Optimal Monetary Policy under Firms’ Rational Inattention*

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ABSTRACT

This paper derives optimal monetary policy under a framework where monetary policy reallocates firms’ attention to different shocks and change their beliefs about economic fundamentals. We label this effect of monetary policy as its “informational effect”. Effectiveness of monetary policy is jointly determined by its informational effect and direct effect. An aggressive response from the central bank may draw firms’ attention and dampen its real effect as firms are less surprised. Internalizing the attention-allocation mechanism, optimal monetary policy balances its direct and informational effect and is contingent on the uncertainty of the fundamental shocks and the total information capacity. With this new informational effect, there is a new source for trade-off between output gap stabilization and price dispersion stabilization: An aggressive response to stabilize output gap may generate high price dispersion, which harms welfare.

Keywords: Optimal monetary policy; Incomplete information; Rational inattention; Shannon capacity; Nominal demand management

JEL Classification: E3; E5; D8

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1 INTRODUCTION

When there is incomplete information, policy actions taken by central banks play a dual role: it directly affects the economic outcomes and indirectly affects the economy through an informational channel: changes in policy could shift private sectors’ beliefs about other economic fundamentals. It’s important to understand how this informational channel and its relationship with the direct channel influence the effects of monetary policy and add new implications for optimal monetary policy.

Following the class of noisy-information models as in Woodford (2001), Sims (2003), and Maćkowiak and Wiederholt (2009), this paper considers an information structure where firms only have limited capacity\(^1\) to process information and need optimally decide how much attention to allocate on each information source including monetary policy. Under different monetary policies, firms process information differently and allocate their attention on received signals differently. This additional informational effect of monetary policy makes its effectiveness state-dependent on firms’ total information capacity and the uncertainty of fundamentals in the economy.\(^2\) Under some states, a large response from the central bank might be as effective as a small one in influencing real output since the larger response attracts more attention from firms and comes out as less surprising. Furthermore, the additional informational effect provides another source for the trade-off between output gap stabilization and price dispersion stabilization. Aggressive responses from the central bank aiming at stabilize output gap increases the uncertainty of monetary policy and might generate high price dispersion, which harms welfare.

The normative analysis of this paper builds upon the model set-up in Adam (2007).\(^3\) In the economy, price is flexible and there are three types of agents: one representative household, a continuum of firms, and a policy-maker maximizing the welfare of the representative household. Uncertainty in the model comes from two kinds of fundamental shocks in the economy: one supply shock and one demand shock, with different implications for welfare.

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\(^1\)modeled by a finite Shannon (1948) capacity.

\(^2\)There has been many papers providing empirical evidence on the state-dependence of the effects of monetary policy. For example, Peersman and Smets (2001), Lo and Piger (2005), Caggiano et al. (2014) show that effects of monetary policy are different during recessions and blooms. and Tenreyro and Thwaites (2016). Also there are a few papers showing the effectiveness of monetary policy depend on macroeconomic uncertainty as in Aastveit et al. (2013) and disagreement about inflation expectation as in Falck et al. (2017). The result in this paper quantitatively relates to the latter category showing the effects of monetary policy depend some second order moments in the economy.

\(^3\)Differences from it will be discussed later in literature review.
The supply shock leads to fluctuations in both equilibrium output and efficient level of output, whereas the demand shock only affects the equilibrium output. Following Adam (2007), central bank’s policy instrument is nominal demand and responds to the above two fundamental shocks. Both the household and the central bank observe the fundamental shocks without noises. However firms have only limited information process capacity. This paper deviates from Adam (2007) on firms’ information set. In this paper, firms perceive the monetary policy as a shock and treat it equivalently as the other two fundamental shocks when allocating attention. Under the endogenous attention shifting mechanism, how firms process information and form expectations is jointly determined by the firms’ total information capacity, uncertainty of underlying fundamental shocks, and monetary policy. Different monetary policies indicate different levels of uncertainty of the policy, leading to firms’ different attention allocations. The major principle in allocating attention is the same as Maćkowiak and Wiederholt (2009): firms allocate more attention to the signals whose underlying processes are more uncertain. While deriving the optimal policy, central bank solves a Ramsey-type problem, internalizing how its policy interacts with this attention-allocation mechanism.

Under this set up, monetary policy affects the output gap through two channels. Firstly, the nominal demand directly affects the equilibrium output and this is labeled as “direct effect”. Secondly, monetary policy affects how firms allocate attention and thus how they set their prices optimally and this is labeled as “informational effect”. When firms pay more attention to one signal, their pricing decisions respond more to it. Combining these two channels, the effectiveness of monetary policy could be dampened if the central bank responds to the fundamental shocks aggressively. Intuitively, larger responses to the fundamental shocks lead to larger uncertainty about monetary policy in the private sector. Firms then pay more attention to the policy signal and the policy doesn’t come out as a large surprise and the real effect is weakened compared with its actual response. Furthermore, the ability of the central bank to stabilize output gap is determined by two factors: 1) Whether the uncertainty of the economy comes from the demand shock or the supply shock; 2) How large is the firms’ information capacity. Generally, if the uncertainty is mostly from the demand shock and the information capacity is high, it’s harder for the central bank to stabilize output gap. This is because the supply shock influences both equilibrium output and efficient level of output, while the demand shock only influences the equilibrium output. When the supply shock is very uncertain and firms pay much attention to it, the equilibrium output closely tracks the
efficient level of output, thus the output gap would be small. But when the demand shock is very uncertain and firms pay much attention to it, the equilibrium output responses to the demand shock actively nonetheless the efficient level of output stays the same, resulting in a large output gap. Thus from the central bank’s perspective, drawing firms’ attention away from the demand shock helps to stabilize output gap. And this is a harder task when the uncertainty from the demand shock is very large and firms stick their attention with the demand shock or the information capacity is high so that firms could easily spare some attention to other signals without decreasing their attention to the demand shock.

However, monetary policy affects price dispersion only through its informational effect. This is because without nominal rigidity in the model, price dispersion stems from information dispersion only. Since each firm processes information in a different way, limited information process capacity leads to idiosyncratic processing errors. These errors are taken into firms’ optimal price setting and are exactly how individual pricing deviates from aggregate price level, which is the price dispersion. Due to this feature, any monetary policy such that firms don’t pay attention to policy signal at all is optimal to stabilize price dispersion. If firms don’t pay attention to policy signal at all, firms don’t make idiosyncratic processing errors about the policy signal. Moreover, firms could save their information process capacity to achieve smaller idiosyncratic processing errors for other signals. In this way, price dispersion is optimally stabilized.

Generally there is trade-off between output gap stabilization and the price dispersion stabilization. And this trade-off stems from the informational effect of monetary policy: To stabilize output gap, the central bank might want to respond to shocks aggressively, but this could generate high price dispersion due to its information effect, which harms welfare.

This is not the first paper studying how monetary policy affects the incentives faced by the firms in allocating attention. Paciello (2012) considers a monetary policy rule with feedback on inflation and output and shows how the feedback component of monetary policy affects the attention allocation decision by firms. It uses the attention allocation mechanism to explain why inflation adjusts faster to aggregate technology shocks than to monetary policy shocks: firms pay more attention to the aggregate technology shock because it’s more volatile. It also emphasizes that the feed back of monetary policy to inflation and output amplifies this mechanism. Although sharing the similar attention allocation mechanism, the role of monetary policy in this paper is different. Firms in this paper don’t know the policy
rule and perceive the policy as an additional shock. When the coefficients of the rule change, the uncertainty with respect to monetary policy changes and shifts firms’ attention. Plus, this paper studies a different question and concentrates on the normative implication for policy analysis.

This paper also relates to a strand of literature following Adam (2007) to study optimal monetary policy under incomplete information. Adam (2007) studies the optimal monetary policy when firms have limited attention. He shows that under strong strategic complementarities and small persistence of economic shocks, monetary policy has strong real effects, making it optimal to stabilize the output gap; while under weak complementariness or sufficient large persistence, it’s optimal for the central bank to stabilize price level. This paper is mainly developed from Adam (2007) whereas the information structure here is endogenous. In that paper, the policy signal firms receive is an announcement from central bank independent of its policy action. This independence leaves the information structure exogenous from the policy. The Kalman gain in firm’s signal extraction problem is constant and independent of the policy. In this paper, policy actions affect how firms’ allocate attention and it has different welfare implications. Later on, Baeriswyl and Cornand (2010) combines the economy in Adam (2007) with the sticky information environment as in Mankiw and Reis (2002) to study optimal monetary policy when central bank explicitly take the policy’s signaling effects into effects. They show that the central banks need to distort its policy response to labor supply and mark-up shocks to optimally control the information it conveys and stabilizes the economy. The central banks have no incentive to provide more information about the mark-up shocks, because those shocks cannot by neutralized. The last result is similar to the result in this paper although the information environments are different: central banks have an incentive to draw firms’ attention away from the demand shock (same as the mark-up shock in their set-up). But again their information structure is exogenous. More recently, Paciello and Wiederholt (2013) extends Adam (2007) and studies the optimal monetary policy when the firms can choose how much attention they devote to different fundamental shocks. The information structure is thus endogenous. They show that under endogenous information set-up, complete price stabilization is optimal also in response to shocks that cause inefficient fluctuations under perfect information, which is different from the result under exogenous

\footnote{The exogeneity in information contrasts with the empirical finding of Andrade and Le Bihan (2013) that when agents update forecasts, they used different information sets.}
information structure where complete price stabilization is optimal only in response to shock that cause efficient fluctuations under perfect equilibrium. This paper differs from the last one in the sense that it explicitly considers the informational effect of the monetary policy by letting the firms receive an explicit signal about the policy and react to that policy. By developing this mechanism, the central bank’s policy design is constrained and distorted by how firms’ attend their attention.

Additionally, this paper is also related to a broader literature studying optimal monetary policy in incomplete information. Most of them are under an exogenous information structure, although the sources of incomplete information are different from each other. The first category studies optimal monetary policy when the central bank can not perfectly observe the current state or the economic variable such as Aoki (2003), Orphanides (2003), and Svensson and Woodford (2003). The second category studies optimal monetary policy when private agents have incomplete information. For example, Ball et al. (2005) studies the optimal monetary policy when firms can only update their information infrequently with an exogenous probability. Lorenzoni (2010) studies the optimal monetary policy under an informational set-up where the private agents receive noisy private and public signals about aggregate technology and the central bank doesn’t have superior information to tell apart shocks to fundamentals and shocks to the public noise (belief). Angeletos and La’O (2011) studies the optimal monetary policy under an environment where the information friction can lead to real rigidity.

2 THE ECONOMY

The economy in this paper is similar to the one in Adam (2007). There exists a representative agent, a continuum of monopolistically competitive firms, and a central bank. The price is flexible. The central bank is benevolent and maximizes the utility of the representative agent via controlling the nominal demand. Deviation from Adam (2007) is the information structure, which will be discussed in detail in section 3.

2.1 REPRESENTATIVE HOUSEHOLD

The representative household has complete information about this economy and chooses aggregate composite good $Y$ and labor supply $L$ in order to maximize his utility. The opti-
mization problem is given by:

\[ U(Y) - \nu V(L) \]
\[ \text{s.t. } WL + \Pi = PY + T \]

where \( W \) is the competitive wage, \( \Pi \) the monopoly profits from firms, and \( T \) the nominal transfers from the central bank.

\( Y \) is a composite good that is produced with the Dixit-Stiglitz aggregator:

\[ Y = \left[ \int_0^1 (Y_i)^{(\theta - 1)}/\theta d\theta \right]^{\theta/(\theta - 1)} \]

where \( \theta > 1 \) measures the price elasticity of demand and \( Y_i \) is the good produced by firm \( i \). \( P \) is the appropriate price index for the aggregate consumption which solves \( PY = \int_0^1 P_iY_idi \).

The price elasticity of demand, \( \theta \), is a random variable with \( E(\theta) = \bar{\theta} \) and induces variations in the desired mark-up of firms. The parameter \( \nu > 0 \) is a stochastic labor supply shifter with \( E[\nu] = 1 \). This shock will result in fluctuations in the efficient level of output.

Furthermore, \( U' > 0, U'' < 0, \lim_{Y \to \infty} U'(Y) = 0, V' > 0, V'' > 0, \text{ and } V'(0) < U'(0) \). The optimality condition from the household is the Euler equation between labor and output and as follows:

\[ W = \frac{\nu V'(L)}{U'(Y)} P \quad (2.1) \]

2.2 Firms

The production sector consists of a continuum of monoplistically competitive firms with incomplete information which are indexed by \( i \in [0,1] \).

Each firm \( i \) produces an intermediate good \( Y_i \) with labor input \( L_i \) according to a linear production function:

\[ Y_i = L_i \quad (2.2) \]

and chooses the price \( P_i \) to maximize its profit conditional on information set \( I_i \):

\[ \max_{P_i} E[(1 + \tau)P_iY_i(P_i) - WY_i(P_i)|I_i] \]
\[ \text{s.t. } Y_i(P_i) = \left( \frac{P_i}{P} \right)^{-\theta} Y \]
where $\tau$ is an output subsidy.

The first order condition is given by:

$$E \left[ (1 + \tau)(1 - \theta)\left(\frac{P_i}{P}\right)^{-\theta} + \theta W\left(\frac{P_i}{P}\right)^{-\theta-1} \mid I_i \right] = 0 \quad (2.3)$$

Combining the first order condition of firm $i$’s problem equation (2.3) with household’s optimality condition (2.1) as well as the production function (2.2) and linearizing it around the deterministic steady state delivers the firms’ optimal pricing rule $1$:

$$p_i = E[p + \xi(y - y^*) + u \mid I_i] \quad (2.4)$$

where lower case letters denote percentage deviation from the deterministic steady state.

The pricing rule says given the information set $I_i$, each firm $i$ sets price $p_i$ according to the aggregate price $p$, the output gap $y - y^*$, and the mark-up shock $u$, where the fluctuation in the efficient level of output $y^*$ is determined by the stochastic labor supply shifter $\nu$ and the fluctuation in the mark-up $u$ is a function of the demand shock $\theta$:

$$u \sim - (\theta - \bar{\theta}).$$

Firms will charge a higher mark-up ($u > 0$) whenever the price elasticity of demand $\theta$ is lower than its mean $\bar{\theta}$. A positive value of $u$ shows that the price becomes less inelastic. Further, $\xi = -U''(\bar{Y})\bar{Y} / U'(\bar{Y}) + V''(\bar{Y})\bar{Y} / V'(\bar{Y})$ measures how sensitive the optimal price is to output gap, where $\bar{Y}$ is the deterministic steady state of $Y$.

Since the nominal aggregate demand $q$ can be expressed as $q = p + y$, the firm $i$’s price-setting rule can thus be rearranged to:

$$p_i = E[(1 - \xi)p + \xi q - \xi y^* + u \mid I_i]; \quad (2.5)$$

For simplicity, we assume that the labor supply shock and the mark-up are independent

$1$See Appendix A.1 in Adam (2007) for detailed derivation.
and follow normal distributions:

\[ y^* \sim \mathcal{N}(0, \sigma^2_{y^*}) \]
\[ u \sim \mathcal{N}(0, \sigma^2_u) \]

In this paper, I follow Adam (2007) and refer \( y^* \) as supply shock and refer \( u \) as (real) demand shock, respectively. Also, we further assume that \( \zeta = 1 \) to avoid higher order beliefs caused by strategic interaction among the firms.\(^1\) So the pricing rule (2.5) becomes:

\[ p_i = E[q - y^* + u | I_i] \quad (2.6) \]

2.3 Central Bank

The central bank observes the fundamental shocks without noises and maximizes the utility of the representative household by adjusting the nominal demand \( q \) as a linear combination of the fundamental shocks. Accordingly, the variance of the policy also changes as the policy responses change. Its objective function is a second order approximation to the representative household’s utility function and it takes the household’s and firms’ decisions as given. The central bank’s problem is given by:

\[
\min_q E[(y - y^*)^2 + \gamma \int_0^1 (p_i - p)^2 di]
\]

subject to
\[
q = y + p \\
p = \int_0^1 p_i di \\
p_i = E[q - y^* + u | I_i]
\]

The central bank minimizes the loss from fluctuations in output gap and price dispersion. The weight on price dispersion \( \gamma \) is given by the mean of price elasticity of demand \( \bar{\theta} \).

\(^1\)This assumes away the strategic actions among firms as in the seminal paper by Morris and Shin (2002) and monetary policy cannot act as a coordination device, but guarantees the uniqueness of the firms’ attention allocation problem and the equilibrium. The unique equilibrium serves the constraints of a Ramsey-type problem for the central bank and allows for analytical optimal policy analysis. Also as to be introduced later, information structure in this paper assumes central bank communication is not common knowledge and thus doesn’t emphasize the difference between private signal and public signals.
2.4 TIMING OF EVENTS

The sequence of events taking place in the economy is as follows: (1) The central bank determines monetary policy rule; (2) Supply and demand shocks realize; (3) The central bank sets the policy according to its rule; (4) Firms process information about demand shock, supply shock, and the monetary policy and then set prices according to the rule (2.6); (5) Consumers demand products and productions take place.

3 INFORMATION STRUCTURE

In this section we describe the information structure of firms, which is the key part of model set-up that is different from Adam (2007).

3.1 FIRMS’ INFORMATION SET

We assume firms don’t know the policy rule and perceive it as a shock like the fundamental shocks.\(^1\) Although de facto the policy is the combination of the supply shock and the demand shock, private agents fail to build the connection when solving the signal extraction problem and making decisions. Each firm \(i\) receives noisy signals of the monetary policy \(^2\), the supply shock (efficient level of output), and the demand shock (mark-up shock):

\[
q_i = q + \epsilon^q_i \quad \text{where } q \sim N(0, \sigma^2_q) \quad \epsilon^q_i \sim N(0, \sigma^2_{eq})
\]

\[
y^*_i = y^* + \epsilon^{y^*}_i \quad \text{where } y^* \sim N(0, \sigma^2_{y^*}) \quad \epsilon^{y^*}_i \sim N(0, \sigma^2_{ey^*})
\]

\[
u_i = u + \epsilon^u_i \quad \text{where } u \sim N(0, \sigma^2_u) \quad \epsilon^u_i \sim N(0, \sigma^2_{eu})
\]

More specifically, firm \(i\)’s information set \(I_i = \{q_i, y^*_i, u_i; \sigma^2_q, \sigma^2_{y^*}, \sigma^2_u, \sigma^2_{eq}, \sigma^2_{ey^*}, \sigma^2_{eu}\}\). Firms fist

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\(^1\)This assumption guarantees the uniqueness of the firms’ attention allocation solution by assuming away correlations in signals and ensuring convexity of the information constraint (3.3). The economic consequence is that it prevents firms extracting information about the fundamentals from the policy rule which is labeled as “signaling effect” of monetary policy and studied in Baeriswyl and Cornand (2010) and Melosi (2016). Although abstracting from the signaling effect, this paper shows there is still an informational role played by the central bank through an attention-allocation mechanism.

\(^2\)The potential drawback of this assumption is that monetary policy is common knowledge. So it is directly observable or can be acquired with a very low cost. But private agents generally do not know the objectives of central bank and figuring out what monetary policy means might be hard and costly. This noisy signal set up could be understood as an abstraction to imperfect information process rather than information acquisition. At the same time, the instrument in this paper is the nominal demand instead of the nominal interest rate we usually use and it’s not directly observed.
use their signals to infer the shocks in the economy and then set the optimal price:

\[ p_i = E[q - y^* + u | I_i] \]

\[ = \frac{\sigma_q^2}{\sigma_q^2 + \sigma_{eq}^2} q_i - \frac{\sigma_{y^*}^2}{\sigma_{y^*}^2 + \sigma_{ey^*}^2} y_i^* + \frac{\sigma_u^2}{\sigma_u^2 + \sigma_{eu}^2} u_i \]  

(3.1)

Following the spirit of Maćkowiak and Wiederholt (2009), we assume the firms can decrease \( \sigma_{eq}^2, \sigma_{ey^*}^2, \) and \( \sigma_{eu}^2 \) to receive more accurate signals, but their ability of doing so is limited by an information constraint. We borrow the concept of mutual information from information theory to quantify this constraint.

Consider a general environment, if the information set is \( I_i \) and the vector of underlying shocks is \( \{Z\} \), the information constraint is defined as:

\[ I(Z; I_i) = H(Z) - H(Z | I_i) \leq K \]  

(3.2)

where \( I(\cdot) \) is the mutual information between \( Z \) and \( I_i \). It denotes how much information about \( Z \) can be inferred by observing \( I_i \). \( H(\cdot) \) is the entropy of a random vector, thus the mutual information can also be understood as the uncertainty about \( Z \) reduced by observing \( I_i \).

From the above specification of the firms’ information set, (3.2) becomes:

\[ \frac{1}{2} \log_2 \left( \frac{\sigma_q^2 + \sigma_{eq}^2}{\sigma_{eq}^2} \right) + \frac{1}{2} \log_2 \left( \frac{\sigma_{y^*}^2 + \sigma_{ey^*}^2}{\sigma_{ey^*}^2} \right) + \frac{1}{2} \log_2 \left( \frac{\sigma_u^2 + \sigma_{eu}^2}{\sigma_{eu}^2} \right) \leq K \]  

(3.3)

as long as there is limited information capacity (i.e., \( K \) is finite), the firms must be receiving noisy signals.

Larger \( k \)'s are paired with smaller \( \sigma_e^2 \)'s. This can be illustrated as: when firms pay more attention to one signal, that signal becomes less noisy to the firms. Or equivalently, firms have a more accurate signal. The small \( k \)'s can also be understood as the information flow between a single signal and its corresponding shock. The information constraint states that when the shocks are independent and normally distributed, the information flow between the set of signals and the shocks is the sum of that between each individual signal and the corresponding shock.
Furthermore, with $k_q$, $k_{y^*}$, and $k_u$ defined as above, the price optimally set by a firm (3.1) can be rewritten in a more intuitive and compact way:

$$p_i = (1 - 2^{-2k_q})q_i - (1 - 2^{-2k_{y^*}})y^*_i + (1 - 2^{-2k_u})u_i$$

which says when firms pay more attention to one signal, their prices respond more to the corresponding signals. And this then transmits into the real economy and will be discussed at length in Section 4.

### 3.2 Firms’ attention allocation

The firms can choose the optimal allocation of $k_{y^*}$, $k_u$, and $k_q$ on different signals within the information constraint (3.3). Depending on the uncertainty about the shocks and the policy, firms choose the attention allocation that generates the highest profit. Firms solves the following optimization problem to allocate their attention:

$$
\begin{align*}
\min_{k_{y^*}, k_u, k_q} & \quad E[(p_i - p^*)^2] \\
\text{subject to} & \quad k_{y^*} + k_u + k_q \leq K \\
& \quad p_i = (1 - 2^{-2k_q})q_i - (1 - 2^{-2k_{y^*}})y^*_i + (1 - 2^{-2k_u})u_i \\
& \quad k_{y^*} \geq 0 \\
& \quad k_u \geq 0 \\
& \quad k_q \geq 0
\end{align*}
$$

where $p^* = q - y^* + u$ is the price firms set under complete information. The objective is a second order approximation of the firms’ profit maximization problem. To maximize its profit, each firm optimally allocates attention by setting the price as close as the optimal price under complete information.

**Proposition 1.** Given the uncertainty about policy $\sigma_q^2$, uncertainty about supply shock $\sigma_{y^*}^2$, and uncertainty about mark up shock $\sigma_u^2$, firms’ optimal attention allocation is unique.

**Proof.** We rewrite the above optimization problem (3.5) by plugging $p_i$ and $p^*$ into the objec-
tive and have:

$$\min_{k_y, k_u, k_q} \sigma_q^2 2^{-2k_y} + \sigma_y^2 2^{-2k_y} + \sigma_u^2 2^{-2k_u}$$

subject to

$$k_y + k_u + k_q \leq K$$

$$k_y \geq 0$$

$$k_u \geq 0$$

$$k_q \geq 0$$

(3.6)

The proof is then straightforward. Since objective function is continuous and strictly convex and the constraint set is convex, the solution to the optimization problem (3.6) is unique. Thus the solution to the equivalent problem (3.5) is also unique.

Based on how many signals firms are paying attention to and the partitioned space of the exogenous variables ($K, \sigma_y^2, \text{and } \sigma_u^2$) and the policy ($\sigma_q^2$), we can divide the solution to (3.6) into seven cases. The complete solution to firms’ attention allocation problem is found in Appendix A. Figure 3.2 shows all seven cases. The horizontal axis is the ratio between uncertainty of mark-up shock and uncertainty of monetary policy ($\sigma_u^2 / \sigma_q^2$) and the vertical axis is the ratio between uncertainty of supply shock and uncertainty of monetary policy ($\sigma_y^2 / \sigma_q^2$). On the left panel, information capacity $K = 1$ and on the right panel, information capacity $K = 2$. The major principle of attention allocation is that firms allocate more information to the more uncertain underlying processes as shown in Maćkowiak and Wiederholt (2009). Case 1 is the case where firms pay attention to all three signals, since these three underlying processes are of similar uncertainty given the state space. Under case 2, 3, and 4, firms only pay attention to two signals since the underlying shocks to those two signals are much more uncertain than the omitted one. Under case 5, 6, and 7, the firms allocate all of their attention to one signal only because the underlying process (monetary policy, mark-up shock, and supply shock respectively) is extremely uncertain compared with the other two.

Figure 3.2 also shows how the information capacity $K$, the relative uncertainty of the two fundamental shocks $\sigma_y^2 / \sigma_u^2$, and the uncertainty of monetary policy $\sigma_q^2$ jointly affect firms’ attention. Information capacity $K$ determines the intersections of those curves: a larger $K$

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1 Throughout the paper, I use Arabic numbers to denote the firms attention allocation solution cases. I later use Roman numbers to denote the partition about the exogenous state space to discuss the optimal policy.
increases the interior solution space (case 1) as shown in the right panel. The relative uncertainty of the two fundamental shocks $\sigma_{y^*}^2 / \sigma_{u}^2$ determines possible cases for a firm to allocate attention. For example, if $\sigma_{y^*}^2 = \sigma_{u}^2$, the unique solution of firms attention allocation problem must be case 5, case 1, or case 4. What eventually pins down firms’ attention allocation is monetary policy. This is the informational role played by monetary policy: it re-allocates firms’ attention to different signals and as a result firms’ optimal price changes accordingly. However, the ability of monetary policy to shift firms’ attention is restricted and depends on the nature of the economy. Following the previous case when $\sigma_{y^*}^2 / \sigma_{u}^2 = 1$, it’s impossible for monetary policy to adjust so that firms only pay attention to mark-up shock as in case 6 or only pay attention to supply shocks as in case 7.

Figure 1: The role of monetary policy in determining firms’ attention allocation

Note: $K$ determines the intersections of curves - a larger $K$ increases the interior solution space as shown in the right panel; $\sigma_{y^*}^2 / \sigma_{u}^2$ determines the candidate cases for policy, for example if $\sigma_{y^*}^2 / \sigma_{u}^2 = 1$, the possible cases are case 5, case 1, and case 4; Policy $\sigma_{y^*}^2$ pins down the final optimal attention allocation. If the policy is very uncertain, then firms end up paying attention as in case 5 and if it’s quite certain, firms pay attention according to case 4. If it’s comparable to other shocks, firms’ attention allocation falls in the interior solution case 1.

\footnote{Note the scales are very different in the two panels.}
4 EQUILIBRIUM

Denote the solution to the optimal attention allocation as \( k_y^*, k_u^*, k_q^* \), each firm sets its price as follows:

\[
p_i = (1 - 2^{-2k_q^*})q_i - (1 - 2^{-2k_y^*})y_i^* + (1 - 2^{-2k_u^*})u_i
\] (4.1)

Then the aggregate price follows:

\[
p = \int_0^1 p_i di = (1 - 2^{-2k_q^*})q - (1 - 2^{-2k_y^*})y^* + (1 - 2^{-2k_u^*})u
\] (4.2)

Combine the price with \( q = y + p \), we have the aggregate output:

\[
y = 2^{-2k_q^*}q + (1 - 2^{-2k_y^*})y^* - (1 - 2^{-2k_u^*})u
\] (4.3)

**Proposition 2.** Together with some monetary policy rule \( q = \phi_y y^* + \phi_u u \), (4.2) and (4.3) uniquely characterize the equilibrium of the economy.

**Proof.** The proposition follows from proposition 1. Solving firms’ attention allocation problem gives unique \( k_y^*, k_u^*, \) and \( k_q^* \), thus the equilibrium can be solved as a unique function of the exogenous fundamental shocks, given some policy rule. \( \square \)

5 OPTIMAL RESPONSE TO SHOCKS

In an endogenous information environment, the central bank’s problem is no longer linear-quadratic since the policy \( q \) can affect \( k_y^*, k_u^*, \) and \( k_q^* \). Thus the genuine optimal policy is not a linear combination of the shocks any more. The rest of the paper will concentrate on the optimal responses to the fundamental shocks only, specifying a rule in the form of \( q = \phi_y y^* + \phi_u u \).

\[1\] As discussed previously, we assume that firms don’t know the exact rule and only know \( \sigma_q^2 \). Adjustments in the policy rule lead to changes in the uncertainty of policy and alter the firms’ attention allocation.
With the specification, the central bank’s problem becomes

\[
\min_{\phi_y, \phi_u} E[(y - y^*)^2 + \gamma \int_0^1 (p_i - p)^2 di]
\]

subject to
\[
q = y + p
\]
\[
p = \int_0^1 p_i di = (1 - 2^{-2k_i^q})q - (1 - 2^{-2k_i^y})y^* + (1 - 2^{-2k_i^u})u
\]
\[
q = \phi_y y^* + \phi_u u
\]

Discussion about the role of monetary policy

The central bank has two channels to affect the equilibrium of the economy. First, it works through a traditional direct channel: it directly affects aggregate price level and real output as shown in (4.2) and (4.3). More importantly, there is an additional informational channel of monetary policy. Different responses from the central bank to exogenous shocks embed different levels of policy uncertainty, which determine how firms allocate their attention when they cannot perfectly observe the state of the economy and have to solve the signal extraction problem. Monetary policy changes firms’ signal extraction problem and pricing behavior, which eventually transmits into the equilibrium.

Furthermore, these two channels have different implication to achieve central bank’s objective. While both direct effect and informational effect influence output stability, the price dispersion stability is only influenced by the informational channel. The following sections discuss how these two channels affect the output stability, the price dispersion stability, and their trade-off in details.

5.1 The limiting complete information case: \( K = \infty \)

Before we go into the incomplete information, we first look at the limiting complete information case and this serves as a natural benchmark. When the economy converges to the complete information case as \( K \) goes to \( \infty \), firms pay perfect attention to every signal, i.e., \( k_{y_i}^* = \infty, k_u^* = \infty, \) and \( k_q^* = \infty \). The equilibrium is as follows:

\[
p = q - y^* + u \quad (5.1)
\]
\[
y = y^* - u \quad (5.2)
\]
Then we are back in the conventional result: without nominal rigidity and incomplete information, there is no real effect of monetary policy. Meanwhile, there is complete information and all firms price in the same way. Price dispersion is then zero and monetary policy doesn’t play a role to stabilize price dispersion either. The minimized loss from central bank is \( E(y - y^*)^2 = \sigma_u^2 \) and it’s purely from output gap fluctuations\(^1\).

5.2 The incomplete information case: \( K \) is finite

In this section, we first discuss the optimal monetary policy purely aiming at stabilizing output or stabilizing price dispersion and then combine the two objectives. This allows us to understand how monetary policy works through different channels to achieve different objectives and where the trade-off between stabilizing output and price dispersion comes from when there is an additional informational effect of monetary policy.

5.2.1 Output Stability Only

The output gap is given by

\[
y - y^* = 2^{-2k_q}q - 2^{-2k_q^*}y^* - (1 - 2^{-2k_q^*})u
\]

\[
= 2^{-2k_q} (\phi_y y^* + \phi_u u) - 2^{-2k_q^*} y^* - (1 - 2^{-2k_q^*})u
\]

\[
= (2^{-2k_q} \phi_y - 2^{-2k_q^*}) y^* + (2^{-2k_q^*} \phi_u - (1 - 2^{-2k_q^*})) u
\]

Ceteris paribus, more attention to the supply shock and less attention to the demand shock help stabilize output gap. This is because the supply shock affects both equilibrium output and efficient level of output, while the demand shock affects only equilibrium output. This is consistent with Angeletos and Pavan (2007)’s result: Providing more information improves welfare to the extent that the equilibrium and the efficient level of output are symmetrically affected by shocks.

As for the effect of the policy, both direct channel and informational channel matter for output gap stability. More specifically, larger \(|\phi_y|\) and \(|\phi_y|\) indicate stronger reactions of the central bank to shocks. These large coefficients lead to larger uncertainty in monetary policy, driving firms’ attention to the policy signal (higher \(k_q^*\)). However with higher \(k_q^*\), the coefficient before the policy: \(2^{-2k_q}\) decreases. The effectiveness of monetary policy or the real effect

\(^1\)However, if the objective of central bank is to stabilize aggregate price level itself instead of price dispersion, then the optimal monetary policy is \( q = y^* - u \) under which the price level is also completely stabilized.
of monetary policy is dampened. This is due to the endogenous information structure. When there is more uncertainty about policy, the firms pay more attention to the policy signals. As a result, policies don’t come out as a large surprise for the firms, the real effect of the policy is largely weakened compared with the central bank’s actual response. In this sense, the central bank’s ability of affecting the real equilibrium is constrained by its own informational effect due to firms’ attention shifts. On the other hand, if the central bank responds to shocks moderately with smaller $|\phi_y|$ and $|\phi_u|$, the firms allocate a smaller attention to the policy signal (lower $k^*_q$). Still, the real effect of the policy is smaller than the central bank’s actual response, but it’s larger than the situation where firms pay more attention. This indicates that policy makers’ weaker reactions are paired with larger real effects. In the extreme case of firms paying no attention to the policy signal ($k^*_q = 0$), response from central bank is reflected in real output on a one-for-one scale and the real effect is maximized.

Because of the dual role of policy on equilibrium output, the optimal response to stabilize output gap should take both channels into consideration and achieve the optimal balance. The following proposition states the optimal response to stabilize output gap.

**Proposition 3.** When output gap stabilization is the only objective, central bank can always fully stabilize the output gap for any given parameter combination of $K$, $\sigma^2_u$, and $\sigma^2_y$*. Also, when firms are informationally small and the uncertainty is mostly from the efficient level of output gap instead of the mark-up shock, there exists multiple optimal responses for the central bank to fully stabilize output gap by manipulating its informational effect and direct effect.

**Proof.** See Appendix B. □

The complete characterization of the optimal responses from central bank to fully stabilize output gap is summarized by Table 1. From Case III to Case I when firms are more informationally constrained (smaller $K$) or the uncertainty is mostly from the efficient level of output (larger $\sigma^2_y$*) instead of the mark up shock (smaller $\sigma^2_u$), there are more ways for the central bank to fully stabilize the output gap or equivalently the central bank is more flexible in stabilizing the output gap. When firms are informationally large (big $K$) or the uncertainty is mostly from the mark up shock (big $\sigma^2_u$) instead of the efficient level of output (small $\sigma^2_y$) as in Case III, there is a unique response from the central bank to fully stabilize the output gap and we say the central bank is less flexible in stabilizing output gap or it’s more constrained to do so.
Table 1: Optimal Responses from Central bank to Fully Stabilize Output Gap

<table>
<thead>
<tr>
<th>State Space Partition</th>
<th>Optimal Responses from Central Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I ( \sigma^2_y / \sigma^2_u \in [2^{2K}, \infty) )</td>
<td>( \phi_y = 2^{-2K}, \phi_u = 0 )</td>
</tr>
<tr>
<td></td>
<td>( \phi_y = 2^{2K}, \phi_u = 0 )</td>
</tr>
<tr>
<td></td>
<td>( \phi_y = 1, \phi_u = 0 )</td>
</tr>
<tr>
<td>Case II ( \sigma^2_y / \sigma^2_u \in [2^{-2K}, 2^{2K}] )</td>
<td>( \phi_y = 2^{2K}, \phi_u = 0 )</td>
</tr>
<tr>
<td></td>
<td>( \phi_y = 2^{-K} \frac{\sigma_u}{\sigma_y}, \phi_u = 1 - 2^{-K} \frac{\sigma_u}{\sigma_y}, ) if</td>
</tr>
<tr>
<td></td>
<td>( 2^{-2K} - 2^{-K} \frac{\sigma_u}{\sigma_y} + (1 - 2^{-K} \frac{\sigma_u}{\sigma_y})^2 \leq 0 )</td>
</tr>
<tr>
<td>Case III ( \sigma^2_y / \sigma^2_u \in (0, 2^{-2K}] )</td>
<td>( \phi_y = 2^{2K}(1 - \sqrt{\Phi}), \phi_u = 2^{2K}(\sqrt{\Phi} - \Phi) )</td>
</tr>
<tr>
<td></td>
<td>where ( \Phi \equiv \max{2^{-2K} - \sigma^2_y / \sigma^2_u, (1 - 2^{-2K})^2} )</td>
</tr>
</tbody>
</table>

Note: From Case III to Case I when firms are more informationally constrained (smaller \( K \)) or the uncertainty is mostly from the efficient level of output (larger \( \sigma^2_y \)) instead of the mark up shock (smaller \( \sigma^2_u \)), there are more ways for the central bank to fully stabilize the output gap.

Specifically, under Case I when firms are informationally small (small \( K \)) and the uncertainty is mostly from the efficient level of output (large \( \sigma^2_y \)) instead of the mark up shock (small \( \sigma^2_u \)), there are three ways for the central bank to fully stabilize the output gap. Under this case, the efficient level of output is so uncertain and the firms are so informationally constrained that they would pay all of their attention to the efficient level of output without the influence of policy. The central bank could then react very gently to accommodate the fluctuation in efficient level of output. With this policy, firms’ attention remains on the efficient level of output. Thus firms themselves closely track the efficient level of output and ignore the mark-up shock. That’s why a policy with only a small response to the efficient level of output and no response to mark-up shock is enough to fully stabilize output gap. Alternatively, central bank could react very aggressively and draw all of firms’ attention to the policy signal and accommodates the shock at the same time by its direct channel. The last way for the central bank to fully stabilize the output gap is to respond one for one to the supply shock. This response is not as aggressive as the former one and only draws half of firm’s attention to the policy shock. Firms track the supply shock using the other half attention. Again, there is no need to accommodate the mark-up shock since firms are informationally small and the mark-up shock is relatively certain. Thus a moderate accommodation from central bank can fully stabilize output gap as well. This is an concrete example of the discussion above: Large responses to shocks from the central bank might draw “too much” attention from the firms.
and could end up being as effective as small responses.

Under Case II, firms are now relatively informationally larger and neither the uncertainty from the efficient level of output nor the uncertainty from the mark-up shock is dominating. Given the nature of uncertainty, firms now pay attention to both shocks. Thus the first way described above in Case I no longer works. But still, central bank could simply react aggressively and draw all of firms’ attention to the policy signal and accommodate the supply shock only as in Case I. Besides, under some situations, the central bank could purely accommodate both shocks to fully stabilize output without shifting firms’ attention. This is the case where only the direct channel works. And this is an example that sometimes central bank is able to fully stabilize the output gap without triggering its informational effect. However this approach is limited by the following constraint:

\[ 2^{-2K} - 2^{-K} \frac{\sigma_y^*}{\sigma_u^2} + (1 - 2^{-K} \frac{\sigma_y^*}{\sigma_u^2})^2 \leq 0 \]  

(5.4)

Fix \( K \), the smaller is \( \frac{\sigma_y^2}{\sigma_u^2} \), the more likely the constraint is violated. This is because when the mark up shock is large, firms tend to pay much to it and central bank needs to react more aggressively to accommodate it, which leads to a large informational effect. On the other hand, when fixing the value of relative size of fundamental shocks, the higher total information capacity \( K \) is, the more likely the constraint is violated. This is because, when the allocatable information for the firms is higher, it would be easier for firms to spare some extra attention to the policy signal even the policy is not aggressive. The above analysis reflects the fact that central bank is essentially constrained by the firms’ endogenous attention allocation mechanism, or its own informational effect.

Under Case III where firms are informationally large and the uncertainty is mostly from the mark-up shock, there is a unique way to fully stabilize the shock. That is to accommodate both and the exact magnitude of the response to each shock depends on total information capacity and the nature of uncertainty. Now the aggressive response way to draw all firms’ attention to the policy signal in the above two cases doesn’t work any more. Because when the uncertainty is mostly from the mark-up shock, to distract firms’ attention from the mark-up shock the central bank needs to react extremely aggressively. Furthermore, firms’ high information capacity amplifies the effect because firms have more attention available to allocate. Thus stopping firms from paying attention to the mark up shock would not be optimal.
any more. This result is also consistent with the limiting case where information is complete as shown in Section 5.1, the output gap can never be fully stabilized.

To conclude, proposition 3 says that the central bank can fully stabilize the output gap by carefully combining its direct channel and the informational channel. A large response to the shocks shifts the firms’ attention to the policy signal and dampens the real effect of the policy. A small response to the shocks eventually can be as effective as a large response to shocks as due to its relatively large real effect. Furthermore, the capability and the flexibility of the central bank to fully stabilize the output gap depends on the total information capacity and the source of the uncertainty in the economy.

5.2.2 Price dispersion stability only

In this section we concentrate on the case when central bank’s objective is to stabilize output gap only. The price dispersion and the loss from price dispersion are as follows:

\[
 p_i - p = (1 - 2^{-2k_q^*})\varepsilon_i^q - (1 - 2^{-2k_y^*})\varepsilon_i^y + (1 - 2^{-2k_u^*})\varepsilon_i^u \tag{5.5}
\]

\[
 E(p_i - p)^2 = (2^{-2k_q^*} - 2^{-4k_q^*})\sigma_q^2 + (2^{-2k_y^*} - 2^{-4k_y^*})\sigma_y^2 + (2^{-2k_u^*} - 2^{-4k_u^*})\sigma_u^2 \tag{5.6}
\]

The above equations say without nominal rigidity, the only source of price dispersion in the economy is dispersed information. Since firms have limited information processing capacity, they receive noisy signal and their pricing decisions based on these noisy signals are different from each other. As for the role of monetary policy, different from output stabilization, only the informational channel works for price dispersion stabilization objective. Specifically, monetary policy affects the loss from price dispersion only through its indirect influence on \(\sigma_q^2\), but not the direct effect of \(\phi_y\) and \(\phi_u\).\(^1\)

**Proposition 4.** When price dispersion stabilization is the only objective, any responses to shocks from the central bank such that the firms don’t pay attention to policy signal are optimal. Attention allocation between supply shock and mark-up shock is determined by the relative size of those shocks.

\(^1\)It’s also worth noting that the objective of the central bank is to stabilize price dispersion instead of the aggregate price level. This is different from Woodford and Walsh (2005) where with Calvo pricing, stabilizing price dispersion is the same as stabilizing the aggregate price level. Now stabilizing aggregate price level cannot be approximated from stabilizing price dispersion which is the direct result from second order approximation of the representative household’s utility function. We will stick with the micro-founded welfare objective and stabilize price dispersion as in Paciello and Wiederholt (2013). Also, although discussing this difference is beyond the scope of this paper, there is some empirical evidence from Sheremirov (2015) and Nakamura et al. (2016) showing that high inflation doesn’t always co-exist with high price dispersion.
Moreover, price dispersion can not be fully stabilized and the optimal response is not unique.

Proof. See Appendix C. \hfill \Box

When price dispersion stabilization is the only objective, Figure 5.2.2 describes how firms’ attention is allocated under the optimal policy. Total information capacity $K$ in the graph is 1. The x-axis is the ratio between the variance of the efficient level of output and the variance of the mark up shock $\sigma^2_y / \sigma^2_u$.

When $\sigma^2_y / \sigma^2_u \leq 2^{-2K}$ where the uncertainty is mostly from mark-up shock, any responses from the central bank such that firms pay all their attention to the mark-up shock are optimal. The loss under optimal response is $(2^{-2K} - 2^{-4K})\sigma^2_y$.

When $\sigma^2_y / \sigma^2_u \in [2^{-2K}, 2^{2K}]$ where the uncertainty of neither shock is dominating, any responses such that the firms don’t pay attention to the policy signal are optimal. In this case, the allocation of attention between the efficient level of output and the mark-up shock is determined by their relative uncertainty. When mark-up shock is more uncertain, under the policy firms pay more attention to the mark up shock and vice versa. The minimized loss under optimal responses in this case is $2 \cdot 2^{-2K} \sigma_u \sigma_y - 2^{-2K} \sigma^2_u - 2^{-2K} \sigma^2_y$.

When $\sigma^2_y / \sigma^2_u \geq 2^{2K}$ where the uncertainty is mostly from the supply shock, responses such that the firms pay full attention to the supply shock are optimal. The loss under optimal response is $(2^{-2K} - 2^{-4K})\sigma^2_u$.

To sum up, proposition 4 says for price dispersion stabilization, the necessary condition is firms don’t pay attention to the policy signal at all. This is because in this environment without nominal rigidity, the only source for price dispersion is information dispersion. As long as information is not dispersed, even when firms process information very inaccurately (but every firm is inaccurate in the same way), it still results in a small price dispersion. Thus to stabilize the price dispersion, the optimal strategy for the central bank is to respond very weakly to the fundamental shocks so that firms don’t pay attention to the policy shock. Consequently, there is no large uncertainty about the policy and firms can invest all of their attention to signals about the economic fundamentals to achieve a small price dispersion. The optimal policy to stabilize price dispersion thus is the one with minimized informational effect.

\begin{align*}
1 & k^*_q = 0, k^*_y = 0, \text{ and } k^*_u = K \\
2 & k^*_q = 0, k^*_y = \frac{1}{2} K + \frac{1}{4} \log 2(\sigma^2_y / \sigma^2_u), \text{ and } k^*_u = \frac{1}{2} K - \frac{1}{4} \log 2(\sigma^2_y / \sigma^2_u) \\
3 & k^*_q = 0, k^*_y = K \text{ and } k^*_u = 0
\end{align*}
Figure 2: Firms’ Attention Allocation under Optimal Policy to Stabilize Price Dispersion

Note: $K = 1, \sigma^2_y / \sigma^2_u$ on x-axis. When the mark up shock is very uncertain $(\sigma^2_y / \sigma^2_u \leq 2^{-2K})$, monetary policy such that firms paying full attention to the mark up shock is optimal for price dispersion. When the supply shock is very uncertain $(\sigma^2_y / \sigma^2_u \geq 2^{-2K})$, monetary policy such that firms paying full attention to the supply shock is optimal for price dispersion. For the cases in the middle, how firms allocate attention between the supply shock and the mark up shock is determined by its relative uncertainty. But overall, monetary policy that doesn’t attract any attention is optimal for stabilizing price dispersion.

Also, as stated in the proposition, the response to optimally stabilize price dispersion from central bank is not unique. Multiple solutions appear here for a different reason from the previous case of output stability only. The informational effect of monetary policy determines price dispersion via the influence of $\sigma^2_q$, but $\sigma^2_q$ itself cannot pin down $\phi_y$ and $\phi_u$ uniquely. Moreover, there exists a range for $\sigma^2_q$ (as long as it’s small enough) such that the informational effect is minimized.

The following lemma considers the loss(welfare) from price dispersion under the optimal policy and shows it’s not monotone in total information capacity $K$.

**Lemma 1.** Under the optimal policy, there exists a level of information capacity $K^*$ such that the loss from price dispersion is maximized. When $K < K^*$, more information availability increases the loss under optimal policy; when $K > K^*$, more information availability decreases the loss under optimal policy.

**Proof.** For Case II, $\text{Loss}|_{\sigma_q} = 2^{-K}(2\sigma_u \sigma_y - 2^{-K} \sigma_u^2 - 2^{-K} \sigma_y^2)$. To minimize the loss, we take derivative w.r.t. $K$ and the first order condition says $2^K = \frac{\sigma^2_u + \sigma^2_y}{\sigma_u \sigma_y}$. Furthermore $\frac{\partial \text{Loss}|_{\sigma_q}}{\partial K} > 0$, if $2^K < \frac{\sigma^2_u + \sigma^2_y}{\sigma_u \sigma_y}$, $\frac{\partial \text{Loss}|_{\sigma_q}}{\partial K} < 0$, if $2^K > \frac{\sigma^2_u + \sigma^2_y}{\sigma_u \sigma_y}$. 

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For Case I and Case III, \( K^* = \frac{1}{2} \). It’s directly from the FOC.

### 5.2.3 Output Stability and Price Dispersion Stability Combined

In this section, we put together the above discussions together. When the central bank’s objective consists of both price dispersion stabilization and output gap stabilization, whether there is a trade-off between price dispersion stabilization and output gap stabilization depends on the information capacity of the firms and the source of uncertainty.

**Proposition 5.** Generally, there is trade-off between price dispersion stabilization and output gap stabilization. The exceptions are the cases where either the efficient level of output or the mark-up shock is extremely uncertain, then optimal price dispersion stabilization and fully output gap stabilization can be achieved simultaneously.

**Proof.** See Appendix D.

In the following part, we discuss the trade-off between output stabilization and price dispersion stabilization in details.

When \( \frac{\sigma_y^2}{\sigma_u^2} \geq 2^{2K} \) that is uncertainty is mostly from the efficient level of output, full output gap stabilization and optimal price dispersion stability can be achieved simultaneously by setting \( \phi_y = 2^{-2K} \) and \( \phi_u = 0 \). Under this policy, firms pay all of their attention to the efficient level of output and ignore both the policy signal and the mark-up shock. Because they don’t pay attention to the policy signal at all, the price dispersion is minimized. Meanwhile, the firms themselves closely track the efficient level of output and the output gap is also minimized.

On the other end when \( \frac{\sigma_y^2}{\sigma_u^2} \leq 2^{-2K} - (1 - 2^{-2K})^2 \) that is uncertainty is mostly from the mark up shock, full output gap stabilization and optimal price dispersion stability can also be achieved simultaneously by setting \( \phi_y = 1 \) and \( \phi_u = 1 - 2^{-2K} \). Under this optimal policy, firms pay all of their attention to the mark up shock. Thus central bank needs to react to both shocks to stabilize output gap. However since the mark-up shock is extremely uncertain, firms tend to pay much attention to it, which makes it easier for central banks not to draw attention to the policy signal and guarantees price dispersion stability.

The above two cases show why when the uncertainty is mostly from one shock in the economy, there is no trade-off between output stability and price dispersion stability. Furthermore, when the firms are more informationally constrained, the above mechanism is re-
inforced. Because when firms are informationally small and the uncertainty is mostly from one shock, it’s natural for firms to stick to that shock, making it easier for the central bank to stabilize the output gap without attracting their attention.

As for the cases in the middle where the uncertainty from the two shocks is comparable (neither of them is too uncertain), there is generally trade-off between output gap stability and price dispersion stability. This is because now firms pay attention to both shocks, monetary policy trying to stabilize output gap easily attract their attention and generates high price dispersion. Again this mechanism is amplified when the total information capacity is high, since the opportunity cost for firms to spare some attention to the policy signal decreases.

A special case is when the following constraint is satisfied:

\[ 2^{-2K} - 2^{-K} \frac{\sigma_y^*}{\sigma_u^*} + (1 - 2^{-K} \frac{\sigma_y^*}{\sigma_u^*})^2 \leq 0 \]  

(5.7)

Then full output gap stability and optimal price dispersion stability can be achieved simultaneously by setting \( \phi_y = 2^{-K} \frac{\sigma_y^*}{\sigma_u^*} \) and \( \phi_u = 1 - 2^{-K} \frac{\sigma_y^*}{\sigma_u^*} \). The argument for why under this constraint, there is no trade-off is the same as in the previous Section 5.2.1.

Appendix E presents an numerical example of the complete description on central bank’s optimal policy when \( K = 1, \gamma = 1 \) and \( \sigma_y^2 / \sigma_u^2 \) varies. The result and the intuition is the same as the discussion above. Here in the main body, instead of fixing \( K \) and varying \( \sigma_y^2 / \sigma_u^2 \), I present Figure 3 and Figure 4 where \( \sigma_y^2 / \sigma_u^2 \) is fixed and show how total information capacity \( K \) changes the source of loss under optimal policy.

Figure 3 and Figure 4 show the cases where \( \sigma_y^2 / \sigma_u^2 = 5 \) and \( \sigma_y^2 / \sigma_u^2 = 1/5 \) respectively. \( K \) varies from 0.1 to 1. We exclude \( K = 0 \) to avoid the situation when firms have no information at all and set price at unconditional mean of the shocks 0.

The conclusions and mechanisms are the same as described in above. In Figure 3, when \( K \) is small, output stabilization and price dispersion stabilization are consistent. So the loss of price dispersion is the only loss and it’s maximized at the value at \( K^* \) as discussed in Lemma 1. When \( K \) gets large, there is a trade-off between stabilizing output and price dispersion. Fully output gap stabilization comes with a large cost from price dispersion. To minimize the total cost, price dispersion stabilization dominates and the loss mainly comes from output gap fluctuations. The loss from price dispersion decays fast when \( K \) increases. This is the effect of increasing \( K \), not because the central bank can now fully stabilize price dispersion. When
the firms’ total information capacity increases, they can process all signals more precisely at the same time and thus loss from price dispersion becomes smaller. The loss from output stability grows as $K$ increases. This is because when the information capacity is high, to achieve the same output gap stability, central bank needs to react more aggressively. However, it’s not worthwhile for the central bank to do so since aggressive response is paired with large informational effect and generates an very high welfare loss due to high price dispersion. When $K = \infty$, the central bank can do nothing at all for real output. Figure 4 shows similar results under the case when mark-up shock is more uncertain. Putting Figure 3 and Figure 4 together, we see that when mark-up shock is more uncertain, the trade off between output stabilization and price dispersion stabilization appears at a smaller level of total information capacity and the general loss is much larger.

![Figure 3: Optimal Responses and Loss Decomposition($\sigma_y^2 = 5, \sigma_u^2 = 1$)](image)

Notes: The graph is evaluated at the following parameter values: $\sigma_y^2 = 5, \sigma_u^2 = 1$, and $\gamma = 1$. $K$ on x-axis varies from 0.1 to 1. The upper panels present the optimal responses to the efficient level of output and the mark-up shock respectively. The lower left panel shows how firms allocate their attention to different signals under optimal monetary policy and is denoted in percentage of the total information capacity $K$. The lower right panel decomposes the total loss under optimal policy into loss from output gap and loss from price dispersion.
6 Conclusion

This paper uses a simple static model to analytically understand the informational role of monetary policy in forming private sectors’ expectation, its interaction with the traditional effect of monetary policy, and the corresponding implication for optimal monetary policy. The key information assumption is that firms can reallocate their attention on different shocks in response to policy actions. Internalizing this attention-allocation mechanism changes central bank’s optimal monetary policy substantially. When the central bank tries to accommodate the shocks, they need to take into consideration how its informational effect changes the effectiveness of its policy as well as the price dispersion. And this mechanism generates a new channel for trade-off between output stability and price dispersion stability.

However, there are a few drawbacks of this model. Firstly, it is a static. Secondly, this paper assumes $\xi = 1$ and is abstract from strategic actions among firms. This assumption makes the model disable of evaluating how policy signals affect the coordination among the firms. Third, it assumes that the firms don’t know the form of the policy rule. This limits the informational channel to working only through the attention-shifting mechanism, but not through signaling. The obstacle of incorporating the signaling effect into the model is that
correlated signals make the information constraint (3.3) lose convexity and the solution to firms’ optimal attention allocation lose uniqueness. Overcoming the above drawbacks and understanding this mechanism in a richer set-up is the direction of future work.
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Appendices

A Solution to Firms’ Attention Allocation Problem

Case 1: $2^{-4K} < \frac{\sigma_y^2}{\sigma_q^2} < 2^{2K}$, $2^{-2K} < \frac{\sigma_q^2}{\sigma_y^2} < 2^{4K}$, and $2^{-2K} < \frac{\sigma_y^2}{\sigma_q^2} < 2^{4K}$

\[
\begin{align*}
k_y^* &= \frac{1}{3} K + \frac{1}{3} \log 2 \frac{\sigma_y^2}{\sigma_q^2} - \frac{1}{6} \log 2 \frac{\sigma_u^2}{\sigma_q^2} \\
k_u^* &= \frac{1}{3} K + \frac{1}{3} \log 2 \frac{\sigma_u^2}{\sigma_y^2} - \frac{1}{6} \log 2 \frac{\sigma_y^2}{\sigma_q^2} \\
k_q^* &= \frac{1}{3} K - \frac{1}{6} \log 2 \frac{\sigma_u^2}{\sigma_y^2} - \frac{1}{6} \log 2 \frac{\sigma_y^2}{\sigma_q^2}
\end{align*}
\]

Case 2: $\frac{\sigma_y^2}{\sigma_q^2} \leq 2^{2K}$ and $2^{-2K} < \frac{\sigma_y^2}{\sigma_q^2} < 2^{2K}$

\[
\begin{align*}
k_y^* &= 0 \\
k_u^* &= \frac{1}{2} K - \frac{1}{4} \log 2 \frac{\sigma_q^2}{\sigma_y^2} \\
k_q^* &= \frac{1}{2} K + \frac{1}{4} \log 2 \frac{\sigma_q^2}{\sigma_u^2}
\end{align*}
\]

Case 3: $\frac{\sigma_u^2}{\sigma_y^2} \leq 2^{-2K}$ and $2^{-2K} < \frac{\sigma_u^2}{\sigma_y^2} < 2^{2K}$

\[
\begin{align*}
k_y^* &= \frac{1}{2} K - \frac{1}{4} \log 2 \frac{\sigma_u^2}{\sigma_y^2} \\
k_u^* &= 0 \\
k_q^* &= \frac{1}{2} K + \frac{1}{4} \log 2 \frac{\sigma_q^2}{\sigma_y^2}
\end{align*}
\]
Case 4: $\frac{\sigma^2_k}{\sigma^2_q} \geq 2^{2K}$ and $2^{-2K} < \frac{\sigma^2_y}{\sigma^2_u} < 2^{2K}$

$$k^*_y = \frac{1}{2}K + \frac{1}{4}\log2\frac{\sigma^2_y}{\sigma^2_u}$$

$$k^*_u = \frac{1}{2}K - \frac{1}{4}\log2\frac{\sigma^2_y}{\sigma^2_u}$$

$$k^*_q = 0$$

Case 5: $\frac{\sigma^2_q}{\sigma^2_u} \geq 2^{2K}$ and $\frac{\sigma^2_y}{\sigma^2_q} \geq 2^{2K}$

$$k^*_y = 0$$

$$k^*_u = 0$$

$$k^*_q = K$$

Case 6: $\frac{\sigma^2_y}{\sigma^2_u} \geq 2^{2K}$ and $\frac{\sigma^2_q}{\sigma^2_y} \geq 2^{2K}$

$$k^*_y = 0$$

$$k^*_u = K$$

$$k^*_q = 0$$

Case 7: $\frac{\sigma^2_y}{\sigma^2_u} \geq 2^{2K}$ and $\frac{\sigma^2_q}{\sigma^2_y} \geq 2^{2K}$

$$k^*_y = K$$

$$k^*_u = 0$$

$$k^*_q = 0$$
The output gap is given by
\[ y - y^* = 2^{-2k_q^*}q - 2^{-2k_y^*}y^* - (1 - 2^{-2k_u^*})u \]
\[ = (2^{-2k_q^*} \phi_y - 2^{-2k_y^*})y^* + (2^{-2k_u^*} \phi_u - (1 - 2^{-2k_u^*}))u \] (B.1)

**Case I:** \( \frac{\sigma_y^2}{\sigma_q^2} \geq 2^{2K} \)

Under this parameter restriction, depending on central bank’s policy, the possible attention allocation by firms are Case 7, Case 5, and Case 3 as shown by figure 3.2. Under each attention allocation case, there is a solution to stabilize output gap completely.

If firms’ attention allocation ends up with Case 5 where \( k_y^* = K \) and \( k_u^* = 0 \), we must have \( \phi_y = 2^{2K} \) and \( \phi_u = 0 \). And we need to check that under this policy, the firms will pay attention in the form of case 5. \( \frac{\sigma_q^2}{\sigma_u^2} = 2^{4K} \frac{\sigma_y^2}{\sigma_y^*} \geq 2^{6K} \) is indeed greater than \( 2^{2K} \). And \( \frac{\sigma_y^2}{\sigma_y^*} = 2^{4K} \) is greater than \( 2^{2K} \).

If firms’ attention allocation ends up with Case 7 where \( k_y^* = K \) and \( k_u^* = k_q^* = 0 \), to fully stabilize the output gap, we must have:
\[ \phi_y - 2^{-2K} = 0 \]
\[ 1 \cdot \phi_u - 0 = 0 \]

The only possible response is \( \phi_y = 2^{-2K} \) and \( \phi_u = 0 \). We again need to check that under this policy response, the firms actually end up with paying attention in the form of case 7. \( \frac{\sigma_q^2}{\sigma_y^*} = 2^{-4K} \) is indeed smaller than \( 2^{-2K} \).

If firms’ attention allocation ends up with Case 3 where \( k_y^* = \frac{1}{2}K - \frac{1}{4} \log 2 \frac{\sigma_q^2}{\sigma_y^*}, k_u^* = 0 \), and \( k_q^* = \frac{1}{2}K + \frac{1}{4} \log 2 \frac{\sigma_u^2}{\sigma_y^*} \), to fully stabilize output, we must have:
\[ \phi_y = 2^{2k_q^* - 2k_y^*} = \frac{\sigma_q^2}{\sigma_y^*} = \phi_y^2 \]
\[ \phi_u = 0 \]

Then we have \( \phi_y = 1, \phi_u = 0 \) or \( \phi_y = 0, \phi_u = 0 \). Again, we need to check under the policy, the firms will end with case 3 attention allocation, i.e., \( \frac{\sigma_q^2}{\sigma_y^*} \frac{\sigma_u^2}{\sigma_y^*} \leq 2^{-2K} \) and \( 2^{-2K} < \frac{\sigma_q^2}{\sigma_y^*} < 2^{2K} \).
should be satisfied and that excludes the case where $\phi_y = 0, \phi_u = 0$. And under the optimal response, $k^*_y = k^*_q = \frac{K}{2}$ and $k^*_u = 0$.

**Case II:** $2^{-2K} \leq \frac{\sigma_y^2}{\sigma_u^2} \leq 2^{2K}$

Under this parameter restriction, the possible attention allocation by firms are Case 5, Case 4, and Case 1.

If the firms’ attention allocation ends up with Case 5 where $k^*_q = K, k^*_u = k^*_y = 0$, setting $\phi_y = 2^{2K}$ and $\phi_u = 0$ again can fully stabilize output gap. We then check under this policy, the firms pay attention according to Case 5: $\frac{\sigma_y^2}{\sigma_u^2} = \phi_y^2 = 2^{4K} \geq 2^{2K}$ and $\frac{\sigma_u^2}{\sigma_y^2} = \phi_u^2 \geq 2^{4K} \cdot 2^{-2K} = 2^{2K}$.

If the firms’ attention allocation ends up with case 4 where $k^*_y = \frac{1}{2}K + \frac{1}{4} \log 2 \frac{\sigma_y^2}{\sigma_u^2}, k^*_u = \frac{1}{2}K - \frac{1}{4} \log 2 \frac{\sigma_y^2}{\sigma_u^2}$, and $k^*_q = 0$, to fully stabilize output gap, we must have

$$\phi_y = 2^{-2k^*_y} = 2^{-k} \frac{\sigma_u}{\sigma_y^*}$$

$$\phi_u = 1 - 2^{-k} \frac{\sigma_y^*}{\sigma_u}$$

To make sure that under this policy, the firms’ attention allocation falls into case 4, $\frac{\sigma_y^2}{\sigma_u^2} \geq 2^{2K}$ should be satisfied. Plug the policy into the inequality, we have the following restriction on the parameter space:

$$2^{-2K} - 2^{-k} \frac{\sigma_y^*}{\sigma_u} + (1 - 2^{-k} \frac{\sigma_y^*}{\sigma_u})^2 \leq 0 \quad (B.2)$$

Thus under Case II, $\phi_y = 2^{2K}, \phi_u = 0$ can always fully stabilize the output gap. Meanwhile with some parameter values, there exists two solutions fully stabilizing output gap.

If the firms’ attention allocation ends up with case 1, existence of the solution to fully output gap stabilization depends on the parameter values. The analytical solution is complex, but it won’t affect the conclusion any way. So it’s not discussed here.

**Case III:** $\frac{\sigma_y^2}{\sigma_u^2} \leq 2^{-2K}$

Under this parameter restriction, the possible firms attention allocation cases are Case 6, Case 5, and Case 2. The solution depends on the parameter values.

If the firms’ attention allocation is in case 5, then the only possible full output stabilization policy should be $\phi_y = 2^{2K}$ and $\phi_u = 0$. However, given this policy the firms won’t allocate attention according to case 5. $\frac{\sigma_y^2}{\sigma_u^2} = \phi_y^2 \frac{\sigma_y^2}{\sigma_u^2} \leq 2^{2K}$ cannot be greater $2^{2K}$. The equality case can be included in Case II.
If the firms’ attention allocation is in case 6 where \( k^*_y = k^*_q = 0 \) and \( k^*_u = K \), to fully stabilize output gap \( \phi_y = 1 \) and \( \phi_u = 1 - 2^{-2K} \). To make sure that under this policy, the firms end up with case 6, the restriction that \( \sigma^2_y / \sigma^2_{u} = \phi_y^2 \sigma^2_{y} / \sigma^2_{u} + \phi_u^2 \leq 2^{-2K} \) needs to be satisfied, which can be rearranged as:

\[
\frac{\sigma^2_y}{\sigma^2_u} \leq 2^{-2K} - (1 - 2^{-2K})^2 \tag{B.3}
\]

Due to the non-negativity of LHS, we have a further restriction to information capacity \( K \):

\[
K \in (0, \frac{1}{2} \log \frac{2 \sqrt{2} - \sqrt{5}}{2}) \tag{B.4}
\]

If the firms’ attention allocation ends up with Case 2 where \( k^*_y = 0, k^*_u = \frac{1}{2} K - \frac{1}{4} \log \frac{\sigma^2_q}{\sigma^2_u} \), and \( k^*_q = \frac{1}{2} K + \frac{1}{4} \log \frac{\sigma^2_q}{\sigma^2_u} \), monetary policy can fully stabilize output by setting

\[
\phi_y = 2^{2k_i} = 2^K \frac{\sigma_q}{\sigma_u} \quad \implies \quad \phi_y^2 (1 - 2^{2K} \frac{\sigma^2_y}{\sigma^2_u}) = 2^{2K} \phi_u^2
\]

\[
\phi_u = 2^{2k_i} - 2^{2(k^*_i - k^*_u)} = \phi_y - \phi_y^2 \frac{\sigma^2_y}{\sigma^2_u} - \phi_u^2
\]

Solve the above system, we get \( \phi_y = 2^{2K} (1 - \sqrt{2^{-2K} - \frac{\sigma^2_y}{\sigma^2_u}}) \) and \( \phi_u = 2^{2K} (\sqrt{2^{-2K} - \frac{\sigma^2_y}{\sigma^2_u}} - 2^{-2K} + \frac{\sigma^2_y}{\sigma^2_u}) \). We then check that under the policy firms pay attention as Case 2. The following condition needs to be satisfied:

\[
2^{-2K} \leq \frac{\sigma^2_y}{\sigma^2_u} = \phi_y^2 2^{-2K} \leq 2^{2K} \tag{B.5}
\]

\[
\frac{\sigma^2_y}{\sigma^2_q} \leq 2^{2K} \tag{B.6}
\]

If (B.5) is satisfied, (B.6) is satisfied automatically due to \( \frac{\sigma^2_y}{\sigma^2_u} \leq 2^{-2K} \), thus the restriction boils down to:

\[
\frac{\sigma^2_y}{\sigma^2_u} \geq 2^{-2K} - (1 - 2^{-2K})^2 \tag{B.7}
\]

Combining the above two situations, we can see under Case III when \( \sigma^2_y / \sigma^2_u \leq 2^{-2K} \), there
always exists a solution for the central bank to stabilize output gap completely. And under any given parameter, the solution is unique. The central bank cannot stabilize output gap as flexibly as in Case I and Case II.

C PROOF OF PROPOSITION 4

Case I: \( \frac{\sigma_y^2}{\sigma_u^2} \geq 2^{2K} \)

Under the parameter restrictions, the firms allocate their attention according to Case 7, Case 5, and Case 3 depending on the policy. To minimize price dispersion, central bank’s optimal policy is to drive firms’ attention to Case 7.

If the policy drives firms’ optimal attention to Case 7, the loss from price dispersion becomes:

\[
E[(p_i - p)^2]_7 = (2^{-2K} - 2^{-4K})\sigma_y^2
\]  
(C.1)

If the policy drives firms’ optimal attention to Case 5, the loss from price dispersion becomes:

\[
E[(p_i - p)^2]_5 = (2^{-2K} - 2^{-4K})\sigma_q^2 \geq (2^{-2K} - 2^{-4K})\sigma_y^2
\]  
(C.2)

where the inequality comes from the restriction of firms’ optimal attention allocation falling into Case 5.

If the policy drives firms’ optimal attention to Case 3, the loss from price dispersion becomes:

\[
E[(p_i - p)^2]_3 = 2 \cdot 2^{-K} \sigma_y \sigma_q - 2^{-2K} \sigma_y^2 - 2^{-2K} \sigma_q^2
\]  
(C.3)

We then show that \( E[(p_i - p)^2]_7 \leq E[(p_i - p)^2]_3 \).

\[
E[(p_i - p)^2]_3 - E[(p_i - p)^2]_7 = 2 \cdot 2^{-K} \sigma_y \sigma_q - 2^{-2K} \sigma_y^2 - 2^{-2K} \sigma_q^2 - (2^{-2K} - 2^{-4K})\sigma_y^2
\]

\[
= - (\sigma_y - 2^{-K} \sigma_q)^2 + (1 - 2^{-2K})^2 \sigma_y^2
\]

\[
= \sigma_y^2 2^{-K} (\frac{\sigma_q}{\sigma_y} - 2^{-K}) (1 - 2^{-2K} + 1 - 2^{-K} \frac{\sigma_q}{\sigma_y}) \geq 0
\]  
(C.4)

Both the second and the third terms are nonnegative due to the restriction for Case 3 attention solution: \( 2^{-K} \leq \frac{\sigma_q}{\sigma_y} \leq 2^K \).

Case II: \( 2^{-2K} \leq \frac{\sigma_y^2}{\sigma_u^2} \leq 2^{2K} \)

Under the parameter restrictions, the firms allocate their attention according to Case 5,
Case 5, and Case 1 depending on the policy. To minimize price dispersion, central bank’s optimal policy is to drive firms’ attention to Case 4.

If the policy drives firms’ optimal attention to Case 4, the loss from price dispersion becomes:

$$E[(p_i - p)^2]_4 = 2 \cdot 2^{-K} \sigma_u \sigma_y^* - 2^{-2K} \sigma_u^2 - 2^{-2K} \sigma_y^2$$

(C.5)

If the policy drives firms’ optimal attention to Case 1, the loss from price dispersion in as follows:

$$E[(p_i - p)^2]_1 = 3 \cdot (2^{-K} \sigma_u \sigma_y^* \sigma_q) \frac{3}{4}$$

$$- 2^{-\frac{3}{4}K} (\sigma_u^3 \sigma_y^* \sigma_q - \frac{3}{4} \sigma_u^3 \sigma_q + \frac{3}{4} \sigma_u \sigma_y^* \sigma_q^2)$$

$$\geq 3 \cdot 2^{-K} \sigma_u \sigma_y^* - 2^{-K} \sigma_u \sigma_y^* - 2^{-2K} \sigma_y^2 - 2^{-2K} \sigma_u^2 = E[(p_i - p)^2]_4$$

(C.6)

The last inequality uses the restriction $2^{-K} \sigma_u \sigma_y^* \leq \sigma_q^2 \leq 2^{-2K} \sigma_u \sigma_y^*$ so that the firms’ attention is according to Case 1.

If the policy drives firms’ optimal attention to Case 5, the loss from price dispersion becomes:

$$E[(p_i - p)^2]_5 = (2^{-2K} - 2^{-4K}) \sigma_q^2$$

(C.7)

We then show that $E[(p_i - p)^2]_4 \leq E[(p_i - p)^2]_5$.

$$E[(p_i - p)^2]_4 = -(2^{-K} \sigma_u - \sigma_y^*)^2 + (1 - 2^{-2K}) \sigma_y^2$$

$$\leq -(2^{-K} \sigma_u - \sigma_y^*)^2 + (1 - 2^{-2K}) 2^{-2K} \sigma_q^2$$

$$\leq E[(p_i - p)^2]_5$$

(C.8)

The second line is from the restriction for firms attention allocation being case 5.

**Case III:** $\frac{\sigma_y^*}{\sigma_q} \leq 2^{-2K}$

This case is symmetric to Case I. Under the restriction, the firms allocate their attention according to Case 6, Case 5, and Case 2. To minimize price dispersion, central bank’s optimal policy is to drive firms’ attention to Case 6.

If the policy drives firms’ optimal attention to Case 6, the loss from price dispersion becomes:

$$E[(p_i - p)^2]_6 = (2^{-2K} - 2^{-4K}) \sigma_u^2$$

(C.9)
If the policy drives firms’ optimal attention to Case 5, the loss from price dispersion becomes:

\[ E[(p_i - p)^2]_5 = (2^{-2K} - 2^{-4K})\sigma_q^2 \geq (2^{-2K} - 2^{-4K})\sigma_u^2 \]  
(C.10)

since \( \frac{\sigma_q^2}{\sigma_u^2} \geq 2^{2K} \) must be satisfied for Case 5.

If the policy drives firms’ optimal attention to Case 2, the loss from price dispersion becomes:

\[ E[(p_i - p)^2]_2 = 2 \cdot 2^{-K} \sigma_u \sigma_q - 2^{-2K} \sigma_u^2 - 2^{-2K} \sigma_q^2 \]  
(C.11)

We then show that \( E[(p_i - p)^2]_2 - E[(p_i - p)^2]_6 \geq 0 \).

\[
E[(p_i - p)^2]_2 - E[(p_i - p)^2]_6 = 2 \cdot 2^{-K} \sigma_u \sigma_q - 2^{-2K} \sigma_u^2 - 2^{-2K} \sigma_q^2 - (2^{-2K} - 2^{-4K})\sigma_u^2 \\
= -\sigma_u^2 (2^{-K} \sigma_q^2) + (1 - 2^{-2K})^2 \sigma_u^2 \\
= \sigma_u^2 (2^{-K} \left( \frac{\sigma_u^2}{\sigma_q^2} - 2^{-K} \right) (1 - 2^{-2K} + 1 - 2^{-K} \frac{\sigma_q^2}{\sigma_u^2}) \geq 0 
\]  
(C.12)

The second and third terms are nonnegative due to restriction for Case 3 attention solution: \( 2^{-K} \leq \frac{\sigma_q^2}{\sigma_u^2} \leq 2^K \).

**D  PROOF OF PROPOSITION 5**

Compare the results from proposition 3, 4, and 5 can be derived quickly. Under the same parameter space, if the policy achieving fully output stabilization and optimal price dispersion, then there is no trade-off between output gap and price dispersion. Since output gap can always be completely stabilized, whenever the two objectives are not consistent, there is a trade-off.

**E  FULL DESCRIPTION OF OPTIMAL POLICY: AN EXAMPLE**

Table 2 shows the optimal responses for a central bank facing an objective with both output gap stabilization and price dispersion stabilization. The solution is under the circumstance where \( K = 1 \) and \( \gamma = 1 \). We vary \( \frac{\sigma_y^2}{\sigma_u^2} \) to cover the above three cases. The first eight rows show the solution information. For comparison, the last two rows display which case the firms’ attention allocation would be under optimal response if the objective is price dispersion stabilization and output gap stabilization only respectively. Case I is shown in the
Table 2: Optimal response to shock when $K = 1, \gamma = 1$

<table>
<thead>
<tr>
<th>$\sigma^2_y / \sigma^2_u$ (with $\sigma^2_u = 1$)</th>
<th>$1 \over 5$</th>
<th>$1 \over 4$</th>
<th>$1 \over 2$</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_y$</td>
<td>0.5000</td>
<td>0.6000</td>
<td>0.5500</td>
<td>0.5000</td>
<td>0.3500</td>
<td>0.2500</td>
<td>0.2500</td>
</tr>
<tr>
<td>$\phi_u$</td>
<td>0.4500</td>
<td>0.4000</td>
<td>0.4500</td>
<td>0.5000</td>
<td>0.3000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$k^*_y$</td>
<td>0</td>
<td>0</td>
<td>0.2499</td>
<td>0.5000</td>
<td>0.7500</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$k^*_u$</td>
<td>0.9964</td>
<td>1.0000</td>
<td>0.7499</td>
<td>0.5000</td>
<td>0.2500</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$k^*_q$</td>
<td>0.0036</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Loss $Y$</td>
<td>0.1411</td>
<td>0.1625</td>
<td>0.0510</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Loss $P$</td>
<td>0.1894</td>
<td>0.1875</td>
<td>0.3322</td>
<td>0.5000</td>
<td>0.6642</td>
<td>0.7500</td>
<td>0.9375</td>
</tr>
<tr>
<td>Attention allocation</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Optimal price stability</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Full output stability</td>
<td>$2^<em>, 6^</em>$</td>
<td>$2^<em>, 6^</em>$</td>
<td>$5, 4^*$</td>
<td>$5, 4^*$</td>
<td>$5, 4^*$</td>
<td>$3, 5, 7$</td>
<td>$3, 5, 7$</td>
</tr>
</tbody>
</table>

Note: $^*$ means only when certain constraints on the parameters are satisfied, the objective can be reached.

last two columns. Under the optimal response $\phi_y = 2^{-2K}$ and $\phi_u = 0$, the output gap is fully stabilized and the price dispersion is optimally stabilized. Column 7 displays larger loss from price dispersion than column 6 under optimal response because the fundamentals are more uncertain and the prices will be more dispersed. Case II are shown from column 3 to 5. While $\sigma^2_y / \sigma^2_u = 1$ and 2, the constraint (5.7) is satisfied. That’s why output gap stabilization can be fully achieved and price dispersion can be optimally stabilized. The intuition is that when the economy’s information capacity is small relative to the supply shock uncertainty (either because of a small $K$, a large $\sigma^2_y$, or a small $\sigma^2_u$), the central bank can flexible fully optimize output gap, leaving a large choice set for stabilizing price dispersion. However, when the information capacity of the economy is relatively large or the mark-up shock becomes relatively large, the central bank’s policy can not be as flexible as before. Polices that fully stabilize the output gap harm the price dispersion stabilization. This is where the central bank starts to face a trade-off between output gap stabilization and price dispersion stabilization. Case III is shown in the first two columns.