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Abstract

We study an optimal pricing problem for an intermediary through which transactions between a monopoly and the consumers take place and consumers receive information about the commodity. The intermediary can provide information to the consumers and charge the monopoly accordingly. We characterize the optimal menus and show that a menu consisting of (garbled) upward censorship that displays negative targeting feature is optimal and that surplus reduces comparing to a benchmark where the monopoly has control of the information technology.

Keywords: Monopolistic pricing, advertising, screening, negative targeting, non-linear pricing, information design.

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1 Introduction

When making purchasing decisions, consumer’s information about the commodity plays a central role, which in turns affect a producer or a seller’s profit that can be generated by

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the sale. How well a consumer is informed about the quality of commodities and what kinds of information does a consumer possess greatly affects how much profit a producer can make by selling these commodities. Moreover, due to rapid development of information technology in the past few decades, the channels and platforms through which consumers receive information about commodities have been widely expanded. However, under many real-world contexts, the channel and platforms through which consumers receive information are neither owned by the producer nor the consumers themselves. Instead, it is often the case that a centralized third party owns the technology and the platform so that it has the ability to provide various information to the consumers of different commodities. For instance, Online platforms such as eBay or Amazon have a well-publicized website on which the commodities are presented to the consumers and different information about the commodities are provided to the consumers via photos, certificates, and descriptions of the product. Or, platforms such as Google or Facebook often publish sellers’ advertisement and relevant information on their search engine or social media so that potential consumers of these commodities and be (possibly differently) informed. Likewise, advertising agencies help sellers design advertisements and use them to publicize the commodities and inform the potential consumers. In these scenarios, the third party who owns the technology or platform for information provision are often not involved directly in the trade between the consumers and the producers, nor does it have direct interests in the profit that the producers can make (e.g. The third party often do not own shares of the seller’s firm). Instead, such third party only has interests in monetary payments made by the producers. In other words, the owner of the information technology or platform often does not care about the outcome of the trade, rather, it wishes to "sell" such technology or platform—mostly through the form of advertisements or advertising spaces—to the producers who seek to maximize profit by selling a commodity to consumers. In the era of vast development of informational technology and computational abilities, it is then of great importance to understand the incentives for such platform or information technology owners and how would such centralized informational intermediary, together with the flexibility of the information they can convey, affect the market. In this article, our main goal is to understand the incentive for such informational intermediary. Specifically, we examine the optimal way for such intermediary to price and
"sell" to the producers its technology or platform and the ways to convey information to the consumers. Furthermore, we investigate the impact on welfare of such information provision process that is concentrated to an intermediary which is separated from the producers and consumers.

To fix ideas, consider a simple parameterized model that illustrates why information conveyed to the consumer is crucial in determining seller’s profit and why selling such information technology would be profitable for a intermediary. Suppose that a seller is trying to sell an indivisible good to a buyer. The buyer’s valuation of the good is \( v \in [0, \bar{v}] \), for some \( \bar{v} \geq 3 \). Consider a seller with cost \( c \in [0, 1] \) who does not know the exact value of \( v \) but only that \( v \) is drawn from a uniform distribution on \([0, \bar{v}]\). If the buyer knows exactly the value of \( v \), then the seller will choose a posted price that solves \( \max_{v \in [0, \bar{v}]} (v - c)(1 - v/\bar{v}) \), which gives profit of \(( (a - c)/2 )^2 \). On the other hand, if the buyer knows nothing but the fact that the valuation is drawn from a uniform distribution, any seller with cost less than or equal to the expectation of \( v \), \( \bar{v}/2 \) will set of price at \( \bar{v}/2 \) and the seller with cost greater than \( \bar{v}/2 \) will not sell at all. As such, the seller’s profit is \(( (\bar{v} - 2c)/2 )^+ \). Notice that with different costs, the seller would prefer different information that the buyer has. For sellers whose costs are low, they will prefer the buyer to be not informed than to be fully informed about \( v \), whereas for sellers whose costs are high, they will prefer otherwise. Therefore, as an intermediary who has the technology to provide the buyer different information, it could then benefit from setting up a menu of different information that is going to be provided to the buyer and charge the seller with different prices.

As a preview of our main result, our analysis (Theorem 1) suggests that if the sellers are uniformly distributed in terms of their costs, the following menu will be optimal. There is a continuum of items in this menu, each of them is indexed by a cutoff \( k \in [0, 2] \). For each item indexed by \( k \), the intermediary provides information to the buyer so that the buyer perfectly learns about \( v \) whenever it is below \( k \) and learns nothing else whenever it is above \( k \) and charges a price \( a + b(1 - k^2/4) \), where \( a, b > 0 \) are constants that depend on \( \bar{v} \). More generally, we show that an upward censorship menu—menu consisting of items that perfectly inform the buyer when the value is below a certain cutoff and nothing else when it is above the cutoff with a price that depend on the level of the cutoff—is optimal.
under an assumption on the distribution that requires the *information rent* of seller to be relatively small comparing to that of the buyer under full information, to which uniform distributions introduced above is an example. Furthermore, we also argue that in general (Theorem 2), without additional assumption on the distribution of valuation and costs, a *garbled upward censorship menu* that resembles upward censorship menus, except that the information buyer gets when the valuation is below the cutoff is possibly garbled and that the largest cutoff in the menu becomes lower. Such optimal menus reflects two interesting features. First, it involves non-linear pricing and contains infinitely many items in the menu. As it is the *information structure* that is sold by the intermediary, a constant per-unit price cannot be well-defined. Furthermore, even if we can index the items in this menu by a one dimensional variable $k$, the price depends on $k$ in a non-linear way in general. Second, any element in this optimal menu displays a *negative targeting* feature that has been discussed and documented in the advertising literature (see, for example Pancras & Sudhir (2007) and Blake, Nosko & Tadelis (2013)). That is, in order to provide the upward (or garbled upward) censorship, the intermediary has to have detailed information about the consumer whose value is low in order to inform them perfectly (or give relatively more information) while the information provided to high-value consumers is relatively coarse. This optimal menu prescribes the intermediary to adopt (garbled) upward censorship that display negative targeting and the scope of targeting is determined by the cutoff of each items. For items with higher cutoff, the consumer receives more information in the sense of Blackwell order, the scope of targeting is larger.

The rest of this paper is structured as follows: In the next section, we summarize the related literature and mark the connections and differences between the literature and our paper. In section 3, we present the model and some preliminary analyses. In section 4, we provide characterizations of optimal menus under the baseline model. Section 5 includes an extension in which we allow the intermediary to also contract on publicity of the advertisement. Section 6 concludes.
2 Related Literature

This paper is related to several branches of literature in interplay between monopolistic pricing and information structure, selling information, and Bayesian persuasion. In the monopolistic pricing literature, Lewis & Sappington (1994) also examines how the change in consumer’s information affects a monopolist’s profit. They show that for a given monopolist with a constant production cost, the buyer having either full information or no information is optimal for the monopolist. Our model distinct from theirs in two major aspects. First, we examine how a third party would price information structures for a monopolist to purchase, instead of examining optimal information structure from a monopolist’s perspective directly. Second, although the setting of Lewis & Sappington (1994) is close to a benchmark of our model in which the monopolist can choose information structure that the buyer has directly, we maintain an assumption that the commodity is indivisible so that buyers have 0-1 demand while the consumer’s demand in their model can be more general. On the other hand, Lewis and Sappington (1994) restrict the information structures to vary within a one-dimensional family by assuming a particular disclosure rule, whereas out model allows full flexibility of the choice of information structure. Johnson & Myatt (2006) also studies how change in the distribution of buyers’ valuation, which is equivalent to the information that buyer has in our model, affects a monopolist’s profit. In particular, they show that when the distributions are ordered by the rotational ordering, the monopolist will prefer two extremes of the order. Under our model, this result is similar to Lewis & Sappington (1994) in that it implies that under a particular one dimensional (and hence, totally-ordered) family of information structures, a monopolist will either prefer the most informative one or the least informative one. Again, our model differs from theirs in that we focus on a third party’s optimal menu and that we allow complete flexibility in providing information structures. Recent developments, on the other hand, have adopted flexible information structures. Bergemann, Brooks & Morris (2015) characterizes all the possible surplus division that can arise by giving the monopolist different information about the buyer’s valuation. Roesler & Szentesz (2017) examines the buyer-optimal information structure when facing a monopoly.

There are several works that also study a problem of pricing information. Bergemann
& Bonatti (2011) studies a pricing problem of a data provider who can provide information about the match value for a seller whose profit from trade depends on the match value of each consumer and the amount of investments the seller makes in each consumer. Although having a similar title, the model Horner & Skrzypacz (2016) is about disclosing an agent’s private information to a decision maker who can choose whether to hire the agent to make the decision. The sell of information in their model is endogenous in the sense that it is through transfers that induces to agent to take proper tests to reveal their information in a selected equilibrium in their dynamic setting. Bergemann, Bonatti and Smolin (2018) studies an optimal menu for a data provider to sell experiments to a decision maker who has a private estimate about the state. Our model differs from the models above in that our intermediary sells information to affect the information of the buyer to the seller, which affects the seller’s value function on different information indirectly, whereas the models above focus on selling information structure to a decision maker whose value function depends on the information she purchases directly.

Furthermore, our screening framework of selling information structure is analogous to standard monopolistic screening problems as in Mussa & Rosen (1978), Myerson (1981) and Maskin & Riely (1984). The screening problem in our model is more complicated in that it is essentially a mixture of adverse selection and moral hazard problem from the intermediary’s perspective. Also, our assumption about the intermediary’s ability to commit to a menu and the characterization of information structure follows from the the Bayesian persuasion literature, as Kamenica & Gentzkow (2011), Gentzkow & Kamenica (2016).

3 Model

There is a buyer (he), a seller (she) and an information intermediary (it). The seller is selling an indivisible object to the buyer. The buyer has quasi-linear preference with $v \in [0, \bar{v}]$, for some $\bar{v} \in \mathbb{R}_+$ being the buyer’s valuation of the object. The buyer does not know about his valuation a priori, rather, he has to learn about his valuation, which follows a common prior $F$, through the information provided by the intermediary. More precisely, the intermediary has the technology to design (and commit to) information structures in order to inform
the buyer. After privately learning about the valuation, the seller then interacts with the buyer by designing selling mechanisms to maximize profit. The seller has private information about her constant marginal cost of production $c \in [0, \bar{c}]$, for some $\bar{c} \in \mathbb{R}_+$. This private cost is drawn from a common prior $G$. As such, the intermediary can “sell” the information technology to the seller by posting a menu of information structures and the associated payments. We assume that the intermediary has no direct interest in the allocation of the object buy only revenue. For the baseline model, we also assume that the seller becomes visible to the buyer only if she uses the intermediary’s technology. That is, if the seller does not buy from any item in the menu that the intermediary provides, she will then not be able to interact with the buyer and thus will always receive zero profit. To sum up, the timing of the model is described as below:

1. Nature draws valuation $v \sim F$ and cost $c \sim G$.

2. The intermediary posts a menu of information structures and payments to the seller.

3. The seller chooses whether and what item to buy from the menu posted by the intermediary.

4. Based on the selected item (if any), the seller pays the payment to the intermediary and the intermediary implements the information structure.

5. The buyer receives private signals from the information structure implemented by the intermediary (if any) and update beliefs about his valuation.

6. The seller (if possible) then designs selling mechanism to sell the object. When the buyer is indifferent, he breaks ties in favor of seller.

Since the buyer has quasi-linear preference, the interim expected value is the only payoff relevant statistic for a given information structure. As such, the marginal distribution of the interim expected value conveys all the payoff-relevant aspect of a given information structure. As in Gentzkow & Kamenica (2016),\textsuperscript{1} we may represent the information structures by the

\textsuperscript{1}Similar characterizations appear in many recent developments in the literature of mechanism and information design, see for instance Neeman (2003), Bergenamm and Pesendorfer (2007), Shi (2012), Roseler & Szentes (2017), Kolotilin et al., Bergemann, Brooks and Morris (2017a), Du (2017)
collection of CDFs of which the prior $F$ is a mean preserving spread. That is, the collection of information structures can be represented by the set

$$H_F := \left\{ H : [0, \bar{v}] \to [0, 1] \middle| \int_0^x H(z)dz \leq \int_0^x F(z)dz, \forall x \in [0, \bar{v}] \right\}. $$

On the other hand, by quasi-linearity of the buyer’s preference again, given any information structure $H \in H_F$, since the interim expected value follows the distribution $H$, it is well-known that posted price mechanisms always achieves the maximal profit for the seller with any cost $c \in [0, \bar{c}]$. Thus, given any information structure $H \in H_F$, the seller’s profit maximization problem can be reduced to

$$\max_{x \in [0, \bar{v}]} (x - c) (1 - H(x^-)) .$$

Finally, by the standard revelation principle arguments, it is without loss to restrict the intermediary to post direct menus that are incentive compatible and individually rational: Menus that ask the seller to report her private cost $c$ and assign an information structure $H^c \in H_F$ and an amount of payment $t(c) \in \mathbb{R}$ to each report in a way that the seller will always participate and report truthfully. Formally, the intermediary posts incentive compatible and individually rational direct menu that takes form of $(H^c, t(c))_{c \in [0, \bar{c}]}$ such that $H^c \in H_F$, $t(c) \in \mathbb{R}$ for all $c \in [0, \bar{c}]$ and that

$$\max_{x \in [0, \bar{v}]} (x - c) (1 - H^c(x^-)) - t(c) \geq \max \left\{ \max_{x \in [0, \bar{v}]} (x - c) (1 - H^{c'}(x^-)) - t(c'), 0 \right\} .$$

for all $c, c' \in [0, \bar{c}]$, to maximize expected revenue

$$\mathbb{E}_G[t(c)] = \int_0^{\bar{c}} t(c) G(dc).$$

Before proceeding in characterizing the incentive compatible and individually rational menus, we first observe that given a menu $(H^c, t(c))_{c \in [0, \bar{c}]}$, for a seller with cost $c$, if the implemented information structure is $H^{c'}$, the seller has profit

$$\Pi(c, c') := \max_{x \in [0, \bar{v}]} (x - c) (1 - H^{c'}(x^-)) .$$

As conventional, $H(x^-) := \lim_{\delta \downarrow 0} H(x - \delta)$ is the left-limit of $H$ at $x$. The left limit is taken since the buyer breaks ties in favor of the seller and thus always buys when the price is equal to his expected value.
and thus, if a seller with cost $c$ reports her cost to be $c'$, the net profit is given by

$$V(c, c') := \Pi(c, c') - t(c'),$$

which resembles the payoff functions as in standard screening problem (see, for example, Mussa & Rosen (1978), Myerson (1981), Maskin & Riley (1984)) with agents that have quasi-linear preference. However, in this model, a complication arises as the function $\Pi$ is endogenous—it is derived from an optimization problem of the seller and depends on the information structure through a seller’s optimal pricing strategy. In other words, from the intermediary’s perspective, our model is in fact a mixture of screening—inducing the seller to report truthfully and moral hazard—inducing the seller to set prices in a desirable way. Nevertheless, as the object is indivisible, the cost of production affects seller’s profit in an affine fashion. This gives the problem enough of structure so that the standard envelope characterization can be modified to accommodate our setting. This is given by the following Lemma.

**Lemma 1.** Suppose that a menu $(H^c, t(c))_{c \in [0, \bar{c}]}$ is incentive compatible and individually rational. Then for any selection

$$x(c, c') \in \arg\max_{x \in [0, \bar{v}]} (x - c)(1 - H^c(x^-)),$$

1. $t(c) = t(\bar{c}) + \max_{x \in [0, \bar{v}]} (x - c)(1 - H^c(x^-)) - \int_{\bar{c}}^{\bar{v}} (1 - H^z(x, z^-))dz.$

2. $t(\bar{c}) \leq 0.$

3. The function $c \mapsto (1 - H^c(x(c, c^-)))$ is nonincreasing.

4. $\int_{\bar{c}}^{\bar{c}} (H^c(x(z, c')^-) - H^z(x(z, z^-))dz \geq 0$ for any $c', c \in [0, \bar{c}].$

Conversely, suppose that for a menu $(H^c, t(c))_{c \in [0, \bar{c}]}$, there exists a selection

$$x(c, c') \in \arg\max_{x \in [0, \bar{v}]} (x - c)(1 - H^c(x^-))$$

satisfying:

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3 As conventional, for $a, b \in \mathbb{R}, b < a$ and any measurable function $f$, we define

$$\int_{a}^{b} f(x)dx := -\int_{b}^{a} f(x)dx.$$
1. \( t(c) = \max_{x \in [0,\bar{v}]} (x - c)(1 - H^c(x^-)) - \int_c^{\bar{c}} (1 - H^z(x(z,z^-)))dz. \)

2. \( \int_c^{c'} (H^c(x,z,c')^- - H^z(x(z,z^-)))dz \geq 0, \) for any \( c', c \in [0,\bar{c}], \) for any \( c', c \in [0,\bar{c}]. \)

Then \((H^c, t(c))_{c \in [0,\bar{c}]}\) is incentive compatible and individually rational.

The proof of Lemma 1 can be found in the Appendix. Lemma 1 is similar to standard envelope characterization of inventive compatibility, where the induced probability of trade for a given information structure, \((1 - H^c(x(c,c')^-)),\) is analogous to the role of "allocation" in standard problems. However, since the intermediary is facing a mixture problem rather than a one-dimensional screening problem, local incentive constraints will not be sufficient for global incentive compatibility, even with monotonicity of the (on path) probability of trade, as the possibility of double deviations complicates the incentive constraints. As a result, the characterization in Lemma 1 involves a family of inequalities that rules out all incentives to misreport, with or without double deviations, rather than a compact monotonicity condition as in standard screening problems.

By Lemma 1, for any incentive compatible menu \((H^c, t(c))_{c \in [0,\bar{c}]},\) by Fubini’s theorem, expected revenue can then be written as:

\[
t(\bar{c}) + \int_0^{\bar{c}} \left( \max_{x \in [0,\bar{v}]} (x - c)(1 - H^c(x^-)) - (1 - H^c(x(c,c^-))) \frac{G(c)}{g(c)} \right) G(dc),
\]

where \(x(c, c)\) is a selection of \(\max_{x \in [0,\bar{v}]} (x - c)(1 - H^c(x^-))\). Lemma 1 and individual rationality then implies that optimal menus can be found by solving the problem:

\[
\sup_{\{H^c\}_{c \in [0,\bar{c}]}} \int_0^{\bar{c}} \left( \max_{x \in [0,\bar{v}]} (x - c)(1 - H^c(x^-)) - (1 - H^c(x(c,c^-))) \frac{G(c)}{g(c)} \right) G(dc)
\]

\[
\text{s.t. } \int_c^{c'} (H^c(x(z,c')^-) - H^c(x(z,z^-)))dz \geq 0, \forall c, c' \in [0,\bar{c}],
\]

where \(x(z, c')\) is the largest element in \(\max_{x \in [0,\bar{v}]} (x - z)(1 - H^c(x^-))\) for all \(z, c' \in [0,\bar{c}].\)

In the next section, we aim to solve the problem defined in (1) and derive the optimal menu for the intermediary.

### 4 Optimal Menu for the Intermediary

To facilitate the derivation and stress the main intuition, we maintain regularity assumptions in this section. Specifically, we assume that both \(F\) and \(G\) are absolutely continuous and
admit densities $f$ and $g$, respectively. Furthermore, the virtual valuation under the prior $F$, 
$\phi(v) := v - \frac{1-F(v)}{f(v)}$, and the virtual cost given by $G$, $\psi(c) := \min\{c + \frac{G(c)}{g(c)}, \bar{v}\}$ are both strictly increasing.$^4$

As in standard screening problems, if we can find a family of information structures 
$\{H^c\}_{c \in [0,\bar{c}]}$ to maximize the integrand of the objective function in (1) pointwisely for all 
$c \in [0,\bar{c}]$ and find a transfer $t : [0,\bar{c}] \to \mathbb{R}$ such that the menu $(H^c, t(c))_{c \in [0,\bar{c}]}$ is incentive 
compatible and individually rational, then the problem is solved. However, due to the double 
deviation concerns as noted above, such approach might not be valid, as the pointwise maximization 
solution might not be incentive compatible. In this case, a more subtle approach will be needed. Specifically, as the main reason for the failure of pointwise maximization 
approach is that local incentive constraints with monotonicity are not sufficient—due to the 
possibility of double deviations, we need to keep track of all the global incentive constraints 
and the double deviation constraints when solving the problem. In what follows, we will use 
the duality approach to characterize the solution.$^5$ Below, we will first solve the problem 
for the intermediary under an additional assumption on the distributions $F$ and $G$ so that 
pointwise maximization approach is valid and characterize the solution, and then we will solve the problem generally by using duality approach.

4.1 Optimality of Upward Censorship Menu

Below, we first show that under an additional assumption about the distributions $F$ and $G$, 
the solution for the intermediary’s problem takes a simple form—an upward censorship menu 
maximizes the intermediary’s revenue. An upward censorship menu has a simple structure: 
For each reported cost $c$, the intermediary will provide an information structure so that 
whenever the buyer’s value is below a certain cutoff, he learns exactly his value whereas 
when the buyer’s value is above the cutoff, he learns nothing else then the value being above 
the cutoff. We will show that under an assumption that requires the seller’s virtual cost not

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$^4$The regularity assumption can be relaxed and the solution we provide below will not be affected qualitatively. Detailed discussions can be found in the Online Appendix.

$^5$Using duality approach to solve mechanism design problems when local incentive constraints are not sufficient aligns with the recent development in the literature, see Bergemann, Brooks and Morris (2017b), where the approach is implicitly applied and Carroll & Segel (2017) with explicit application.
to be too high relative to the buyer’s virtual valuation under full information, an upward censorship menu with cutoffs given by the virtual cost, $\psi(c)$, is optimal.

Specifically, we assume that for any $c \in [0, \bar{c}]$, $\phi(\psi(c)) \leq c$. That is, the virtual cost of seller with cost $c$ is below the optimal monopolist price for this seller when the buyer has full information and define an upward censorship menu as follows:

**Definition 1.** Let $\tilde{\psi} : [0, \bar{c}] \rightarrow [0, \bar{v}]$ be an increasing function. Fix any $c \in [0, \bar{c}]$. An information structure $H \in \mathcal{H}_F$ is an upward censorship with cutoff $\tilde{\psi}(c)$ if

$$H(x) = \begin{cases} 
F(x), & \text{if } x \in [0, \tilde{\psi}(c)) \\
F(\tilde{\psi}(c)), & \text{if } x \in [\tilde{\psi}(c), \mathbb{E}_F[v|v > \tilde{\psi}(c)]], \forall x \in [0, \bar{v}]. \\
1, & \text{if } x \in [\mathbb{E}_F[v|v > \tilde{\psi}(c)], \bar{v}].
\end{cases}$$

Moreover, we say that a menu $(H^c, t(c))_{c \in [0, \bar{c}]}$ is an upward censorship menu with cutoff $\tilde{\psi}$ if for all $c \in [0, \bar{c}]$, $H^c$ is an upward censorship with cutoff $\tilde{\psi}(c)$.

Our first result can then be formally stated as follows:

**Theorem 1.** Suppose that $\phi(\psi(c)) \leq c$ for all $c \in [0, \bar{c}]$. For each $c \in [0, \bar{c}]$, let $H_u^c$ be an upward censorship with cutoff $\psi(c)$ and let

$$t_u(c) := (v(c) - c)(1 - F(\psi(c))) - \int_c^{\bar{c}} (1 - F(\psi(z)))dz,$$

where $v(c) := \mathbb{E}_F[v|v > \psi(c)]$. Then $(H_u^c, t_u(c))_{c \in [0, \bar{c}]}$ is incentive compatible, individually rational and maximizes the intermediary’s revenue among all the inventive compatible and individually rational menus.

Formal proof of Theorem 1 can be found in the Appendix. We provide a graphical illustration below. First notice that the set of available information structures for the intermediary can be represented by a family of convex functions the are majorized by the function $x \mapsto \int_x^{\bar{v}} (1 - F(z))dz$ and majorizes the function $x \mapsto (\mathbb{E}_F[v] - x)^+$, and share the same values at the end points, as illustrated in Figure 1, in which we plot the integral, from $x$ to $\bar{v}$, as a function of $x$, of the prior $F$ as the blue curve, the degenerate distribution $F_0$ that puts probability 1 on $\mathbb{E}_F[v]$ has the green curve and a generic $H \in \mathcal{H}_F$ as the red curve. Second, consider any $H \in \mathcal{H}_F$ and fix a $c \in [0, \bar{c}]$. For simplicity, assume that $H$ is continuous and...
that $\text{argmax}_{x \in [0,v]} (x - \psi(c))(1 - H(x))$ is a singleton and is denoted by $x^*$. Then the buyer’s surplus is $\int_{x^*}^{\bar{v}} (1 - H(x)) \, dx$ and the seller’s profit is $(x^* - c)(1 - H(x^*))$. These two quantities can be easily represented on the graph introduced above, as illustrated in Figure 2. In general, for any information structure $H \in \mathcal{H}_F$ and any $x^* \in \text{argmax}_{x \in [0,v]} (x - \psi(c))(1 - H(x^-))$, the buyer’s surplus is exactly the value of the convex function associated with $H$ at $x^*$ and the seller’s profit is the difference between the height of the intersection of the vertical line $x = \psi(c)$ and the tangent line segment that is tangent to the convex function associated with $H$ at $x^*$ and the height of buyer’s surplus.\footnote{We thank Doron Ravid for suggesting this graphical representation.}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure1.png}
\caption{Feasible Set $\mathcal{H}_F$.}
\end{figure}

With this graphical representation, it is then convenient to solve the intermediary’s problem (1) by the following procedure. Consider the integrand of the objective in (1) for any fixed $H \in \mathcal{H}_F$. Take any $x_H(c) \in \text{argmax}_{x \in [0,v]} (x - c)(1 - H(x^-))$, we then notice that:

$$
\max_{x \in [0,v]} (x - c)(1 - H(x^-)) - (1 - H(x_H(c^-))) \frac{G(c)}{g(c)}
\leq \max_{x \in [0,v]} (x - \psi(c))(1 - H(x^-)).
$$

(2)
That is, for any $c \in [0, \bar{c}]$, the integrand of the objective in (1) is bounded from above by the maximized profit of a hypothetical seller with cost $\psi(c)$ instead of $c$. As such, if we can find a family of information structures $\{H^c\}_{c \in [0, \bar{c}]}$ that maximizes the optimal profit of this hypothetical seller, $\max_{x \in [0, \bar{c}]}(x - \psi(c))(1 - H^c(x^-))$, for all $c \in [0, \bar{c}]$, together with a transfer $t : [0, \bar{c}] \rightarrow \mathbb{R}$ such that the menu $(H^c, t(c))_{c \in [0, \bar{c}]}$ is incentive compatible and individually rational, then it must be a solution of the intermediary’s problem (1), as it attains an upper bound of its relaxed problem.

We will now argue that the upward censorship menu with cutoff $\psi(c)$ indeed satisfies the criteria above. First, fix any $c \in [0, \bar{c}]$ and suppose that a seller’s cost is $\psi(c)$. Take any information structure $H \in \mathcal{H}_F$ and any $x^* \in \arg\max_{x \in [0, \bar{c}]}(x - \psi(c))(1 - H(x^-))$. Consider an alternative information structure that garbles $H$: Inform the buyer nothing but whether his expected value given by $H$ is above the original price $x^*$, as illustrated by the transition from the dotted curve to the red curve in Figure 3. Under this information structure, every buyer value that would have bought at $x^*$ under the information structure $H$ will still buy at price $x^*$. However, all the buyers will then have valuation $\mathbb{E}_H[v | v > x^*]$ conditional on buying,

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Figure 2: Representing Seller’s Profit.
meaning that the seller can set a higher price to extract surplus. In fact, the optimal price for the seller under this garbled information structure is exactly $E_H[v|v > x^*]$, which allows the seller to extract all the expected surplus under the original information structure $H$. As such, to maximize the seller’s optimal profit, it is without loss to search across information structures that has only two realizations. However, across those information structures, the largest surplus that can be extracted is $\int_{\psi(c)}^{b} (1 - F(x))dx$, the total expected surplus when the seller’s cost is $\psi(c)$. This is achieved by an information structure that discloses whether the buyer’s value is above the cutoff $\psi(c)$, as illustrated by the green curve in Figure 3. Although the information structure that discloses whether the buyer’s value is above a cutoff $\psi(c)$ maximizes optimal profit of the hypothetical seller, $\max_{x \in [0,\bar{v}]}(x - \psi(c))(1 - H_c(x^-))$, pointwisely, it may fail to be incentive compatible in two senses. First, the seller with costs $c$ may not be willing to set the same optimal price as the hypothetical seller with cost $\psi(c)$. Second, the seller may have incentive to misreport. The intermediary may address the first issue by further giving the buyer all the information about his valuation whenever his value is below the cutoff $\psi(c)$ so that the information structure becomes an upward

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3}
\caption{Upper-bound on Revenue.}
\end{figure}
censorship with cutoff $\psi(c)$. Due to the assumption $\phi(\psi(c)) \leq c$, the seller with cost $c$ would never optimally set prices at any point below $\psi(c)$, as $\psi(c)$ is below the optimal monopolist price under full information and the monopolist’s profit function is single-peaked due to regularity. Therefore, for the seller with cost $c$, her optimal monopoly price under this information structure must coincide with the hypothetical seller with cost $\psi(c)$, namely $v(c)$. This shows that the upward censorship is able to incentivize the seller to set prices in a desirable way so that the pointwise upper bound in (2) can be attained. Finally, for incentive compatibility, using the assumption $\phi(\psi(c)) \leq c$ again, it can be verified that the inequality given by Lemma 1 is satisfied for all $c, c'$. Together, this shows that an upward censorship menu with cutoff $\psi$ is optimal.

To better understand the implication of Theorem 1, we may consider a benchmark in which the seller has full control of the technology to provide information for buyer to learn about his valuation. In this case, previous arguments imply that the seller with cost $c \in [0, \bar{c}]$ would prefer an upward censorship with cutoff $c$, as such information structure concentrates all the posterior expected value that are profitable for the seller (i.e. whenever $v > c$) to a singleton $\mathbb{E}[v|v > c]$ so that the seller can extract all the expected surplus by setting a price at $\mathbb{E}[v|v > c]$ and obtain profit $(\mathbb{E}[v|v > c] - c)(1 - F(c))$.

Being implicit in the statement of Theorem 1, under this optimal upward censorship, a seller with any cost $c \in [0, \bar{c}]$ would not only be willing to report truthfully and then face a buyer with distribution of posterior given by $H^c_u$, but would also optimally set a price at the highest possible posterior valuation $v(c)$. As such, for seller with cost $c \in [0, \bar{c}]$, profit from selling the object to the buyer is $(v(c) - c)(1 - F(\psi(c)))$. By comparing the intermediary-optimal information structure and the seller-optimal information structure, we can then see that the upward censorship is in fact the seller-optimal information structure had her cost $c$ been replaced by her virtual cost $\psi(c)$. Intuitively, the optimal upward censorship given in Theorem 1 is chosen so that the seller with cost $c$ is treated as if she has the technology to provide information to the buyers and has cost $\psi(c)$ and then internalizes all the information rents for having private cost when making pricing decisions. By providing such information structure, together with a properly designed transfer rule to extract profits from the seller, the menu $(H^c_u, t_u(c))_{c \in [0, \bar{c}]}$ will then be optimal. In other words, the upward censorship menu
given in Theorem 1 grants the technology of providing information to the seller but with an adjustment of cost to incorporate information rents and then extract the sellers surplus via a transfer that is paid up-front.

4.2 Optimal Menu with General Distributions

Although the assumption \( \phi(\psi(c)) \leq c \) guarantees optimality of a simple upward censorship menu, this assumption may not be satisfied for many reasonable applications. Qualitatively, it requires the information rent of the seller to be sufficiently small comparing to the information rent of the buyer under the prior, which, under regularity, is equivalent to requiring that the seller’s virtual cost \( \psi(c) \) is always below her optimal monopoly price when facing a fully-informed buyer. However, there are reasonable scenarios in which the seller’s information rent is considerably high relative to the buyer’s information rent under full information, such as cases when the prior distribution has low variances. To complete the analysis, we will now solve for the optimal menu without any assumptions on the distributions \( F \) and \( G \) other than regularity.

As a preview of the result, the optimal menu in this general case has a similar structure. Specifically, recall that with \( \phi(\psi(c)) \leq c \), the upward censorship menu with cutoff \( \psi \) has three critical features: 1) Given any reported \( c \in [0, \bar{c}] \), it fully informs the buyer when his valuation is below the cutoff \( \psi(c) \) and nothing else when his valuation is above the cutoff. 2) For seller with any cost \( c \in [0, \bar{c}] \), the optimal monopoly price under this upward censorship is the buyer’s expected value given that it is above the cutoff \( \psi(c) \) and 3) Truthful report is optimal for the seller. The solution in the general environment has similar features. In particular, for feature 1), it also gives no further information to the buyer when his valuation is above a certain cutoff. The difference is that the cutoff level may not be \( \psi(c) \) and that when the buyer’s valuation is below the cutoff, the information that he receives might be garbled rather than his true value. On the other hand, both feature 2) and feature 3) are preserved.

More formally, the solution in the general environment has the following properties:

**Definition 2.** Let \( \tilde{\psi} : [0, \bar{c}] \to [0, \bar{v}] \) be an increasing function. Fix any \( c \in [0, \bar{c}] \). An
information structure $H \in \mathcal{H}_F$ is a \textit{garbled upward censorship} with cutoff $\tilde{\psi}(c)$ if

$$H(x) = \begin{cases} F(\tilde{\psi}(c)), & \text{if } x \in [\tilde{\psi}(c), \mathbb{E}_F[v|v > \tilde{\psi}(c)]] \setminus \mathbb{E}_F[v|v > \tilde{\psi}(c)], \\ 1, & \text{if } x \in \mathbb{E}_F[v|v > \tilde{\psi}(c)], \forall x \in [\tilde{\psi}(c), \bar{v}] \end{cases}$$

and

$$\int_{\tilde{\psi}(c)}^{\bar{v}} H(x)dx = \int_{\tilde{\psi}(c)}^{\bar{v}} F(x)dx.$$ Furthermore, a garbled upward censorship with cutoff $\tilde{\psi}(c)$ is said to be \textit{responsive} if

$$\mathbb{E}_F[v|v > \tilde{\psi}(c)] \in \arg\max_{x \in [0, \bar{v}]} (x - c)(1 - H(x)).$$

Also, a menu $(H^c, t(c))_{c \in [0, \bar{c}]}$ is called a responsive garbled upward censorship menu with cutoff $\tilde{\psi}$ if $H^c$ is a responsive garbled upward censorship with cutoff $\tilde{\psi}(c)$ for all $c \in [0, \bar{c}]$.

Using this formal definition, the above description for the optimal menu means that a responsive upward censorship menu with cutoff $\tilde{\psi}$ is optimal, for some increasing function $\tilde{\psi} : [0, \bar{c}] \to [0, \bar{v}]$. Indeed, as Lemma 3 in the Appendix shows, one of the responsive garbled upward censorship menus must be optimal for the intermediary. It then remains to describe the cutoff function $\psi^*$ that gives an optimal menu. Before stating the formal result, we first examine how the intermediary’s problem (1) depend on the choice of cutoff and how to understand the incentive constraints and responsiveness.

In Figure 4 below, we fix an increasing function $\tilde{\psi}$ and plot the CDF of an upward censorship menu with cutoff $\tilde{\psi}(c) \in (0, \bar{v})$ as the blue curve, where the jump point is given by $\tilde{v}(c) := \mathbb{E}_F[v|v > \tilde{\psi}(c)]$. This upward censorship is clearly a garbled upward censorship. For it to be responsive, a seller with cost $c$ has to be willing to set the price at $\tilde{v}(c)$. One way to understand this constraint is through the following graphical approach. Let $\bar{\pi}(c) := (\tilde{v}(c) - c)(1 - F(\tilde{\psi}(c)))$ be the seller’s profit when setting a price at $\tilde{v}(c)$. For the price $\tilde{v}(c)$ to be optimal under the upward censorship menu, it has to be that

$$(x - c)(1 - F(x)) \leq \bar{\pi}(c) \iff F(x) \geq \left(1 - \frac{\bar{\pi}(c)}{(x - c)^+}\right)^+, \forall x \in [0, \tilde{\psi}(c)].$$

That is, the CDF $F$ has to be above the CDF of a \textit{Pareto distribution} with parameters $\bar{\pi}(c)$ and $c$ for all $x \in [0, \tilde{\psi}(c)]$, as illustrated by the green curve in Figure 4. If, on the other hand, the Pareto CDF is above $F$, as illustrated by the red curve in Figure 4, then setting a price
at $\tilde{v}(c)$ would not be optimal under the upward censorship for the seller with cost $c$. In fact, the graphs of Pareto distributions can be regarded as *iso-profit curves* for the seller and the direction of increment is toward the button-right corner. Responsiveness is then equivalent to requiring that the graph of the CDF of an information structure must be always above the graph of the Pareto CDF with parameters $\tilde{\pi}$ and $c$ on $[0, \tilde{\psi}(c)]$.

Figure 4: Upward Censorship with Cutoff $\tilde{\psi}(c)$.

With the observation above, we then know that for any information structure $H$ to be a responsive garbled upward censorship with cutoff $\tilde{\psi}$, it must be that: 1) The graph of $H$ is always above the graph of the Pareto CDF with parameters $\tilde{\pi}(c)$ and $c$. 2) The conditional CDF $F(x|x \leq \tilde{\psi}(c))$ is a mean preserving spread of $H(x|x \leq \tilde{\psi}(c))$. As such, a necessary condition for a garbled upward censorship menu $(H^c, t(c))_{c \in [0, \bar{c}]}$ with cutoff $\tilde{\psi}$ to be responsive and incentive compatible is that the Pareto CDF $(1 - \tilde{\pi}(c)/(x - c)^+)^+$ satisfies the second-order stochastic dominance constraint at $x = P(\tilde{\psi}(c), c)$, where, due to regularity this Pareto CDF crosses with $F$ at at most two points other than zero and $P(\tilde{\psi}(c), c)$ denotes the largest
crossing point. That is: For all \( c \in [0, \bar{c}] \),
\[
\int_0^{P(\tilde{\psi}(c), c)} \left(1 - \frac{\tilde{\pi}(c)}{(x - c)^+}\right)^+ \, dx \leq \int_0^{P(\tilde{\psi}(c), c)} F(x) \, dx.
\] (3)

Since if not, for the garbled upward censorship \( H^c \) to be responsive, as observed above, it must be that \( H^c(x) \geq (1 - \tilde{\pi}(c)/(x - c)^+) \) for all \( x \in [0, \tilde{\psi}(c)] \) and thus
\[
\int_0^{\tilde{\psi}(c)} H^c(x) \, dx \geq \int_0^{\tilde{\psi}(c)} \left(1 - \frac{\tilde{\pi}(c)}{(x - c)^+}\right)^+ \, dx > \int_0^{\tilde{\psi}(c)} F(x) \, dx,
\]
so that \( F \) cannot be a mean preserving spread of \( H^c \).

Figure 5: Necessary Condition for Responsiveness.

However, (3) is not sufficient for a garbled upward censorship menu \( (H^c, t(c))_{c \in [0, \bar{c}]} \) with cutoff \( \tilde{\psi} \) to be responsive and incentive compatible at the same time, due to the possibility of double deviations. To see this, suppose that \( H^z \) is responsive for all \( z \in [0, \bar{c}] \) and that (3) holds with equality for some \( c \in [0, \bar{c}] \). Then the only possible way for \( H^c \) to be responsive is that \( H^c(x) = (1 - \tilde{\pi}(c)/(x - c)^+) \) for all \( x \in [0, P(\tilde{\psi}(c), c)] \). As such, for any \( c' \in [0, c) \), the associated Pareto distribution (i.e. the iso-profit curve for the seller with cost \( c' \) at the price \( \tilde{v}(c) \), \( (1 - \tilde{\pi}(c')/(x - c')^+) \) is always above \( H^c \), as illustrated in Figure 5, where the
red curve represents the iso-profit curve $(1 - \tilde{\pi}(c')/(x - c')^+) + \pi^*$ and the green curve represents such $H^c$. This implies that whenever the seller’s cost is below $c$, she would optimally set a price at $\tilde{\pi}(c) + c$, which gives $H^c((\tilde{\pi}(c) + c)^-) = 0$. Consequently, if the seller’s cost is $c'$ and she misreports to be of cost $c$, the deviation gain is

$$\int_{c'}^{c} [F(\tilde{\psi}(z)) - H^c((\tilde{\pi}(c) + c)^-)] dz = \int_{c'}^{c} F(\tilde{\psi}(z)) dz > 0$$

and thus there exits no transfers $t : [0, \bar{c}] \to \mathbb{R}$ such that menu $(H^c, t(c))_{c \in [0, \bar{c}]}$ is incentive compatible.

This problem occurs because (3) only accounts for the incentives for a truth-telling seller to set prices correctly so that the garbled upward censorship $H^c$ could be responsive, but fail to account the misreporting seller to set prices in a desirable way and thus creates incentives for misreporting. The following menu, however, accounts for both by adjusting the cost for which we plot the iso-profit curves. Fix any increasing function $\tilde{\psi} : [0, \bar{c}] \to [0, \bar{v}]$, For each $c \in [0, \bar{c}]$, let

$$\tilde{k}(c) := c - \frac{1}{F(\tilde{\psi}(c))} \int_{0}^{c} F(\tilde{\psi}(z)) dz$$

and construct the Pareto iso-profit curve for seller with cost $\tilde{k}(c)$ when she sets a price at $\tilde{v}(c)$. That is, the CDF of $(1 - \tilde{\pi}(\tilde{k}(c)))/(x - \tilde{k}(c))^+$ again, by regularity of $F$, this iso-profit curve crosses with $F$ at at most two points other than zero, as illustrated by the green curve in Figure 6. Now if

$$\int_{0}^{P(\tilde{\psi}(c),\tilde{k}(c))} \left(1 - \frac{\tilde{\pi}(\tilde{k}(c))}{(x - \tilde{k}(c))^+} \right)^+ dx = \int_{0}^{P(\tilde{\psi}(c),\tilde{k}(c))} F(x) dx,$$

we may then take $H^c(x) = (1 - \tilde{\pi}(\tilde{k}(c)))/(x - \tilde{k}(c))^+$ for all $x \in [0, P(\tilde{\psi}(c),\tilde{k}(c))$ and $H^c(x) = F(x)$ for any $x \in [P(\tilde{\psi}(c),\tilde{k}(c)), \tilde{v}(c)]$. Then clearly, as the iso-profit curve of setting price at $\tilde{v}(c)$ for the seller with cost $c > \tilde{k}(c)$ is below $H^c$, $H^c$ is responsive. Furthermore, since the iso-profit curve crosses with $F$ at two points other than zero, $F$ is indeed a mean preserving spread of $H^c$. Finally, notice that for seller with any cost $c' \in (\tilde{k}(c), c]$, optimal monopoly price under $H^c$ is $\tilde{v}(c)$, whereas for seller with cost $c' \in [0, \tilde{k}(c))$, optimal monopoly price under $H^c$ gives zero probability of trade. Therefore, by construction of $\tilde{k}$, for any $c' \in [0, c]$, the deviation gain from misreporting to be of cost $c$ is

$$\int_{c'}^{c} F(\tilde{\psi}(z)) dz - (c - \max\{c', \tilde{k}(c)\}) F(\tilde{\psi}(c)) \leq \int_{0}^{c} F(\tilde{\psi}(z)) dz - (c - \tilde{k}(c)) F(\tilde{\psi}(c)) = 0.$$
On the other hand, for any cost $c' \in (c, \bar{c}]$, setting price at $\tilde{v}(c)$ is always optimal and therefore deviation gain is
\[ \int_{c}^{c'} [F(\tilde{v}(c)) - F(\tilde{v}(z))] dz \leq 0, \]
which then ensures that there is no incentive for any other costs $c'$ to misreport to be of cost $c$.

In fact, even if
\[ \int_{0}^{P(\tilde{\psi}(c), \tilde{k}(c))} \left( 1 - \frac{\tilde{\pi}(\tilde{k}(c))}{(x - \tilde{k}(c)^+)} \right)^+ dx < \int_{0}^{P(\tilde{\psi}(c), \tilde{k}(c))} F(x) dx, \]
an "ironing" procedure as illustrated by the red curve in Figure 6 can be applied and yields another feasible information structure $H^c$ that is responsive and gives non-positive deviation gain for any cost $c'$ to misreport to be of cost $c$.\(^7\) Together, as long as
\[ \int_{0}^{P(\tilde{\psi}(c), \tilde{k}(c))} \left( 1 - \frac{\tilde{\pi}(\tilde{k}(c))}{(x - \tilde{k}(c)^+)} \right)^+ dx \leq \int_{0}^{P(\tilde{\psi}(c), \tilde{k}(c))} F(x) dx, \]
for all $c \in [0, \bar{c}]$, we can construct an incentive compatible, individually ration and responsive menu $(H^c, t(c))_{c \in [0, \bar{c}]}$ with cutoff $\tilde{\psi}$.

In fact, by selecting a proper cutoff function $\tilde{\psi}$, the menu constructed above will be optimal for the intermediary. To construct such function. We first notice that as $c$ increases,
\[ \int_{0}^{P(\psi(c), k(c))} \left( 1 - \frac{\pi(k(c))}{(x - k(c)^+)} \right)^+ dx - \int_{0}^{P(\psi(c), k(c))} F(x) dx \]
also increases. As such, if we define
\[ c^* := \sup \left\{ c \in [0, \bar{c}] \left| \int_{0}^{P(\psi(c), k(c))} \left( 1 - \frac{\pi(k(c))}{(x - k(c)^+)} \right)^+ dx \leq \int_{0}^{P(\psi(c), k(c))} F(x) dx \right\} , \]
where
\[ k(c) := c - \frac{1}{F(\psi(c))} \int_{0}^{c} F(\psi(z)) dz \]
and
\[ \pi(c) := (v(c) - c)(1 - F(\psi(c))), \]
\(^7\)Formally, we find the convex hull of the minimum between the integral of prior $F$ and such Pareto distribution and take the smallest sub-differential pointwisely. See the last step of the proof of Theorem 2 in the Appendix.
for all \( c \in [0, \bar{c}] \), then
\[
\int_0^{P(\psi(c), k(c))} \left(1 - \frac{\pi(k(c))}{(x-k(c))^+}\right)^+ dx \leq \int_0^{P(\psi(c), k(c))} F(x) dx
\]
if and only if \( c \in [0, c^*] \). Now let \( \psi^*(c) := \min\{\psi(c), \psi(c^*)\} \), such \( \psi^* \) is then in fact optimal, which is stated in the following Theorem.

\[\Xi(x)\]

\[\Xi(x) = (1 - \frac{\pi(k(c))}{(x-k(c))^+})^+\]

\[\Xi(x) = F(x)\]

\[P(\tilde{\psi}(c), \tilde{k}(c)) \quad \tilde{v}(c)\]

\[x \quad \bar{v}\]

**Figure 6: Optimal Responsive Garbled Upward Censorship.**

**Theorem 2.** For each \( c \in [0, \bar{c}] \), let
\[
t_{gu}(c) := (v^*(c) - c)(1 - F(\psi^*(c))) - \int_{c}^{\bar{c}} (1 - F(\psi^*(z))) dz,
\]
where \( v^*(c) := \mathbb{E}_F[v|v > \psi^*(c)] \). Then there exists a family of information structures \( \{H_{gu}^c\}_{c \in [0, \bar{c}]} \subset \mathcal{H}_F \) such that \( (H_{gu}^c, t_{gu}(c))_{c \in [0, \bar{c}]} \) is an incentive compatible, individually rational and responsive garbled upward censorship menu with cutoff \( \psi^* \) that maximizes the intermediary’s revenue among all the incentive compatible and individually rational menus.

Formal proof of Theorem 2 can be found in the Appendix. We sketch the steps and stress the intuition here. As noted above, the main difficulty of the proof is the possibility
of double deviation and hence local incentive constraint fails to imply global incentive constraints. We address this problem by examine the dual. Specifically, we first characterize the incentive compatible, individually rational and responsive garbled upward censorship menus by a family of inequalities and further identify the critical ones that must be binding under any optimal menu. As such, the intermediary’s problem (1) then becomes a constraint maximization problem, to which we can write a dual problem. Then, we find the Lagrange multipliers under which the garbled upward censorship constructed above, together with the associated transfer induced by Lemma 1, is a solution of the dual problem and is incentive compatible and individually rational as well. By weak duality, we are then ensured that such menu is indeed optimal.

\[
\Xi(x) = H^c(x)
\]

\[
\Xi(x) = \left( 1 - \frac{\pi(k(c))}{(x-k(c))^+} \right)^+
\]

Figure 7: Optimality of Pareto-shape Garbling.

Qualitatively, the optimal garbled upward censorship \( \{H^c_{gu}\}_{c \in [0,\bar{c}]} \) is based on the Pareto distribution with parameters \( \pi(k(c)) \) and \( k(c) \), possibly with some "ironing procedures" to ensure mean-preserving spread property. Under a particular garbled upward censorship \( H^c_{gu} \), different costs of the seller are grouped into two classes, one that will optimally set prices at \( v^*(c) \) as the truthfully-reporting type, while the others will set a price that gives zero
probability of trade. The reason for using this Pareto-shape garbling can be seen below: Fix any increasing function $\tilde{\psi} : [0, \bar{c}] \to [0, \bar{v}]$ and consider any incentive compatible, individually rational and responsive garbled upward censorship menu $(H^c, t(c))_{c \in [0, \bar{c}]}$ with cutoff $\tilde{\psi}$. Recall that for any $c \in [0, \bar{c}]$ a family of Pareto-shaped distributions represent the iso-profit curves for each costs $c' \in [0, c]$ and must be everywhere below a CDF. As illustrated in Figure 7, where we take $H^c \equiv F$ on $[0, \tilde{\psi}(c)]$, the distribution $H^c$ must be above the upper envelope of a family of Pareto distributions. Indeed, each dotted curve in Figure 7 represents a Pareto distribution induced by a particular cost and the tangent point represents the optimal monopoly price for the seller with a given cost. Furthermore, notice that since $\tilde{\psi}(c) > \bar{\psi}(c)$, there exists some $\hat{k}(c) \in [0, c)$ such that for all sellers with cost $c' \in [\hat{k}(c), c]$, setting price at $\tilde{\psi}(c)$ as the truthfully-reporting type $c$ does is optimal. Whenever such cutoff type $\hat{k}(c)$ is positive, some mis-reporting types would set prices differently and thus monotonocity of $\psi$ would not be sufficient for global incentive compatibility. Such cutoff type $\hat{k}(c)$ must induce a Pareto distribution that crosses $(\tilde{\psi}(c), F(\tilde{\psi}(c)))$ and tangents to the distribution $H^c$ at some lower points, as illustrated by the red curve in Figure 7. Clearly, since $H^c \in \mathcal{H}_F$, the Pareto distribution induced by $\hat{k}(c)$ must satisfy the second order stochastic dominance constraint but with

$$\int_0^{P(\tilde{\psi}(c), \hat{k}(c))} \left( 1 - \frac{\pi(\hat{k}(c))}{(x - \hat{k}(c)^+) - \tilde{\psi}(c)} \right) dx < \int_0^{P(\tilde{\psi}(c), \hat{k}(c))} F(x) dx. \quad (4)$$

Therefore, we may reduce the cutoff cost $\hat{k}(c)$ so that (4) holds with equality. Notice that by reducing $\hat{k}(c)$ locally, the first order effect on deviation gain for any type $c' \in [0, \hat{k}(c)]$ is

$$H^c(x(\hat{k}(c), c)) - F(\tilde{\psi}(c)) < 0$$

and thus incentive compatibility can still be preserved by reducing the cutoff $\hat{k}(c)$. As a result, any incentive compatible, individually rational and responsive garbled upward censorship menu $(H^c, t(c))_{c \in [0, \bar{c}]}$ induces another Pareto-shaped garbled upward censorship menu with the same cutoff $\tilde{\psi}$. Moreover, since under any responsive menu with cutoff $\tilde{\psi}$, the objective for the intermediary in (1) is

$$\int_0^{\bar{c}} (\tilde{\psi}(c) - \psi(c))(1 - F(\tilde{\psi}(c)))G(dc),$$
which only depends on the cutoff $\tilde{\psi}$, this illustrates the reason why Pareto-shaped garbling as described above can be optimal.\footnote{When examining the dual problem, there is a more convenient way to establish optimality of Pareto-shaped garbling by observing that the objective in the dual problem is a convex functional on family of uniformly bounded increasing functions, which ensures that one of the extreme points must be optimal. Under our characterization, such extreme points correspond to Pareto-shape garbling. See detailed discussions in the Appendix.}

![Cutoff Functions](image)

**Figure 8: Cutoff Functions.**

Another way to understand the optimal garbled upward censorship menu in Theorem 2 is through the cutoff function $\psi^*$. In Figure 8, we plot the virtual cost $\psi$ (blue curve) and the cutoff function $\psi^*$ (red curve). Notice that if we drop the incentive constraints for truthfully-reporting costs and consider only on the pricing constraint for responsiveness. In this case, (3) is in fact both necessary and sufficient for existence of a responsive garbled upward censorship with cutoff $\tilde{\psi}$. This relation pins down a largest possible cutoff function below $\psi$ that can be supported by a responsive garbled upward censorship menu, denoted by $\hat{\psi}$ and illustrated by the green curve in Figure 8. Since the objective for the intermediary depends only on the cutoff function under responsive garbled upward censorship menus, $\hat{\psi}$ would then be the optimal cutoff function when ignoring the truthfully-reporting con-
straints. To incorporate the truthfully-reporting constraints, we must further reduce \( \hat{\psi} \) to rule out mis-reporting incentives. Since the definition of function \( k \) gives binding incentive constraints for the lowest type 0, it is the "cheapest" way, in the sense that least adjustment for \( \hat{\psi} \) is needed, to accommodate truthfully-reporting constraints among all the Pareto-shape garblings. This then gives the cutoff function \( \psi^* \). Notice that at \( c^* \), the Pareto iso-profit curve \((1 - \pi(k(c^*)))/(x - k(c^*))^+\) agrees with the distribution \( H_{gu}^c \) on \([0, \psi(c^*)]\), meaning that the constraint for responsiveness is binding at \( c^* \). Furthermore, type 0 is indifferent between truthfully reporting and reporting to be of any cost \( c \in [c^*, \bar{c}] \). However, since there is no distortion at the bottom under the menu \((H_{gu}^c, t_{gu}(c))_{c \in [0, \bar{c}]}\), the intermediary cannot further improve by relaxing the incentive constraint for type 0 in order to gain more for type \( c^* \), which suggests optimality of the menu \((H_{gu}^c, t_{gu}(c))_{c \in [0, \bar{c}]}\) as well.

Finally, we notice that from the payoff perspective, the only difference between the optimal upward censorship menu in Theorem 1 and the optimal garbled upward censorship menu in Theorem 2 is when the seller’s realized cost \( c \) is above the threshold \( c^* \). It is straightforward to show that when \( \phi(\psi(c)) \leq c \) for all \( c \in [0, \bar{c}] \), \( c^* = \bar{c} \) and therefore \( \psi \equiv \psi^* \), meaning that the two menus give the same revenue when the condition in Theorem 1 is satisfied. On the other hand, when \( c^* < \bar{c} \), the optimal garbled upward censorship menu in Theorem 2 will give all the sellers with cost above \( c^* \) the same information structure at the same price, which reflects a bundling property of this optimal menu. The intuition is that, when the virtual costs of the seller is too high, granting information rents to these sellers will be too costly since there is not enough of leverage to create profit from the sell via manipulating the buyer’s information. The intermediary would then have to sacrifice the additional gains from trade for high-virtual cost sellers to avoid paying too much information rent.

### 4.3 Welfare Analysis and Comparative Statics

We end this section by providing some welfare analysis and comparative statics. For simplicity, we focus on the case when the condition of Theorem 1 holds. Analyses with general regular distributions have qualitatively similar implications and can be found in the Online Appendix. To begin with, assume that \( F \) and \( G \) satisfy the condition \( \phi(\psi(c)) \leq c \) for all \( c \in [0, \bar{c}] \) so that the upward censorship menu with cutoff \( \psi \) is optimal. By Theorem 1, the
total surplus generated by the sell is

$$(v(c) - c)(1 - F(\psi(c))) = \int_{\psi(c)}^{\bar{\psi}} (1 - F(x))dx + (\psi(c) - c)(1 - F(\psi(c))).$$

With the probability of trade being $(1 - F(\psi(c)))$, for all $c \in [0, \bar{c}]$. Comparing this with the benchmark case in which the seller has the technology to provide information to the buyer, where the optimal information structure, as noted above, gives total surplus generated by trade

$$(\mathbb{E}_F[v|v > c] - c)(1 - F(c)) = \int_{c}^{\bar{\psi}} (1 - F(x))dx$$

and a probability of trade $1 - F(c)$, we have the following observation:

**Proposition 1** (Welfare Comparison). Suppose that $\phi(\psi(c)) \leq c$. Then the expected total surplus generated by trade and the probability of efficient trade are larger when the seller has control of the information technology than when the intermediary has control of the information technology.

In brief, Proposition 1 shows that when the seller does not have the technology to provide information to the buyer directly but has to do so by interacting with an intermediary who has this technology, since the seller has private information about production cost, the ownership of such information technology matters. Indeed, when the seller has to buy such information technology from the intermediary, due to the presence of incomplete information, the seller would demand information rent from the intermediary and total surplus will be reduced since the intermediary has to provide information structures so that the seller would be willing to internalize her information rent when making pricing decisions.

Finally, we examine how the shifts of the distribution of valuation and the distribution of production cost affects the intermediary’s revenue and the total surplus generated by trade. Specifically, take any two pairs of distributions $F_1, G_1$ and $F_2, G_2$ such that $\phi_i(\psi_j(c)) \leq c$, for all $c \in [0, \bar{c}]$, all $i, j \in \{1, 2\}$, where $\phi_i, \psi_i$ are the induced virtual value and virtual cost of $F_i, G_i$, respectively, for all $i \in \{1, 2\}$. The previous observation then gives us the following comparative statics analysis:

**Proposition 2** (Comparative Statics).
1. Suppose that $F_1$ first order stochastic dominates $F_2$. That is, $F_1 \leq F_2$. Then the total surplus, the intermediary’s revenue and seller’s expected net profit under $(F_1, G_i)$ are larger than those under $(F_2, G_i)$, $i \in \{1, 2\}$.

2. Suppose that $F_1$ is a mean preserving spread of $F_2$. Then intermediary revenue under $(F_1, G_i)$ is larger than that under $(F_2, G_i)$, $i \in \{1, 2\}$.

3. Suppose that $G_2$ dominates $G_1$ in the hazard rate order. That is $g_1/G_1 \geq g_2/G_2$. Then the total surplus, the intermediary’s revenue and the seller’s expected revenue are larger under $(F_i, G_1)$ than those under $(F_i, G_2)$, $i \in \{1, 2\}$.

To summarize, when the buyer’s value becomes higher, in the sense of first order stochastic dominance, it becomes easier for the intermediary to generate trade surplus by providing proper information to the buyer and therefore total surplus, the intermediary’s revenue and the seller’s net profit all increase. When the distribution of valuation becomes more spread-out, in the sense of mean preserving spread, the informational tools for the intermediary becomes more flexible and therefore revenue increases. Finally, when the seller’s cost shifts in hazard rate order, causing a reduction of information rent and the costs, in the sense of first order stochastic dominance, the seller retains less information rent and the distortion on information structure for her to internalize pricing decision reduces. These two factors jointly increase total surplus and intermediary’s revenue as well. Furthermore, although the reduction of information rent and reduction of production has opposite effects on the seller’s net profit, Proposition 2 shows that the gain in total surplus offsets the loss of information rent of the seller and hence also increases seller’s expected net profit.

5 Extension: Contracting Publicity

In the baseline model analyzed above, we assumed that the buyer becomes aware of the seller’s object only through the intermediary’s technology so that the seller gets zero profit if she does not buy from any of the items in the menu that the intermediary offers. This often occurs in situations where the seller does not have significant publicity on the market and the intermediary owns a platform that is well known to the buyer, on which the seller’s object
can be presented. For instance, when a seller wants to sell an object on Online platforms such as Amazon, or when a seller wants to advertise her object through the search engine on the Internet and buys advertisement from Google. However, in such scenarios, it is reasonable to argue that the intermediary—owning a platform that can not only provide information, but can also publicize the seller’s object—can also screen the seller on the extensive margin by controlling how public it can make the seller’s object be to the buyer, in addition to the intensive margin by providing different information to the buyer, as modeled above. In this section, we introduce this extra leverage to the intermediary. Specifically, we now consider a model in which the seller’s menu contains not only information structures that will be use to inform the buyer but also the level of publicity the seller’s object is going to receive.

Formally, we include an other component \( \alpha : [0, \bar{c}] \rightarrow [0, 1] \) in the intermediary’s (direct) menu, where \( \alpha(c) \) stands for the probability that the buyer will be aware of the seller’s object when the reported cost is \( c \), which we will refer as publicity policy hereafter. Again, as the buyer has quasi-linear preference, a menu can be represented by a tuple \( (H^c, t(c), \alpha(c))_{c \in [0, \bar{c}]} \) with \( H^c \in \mathcal{H}_F \), \( t(c) \in \mathbb{R} \), \( \alpha(c) \in [0, 1] \) for all \( c \in [0, \bar{c}] \) and incentive compatibility and individual rationality constraints become:

\[
\alpha(c) \left[ \max_{x \in [0, \bar{v}]} (x - c)(1 - H^c(x^-)) \right] - t(c) \geq \max \left\{ \alpha(c') \left[ \max_{x \in [0, \bar{v}]} (x - c)(1 - H^{c'}(x^-)) \right] - t(c'), 0 \right\}.
\]

Therefore, by using the same argument, an analogous characterization as Lemma 1 can be found as below:

**Lemma 2.** Suppose that a menu \( (H^c, t(c), \alpha(c))_{c \in [0, \bar{c}]} \) is incentive compatible and individually rational. Then for any selection

\[
x(c, c') \in \arg\max_{x \in [0, \bar{v}]} (x - c)(1 - H^{c'}(x^-)),
\]

1. \( t(c) = t(\bar{c}) + \alpha(c) \max_{x \in [0, \bar{v}]} (x - c)(1 - H^c(x^-)) - \int_0^c \alpha(z)(1 - H^z(x(z, z^-)))dz \).
2. \( t(\bar{c}) \leq 0 \).
3. The function \( c \mapsto \alpha(c)(1 - H^c(x(c, c^-))) \) is nonincreasing.
4. \( \int_c^{c'} (\alpha(z)(1 - H^z(x(z, z^-))) - \alpha(c')(1 - H^{c'}(x(z, c'^-))))dz \geq 0 \) for any \( c', \bar{c} \in [0, \bar{c}] \).
Conversely, suppose that for a menu \((H, t(c), \alpha), c \in [0, \bar{c}]\), there exists a selection 

\[ x(c, c') \in \arg\max_{x \in [0, \bar{x}]} (x - c)(1 - \phi'(x)) \]

satisfying:

1. \( t(c) = \alpha(c) \max_{x \in [0, \bar{x}]} (x - c)(1 - \phi(x)) - \int_{c}^{\bar{c}} \alpha(z)(1 - \phi(z))dz \).
2. \( \int_{c}^{\bar{c}} (\alpha(z)(1 - \phi(z)) - \alpha(c')(1 - \phi'(z)))dz \geq 0 \) for any \( c', c \in [0, \bar{c}] \).

Then \((H, t(c), \alpha(c)), c \in [0, \bar{c}]\) is incentive compatible and individually rational.

By the characterization of Lemma 2, the intermediary’s revenue maximization can then be similarly written as:

\[
\sup_{\{H_{c}, t(c), \alpha(c)\} \in [0, \bar{c}], \alpha(c)} \int_{c}^{\bar{c}} \alpha(c) \left( \max_{x \in [0, \bar{x}]} (x - c)(1 - \phi(x)) - (1 - \phi(x)) \right) \frac{G(c)}{g(c)} G(d) \\
\text{s.t.} \int_{c}^{\bar{c}} (\alpha(z)(1 - \phi(z)) - \alpha(c')(1 - \phi'(z)))dz \geq 0, \forall c', c \in [0, \bar{c}],
\]

It is rather straightforward to see that under the assumption of Theorem 1, \( \phi(\phi(c)) \leq c \) for all \( c \in [0, \bar{c}] \), the upward censorship menu with cutoff \( \psi \) as in Theorem 1, together with an always-publicizing policy \( \alpha_u \equiv 1 \) maintains to be optimal. To see this, analogous to (2), we have that for any information structure \( H \in \mathcal{H}_P \), any selection \( x_H(c) \in \arg\max_{x \in [0, \bar{x}]} (x - c)(1 - \phi(x)) \) and any publicizing policy \( \alpha \),

\[
\alpha(c) \left[ \max_{x \in [0, \bar{x}]} (x - c)(1 - \phi(x)) - (1 - \phi(x)) \right] \frac{G(c)}{g(c)} \leq \max_{x \in [0, \bar{x}]} (x - \psi(c))(1 - \phi(x)).
\]

As the upward censorship menu maximizes the optimal profit for the hypothetical seller with cost \( \psi(c) \), by (6), the revenue given by the upward censorship menu with cutoff \( \psi \) and publicizing policy \( \alpha_u \equiv 1 \) attains an upper bound of the seller’s revenue. As this menu is incentive compatible and individually rational by arguments above and by Lemma 2, it also solves the intermediary’s problem. As such, whenever the upward censorship menu with cutoff \( \psi \) is incentive compatible and individually rational, fully-publicizing upward censorship menu with cutoff \( \psi \) is optimal.
On the other hand, when we do not have the assumption \( \phi(\psi(c)) \leq c \) for all \( c \in [0, \bar{c}] \), the fully-publicized upward censorship menu with cutoff \( \psi \) will not be incentive compatible, as noted in the previous section. To understand the optimal menu in the general case, we first notice that optimality of responsive garbled upward censorship menus can still be analogously established by using similar arguments as in Lemma 3. Given any publicizing policy \( \alpha : [0, \bar{c}] \rightarrow [0, 1] \), under any responsive garbled upward censorship, the intermediary’s expected revenue is given by

\[
\int_0^{\bar{c}} \alpha(c)(\bar{v}(c) - \psi(c))(1 - F(\tilde{\psi}(c)))G(dc).
\]

Notice that under the optimal responsive garbled upward censorship menu with cutoff \( \psi^* \) introduced in the previous section, there exists some \( \hat{c} \in [c^*, \bar{c}] \) such that integrand in (7),

\[
\alpha(c)(v^*(c) - \psi(c))(1 - F(\psi^*(c))) < 0
\]

for all \( c \in (\hat{c}, \bar{c}] \) whenever \( \alpha(c) > 0 \). Indeed, as \( \psi^*(c) = \psi(c^*) \) for all \( c \in [c^*, \bar{c}] \), when \( c \) is large enough, we will have \( (v^*(c) - \psi(c)) < 0 \). This means that when the intermediary cannot control publicity, Theorem 2 indicates that when the seller’s information rent is large, the intermediary would rather shut down the publicity of these sellers, instead of including them in the menu and pay a significant amount of information rent. Indeed, by setting \( \alpha(c) = 0 \) whenever \( (v^*(c) - \psi(c))(1 - F(\psi^*(c))) < 0 \), the intermediary can increase the expected surplus since the integrand of (7) would be increased pointwisely and the incentive constraints will be relaxed. Formal statement of the result and the proof under this environment can be found in the Online Appendix. In brief, combining the argument for optimality of responsive garbled upward censorship menu with cutoff \( \psi^* \) in the previous section and in the proof of Theorem 2 and this observation, we may conclude that the optimal menu in this environment is given by \( (\alpha_{gu}(c), H^c_{gu}, t_{gu}(c))_{c \in [0, \bar{c}]} \), where \( (H^c_{gu}, t_{gu}(c))_{c \in [0, \bar{c}]} \) is the responsive garbled upward censorship with cutoff \( \psi^* \) described in the previous section and \( \alpha(c) = 1\{c \leq \hat{c}\} \) and \( \hat{c} \) is the unique solution of

\[
E_F[v|v > \psi(c^*)] = \psi(\hat{c}).
\]

That is, in this optimal menu, the intermediary rules out the sellers with information rents that are too high by shutting down their publicity while maintaining full publicity for the
remaining sellers and provided a responsive garbled upward censorship menu with cutoff $\psi^*$ as in the baseline model.

6 Conclusion

We studied an optimal pricing problem for an informational intermediary to sell information structures that informs the consumers of a monopolist. We found that the optimal menu displays an upward censorship feature consisting of information structures that discloses all—or a partially garbled, in general—information to the buyer when the valuation is low and no information when the valuation is high, which displays a negative targeting feature. Such menu contains a continuum of items that prescribes the range of consumer values that will be given full (or partially garbled, in general) information and prices for each information structure. A welfare analysis suggests that total surplus generated by the sell reduces comparing to a benchmark where the monopolist can disclose information themselves, due to the presence of incomplete information of production cost. We also show that both the intermediary’s revenue and the monopolist’s net profit increases when either the distribution of valuation shifts up in the sense of first order stochastic dominance or when the distribution of costs shifts up in the sense of hazard rate dominance. Moreover, the intermediary’s revenue increases when the distribution of valuation becomes more dispersed. Finally, we extend our result to a situation when the intermediary can also contract on how well the monopolist is going to be perceived by the consumers and find that the intermediary will only include items that fully publicize the monopolist in its optimal menu and some of the monopolists will be left out. Allowing the publicity to be consumer-value specific will extend our model to capture both the intensive margin (as in our model) and the extensive margin of targeting can be a direction for future research.

Although our model is a revenue maximizing problem of an informational intermediary who owns the technology to publicize a commodity and provide information to the consumers, the techniques that we developed in this framework—the characterization of inventive compatible menu, the pointwise maximization approach and the duality approach that solves for optimal menu—seem to be applicable to a broader class of setting where the interme-
diary might have different objectives and the outside options of the buyer of information structures might be different, including regulatory policies on information disclosure of new products (e.g. financial products or drugs) that a government can implement to improve social welfare. These can also be topics for future studies.

References


Appendix

A. Proof of Lemma 1.

Proof of Lemma 1. For necessity, consider any incentive compatible and individually menu \((H^c, t(c))_{c \in [0, \bar{c}]}\).

Let

\[ \Pi(c, c') := \max_{x \in [0, \bar{v}]} (x - c)(1 - H^c(x^-)) \]

be the seller’s expected profit under the information structure \(H^c\) and cost \(c\). By the envelope theorem (Milgrom & Segal, 2002), since the function

\[ c \mapsto (x - c)(1 - H^c(x^-)) \]

is absolutely continuous with value uniformly bounded by \(-\bar{c}\) and \(\bar{v}\) for any fixed \(x \in [0, \bar{v}], c' \in [0, \bar{c}]\), \(\Pi(\cdot, c')\) is absolutely continuous for all \(c' \in [0, \bar{c}]\) and its derivative exists and equals to

\[ \Pi_1(c, c') = -(1 - H^c(x(c, c')^-)) \] (8)

for any selection \(x(c, c') \in \text{argmax}_{x \in [0, \bar{v}]} (x - c)(1 - H^c(x^-))\), for (Lebesgue) almost all \(c \in [0, \bar{c}]\).

Now let

\[ V(c, c') := \Pi(c, c') - t(c') \]

be the seller’s profit net of transfer if the cost is \(c\) and the (mis)report \(c'\). Incentive compatibility then implies

\[ V^*(c) := V(c, c) = \max_{c' \in [0, \bar{c}]} V(c, c'). \]

Since \(\Pi(\cdot, c')\) is absolutely continuous and uniformly bounded by \(-\bar{c}\) and \(\bar{v}\), by the envelope theorem again,

\[ V^*(c) = V(\bar{c}) - \int_c^{\bar{c}} \Pi_1(z, z)dz = V(\bar{c}) + \int_c^{\bar{c}} (1 - H^z(x(z, z^-)))dz. \]

Rearranging, we have:

\[ t(c) = t(\bar{c}) + \max_{x \in [0, \bar{v}]} (x - c)(1 - H^c(x^-)) - \int_c^{\bar{c}} (1 - H^z(x(z, z^-)))dz, \]

which established assertion 1.

Furthermore, since \(V^*\) is nonincreasing, individual rationality implies that \(-t(\bar{c}) = V^*(\bar{c})\geq 0\), which established assertion 2.
In addition, by assertion 1, for any \(c, c' \in [0, \bar{c}]\),

\[
\int_{c}^{c'} [H^c(x(z, c')) - H^z(x(z, z^-))] dz
= \int_{c}^{c'} (1 - H^z(x(z, z^-))) dz - \int_{c}^{c'} \Pi_1(z, c') dz
= \int_{c}^{c'} (1 - H^z(x(z, z^-))) dz - (\Pi(c, c') - \Pi(c', c'))
= \int_{c}^{c'} (1 - H^z(x(z, z^-))) dz - (\Pi(c, c') - \Pi(c', c')) - \int_{c'}^{\bar{c}} (1 - H^z(x(z, z^-))) dz
= V(c, c) - V(c, c')
\geq 0,
\]

where the first equality follows from (8), the second equality follows from the fundamental theorem of calculus and the last equality follows from assertion 1. This establishes assertion 3.

Finally, notice that for all \(c' \in [0, \bar{c}]\), \(\Pi(\cdot, c')\) is a pointwise supremum of a family of affine functions and this is convex. Therefore, \(V^*\) is also convex as it is a pointwise supremum of a family of convex functions. Together, its (almost everywhere) derivative \(-(1 - H^c(x(c, c)))\) is nondecreasing in \(c\). This proves assertion 4.

Conversely, take any menu \((H^c, t(c))_{c \in [0, \bar{c}]}\) and any selection \(x(c, c') \in \text{argmax}_{x \in [0, \bar{c}]} (x - c)(1 - H^{c'}(x^-))\) that satisfy conditions 1 and 2. Again, let \(\Pi(c, c') := \max_{x \in [0, \bar{c}]} (x - c)(1 - H^{c'}(x^-))\) and let \(V(c, c') := \Pi(c, c') - t(c')\). By condition 1 and 2, (8) and (9), for any \(c, c' \in [0, \bar{c}]\),

\[
V(c, c) - V(c, c') = \int_{c}^{c'} [H^c(x(z, c')) - H^z(x(z, z^-))] dz \geq 0,
\]

where the inequality follows from condition 2. This completes the proof. 

\[\blacksquare\]

\section*{B. Proofs of Main Results}

\subsection*{B1. Proof of Theorem 1}

\textit{Proof of Theorem 1.} We first construct an upper bound of the intermediary’s revenue given by (1), denoted by \(R^*\) and then show that the proposed upward censorship menu \((H^c, t(c))_{c \in [0, \bar{c}]}\) is incentive compatible, individually rational and attains the upper bound \(R^*\). First recall that given any incentive compatible and individually menu \((H^c, t(c))_{c \in [0, \bar{c}]}\). Lemma 1 gives the intermediary’s revenue as:

\[
t(\bar{c}) + \int_{0}^{\bar{c}} \left( (x(c, c) - c)(1 - H^c(x(c, c^-))) - (1 - H^c(x(c, c^-)) \frac{G(c)}{g(c)} \right) G(dc),
\]
where for each \( c \in [0, \bar{c}] \), \( x(c, c) \) is the largest selection of \( \arg\max_{x \in [0, \bar{c}]} (x - c)(1 - H^c(x^-)) \) be the largest selection. Then for each \( c \in [0, \bar{c}] \),

\[
(x(c, c) - c)(1 - H^c(x(c, c)^-)) - (1 - H^c(x(c, c)^-)) \frac{G(c)}{g(c)} \leq \max_{x \in [0, \bar{c}]} (x - \psi(c))(1 - H^c(x^-)),
\]

and thus for any incentive compatible and individually rational menu \((H^c, t(c))_{c \in [0, \bar{c}]}\), the intermediary’s revenue is bounded from above by

\[
\int_0^\bar{c} \max_{x \in [0, \bar{c}]} (x - \psi(c))(1 - H^c(x^-)) G(dc).
\]

On the other hand, since \( H^c \in \mathcal{H}_F \) for all \( c \in [0, \bar{c}] \), for any \( \tilde{x}(c, c) \in \arg\max_{x \in [0, \bar{c}]} (x - \psi(c))(1 - H^c(x^-)) \),

\[
\max_{x \in [0, \bar{c}]} (x - \psi(c))(1 - H^c(x^-)) \
\leq (\tilde{x}(c, c) - \psi(c))(1 - H^c(\tilde{x}(c, c)^-)) + \int_0^\bar{c} (1 - H^c(z)) dz \
\leq \int_{\psi(c)}^\bar{c} (1 - H^c(z)) dz \
\leq \int_{\psi(c)}^\bar{c} (1 - F(z)) dz \
= (\mathbb{E}_F[v | v > \psi(c)] - \psi(c))(1 - F(\psi(c))),
\]

where second inequality follows from monotonicity of \( H^c \), the third inequality follows from the fact that \( F \) is a mean-preserving spread of \( H^c \) and the last equality follows from integration by parts. Together, we have that for any incentive compatible and individually rational menu \((H^c, t(c))_{c \in [0, \bar{c}]}\),

\[
R^* := \int_0^\bar{c} (v(c) - \psi(c))(1 - F(\psi(c))) G(dc) \
\geq \int_0^\bar{c} \max_{x \in [0, \bar{c}]} (x - \psi(c))(1 - H^c(x^-)) G(dc). \
\geq \int_0^\bar{c} \left( (x(c, c) - c)(1 - H^c(x(c, c)^-)) - (1 - H^c(x(c, c)^-)) \frac{G(c)}{g(c)} \right) G(dc)
\]

and therefore the intermediary’s revenue given by any incentive compatible and individually rational menu \((H^c, t(c))_{c \in [0, \bar{c}]}\) must be no greater than \( R^* \).

Now notice that under the upward censorship menu \((H^c_u, t_u(c))_{c \in [0, \bar{c}]}\), if each truthful-reporting seller whose cost is \( c \in [0, \bar{c}] \) sets price optimally at \( v(c) \), then for all \( c \in [0, \bar{c}] \),

\[
\max_{x \in [0, \bar{c}]} (x - c)(1 - H^c_u(x^-)) = (v(c) - \psi(c))(1 - F(\psi(c))).
\]
Therefore, it suffices to show that (10) holds for the upward censorship menu \((H_u^c, t_u(c))_{c \in [0, \bar{c}]}\) and that this menu is incentive compatible and individually rational, as this would imply that the menu \((H_u^c, t_u(c))_{c \in [0, \bar{c}]}\) is feasible and attains the upper bound \(R^*\) of problem (1).

Indeed, first notice that \(\phi(\psi(c)) \leq c\) for all \(c \in [0, \bar{c}]\) is equivalent to:

\[
\psi(c) \leq x_F(c), \ \forall c \in [0, \bar{c}],
\]

where \(x_F(c)\) is the unique element of \(\arg\max_{x \in [0, \bar{v}]} (x - c)(1 - F(x))\). Take and fix any \(c \in [0, \bar{c}]\), under the upward censorship \(H_u^c\), for a seller with cost \(c\), setting price at any \(x \in [0, \psi(c)]\) gives profit

\[
(x - c)(1 - F(x)) \leq (\psi(c) - c)(1 - F(\psi(c))),
\]

since the function \(x \mapsto (x - c)(1 - F(x))\) is single-peaked by regularity and since \(x \leq \psi(c) \leq x_F(c)\). Furthermore, setting any prices in \([\psi(c), v(c)]\) must be worse than setting price at \(v(c)\) since \(H_u^c\) is a constant on \((\psi(c), v(c))\). Finally, for any \(x \in (v(c), \bar{v}]\), the seller gets zero profit by setting a price at \(x\). Together, for the truthfully-reporting seller with cost \(c\), setting price at \(v(c)\) is indeed optimal.

Moreover, for any \(c, c' \in [0, \bar{c}]\), if \(c' \leq c\),

\[
\int_{c'}^c [H_u^c(x(z)^-) - H_u^c(x(z), z^-)] dz = \int_{c'}^c [F(\psi(c)) - F(\psi(z))] dz + \int_0^{c} [F(x_F(z)) - F(\psi(z))] dz \geq 0,
\]

for some \(\hat{c} \in [0, c]\), where the inequality follows from monotonocity of \(\psi\) and that \(x_F(c) \geq \psi(c)\) for all \(c \in [0, \bar{c}]\). Finally, if \(c' > c\), then for any \(z \in [c, c']\), \(H_u^c(x(z), c^-) = F(\psi(c)) \leq F(\psi(z))\) by construction of \(H_u^c\) and by monotonocity of \(\psi\). As such,

\[
\int_{c}^{c'} [H_u^c(x(z)^-) - H_u^c(x(z), c^-)] dz = \int_{c}^{c'} [F(\psi(z)) - F(\psi(c))] dz \geq 0.
\]

Together with Lemma 1, the upward censorship menu \((H_u^c, t_u(c))_{c \in [0, \bar{c}]}\) is indeed incentive compatible and individually rational. This completes the proof.

### B2. Proof of Theorem 2

Before stating the formal proof of Theorem 2, we first outline each step and sketch the structure of the proof. First, we will show that to maximize the expected revenue by choosing among all the possible incentive compatible and individually rational menu, it is without loss to restrict attention
to incentive compatible, individually rational and responsive garbled upward censorship menus. This will be done by Lemma 3. Second, we develop a characterization for incentive compatible, individually rational and responsive garbled upward censorship menus that allows us to represent them by a family of inequalities and equalities so that the intermediary’s problem can then be expressed as a constraint optimization problem. This will be done by Lemma 4 and Lemma 5. Next, we will then identify a class of critical constraints for the constraint optimization problem just obtained and show that for any feasible choice in the constraint optimization, two types of constraints must be met with equality. This is the content of Lemma 6. Finally, we will use the critical constraints to write down the dual of the constraint optimization problem and find the proper Lagrange multipliers so that the proposed menu indeed solves the dual problem, which effectively closes the duality gap and establishes optimality.

Proof of Theorem 2.

Step 1: We first show that it is without loss to restrict attention to the family of incentive compatible, individually rational and responsive upward censorship menus. This is implied by the following Lemma.

Lemma 3. For any incentive compatible and individually rational menu \((H^c, t(c))_{c \in [0, \bar{c}]\)}, there exists an increasing function \(\tilde{\psi} : [0, \bar{c}] \to [0, \bar{v}]\) and an incentive compatible and individually rational menu \((\tilde{H}^c, \tilde{t}(c))_{c \in [0, \bar{c}]\)} such that \(\tilde{H}^c\) is a responsive garbled upward-censorship with cutoff \(\tilde{\psi}(c)\) and that

\[
\int_0^{\bar{c}} t(c)G(dc) \leq \int_0^{\bar{c}} \tilde{t}(c)G(dc).
\]  

Proof. Let \((H^c, t(c))_{c \in [0, \bar{c}]\)} be an incentive compatible and individually rational menu. For each \(c, z \in [0, \bar{c}]\), denote \(x(z, c)\) by the largest element of

\[
\text{argmax}_{x \in [0, \bar{v}]} (x - z)(1 - H^c(x^-))
\]

and let \(x(c) := x(c, c)\). By Lemma 1 and Fubini’s theorem,

\[
\int_0^{\bar{c}} t(c)G(dc) = t(\bar{c}) + \int_0^{\bar{c}} (x(c) - \psi(c))(1 - H^c(x(c^-)))G(dc)
\]

\[
\leq \int_0^{\bar{c}} (x(c) - \psi(c))(1 - H^c(x(c^-)))G(dc),
\]

where the inequality follows from individual rationality.
For any \( c \in [0, \bar{c}] \), let \( \tilde{\psi}(c) := F^{-1}(H^c(x(c)^-)) \). Notice first that by Lemma 1, \( \tilde{\psi} \) is increasing. Now define an information structure \( \hat{H}^c \) as follows:

\[
\hat{H}^c(x) := \begin{cases} 
H^c(x), & \text{if } x \in [0, \psi(c)) \\
F(\tilde{\psi}(c)), & \text{if } x \in [\psi(c), \bar{v}(c)) \\
1, & \text{if } x \in [\bar{v}(c), \bar{v}]
\end{cases}
\]

where \( \psi(c) \) is uniquely determined by the equation

\[
\int_0^{\psi(c)} F(x)dx = \int_0^{\psi(c)} H^c(x)dx + (\tilde{\psi}(c) - \psi(c))F(\tilde{\psi}(c))
\]

and

\[
\bar{v}(c) := \mathbb{E}_F[v|v > \tilde{\psi}(c)].
\]

We claim that \( \bar{v}(c) \) is the largest optimal price for a seller with cost \( c \) under the information structure \( \hat{H}^c \). That is,

\[
\bar{v}(c) = \max \left\{ \arg\max_{x \in [0, \bar{v}]} (x - c)(1 - \hat{H}^c(x^-)) \right\}.
\]

Indeed, any \( x \in (\psi(c), \bar{v}(c)) \) cannot be optimal since the function is strictly increasing on \( (\psi(c), \bar{v}(c)) \). Also, any \( x \in (\bar{v}(c), \bar{v}] \) cannot be optimal either since it gives zero profit to the seller. Finally, for any \( x \in [0, \psi(c)] \),

\[
(x - c)(1 - \hat{H}^c(x^-)) = (x - c)(1 - H^c(x^-))
\]

\[
\leq (x(c) - c)(1 - H(x(c)^-))
\]

\[
\leq (\mathbb{E}_{H^c}[v|v > x(c)] - c)(1 - H(x(c)^-))
\]

\[
\leq (\mathbb{E}_{F}[v|v > \tilde{\psi}(c)] - c)(1 - F(\tilde{\psi}(c)));
\]

where the first equality follows from the fact that \( x \in [0, \psi(c)] \), the first inequality follows from optimality of \( x(c) \) under \( H^c \) and the last inequality follows from the construction of \( \tilde{\psi} \) do that \( F(\tilde{\psi}(c)) = H^c(x(c)^-) \) and that \( \mathbb{E}_{H^c}[v|v > x(c)] \leq \mathbb{E}_F[v|v > \tilde{\psi}(c)] \), which follows from \( F(\tilde{\psi}(c)) = H^c(x(c)^-) \) and the fact that \( H^c \in \mathcal{H}_F \).

We now verify that the menu \( \{\hat{H}^c\}_{c \in [0, \bar{c}]} \) is implementable by some transfer \( \hat{t} : [0, \bar{c}] \to \mathbb{R} \). By Lemma 1, it suffices to show that for any \( c', c \in [0, \bar{c}] \) with \( c' < c \),

\[
\int_{c'}^c [\hat{H}^c(\hat{x}(z, c)^-) - \hat{H}^c(\hat{x}(z, c)^-)]dz \geq 0,
\]

where \( \hat{x}(z, c) \) is the largest element of

\[
\arg\max_{x \in [0, \bar{v}]} (x - z)(1 - \hat{H}^c(x^-)).
\]
Take and fix any $c', c \in [0, \bar{c}]$ with $c' < c$. Notice that as argued above, since $\tilde{v}(c)$ is the largest optimal price for a seller with cost $c$ under $\tilde{H}^c$, $\tilde{H}^c(\tilde{x}(c, c)^-) = F(\tilde{v}(c)) = H^c(x(c))$ for all $c \in [0, \bar{c}]$. On the other hand, notice that by construction of $\tilde{H}^c$, for any $z \in [c', c]$, either $\tilde{x}(z, c) = \tilde{v}(c)$ or $\tilde{x}(z, c) = x(z, c) \in [0, \tilde{v}(c))$. In addition, since the original menu $(H^c, t(c))_{c \in [0, \bar{c}]}$ is incentive compatible, Lemma 1 implies that

$$\int_{c'}^{c} [H^c(x(z, c)^-) - H^z(x(z, z)^-)] dz \geq 0.$$ 

Together,

$$\int_{c'}^{c} [\tilde{H}^c(\tilde{x}(z, c)^-) - \tilde{H}^z(\tilde{x}(z, z)^-)] dz = \int_{c'}^{c} [\tilde{H}^c(\tilde{x}(z, c)^-) - F(\tilde{v}(z))] dz$$

$$\geq \int_{c'}^{c} [H^c(x(z, c)^-) - F(\tilde{v}(z))] dz$$

$$= \int_{c'}^{c} [H^c(x(z, c)^-) - H^z(x(z, z)^-)] dz$$

$$\geq 0,$$

where the two equalities follows from the constructions of $\tilde{H}^z$ and $\tilde{v}$ so that $\tilde{H}^z(\tilde{x}(z, z)^-) = F(\tilde{v}(z)) = H^z(x(z, z)^-)$, the first inequality follows from the observation that $\tilde{H}^c(\tilde{v}(c)^-) = F(\tilde{v}(c)) \geq H^c(x)$ for any $x \in [0, \tilde{v}(c))$.

Finally, since as noted above, $F(\tilde{v}(c)) = H^c(x(c)^-) \text{ and } \mathbb{E}_{H^c}[v | v > x(c)] \leq \mathbb{E}_F[v | v > \tilde{v}(c)]$, for any $c \in [0, \bar{c}]$, we have:

$$(x(c) - \psi(c))(1 - H(x(c)^-)) \leq (\mathbb{E}_{H^c}[v | v > x(c)] - \psi(c))(1 - H(x(c)^-))$$

$$\leq (\mathbb{E}_F[v | v > \tilde{v}(c)] - \psi(c))(1 - F(\tilde{v}(c)))$$

$$= (\tilde{v}(c) - \psi(c))(1 - F(\tilde{v}(c))) \quad (12)$$

Together, by Lemma 1, let

$$\tilde{t}(c) := (\tilde{v}(c) - c)(1 - F(\tilde{v}(c))) - \int_{c}^{\tilde{c}} (1 - F(\tilde{v}(z))) dz.$$ 

Then $(\tilde{H}^c, \tilde{t}(c))_{c \in [0, \bar{c}]}$ is an incentive compatible and individually rational menu, $\tilde{H}^c$ is a responsive garbled upward-censorship with cutoff $\tilde{v}$ and

$$\int_{0}^{\tilde{c}} \tilde{t}(c)G(dc) = \int_{0}^{\tilde{c}} (\tilde{v}(c) - \psi(c))(1 - F(\tilde{v}(c)))G(dc)$$

$$\geq \int_{0}^{\tilde{c}} (x(c) - \psi(c))(1 - H(x(c)^-))G(dc)$$

$$\geq \int_{0}^{\tilde{c}} t(c)G(dc),$$

where the last two inequalities follow from (11) and (12). This completes the proof.
Step 2: We now characterize the family of incentive compatible, individually rational and responsive garbled upward censorship menus by a set of inequalities and equalities. This is the result of Lemma 5. To show Lemma 5, we first need Lemma 4 to simplify the procedure.

Lemma 4. Fix any \( \hat{c} \in [0, \bar{c}] \). Given any functions \( p : [0, \hat{c}] \rightarrow [0, \bar{v}] \) and \( q : [0, \hat{c}] \rightarrow [0, 1] \). Then

\[
(p(c) - c)(1 - q(c)) \geq (p(c') - c)(1 - q(c')), \forall c, c' \in [0, \hat{c}]
\]

if and only if

1. \( (p(c) - c)(1 - q(c)) = (p(\hat{c}) - \hat{c})(1 - q(\hat{c})) + \int_{c}^{\hat{c}} (1 - q(z))dz \) for all \( c \in [0, \hat{c}] \).

2. \( q \) is nondecreasing.

Proof. For necessity, consider and pair of functions \( p, q \) satisfying (13). Let \( \pi(c, c') := (p(c') - c')(1 - q(c')) \), for any \( c, c' \in [0, \hat{c}] \). Notice that for each \( c' \in [0, \hat{c}] \), the function \( \pi(\cdot, c') \) is absolutely continuous and uniformly bounded by \(-\hat{c}\) and \(\bar{v}\). By the envelope theorem, the function

\[
\pi^*(c) := \pi(c, c) = \max_{c' \in [0, \hat{c}]} \pi(c, c')
\]

is also absolutely continuous and the derivative exists and equals to \(-(1 - q(c))\) for almost all \( c \in [0, \hat{c}] \). Thus,

\[
(p(c) - c)(1 - q(c)) = \pi^*(c) = \pi^*(\hat{c}) + \int_{c}^{\hat{c}} (1 - q(z))dz,
\]

which establishes assertion 1. Moreover, since \( \pi(\cdot, c') \) is an affine function, \( \pi^* \) is a pointwise supremum of a family of affine functions and thus is convex. As a result, its derivative, \(-(1 - q)\), is nondecreasing.

For sufficiency, consider a pair of functions \( p, q \) satisfying 1 and 2. Then, for any \( c, c' \in [0, \hat{c}] \),

\[
(p(c) - c)(1 - q(c)) - (p(c') - c)(1 - q(c'))
\]

\[
= (p(c) - c)(1 - q(c)) - (p(c') - c')(1 - q(c')) - (c' - c)(1 - q(c'))
\]

\[
= \int_{c}^{c'} (1 - q(z))dz - (c' - c)(1 - q(c'))
\]

\[
= \int_{c}^{c'} (q(c') - q(z))dz
\]

\[
\geq 0,
\]

where the second equality follows from 1 and the inequality follows from 2. ■
Lemma 5. For any nondecreasing function \( \tilde{\psi} : [0, \bar{c}] \rightarrow [0, \tilde{v}] \), let \( \tilde{v}(c) := E_F[v|v > \tilde{\psi}(c)] \), \( \tilde{\pi}(c) := (\tilde{v}(c) - c)(1 - F(\tilde{\psi}(c)), \forall c \in [0, \bar{c}] \). Then there exists \( t : [0, \bar{c}] \rightarrow \mathbb{R} \) such that \( (H^c, t(c))_{c \in [0, \bar{c}]} \) is an incentive compatible, individually rational and responsive upward censorship menu with cutoff \( \tilde{\psi} \) if and only if:

1. \( H^c(\tilde{\psi}(c)^-) = F(\tilde{\psi}(c)) \), \( \int_0^c [H^c(z) - F(z)]dz \geq 0 \), for all \( x \in [0, \tilde{\psi}(c)] \), with equality at \( x = \tilde{\psi}(c) \), for all \( c \in [0, \bar{c}] \).

2. \( \int_0^c [q(z|c) - F(\tilde{\psi}(z))] \geq 0 \), for all \( c, c' \in [0, \bar{c}] \) with \( c' \leq c \).

3. For any \( c \in [0, \bar{c}] \), \( x \in [0, \tilde{\psi}(c)] \), \( H^c(x^-) \geq \max_{c' \in [0, \bar{c}]} [1 - (\tilde{\pi}(c) + \int_{c'}^c (1 - q(z|c))dz)/(x - c') \] with equality if and only if \( (x - c')(1 - q(c'|z)) = \tilde{\pi}(c) + \int_{c'}^c (1 - q(z|c))dz \) for some \( c' \in \arg\max_{c' \in [0, \bar{c}]} [1 - (\tilde{\pi}(c) + \int_{c'}^c (1 - q(z|c))dz)/(x - c') \].

4. \( q(\cdot|c) \) is nondecreasing and \( q(c|c) = F(\tilde{\psi}(c)) \) for all \( c \in [0, \bar{c}] \).

Proof. For necessity, consider any incentive compatible, individually rational and responsive upward censorship menu \( (H^c, t(c))_{c \in [0, \bar{c}]} \) with cutoff \( \tilde{\psi} \). Since \( H^c \) is an upward censorship, assertion 1 is clearly satisfied.

Since \( H^c \) is responsive, for any \( c \in [0, \bar{c}] \),

\[
(x - c)(1 - H^c(x^-)) \leq (\tilde{v}(c) - c)(1 - F(\tilde{\psi}(c))) = \tilde{\pi}(c), \forall x \in [0, \tilde{v}]
\]

and \( H^c(x^-) = 1 \) for all \( x \in (\tilde{v}(c), \tilde{v}] \). Therefore,

\[
\tilde{v}(c) \in \arg\max(x - c)(1 - H^c(x^-)), \forall c \in [0, \bar{c}].
\]

On the other hand, for any \( c', c \in [0, \bar{c}] \) with \( c' \leq c \), take any selection

\[
x(c', c) \in \arg\max_{x \in [0, \bar{c}]}(x - c')(H^c(x^-))
\]

and let

\[
p(c'|c) := x(c', c)
\]

\[
q(c'|c) = H^c(x(c', c)^-).
\]

Since \( (H^c, t(c)) \) is incentive compatible, Lemma 1 then ensures that

\[
\int_{c'}^c [q(z|c) - F(\tilde{\psi}(z))]dz = \int_{c'}^c [H^c(x(z, c)) - F(\tilde{\psi}(z))]dz \geq 0,
\]

Thus, \( \tilde{v}(c) \) is incentive compatible, individually rational and responsive upward censorship menu with cutoff \( \tilde{\psi} \).
which establishes assertion 2.

Now notice that as shown above, \( q(c|c) = F(\tilde{\psi}(c)) \) and \( (p(c|c) - c)(1 - q(c|c)) = \tilde{\pi}(c) \) for all \( c \in [0, \bar{c}] \). Since \( x(c', c) \in \text{argmax}_{x \in [0, \bar{c}]} (x - c')(H^c(x^-)) \), for any such \( c', c \) we have

\[
(x - c')(1 - H^c(x^-)) \leq (p(c'|c) - c')(1 - q(c'|c)), \forall x \in [0, \bar{v}].
\] (14)

Rearranging, we have:

\[
H^c(x^-) \geq 1 - \frac{(p(c'|c) - c')(1 - q(c'|c))}{(x - c')^+}, \forall x \in [0, \bar{v}].
\] (15)

Notice that the right hand side of (15) does not depend on \( c' \), we thus have

\[
H^c(x^-) \geq \max_{c' \in [0, \bar{c}]} \left[ 1 - \frac{(p(c'|c) - c')(1 - q(c'|c))}{(x - c')^+} \right]^+, \forall x \in [0, \tilde{\psi}(c)], \forall c \in [0, \bar{c}].
\] (16)

Moreover, by \( x(c', c) \in \text{argmax}_{x \in [0, \bar{c}]} (x - c')(H^c(x^-)) \) again, for any \( c', c'' \in [0, c] \) and for any \( c \in [0, \bar{c}] \),

\[
(p(c'|c) - c')(1 - q(c'|c)) \geq (p(c''|c) - c')(1 - q(c''|c)).
\] (17)

By Lemma 4, \( \hat{q}(\cdot|c) \) is nondecreasing, which establishes assertion 4.

Furthermore, also by Lemma 4,

\[
(p(c'|c) - c')(1 - q(c'|c)) = (p(c|c) - c)(1 - q(c|c)) + \int_{c'}^c (1 - q(z|c))dz = \tilde{\pi}(c) + \int_{c'}^c (1 - q(z|c))dz,
\] (18)

for any \( c \in [0, \bar{c}] \) and any \( c' \in [0, c] \). Combining (16) and (18), we obtain

\[
H^c(x^-) \leq \max_{c' \in [0, \bar{c}]} \left[ 1 - \frac{\tilde{\pi}(c) + \int_{c'}^c (1 - \hat{q}(z|c))}{(x - c')^+} \right]^+, \forall x \in [0, \bar{v}].
\]

Moreover, by (18), for any \( c \in [0, \bar{c}] \), any \( x \in [0, \tilde{\psi}(c)] \),

\[
(x - c')(1 - q(c'|c)) = \tilde{\pi}(c) + \int_{c'}^c (1 - q(z|c))dz
\]

for some \( c' \in \text{argmax}_{c' \in [0, c]} [1 - (\tilde{\pi}(c) + \int_{c'}^c (1 - q(z|c))dz)/(x - c')]^+ \), if and only if \( x = x(c', c) = p(c'|c) \) and

\[
H^c(x^-) = \max_{c' \in [0, \bar{c}]} \left[ 1 - \frac{(p(c'|c) - c')(1 - q(c'|c))}{(x - c')^+} \right]^+,
\]

which, together with (17), establishes assertion 3.

Conversely, for sufficiency, take any \( q : [0, \bar{c}]^2 \to [0, 1] \) and \( \{H^c\}_{c \in [0, \bar{c}]} \) satisfying conditions 1, 2, 3 and 4. Notice that if \( H^c \) is not an upward censorship, then for each \( c \in [0, \bar{c}] \), \( x \in [0, \bar{v}] \), let

\[
\hat{H}^c(x) := \begin{cases} 
H^c(x), & \text{if } x \in [0, \tilde{\psi}(c)] \\
F(\tilde{\psi}(c)), & \text{if } x \in [\tilde{\psi}(c), \tilde{v}(c)] \\
1, & \text{if } x \in [\tilde{v}(c), \bar{v}]
\end{cases}
\]
As such, for any $c \in [0, \tilde{c}]$ and that
\[
\tilde{H}^c = \bar{H}^c \equiv H^c
\] on $[0, \tilde{v}]$. Thus suffices to take $H^c$ as an upward censorship and verify that there exists $t : [0, \tilde{c}] \to \mathbb{R}$ such that $(H^c, t(c))_{c \in [0, \tilde{c}]}$ is an incentive compatible, individually rational and responsive menu.

For responsiveness, notice that by assertion 3, for any $x \in [0, \tilde{v}(c)]$,
\[
H^c(x^-) \geq \max_{c' \in [0, c]} \left[ 1 - \frac{\pi(c) + \int_c^c (1 - q(z|c))dz}{(x - c')^+} \right]^+ \geq \left[ 1 - \frac{\tilde{\pi}(c)}{(x - c)^+} \right]^+.
\]
Rearranging, we have:
\[
(x - c)(1 - H^c(x^-)) \leq \tilde{\pi}(c) = (\tilde{v}(c) - c)(1 - H^c(\tilde{v}(c)^-)).
\]

On the other hand, any $x \in (\tilde{v}(c), \tilde{v}]$ gives zero profit under $\tilde{H}^c$. Thus,
\[
\tilde{v}(c) \in \argmax_{x \in [0, \tilde{c}]} (x - c)(1 - H^c(x^-)), \forall c \in [0, \tilde{c}].
\]
Therefore, $\{H^c\}_{c \in [0, \tilde{c}]}$ is indeed responsive.

For incentive compatibility and individual rationality, by Lemma 4, since $q(\cdot|c)$ is increasing and $q(c|c) = F(\tilde{\psi}(c))$ for all $c \in [0, \tilde{c}]$, there exists $p : [0, \tilde{c}]^2 \to [0, \tilde{v}]$ such that
\[
(p(c'|c) - c')(1 - q(c'|c)) = \pi(c) + \int_c^c (1 - q(z|c))dz, \forall c' \in [0, c], \ c \in [0, \tilde{c}]
\]
and that
\[
(p(c'|c) - c')(1 - q(c'|c)) \geq (p(c''|c) - c')(1 - q(c''|c)), \forall c', c'' \in [0, c], \ c \in [0, \tilde{c}].
\]
As such, for any $c \in [0, \tilde{c}]$, $x \in [0, \tilde{v}(c)]$ and any $c' \in [0, c]$, whenever
\[
H^c(x^-) = \max_{c' \in [0, c]} \left[ 1 - \frac{\pi(c) + \int_c^c (1 - q(z|c))dz}{(x - c')^+} \right]^+.
\]
by assertion 3 and (4), there exists $c'$ such that $H^c(x) = q(c'|c)$
\[
(x - c')(1 - H^c(x^-)) \geq \pi(c) + \int_c^{c'} (1 - q(z|c)) = (p(c''|c) - c')(1 - q(c''|c)), \forall c'' \in [0, c],
\]
for some $c' \in [0, \tilde{c}]$. Assertion 3 then implies that whenever (19) holds for some $x \in [0, \tilde{v}(c)]$, $c' \in [0, \tilde{c}]$
\[
(x - c')(1 - H^c(x^-)) \geq (y - c')(1 - H^c(y^-))
\]
for any $y$ such that (19) holds for some (possibly distinct) $c'' \in [0, c]$. 
On the other hand, for any $x \in [0, \tilde{\psi}(c)]$ such that (19) does not hold, there must be some $c' \in [0, \tilde{\psi}(c)]$ such that
\[
(x - c')(1 - H^c(x^-)) < \tilde{\pi}(c) + \int_{c'}^c (1 - q(z|c))dz
\]
and therefore
\[
(x - c')(1 - H^c(x^-)) < (y - c')(1 - H^c(y^-))
\]
for $y = p(c'|c)$. Together, it must be that $\arg\max_{x \in [0, \tilde{\psi}(c)]} (x - c')(1 - H^c(x^-))$ is exactly the set where (19) holds for some $c' \in [0, \tilde{\psi}(c)]$. Thus, we may take a selection $x(c', c) \in \arg\max_{x \in [0, \tilde{\psi}(c)]} (x - c')(1 - H^c(x^-))$ such that $H^c(x(c', c^-)) = q(c'|c)$ for all $c \in [0, \tilde{c}], c' \in [0, c]$. Then, by Lemma 1 and assertion 2, there exists $t : [0, \tilde{c}] \to \mathbb{R}$ such that $(H^c, t(c))_{c \in [0, \tilde{c}]}$ is incentive compatible and individually rational.

**Step 3:** Now, we identify two critical constraints for the intermediary’s problem. Lemma 6, together with Lemma 5, show that for any incentive compatible, individually rational and responsive garbled upward censorship menu, two families of equality constraints must be satisfied, which will later be used in forming the dual problem.

**Lemma 6.** For any nondecreasing function $\tilde{\psi} : [0, \tilde{c}] \to [0, \tilde{v}]$, any $\{H^c\}_{c \in [0, \tilde{c}]} \subset \mathcal{H}_F$ and any $q : [0, \tilde{c}]^2 \to [0, 1]$ satisfying conditions 1-4 in Lemma 5, there exists some $\hat{q} : [0, \tilde{c}]^2 \to [0, 1]$ such that $\hat{q}(c|c) = F(\tilde{\psi}(c))$, $\hat{q}(\cdot|c)$ is increasing and
\[
\int_0^{\tilde{\psi}(c)} [\Gamma_{q, \tilde{\psi}}^c(x) - F(x)]dx = 0
\]
\[
\int_0^c [\hat{q}(z|c) - F(\tilde{\psi}(z))]dz = 0,
\]
where
\[
\Gamma_{q, \tilde{\psi}}^c(x) := \begin{cases} 
q(0|c), & \text{if } x \in [0, \underline{x}_q(c)) \\
\max_{c' \in [0, \tilde{c}]} \left[ 1 - \frac{\tilde{\pi}(c) + \int_{c'}^c (1 - q(z|c))dz}{(x-c')^+} \right] + F(\tilde{\psi}(c)), & \text{if } x \in \left[ \underline{x}_q(c), \bar{x}_q(c) \right] \\
F(\tilde{\psi}(c)), & \text{if } x \in \left[ \bar{x}_q(c), \tilde{\psi}(c) \right] 
\end{cases}
\]
for any \( q : [0, \tilde{c}]^2 \rightarrow [0, 1] \) with \( q(-c) \) being nondecreasing, any \( c \in [0, \tilde{c}] \) and any \( x \in [0, \tilde{\psi}(c)] \), with

\[
\bar{z}_q(c) := \frac{1}{(1 - q(0|c))} \left[ \bar{\pi}(c) + \int_0^c (1 - q(z|c))dz \right] + c
\]

\[
\bar{x}_q(c) := \frac{1}{(1 - q(z_q(c)|c))} \left[ \bar{\pi}(c) + \int_{z_q(c)}^c (1 - q(z|c))dz \right] + c
\]

\[
\bar{z}_q(c) := \inf \{ z \in [0, c] \} q(z|c) = F(\tilde{\psi}(c))
\]

**Proof.** Take any \( \tilde{\psi} \), \( \{H^c\}_{c \in [0, \tilde{c}]} \) and \( q \) satisfying conditions 1-4 in Lemma 5. Fix any \( c \in [0, \tilde{c}] \). By Lemma 5, we have

\[
\int_0^\tilde{\psi}(c) [H^c(x) - F(x)]dx = 0.
\]

Moreover, since \( \tilde{\psi}(c) > \tilde{\psi} \), we must have \( \bar{z}_q(c) < c \) and \( F(\tilde{\psi}(c)) = F(\tilde{\psi}(c)) > q(\bar{z}_q(c)|c) \).

Consider first the case where

\[
\int_0^c [q(z|c) - F(\tilde{\psi}(z))] > 0,
\]

first define \( \bar{z}_q(c) \) as the unique \( z \) that solves to the equation

\[
\int_z^c q(t|c)dt = \int_0^c F(\tilde{\psi}(t))dx.
\]

For this \( \bar{z}_q(c) \), consider the following construction: Define

\[
\bar{q}(z|c) := \begin{cases} 
0, & \text{if } z \in [0, \bar{z}_q(c)) \\
q(z|c), & \text{if } z \in [\bar{z}_q(c), c]
\end{cases}
\]

Notice that we then have

\[
\int_0^c [\bar{q}(z|c) - F(\tilde{\psi}(z))]dz = 0.
\]

If, furthermore,

\[
\int_{z_q(c)}^{\bar{z}_q(c)} \left[ H^c(x) - \left( 1 - \frac{\bar{\pi}(c) + \int_{z_q(c)}^c (1 - \bar{q}(z|c))dz}{x} \right)^+ \right] dx
\]

\[
\leq \int_{z_q(c)}^{\bar{z}_q(c)} \left[ F(\tilde{\psi}(c)) - \left( 1 - \frac{\bar{\pi}(c) + \int_{z_q(c)}^c (1 - \bar{q}(z|c))dz}{x - \bar{z}_q(c)} \right)^+ \right] dx,
\]

then there exists some \( q_0 \in [q(\bar{z}_q(c)|c), F(\tilde{\psi}(c))] \) such that

\[
\int_{z_q(c)}^{\bar{z}_q(c)} \left[ H^c(x) - \left( 1 - \frac{\bar{\pi}(c) + \int_{z_q(c)}^c (1 - \bar{q}(z|c))dz}{x} \right)^+ \right] dx
\]

\[
= (\tilde{\psi}(c) - \bar{x}) F(\tilde{\psi}(c)) - \int_{z_q(c)}^{\bar{z}_q(c)} \left( 1 - \frac{\bar{\pi}(c) + \int_{z_q(c)}^c (1 - \bar{q}(z|c))dz}{x - \bar{z}_q(c)} \right)^+ dx,
\]
where
\[ \dot{x} := -\frac{1}{1 - q_0} \left[ \tilde{\pi}(c) + \int_{\tilde{z}_q}^c (1 - \tilde{q}(z|c))dz \right] + c. \]

Thus, by possibly redefining \( \tilde{q}(\tilde{z}_q|c) \) as \( q_0 \), we then have
\[ \int_0^\tilde{\psi}(c) \Gamma_q^c \tilde{\psi}(x)dx = \int_0^{\tilde{\psi}(c)} H^c(x)dx = \int_0^{\tilde{\psi}(c)} F(x)dx. \]

Furthermore, since the set of maximizers admits a continuous selection \( \tilde{\psi}(\tilde{x}_q|c) \) has Lebesgue measure zero, we still have
\[ \int_0^c [\tilde{q}(z|c) - F(\tilde{\psi}(z))]dz = 0. \]

On the other hand, if
\[ \int_{\tilde{x}_q(c)}^{\tilde{x}_q(c)} \left[ \tilde{\psi}(c) - \left( 1 - \frac{\tilde{\pi}(c) + \int_{\tilde{z}_q}^c (1 - \tilde{q}(z|c))dz}{(x - c')^+} \right) \right] dx, \]
we construct a similar procedure for any \( x \in [\tilde{x}_q(c), \tilde{x}_q(c)] \). Specifically, since the mapping
\[ c' \mapsto \left[ 1 - \frac{\tilde{\pi}(c) + \int_{\tilde{z}_q}^c (1 - q(z|c))dz}{(x - c')^+} \right] \]
is continuous on \([0, c]\), its set of maximizers admits a continuous selection
\[ c_q(x) \in \arg\max_{c' \in [0, c]} \left[ 1 - \frac{\tilde{\pi}(c) + \int_{\tilde{z}_q}^c (1 - q(z|c))dz}{(x - c')^+} \right] \]
Observe that for any \( x \in [\tilde{x}_q(c), \tilde{x}_q(c)] \), \( c_q(x) \geq \tilde{x}_q(c) \) and therefore there exists \( q_x \geq 0 \) such that
\[ \int_{c_q(x)}^c [q(z|c) - F(\tilde{\psi}(z))]dz + \int_0^{c_q(x)} [q_x - F(\tilde{\psi}(z))]dz = 0 \]
Now define
\[ q_x(z|c) := \begin{cases} 
q_x, & \text{if } z \in [0, c_q(x)) \\
q(z|c), & \text{if } z \in [c_q(x), c] 
\end{cases} \]
and let
\[ \dot{x}(x) := \frac{1}{1 - q_x} \left[ \tilde{\pi}(c) + \int_{c_q(x)}^c (1 - q_x(z|c))dz \right]. \]

Then, as long as
\[ \int_{\tilde{x}_q(c)}^{\tilde{x}_q(c)} \left[ \tilde{\psi}(c) - \left( 1 - \frac{\tilde{\pi}(c) + \int_{\tilde{z}_q}^c (1 - \tilde{q}(z|c))dz}{(x - \tilde{z}_q(c))^+} \right) \right] dx, \]
by the intermediate value theorem, since the selection $c_q$ is continuous and $c_q(\bar{x}_q(c)) = \bar{z}_q(c)$ by construction, there must be some $x^* \in [\bar{x}_q(c), \bar{x}_q(c)]$ such that

$$
\int_0^{\bar{x}(x^*)} \left[ H^c(x) - \left( 1 - \frac{\bar{\pi}(c) + \int_{c_q(x^*)}^{c_q(c)} (1 - q_x(z|c))dz}{(x - c_q(x^*))^+} \right) \right] dx 
= \int_{\bar{x}_q(c)}^{\bar{\psi}(c)} \left[ F(\tilde{\psi}(c)) - \left( 1 - \frac{\bar{\pi}(c) + \int_{c_q(c)}^c (1 - q_x(z|c))dz}{(x - \bar{z}_q(c))^+} \right) \right] dx.
$$

We then have

$$
\int_0^{\bar{\psi}(c)} \Gamma_{q_u,\bar{\psi}}^c(x) dx = \int_0^{\bar{\psi}(c)} H^c(x) dx = \int_0^{\bar{\psi}(c)} F(x) dx.
$$

Therefore, by defining $q(\cdot|c) \equiv q_u(\cdot|c)$, we then have the desired $\tilde{q}$.

Finally, if

$$
\int_0^{\bar{x}(x_q)} \left[ H^c(x) - \left( 1 - \frac{\bar{\pi}(c) + \int_{x_q(c)}^{c_q(c)} (1 - q_x(z|c))dz}{(x - \bar{z}_q(c))^+} \right) \right] dx 
< \int_{x_q(c)}^{\bar{\psi}(c)} \left[ F(\tilde{\psi}(c)) - \left( 1 - \frac{\bar{\pi}(c) + \int_{c_q(c)}^c (1 - q_x(z|c))dz}{(x - \bar{z}_q(c))^+} \right) \right] dx,
$$

then there exists $x^* \in [0, \bar{x}_q(c)]$ such that

$$
\int_0^{x^*} \max \left\{ q_{\bar{x}_q(c)}, \left( 1 - \frac{\bar{\pi}(c) + \int_{\bar{x}_q(c)}^{c_q(c)} (1 - q(z|c))dz}{(x - \bar{z}_q(c))^+} \right)^+ \right\} dx + (\bar{v}(c) - x^*) F(\tilde{\psi}(c)) = \int_0^{\bar{\psi}(c)} H^c(x) dx.
$$

Then, by defining

$$
\tilde{q}(z|c) := \begin{cases} 
q_{\bar{x}_q}, & \text{if } z \in [0, \bar{z}_q(c)] \\
\tilde{F}(\tilde{\psi}(c)), & \text{if } z \in (\bar{z}_q(c), c]
\end{cases},
$$

we then have

$$
\int_0^{\bar{\psi}(c)} \Gamma_{\tilde{q}_{\tilde{\psi}}}^c(x) dx = \int_0^{\bar{\psi}(c)} H^c(x) dx = \int_0^{\bar{\psi}(c)} F(x) dx,
$$

as desired.

Finally, if, on the other hand,

$$
\int_0^c [q(z|c) - F(\tilde{\psi}(z))]dz = 0,
$$

then it must be that $q(0|c) = 0$ since by Lemma 5,

$$
\int_c^e [q(z|c) - F(\tilde{\psi}(z))]dz \geq 0
$$
and $F(\bar{\psi}(0)) = 0$. Thus, by selecting a proper $\tilde{q} \in (0, q(\tilde{z}_q(c) + |c|))$ and letting $\tilde{q}(z|c) := \tilde{q}$ for all $z \in [0, \tilde{z}_q]$ and $\tilde{q}(z|c) := q(z|c)$ otherwise, we will have

$$\int_0^c [\tilde{q}(z|c) - F(\tilde{\psi}(z))]dz > 0,$$

where

$$\tilde{H}^c(x) := \begin{cases} \tilde{q}, & \text{if } x \in [0, \tilde{x}) \\ H^c(x), & \text{if } x \in [\tilde{x}, \tilde{\psi}(c)] \end{cases},$$

and

$$\tilde{x} := \frac{1}{1 - \tilde{q}} \left[ \tilde{\pi}(c) + \int_{\tilde{z}_q(c)}^c (1 - \tilde{q}(z|c))dz \right].$$

Since the previous arguments hinge only on the property that

$$\int_0^{\tilde{\psi}(c)} H^c(x)dx = \int_0^{\tilde{\psi}(c)} F(x)dx$$

and that $H^c(x) \geq \max_{c' \in [0, \tilde{c}] [1 - (\tilde{\pi}(c) + \int_{\tilde{z}_q(c)}^c (1 - q(z|c))dz)/(x - c')^+]$ for all $x \in [0, \tilde{\psi}(c)]$, by replacing $H^c$ with $\tilde{H}^c$, $q$ with $\tilde{q}$ and repeating the procedures, we can then find the desired $\hat{q}$. \hfill \blacksquare

**Step 4:** Based on the characterizations above, we now form a dual for the original problem. Fix any Borel measures $\mu, \nu$ on the measurable space $[0, \tilde{c}]$ (endowed with the Borel algebra). Let

$$D(\mu, \nu) := \sup_{\tilde{\psi}, q} \left[ \int_0^c (\tilde{\psi}(c) - \psi(c))(1 - F(\tilde{\psi}(c)))G(dc) - \int_0^c \left( \int_0^c (1 - \tilde{\Gamma}_q^c(x)) - (1 - F(x)) \right)dx \mu(dc) \\
- \int_0^c \left( \int_0^c [q(z|c) - F(\tilde{\psi}(z))]dz \right) \nu(dc) \right],$$

where the supremum is taken over all nondecreasing function $\tilde{\psi} : [0, \tilde{c}] \to [0, \tilde{v}]$ and all $q : [0, \tilde{c}] \to [0, 1]$ such that $q(z|c)$ is nondecreasing and $q(c|c) = F(\tilde{\psi}(c))$, for all $c \in [0, \tilde{c}]$.

Notice that for any incentive compatible, individually rational and responsive garbled upward censorship menu $(H^c, t(c))_{c \in [0, \tilde{c}]}$ with cutoff $\tilde{\psi}$, the expected revenue for the intermediary is

$$R(\tilde{\psi}) := \int_0^\tilde{\psi} (\tilde{\psi}(c) - \psi(c))(1 - F(\tilde{\psi}(c)))G(dc).$$
By Lemma 5 and Lemma 6, for any such menu, there exists $q_{\psi,H} : [0, \tilde{c}]^2 \rightarrow [0, \tilde{c}]$ such that $q_{\psi,H}(\cdot|c)$ is nondecreasing, $q_{\psi,H}(c|c) = F(\tilde{\psi}(c))$,

$$\int_0^{\tilde{c}} [(1 - \Gamma^c_{\psi,H}\tilde{\psi}(x)) - (1 - F(x))] dc = 0,$$

$$\int_0^{\tilde{c}} [q_{\psi,H}(z|c) - F(\tilde{\psi}(z))] dz = 0,$$

for all $c \in [0, \tilde{c}]$.

As such, for any such menu, for any Borel measures $\mu, \nu$ on $[0, \tilde{c}]$,

$$R(\tilde{\psi}) = \int_0^{\tilde{c}} (\tilde{\psi}(c) - \psi(c))(1 - F(\tilde{\psi}(c)))G(dc).$$

$$= \int_0^{\tilde{c}} (\tilde{\psi}(c) - \psi(c))(1 - F(\tilde{\psi}(c)))G(dc) - \int_0^{\tilde{c}} \left( \int_0^{\tilde{c}} (1 - \Gamma^c_{\psi,H}\tilde{\psi}(x)) - (1 - F(x)) \right) dx \mu(dc)$$

$$- \int_0^{\tilde{c}} \left( \int_0^{\tilde{c}} [q_{\psi,H}(z|c) - F(\tilde{\psi}(z))] dz \right) \nu(dc)$$

$$\leq \sup_{\tilde{\psi},q} \left[ \int_0^{\tilde{c}} (\tilde{\psi}(c) - \psi(c))(1 - F(\tilde{\psi}(c)))G(dc) - \int_0^{\tilde{c}} \left( \int_0^{\tilde{c}} (1 - \Gamma^c_{\psi,H}\tilde{\psi}(x)) - (1 - F(x)) \right) dx \mu(dc)$$

$$- \int_0^{\tilde{c}} \left( \int_0^{\tilde{c}} [q(z|c) - F(\tilde{\psi}(z))] dz \right) \nu(dc) \right]$$

$$= D(\mu, \nu),$$

and therefore, if $R^*$ is the supremum of the expected revenue among all possible incentive compatible, individually rational and responsive garbled upward censorship menus, we have

$$R^* \leq D(\mu, \nu).$$

It then suffices to show that the exists Borel measures $\mu^*, \nu^*$ and a function $q^* : [0, \tilde{c}]^2 \rightarrow [0, 1]$ with $q^*(\cdot|c)$ being nondecreasing and $q^*(c|c) = F(\psi^*(c))$ such that $(\psi^*, q^*)$ solves the dual problem (20), that

$$\int_0^{\tilde{c}} \left( \int_0^{\tilde{c}} [(1 - \Gamma^c_{q^*\psi^*}(x)) - (1 - F(x)) dx \right) \mu^*(dc) = 0,$$

$$\int_0^{\tilde{c}} \left( \int_0^{\tilde{c}} [q^*(z|c) - F(\psi^*(z))] dz \right) \nu^*(dc) = 0$$

for all $c \in [0, \tilde{c}]$ and that there exists an incentive compatible, individually rational and responsive garbled upward censorship menu with cutoff $\psi^*$, as this would imply:

$$R^* \leq D(\mu^*, \nu^*) = R(\psi^*) \leq R^*.$$
Together, we will then have

$$R^* = R(\psi^*),$$

as desired.

Indeed, for any Borel measures $\mu, \nu$ on $[0, \bar{c}]$, (20) can be written as:

$$
\sup_{\psi} \left[ \int_0^c (\bar{v}(c) - \psi(c))(1 - F(\tilde{\psi}(c)))G(dc) + \sup_q \left( \int_0^c \left( \int_0^x [\Gamma^c_{q,\tilde{\psi}}(x) - F(x)]dx \right) \mu(dc) - \int_0^c \left( \int_0^c [q(z|c) - F(\tilde{\psi}(z))]dz \right) \nu(dc) \right) \right].
$$

Notice that for any fixed $\tilde{\psi}$, the functional

$$q \mapsto \Gamma^c_{q,\tilde{\psi}}$$

is convex, as it is essentially a pointwise supremum of a family of affine functionals. Also, since for each $c \in [0, \bar{c}]$, the collection of nondecreasing functions $q(\cdot|c) : [0,c] \to [0,\tilde{\psi}(c)]$, $Q(c)$, is convex, for each $c \in [0, \bar{c}]$, and any fixed nondecreasing function $\tilde{\psi} : [0, \bar{c}] \to [0, \bar{v}]$, the maximization problem

$$\max_{q(\cdot|c) \in Q(c)} \int_0^c [\Gamma^c_{q,\tilde{\psi}}(x) - F(\tilde{\psi})]dx - \int_0^c [q(z|c) - F(\tilde{\psi}(z))]dz$$

has a solution and one of the extreme points of $Q(c)$, which take form of $q(z|c) \in \{0,F(\tilde{\psi}(c))\}$ for all $z \in [0, c]$, attains the maximum. Therefore, (20) can be reduced to choosing cutoff points of the extreme points of $Q(c)$, denoted as $k(c)$, instead of choosing among all nondecreasing functions for each $c \in [0, \bar{c}]$. That is:

$$D(\mu, \nu) = \sup_{\psi} \left[ \int_0^c (\bar{v}(c) - \psi(c))(1 - F(\tilde{\psi}(c)))G(dc) + \sup_{k: [0,\bar{c}] \mapsto [0,\bar{c}], k(c) \leq c} \left( \int_0^c \left( \int_0^x [\Gamma^c_{1\{z \geq k(c)\},\tilde{\psi}}(x) - F(x)]dx \right) \mu(dc) - \int_0^c \left( \int_0^c [F(\tilde{\psi}(z))1\{z \geq k(c)\} - F(\tilde{\psi}(z))]dz \right) \nu(dc) \right) \right].$$

Notice that by definition of $\Gamma^c_{1\{z \geq k(c)\},\tilde{\psi}}$, fix any $\tilde{\psi}$, and $k$,

$$\int_0^\theta [\Gamma^c_{1\{z \geq k(c)\},\tilde{\psi}}(x) - F(x)]dx \geq \int_0^{P(\tilde{\psi}(c),k(c))} [\gamma(x, \tilde{\psi}(c), k(c)) - F(x)]dx,$$

where

$$\gamma(x, \tilde{\psi}(c), k(c)) := \left( 1 - \frac{(\bar{v}(c) - k(c))(1 - F(\tilde{\psi}(c)))}{(x - k(c))^+} \right)^+ , \forall x \in [0, \tilde{\psi}(c)]$$
and
\begin{align*}
P(\tilde{\psi}(c), k(c)) := \begin{cases} \max \Phi_{k(c)}^{-1}((\tilde{\psi}(c) - k(c))(1 - F(\tilde{\psi}(c)))) & \text{if } \Phi_{k(c)}^{-1}((\tilde{\psi}(c) - k(c))(1 - F(\tilde{\psi}(c)))) \neq \emptyset \\ 0, & \text{if } \Phi_{k(c)}^{-1}((\tilde{\psi}(c) - k(c))(1 - F(\tilde{\psi}(c)))) = \emptyset, \end{cases}
\end{align*}

with \( \Phi_k(x) := (x - z)(1 - F(x)) \) being the profit function of a seller with a cost \( z \in [0, c] \).

Now fix Borel measures \( \mu, \nu \) on \([0, c]\) that are absolutely continuous with respect to the Lebesgue measure with densities \( m, n \), respectively, and consider an auxiliary problem:

\[
D'(\mu, \nu) := \sup_{\tilde{\psi}, k} \int_0^c \left( (\tilde{\psi}(c) - \psi(c))(1 - F(\tilde{\psi}(c)))g(c) - \left( \int_0^{P(\tilde{\psi}(c), k(c))} [1 - \gamma(x, \tilde{\psi}(c), k(c))] - (1 - F(x)) \right) m(c) \right.
\]

\[
- \left. \left( (c - k(c))F(\tilde{\psi}(c)) - \int_0^c F(\tilde{\psi}(z)) \right) n(c) \right) dc. \tag{21}
\]

Notice that for any fixed \( \tilde{\psi} \), (21) is a (concave) pointwise maximization problem of choosing \( k(c) \leq c \), whereas for any fixed \( k \), (21) is a (concave) variational problem.\(^9\) As such, for \( \psi^*, k^* \) to be optimal under the Borel measures \( \mu, \nu \), it is equivalent to that \( k^*(c) \) solves the pointwise first order condition given \( \psi^* \)

\[
m(c) \int_0^{P(\psi^*(c), k^*(c))} \gamma_3(x, \psi^*(c), k^*(c))dx = -F(\psi^*(c))n(c)
\]

\[
\iff m(c) = -\frac{F(\psi^*(c))}{\int_0^{P(\psi^*(c), k^*(c))} \gamma_3(x, \psi^*(c), k^*(c))dx} n(c), \tag{22}
\]

whenever \( P(\psi^*(c), k^*(c)) > 0 \) and that \( \psi^* \) solves the Euler-Lagrange equation given \( k^* \):

\[
n(c) = \frac{d}{dc} \left[ (\psi(c) - \psi^*(c))g(c) + m(c) \int_0^{P(\psi^*(c), k^*(c))} \gamma_2(x, \psi^*(c), k^*(c))dx - (c - k^*(c))n(c) \right], \tag{23}
\]

for (Lebesgue) almost all \( c \in [0, c] \).

Substituting (22) into (23) and integrating both sides with the fact that \( \psi^*(0) = 0 \), we then have:

\[
N(c) := \int_0^c n(z)dz = (\psi(c) - \psi^*(c))g(c) + \Omega^s(c)n(c), \tag{24}
\]

where

\[
\Omega^s(c) := \frac{\int_0^{P(\psi^*(c), k^*(c))} \gamma_2(x, \psi^*(c), k^*(c))dx F(\psi^*(c))}{\int_0^{P(\psi^*(c), k^*(c))} \gamma_3(x, \psi^*(c), k^*(c))dx f(\psi^*(c))} - (c - k^*(c))
\]

\(^9\)This can be seen by observing that \( (\tilde{\psi}(c) - \psi(c))(1 - F(\tilde{\psi}(c))) = \int_{\tilde{\psi}(c)}^c (1 - F(x))dx - (\psi(c) - \tilde{\psi}(c))(1 - F(\tilde{\psi}(c))), (\tilde{\psi}(c) - k(c))(1 - F(\tilde{\psi}(c))) = \int_{\tilde{\psi}(c)}^c (1 - F(x))dx + (\tilde{\psi}(c) - k(c))(1 - F(\tilde{\psi}(c))) \) and by letting \( \eta(c) := \frac{1}{F(\psi(c))} \int_0^c F(\tilde{\psi}(z))dz \).
CLAIM 1: $\Omega^*$ is strictly decreasing on $[c^*, \bar{c}]$ and $\Omega^*(c) \geq 0$ for all $c \in [c^*, \bar{c}]$.

Notice that (24) is a first order linear differential equation and therefore the initial value problem with $N(c^*) = 0$ has a unique solution

$$N^*(c) = \frac{\zeta(c) \left( \int_{c^*}^{c} \left( \psi(z) - \psi(c^*) \right) \frac{g(z)}{g(z)} dz \right)}{\zeta(c)}$$

where

$$\zeta(c) := \exp \left( \int_{0}^{c} \frac{1}{\Omega^*(c)} \right), \forall c \in [0, \bar{c}].$$

It can be verified that $N^*$ is increasing and therefore is indeed a CDF of a Borel measure with density $n^*$. As such, let

$$m^*(c) := -\frac{F(\psi^*(c))}{\int_{0}^{P(\psi^*(c), k^*(c))} \gamma_3(x, \psi^*(c), k^*(c)) dx} n^*(c), \forall c \in [0, \bar{c}],$$

and let $\mu^*, \nu^*$ be the Borel measures induced by $m^*$ and $n^*$. Notice that $\text{supp}(\mu^*) = \text{supp}(\nu^*) = [c^*, \bar{c}]$ and that $\psi^*, k^*$ solves the auxiliary problem (21). Moreover, by construction, for any $c \in [c^*, \bar{c}]

$$\int_{0}^{\bar{c}} \left[ \int_{0}^{c} \left[ (1 - \Gamma_{\psi^*,k^*}(x)) - (1 - F(x)) \right] \right] \mu^*(dc) = 0,$$

$$\int_{0}^{\bar{c}} \left[ \int_{0}^{c} [q^*(z|c) - F(\psi^*(z))] \right] \nu^*(dc) = 0$$

and therefore,

$$D'(\mu^*, \nu^*) = D(\mu^*, \nu^*).$$

Furthermore, let $q^*(z|c) := 1\{z \geq k^*(c)\}$. Then $(\psi^*, q^*)$ indeed solves the dual problem (20) and

$$\int_{0}^{\bar{c}} \left[ \int_{0}^{c} \left[ (1 - \Gamma_{\psi^*,k^*}(x)) - (1 - F(x)) \right] \right] \mu^*(dc) = 0,$$

$$\int_{0}^{\bar{c}} \left[ \int_{0}^{c} [q^*(z|c) - F(\psi^*(z))] \right] \nu^*(dc) = 0$$

for all $c \in [0, \bar{c}]$.

Finally, for any $c \in [0, \bar{c}]$, any $x \in [0, \bar{x}]$, let

$$\hat{H}^{c}(x) := \begin{cases} \left[ 1 - \frac{(\psi^*(c) - k^*(c))(1 - F(\psi^*(c)))}{(x - k^*(c))^+} \right]^{+}, & \text{if } x \in [0, P(\psi^*(c), k^*(c))] \\ F(x), & \text{if } x \in [P(\psi^*(c), k^*(c)), \psi^*(c)) \\ F(\psi^*(c)), & \text{if } x \in [\psi^*(c), v^*(c)) \\ 1, & \text{if } x \in [v^*(c), \bar{x}] \end{cases}$$
where \( v^*(c) := \mathbb{E}_F[v|v \geq \psi^*(c)] \). On the other hand, for any \( H \in \mathcal{H}_F \) denote the integral of \( H \) by \( I_H(x) := \int_x^\theta (1 - H(z))dz \) for all \( x \in [0, \bar{v}] \). Now let

\[
I_{gu}^c(x) := \text{conv}\left( \min\{I_F(x), I_{\hat{H}}(x)\} \right).
\]

By construction, for all \( c \in [0, \bar{c}] \) \( I_{gu}^c \) is convex and thus its subdifferential, \( \partial I_{gu}^c(x) \), is nonempty for all \( x \in [0, \bar{v}] \). Finally, let

\[
H_{gu}^c(x) := \inf \partial I_{gu}^c(x), \forall x \in [0, \bar{v}], c \in [0, \bar{c}].
\]

It can be verified that \( H_{gu}^c \in \mathcal{H}_F \) for all \( c \in [0, \bar{c}] \) and that \( \{H_{gu}^c\}_{c \in [0, \bar{c}]} \) satisfies the sufficient conditions 1 and 2 in Lemma 1. Then by Lemma 5 and Lemma 1, there exists a transfer \( t_{gu} : [0, \bar{c}] \to \mathbb{R} \) such that \( (H_{gu}^c, t_{gu}(c))_{c \in [0, \bar{c}]} \) is indeed an incentive compatible, individually rational and responsive upward censorship menu with cutoff \( \psi^* \). This completes the proof. \( \blacksquare \)

C. Proofs for Welfare Analysis and Comparative Statics

Proof of Proposition 1. Notice that for each \( c \in [0, \bar{c}] \), probability of efficient trade is the probability of the event that trade occurs when the buyer’s value is greater than the seller’s cost. Since \( \psi(c) > c \) for all \( c \in [0, \bar{c}] \),

\[
\int_0^\bar{c} (1 - F(c))G(dc) > \int_0^\bar{c} (1 - F(\psi(c)))G(dc),
\]

which implies that the probability of efficient trade is larger when the seller has control of the information technology.

On the other hand, since \( \psi \) is increasing and \( \psi(c) > c \) for all \( c \in [0, \bar{c}] \),

\[
\int_{\psi(c)}^\bar{c} (1 - F(x))dx + (\psi(c) - c)(1 - F(\psi(c)))\\< \int_{\psi(c)}^\bar{c} (1 - F(x))dx + \int_c^{\psi(c)} (1 - F(x))dx\\= \int_c^{\bar{c}} (1 - F(x))dx,
\]

for all \( c \in [0, \bar{c}] \). Thus,

\[
\int_0^\bar{c} (v(c) - c)(1 - F(\psi(c)))G(dc) < \int_0^\bar{c} (\mathbb{E}_F[v|v > c] - c)(1 - F(c))G(dc).
\]

This completes the proof. \( \blacksquare \)
**Proof of Proposition 2.** For 1., notice that the intermediary’s revenue is given by

\[
\int_0^c (v(c) - \psi(c))(1 - F(\psi(c)))G(dc),
\]

and total surplus is

\[
\int_0^c (v(c) - c)(1 - F(\psi(c)))G(dc),
\]

and the seller’s expected net profit is

\[
\int_0^c \left( \int_c^\phi (1 - F(\psi(z)))dz \right) G(dc)
\]

for any distributions \(F, G\) satisfying \(\phi(\psi(c)) \leq c\) for all \(c \in [0, \phi]\). As such for any \(i \in \{1, 2\}\),

\[
(v_1(c) - \psi_i(c))(1 - F_1(\psi_i(c))) = \int_{\psi_i(c)}^\phi (1 - F_1(x))dx
\]

\[
\geq \int_{\psi_i(c)}^\phi (1 - F_2(x))dx = (v_1(c) - \psi_i(c))(1 - F_1(\psi_i(c))),
\]

and

\[
(v_1(c) - c)(1 - F_1(\psi_i(c))) = \int_{\psi_i(c)}^\phi (1 - F_1(x))dx + (\psi_i(c) - c)(1 - F_1(\psi_i(c)))
\]

\[
\geq (v_1(c) - c)(1 - F_2(\psi_i(c))) = \int_{\psi_i(c)}^\phi (1 - F_2(x))dx + (\psi_i(c) - c)(1 - F_2(\psi_i(c))),
\]

and also

\[
\int_c^\phi (1 - F_1(\psi_i(z)))dz \geq \int_c^\phi (1 - F_2(\psi_i(z)))dz,
\]

for all \(c \in [0, \phi]\) and therefore

\[
\int_0^\phi (v_1(c) - \psi_i(c))(1 - F_1(\psi_i(c)))G_i(dc) \geq \int_0^\phi (v_2(c) - \psi_i(c))(1 - F_2(\psi_i(c)))G_i(dc)
\]

and

\[
\int_0^\phi (v_1(c) - c)(1 - F_1(\psi_i(c)))G_i(dc) \geq \int_0^\phi (v_2(c) - c)(1 - F_2(\psi_i(c)))G_i(dc),
\]

and also

\[
\int_0^\phi \left( \int_c^\phi (1 - F_1(\psi_i(z)))dz \right) G_i(dc) \geq \int_0^\phi \left( \int_c^\phi (1 - F_2(\psi_i(z)))dz \right) G_i(dc),
\]

for any \(i \in \{1, 2\}\).

For 2., notice that by using integration by parts, for all \(c \in [0, \phi]\), \(i,j \in \{1, 2\}\,

\[
\int_{\psi_i(c)}^\phi (1 - F_j(x))dx = \int_{\psi_i(c)}^\phi 1\{x \geq \psi_i(c)\}(1 - F_j(x))dx = \int_{\psi_i(c)}^\phi (x - \psi_i(c))^+ F_j(dx).
\]
Therefore, since the function $x \mapsto (x - \psi(c))^+$ is convex and since $F_1$ is a mean preserving spread of $F_2$, for any $i \in \{1, 2\}$,

\[
\int_0^e (v_1(c) - \psi_i(c))(1 - F_i(\psi_i(c)))G_i(dc)
= \int_0^e \left( \int_0^c (x - \psi_i(c))^+ F_i(dx) \right) \geq \int_0^e \left( \int_0^c (x - \psi_i(c))^+ F_1(dx) \right) = \int_0^e (v_2(c) - \psi_i(c))(1 - F_2(\psi_i(c)))G_i(dc)
\]

For 3., first notice that the hazard rate dominance implies that $\psi_1 \leq \psi_2$ and that

\[
G_1(c) = \exp \left( - \int_c^e \frac{1}{\psi_1(z)} dz \right) \geq \exp \left( - \int_c^e \frac{1}{\psi_2(z)} dz \right) = G_2(c).
\]

That is, $G_2$ first order stochastic dominates $G_1$. As such, for each $i \in \{1, 2\}$,

\[
\int_0^e (v_1(c) - \psi_1(c))(1 - F_i(\psi_1(c)))G_1(dc)
= \int_0^e \left( \int_{\psi_1(c)}^e (1 - F_i(x)) dx \right) G_1(dc)
\geq \int_0^e \left( \int_{\psi_2(c)}^e (1 - F_i(x)) dx \right) G_1(dc)
\geq \int_0^e \left( \int_{\psi_2(c)}^e (1 - F_1(x)) dx \right) G_2(dc)
= \int_0^e (v_2(c) - \psi_2(c))(1 - F_i(\psi_1(c)))G_2(dc),
\]

where the first inequality follows from $\psi_1 \leq \psi_2$ and the second inequality follows from the fact that $G_2$ first order stochastic dominates $G_1$ and that $\psi_2$ is increasing. Similarly, for each $i \in \{1, 2\}$,

\[
\int_0^e (v_1(c) - c)(1 - F_i(\psi_1(c)))G_1(dc)
= \int_0^e \left( \int_{\psi_1(c)}^e (1 - F_i(x)) dx + (\psi_1(c) - c)(1 - F(\psi_1(c))) \right) G_1(dc)
\geq \int_0^e \left( \int_{\psi_2(c)}^e (1 - F_i(x)) dx + (\psi_2(c) - c)(1 - F(\psi_2(c))) \right) G_1(dc)
\geq \int_0^e \left( \int_{\psi_2(c)}^e (1 - F_1(x)) dx + (\psi_2(c) - c)(1 - F(\psi_2(c))) \right) G_1(dc)
= \int_0^e (v_2(c) - c)(1 - F_i(\psi_2(c)))G_2(dc).
\]
Finally, for the same reasons,
\[ \int_0^c \left( \int_c^\epsilon (1 - F_i(\psi_1(z)))dz \right) G_1(dc) \geq \int_0^c \left( \int_c^\epsilon (1 - F_i(\psi_2(z)))dz \right) G_2(dc) \]
This completes that proof. ■

D. Proof of Claim

*Proof of Claim 1.* Take a sequence of strictly increasing and differentiable functions \( \psi_n \) such that \( \{\psi_n\} \to \psi^\ast \) pointwisely. For each \( n \in \mathbb{N} \), let
\[
k_n(c) := c - \frac{1}{F(\psi_n(c))} \int_0^c F(\psi_n(z))dz
\]
and let
\[
\pi_n(c) := \int_{\psi_n(c)}^0 (1 - F(x))dx + (\psi_n(c) - k_n(c))(1 - F(\psi_n(c))),
\]
for all \( c \in [0, \bar{c}] \). Notice that since \( \{\psi_n\} \to \psi^\ast \) pointwisely, by the dominated convergence theorem, \( \{\pi_n\} \to \{\pi^\ast\} \) pointwisely as well and \( (\psi_n(c) - k_n(c)) \geq 0 \) for all \( c \in [0, \bar{c}] \) for \( n \) large enough. As such, for \( n \) large enough,
\[
\pi'_n(c) = -(\psi_n(c) - k_n(c))(1 - F(\psi_n(c))) - \frac{(1 - F(\psi_n(c)))}{F(\psi_n(c))^2} \int_0^c F(\psi_n(z))dz < 0, \forall c \in [0, \bar{c}]
\]
This then implies that for \( n \) large enough,
\[
\frac{d}{dc} \left[ \frac{P(\psi_n(c), k_n(c))}{P(\psi_n(c), k_n(c))} \right] (1 - \gamma(x, \psi_n(c), k_n(c)) - (1 - F(x))) \leq 0, \forall c \in [0, \bar{c}].
\]
Therefore,
\[
0 \geq \frac{d}{dc} \left[ \frac{P(\psi_n(c), k_n(c))}{P(\psi_n(c), k_n(c))} \right] (1 - \gamma(x, \psi_n(c), k_n(c)) - (1 - F(x))) \]
\[
= f(\psi_n(c)) \psi'_n(c) \left[ - \frac{1}{f(\psi_n(c))} \int_0^{P(\psi_n(c), k_n(c))} \gamma_2(x, \psi_n(c), k_n(c))dx \right.
\]
\[
- \int_0^{P(\psi_n(c), k_n(c))} \gamma_3(x, \psi_n(c), k_n(c))dx \frac{\int_0^c F(\psi_n(z))dz}{F(\psi_n(c))^2} \left. \right]
\]
\[
= f(\psi_n(c)) \psi'_n(c) \left[ - \frac{f(\psi_n(c))}{\int_0^{P(\psi_n(c), k_n(c))} \gamma_2(x, \psi_n(c), k_n(c))dx} \gamma_2(x, \psi_n(c), k_n(c))dx F(\psi_n(c)) - (c - k_n(c)) \right]
\]
\[
\times \frac{\int_0^{P(\psi_n(c), k_n(c))} \gamma_3(x, \psi_n(c), k_n(c))dx}{F(\psi_n(c))},
\]
for all $c \in [0, \bar{c}]$, $n \in \mathbb{N}$. Furthermore, direct calculation shows that
\[
\int_0^{P(\psi_n(c), k_n(c))} \frac{\gamma_3(x, \psi_n(c), k_n(c))dx}{F(\psi_n(c))} < 0, \forall c \in [0, \bar{c}], n \in \mathbb{N}.
\]
Together, since $f(\psi_n(c))\psi'_n(c) > 0$ for all $c \in [0, \bar{c}]$ and $n \in \mathbb{N}$, for $n$ large enough,
\[
\Omega_n(c) := -\frac{\int_0^{P(\psi_n(c), k_n(c))} \gamma_2(x, \psi_n(c), k_n(c))dx F(\psi_n(c))}{\int_0^{P(\psi_n(c), k_n(c))} \gamma_3(x, \psi_n(c), k_n(c))dx f(\psi_n(c))} - (c - k_n(c)) \geq 0,
\]
for all $c \in [0, \bar{c}]$. Finally, by the dominance convergence theorem, $\{\Omega_n\} \to \Omega^*$ pointwisely and therefore
\[
\Omega^*(c) \geq 0, \forall c \in [0, \bar{c}].
\]
Direct computation then shows that $\Omega^*$ is strictly decreasing on $[c^*, \bar{c}]$. This completes the proof. $\blacksquare$