Wealth, Price Dispersion and Risk Sharing

Eungsik Kim∗  Stephen Spear†
Tepper School of Business  Tepper School of Business
Carnegie Mellon University  Carnegie Mellon University

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Abstract

In this paper, we study how the market power of buyers can generate a dispersion over effective prices depending on wealth in an economy with imperfect competition. We ask how the imperfect competition brings about the consumption/income parallel without capital market imperfection or impatient consumers. More importantly, we examine whether the strategic interaction reduces the risk sharing and increases the consumption volatility cross-sectionally in an incomplete market. In numerical analysis, we quantify the welfare loss from the complementary effect between the two friction: incomplete market and imperfect competition. We find out the supermodular welfare loss takes about 2% out of the benchmark expected utility. Lastly, the structure of exogenous shocks determines the additional consumption inequality via imperfect competition. When the endowment of overlapped generations are positively correlated under the exogenous uncertainty, the consumption volatility does not grow as much as an economy with an idiosyncratic shock where some agents are favorable by the shock and others are not.

JEL classification: C72; C72; D51; D52; L16

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∗Email address: eungsikk@andrew.cmu.edu
†Email address: ss1f@andrew.cmu.edu
1 Introduction

The consumption risk-sharing literature often assumes the good markets are competitive where agents purchase goods at the same prices and cannot affect them. Putting differently, it ignores the imperfectly competitive good markets where traders can change prices and thus, prices could be different for the different types of agents. However, we can find anecdotal evidence to show the prices are dispersed over the wealth of buyers because of bulk discounts, bargaining power, and many other reasons. For example, savers have a higher interest rate if saving large deposits and buyers can get discounts if purchasing more than a certain amount in stores. If prices are different according to the wealth of agents, then consumptions will be allocated more to those facing favorable prices in the market. Therefore, the imperfect competition in good markets can affect the consumption risk sharing and its inequality.

Therefore, we, in this paper, ask how the price dispersion under the imperfect competition affects the risk sharing. We examine whether the imperfect competition will increase consumption volatility and thus reduce the risk sharing. We also check whether the welfare loss from the additional consumption volatility is significant.

In the literature on the Permanent Income Hypothesis, a well-known stylized fact is consumption growth closely parallels income growth (see Carroll and Summers 1991; Carroll 1997 and many others). There are two strands of models to explain the empirical fact. One is the Life Cycle/Permanent Income Hypothesis model of saving under liquidity constraint and precautionary saving motive. The other one is the Keynesian model inhabited by impatient and rule of thumb consumers who consume their current disposable income in every period. We inspect how the model incorporating the imperfect competition as an alternative gives rise to the consumption/income parallel over the life-cycle without any other frictions.

To answer these questions, we use a stochastic three-period lived overlapping generations (OLG) model incorporating the Shapley-Shubik market game (see Shapley and Shubik 1977 and Dubey and Shubik 1978). Under the Shapley-Shubik market game, agents have market powers to affect the price of a good and thus allocations as well.¹ For a tractable model with the financial market incompleteness, we assume the economy has only the fiat money as assets to insure against risk. Thus, the fiat money in this model is used not only as a medium of exchange and a store of value but also as an instrument to hedge risk partially and to exercise the market power on the price of a good. Unlike the infinitely-lived agent model, the overlapping generations model involves a birth-date risk that the asset market in the sequential economy cannot hedge unless agents are allowed to trade contingent on the state of nature before it realizes. A three-period OLG model captures some interesting features that cannot be included in a two-period OLG model. First, we can model an inverse-V shape endowment distribution consistent with the life-cycle income structure in the data. There is an income effect from the state-dependent endowment for the middle-aged which can affect the portfolio rebalancing choice. As proved in Henriksen and Spear (2012), the income effect is the main cause of the non-existence of the stochastic steady state equilibria (or strongly stationary equilibria in their paper).

¹ We study an economy with a single perishable good in which the price of a good is the inverse of the price of money. Thus, if agents have the market power in the commodity market, then they also have the market power on the price of money.
the non-existence of the stochastic steady state equilibria in the OLG model with the strategic interaction. Thus, we focus on the pure strategy Markov Nash equilibria as an equilibrium concept. Note that we assume there is no monitoring for the past actions so that agents do not condition their decisions on the past choices of other players. In the Shapley-Shubik market game, the trading mechanism is as follows. In a centralized trading post, players with anonymous players by making offers of consumption goods and bids of money. Here, the fiat money is used to bid for an exchange for the single good. Each consumer is allocated a proportion of the aggregate offer of the commodity in the proportion that her bid bears to the aggregate bid. Similarly, she is assigned a share of the aggregate fiat money bid in the proportion that her offer has in the aggregate offer of the good. Lastly, the price is explicitly determined by dividing the total money bid with the total offer i.e. the total demand measured in money / the total supply of a good.

In a deterministic economy, the perfect consumption smoothing fails under the strategic interaction because agents face different effective prices depending on their return rates of bidding, unlike the perfect competition. Agents transfer money into the age when they face a lower effective price to consume more by exercising a higher bid. Even when agents face the same effective prices over two ages, they might consume different amounts according to the money holdings on their hands which differentiate the bidding power of agents. In the perfectly competitive economy, agents face the same price and price adjusts to balance the purchasing power of different generations so that the perfect consumption smoothing can be attained. However, the trading mechanism via bidding in the Shapley-Shubik market game lets the market power determine prices rather than prices change to balance the market power. Therefore, the strategically interacted economy cannot achieve the smoothed consumption over ages. Under the inverse-V shape endowment structure, the middle-aged faces the lowest effective price because of the highest bidding power induced by the most endowment in that period. Thus, the consumption pattern is consistent with the endowment stream. This result provides a distinctive explanation for the consumption/income parallel stylized fact unlike the model based on capital market imperfections such as liquidity constraints.

In a stochastic economy with an idiosyncratic shock, the imperfect competition reduces the risk sharing and increases consumption inequality between states. The consumption risk-sharing indeed requires a transfer from the poor to the rich. However, under the idiosyncratic shock, the rich have a lower effective price of good due to a higher return rate of bids. Compared to the competitive economy, the rich group rather consumes more and the poor group consumes less. Hence, the imperfect competition in the incomplete market worsens the risk sharing and increase consumption volatility. In a parameterized version of the model, the within-age consumption volatility increases roughly 8% for the young and 6% for the middle-aged and the old when adding the imperfect competition on the top of the incomplete market. As another interpretation of this result, there is a complementarity effect of the two friction regarding the welfare loss via additional consumption volatility induced by the imperfect competition. In the numerical analysis, the supermodular welfare loss takes 2% out of the benchmark utility level. We find out that the structure of the shock matters for the additional consumption volatility: positively or negatively correlated endowment shocks. If the endowments of the overlapped generations are affected in the same direction by the shock, the consumption volatility is not much enlarged because the effective prices move similarly and thus there are no
specific groups to which the shock is mostly favorable. Therefore, the within-age consumption volatility increases roughly only 3% for the young, 1% for the middle-aged and 1.5% for the old.

We organized this paper as follows. Section 2 briefly describes the three-period stochastic OLG model incorporating the Shapley-Shubik market game. In this section, we show the short memory equilibria do not exist which implies we need to compute the recursive Markovian equilibria for the numerical analysis. In Section 3, we examine first the steady-state equilibria in the deterministic economy to understand how the imperfect competition makes the consumption allocations deviate from the perfect consumption smoothing. Section 4 runs diverse numerical analyses to study how the imperfect competition reduces the risk sharing and increases the consumption volatility and check the welfare loss from the complementarity effect between the two friction.

2 Model

In this section, we develop a closed, pure exchange overlapping generation model incorporating the Shapley-Shubik market game. Time is discrete and indexed by $t$ from 1 to infinity. In each period, $n > 0$ agents are born and live three-periods labeled as young, middle-aged and old. We assume the agents in the same cohort are identical and thus, we focus on the symmetric Nash equilibria. Each generation overlaps with the previous generations for two periods. They are endowed with homogeneous, non-storable, stochastic consumption goods. There is no population growth. In period 1, there are $n$ middle-aged consumers who live in period 1 and 2 and $n$ old consumers who live in period 1.

In every period, there are a single perishable commodity and fiat money. Agents are given stationary endowments defined by a nonnegative vector $\omega_s = (\omega_{s1}, \omega_{s2}, \omega_{s3})$ where $\omega_{s1}$ is the endowment when young, $\omega_{s2}$ is the endowment when middle-aged and $\omega_{s3}$ is the endowment when old in state $s$. We assume $\omega_{si} \gg 0$ for $\forall i$.

There is an exogenous uncertainty which is governed by two states of nature, $s \in \{\alpha, \beta\}$. The process of nature is assumed to an independent and identically distributed (I.I.D.) with the probability given by $0 < \pi_s < 1$ for $s \in \{\alpha, \beta\}$, where $\pi^\alpha + \pi^\beta = 1$.

The consumption plan of the representative households born in time $t$ is denoted by $(c_{s1,t}, c_{s2,t+1}, c_{s3,t+2})$ where $c_{s1,t}$ is the first-period consumption given the state realization of $s$, $c_{s2,t+1}$ is the second-period consumption given the state realization of $s'$, and $c_{s3,t+2}$ is the last period consumption the state realization of $s''$. Hereafter, we denote $x_{i,j}^{s'}$ as the value of $x$ for the agents in the $i$th stage of life at time $j$ given state the state realization of $s$.

Lifetime preferences are time-additively separable described by a von Neumann-Morgenstern utility function $U: \mathbb{R}_+^7 \to \mathbb{R}$ and are smooth functions. $U$ is specified by:

$$U(c_{s1,t}, c_{s2,t+1}, c_{s3,t+2}) = u(c_{s1,t}) + \delta \sum_{s' \in \{\alpha, \beta\}} \pi^{s'} u(c_{s2,t+1}) + \delta^2 \sum_{s'' \in \{\alpha, \beta\}} \pi^{s''} u(c_{s3,t+2})$$

where $u(\cdot)$ is strictly increasing, strictly concave and satisfies the Inada condition and $0 < \delta \leq 1$ is the time discount factor.
Agents trade the single commodity with a costlessly storable outside money in a single trading post to transfer income over time under strategic interactions. The identical agents make non-negative lifetime offers of consumption goods \( q^s = (q_{1,t}, q_{2,t+1}, q_{3,t+2}) \) and non-negative lifetime bids of money \( b^s = (b_{1,t}, b_{2,t+1}, b_{3,t+2}) \). Offers are restricted by the endowments because they are made in terms of the commodity: \( q^s_i \geq q^s_t \).

In period 1, only the initial identical middle-aged and old receive fixed money endowments \( \tilde{m}_{1,0} \) and \( \tilde{m}_{2,0} \) where \( n \cdot (\tilde{m}_{1,0} + \tilde{m}_{2,0}) = n \cdot \tilde{m} \). The identical agents born in time \( t \) demand \( m_{1,t}\) amount of money today and \( m_{2,t+1} \) tomorrow to bid for goods in the next period. The aggregate supply of money is fixed at \( n \cdot \tilde{m} \) for all times.\(^2\) The representative household’s strategy set is \( \{ (q_{1,t}, q_{2,t+1}, q_{3,t+2}, b_{1,t}, b_{2,t+1}, b_{3,t+2}, m_{1,t}, m_{2,t+1}) \} \in \mathbb{R}^3 \) if \( \omega_t \geq q_t \gg 0 \).

Given offers and bids, the trading process runs as follows. Each consumer is allocated a proportion of the aggregate offer of the commodity in the proportion that her bid bears to the aggregate bid. Similarly, each consumer is assigned a share of the aggregate fiat money bid in the proportion that her offer has in the aggregate offer of the good.

We denote \( (Q_{1,t}, Q_{2,t+1}, Q_{3,t+2}) = (nq_{1,t}, nq_{2,t+1}, nq_{3,t+2}) \) as the aggregate offer vector of the \( n \) agents born in \( t \) and offering in time \( t, t+1 \) and \( t+2 \) and \( (B_{1,t}, B_{2,t+1}, B_{3,t+2}) = (nb_{1,t}, nb_{2,t+1}, nb_{3,t+2}) \) as the aggregate bid vector of the \( n \) agents born in \( t \) and bidding in time \( t, t+1 \) and \( t+2 \). Note that the identical agents in the same cohort offer and bid equal amount since this paper studies a symmetric market game.

The aggregate offer of the consumption good in time \( t \) is the sum of the offers in that period made by the consumers born in period \( t-2, t-1 \) and \( t \): \( Q^s_t = Q^s_{3,t-2} + Q^s_{2,t-1} + Q^s_{1,t} \). Likewise, The aggregate bid of the fiat money in time \( t \) is the sum of the bids in that period made by the consumers born in period \( t-2, t-1 \) and \( t \): \( B^s_t = B^s_{3,t-2} + B^s_{2,t-1} + B^s_{1,t} \).

The budget constraints that a representative young agent faces, given a share of the aggregate bids, are:

\[
\begin{align*}
\frac{b^s_{1,t} + m^s_{1,t}}{Q^s_t} & = \frac{q^s_{1,t}}{Q^s_t} B^s_t \\
\frac{b^s_{2,t+1} + m^s_{2,t+1} - m^s_{1,t}}{Q^s_{t+1}} & = \frac{q^s_{2,t+1}}{Q^s_{t+1}} B^s_{t+1} \\
\frac{b^s_{3,t+2} - m^s_{2,t+1}}{Q^s_{t+2}} & = \frac{q^s_{3,t+2}}{Q^s_{t+2}} B^s_{t+2}
\end{align*}
\]

where \( \frac{B^s_t}{Q^s_t} \) can be interpreted as the price of the consumption good in period \( t \) with the \( s \) state of nature in terms of the outside money.

Under the trading mechanism of the market game, a household born in time \( t \) earns the

\(^2\) Note that we multiply by \( n \) for simplicity in calculations later.
following allocations:

\[ c_{1,t}^s = \omega_1^s - q_{1,t}^s + \frac{b_{1,t}^s}{B_t^s} Q_t^s \]
\[ c_{2,t+1}^{s'} = \omega_2^{s'} - q_{2,t+1}^{s'} + \frac{b_{2,t+1}^{s'}}{B_{t+1}^{s'}} Q_{t+1}^{s'} \]
\[ c_{3,t+2}^{s''} = \omega_3^s - q_{3,t+2}^{s''} + \frac{b_{3,t+2}^{s''}}{B_{t+2}^{s''}} Q_{t+2}^{s''} \]

We are interested in the pure strategy Nash equilibria of this game and thus, we express the price ratio in (2) in terms of the offers and the bids of the other agents. For this, we introduce the following notations: \( B^s_{i,i} = B_i^s - b^s_{i,t} \) and \( Q^s_{i,i} = Q_i^s - q^s_{i,t} \) for \( \forall i \in \{1,2,3\} \). Thus, we can rewrite (2) as:

\[ b_{1,t}^s + m_{1,t}^s = \left( \frac{B_{t-1}^s - m_{1,t}^s}{Q_{t-1}^s} \right) q_{1,t}^s \]
\[ b_{2,t+1}^{s'} + m_{2,t+1}^{s'} - m_{1,t}^s = \left( \frac{B_{t+1,t-2}^{s'} - m_{2,t+1}^{s'} + m_{1,t}^s}{Q_{t+1,-2}^s} \right) q_{2,t+1}^{s'} \]
\[ b_{3,t+2}^{s''} - m_{2,t+1}^{s'} = \left( \frac{B_{t+2,-3}^{s''} + m_{2,t+1}^{s'}}{Q_{t+2,-3}^s} \right) q_{3,t+2}^{s''} \]

By equating the right-hand sides of (2) and (4), we obtain the following equations:

\[ \frac{Q_t^s}{B_t^s} = \frac{Q_{t-1}^s}{B_{t-1}^s - m_{1,t}^s}, \quad \frac{Q_{t+1}^{s'}}{B_{t+1}^{s'}} = \frac{Q_{t+1,-2}^{s'}}{B_{t+1,-2}^{s'} - m_{2,t+1}^{s'} + m_{1,t}^s} \quad \text{and} \quad \frac{Q_{t+2}^{s''}}{B_{t+2}^{s''}} = \frac{Q_{t+2,-3}^{s''}}{B_{t+2,-3}^{s''} + m_{2,t+1}^{s'}} \]

Substituting these equations with (4) into (3), we get the following expressions:

\[ c_{1,t}^s = \omega_1^s - \left( \frac{Q_{t-1}^s}{B_{t-1}^s - m_{1,t}^s} \right) m_{1,t}^s \]
\[ c_{2,t+1}^{s'} = \omega_2^{s'} - \left( \frac{Q_{t+1,-2}^{s'}}{B_{t+1,-2}^{s'} - m_{2,t+1}^{s'} + m_{1,t}^s} \right) (m_{2,t+1}^{s'} - m_{1,t}^s) \]
\[ c_{3,t+2}^{s''} = \omega_3^s + \left( \frac{Q_{t+2,-3}^{s''}}{B_{t+2,-3}^{s''} + m_{2,t+1}^{s'}} \right) m_{2,t+1}^{s'} \]

A representative agent should maximize the following utility functions given the offers and bids of all other agents:

\[ \max_{m_{1,t}^s, m_{2,t+1}^{s'}} U \left( c_{1,t}^s, c_{2,t+1}^{s'}, c_{3,t+2}^{s''} \right) = u \left( \omega_1^s - \left( \frac{Q_{t-1}^s}{B_{t-1}^s - m_{1,t}^s} \right) m_{1,t}^s \right) \]
\[ + \delta \sum_{s' \in \{\alpha, \beta\}} \pi^{s's} u \left( \omega_2^{s'} - \left( \frac{Q_{t+1,-2}^{s'}}{B_{t+1,-2}^{s'} - m_{2,t+1}^{s'} + m_{1,t}^s} \right) (m_{2,t+1}^{s'} - m_{1,t}^s) \right) \]
\[ + \delta^2 \sum_{(s', s'') \in \{\alpha, \beta\}^2} \pi^{s's''} u \left( \omega_3^s + \left( \frac{Q_{t+2,-3}^{s''}}{B_{t+2,-3}^{s''} + m_{2,t+1}^{s'}} \right) m_{2,t+1}^{s'} \right) \]
The first order conditions with respect to $m_{1,t}$ and $m_{2,t+1}'$ are:

(8)

$$u'(c_{1,t}') \left( \frac{Q_{t-1}^s}{B_{t-1}^s - m_{1,t}'} + \frac{Q_{t-1}^s}{B_{t-1}^s - m_{1,t}^s} m_{1,t}' \right)$$

$$= \delta \sum_{s' \in \{a,b\}} \pi_{ss'} u'(c_{2,t+1}') \left( \frac{Q_{t+1}^{s'}}{B_{t+1}^{s'} - m_{2,t+1}' + m_{1,t}'} + \frac{Q_{t+1}^{s'}}{B_{t+1}^{s'} - m_{2,t+1}' + m_{1,t}^s} m_{2,t+1}' \right)$$

and

(9)

$$u'(c_{2,t+1}') \left( \frac{Q_{t+1}^{s'}}{B_{t+1}^{s'} - m_{2,t+1}' + m_{1,t}'} + \frac{Q_{t+1}^{s'}}{B_{t+1}^{s'} - m_{2,t+1}' + m_{1,t}^s} m_{2,t+1}' \right)$$

$$= \delta \sum_{s'' \in \{a,b\}} \pi_{ss''} u'(c_{3,t+2}') \left( \frac{Q_{t+2}^{s''}}{B_{t+2}^{s''} + m_{2,t+1}'} - \frac{Q_{t+2}^{s''}}{B_{t+2}^{s''} + m_{2,t+1}^s} m_{2,t+1}' \right)$$

Note that once the money demands are determined by (8) and (9), either the offers or the bids of agents are indeterminate at the Nash equilibrium in (4) (see Peck et al. 1992). For example, when increasing the amount of offers, raising the bids buys back the additional offer so that (4) is satisfied. The economic intuition for the indeterminacy is a household’s net trade can be achieved in an infinite number of ways by combining offers and bids given the offers and bids of other agents. Hence, we focus on equilibrium for the game in which offers are exogenously given. As shown in Peck et al. (1992), the equilibria of the offer constrained game are equilibria of the unconstrained game as well, as long as offers yield interior equilibrium bids. Finally, we concentrate on monetary Nash equilibria where the aggregate holdings of the fiat money are positive. Depending on the endowment structure, the private savings of the young and the middle-aged agents can be negative (borrowing money) to smooth consumption. We assume appropriate endowment structures such that the resulting equilibrium aggregate savings are positive.

**Definition 1.** The market game with symmetric information and exogenous uncertainty is the strategic game described by

(a) 3n players in each period

(b) A finite set $\Omega = \{a, b\}$ of states of nature. The nature follows a stationary first-order Markovian process characterized by the following probabilities, $0 < \pi^s < 1$ for $s \in \{a, b\}$ where $\pi^a + \pi^b = 1$

(c) Stationary endowments $\omega_t^s = (\omega_1^s, \omega_2^s, \omega_3^s)$ where $s \in \{a, b\}$ for all times

(d) The lifetime von Neumann-Morgenstern utility function $U$

(e) The strategy set $\left\{ (q_{1,t}', q_{2,t+1}', q_{3,t+2}', b_{1,t}', b_{2,t+1}', b_{3,t+2}', m_{1,t}', m_{2,t+1}') \in \mathbb{R}^{44} \mid \omega_t \geq q_t \gg 0 \right\}$
Definition 2. For the offer constrained market game in the three-period overlapping generations models, a monetary Nash equilibrium in pure strategies is a sequence of bids and money demands \( \left( b_{2,1}, b_{3,1}, b_{3,2}, m_{2,1}, b_{1,1}, b_{2,1}, b_{2,2}, m_{1,1}, m_{2,1} \right)_{t=1,2,...} \), where \( s_t \) is the state realization in period \( t \), such that

(a) Offers are exogenously given by \( \left( q_{2,1}^s, q_{3,1}^s, q_{3,2}^s, q_{2,t+1}^s, q_{3,t+1}^s \right)_{t=1,2,...} \)

(b) Every agent’s strategies \( \left( b_{2,1}, b_{3,1}, b_{3,2}, m_{2,1}, b_{1,1}, b_{2,1}, b_{2,2}, m_{1,1}, m_{2,1} \right)_{t=1,2,...} \) are the best response to the bids of other agents, taken as given.

(c) For \( \forall t \left( m_{1,t} + m_{2,t} \right) = \bar{m} \), where \( \bar{m} \) is the externally specified stock of fiat money.

2.1 Stochastic Steady States and Recursive Equilibria

In this section, we will show the non-existence of the short memory monetary Nash equilibria. For this, we simplify the parentheses in the consumption good allocations and the first-order conditions with (5):

\[
c_{1,t}^s = \omega_1^s - \frac{Q_1^s}{B_t^s} m_{1,t}^s
\]

\[
c_{2,t+1}^s = \omega_2^s - \frac{Q_{t+1}^s}{B_{t+1}^s} \left( m_{2,t+1}^s - m_{1,t}^s \right)
\]

\[
c_{3,t+2}^s = \omega_3^s + \frac{Q_{t+2}^s}{B_{t+2}^s} m_{2,t+1}^s
\]

\[
u' \left( c_{1,t}^s \right) \frac{B_{t-1}^s}{Q_{t-1}^s} \left( \frac{Q_t^s}{B_t^s} \right)^2 = \delta \sum_{s' \in \{\alpha, \beta\}} \pi^{s,s'} u' \left( c_{2,t+1}^{s'} \right) \frac{B_{t+1-2}^{s'}}{Q_{t+1-2}^{s'}} \left( \frac{Q_{t+1}^{s'}}{B_{t+1}^{s'}} \right)^2
\]

and

\[
u' \left( c_{2,t+1}^{s'} \right) \frac{B_{t+1-2}^{s'}}{Q_{t+1-2}^{s'}} \left( \frac{Q_{t+1}^{s'}}{B_{t+1}^{s'}} \right)^2 = \delta \sum_{s'' \in \{\alpha, \beta\}} \pi^{s',s''} u' \left( c_{3,t+2}^{s''} \right) \frac{B_{t+2-3}^{s''}}{Q_{t+2-3}^{s''}} \left( \frac{Q_{t+2}^{s''}}{B_{t+2}^{s''}} \right)^2
\]

where \( \frac{Q_t^s}{B_t^s}, \frac{Q_{t+1}^{s'}}{B_{t+1}^{s'}} \) and \( \frac{Q_{t+2}^{s''}}{B_{t+2}^{s''}} \) are the inverse of good prices in time \( t, t+1 \) and \( t+2 \) respectively. As the number of agents in each generation converges to infinity, \( \frac{B_{t-1}^s}{Q_{t-1}^s} \) and \( \frac{Q_t^s}{B_t^s} \) will be canceled.
out in (11). Likewise, \( \frac{B_{t+1-2}}{Q_{t+1-2}} \) and \( \frac{Q_{t+1}'}{B_{t+1}} \) are canceled and \( \frac{B_{t+2-3}}{Q_{t+2-3}} \) and \( \frac{Q_{t+2}''}{B_{t+2}} \) are canceled in (11) and (12). Thus, the first-order conditions in the overlapping generation economy with the strategic interaction degenerate to the usual conditions in the perfectly competitive model in the limit. We call \( \frac{B_{t-i}}{Q_{t-i}} \left( \frac{Q_{t-i}}{B_{t}} \right)^2 \) as the effective prices of money for age \( i, i \in \{1, 2, 3\} \) and the inverses of them are the effective prices of the commodity that each age pays to purchase the good. The derivation of the effective prices are straightforward from the allocation rule and budget constraints. To get an additional consumption up to \( \triangle c_{i,t}^s \) for age \( i \) in time \( t \) and state \( s \), the age \( i \) should bid \( \frac{\triangle c_{i,t}^s (B_{i-t})^2}{Q_{t}B_{t-i-1} - \triangle c_{i,t}^s B_{t}^2} \) from the allocation rule in (3). The budget constraint (2) tells the age \( i \) returns back its own bid partially by \( \frac{Q_{t}}{Q_{t}'} \) via offer. Thus, the effective money requirement should be less than 1 to bid 1 more. Indeed, the age \( i \) needs to give up \( \frac{Q_{t-i}}{Q_{t}} \) amount of money to increase the bid up to 1. The return rates depending on offers are the main cause of the diverse effective prices that agents need to pay. Combining the results above, the agents have to essentially bid \( \frac{\triangle c_{i,t}^s (B_{i-t})^2}{Q_{t}B_{t-i-1} - \triangle c_{i,t}^s B_{t}^2} \cdot \frac{Q_{t-i}}{Q_{t}} \) to receive an additional consumption by \( \triangle c_{i,t}^s \). After calculations, \( \frac{\triangle c_{i,t}^s (B_{i-t})^2}{Q_{t}B_{t-i-1} - \triangle c_{i,t}^s B_{t}^2} \cdot \frac{Q_{t-i}}{Q_{t}} = \triangle c_{i,t}^s \cdot \left( \frac{B_{t}}{Q_{t}} \right)^2 \frac{Q_{t}B_{t-i-1}}{Q_{t-i}B_{t-i-1} - \triangle c_{i,t}^s B_{t}^2} \). Thus, the effective prices of the good are \( \frac{Q_{t-i}}{B_{t-i}} \cdot \left( \frac{B_{t}}{Q_{t}} \right)^2 \frac{Q_{t}B_{t-i-1}}{Q_{t-i}B_{t-i-1} - \triangle c_{i,t}^s B_{t}^2} \) for age \( i, i \in \{1, 2, 3\} \). As \( \triangle c_{i,t}^s \longrightarrow 0 \), they reduce to \( \frac{Q_{t-i}}{B_{t-i}} \cdot \left( \frac{B_{t}}{Q_{t}} \right)^2 \).

**Proposition 1.** The non-existence of short memory equilibria. For an open and dense set of the offer constrained market game in the three-period overlapping generations economy, there is no short memory (T-memory) monetary Nash equilibria.

**Proof.** We will outline the proof for the non-existence of a strongly stationary monetary Nash equilibrium (no-recall equilibria). The extension to the short memory monetary Nash equilibria is straightforward. So, assume, for the moment, that there is a strongly stationary monetary Nash equilibrium where offers, bids and money holdings only depend on the current realization of the exogenous shock. Then the assumption that the exogenous uncertainty \( s \in \{\alpha, \beta\} \) and the shock follows an I.I.D. process, allows us to write the consumption good allocations and the first-order conditions as:

\[
\begin{align*}
c_1^s &= \omega_1^s - \left( \frac{Q_{s-1}^s}{B_{s-1} - m_1^s} \right)m_1^s \\
\end{align*}
\]

(13)

\[
\begin{align*}
c_2^{ss'} &= \omega_2^s - \left( \frac{Q_{s-2}^{s'}}{B_{s-2} - m_2^s + m_1^s} \right)(m_2^s - m_1^s) \\
\end{align*}
\]

\[
\begin{align*}
c_3^{ss''} &= \omega_3^s + \left( \frac{Q_{s-3}^{s''}}{B_{s-3}^s + m_2^s} \right)m_2^s \\
\end{align*}
\]
where \{s, s', s''\} ∈ \{α, β\}³.

\[
\begin{align*}
&u' \left( c_1^{s} \right) \frac{B^s_{-1}}{Q^s_{-1}} \left( \frac{Q^a}{B^a} \right)^2 \\
&= \delta \sum_{s' \in \{α, β\}} \pi^{as'} u' \left( c_2^{as'} \right) \frac{B^{s'}_{-2}}{Q^{s'}_{-2}} \left( \frac{Q^{a'}}{B^{a'}} \right)^2 \\
&= \delta \sum_{s' \in \{α, β\}} \pi^{bs'} u' \left( c_2^{bs'} \right) \frac{B^{s'}_{-2}}{Q^{s'}_{-2}} \left( \frac{Q^{a'}}{B^{a'}} \right)^2 \\
&u' \left( c_2^{sa} \right) \frac{B^s_{-2}}{Q^s_{-2}} \left( \frac{Q^a}{B^a} \right)^2 \\
&= \delta \sum_{s'' \in \{α, β\}} \pi^{as''} u' \left( c_3^{as''} \right) \frac{B^{s''}_{-3}}{Q^{s''}_{-3}} \left( \frac{Q^{a''}}{B^{a''}} \right)^2 \\
&u' \left( c_2^{sb} \right) \frac{B^b_{-2}}{Q^b_{-2}} \left( \frac{Q^b}{B^b} \right)^2 \\
&= \delta \sum_{s'' \in \{α, β\}} \pi^{bs''} u' \left( c_3^{bs''} \right) \frac{B^{s''}_{-3}}{Q^{s''}_{-3}} \left( \frac{Q^{b''}}{B^{b''}} \right)^2
\end{align*}
\]

where \(s ∈ \{α, β\}\).

Here, we index the second and third-period consumptions with both the current and lagged shock realizations because agents’ money holdings will generally depend on the state in which the asset was purchased. Market clearing requires that:

\[
(m_1^s + m_2^s) = \hat{m} \text{ for } s ∈ \{α, β\}
\]

These equations have several implications. Note first that since the expected marginal utility expressions in each of the first-order conditions of the middle-aged in (16) and (17) are independent of the lagged state, \(s\), this implies that \(c_2^{αα} = c_2^{βα} = c_2^a\) and \(c_2^{αβ} = c_2^{ββ} = c_2^b\). Thus, there are four independent first-order conditions. The first-period consumptions only depend on the current state. By summing the consumptions of the three overlapping generations in (3), we obtain \(c_1^{s} + c_2^{s} + c_3^{ss'} = \omega_1^s + \omega_2^s + \omega_3^s\) for \(s', s ∈ \{α, β\}\). Therefore, \(c_2^{αα} = c_2^{βα} = c_3^a\) and \(c_2^{αβ} = c_2^{ββ} = c_3^b\). Through the relationship between consumption goods and money holdings in (6), we can demonstrate that the money holdings must be state independent in the model, i.e. \(m_1^α = m_1^β = m_1\) and \(m_2^α = m_2^β = m_2\). Since \(c_2^{as} = c_2^{bs}\) and \(c_3^{as} = c_3^{bs}\) for \(s ∈ \{α, β\}\),

\[
\omega_2^s - \left( \frac{Q_{-2}^s}{B_{-2}^s - m_2^s + m_1^s} \right) (m_2^s - m_1^s) = \omega_2^s - \left( \frac{Q_{-2}^s}{B_{-2}^s - m_2^s + m_1^s} \right) (m_2 - m_1^s)
\]
and

\[ \omega^s_3 + \left( \frac{Q^s_{-3}}{B^s_{-3} + m^\alpha_2} \right) m^\alpha_2 = \omega^s_3 + \left( \frac{Q^s_{-3}}{B^s_{-3} + m^\beta_2} \right) m^\beta_2 \]

Thus, \( m^\alpha_1 = m^\beta_1 = m_1 \) and \( m^\alpha_2 = m^\beta_2 = m_2 \). These results simplify the budget constraints as follows:

\[ b^s_1 + m_1 = \frac{q^s_1}{Q^s} B^s \]

\[ b^s_2 + m_2 - m_1 = \frac{q^s_2}{Q^s} B^{s'} \]

\[ b^{s''}_3 - m_2 = \frac{q^{s''}_3}{Q^{s''}} B^{s''} \]

where \( \{s, s', s''\} \in \{\alpha, \beta\}^3 \). The budget constraints in (21) are linearly dependent since we obtain an identical equation by summing the budget constraints multiplied by \( n \) over the three periods in the same state realization:

\[ nb^s_1 + nb^s_2 + nb^s_3 = \frac{nq^s_1}{Q^s} B^s + \frac{nq^s_2}{Q^s} B^{s'} + \frac{nq^s_3}{Q^s} B^{s''} = B^s \]

where \( s \in \{\alpha, \beta\} \). Thus, there are four independent budget constraints. We are left with a system of nine equations: four from the first order conditions, four from the budget constraints and one from the money market clearing condition. However, there are eight variables, \( b^s_1, b^s_2, b^s_3, b^\alpha_1, b^\alpha_2, b^\alpha_3, m_1 \) and \( m_2 \). There are more equations than variables and thus, it is possible to show that generically there cannot be an equilibrium using techniques similar to those developed by Citanna and Siconolfi (2007).

We have shown the non-existence of a strongly stationary monetary Nash equilibrium in a three-period overlapping generations economy under an exogenous uncertainty. Extending this result to all for short memory equilibria requires noting that the only way allocations could depend on two or more lagged exogenous shocks would be if the prices depended on these lagged shocks. In this case, if consumptions depend on \( n \) lagged shock realizations, then a middle-aged agent looking forward one period will integrate out the next period shock, so that the expected marginal utility component of the first-order condition will be independent of the first lagged shock in the middle-aged agent’s consumption, so that this consumption must, in fact, not depend on this shock. From this observation, we can unravel the shock dependence back to the strongly stationary case.

The non-existence result implies any competitive rational expectations equilibrium should include lagged, endogenous variables as state variables. Here, the endogenous state variables are mostly the distribution of wealth across agents. This type of equilibrium is generally referred to as recursive Markovian equilibria. In our companion paper, we will show such equilibria exist for an open and dense in the space of OLG economies. Thus, in this paper, we restrict our attention to this equilibrium concept. Note that in this equilibrium concept we assume there is no trigger strategy monitoring the past actions in the market game of this paper.
**Definition 3.** For the offer constrained market game in the three-period overlapping generations models, recursive Markovian monetary Nash equilibria in pure strategies are the stationary policy functions of bids and money demands which depend on the realization of the state and lagged money holdings by the representative young agent. By the money market clearing condition, the money holdings of the middle-aged can be ignored in the space of the endogenous state variables. Thus, the vector \( \sigma_t = [m_{1,t-1}, s_t] \in \Sigma \subset R^2 \) is the state of the economy. The stationary policy functions denoted by \( \{b_1(\sigma_t), b_2(\sigma_t), b_3(\sigma_t), m_1(\sigma_t), m_2(\sigma_t)\} \) should satisfy the following conditions given offers exogenously by \( (q_{2,t}^{s_1 t}, q_{3,t}^{s_1 t}, q_{3,t}^{s_2 t}, q_{1,t}^{s_1 t}, q_{2,t+1}^{s_1 t+1}, q_{3,t+2}^{s_1 t+2}, t=1,2,...) \):

(a) Every agent’s policy functions of bids and money demands are the best response to the bids of other agents, taken as given.

(b) For \( \forall t, (m_1(\sigma_t) + m_2(\sigma_t)) = \bar{m} \), where \( \bar{m} \) is the externally specified stock of fiat money.

**3 Analytical Analysis**

In this section, we show analytically how the market power of agents can generate consumption/income parallel in a simplified version of the model above. We first study the deterministic economy case and then extend the result to the stochastic case.

**3.1 Deterministic Economy**

For the analytical tractability, we assume \( \delta = 1 \). In a competitive economy with the fiat money, the consumption allocations are equivalent over ages at the deterministic steady state, i.e. perfectly smoothed consumption. However, in an economy under the strategic interaction, there is a consumption volatility over ages. The consumption is higher in the age endowed with a higher amount of good, assuming sell-all strategy under which agents offer all their endowments following the literature on the market game. Thus, with an inverse-V shape endowment distribution given by \( (\omega_1, \omega_2, \omega_3) \) where \( \omega_2 > \omega_1 > \omega_3 = 0 \), the consumption allocation is defined by \( c_2 > c_1 > c_3 \) which captures a well-known stylized fact of consumption/income parallel over the life-cycle as noted in Carroll and Summers (Carroll and Summers, 1991), Carroll (Carroll, 1997) and many other papers. This result implies the imperfect competition generates a welfare loss via a distortion in the allocation distribution deviating from the perfectly smoothed consumption over the lifetime since we study an exchange economy. We summarize these findings in the following proposition and corollary.

**Proposition 2.** The perfect consumption smoothing cannot be optimal at the deterministic steady state in an economy with the strategic interaction. Instead, consumption is higher in the age endowed with a higher amount of good assuming the sell-all strategy.

**Proof.** Assume the perfectly smoothed consumption allocations are optimal at the deterministic steady state, i.e. \( c_1 = c_2 = c_3 \). Then, the Euler equation at the deterministic steady state implies \( \frac{B_{-1}}{Q_{-1}} = \frac{B_{-2}}{Q_{-2}} = \frac{B_{-3}}{Q_{-3}} \). Let \( k = \frac{B_{-1}}{Q_{-2}} = \frac{B_{-2}}{Q_{-3}} \). Then, \( B_{-1} = Q_{-1}k, B_{-2} = Q_{-2}k, \) and \( B_{-3} = Q_{-3}k \). By summing these three equations and dividing both sides with \( (3n - 1) \), we get \( B = Qk \). Combining \( B_{-1} = Q_{-1}k \) and \( B = Qk \) indicates \( b_1 = q_1k \). Likewise, \( b_2 = q_2k \) and
\[ b_3 = q_3k. \] Therefore, \( \frac{q_1}{Q} = \frac{b_1}{B}, \frac{q_2}{Q} = \frac{b_2}{B} \) and \( \frac{q_3}{Q} = \frac{b_3}{B} \). By plugging these last equations into the budget constraints at the deterministic steady-state, we obtain \( b_1 + m_1 = \frac{b_1}{B}B \) and \( b_3 - m_2 = \frac{b_3}{B}B \) which requires \( m_1 = m_2 = 0 \). This is a contradiction to the money market clearing condition where \( m_1 + m_2 = \bar{m} \). Hence, there should be a consumption volatility over ages at the optimal deterministic steady state equilibrium in an economy with the strategic interaction.

Under the sell-all strategy, the Euler equations can be written as

\[
\frac{u'}{\frac{b_1}{B} \Omega} \frac{B_{-1}}{\Omega_{-1}} = \frac{u'}{\frac{b_2}{B} \Omega} \frac{B_{-2}}{\Omega_{-2}} = \frac{u'}{\frac{b_3}{B} \Omega} \frac{B_{-3}}{\Omega_{-3}}
\]

where \( \Omega = n \cdot (\omega_1 + \omega_2 + \omega_3) \) and \( \Omega_{-i} = \Omega - \omega_i \) for \( i \in \{1, 2, 3\} \). With the assumption that all bids are identical, i.e. \( b_1 = b_2 = b_3 \), the normalized effective prices of the good are \( \Omega_{-i} \) for each age \( i \in \{1, 2, 3\} \). Thus, an age with a higher endowment faces a lower effective price and thus, agents bid more and consume more when they can purchase the good in a cheaper price to maximize the lifetime utility. Therefore, the consumption is parallel to the endowment at the optimal stationary equilibria.

Assuming the sell-all strategy, agents face a higher return rate for their own bids in the money budget constraint when having a larger endowment. Since agents make both bids and offers, they return back their own bids partially. A share of the aggregate fiat money bid is in the proportion that an offer has in the aggregate offer of the good. Thus, agents with larger offers experience higher return rates and those having higher return rates are who endowed with larger endowments under the sell-all strategy. Having a higher return rate for bids implies one needs less amount of money to buy the same amount of good. In other words, those with a higher return rate can purchase a good at a lower effective price. This result implies intertemporally optimizing consumers will transfer wealth to the age with the highest endowment and the lowest effective price to increase the consumption. Thus, the consumption growth closely parallels income growth at the optimal stationary equilibria. Note that even when agents face the same effective prices over two ages, they might consume different amounts because of the money holdings on their hands (required by clearing the money market) which differentiates the bidding power of agents. Looking at the allocation cross-sectionally in a fixed time, ages with higher endowments are given lower effective prices compared to other overlapped generations and thus, they receive a larger share of consumption.

It is worth recognizing how the price works differently in the economy with the strategic interaction compared to the competitive economy. In the latter one, agents face the same price and price adjusts to balance the purchasing power of different generations so that the perfect consumption smoothing can be attained. However, the trading mechanism via bidding in the Shapley-Shubik market game lets the market power determine prices rather than prices change to balance the market power.

To be consistent with the age/income profile in the data, we restrict our attention to an inverse-V shape endowment distribution given by \( (\omega_1, \omega_2, \omega_3) \) where \( \omega_2 > \omega_1 > \omega_3 = 0 \). The following corollary summarizes some interesting features under such endowment structure.

**Corollary 1.** With an inverse-V shape endowment distribution, the consumption allocation also shows the inverse-V shape distribution. \( c_2 > \frac{1}{3} (\omega_1 + \omega_2) \), \( c_3 < \frac{1}{3} (\omega_1 + \omega_2) \) and \( c_1 \geq \frac{1}{3} (\omega_1 + \omega_2) \).
Proof. According to Proposition 2, the consumption is proportional in the endowment at the optimal stationary equilibria. Thus, when \( \omega_2 > \omega_1 > \omega_3 \), then \( c_2 > c_1 > c_3 \). Assuming \( c_2 \leq \frac{1}{3} (\omega_1 + \omega_2) \), \( c_1 < \frac{1}{3} (\omega_1 + \omega_2) \) and \( c_3 < \frac{1}{3} (\omega_1 + \omega_2) \). This is a contradiction because \( c_1 + c_2 + c_3 < \omega_1 + \omega_2 \) which means the good market clearing condition is violated. Likewise, \( c_3 < \frac{1}{3} (\omega_1 + \omega_2) \). If not, the good market clearing condition is violated as well. \( c_1 \) can be larger and smaller than \( \frac{1}{3} (\omega_1 + \omega_2) \) which depends on \( \omega_1 \) and the utility function, \( u(c) \).

This result implies the consumption/income parallel arises in the overlapping generations model with patient intertemporally optimizing consumers and without capital market imperfections such as liquidity constraint and precautionary saving motive if there instead exists the imperfect competition on the good market.

4 Quantitative Evaluation

In this section, we quantitatively show how the imperfect competition reduces the risk-sharing between states among overlapped generations when the financial market is incomplete against an idiosyncratic shock. The consumption inequality is more volatile than the incomplete market with the perfect competition which adds additional welfare loss. Then, we study the risk-sharing with an aggregate shock where overlapped generations are affected by the exogenous uncertainty in the same direction.

4.1 The Economy with an Idiosyncratic Shock

The reason why we focus first on the idiosyncratic shock is that, under the shock, the perfectly smoothed consumptions over ages and states are the optimal allocation of the social planner under the utilitarian social welfare function. Thus, we can easily analyze the welfare loss in three cases compared to the benchmark allocation: an economy with the only incomplete market, only imperfect competition, and both friction. This analysis helps us to examine whether there is a complementarity effect between the two friction in terms of the welfare loss.

For a numerical analysis, we study a parameterized version of the model in order to investigate the nature of the recursive Markovian Nash monetary equilibria in this model. (We will extend our analysis to several parameterized versions to check the robustness of the results here.) The utility function is assumed to be the constant relative risk aversion utility function, \( u(c) = \frac{c^{1-\sigma}}{1-\sigma} \). The time-preference parameter \( \delta = 1 \). In the deterministic version of the model, we assume the total endowment is 1. To be consistent with the age-income profile in the data, the shares of the endowment are \( \frac{3}{8}, \frac{5}{8} \) and 0 for the young, the middle-aged and the old respectively. We assume there are only one agent in each generation to reveal the effect of market power on the risk-sharing, i.e. \( n = 1 \). The total money quantity, \( \bar{m} \), is normalized to be 1. The idiosyncratic shock on the endowment follows a simple I.I.D. Bernoulli process with \( \pi^\alpha = 0.5 \) and \( \pi^\beta = 0.5 \). The shock is an additive shock to the endowment where \( \{\theta^\alpha_1, \theta^\alpha_2, \theta^\alpha_3\} = \{-\frac{1}{8}, \frac{1}{8}, 0\} \) and \( \{\theta^\beta_1, \theta^\beta_2, \theta^\beta_3\} = \{\frac{1}{8}, -\frac{1}{8}, 0\} \). Thus, \( \{\omega^\alpha_1, \omega^\alpha_2, \omega^\alpha_3\} = \{\frac{2}{8}, \frac{6}{8}, 0\} \) and \( \{\omega^\beta_1, \omega^\beta_2, \omega^\beta_3\} = \{\frac{4}{8}, \frac{4}{8}, 0\} \). Note in the structure of the idiosyncratic shock, the overlapped young and middle-aged are inversely affected by the exogenous uncertainty.
For a numerical welfare analysis, we compute the recursive Markovian Nash equilibria under the parametrization (and the algorithm computing suboptimal dynamic equilibria will be written in the appendix). Specifically, we simulate the economy for 21,000 periods and ignore the first 1000 periods to avoid the effect of initial conditions on the results. The distribution of allocations generated by the competitive equilibrium is ergodic, and thus time averages and cross-sectional averages will be the same. This allows us to calculate the ex-ante expected utility of the Markovian equilibria with the simulation data. There are two things to stress out in the tables below. The consumption values in each state under the shock are mean values over the endogenous state variables conditional on the current state. However, we use the entire simulation data when computing the mean and standard deviation of consumptions in each age.

Table 1: The basic economy with $\sigma = 2$

<table>
<thead>
<tr>
<th>Age</th>
<th>State $\alpha$</th>
<th>State $\beta$</th>
<th>Mean</th>
<th>S.D/Mean (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark optimal equilibrium</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.0000%</td>
</tr>
<tr>
<td>Middle-aged</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.0000%</td>
</tr>
<tr>
<td>Old</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.0000%</td>
</tr>
<tr>
<td>Equilibrium consumption only with imperfect competition</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young</td>
<td>0.3286</td>
<td>0.3286</td>
<td>0.3286</td>
<td>0.0000%</td>
</tr>
<tr>
<td>Middle-aged</td>
<td>0.4007</td>
<td>0.4007</td>
<td>0.4007</td>
<td>0.0000%</td>
</tr>
<tr>
<td>Old</td>
<td>0.2707</td>
<td>0.2707</td>
<td>0.2707</td>
<td>0.0000%</td>
</tr>
<tr>
<td>Equilibrium consumption only with incomplete market</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young</td>
<td>0.2884</td>
<td>0.3537</td>
<td>0.3211</td>
<td>10.17%</td>
</tr>
<tr>
<td>Middle-aged</td>
<td>0.3970</td>
<td>0.2834</td>
<td>0.3402</td>
<td>16.70%</td>
</tr>
<tr>
<td>Old</td>
<td>0.3147</td>
<td>0.3629</td>
<td>0.3388</td>
<td>7.11%</td>
</tr>
<tr>
<td>Equilibrium consumption with both friction</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young</td>
<td>0.2612</td>
<td>0.3764</td>
<td>0.3188</td>
<td>18.07%</td>
</tr>
<tr>
<td>Middle-aged</td>
<td>0.5034</td>
<td>0.3167</td>
<td>0.4101</td>
<td>22.77%</td>
</tr>
<tr>
<td>Old</td>
<td>0.2354</td>
<td>0.3069</td>
<td>0.2712</td>
<td>13.18%</td>
</tr>
<tr>
<td>Benchmark</td>
<td>0.3333</td>
<td>0.3249</td>
<td>0.3237</td>
<td>0.3104</td>
</tr>
<tr>
<td>Imp. Comp</td>
<td>100%</td>
<td>97.12%</td>
<td>97.48%</td>
<td>93.13%</td>
</tr>
<tr>
<td>Inc. Mk</td>
<td>100%</td>
<td>97.03%</td>
<td>97.41%</td>
<td>92.63%</td>
</tr>
</tbody>
</table>

- S.D/Mean (%) is the standard deviation of consumption between states divided by its mean measured in %
- C.E. represents certainty equivalent

Table 1 exhibits the competitive equilibria for the parameterized version of this model. Several features of the monetary equilibria in Table 1 are worth discussing. The benchmark optimal
allocation of the social planner is identified by the perfect consumption smoothing over ages and states since there are no aggregate fluctuations in the economy. Welfare losses in an economy with friction are evaluated with two measures. One is the certainty equivalent consumption loss as the percentage of the benchmark allocation. The other one is the ex-ante expected utility loss as the percentage of the benchmark allocation.

Looking at the equilibrium consumption under the imperfect competition, there is a consumption volatility over ages which is consistent with the analytical result in Section 3. Indeed, the consumption is parallel to the income, not smoothed over ages. Since we focus on the inverse-V shape endowment, the consumption is peaked when middle-aged and it is lower in the other two periods. Such consumption volatility over ages arises because agents face different effective prices in different ages depending on their endowment offered to the central market. When middle-aged, the effective price of the good is the lowest and thus, the young agents transfer money to increase the consumption in the next period. The deviation from the perfect consumption smoothing yields a welfare loss about 3% under the two measures in this parameterized version of the model.

In contrast, the average consumptions conditional on the ages are quite stable in an economy with the incomplete market since different ages face the same prices for the good in this economy. However, the consumption allocations are volatile between states because the fiat money alone cannot hedge the birth-date risk and successive exogenous uncertainty. We calculate the consumption volatility measure in the fifth column for each age by dividing the standard deviation of consumption in the simulation data conditioning on age with its mean. For the young, the consumption is lower in state \( \alpha \) but higher in state \( \beta \) because the idiosyncratic shock is unfavorable to the young in state \( \alpha \) and the middle-aged in state \( \beta \). Thus, the consumption for the middle-aged is higher in state \( \alpha \) but lower in state \( \beta \). This imperfect risk sharing under the incomplete market generates about 2.6% of welfare loss measured in both the certainty equivalent consumption loss and the loss of the expected utility. Note that the consumption volatility between states is the highest when middle-aged and is the lowest when old.

In an economy with both frictions, the equilibrium consumptions are volatile over both ages and states as expected. The most interesting part of this case is the consumption inequality between states fixing ages is larger than its counterpart in the economy with only the incomplete market. Specifically, the within-age consumption volatility increases roughly 8% for the young and 6% for the middle-aged and the old when adding the imperfect competition on the top of the incomplete market. This additional volatility in consumption increases the welfare loss by reducing the risk sharing between states among overlapped generations. Thus, the total welfare loss in this economy is approximately 7.4% which is 1.8% larger than the sum of welfare losses from each friction. This numerical exercise shows the evidence of the supermodularity between the two friction in terms of welfare loss.

The imperfect competition makes the equilibrium consumption more volatile by setting the state-dependent effective prices in more favorable to the richer and less favorable to the poorer. In other words, the rich age can purchase the goods at a cheaper price whereas the poor age should pay more to buy the same amount of goods. Thus, the richer consume more and the poorer consume less in the economy with both friction than the economy only with the incomplete market where the poor and rich groups face the same price of the goods.

Under the sell-all strategy in the Shapley-Shubik market game, agents return back their own
bids partially and the return rates are proportional to their endowment share out of the aggregate endowment as seen in the budget constraints. Thus, the market mechanism essentially assigns a lower effective price of the good to those contributing more to the market via offering more endowments. The idiosyncratic shock negatively affects the endowments of the overlapped young and middle-aged. There are winning ages and losing ages in each state in terms of effective prices. The losing ages with a negative endowment shock face a higher effective price and consume less which is conflicting with the risk-sharing since they need a transfer from the winning ages to compensate their low endowments.

If agents are affected by the exogenous shock in the same direction, the effective prices of overlapped generations will move similarly. Thus, the risk-sharing will not reduce in this case as much as the economy with a negatively correlated shock to different ages.

4.2 The Economy with an Aggregate Shock

In this subsection, we examine the effective prices are not risk-diverging if the exogenous uncertainty impacts the endowments of the overlapped generations in the same direction. For this, we only alter the endowment structure as follows remaining other structural parameters:

\[
\{\theta_1^\alpha, \theta_2^\alpha, \theta_3^\alpha\} = \left\{-\frac{1}{8}, -\frac{1}{8}, 0\right\} \quad \text{and} \quad \{\theta_1^\beta, \theta_2^\beta, \theta_3^\beta\} = \left\{\frac{1}{8}, \frac{1}{8}, 0\right\}. \quad \text{Thus,} \quad \{\omega_1^\alpha, \omega_2^\alpha, \omega_3^\alpha\} = \left\{\frac{2}{8}, \frac{4}{8}, 0\right\} \quad \text{and} \quad \{\omega_1^\beta, \omega_2^\beta, \omega_3^\beta\} = \left\{\frac{4}{8}, \frac{6}{8}, 0\right\}.
\]

<table>
<thead>
<tr>
<th>Age</th>
<th>State $\alpha$</th>
<th>State $\beta$</th>
<th>Mean</th>
<th>S.D/Mean (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young</td>
<td>0.2505</td>
<td>0.3940</td>
<td>0.3223</td>
<td>22.27%</td>
</tr>
<tr>
<td>Middle-aged</td>
<td>0.2804</td>
<td>0.3577</td>
<td>0.3191</td>
<td>12.11%</td>
</tr>
<tr>
<td>Old</td>
<td>0.2191</td>
<td>0.4983</td>
<td>0.3587</td>
<td>38.92%</td>
</tr>
</tbody>
</table>

Equilibrium consumption with both friction

<table>
<thead>
<tr>
<th>Age</th>
<th>State $\alpha$</th>
<th>State $\beta$</th>
<th>Mean</th>
<th>S.D/Mean (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young</td>
<td>0.2383</td>
<td>0.3998</td>
<td>0.3191</td>
<td>25.31%</td>
</tr>
<tr>
<td>Middle-aged</td>
<td>0.3365</td>
<td>0.4369</td>
<td>0.3867</td>
<td>12.98%</td>
</tr>
<tr>
<td>Old</td>
<td>0.1751</td>
<td>0.4134</td>
<td>0.2943</td>
<td>40.49%</td>
</tr>
</tbody>
</table>

- S.D/Mean (%) is the standard deviation of consumption between states divided by its mean measured in %
- C.E. represents certainty equivalent

<table>
<thead>
<tr>
<th>Age</th>
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<th>State $\beta$</th>
<th>Mean</th>
<th>S.D/Mean (%)</th>
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<td>0.4983</td>
<td>0.3587</td>
<td>38.92%</td>
</tr>
</tbody>
</table>

Under the positively correlated aggregate shock, we also observe the consumption allocations are parallel to the endowment when agents have market power on the good price. Thus, the consumptions are volatile across ages. However, the imperfect competition does not enlarge the consumption inequality in each age as much as the economy with an idiosyncratic shock above. Indeed, the within-age consumption volatility increases roughly 3% for the young, 1% for the middle-aged and 1.5% for the old when adding the imperfect competition on the top of the incomplete market. These numerical results verify our conjecture that the positively correlated aggregate shock do not differentiate the effective prices of overlapped generations and
thus, the risk-sharing will not reduce severely when modeling the strategic interaction in this case.

5 Conclusion

In this paper, we study how the market power of buyers can generate a dispersion over effective prices depending on wealth in an economy with imperfect competition. We ask how the imperfect competition brings about the consumption/income parallel without capital market imperfection or impatient consumers. More importantly, we examine whether the strategic interaction reduces the risk sharing and increases the consumption volatility cross-sectionally in an incomplete market. In a numerical analysis, we quantify the welfare loss from the complementary effect between the two friction: incomplete market and imperfect competition. We find out the supermodular welfare loss takes about 2% out of the benchmark expected utility. Lastly, the structure of exogenous shocks determines the additional consumption inequality via imperfect competition. When the endowment of overlapped generations are positively correlated under the exogenous uncertainty, the consumption volatility does not grow as much as an economy with an idiosyncratic shock where some agents are favorable by the shock and others are not.
References


