes the long-run impact of immigration on employment and productivity in the US. Using the distance from the Mexican border as an instrument, he finds that immigration does not crowd out employment of natives, while having a strong, positive effect on productivity.
3.2 Offshoring Policies

In this section, I investigate how further exposure to offshoring through a reduction in the variable offshoring cost $\beta_o$ affects the Home economy.\textsuperscript{16} According to Lemma 1, a reduction in $\beta_o$ increases the set of tasks offshored, but has no impact on the set of tasks performed by immigrant workers at non-offshoring firms. Consequently, the aggregate wage index $W_o$ decreases, but $W_d$ remains the same. Note that native (immigrant) workers employing at offshoring firms now perform more (less) complex tasks.

Reducing $\beta_o$ clearly makes offshoring more profitable, which in turn induces more entrepreneurs to offshore (i.e., $a_o \downarrow$). Since offshoring entails fixed entry costs, the increased demand for good 1 by firms wishing to offshore increases the relative price of good 1 (i.e., $p \downarrow$), and thus forces less able entrepreneurs to become workers (i.e., $a_d \uparrow$). In sum, reducing the variable offshoring cost $\beta_o$ decreases the mass of entrepreneurs, but increases the mass of offshoring entrepreneurs (see Appendix A.4 for formal proofs).

**Lemma 5.** In the open-economy equilibrium, $da_d/d\beta_o < 0$ and $da_o/d\beta_o > 0$.

As the relative price of good 1 rises, acquiring managerial capital becomes more expensive; consequently, non-offshoring firms acquire less managerial capital and generate lower entrepreneurial income as indicated by (14) and (15), respectively. For offshoring firms, however, as $\beta_o$ falls, the unit-labor cost falls as well ($W_o \downarrow$). As argued by Grossman and Rossi-Hansberg (2008), this cost reduction can be thought much the same as an increase in the productivity of workers. In the present context, reducing $\beta_o$ also directly increases the productivity of offshoring firms, because additional profits obtained from lower production costs induce these firms to increase their managerial capital $z$. With improved productivity, incumbent offshoring firms generate higher entrepreneurial income as well.\textsuperscript{17}

\textsuperscript{16} Effects of reducing the fixed offshoring cost $f_o$ are mostly similar to those of a reduction in $\beta_o$, and will be discussed in Appendix A.5.

\textsuperscript{17} Differentiating $T = a_d W_o^{2\eta/(1-\eta)}$ with respect to $\beta_o$ and using (A.13) and (A.15) from the appendix yields

$$\frac{dT}{d\beta_o} = \frac{2\eta(j_3 - j_1)(1 - A^{1-k})T}{(1-\eta)(1 + f_o A^{1-k})\beta_o} > 0.$$
Figure 3: The Impact of Reducing $\beta_o$ on Income and Welfare Distributions

Figure 2.a shows the impact of reducing $\beta_o$ on income distribution in Home. There are two important points related to the distributional change. First, the policy leads to job polarization in the sense that two ends of the ability distribution (i.e., workers and offshoring entrepreneurs) increase their shares in labor supply, whereas the middle group in the distribution experiences a reduction in its share..dd Second, since a reduction in $\beta_o$ reduces $A$ as shown in Appendix A.4, equations in (21) imply that the average income of all entrepreneurs increases, but the average income of offshoring entrepreneurs decreases. Thus, the income inequality between all entrepreneurs and workers increases, but that between offshoring entrepreneurs and workers decreases. The latter one decreases because the supply of offshoring entrepreneurs increases and the entrants have lower income than the incumbents.

Figure 2.b shows the impact of this policy on welfare distribution. Note that there is a welfare polarization: individuals with medium-level ability (i.e., $a \in (\bar{a}_1, \bar{a}_s)$) become worse off, while those at the two ends of the ability distribution become better off. Since $v \propto p^{2/(1-\eta)-\theta}$ for non-offshoring firms and $2/(1-\eta) - \theta > 0$, their welfare decreases as $p$ falls. A reduction in the relative price $p$ clearly makes everyone in Foreign better off.

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18 The job-polarization is mainly driven by the general-equilibrium structure of the model. If Home were a small, open economy so that the relative price $p$ is exogenously fixed, equation (16) implies that $a_d$ is independent of the offshoring costs; as a result, lowering $\beta_o$ has no impact on the ability cutoff $a_d$.

19 Since $v \propto p^{2/(1-\eta)-\theta}$ for non-offshoring firms and $2/(1-\eta) - \theta > 0$, their welfare decreases as $p$ falls.

20 It follows from (28) that Home’s national income is given by $E^n = E - w_o M$ is independent of $\beta_o$. Since $\nabla^n = p^{-\theta} E^n$ and $dp/d\beta_o < 0$, it then follows that national welfare $\nabla^n$ increases.
**Proposition 3.** Consider the two trading economies as described. A reduction in the variable cost of offshoring $\beta_0$ has the following effects in Home. It

a. leads to task upgrading (downgrading) of native (immigrant) workers employing at offshoring firm;

b. generates job polarization by increasing the mass of workers and offshoring entrepreneurs;

c. increases (decreases) the productivity of offshoring (non-offshoring) firms;

d. increases (decreases) the inequality between workers and all (offshoring) entrepreneurs;

e. improves aggregate welfare.

The prediction that lowering variable offshoring costs leads to task upgrading of native workers and task downgrading for immigrants in offshoring firms is in line with Ottaviano et al. (2013). The prediction that reducing the variable offshoring cost $\beta_0$ induces offshoring firms to import more intermediate goods is consistent with Goldberg et al. (2010) who, using firm-level data from India, show that lower input tariffs increases new input varieties. Topalova and Khandelwal (2011) analyze India’s externally imposed trade reform in 1991, and show that lower input tariffs appear to have increased firm-level productivity. Similarly, using Hungarian micro data, Halpern et al. (2015) find that using more imported inputs increases firm productivity significantly. In particular, they attribute a quarter of Hungarian productivity growth over the 1993–2002 period to imported inputs.

The prediction that lowering the variable offshoring cost $\beta_0$ generates job polarization in Home is interesting, because it recalls to mind what has happened in the U.S. and European labor markets over the past three decades. Specifically, recent empirical studies have documented a substantial increase in the share of high-skill and low-skill employment in the US and many other developed countries at the expense of middle-skill employment (Goos et al. 2009, Acemoglu and Autor 2011, Autor and Dorn 2013). Technology and globalization are argued to be the main forces behind this job polarization (e.g., Goos et al. 2009, Acemoglu and Autor 2011). The

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21Similarly, Gopinath and Neiman (2014) show that the within-firm drop in imported varieties explains about 45 percent of the decline in imports during the Argentine crisis in 2001-2002.
present model theoretically complements these studies by showing that offshoring indeed can generate job-polarization, although moderately skilled individuals here are entrepreneurs not workers.\(^{22}\)

Egger et al. (2015) develop a monopolistic competition model of offshoring with occupational choice, where the final good production is subject to external increasing returns (EIR) to scale. They show that exposure to offshoring may decrease the welfare if EIR are weak and the share of offshoring firms is sufficiently small. In my model, the production technology exhibits constant returns to scale, the final good is produced using a fixed set of intermediate goods, and markets are perfectly competitive.

### 4 Conclusion

This paper develops a model of task trade with occupational choice and immigration to investigate the impact of offshoring and immigration on entrepreneurial activity, income inequality, and welfare. Tasks can be performed by natives, immigrants, or offshore workers, and offshoring entails variable and fixed costs. Furthermore, markets are perfectly competitive and firms can invest to improve their productivity.

I find that an immigration policy that makes immigrants more integrated to the economy expands the supply of entrepreneurs, increases (decreases) the productivity of non-offshoring (offshoring) firms, lowers the income inequality between workers and entrepreneurs, and improves welfare. My analysis also indicates that immigration is beneficial for the economy in the sense that it increases the number of entrepreneurs and their firm productivity, does not affect the inequality between workers and entrepreneurs, and again improves the welfare.

Reducing offshoring costs leads to job polarization by increasing the supply of workers and offshoring entrepreneurs, while reducing the mass of entrepreneurs who produce domestically. It also increases the income inequality between entrepreneurs and workers. Although lowering offshoring costs also leads to a welfare polarization, it improves the aggregate welfare.

\(^{22}\)Dinopoulos and Unel (2017) develop a trade model where individuals choose to become entrepreneurs or workers, and their model also generates job-polarization. In their model countries are symmetric, firms compete with each other monopolistically, and there are no offshoring and immigration.
A Appendix

A.1 Proof of Lemma 1

Differentiating equations (7) and (9) with respect to \( \beta_m \) yields

\[
\frac{dj_1}{d\beta_m} = \frac{t_m(j_1)}{\beta_m t'_m(j_1) - \beta_o t'_o(j_1)} < 0, \quad \frac{dj_2}{d\beta_m} = -\frac{t_m(j_2)}{\beta_m t'_m(j_2)} < 0, \quad \frac{dj_3}{d\beta_m} = 0, \tag{A.1}
\]

where the first inequality follows from \( \beta_m t'_m(j_1) > \beta_o t'_o(j_1) \), because at \( j_1 \) the green curve in Figure 1 is steeper than the red one.

Similarly, differentiating equations in (7) with respect to \( \beta_o \) yields

\[
\frac{dj_1}{d\beta_o} = \frac{t_o(j_1)}{\beta_m t'_m(j_1) - \beta_o t'_o(j_1)} > 0, \quad \frac{dj_2}{d\beta_o} = 0, \quad \frac{dj_3}{d\beta_o} = -\frac{t_o(j_3)}{\beta_o t'_o(j_3)} < 0, \tag{A.2}
\]

where the first inequality follows again from \( \beta_m t'_m(j_1) > \beta_o t'_o(j) \). Finally, according to equations (7) and (9), \( j_1, j_2, \) and \( j_3 \) are clearly independent of \( z, M, \) and \( f_o \).

A.2 Aggregation of Variables

Maximizing profit function (11) with respect to \( l_v(j) \) and using \( W_v \) yields

\[
l_v(j; a) = \frac{\eta p y_2(v) a}{c_v(j)} = \frac{2\eta a}{(1 - \eta) a_d c_v(j)} \left( \frac{W_d}{W_v} \right)^{\frac{2\eta}{1 - \eta}},
\]

where the second equality follows from (19b). Aggregating \( l_v(j; a) \) over \([j_2, 1]\) and \([j_3, 1]\) yields the amount of native workers used by an entrepreneur with ability \( a \)

\[
\ell(a) = \frac{2\eta a}{(1 - \eta) a_d} \times \begin{cases} (1 - j_2) & a \in [a_d, a_o) \\ (1 - j_3) \left( \frac{W_d}{W_o} \right)^{\frac{2\eta}{1 - \eta}} & a \geq a_o \end{cases} \tag{A.3}
\]

Integrating (A.3) across all entrepreneurs yields

\[
L_2 = \frac{b}{a_d^k} \left[ (1 - A^{1-k})(1 - j_2) + (1 - j_3)A^{1-k} \left( \frac{W_d}{W_o} \right)^{\frac{2\eta}{1 - \eta}} \right], \tag{A.4}
\]

where \( b = 2\eta / [(k - 1)(1 - \eta)] \).

Since production of one unit of task \( j \) requires \( \beta_m t_m(j)/w_m \) units of immigrants, aggregating \( \beta_m t_m(j)l_v(j; a)/w_m \) over \([0, j_1]\) and \([0, j_2]\) yields the amount of immigrant workers used by an entrepreneur with ability \( a \)

\[
m(a) = \frac{2\eta a}{(1 - \eta) w_m a_d} \times \begin{cases} j_2 & a \in [a_d, a_o) \\ j_1 \left( \frac{W_d}{W_o} \right)^{\frac{2\eta}{1 - \eta}} & a \geq a_o \end{cases} \tag{A.5}
\]
Integrating (A.5) across all entrepreneurs yields
\[
M_2 = \frac{b}{w_m a_d^k} \left[ j_2(1 - A^{1-k}) + j_1 A^{1-k} \left( \frac{W_d}{W_o} \right)^{\frac{2\eta}{1-\eta}} \right], \quad (A.6)
\]

The number of native workers in sector 1 is \( L_1 = G(a_d) - L_2 \), and the number of immigrant workers in sector 1 is \( M_1 = M - M_2 \), where \( M \) is the total population of immigrants in Home.

If a Home firm offshores task \( j \), its unit production requires \( \beta_o t_o(j) / w^* \) units of workers in Foreign. Aggregating \( \beta_o t_o(j) l_o(j; a) / w^* \) over \([j_1, j_3]\) yields the amount of offshore workers used by an entrepreneur with ability \( a \ell^* = 2\eta a\).

Good 1 is used in acquiring managerial capital and entry to Foreign. The cost of acquiring \( z \) units of managerial capital is given by \( \lambda z^2 / 2a \). Substituting \( z \) from (A.4) into the latter, and then aggregating across all entrepreneurs and using \( A \) from (18) yields
\[
C_1z = \frac{k[1 + f_o A^{-k}a_d^{-k}]}{k - 1}.
\]
The total amount of good 1 used in entry to Foreign is \( C_1e = f_o A^{-k}a_d^{-k} \). It then follows that
\[
C_1 = C_1z + C_1e = \frac{k + (2k - 1)f_o A^{-k}}{(k - 1)a_d^k}.
\]

### A.3 Proof of Lemma 3

To determine the impact of a reduction in \( \beta_m \) on ability cutoffs \( a_d \) and \( a_o \), I first need to determine its impact on aggregate wages \( W_o \) and the world relative price \( p \). Using Leibniz rule, differentiating \( W_d \) from (8) and \( W_o \) from (9) with respect to \( \beta_m \) yields
\[
\frac{dW_d}{d\beta_m} = \frac{j_2 W_d}{\beta_m} > 0, \quad \frac{dW_o}{d\beta_m} = \frac{j_1 W_o}{\beta_m} > 0, \quad (A.8)
\]
which further imply
\[
\frac{W_o d(W_d / W_o)}{W_d d\beta_m} = \frac{j_2 - j_1}{\beta_m} > 0. \quad (A.9)
\]
To determine \( da_o / d\beta_m \), first note that
\[
\frac{dA}{d\beta_m} = -\frac{2\eta(j_2 - j_1)}{1 - \eta} < 0. \quad (A.10)
\]

23
Now differentiating $a_d$ from (26) with respect to $\beta_m$ and (A.10) yields
\[
\frac{d a_d}{d \beta_m} = \frac{2\eta(j_2 - j_1)(A^{1-k} + f_o A^{-k})a_d}{(1 - \eta)(1 + f_o A^{-k})\beta_m} > 0, \tag{A.11}
\]
and differentiating $a_o = Aa_d$ with respect to $\beta_m$ and using (A.10) and (A.11) yields
\[
\frac{d a_o}{d \beta_m} = \frac{2\eta(j_2 - j_1)(1 + Af_o^{-1})a_o}{(1 - \eta)(1 + f_o A^{-k})\beta_m} < 0.
\]
Finally, differentiating $p$ from (27) with respect to $\beta_m$ and using (A.10) yields
\[
\frac{d p}{d \beta_m} = \frac{(j_2 - (j_2 - j_1)A^{1-k} + j_1 f_o A^{-k})\eta p}{(1 + f_o A^{-k})\beta_m} > 0, \tag{A.12}
\]
since $j_2 > j_2 - j_1$ and $A^{1-k} < 1$.

### A.4 Proof of Lemma 5

Note that $W_d$ does not depend on $\beta_o$. Using Leibniz rule, differentiating $W_o$ from (12b) with respect to $\beta_o$ yields
\[
\frac{d W_o}{d \beta_o} = \frac{(j_3 - j_1)W_2}{\beta_o} > 0. \tag{A.13}
\]
Differentiating $A$ from (18) with respect to $\beta_o$ yields
\[
\frac{d A}{d \beta_o} = \frac{2\eta(j_3 - j_1)A}{(1 - \eta)\beta_o \left[ 1 - (W_d/W_o)^{-\frac{2\eta}{1-\eta}} \right]} > 0, \tag{A.14}
\]
$dW_o/d\beta_o$ is given by (A.13). Differentiating (26) with respect to $\beta_o$ yields
\[
\frac{d a_d}{d \beta_o} = -\frac{f_o A^{-k-1}a_d}{1 + f_o A^{-k}} \frac{d A}{d \beta_o} < 0. \tag{A.15}
\]
Differentiating $a_o = Aa_d$ with respect to $\beta_o$ yields
\[
\frac{d a_o}{d \beta_o} = \frac{a_d}{1 + f_o A^{-k}} \frac{d A}{d \beta_o} > 0.
\]
Finally, using (16) and (A.15) yields
\[
\frac{d p}{d \beta_o} = \frac{(1 - \eta)f_o A^{-k-1}p}{2(1 + f_o A^{-k})} \frac{d A}{d \beta_o} > 0, \tag{A.16}
\]
where $dA/d\beta_o$ is given by (A.14).
A.5 Reducing Fixed Cost of Offshoring $f_o$

According to Lemma 1, a reduction $f_o$ has no impact on any of the task cutoffs; as a result, it has no impact on the aggregate price indexes $W_d$ and $W_o$. This policy increases $a_d$ and decreases $a_o$, and thus leads to a job polarization (see eqs. (17) and (26)).

Since a reduction in $f_o$ increases the relative price of good 1, the cost of acquiring managerial capital goes up; as a result, all entrepreneurs acquire less managerial capital. The incomes of entrepreneurs who continue on producing domestically decline. The impact of this policy on incomes of offshoring entrepreneurs is ambiguous. To see this, differentiating income function (20) with respect to $f_o$ yields

$$\frac{\partial e_{2o}(a)}{\partial f_o} = \frac{(a^{1-k} + f_oA^{-k})a}{(1 + f_oA^{-k})a_o} - 1,$$

where $A$ is given by (18). Note that $\partial e_2/\partial f_o > 0$ for sufficiently high values of $a$, i.e. entrepreneurs earning very top-income experience an income loss due to lower firm productivity. Although the policy has an ambiguous impact on the welfare distribution, it increases the aggregate welfare.
References


