Liability Insurance: Equilibrium Contracts under Monopoly and Competition.*

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Abstract

In third-party liability lawsuits (e.g. product liability or patent infringement), a third party demands compensation from a firm. Verifying that the firm harmed the third party is costly and parties often negotiate settlement agreements. In this setting, liability insurance is valuable for the firm because it improves its bargaining position when negotiating a settlement. We show that equilibrium contracts for liability insurance under adverse selection differ dramatically from existing results on first-party insurance: in a competitive market, only a pooling equilibrium may exist; in a monopolistic setting, the insurer offers at most two contracts which under-insure low-risk types and may inefficiently induce high-risk types to litigate.

JEL Codes: D82, G22, K1, K4.

Keywords: insurance, adverse selection, liability, litigation, ex-post moral hazard, competitive equilibrium, monopoly.

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1 Introduction

Third-party liability insurance is fundamentally different from first-party insurance: in the former setting, insurance protects against liability for harm caused to a third party (e.g., patent infringement, employment-related, product, or environmental liability); whereas in the latter setting, insurance protects against own losses (e.g., health, life, or property insurance). In liability insurance, claims for compensation require costly assignment of responsibility between the policy holder and a third party—a court must determine who is responsible for the loss incurred by the third party. A large theoretical literature studies first-party insurance and adverse selection, including the seminal work of Rothschild and Stiglitz (1976) and Stiglitz (1977). In reality, however, third-party insurance markets are pervasive and nonetheless are not well understood.

We analyze markets for liability insurance contracts, where buyers are usually risk neutral firms. In first-party insurance markets, agents are willing to pay for insurance because they are risk averse and first-party insurance reduces risk. In contrast, liability insurance is valuable even for risk neutral agents. The source of this value is an improved bargaining position: most liability lawsuits are settled out of court to avoid the costs involved in the legal process and liability insurance is valuable in part because it improves the insured agent’s payoff from negotiating a settlement with the third party.

We consider a setting in which an agent buys insurance that covers litigation costs and/or damages. At the time of contracting, the probability that the agent will be liable to a third party for damages may be imperfectly known—this is the agent’s type. If and when a third party subsequently sues the agent for damages, the agent and third party bargain over a settlement or litigate. Agents that settle introduce no costs to the insurer, whereas agents that litigate introduce strictly positive costs to the insurer. The ex-post decision to settle or to litigate creates a discontinuity in the insurer’s cost function, which dramatically changes the equilibrium contracts compared to those in the existing literature on first-party insurance under adverse selection.

We study two canonical market structures: a perfectly competitive market for liability insurance with free entry, following Rothschild and Stiglitz (1976); and a mechanism design setting in which a monopolist designs and prices insurance contracts. We study two information environments: symmetric information, where neither the agent nor the insurer know the agent’s probability of liability; and asymmetric information, where the agent alone is privately informed. For both market structures, we find that contracts for third-party insurance differ significantly from standard first-party insurance contracts. First, in a competitive market under asymmetric information, we find that for any distribution of types there can only be pooling equilibria, and any such equilibrium never induces litigation and features
under-insurance. Second, with a single seller and regular type distributions, we show that in any optimal mechanism at most two contracts are offered in equilibrium—one that covers legal costs only, and one that covers legal costs and partially covers damage payments. We also show that damage insurance is more generous, and induces more litigation, under symmetric information than under asymmetric information.

Our results for the equilibrium of a competitive market for third-party liability insurance contrast sharply with the seminal work of Rothschild and Stiglitz (1976), where only separating contracts are offered in equilibrium, due to “cream skimming.” With first-party insurance, in a candidate pooling equilibrium, an insurer is able to profitably deviate by offering a contract that only attracts types who generate positive surplus, which undermines the cross-subsidization needed to sustain the pooling equilibrium. In contrast, with third-party insurance, cross-subsidization is not necessary as long as insurance does not induce litigation by any type that buys it. This enables pooling to survive in equilibrium. In addition, the cream skimming effect is reversed. A separating equilibrium in a competitive market for liability insurance requires that contracts be sold at different prices, because otherwise types would pool on the more generous insurance. But for a contract to sell for a positive price and yield zero profit, it must attract types that settle and types that litigate. Such a contract cannot survive in equilibrium, because it requires cross-subsidization and is therefore cream-skimmed by another contract that only attracts types that settle. This implies that a separating equilibrium does not exist. Similar to Rothschild and Stiglitz (1976), however, we find that adverse selection destroys the possibility of equilibrium altogether, when there are too few high-risk types of agents. For the most part, our findings indicate that the canonical model of adverse selection in markets for insurance applies only to first-party insurance. In particular, third-party liability insurance requires a richer model that also considers the effect of insurance on an agent’s ex post actions.

Our results on the optimal mechanism with a single seller also differ from existing results on insurance contracts, such as in Chade and Schlee (2012), where the optimal menu discriminates among different agent types. In sharp contrast we find that the insurer will offer at most one contract that covers damages. In fact, the insurer’s problem of designing a menu of liability insurance contracts is one of mechanism design with a non-differentiable value function, where the non-differentiability arises because the agent has a non-contractible ex-post action—to settle or litigate. This choice introduces a novel type of ex-post moral hazard that does not appear in first-party insurance, because the insurance changes the agent’s incentives to settle, which enters the seller’s mechanism design problem as an additional ex post incentive constraint. We find that in general the insurer wants to fully cover the legal costs of all agent types, and to partially cover the damage payments of a subset of relatively high (“riskier”) types. The solution generally features distortions at the top, in addition to the more familiar type of distortion at the bottom, and in fact the optimal mechanism does
not necessarily allocate perfect insurance to the highest type. In some cases, the optimal contract may induce inefficient litigation in equilibrium, where in the absence of insurance there would have been no litigation. This points to novel potentially negative welfare effects of liability insurance.

The monopoly insurer’s problem of choosing the level of damages relates to product-quality choice (Spence, 1975, 1976). Higher coverage for damages raises the willingness-to-pay of all agents that buy insurance, and raises the insurer’s costs by inducing more litigation. We find that the level of damages covered under asymmetric information is (weakly) lower than when information is symmetric. Intuitively, a monopolist insurer selling to uninformed agents cares about how the level of damage insurance affects the willingness-to-pay conditional on each type. Under asymmetric information, in contrast, a monopolist insurer cares about how the level of damages affects the willingness-to-pay of the marginal type of agent that buys insurance. We find that the marginal effect of increasing damage coverage is higher for agents with a higher willingness-to-pay, so the monopolist optimally chooses higher damage coverage under symmetric information.

Motivating Examples and Applications

In most liability insurance settings the insured agent is a firm (and hence risk neutral) and the decision to settle or litigate rests with the agent—features that are captured by our modeling assumptions. These settings include well-established markets for insurance for product liability (e.g. product recalls such as the Samsung Note 7 recall and compensation cases in 2016, or the longest-running liability litigation in U.S. history, involving asbestos products (Carroll et al., 2005)), employment liability (e.g. lawsuits for employment discrimination or negligent care in nursing homes), and environmental liability. Other settings include relatively newer insurance markets for intellectual property infringement or for professional negligence such as cyber attacks (e.g. the recent data-breach liability cases against Equifax, Target, Anthem, Home Depot and others). The relevant features in these examples are captured well in our setting.

Our discussions with insurance industry and litigation consulting experts confirm that in most of the liability cases mentioned above, the defendant’s own in-house counsel and external lawyers generally negotiate and agree on a settlement or try the case to verdict. The insurer then handles the reimbursement of expenses and payments, depending on what the defendant

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1 Risk aversion and control may be relevant for other applications. For instance, risk aversion is relevant for medical malpractice and delegation of control is relevant in third-party car insurance. In Appendix B.3 we incorporate risk aversion. This extension is less tractable, but most of the economic insights from our main model remain valid. In Appendix B.5 we consider a setting where the agent may delegate control over the settlement decision to the insurer. We show that, unless the insurer has a strong advantage in settlement negotiations, such delegation is generally not optimal.
agreed to or what the court determined. In settings where the insured agent is prone to certain types of tort lawsuits (e.g. they produced asbestos products in the past and regularly receive injury or death lawsuits), the insured and the insurer periodically process reimbursements based on verdicts or settlements that the insured’s defense agreed to. In some cases conflicts may subsequently also arise between the defendant and the insurer, if the insurer believes the defendant is not putting sufficient efforts in the process of settlement. This also highlights the fact that both insurers and the insured agents recognize that insurance affects the insured’s settlement negotiation strategy and the incentives to litigate.

Insurance for intellectual property infringement is another setting where our model and predictions fit remarkably well. Patent litigation is costly (Bessen et al., 2011) and the recent increase in patent litigation has spurred more growth and activity in this market. Firms such as RPX Corporation, IPISC, Trilogy, and InsureCast offer insurance to cover some fraction of the legal costs or damages related to a patent infringement lawsuit. These companies offer both offensive and defensive insurance contracts. The former pays for the cost of enforcing patents, whereas the latter is used by firms accused of patent infringement to cover the legal costs and penalties imposed by a court following a lawsuit. A cornerstone feature of these contracts is the freedom of the policy holder to decide whether to settle or to litigate.

“The Policy Holder controls the lawsuit. The Company may suggest reliable and preferred counsel to the Insured but the Insured ultimately chooses [...] The Insured dictates the settlement terms, if any, not the Company.”

The market for patent insurance has also been active in Europe. A 2006 study for the European Commission proposed to make patent insurance mandatory for small-to-medium-sized enterprises. Fuentes (2009) studies the trade-offs of this proposal.

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2Patent litigation in the U.S. increased after the establishment of the Court of Appeals for the Federal Circuit, in 1982, and further surged after 2004 (Bessen et al., 2015; Tucker, 2016). This surge—which doubled the number of cases per year—has largely been driven by litigation initiated by patent assertion entities (“PAEs”), also called “patent trolls” (Chien, 2009; Tucker, 2016).


5Trilogy Insurance: http://www.trilogyinsurancegroup.com/services/defense-insurance

6http://jolt.law.harvard.edu/digest/patent/insuring-patents

2 Literature Review

To the best of our knowledge, our paper is the first to study third-party liability insurance markets under adverse selection, and it relates to work on insurance in both law (Schwarz and Siegelman, 2015b) and economics (Dionne, 2013). First-party insurance markets with perfect competition have been extensively studied beginning with Rothschild and Stiglitz (1976), who show that in their framework pooling equilibria do not exist. Subsequent work—e.g., Wilson (1977); Miyazaki (1977); Riley (1979); Crocker and Snow (1985); Azevedo and Gottlieb (2017); Farinha Luz (2017)—shows that alternative equilibrium concepts change both the set of equilibrium contracts and welfare implications. In our setting, pooling equilibria exist under perfect competition and free entry, the equilibrium concept in Rothschild and Stiglitz (1976).

The framework of Stiglitz (1977), which studies the problem of a monopoly insurer under adverse selection, is generalized by Chade and Schlee (2012). We use mechanism design tools to derive the optimal monopoly menu of contracts in our setting.

The literature on optimal contracting under adverse selection and moral hazard (Picard, 1987; Guesnerie et al., 1989) is also related. The key driving force in our model is the improved bargaining position of an insured agent. Kirstein (2000), Van Velthoven and van Wijck (2001), Kirstein and Rickman (2004), and Llobet and Suarez (2012) have shown that risk-neutral buyers may value insurance because it makes litigation credible or it improves the policy holder’s bargaining position. However, none of these papers study equilibrium under adverse selection or the optimal monopoly contract. Townsend (1979) also shows that contracts change when it is costly to verify an agent’s private information. In an insurance context, this work helps explain why contracts include deductibles, which reduces the frequency of an agent filing a claim. In our setting, the agent is not privately informed about the true state of liability. Verification requires litigation. As a result, insurance in our context provides value even when the agent chooses to settle out of court rather than verify the state.

Shavell (1982) studies the effect of liability insurance on ex-ante moral hazard (demand for care) in a model without ex-post bargaining. In contrast, we focus on ex-post bargaining given the equilibrium contracts under different market structures. Meurer (1992) investigates why it may be optimal for the insurer to offer a contract where it controls the litigation and settlement process on behalf of the insured, despite a potential conflict of interest. We focus instead on the case where the insured controls litigation and settlement. Veiga and Weyl (2016) study insurance with multidimensional types and an endogenous product quality. Our model also has a product quality interpretation, albeit in a different setting.

Some existing literature studies third-party funding of plaintiffs and its effect on settlement.
For example, Daughety and Reinganum (2014) adapt the signaling model of Reinganum and Wilde (1986) to show that optimal loans to privately-informed plaintiffs may both eliminate the possibility of litigation and extract very favorable settlement terms. Related to this, the literature on offensive patent insurance shows that some litigation threats become credible under insurance, which increases the entry deterrence value of patents. Llobet and Suarez (2012) and Buzzacchi and Scellato (2008) study insurance that covers a fraction of the patentee’s litigation costs. Duchene (2015) shows that with private information, patent holders may opt not to buy insurance because of an inability to sharply signal and avoid pooling equilibria. In our setting, by contrast, there is no gain to the insuring party from making litigation threats credible and the insurer is exposed to significant losses when litigation occurs. Both factors affect equilibrium contracting.

Historically, markets for third-party insurance have been more volatile than first-party insurance markets. In 1986 in the United States, for example, premiums rose sharply and some insurers declined to sell certain types of coverage. In the wake of this crisis, Priest (1987), Winter (1991) and Harrington and Danzon (1994) analyze how liability insurance differs from other kind of insurance—in particular, the difficulty insurers have in forecasting liability losses. Unlike our setting, these papers do not focus on the role of insurance in shaping subsequent bargaining.

Our work also relates to the literature on lawyers’ contingent fees. Under such contracts, lawyers charge lower upfront fees but keep part of any payments awarded. The agent that hired the lawyer may not receive the full litigation outcome. Dana and Spier (1993) show that contingency fees help solve an agency problem. Rubinfeld and Scotchmer (1993) study a Rothschild-Stiglitz-style competition model, and make the point that clients with high-quality cases can signal their cases’ strength by selecting hourly fees, while attorneys can signal their ability by requesting contingency fees. Gravelle and Waterson (1993) make similar points. Finally, Hay and Spier (1998) and Spier (2007) review the large literature on litigation and settlement.

3 Model

Consider a risk-neutral agent (A), or a firm, who sells a product or provides a service. The agent may harm a risk-neutral third party (TP), thereby creating a legal liability. Only a court can verify whether or not the harm has occurred. To cover the legal costs and damages, if the court determines that harm in fact has occurred, the agent may purchase third-party liability insurance from a risk-neutral insurer (I). Going to trial is costly: A pays a cost $c_A > 0$ and TP pays $c > 0$. If the court determines that the agent is liable, the agent must
make a payment \( d \) to the third party. The agent’s type is \( p \in [0, 1] \), which is the probability that the agent is found liable. In our setting, this probability is unknown by the insurer and it may or it may not be known by the agent at the time of contracting. After contracting, \( p \) is revealed to all parties. Figure 1 describes the timing of the model.

Figure 1: Timing of the events in the model.

At \( t = 1 \), the risk-neutral agent considers buying liability insurance. Insurance contracts are defined by \( \alpha = (\alpha_L, \alpha_D) \), where the insurer will pay \( \alpha_L \) to cover the litigation costs and \( \alpha_D \) to cover damages. The set of contracts that the insurer can offer is

\[
\mathcal{A} = \{ (\alpha_L, \alpha_D) : \alpha_L \in [0, c_A], \ \alpha_D \in [0, d] \}.
\]

We assume that the insurer has commitment and it cannot renegotiate the contract signed at \( t = 1 \)—this is a natural assumption in a setting with contractual commitments, and is also justified by the fact that insurers generally also have reinsurance contracts with other insurers, based on contracts that have already been sold. Furthermore, if the contract were renegotiable, then the solution is analogous to the complete information case discussed in subsection 3.1.

At \( t = 2 \), after observing \( p \), the third party has a credible litigation threat if and only if \( pd \geq c \). If \( pd < c \), the game ends. If and when a lawsuit is filed, the agent and the third party bargain at \( t = 3 \) under complete information. The agent’s bargaining payoff at \( t = 3 \) depends on the probability of liability \( p \), the insurance contract it has bought, and the decision to settle or to go to trial. If parties go to litigation, at \( t = 4 \), the agent’s expected payoff is

\[
V_L(p, \alpha) = -(c_A - \alpha_L) - p(d - \alpha_D).
\]

Notice the importance of the litigation costs in Equation 1: if \( c_A = c = 0 \), this is precisely the value of insurance in Rothschild and Stiglitz (1976) for a risk neutral agent with wealth normalized to \( W = 0 \).

At \( t = 3 \), the agent and the third party Nash-bargain over a fee to settle the lawsuit. The agent’s bargaining power is \( \theta \in [0, 1] \). When the agent does not have insurance, the joint surplus between the agent and the third party increases by \( c_A + c \), so settlement always occurs. However, when the agent is covered by insurance policy \( \alpha \), the change in joint surplus from
settlement, relative to litigation, is

\[ S_B = c + c_A - \alpha_L - p\alpha_D, \]

which can be positive or negative, depending on the insurance policy and the agent’s type. If \( \alpha_D = 0 \), \( S_B \) is positive and independent of the liability probability, so there is always settlement. However, because the settlement fee is proportional to the joint surplus, the agent pays a lower settlement fee when it is covered by insurance—having insurance improves the agent’s bargaining position. Within the class of contracts where \( \alpha_D = 0 \), the contract that maximizes the value of insurance for the agent is \( \alpha_L = c_A \).

If \( \alpha_D > 0 \), then \( S_B \) could be negative in which case the parties go to trial. In particular, \( S_B \) is negative for agents with a probability of liability, \( p \), larger than

\[ p^* \equiv \frac{c + c_A - \alpha_L}{\alpha_D}. \]  

(2)

If \( \frac{c}{d} \leq p \leq p^* \), settlement increases the joint surplus and the agent’s payoff is

\[ V_S(p, \alpha) = -(c_A - \alpha_L) - p(d - \alpha_D) + \theta S_B. \]  

(3)

If \( p > p^* \), settlement decreases the joint surplus, so litigation becomes unavoidable. In this case, the agent’s payoff is given by Equation 1.

Insurance allows an agent that settles to capture more of the bargaining surplus: it increases the payoff of a low-risk type by improving its bargaining position. The agent only captures a fraction \((1 - \theta)\) of the savings induced by a better bargaining position. High-risk agents go to trial and part of their expenses are covered by insurance. The cost of insurance jumps discontinuously at \( p = p^* \), because the insurer pays no claims under settlement but pays strictly positive claims when litigation occurs. Figure 2 summarizes the effects of insurance on the decision to reach a settlement.\(^8\)

\[ \text{Figure 2: The effect of insurance on litigation for different types of agents.} \]

Lemma 1. Consider an insurance policy \( \alpha = (\alpha_L, \alpha_D) \in A \) and \( p^* \) as defined in (2). The agent’s willingness to pay for insurance, \( W(p, \alpha) \), and the expected cost for the insurer of

\[^8\text{The agent faces no threat for } p < \frac{c}{d}. \text{ For the remainder of the paper we restrict attention to } p \geq \frac{c}{d}. \]
providing policy $\alpha$ to an agent of type $p$, $K(p, \alpha)$, are given by

\[
W(p, \alpha) = \begin{cases} 
(1 - \theta)(c + c_A) + (p - p^*)\alpha_D(1 - \theta) & \text{if } p \leq p^* \\
(1 - \theta)(c + c_A) + (p - p^*)\alpha_D & \text{if } p > p^*
\end{cases},
\]

\[
K(p, \alpha) = \begin{cases} 
0 & \text{if } p \leq p^* \\
c + c_A + (p - p^*)\alpha_D & \text{if } p > p^*
\end{cases}.
\]

All the proofs omitted in the text are in Appendix A.

Equation (4) shows that the willingness to pay is a continuous and convex function of $p$ with a kink at $p^*$. Also, it depends on $\alpha_L$ implicitly through the definition of $p^*$. From equations (4) and (5) it is easy to see that the willingness to pay for insurance is always less than the cost of providing it for high risk types that choose to litigate, i.e., for types $p > p^*$. In fact, the difference between the willingness to pay and cost is exactly $\theta(c + c_A)$ for $p > p^*$. Figure 3 depicts the willingness to pay and the cost of providing an insurance contract $\alpha$ to an agent of type $p$.

![Figure 3: $W(p, \alpha)$ is type $p$’s willingness to pay for insurance policy $\alpha$. The cost to the insurer of providing the coverage prescribed by policy $\alpha$ for an agent of type $p$ is given by $K(p, \alpha)$. Type $p^*$ is indifferent between settlement and litigation.](image)

Corollary 1. We have:

1. The willingness to pay for contract $(\alpha_L, \alpha_D) = (c_A, 0)$ is $(1 - \theta)c_A$.

2. For any $p > p^*$ and for any policy $\alpha$ we have $K(p, \alpha) - W(p, \alpha) = \theta(c + c_A)$.

The intuition for Corollary 1 is the following. A contract that fully covers litigation costs but does not cover damages always induces the agent to settle. From the third party’s perspective, when the agent does not pay for its own litigation costs, the agent has litigation costs equal
to zero at the time of negotiating a settlement. This improves the agent’s bargaining position and the third party is unable to capture \((1 - \theta)c_A\) in bargaining rents. The reduction in the bargaining surplus lowers the settlement fee that the agent pays to the third party, which is precisely the amount the agent is willing to pay for an insurance policy that fully covers litigation costs but does not cover damages. The second part of Corollary 1 shows that when the agent litigates, the joint surplus of the insurer and the agent decreases by \(\theta(c + c_A)\), which corresponds to the bargaining surplus captured by the agent in a settlement negotiation.

Although the insurance contracts we consider are generally characterized by two parameters, some contracts are weakly dominated from the insurer’s perspective. Most notably, if coverage for litigation costs is incomplete \((\alpha_L < c_A)\), then the insurer could increase coverage for litigation costs, lower coverage for damages, and keep the cutoff type \(p^*\) the same. This would increase willingness-to-pay for insurance for the \(p \leq p^*\) types that settle. Because insurance is costless for types that settle, this increases the insurer’s profit. Meanwhile, for the \(p > p^*\) types that litigate, the gap between willingness-to-pay and the insurer’s cost is \(K(p, \alpha) - W(p, \alpha) = \theta(c + c_A)\), independent of \(\alpha\). Hence, it pays to increase \(\alpha_L\) as much as possible.

**Proposition 1.** Any insurance contract \(\alpha = (\alpha_L, \alpha_D)\) with \(\alpha_L < c_A\) is weakly dominated by an alternative contract \(\alpha' = (c_A, \alpha_D')\).

By Proposition 1, the space of contracts is characterized by \(p^* \in \left[\frac{c_A}{\theta}, \infty\right]\), representing the contract \((\alpha_L, \alpha_D) = (c_A, \frac{c_A}{p^*})\).\(^9\) We now re-write equations (4) and (5), the value of a contract \(p^*\) to an agent of type \(p\), and the insurer’s cost of providing a contract \(p^*\) for an agent of type \(p\), using the single parameter \(p^*\) to characterize different contracts,

\[
W(p, p^*) = \begin{cases} 
(1 - \theta) \left[ c_A + \frac{c_p}{p^*} \right] & \text{if } p \leq p^* \\
\left[ c_A + \frac{c_p}{p^*} \right] - \theta(c + c_A) & \text{if } p > p^*
\end{cases}, \quad (6)
\]

\[
K(p, p^*) = \begin{cases} 
0 & \text{if } p \leq p^* \\
c_A + \frac{c_p}{p^*} & \text{if } p > p^*
\end{cases}. \quad (7)
\]

With this change in notation, it is easy to see that willingness to pay for insurance \(p^*\) increases faster with \(p\) when insurance is more generous (i.e., \(p^*\) is lower). Figure 4 shows two policy contracts \(p_1^*\) and \(p_2^*\) with \(p_2^* > p_1^*\). For any type \(p\), \(W(p, p_1^*) > W(p, p_2^*)\) and that \(W(p, p_1^*) - W(p, p_2^*)\) is increasing in \(p\).

**Corollary 2.** Let \(\tilde{W}(p, p^*) = W(p, 1 - p^*)\). Then, \(\tilde{W}(p, p^*)\) is supermodular.

\(^9\) When \(p^* = +\infty\) captures limit case corresponding to contract \((c_A, 0)\).
Figure 4: Willingness to pay for two insurance policy contracts indexed by $p_1^*$ and $p_2^*$.

Before we proceed with our main results, we summarize some of the extensions of the model considered in our supplemental material in Appendix B. Our results in the main text are derived for an arbitrary distribution of types with continuous density. For illustrative purposes, in Online Appendix B.1, we derive all of our results for a two-type discrete distribution. In Online Appendix B.2, we allow the insurer to offer contracts that cover settlements and we show that it is not optimal for the insurer to do so. In Online Appendix B.3, we consider a risk averse agent. Risk aversion makes the agent more willing to settle, which affects the willingness to pay for insurance. We show that, under some conditions, the agent’s value for insurance as a function of its type retains properties from our risk neutral model: the value function is continuous, increasing in $p$, and has a kink at a particular point which depends on the contract. Our baseline model focuses on risk neutrality because liability insurance is often bought by firms, rather than individuals. Most of our results are qualitatively preserved in the setting with risk aversion, but it is far less tractable than the risk neutral setting. In Online Appendix B.4 we discuss the case of bargaining under incomplete information, where the agent is privately informed about the probability of liability and the third party is uninformed. We derive the equilibrium contract and results for perfect competition in the two-type case. Finally, in Online Appendix B.5 we discuss the optimal assignment of control over the settlement process between the agent and the insurer.

### 3.1 Complete Information

As a benchmark, we first consider the case of complete information regarding $p$. With complete information, an insurer sells the contract that most improves the bargaining position of the agent without inducing litigation. This is an equilibrium in the case of competition or monopoly, although the prices of the policies differ in the two cases.

**Proposition 2.** For a monopoly or under perfect competition, if the insurer(s) can observe $p$, 

\[ W(p, p_1^*) > W(p, p_2^*) \]

\[ (1 - \theta)(c + c_A) \]

\[ \frac{2}{\theta} \]

\[ p_1^* \]

\[ p_2^* \]

\[ 1 \]
the equilibrium insurance policy is \( \alpha^*(p) = (c_A, \frac{\xi}{p}) \), a contract that fully covers the litigation expenses, partially covers damages, and does not induce litigation. A competitive market offers this policy for free and a monopolist charges \((1 - \theta)(c + c_A)\).

Proof. The equilibrium contract must induce each agent to reach a settlement agreement, because the insurer incurs a loss by selling a policy that induces litigation. When all agents settle, the insurer does not incur costs, hence, either a monopolist or a competitive market offer the contract \( \alpha^*(p) = (c_A, \frac{\xi}{p}) \) that maximizes the agent’s willingness to pay under settlement. The monopolist extracts all the surplus and sells it at price \((1 - \theta)(c + c_A)\). A competitive market offers this policy for free.

We henceforth refer to contract \( \alpha^*(p) \) as perfect insurance for type \( p \), because it generates the most joint surplus to be shared by the agent and insurer.

Under complete information, there is always settlement and the effect of insurance is to reduce the bargaining surplus. By taking the agent’s incentive to litigate to the absolute brink with damages insurance \( \hat{\alpha}_D(p) = \frac{\xi}{p} \), the equilibrium insurance contract extracts all bargaining surplus from the third party. Effectively, insurance under complete information transfers rents from the third party to the insurer (in the case of monopoly) or to the agent (in the case of perfect competition).

Third-party insurance contracts and first-party insurance contracts have significant differences. First-party insurance contracts have no value for risk neutral individuals since all value comes from risk reduction. Third-party insurance contracts, in contrast, are valuable for risk neutral individuals because there is costly verification of the harm. This verification gives rise to settlement negotiations and insurance adds value within that framework, as long as the third party has some bargaining power.

### 3.2 Symmetric Information (No Adverse Selection)

Consider the problem of selling insurance when the insurer and the agent are both uninformed about \( p \) but they know its distribution \( F \).\(^{10}\) In this instance, every agent is ex-ante identical, and because there are no externalities among agents, there is no reason to offer more than a single insurance policy.

The expected willingness to pay for liability insurance contract \( \hat{p} \) is \( E_p[W(p, \hat{p})] \). A monopolist prices this policy at \( P_M = E_p[W(p, \hat{p})] \) and extracts all the ex-ante value from the uninformed

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\(^{10}\) In the context of defensive patent insurance, a firm and an insurer know that the firm potentially infringes on some patents, but they do not know the scope of the threat (patent thickets).
agents. Hence, the profit maximizing contract for the monopolist solves:

\[ p^* \in \arg\max_{\hat{p} \in \left[ \frac{c}{d}, \infty \right]} \Psi_{SI}(\hat{p}) \equiv \mathbb{E}_p[W(p, \hat{p}) - K(p, \hat{p})]. \] (8)

In a perfectly competitive market there is free entry, so any active insurer must break even in equilibrium. If insurance contract \( \hat{p} \) is offered in equilibrium its price must be \( P_C(\hat{p}) = \mathbb{E}_p[K(p, \hat{p})] \). Agents buy this contract as long as \( \mathbb{E}_p[W(p, \hat{p})] \geq P_C(\hat{p}) \). Thus, the only contract that is offered in equilibrium must also be the solution to (8). A perfectly competitive market and a monopolist sell the same contract at different prices. The next proposition characterizes the contract offered to an agent that is uninformed about its type when buying insurance.

**Proposition 3.** Let both the agent and the insurer know \( F(\cdot) \) but be uninformed about \( p \). Then, the liability insurance policy offered by a monopolist or a perfectly competitive market is \( p^* \) characterized by the solution to:

\[ p^* \in \arg\max_{\hat{p} \in \left[ \frac{c}{d}, \infty \right]} \Psi_{SI}(\hat{p}) = (1 - \theta) \int_{c/d}^{\hat{p}} \left[ c_A + \frac{cp}{\hat{p}} \right] dF(p) - \theta(c + c_A)[1 - F(\hat{p})]. \] (9)

The price of the contract under perfect competition is \( P_C(p^*) = \mathbb{E}_p[K(p, p^*)] \) and under monopoly is \( P_M(p^*) = \mathbb{E}_p[W(p, p^*)] \).

Equation (9) in Proposition 3 shows that the optimal contract balances two forces. Only an agent of type \( \hat{p} \) receives perfect insurance under contract \( \hat{p} \). Type \( p \leq \hat{p} \) is under-insured by this contract. The insurer’s marginal cost for these types is zero. Types \( p > \hat{p} \) litigate and their willingness to pay rises more with \( p \) than types below \( \hat{p} \)—there is a kink in the demand at \( \hat{p} \). However, the marginal cost of insurance is positive for these types, and exceeds willingness to pay by \( \theta(c + c_A) \). This amount is exactly what the agent would have captured in a settlement, and therefore cannot be priced by the insurer.

For a given distribution of types, these effects have different weights represented by areas A and B in Figure 5. Area A is the gain in joint surplus from types that settle and corresponds to the term \( (1 - \theta) \int_{c/d}^{\hat{p}} \left[ c_A + \frac{cp}{\hat{p}} \right] dF(p) \) in equation (9). Area B is the loss in joint surplus from types that litigate and corresponds to the term \(-\theta(c + c_A)[1 - F(\hat{p})]\) in equation (9).

To help characterize the solution to this problem, we consider smooth distributions for which the density may equal zero only at the boundaries of the support.

\[ \Psi_{SI}(\hat{p}) \] is upper semi-continuous: it is obvious when \( F(\cdot) \) is continuous; when \( F(\cdot) \) is not continuous (e.g., discrete types), u.s.c. follows from our assumption that the agent settles when indifferent. \( \hat{p} = +\infty \) corresponds to the contract that does not cover damages. However, \( \Psi_{SI}(\cdot) \) decreasing for \( \hat{p} > 1 \), so a solution must lie in the compact interval \( \left[ \frac{c}{d}, 1 \right] \). This guarantees existence of a solution.
Figure 5: The solid area (in blue) represents the gains and the dashed area (in red) represents the losses of contract $\hat{p} < 1$.

Assumption 1. Let $F(\cdot)$ be twice-continuously differentiable, with probability density $f(p) > 0$ for all $p \in (0, 1)$.

Consider the derivative of the objective function in equation (9):

$$
\Psi_{SI}'(p^*) = (1 - \theta)(c + c_A)f(p^*) - (1 - \theta) \frac{c}{(p^*)^2} \int_{c/d}^{p^*} pdF(p) + \theta(c + c_A)f(p^*). \tag{10}
$$

Increasing coverage has an effect on the marginal type and infra-marginal types. First, the marginal type $p^*$ gets perfect insurance and extracts the full bargaining surplus $(c + c_A)$ from the third party. The gain of the marginal type is shown in equation (10) in two different places: a gain from the improved bargaining position of the marginal type $(1 - \theta)(c + c_A)$; and a gain from avoiding a loss of $\theta(c + c_A)$ in bargaining surplus had the marginal type gone to court. Second, the infra-marginal types $p < p^*$ receive a level of insurance further away from the perfect level, inducing a loss in the joint surplus of the insurer and agent.

The optimal contract either precludes litigation entirely ($p^* = 1$) or balances the gain of the marginal type versus the average loss of the infra-marginal types. To further understand when it is optimal to offer a contract that induces litigation, we define the elasticity of density.

Definition 1. For distributions satisfying Assumption 1, the elasticity of density is

$$
\eta(p) = \frac{pf'(p)}{f(p)}.
$$
It is easy to see that the following identity holds
\[ \Psi''_{SI}(p)p^2 + 2\Psi'_{SI}(p)p = \frac{pf(p)}{cA + c} \left[ \eta(p) + 1 + \frac{cA + \theta c}{cA + c} \right]. \]
Thus, if \( p^* \) is an interior solution of problem (9), the first and second order conditions, \( \Psi'_{SI}(p^*) = 0 \) and \( \Psi''_{SI}(p^*) < 0 \), respectively, imply
\[ \eta(p^*) < -\left( 1 + \frac{cA + \theta c}{cA + c} \right). \]
The elasticity of density provides us with a sufficient condition for a unique solution of problem (9).

**Lemma 2.** Under Assumption 1, the solution to problem (9) is unique and equal to \( p^* = 1 \) if for all \( p \in \left[ \frac{a}{d}, 1 \right] \) we have
\[ \eta(p) \geq -\left( 1 + \frac{cA + \theta c}{cA + c} \right). \]

For any convex distribution \( F(\cdot) \), \( \eta(p) \geq 0 \) for all \( p \). By Lemma 2, the unique optimal contract precludes litigation by setting \( p^* = 1 \). When the density function is increasing, the marginal gain dominates the infra-marginal loss, i.e., it is suboptimal to sell insurance generous enough to induce litigation by risky types. Intuitively, it is also optimal to preclude litigation when \( F(p) \) is mildly concave.

There are many distributions where the solution to (9) induces litigation for some types. In such cases, \( \eta(p) \) allows us to provide a sufficient condition for uniqueness.

**Lemma 3.** Under Assumption 1, let \( p^* < 1 \) be such that \( \Psi'_{SI}(p^*) = 0 \) and \( \Psi''_{SI}(p^*) < 0 \). Then, \( p^* \) is the unique interior solution if
\[ \eta(p) \leq -\left( 1 + \frac{cA + \theta c}{cA + c} \right), \quad \text{for all } p \in [p^*, 1] \]

When \( p^* < 1 \), the insurer targets a particular type \( p^* \) with perfect insurance and endures litigation by types \( p > p^* \) and imperfect insurance for types \( p < p^* \). In targeting, the insurer seeks a sufficiently low level of relative litigation risk associated with type \( p^*,12 \) When the elasticity of density falls with \( p \) and the density of a high-risk type is low,13 intuitively, the insurer prefers to induce some litigation. We have the following result.

---

12. \( \eta(\cdot) \) is analogous to the Arrow-Pratt coefficient of relative risk aversion when the Bernoulli utility function is \( u(x) = F(x) \). A large coefficient of relative risk aversion implies that the decision-maker has very little to gain by gambling. In our environment, a large negative \( \eta(p) \) means that the insurer wants a lower \( p \), because it has very little to lose from gambling on relatively unlikely litigation.

13. Note that specifying \( \eta(p) \) as decreasing in \( p \) is a weaker assumption than specifying \( f(p) \) to have decreasing density and to be log-concave in \( p \).
Corollary 3. If $\eta(p)$ is non-increasing and $f(1) < \frac{(1 - \theta)c}{c_A + c} \int_{c/d}^{1} p dF(p)$, there exists a unique $p^* \in \left(\frac{c}{d}, 1\right)$ that solves (9).

Proof. When $\Psi_{SI}'(1) < 0$, there exists $p^* < 1$ that solves (9). Since $\eta(p^*) < -\left(1 + \frac{c_A + \theta c}{c + c_A}\right)$ and $\eta(p)$ is non-increasing, the sufficient condition for uniqueness in Lemma 3 holds.

Figure 6 shows the gains and losses of a contract $p^* < 1$ relative to $p^* = 1$. The gain of $p^* < 1$ comes from offering insurance that is closer to the perfect level, so every type below $p^*$ is willing to pay more for this contract. The losses come from two sources. First, the cost of providing insurance is larger than the willingness to pay for types above $p^*$, thus the insurer incurs a net loss for types above $p^*$. Second, there is an opportunity cost of offering $p^* < 1$ instead of $p^* = 1$. With $p^* = 1$ all types settle and the insurer does not incur costs. The balance, of course, depends on the distribution of types. It is immediate from the figure that if the density of types in a neighborhood of $p = 1$ is small, the gain is larger than the loss and hence $p^* < 1$ dominates $p^* = 1$.

The following two families of distributions help illustrate our results.

Example 1. The unique optimal contract for an uninformed agent is

1. $p^* = 1$ if $F(p) = p^\alpha$, $\alpha > 0$.
2. $p^* < 1$ if $F(p) = 1 - (1 - p)^\alpha$, $\alpha > 1$.
Figure 7 illustrates these families of distributions. Figure 7(a) shows the density of the cdf $F(p) = p^\alpha$, which allocates significant probability mass to the highest-risk types for all $\alpha$. For these distributions, $\eta(p) \geq -1$ for all $p$ and $\alpha$, so by Lemma 2, it is optimal to set $p^* = 1$. Figure 7(b) shows the density of the cdf $F(p) = 1 - (1-p)^\alpha$ for $\alpha > 1$, showing low mass around $p = 1$. For these distributions, it is easy to show that $\Psi_S^\prime(1) < 0$ because $f(1) = 0$. Therefore, the solution must be $p^* < 1$. Moreover, because $\eta(p)$ is decreasing for this distribution, we know the solution must be unique.

Another way to think about the problem is that the insurer wishes to target the dense part of the distribution with perfect insurance. Consider a discrete distribution with only two types.

**Definition 2** (Two-types case). Let $p \in \{p_L, p_H\}$, such that $c/d < p_L < p_H \leq 1$, and suppose the type distribution is $\Pr(p = p_H) = \lambda$ and $\Pr(p = p_L) = 1 - \lambda$.

From Proposition 3 it is easy to see that with two types, the optimal contract is either $p^* = p_L$ or $p^* = p_H$. Which of these contracts is optimal depends on the fraction of types. When the proportion of high-risk types is relatively large,

$$\lambda > \lambda_{SI}^{Lit} \equiv \frac{(1 - \theta)c(p_H - p_L)}{p_H(c + c_A) + (1 - \theta)c(p_H - p_L)},$$

then the optimal contract is $p^* = p_H$ and targets types $p_H$. However, when $\lambda$ is small, the optimal contract is $p^* = p_L$.

Consider now comparative statics. We have the following results.\textsuperscript{15}

\textsuperscript{14}The details of this case is in Online Appendix B.1.

\textsuperscript{15}As the two-type case suggests, problem (9) may have multiple solutions, e.g. with a continuous distribution with non-monotonic $\eta(p)$. If so, the monotonicity of $p^*$ is in the strong set order.
Lemma 4. $p^*$ is non-decreasing in $c_A$ and $\theta$, and is non-increasing in $d$.

Lemma 4 follows from the Topkis monotonicity theorem. An increase in the agent’s litigation cost $c_A$ increases the opportunity cost of litigation. The gain from increasing the number of types that settle is unambiguously higher, so $p^*$ is non-decreasing in $c_A$. An increase in the agent’s bargaining power decreases the insurer’s ability to profit from insurance: the willingness to pay for insurance falls but the cost of insurance is the same. Thus $p^*$ is non-decreasing in $\theta$ because an increase in the agent’s bargaining power does not change the surplus gain of the marginal type, but it reduces the surplus loss of the infra-marginal types. An increase in damages $d$ increases the number of agents exposed to credible liability claims. Thus the number of infra-marginal types increases and therefore $p^*$ weakly decreases. The effect of the third-party’s litigation cost $c$ is ambiguous, because it increases both the surplus gain of the marginal type and the loss in surplus of the infra-marginal types.

3.3 Asymmetric Information (Adverse Selection)

3.3.1 Perfect Competition

Suppose agents are privately informed about the probability of liability, and the market for insurance is perfectly competitive. There is a perfectly elastic supply of potential insurers capable of freely entering and selling insurance. We follow Rothschild and Stiglitz (1976) in specifying that equilibrium requires insurer profit be zero in equilibrium and that there is no possibility of a profitable deviation by an alternative insurer. That is, there is no contract that an entrant could offer that would earn a strictly positive profit.

The equilibrium price depends on how much litigation is induced by the insurance contracts. If an insurance policy induces all types that buy it to settle, its price must be zero in equilibrium, because the insurer providing the policy bears no cost. In contrast, if the insurance induces litigation for some types, then Corollary 1 shows that the insurer earns a negative profit on the group of agents who litigate. Hence, to break even, the insurer must earn a strictly positive profit on the other group of agents. Hence, any pooling contract that induces litigation requires cross-subsidization, and cannot survive in equilibrium.

Proposition 4. For any distribution $F(\cdot)$, a single pooling contract that induces litigation cannot be offered in equilibrium in a perfectly competitive market.

Intuitively, an alternative, slightly less generous contract could be offered to attract only types that settle (which does not impose any cost on the insurer) and could be sold at a slightly lower, but positive price. This intuition is similar to the cream skimming argument.
The intuition of these results is easiest to see with two types. Suppose agents can be low-risk (type $p_1$) or high-risk (type $p_2$), with $p_1 < p_2$. To separate types in equilibrium, an insurer must sell contracts with different damage coverage $p^*$ at different prices. With common prices, all types would buy the more generous coverage. This rules out two contracts that preclude litigation and are sold for a price of zero. Indeed, to earn zero profit with two contracts that each generate trade, some types must litigate, some types must settle, and the types that settle must pay strictly positive prices (while generating no costs). The reason is that the willingness to pay of types that litigate is below the insurer’s cost, so the insurer inevitably loses money on these types. The insurer must therefore earn money from types that settle. But given these requirements, an alternative insurer can then attract some types that settle, by offering a slightly less generous contract at a slightly lower price. This generates positive profits because all switching types settle. This cream-skimming intuition therefore undermines any such separating equilibrium.

The result in Theorem 1 contrasts with Rothschild and Stiglitz (1976), where a separating equilibrium does exist provided there are a sufficiently high number of high-risk types. Also in contrast to Rothschild and Stiglitz (1976), we now show that a simple pooling equilibrium may exist in this market. From Proposition 4 and Theorem 1, the only possible equilibrium is a pooling equilibrium that does not induce litigation.

**Theorem 2.** Let $p^*$ such that $F(p^*) = 1$. A pooling equilibrium exists if and only if

$$\max_{\tilde{p} \in \left[\frac{c_A}{p^*}, \tilde{p}^*\right]} \left\{ \frac{(1 - \theta)c \cdot (p^* - \tilde{p})}{\tilde{p}p^*} \cdot \max_{\tilde{p} \in \left[\frac{c_A}{\tilde{p}}, \tilde{p}\right]} \tilde{p}[1 - F(\tilde{p})] - \int_{\tilde{p}}^{p^*} \left[ c_A + \frac{cp}{\tilde{p}} \right] dF(p) \right\} \leq 0.$$ 

The pooling equilibrium contract is $p^*$ sold at price zero.

Theorems 2 and 1 in combination say that in a perfectly competitive market for liability insurance, only a pooling equilibrium can exist, and its existence will depend on the distribution of types. Intuitively, the condition in Theorem 2 says that a pooling equilibrium exists as long as the distribution of high-risk types is such that any deviation would induce such losses that it is not profitable to offer a contract that induces litigation. This condition is related to the condition for inducing litigation under symmetric information.

**Proposition 5.** If the optimal liability insurance contract under symmetric information, denoted by $p^*$, satisfies $F(p^*) = 1$, then there exists a pooling equilibrium with $F(p^*) = 1$ in a competitive market with adverse selection.
The intuition for this result can be seen in Figure 6. The joint gains from $p^* < 1$ relative to $p^* = 1$ are higher for a monopoly under symmetric information than for a deviating insurer in a competitive market. This is because the monopolist offers only one contract, so the agent’s outside option is to not buy liability insurance. In contrast, when contract $p^* = 1$ is offered in a competitive market, any deviation must take into account that only types that prefer the deviating contract $\tilde{p}$ over $p^* = 1$ will buy it. Therefore, the gain from deviating from $p^* = 1$ in a competitive market is weakly lower than in the case of monopoly. However, the losses are the same and equal to $\theta(c_A + c)[1 - F(\tilde{p})]$. Hence, whenever $p^* = 1$ is optimal for a monopolist under symmetric information, no insurer finds that deviating from $p^* = 1$ is profitable. Proposition 5 paired with Lemma 2 from the previous section, implies that a pooling equilibrium with $p^* = 1$ exists whenever

$$\eta(p) \geq - \left(1 + \frac{c_A + \theta c}{c_A + c}\right).$$

The conditions needed for a pooling equilibrium are weaker than the sufficient conditions for $p^* = 1$ under symmetric information, however.

Consider again the two-type case: there is a mass $\lambda$ of high-risk types $p^H$ and a mass $(1 - \lambda)$ of low-risk types $p^L$. The candidate for pooling equilibrium is to sell contract $p^* = p^H$ to all types at price zero. This contract does not induce litigation. Applying the condition in Theorem 2, it is easy to see that the only deviation to consider is $\tilde{p} = \bar{p} = p^L$. Therefore, in this case the condition is equivalent to

$$\lambda \geq \lambda^{Pool}_{AI} \equiv \frac{(1 - \theta)c(p^H - p^L)p^L}{p^H(c_A p^L + cp^H)}.$$

When the population consists primarily of $p^H$ types, then a free contract that targets these types is an equilibrium. The $p^L$ types will also “buy” this contract. There is no way to “cream skim,” because any better contract offered to $p^L$ types also attracts too many litigious $p^H$ types. Consistent with Proposition 5, it is easy to show that $\lambda^{Lit}_{SI} > \lambda^{Pool}_{AI}$. Hence, if $\lambda$ is high enough so that $p^* = p^H$ under symmetric information, then a pooling equilibrium exists for contract $p^* = p^H$ under competition with asymmetric information.

### 3.3.2 Monopoly

Now consider a monopolist insurer. When agents have private information about their type, a monopolist may offer a menu of contracts, or a mechanism, to maximize profits. By the revelation principle we can restrict attention to direct mechanisms that are incentive compatible.

Our mechanism design problem, however, presents a subtle complication. For a given contract $p^*$, the willingness to pay and the cost for the monopolist are not differentiable at the point $p = p^*$. Carbajal and Ely (2013) study quasi-linear settings with non-differentiable valuations.
In this case, the envelope theorem characterization may lead to a range of possible payoffs as a function of the allocation rule. The problem is that, although the valuation may be non-differentiable at one point (which has zero-measure), the mechanism may allocate a non-zero measure set of types to the non-differentiable point. The marginal valuation is not ‘point-identified’ at the non-differentiable point, because it belongs to an interval (the sub-differential instead of the derivative). In our context, however, the optimal mechanism allocates at most one type to the non-differentiable point; hence, we can apply the envelope theorem to derive the optimal mechanism. Before we present the main result of this section, we derive a series of results that are useful to characterize the optimal menu of contracts.

Instead of indexing contracts by \( p^* \in \left[ \frac{c}{d}, \infty \right] \), we define \( x(p^*) = \frac{1}{p^*} \in \left[ 0, \frac{d}{c} \right] \) to be the allocation, which corresponds to \( x \cdot c = \alpha D \). The insurer offers a direct revelation mechanism such that for each reported type \( p \), the agent receives allocation \( x(p) \) at price \( T(p) \). The payoff for an agent of type \( p \) that reports \( \tilde{p} \) is given by:

\[
U(p, \tilde{p}) = \hat{W}(p, x(\tilde{p})) - T(\tilde{p}),
\]

where \( \hat{W}(p, x) = \begin{cases} (1 - \theta)(cpx + c_A) & px \leq 1, \\ cpx + c_A - \theta(c + c_A) & px > 1. \end{cases} \)

Notice that when \( \theta = 0 \), this is the classic quasilinear environment. When \( \theta > 0 \), the agent’s payoff has a non-differentiable point (a kink) whenever \( xp = 1 \).

The insurer’s cost of serving type \( p \) with allocation \( x \) is

\[
K(p, x) = \begin{cases} 0 & px \leq 1, \\ cpA + cpx & px > 1. \end{cases}
\]

The insurer’s cost has a kink whenever \( xp = 1 \), regardless of the value of \( \theta \).

The problem of the insurer is to choose the functions \( x(\cdot) \) and \( T(\cdot) \) to solve:

\[
\max_{T(\cdot), x(\cdot)} \int_{c/d}^{1} T(p) dF(p) - \int_{p: px(p) > 1} [c_A + cpA/p] dF(p)
\]

subject to

\[
p \in \text{arg max}_{p'} \hat{W}(p, x(p')) - T(p') \quad (\text{IC})
\]

\[
U(p, p) \geq 0 \quad (\text{IR})
\]

As is standard in the mechanism design literature, when the valuation satisfies supermodularity, the allocation features a monotonicity property.

**Lemma 5.** In an incentive compatible mechanism, \( x(\cdot) \) must be non-decreasing.
By Lemma 5, the supermodularity of the willingness to pay implies that incentive compatibility requires high types receive weakly more generous insurance. The next lemma shows that given the non-decreasing property of the allocation, there exists at most one type that receives the perfect amount of damage coverage.\footnote{Interestingly, this will not be in general the type ‘at the top’, but the type at the ‘kink.’}

**Lemma 6.** In the optimal menu of contracts, $px(p) = 1$ for at most one $p \in \left[\frac{\beta}{\theta}, 1\right]$.

**Proof.** Suppose there exist $p_1 > p_2 > 0$ such that $p_1 x(p_1) = p_2 x(p_2) = 1$. Then, $x(p_1) = \frac{1}{p_1} < \frac{1}{p_2} = x(p_2)$. This contradicts Lemma 5. \hfill \Box

Lemma 6 further shows that at most one type will receive perfect damage coverage. We can now use the envelope theorem and derive a unique payoff function for the optimal allocation, because the set of types for which the derivative of the payoff is not defined has measure zero for all incentive compatible contracts. The next lemma shows that the non-decreasing property of the optimal allocation implies that there must be a threshold type, $\hat{p}$, that is indifferent between settlement and litigation.

**Lemma 7.** Suppose that in the optimal allocation $px(p) > 1$. Then, for $p' > p$ we must have $p' x(p') > 1$.

**Proof.** Suppose that $p' > p$, $px(p) > 1$, and that (by contradiction) $p' x(p') \leq 1$. Then, $p' x(p') < px(p)$. This contradicts that $x(p') \geq x(p)$ in the optimal contract. \hfill \Box

Lemmas 6 and 7 allow us to characterize the optimal contract as a threshold strategy: there exists $\hat{p} \in \left[\frac{\beta}{\theta}, 1\right]$ such that for all types $p \leq \hat{p}$ there is settlement and for types $p > \hat{p}$ there is litigation.

**Assumption 2.** Let $G(p) = p - \frac{1 - F(p)}{f(p)}$ and assume that $G(\cdot)$ crosses zero only once and from below.

The class of distributions that satisfy Assumption 2 is larger than the class of regular distributions (i.e., when $G(\cdot)$ is increasing everywhere). The following Theorem characterizes the optimal menu of contracts offered by a monopolist facing an agent with private information regarding the risk of liability.

**Theorem 3.** For any distribution satisfying Assumption 2, let $\bar{p}$ be the solution to $\bar{p} = \frac{1 - F(\bar{p})}{f(\bar{p})}$. Define $p^*$ as

$$p^* \in \arg \max_{\hat{p} \in [\bar{p}, 1]} \Psi_{A1}(\hat{p}) \equiv (1 - \theta) c_A F(\hat{p}) + (1 - \theta) \int_{\hat{p}}^{\bar{p}} \left[ c_A + \frac{c}{\hat{p}} \left( p - \frac{1 - F(p)}{f(p)} \right) \right] f(p) dp$$
\[-\int_{\tilde{p}}^{1} \left[ \theta(c + c_A) + \frac{c}{\tilde{p}} \left( \frac{1 - F(p)}{f(p)} \right) \right] f(p) dp.\]

The optimal menu of contracts offered by a monopolist insurer consist of (at most) two contracts:
1) \((c_A, 0)\) sold at price \(T(p) = (1 - \theta)c_A\) for types \(p \leq \tilde{p}\);
2) Contract \((c_A, \frac{c_A}{p^*})\) sold at price \(T(p) = (1 - \theta) (c_A + c_A \frac{\tilde{p}}{p^*})\) for types \(p > \tilde{p}\).

First, we find a type \(\tilde{p}\) that partitions types into those with positive and negative virtual surplus. Unlike the standard setting, where the mechanism excludes types with negative surplus, in our setting ‘exclusion’ refers to exclusion from covering damages. The insurer can always offer a contract that only covers litigation costs. Agents are willing to pay the type-independent amount \((1 - \theta)c_A\) to purchase this contract and this does not create information rents. The monopolist sells this contract at price \((1 - \theta)c_A\), and receives a profit of \((1 - \theta)c_A F(\tilde{p})\) from these types. This is the first term in \(\Psi_{AI}(\hat{\tilde{p}})\).

For types above \(\tilde{p}\) the insurer wants to offer a contract that covers damages, which corresponds to the perfect contract for some type \(p^*\). Relative to the perfect provision of insurance, described in Section 3.1, the monopolist’s contract distorts the incentives in two different ways. First, types in \([\tilde{p}, \hat{\tilde{p}}]\) settle but do not extract all the bargaining surplus from the third party because they receive less insurance compared to the first-best. Second, types in \((\hat{\tilde{p}}, 1]\) litigate, which generates a loss of \(\theta(c + c_A)\) in joint surplus between the insurer and the agent. By lowering \(\hat{\tilde{p}}\) the insurer increases the willingness to pay of all agents, but induces litigation for a larger set of types. The optimal damage contract, denoted by \(p^*\), maximizes over this trade-off.

Notice the similarity in the monopolist’s problem under adverse selection (Theorem 3) and under symmetric information (Proposition 3). To satisfy incentive compatibility, the insurer must leave information rents to the agents: in Theorem 3 the agent’s virtual type, \(p - \frac{1 - F(p)}{f(p)}\), replaces the agent’s type \(p\) from Proposition 3. The term \(\frac{1 - F(p)}{f(p)}\) reflects the fact that \(p\) is the agent’s private information. Hence the trade-off in these two results is similar, except now the insurer must consider information rents and the fact that some types are excluded from damages insurance.

Figure 8 illustrates the trade-off when choosing the optimal \(\hat{\tilde{p}}\) in Theorem 3. Area E shows the monopolist’s profit from selling litigation cost insurance (and not damages insurance) to types below \(\tilde{p}\). Area D above area C represents the deadweight loss from excluding these types from damages insurance. Area A’ represents the insurer’s revenue from contract \(\hat{\tilde{p}}\) sold to types in \([\tilde{p}, \hat{\tilde{p}}]\). Area C above area A’ represents the information rents these types obtain. Areas B and F represent the total net loss incurred by the insurer, net of the price paid for insurance by types in \([\hat{\tilde{p}}, 1]\): B is the part of the loss due to litigation, while F is the
information rents that types in \([\hat{p}, 1]\) obtain.

\[
(1 - \theta)(c + c_A)
\]

\[
(1 - \theta) \left( c_A + \frac{\hat{p}}{p} \right)
\]

\[
(1 - \theta)c_A
\]

\[
\hat{p} \bar{p}
\]

Figure 8: The solid area (in blue) represents the gain and the dashed area (in red) represents the losses from contract \(\hat{p} < 1\).

3.4 Litigation Frequency with and without Adverse Selection

Under complete information the first best contract never induces litigation. In contrast, with incomplete information the contracts offered in equilibrium may induce litigation.

First, in a perfectly competitive market it is easy to see that adverse selection induces less litigation than a setting in which there is symmetric information between the agent and the insurer. From Proposition 3, the contract offered by a perfectly competitive market may induce litigation, as shown in Example 1. However, when an equilibrium with adverse selection exists, Proposition 2 shows that the only possibility is a pooling equilibrium at the top of the distribution, i.e., without litigation.

Second, consider a monopolist insurer. To compare the level of litigation with symmetric and asymmetric information, we need to compare the solution to the problem in Proposition 3 and Theorem 3. Denote by \(p_{SI}^*\) the optimal contract in Proposition 3 and let \(p_{AI}^*\) the optimal contract in Theorem 3. We can show that \(p_{SI}^* \leq p_{AI}^*\).\(^{17}\)

Proposition 6. Under Assumption 2, the monopoly contract with symmetric information induces weakly more litigation than the contract under asymmetric information.

The intuition for Proposition 6 can be illustrated by the monopolist’s trade-off when choosing \(p^*\) under private information. Consider the choice of \(p^* = \hat{p} < 1\) versus \(p^* = 1\) illustrated in Figure 9.

\(^{17}\)This inequality is in the strong set order when the solutions fail to be unique.
Relative to a monopolist under symmetric information (Figure 6) the gains relative to the losses are smaller when the monopolist faces adverse selection. The gain from deviating to $\hat{p} < 1$ is smaller under adverse selection because only types above $\bar{p}$ receive damages insurance, and also for all $p > \bar{p}$ we have that $W(p, p^*) - W(p, 1) > W(\bar{p}, p^*) - W(\bar{p}, 1)$. The losses for the monopolist facing adverse selection are also higher than a monopolist selling insurance under symmetric information because of the information rents given to types that litigate. Therefore, compared to the case of symmetric information, a monopolist selling insurance to privately informed types obtains smaller gains and larger losses when deviating from $p^* = 1$ to $p^* < 1$.

Proposition 6 and Lemma 2 show that the amount of litigation in equilibrium increases when the insurer and the agent are uninformed, as illustrated in Figure 10. The ranking of the equilibrium level of litigation on the level of information is the same under perfect competition and monopoly.

Figure 10: Equilibrium amount of litigation depending on the information structure.

4 A Product Quality Interpretation

The monopoly problems we study include important features that relate to product quality choice, which have been studied, for example, by Spence (1975, 1976). We focus on damages coverage, because a monopolist always covers litigation costs.
the quality of insurance for the agent, but increases the insurer’s costs by inducing more litigation. Under this interpretation, Proposition 6 shows that the quality of insurance sold is higher under symmetric information than under asymmetric information.

To see the intuition, consider Figure 11, which sorts agents from the highest to the lowest willingness to pay (the x-axis is the probability that liability is absent, $1 - p$), and shows inverse demand and marginal cost for contracts $p^* = 1$ and $p^* = \hat{p}$. When $p^* = 1$, demand is linear and marginal cost is zero (every agent settles). If $p^* = \hat{p}$, the willingness-to-pay of each agent is higher, and moreso for agents with higher willingness to pay. The demand curve has a kink at $1 - p = 1 - \hat{p}$, because willingness to pay for agents that choose to litigate rises faster than for agents that settle. In Figure 11, the insurer’s marginal cost is $c_A + c_p \hat{p}$ for agents that litigate ($1 - p < 1 - \hat{p}$), and zero for agents that settle (i.e., $1 - p \geq 1 - \hat{p}$).

![Figure 11: Demand Curve and Marginal Cost for contracts $p^* = 1$ and $p^* = \hat{p} < 1$.](image)

Under symmetric information, the marginal effect of insurance quality on revenue averages over all marginal shifts in willingness-to-pay (i.e., the entire demand curve in Figure 11). This is the so-called 'average marginal' effect of product quality. Under asymmetric information, in contrast, the marginal effect of insurance quality on revenue is the shift in willingness-to-pay of the marginal buyer only. This is the so-called 'marginal marginal' effect of product quality. Because the marginal effect of product quality is higher for consumers with higher willingness to pay, the 'average marginal' is higher.

The optimal $p^*$ (in both settings) balances the marginal effect of insurance quality on revenue versus the marginal effect of insurance quality on costs. Importantly, quality choice does not affect output. With symmetric information, the agent does not know its type, so all types $p \geq 0$ are sold insurance regardless of $p^*$. With asymmetric information, some agents choose not to buy damages insurance (types below $\bar{p}$ in Theorem 3), but the type that is indifferent between buying damages insurance or not (type $\bar{p}$) depends only on the distribution of types and not on the choice of quality $p^*$. Hence, because the cost of insurance increases in quality.

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and the "average marginal" is higher than the 'marginal marginal," insurance quality is higher under symmetric information.

5 Conclusion

Liability insurance markets are pervasive and not well understood. We contribute to the literature by showing that third-party equilibrium insurance contracts are quite different from first-party insurance contracts, both in a perfectly competitive market and in a monopolistic setting. In a perfectly competitive market for third-party insurance only a pooling equilibrium can exist, in contrast to Rothschild and Stiglitz (1976) where only a separating equilibrium can exist. Separating equilibria do not exist in our setting because in such an equilibrium at least one contract would attract both types that settle and types that litigate. Types that settle impose no cost to the insurer and can be “cream skimmed” by offering an alternative contract. A pooling equilibrium, when it exists, delivers imperfect insurance to all but the highest type. Crucially, in third-party insurance, the insurer’s cost function features a discontinuity because of the costly ex-post verification of liability—the agent’s choice to settle or to litigate—in contrast to the first-party insurance setting (Azevedo and Gottlieb, 2017).

With a monopolist insurer, the optimal contract is qualitatively different from first party insurance studied by Stiglitz (1977) and Chade and Schlee (2012). First, the optimal contract may distort types “at the top”—for some distributions, only an interior type gets perfect insurance—who pursue inefficient litigation. Second, our result differs from the classic discriminating monopolist problem under private information (Mussa and Rosen, 1978). Given the particular characteristics of the insurer’s cost function and the willingness to pay of the agent, there are points of non-differentiability that affect the shape of the optimal contract (Carbajal and Ely, 2013).

In addition to our characterizations of equilibria under different market structures, we compare equilibrium contracts in the cases of symmetric and asymmetric information. We show that in both competition and monopoly, equilibria with symmetrically uninformed parties feature more generous coverage and induce more litigation, compared to equilibria where the agent is privately informed about the probability of liability.

Our setting of risk-neutral agents, and bargaining under perfect information, captures key elements of markets for liability insurance in an analytically tractable way. Of course, some markets may have different features. In an Online Appendix, we consider the following extensions: an application of our results to the classical two-types setting; a setting where the insurer can use contracts that cover settlement transfers; a setting with a risk averse agent; a setting where settlement negotiations happen under incomplete information, with
the uninformed (third) party making a settlement offer; and a setting where control over the decision of whether to settle or litigate is endogenously allocated. The Appendix shows that our main results extend beyond our basic setting.

6 References


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A Appendix: Proofs

Proof of Lemma 1

Proof. The payoff of settlement and litigation for an agent covered by insurance policy \( \alpha = (\alpha_L, \alpha_D) \) are given, respectively, by

\[
\begin{align*}
V_S(p, \alpha) &= -c_A - pd + \theta(c + c_A) + (1 - \theta)(\alpha_L + p\alpha_D), \\
V_L(p, \alpha) &= -c_A - pd + \alpha_L + p\alpha_D.
\end{align*}
\]

Without insurance \( (\alpha = 0) \) the agent settles. The willingness to pay for insurance is then

\[
W(p, \alpha) = \begin{cases} 
V_S(p, \alpha) - V_S(p, 0) & \text{if } p \leq p^* \\
V_L(p, \alpha) - V_S(p, 0) & \text{if } p > p^*
\end{cases}
\]

Notice that \( \alpha_L + \alpha_D p^* = c + c_A \), so we can write \( \alpha_L + p\alpha_D = c + c_A + (p - p^*)\alpha_D \). From these expressions the lemma follows. \( \square \)

Proof of Proposition 1

Proof. Consider a contract \( \alpha = (\alpha_L, \alpha_D) \), with \( \alpha_L < c_A \). Let \( p^* \equiv p^*(\alpha) \) denote the type that is indifferent between settlement and litigation. Consider a contract \( \alpha' = (c_A, \alpha_D') \) such that \( p^*(\alpha') = p^* \). By construction this contract leaves the same type \( p^* \) indifferent, and clearly \( \alpha_D' < \alpha_D \). By Lemma 1, under \( \alpha' \) the willingness to pay for types \( p < p^* \) increases, while for \( p > p^* \) it decreases. By Corollary 1, for \( p > p^* \) the difference between cost and willingness to pay is constant and independent of the contract, \( K(p, \alpha) - W(p, \alpha) = \theta(c + c_A) \). Hence the insurer’s net surplus, evaluated type-by-type, is larger in \( \alpha' \) than in \( \alpha \). Moreover, if the agent has private information regarding \( p \), the reduction in the willingness to pay for high-risk types under the contract \( \alpha' \) implies that fewer types \( p > p^* \) are willing to buy insurance, for a given price, compared to the original contract \( \alpha \). This is good for the insurer since it reduces losses. Therefore, \( \alpha' \) weakly dominates \( \alpha \) from the perspective of the insurer. \( \square \)
Proof of Corollary 2

Proof. Consider \( p' > p \). Let \( g(p^*) = W(p', p^*) - W(p, p^*) \). Then, we have:

\[
g(1 - p^*) = c \frac{(p' - p)}{p^*} = \begin{cases} 
0 & p^* < p \\
\frac{c}{p^*} - \theta (p' - p) & p \leq p^* < p' \\
\frac{c}{p^*} & p^* \geq p'
\end{cases}
\]

It is easy to see that \( g(p^*) \) is decreasing in \( p^* \). Therefore, \( \tilde{g}(p^*) = g(1 - p^*) \) is increasing in \( p^* \) which implies that \( \tilde{W} \) is supermodular.

Proof of Proposition 3

Proof. Replace the expressions from equations (6) and (7) in equation (8) to get

\[
W(p, p^*) - K(p, p^*) = \begin{cases} 
0 & p < \frac{c}{a} \\
(1 - \theta) \left[ c_A + c \frac{p}{p^*} \right] & p \leq p^* \\
-\theta (c + c_A) & p > p^*
\end{cases}
\]

Taking expected value over \( p \) we get the expression in the proposition.

Proof of Lemma 2

Proof. \( p^* \neq \hat{p}' > 1 \) and \( p^* \neq \frac{c}{a} \) because \( \Psi_{SI}(\hat{p}') < \Psi_{SI}(1) \) and \( \Psi_{SI}(\frac{c}{a}) < \Psi_{SI}(1) \). With a continuous distribution \( F(\cdot) \), the objective function is continuous, so a maximum exists (not necessarily unique). With a continuous density, the derivative of the \( \Psi_{SI}(\cdot) \) is also continuous. If there are multiple solutions, then at least one must be an interior local maximum. The density \( f(\cdot) \) is differentiable because \( F \) is twice differentiable, so the first and second order conditions imply

\[
(c + c_A)f'(p^*) + \frac{f(p^*)}{p^*} [2c_A + (1 + \theta)c] < 0. \tag{13}
\]

Then, if for all \( p^* \) condition (13) is violated, we can guarantee that the solution of the problem is \( p^* = 1 \) because in that case there is no interior local maximum of \( \Psi(\cdot) \). Hence, since a solution must exist, it must be that \( p^* = 1 \).
Proof of Lemma 3

Proof. Suppose \( p_1 < p_2 < 1 \) are two points satisfying the FOC, \( \Psi'_SI(p_i) = 0 \), and the SOC, \( \Psi''SI(p_i) < 0 \). We have \( p_i > \frac{c}{\theta} \) because \( \Psi'_SI\left(\frac{c}{\theta}\right) > 0 \). Then, by continuity of \( \Psi' \), there exists \( \xi \in (p_1, p_2) \) such that \( \Psi'_SI(\xi) = 0 \) and \( \Psi''SI(\xi) > 0 \), which implies

\[
(c + c_A)f'(\xi) + \frac{f(\xi)}{\xi}[2c_A + (1 + \theta)c] > 0 \iff \eta(\xi) > -1 - \frac{c_A + \theta c}{c_A + c}.
\]

If this condition does not hold, the existence of both \( p_1 \) and \( p_2 \) is a contradiction.

Proof of Lemma 4

Proof. By Topkis’ monotonicity theorem, \( \frac{\partial^2 \Psi_{SI}}{\partial \hat{p} \partial \eta} \geq 0 \Rightarrow p^*(\cdot) \) non-decreasing in \( \eta \). It is easy to show that \( \frac{\partial^2 \Psi_{SI}}{\partial \hat{p} \partial \eta} > 0 \), and \( \frac{\partial^2 \Psi_{SI}}{\partial \hat{p} \partial d} < 0 \). We have \( \frac{\partial^2 \Psi_{SI}}{\partial \hat{p} \partial c}(p^*) = f(p^*) - \frac{(1-\theta)}{(p^*)^2} \left[ \int_c^{p^*} pf(p)dp - \left(\frac{c}{\theta}\right)^2 f\left(\frac{c}{\theta}\right) \right] \). As \( p^* \to \frac{c}{\theta} \), \( \frac{\partial^2 \Psi_{SI}}{\partial \hat{p} \partial c} \to \theta f\left(\frac{c}{\theta}\right) > 0 \). Moreover, \( \frac{\partial^2 \Psi_{SI}}{\partial \hat{p} \partial c} \) is increasing if \( \eta(p) \geq -1 \).

Proof of Proposition 4

Proof. Consider a distribution of types \( F \sim [0, 1] \). If \( F(p^*) < 1 \), we will show that the contract \( p^* \) cannot be offered in equilibrium in a perfectly competitive market. Suppose \( p^* \) is offered in equilibrium at price \( P \). Since \( F(p^*) < 1 \), then there is a positive mass of types that litigate, for which the insurer incur losses (Corollary 1). To break even in equilibrium, insurers must be selling this contract at a positive price \( P > 0 \). Consider an alternative contract \( \tilde{p} = p^* + \varepsilon \) sold at price \( \tilde{P} \), with \( \varepsilon \) sufficiently small. This new contract offers a lower damages coverage, is cheaper, and preferred by types \( p < \tilde{p} \) over contract \( p^* \) and not preferred for types \( p > \tilde{p} \) as long as \( W(p, p^*) - P < W(p, \tilde{p}) - \tilde{P} \), for all \( p < \tilde{p} \) and \( W(p, p^*) - P > W(p, \tilde{p}) - \tilde{P} \), for all \( p > \tilde{p} \). By Corollary 2, these conditions are satisfied as long as \( \tilde{P} = P + W(\tilde{p}, \tilde{p}) - W(\tilde{p}, p^*) = P - \frac{c}{\theta} \varepsilon \). Thus, for \( \varepsilon \) small enough, contract \( \tilde{p} \) sold at price \( \tilde{P} = P - \frac{\varepsilon}{\theta} > 0 \) only attracts types that settle and it is sold at a positive price, so it is a profitable deviation from selling \( p^* \).
Proof of Theorem 1

Proof. We show it by contradiction. Let $\mathcal{M}$ be the set of contracts offered in equilibrium. In a separating equilibrium, at least two of these contracts must attract a different set of types. Let $p_1^*$ and $p_2^*$ with $p_1^* < p_2^*$, sold at prices $P_1$ and $P_2$, respectively, be such a pair of contracts. Let $D_i \subseteq [0,1]$ the set of types that prefer contract $p_i^*$,

$$D_i = \left\{ p \in \left[ \frac{c}{d}, 1 \right] : W(p, p_i^*) - P_i \geq W(p, p_j^*) - P_j, \text{ for all } p_j^* \in \mathcal{M} \right\}.$$

Let $D_i(S) = D_i \cap [0, p_i^*]$ and $D_i(L) = D_i \cap (p_i^*, 1]$ be the set of types that buy contract $p_i^*$ and that settle and litigate, respectively. If the measure of the set $D_i(L)$ is zero, then $P_i = 0$, since the insurer would not bear any costs by offering $p_i^*$. But it cannot be that $D_1(L)$ and $D_2(L)$ have both measure zero, since they would be sold at price zero and by Corollary 2, types would pool at $p_1^*$ (see Figure 4). This rules out separating equilibrium with any pair of contracts such that litigation is precluded under both, because such a pair would need to be priced at zero in equilibrium and types would pool at the lowest $p_i^*$. So, in any separating equilibrium we must have a positive measure of $D_i(L) > 0$ for some $i \in \{1, 2\}$. Without loss of generality, suppose that $\mu_F(D_1(L)) > 0$. Notice that if $\mu_F(D_1(S)) = 0$, then by Corollary 1 insurers incur losses by selling this contract. Thus, contract $p_1^*$ must attract types that settle and must sell at a positive price $P_1 > 0$. Consider a new contract $\tilde{p}_1 = p^* + \epsilon$ sold at price $\tilde{P}$ as in Proposition 4 to build a profitable deviation from $p_1^*$—by construction, this deviation only attracts types that settle. This profitable deviation implies that $p_1^*$ cannot be offered in equilibrium, because then $p_1^*$ would only attract types that litigate (it would be a money loser). This is a contradiction. \qed

Proof of Theorem 2

Proof. By Proposition 4, there is no pooling equilibrium at $p^*$ such that $F(p^*) < 1$. Hence, the only candidate is $p^*$ such that $F(p^*) = 1$.

A contract $\tilde{p}$ sold a price $\tilde{P}$ is a profitable deviation if attracts enough low-risk types that settle but pay a positive price to compensate the loss of selling insurance to high-risk types that litigate and generate losses for the insurer. Let $\tilde{p}$ the (unique by single crossing) type that is indifferent between $\tilde{p}$ at price $\tilde{P}$ and $p^*$ for free. Then,

$$W(\tilde{p}, p^*) = W(\tilde{p}, \tilde{p}) - \tilde{P} \Rightarrow \tilde{P} = \tilde{p} \left[ \frac{(1 - \theta)c \cdot (p^* - \tilde{p})}{\tilde{p}p^*} \right]$$

Next, we only consider contracts such that $\tilde{p} > \tilde{p}$. In any other case, the insurer loses money.
by offering the deviation. Then, the profit of contract \( \tilde{p} \) at price \( \tilde{P} \) is given by

\[
\tilde{P}[1 - F(\tilde{p})] - \int_{\tilde{p}}^{1} K(p, \tilde{p})dF(p) = \tilde{P}[1 - F(\tilde{p})] - \int_{\tilde{p}}^{1} \left[ c_A + \frac{cp}{\tilde{p}} \right] dF(p)
\]

We can choose the best cutoff point \( \tilde{p} \) for a given \( \tilde{p} \) and then choose the best deviation \( \tilde{p} \). Hence, there is no profitable deviation when the condition in the Theorem holds.

\[\square\]

**Proof of Proposition 5**

**Proof.** Without loss of generality, suppose that \( F(p^*) = 1 \) implies that \( p^* = 1 \). If \( p^* = 1 \) is optimal under symmetric information, then for any \( \tilde{p} \in \left( \frac{\epsilon}{\alpha}, 1 \right) \), we have

\[
\int_{\frac{\epsilon}{\alpha}}^{\tilde{p}} (1 - \theta) \left( c_A + \frac{cp}{\tilde{p}} \right) dF(p) - \int_{\tilde{p}}^{1} \theta \left( c_A + c \right) dF(p) < \int_{\frac{\epsilon}{\alpha}}^{1} (1 - \theta) \left( c_A + cp \right) dF(p).
\]

This implies that

\[
\int_{\frac{\epsilon}{\alpha}}^{\tilde{p}} (1 - \theta)cp \left( \frac{1 - \tilde{p}}{\tilde{p}} \right) dF(p) - \int_{\tilde{p}}^{1} \left\{ c_A + c \left[ \theta + (1 - \theta)p \right] \right\} dF(p) < 0 \tag{14}
\]

for any \( \tilde{p} \). To establish that a pooling equilibrium exists with \( p^* = 1 \) under competition, we need to show that there are no \( \tilde{p} \) and \( \tilde{p} \) such that alternative insurance \( \tilde{p} \) sold for price \( \tilde{P}(\tilde{p}) = (1 - \theta)cp \left( \frac{1 - \tilde{p}}{\tilde{p}} \right) \) attracts all types \( p > \tilde{p} \) and yields a profit. Hence, we must show that

\[
\int_{\tilde{p}}^{1} (1 - \theta)cp \left( \frac{1 - \tilde{p}}{\tilde{p}} \right) dF(p) - \int_{\tilde{p}}^{1} \left( c_A + \frac{cp}{\tilde{p}} \right) < 0
\]

for all \( \tilde{p} \) and \( \tilde{p} \). Let \( \tilde{p} \) maximize this expression conditional on \( \tilde{p} \) and rewrite the expression as

\[
\int_{\tilde{p}}^{\tilde{p}} (1 - \theta)cp \left( \frac{1 - \tilde{p}}{\tilde{p}} \right) dF(p) - \int_{\tilde{p}}^{1} \left\{ c_A + c \left[ \frac{p - (1 - \theta)\tilde{p}(1 - \tilde{p})}{\tilde{p}} \right] \right\} dF(p) < 0. \tag{15}
\]

Because \( \frac{\epsilon}{\alpha} \leq \tilde{p} \leq \tilde{p} < 1 \), it is obvious that

\[
\int_{\tilde{p}}^{\tilde{p}} (1 - \theta)cp \left( \frac{1 - \tilde{p}}{\tilde{p}} \right) dF(p) \leq \int_{\tilde{p}}^{\tilde{p}} (1 - \theta)cp \left( \frac{1 - \tilde{p}}{\tilde{p}} \right) dF(p)
\]

for any \( \tilde{p} \) and \( \tilde{p} \). Thus, the first term in (15) is smaller than the first term in (14). It remains to show that

\[
\int_{\tilde{p}}^{1} \left\{ c_A + c \left[ \frac{p - (1 - \theta)\tilde{p}(1 - \tilde{p})}{\tilde{p}} \right] \right\} dF(p) \geq \int_{\tilde{p}}^{1} \left\{ c_A + c \left[ \theta + (1 - \theta)p \right] \right\} dF(p).
\]
This holds as long as
\[
\frac{p - (1 - \theta)\bar{p}(1 - \bar{p})}{\bar{p}} \geq (1 - \theta)p + \theta
\]
for all \( p > \bar{p} \geq \hat{p} \). This inequality is equivalent to
\[
p \geq (1 - \theta) \left[ \bar{p}p + (1 - \bar{p})\bar{p} \right] + \theta \bar{p}
\]
The RHS is a convex combination of points strictly lower than \( p \), so this inequality always hold (and it is strict). Hence, for any \( \bar{p} \), the left-hand side of (15) is lower than the left-hand side of (14). Thus, whenever \( p^* = 1 \) in the problem with symmetric information, there is no profitable deviation from \( p^* = 1 \) and a pooling equilibrium exists.

Proof of Lemma 5

Proof. Consider \( p_1 > p_2 \). Combining the incentive compatibility constraints we get:
\[
W(p_1, x(p_1)) - W(p_2, x(p_1)) \geq W(p_1, x(p_2)) - W(p_2, x(p_2)).
\]
Let \( g(x) = W(p_1, x) - W(p_2, x) \). It is easy to see (Corollary 2) that \( g(\cdot) \) is an strictly increasing function. Therefore, incentive compatibility is equivalent to \( x(\cdot) \) increasing.

Proof of Theorem 3

Proof. Consider a direct revelation mechanism: \( p \rightarrow (x(p), T(p)) \), where \( x(\cdot) \) and \( T(\cdot) \) are the allocation and price for an agent who reports type \( p \). The insurer chooses \( x(\cdot) \) and \( T(\cdot) \) to solve:
\[
\max_{T(\cdot), x(\cdot)} \int_c^d T(p) dF(p) - \int_{\{p; px(p) > 1\}} [c_A + cpx(p)]dF(p)
\]
subject to \( p \in \arg\max_{p'} \left\{ \hat{W}(p, x(p')) - T(p') \right\} \). Let \( V(p) = \max_{p'} u(p, p') \). By the envelope theorem and incentive compatibility we have:
\[
V'(p) = \begin{cases} 
(1 - \theta)cx(p) & px(p) < 1 \\
x(p) & px(p) > 1 
\end{cases}
\]
By Lemma 5, \( x(\cdot) \) must be weakly increasing for incentive compatibility. Hence \( px(p) \) is strictly increasing when \( x(p) > 0 \) and therefore there exists a unique type \( \hat{p} \) such that \( px(p) > 1 \) for all \( p > \hat{p} \) and \( px(p) \leq 1 \) for all \( p \leq \hat{p} \) (it may be that \( \hat{p} = 1 \)). For \( p \leq \hat{p} \), \( V(p) = \)
Incentive compatibility requires holds under the notion of strong set order. Proposition 3, i.e., $p > \hat{p}$, $V(p) = V(c/d) + \int_{c/d}^{\hat{p}} (1 - \theta) cx(s) ds + \int_{\hat{p}}^{p} cx(s) ds$. For $p > \hat{p}$, $V(p) = V(c/d) + \int_{c/d}^{\hat{p}} (1 - \theta) cx(s) ds + \int_{\hat{p}}^{p} cx(s) ds$. Incentive compatibility requires $V(p) = u(p, p)$, so for $p \leq \hat{p}$,

$$T(p) = (1 - \theta)(cpx(p) + c_A) - V(c/d) - \int_{c/d}^{\hat{p}} (1 - \theta) cx(s) ds$$

and for $p > \hat{p}$,

$$T(p) = cpx(p) + c_A - \theta(c + c_A) - V(c/d) - \int_{c/d}^{\hat{p}} (1 - \theta) cx(s) ds - \int_{\hat{p}}^{p} cx(s) ds$$

It is optimal for the insurer to set $V(c/d) = 0$. Following standard algebra from mechanism design, we can re-write the problem as:

$$\max_{x(\cdot)} \int_{c/d}^{\hat{p}} \left[ (1 - \theta) cx(p) \left( p - \frac{1 - F(p)}{f(p)} \right) \right] dF(p) - \int_{\hat{p}}^{1} \left[ cx(p) \left( \frac{1 - F(p)}{f(p)} \right) \right] dF(p) +$$

$$+ \int_{c/d}^{\hat{p}} c_A dF(p) - \int_{\hat{p}}^{1} \theta cdF(p) - \theta c_A.$$ The final three terms do not depend on $x(\cdot)$. Let $\bar{p}$ such that $\bar{p} = \frac{1 - F(\bar{p})}{f(\bar{p})}$. In the optimal mechanism, we must have $x(\bar{p}) = \frac{1}{\bar{p}}$. For $p > \hat{p}$, the objective function is decreasing in $x(p)$, and given that $x(p)$ is weakly increasing it is optimal to set $x(p) = x(\hat{p})$. For $p \leq \hat{p}$ there are two cases: 1) If $p \leq \bar{p}$, we set $x(p) = 0$, which does not restrict the monotonicity condition for higher values of $p$; 2) If $\bar{p} < p \leq \hat{p}$, we would like to make $x(p)$ as large as possible. However, since incentive compatibility imposes that $x(p)$ must be weakly increasing and $x(\hat{p}) = \frac{1}{\hat{p}}$, the best the insurer can do is to set $x(p) = x(\hat{p})$. Finally, if $\hat{p} < \bar{p}$ we would set $x(p) = 0$ for all $p$. It is easy to see that setting $\hat{p} < \bar{p}$ is not optimal. Then, to satisfy incentive compatibility, the optimal contract we must have: $x(p) = 0$ for $p \leq \bar{p}$ and $x(p) = \frac{1}{\bar{p}}$ for $p > \bar{p}$. The insurer chooses $\hat{p}$ according to the expression in the theorem.

\textbf{Proof of Proposition 6}

\textit{Proof.} Denote by $p^*_S$ the optimal contract in Proposition 3 and let $p^*_AS$ the optimal contract in Theorem 3.\textsuperscript{19} Denote by $\bar{p}$ the solution to $\bar{p} = \frac{1 - F(\bar{p})}{f(\bar{p})}$, and let $H_S$ the objective function in Proposition 3, i.e.,

$$\Psi_{SI}(\hat{p}) = (1 - \theta) \int_{c/d}^{\hat{p}} \left[ c_A + \frac{cp}{\hat{p}} \right] dF(p) - \theta(c + c_A)[1 - F(\hat{p})].$$

\textsuperscript{19}For simplicity, we can assume that the solution of each of these problems is unique. If not, our conclusion holds under the notion of strong set order.
Notice that \( p_{SI}^* \) belongs to the interval \( \left[ \frac{c}{d}, 1 \right] \) and, with a regular distribution, \( \bar{p} \leq p_{SI}^* \bar{p} \). Thus, whenever \( p_{SI}^* \leq \bar{p} \) we have \( p_{SI}^* \leq p_{AI}^* \).

Consider the case \( p_{SI}^* \geq \bar{p} \). Then,

\[
p_{SI}^* \in \arg \max_{\hat{p} \in [\frac{c}{d}, \infty]} \Psi_{SI}(\hat{p}) = \arg \max_{\hat{p} \in [\bar{p}, \infty]} \Psi_{SI}(\hat{p}).
\]

It is easy to see that the objective function in Theorem 3 can be written as \( \Psi_{AI}(\hat{p}) = \Psi_{SI}(\hat{p}) - \Delta(\hat{p}) \), where

\[
\Delta(\hat{p}) = \frac{1 - \theta}{\hat{p}} \int_{c/d}^{\bar{p}} p f(p) dp + \frac{(1 - \theta) c}{\hat{p}} \int_{\bar{p}}^{\bar{p}} (1 - F(p)) dp + \frac{c}{\hat{p}} \int_{\bar{p}}^{1} (1 - F(p)) dp.
\]

Consider the problem

\[
p^*(\beta) = \arg \max_{\hat{p} \in [\bar{p}, \infty]} H_S(\hat{p}) - \beta \Delta(\hat{p}),
\]

so \( p^*(0) = p_{SI}^* \) and \( p^*(1) = p_{AI}^* \). By Topkis theorem, when \( \Delta'(\hat{p}) < 0 \) for all \( \hat{p} \) we have \( p^*(0) \leq p^*(1) \). Notice that

\[
\Delta(\hat{p}) = \frac{(1 - \theta) c}{\hat{p}} \left[ \int_{c/d}^{\bar{p}} p f(p) dp + \int_{\bar{p}}^{1} (1 - F(p)) dp \right] + \theta \frac{c}{\hat{p}} \int_{\bar{p}}^{1} (1 - F(p)) dp.
\]

Denote by \( A \) the expression in the bracket, which is independent of \( \hat{p} \). Then, taking derivative we get

\[
\Delta'(\hat{p}) = -\frac{c}{\hat{p}^2} \left[ (1 - \theta) A + \theta \int_{\bar{p}}^{1} (1 - F(p)) dp \right] - \theta \frac{c}{\hat{p}} (1 - F(\hat{p})) < 0.
\]
Appendix:
Intended For Online Publication

Liability Insurance: Equilibrium Contracts under Monopoly and Competition

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B Online Appendix

B.1 Optimal Contracts in the Two-Types Case

Suppose that a single insurer serves the market as a monopolist, and the agent’s type is \( p \in \{p_L, p_H\} \), as in Definition 2. We next characterize the optimal contract in the symmetric information setting, where the agent and the insurer both do not know \( p \), and the optimal menu of contracts in the private information setting, where the agent privately knows \( p \).

B.1.1 The Two-Types Case with Symmetric Information

Consider the case with symmetric information. The policy \( p^* = \infty \) only covers litigation costs. With this policy all the agents settle and the expected profit for a monopolist is \( \pi_\infty = (1 - \theta) c_A \). This corresponds to the fraction of bargaining surplus that agents are able to capture from their improved bargaining position.

The policy \( p^* = p_H \) is perfect for type \( p_H \) but imperfect for type \( p_L \). By Proposition 3, the profit of this policy is

\[
\pi_H = (1 - \theta) \left[ c_A + c \frac{p_L}{p_H} \right] (1 - \lambda) + (1 - \theta) (c + c_A) \lambda,
\]

which corresponds to the maximal surplus for type \( p_H \) (the second term), plus the maximal surplus for type \( p_L \) minus a reduction from imperfect coverage (\( p_H \)) to types \( p_L \), leading to inefficient settlement (the first term).

Policy \( p^* = p_L \) is perfect for type \( p_L \). There are no lower types, so there is no inefficient settlement. However, types above \( p_L \) are engaging in litigation and therefore losing \( \theta (c + c_A) \), the bargaining surplus they used to capture in the settlement negotiation. By Proposition 3, the profit of this policy is then

\[
\pi_L = (1 - \theta)(c + c_A)(1 - \lambda) - \theta (c + c_A) \lambda.
\]

The first term is the maximal bargaining surplus captured by types \( p_L \), and the second term is the bargaining surplus lost by types \( p_H \) because they litigate instead of settling.

We can see that \( \pi_H > \pi_\infty \) so that contract is dominated. The optimal contract will now depend on the relative mass of high-risk types.

**Corollary 4.** For the two-types case, there exists a threshold \( \lambda_{Lit}^{SI} \) such that for \( \lambda \geq \lambda_{Lit}^{SI} \) the optimal contract \( p^* = p_H \) precludes litigation and for \( \lambda < \lambda_{Lit}^{SI} \) the optimal contract \( p^* = p_L \).
induces litigation by \( p^H \) types.

**Proof.** Writing \( \pi_H \) and \( \pi_L \) as a function of \( \lambda \) we find that \( \pi_H \) increases and \( \pi_L \) decreases. We can find \( \lambda_{SI}^{Lit} \) such that \( \pi_H(\lambda_{SI}^{Lit}) = \pi_L(\lambda_{SI}^{Lit}) \) where

\[
\lambda_{SI}^{Lit} = \frac{(1 - \theta) c(p_H - p_L)}{p_H (c + c_A) + (1 - \theta) c(p_H - p_L)}.
\]

\[\square\]

### B.1.2 The Two-Types Case with Asymmetric Information

Consider the case with asymmetric information, where the agent privately learns \( p \). The optimal menu of contracts consists of at most two contracts from the set \( \{p_L, p_H, \infty\} \). Denote the menu of contracts by \((p_1, p_2)\), where contract \( p_1 \) is selected by type \( p_L \) and contract \( p_2 \) is selected by type \( p_H \).

First, notice that it is never optimal for the insurer to completely exclude type \( p_L \)—the insurer could serve the low type with a contract \( p_1 = \infty \), which does not introduce any information rents for the high type, and the insurer can charge the low type \( (1 - \theta) c_A \). Hence any menu that excludes the low type is dominated by a menu where we add the \( p_1 = \infty \) contract.

Second, notice that menus such that \( p_1 < p_2 \), and type \( p_L \) takes \( p_1 \) while \( p_H \) takes \( p_2 \) are not incentive compatible, by the observation in Corrolary 2 that higher types have increasingly higher valuations for more generous contracts, i.e. contracts with lower \( p^* \). The candidate menus of contracts are therefore \((\infty, p_H)\), \((p_L, p_L)\), and \((p_H, p_H)\). These correspond to the 3 interesting cases of Theorem 3: the menu \((\infty, p_H)\) is where \( p_L < \bar{p} \leq p_H = p^* \); the menu \((p_L, p_L)\) is where \( \bar{p} \leq p_L = p^* < p_H \); the menu \((p_H, p_H)\) is where \( \bar{p} \leq p_L < p_H = p^* \).

Consider the menu with \( p_1 = \infty \) and \( p_2 = p_H \). The optimal price of \( p_1 \) is clearly \( (1 - \theta) c_A \), and for type \( p_H \) to buy the contract \( p_2 = p_H \), the price of the latter must be \( (1 - \theta)(c_A + c) \). The insurer’s total profit is therefore

\[
\pi_{(\infty, p_H)} = (1 - \theta) c_A (1 - \lambda) + (1 - \theta)(c + c_A) \lambda.
\]

Consider the menu with \( p_1 = p_2 = p_L \). Clearly there is a single optimal price in this case, equal to \( (1 - \theta)(c + c_A) \), and the contract induces type \( p_H \) to litigate. The insurer’s profit is

\[
\pi_{(p_L, p_L)} = (1 - \theta)(c + c_A) - (c_A + c \frac{p_H}{p_L}) \lambda.
\]
Consider the menu with \( p_1 = p_2 = p_H \). Clearly there is a single optimal price in this case, equal to \((1 - \theta)(c_{PL}/p_H + c_A)\), and the contract induces both types to settle. The insurer’s profit is
\[
\pi_{(p_H,p_H)} = (1 - \theta)(c_{PL}/p_H + c_A).
\]

**Corollary 5.** For the two-types case, there exist three thresholds, \( \lambda_{AI}^1 = \frac{c(1-\theta)}{c + c A + c_{PL}/p_H} \), \( \lambda_{AI}^2 = \frac{(1 - \frac{p_L}{p_H})c(1-\theta)}{c_A + c_{PL}/p_L} \), and \( \lambda_{AI}^3 = \frac{p_L}{p_H} \), which are ordered either as \( \lambda_{AI}^2 \leq \lambda_{AI}^1 \leq \lambda_{AI}^3 \), or \( \lambda_{AI}^3 \leq \lambda_{AI}^1 \leq \lambda_{AI}^2 \). The optimal menu is then:

1. \((p_1, p_2) = (\infty, p_H)\) at prices \((1 - \theta)c_A\) and \((1 - \theta)(c + c_A)\), if \( \lambda \geq \lambda_{AI}^3 \) and \( \lambda \geq \lambda_{AI}^1 \).
2. \((p_1, p_2) = (p_L, p_L)\) at price \((1 - \theta)(c + c_A)\), if \( \lambda \leq \lambda_{AI}^2 \) and \( \lambda \leq \lambda_{AI}^1 \).
3. \((p_1, p_2) = (p_H, p_H)\) at price \((1 - \theta)(c_{PL}/p_H + c_A)\), if \( \lambda \geq \lambda_{AI}^3 \) and \( \lambda \leq \lambda_{AI}^2 \).

**Proof.** With some algebra, one can show the following:

\[
\pi_{(\infty,p_H)} \geq \pi_{(p_L,p_L)} \iff \lambda \geq \frac{c(1-\theta)}{c(1-\theta) + c_A + c_{PL}/p_H}
\]
\[
\pi_{(p_L,p_L)} \geq \pi_{(p_H,p_H)} \iff \lambda \leq \frac{(1 - \frac{p_L}{p_H})c(1-\theta)}{c_A + c_{PL}/p_L}
\]
\[
\pi_{(\infty,p_H)} \geq \pi_{(p_H,p_H)} \iff \lambda \geq \frac{p_L}{p_H}
\]

These 3 inequalities define \( \lambda_{AI}^1, \lambda_{AI}^2, \) and \( \lambda_{AI}^3 \), respectively, and these cutoffs must be ordered either as \( \lambda_{AI}^2 \leq \lambda_{AI}^1 \leq \lambda_{AI}^3 \) (when \( \pi_{(p_H,p_H)} \leq \max\{\pi_{(\infty,p_H)}, \pi_{(p_L,p_L)}\} \)), or as \( \lambda_{AI}^3 \leq \lambda_{AI}^1 \leq \lambda_{AI}^2 \) (when \( \pi_{(p_H,p_H)} \leq \max\{\pi_{(\infty,p_H)}, \pi_{(p_L,p_L)}\} \)).

The intuition for each of these 3 menus is the following. The menu \((\infty, p_H)\) targets type \( p_H \) and extracts all of its surplus, but offers a very limited contract to type \( p_L \) and extracts less surplus. This is optimal when there are relatively more high types. The menu \((p_L, p_L)\) offers the same contract to both types, and this contract targets type \( p_L \) and extracts all of its surplus, but it induces type \( p_H \) to litigate and does not capture all of its surplus. This is optimal when there are relatively few high types. The menu \((p_H, p_H)\) offers the same contract to both types, this contract targets type \( p_H \), but is priced low to induce both types to buy it, and does not induce any litigation. This is optimal when \( \lambda \) is in some intermediate range, which may be empty depending on the parameters.
B.2 Covering Settlement

Consider a contract that not only covers the legal costs and the damages, but also covers the settlement payment up to an amount \( \hat{\alpha}_S \). Thus, a contract is now defined by three parameters: \( \alpha = (\hat{\alpha}_L, \hat{\alpha}_D, \hat{\alpha}_S) \). There are several possible outcomes: Going to court or agreeing on a settlement fee \( \phi \). Suppose the agent and the third party agree on a settlement fee \( \phi \). Then, the payoff of the agent and the third party are

\[
    u_A = \min\{\hat{\alpha}_S - \phi, 0\}, \quad u_{TP} = \phi,
\]

respectively. The joint surplus for this agreement is \( J = \min\{\hat{\alpha}_S, \phi\} \), which is weakly increasing in \( \phi \). Therefore, the best arrangement between the agent and the third party is to set \( \phi = \hat{\alpha}_S \). Notice that in this case, \( J = \hat{\alpha}_S \).

The disagreement payoff is to go to court. In that case, the third party gets \( pd - c \) and the agent gets \( -pd - c_A + \hat{\alpha}_L + p\hat{\alpha}_D \).

The increase in joint surplus for an agreement is

\[
    S_B = \hat{\alpha}_S + c + c_A - \hat{\alpha}_L - p\hat{\alpha}_D
\]

Thus, the joint surplus between the agent and the third party from settling is larger than the joint surplus from going to court if and only if

\[
    S_B = \hat{\alpha}_S + c + c_A - \hat{\alpha}_L - p\hat{\alpha}_D \geq 0
\]

Types \( p \) below the threshold \( p^* \) settle, where

\[
    p \leq p^* \equiv \frac{\hat{\alpha}_S + c + c_A - \hat{\alpha}_L}{\hat{\alpha}_D}
\]

Then, the third party gets a payoff equal to

\[
    u_{agreement}^{TP} = pd - c + (1 - \theta)S_B
\]

This payoff must equal the payoff of the agreement outcome \( \hat{\alpha}_S + T \), so we have:

\[
    T = pd - c - \hat{\alpha}_S + (1 - \theta)[\hat{\alpha}_S + c + c_A - \hat{\alpha}_L - p\hat{\alpha}_D]
\]

\[
    T = pd - c - \theta\hat{\alpha}_S + (1 - \theta)[c + c_A - \hat{\alpha}_L - p\hat{\alpha}_D]
\]

\textsuperscript{20}This is without loss of generality since setting \( \phi > \alpha_S \) does not increase the joint surplus.
Notice that, compared to the case in which the insurance company does not pay for settlement, the agent pays a lower fee when settling. Hence, there are two effects: The threshold for settlement changes, and the agent pays a lower settlement fee when settling.

\[
\text{Settlement, pays } T(\alpha_S)
\]

\[
\text{Litigation}
\]

\[
\frac{\hat{\alpha}_S}{p^*} 1
\]

**Figure 12:** The effect of insurance contract \( \alpha \) on licensing and litigation for different types of agents.

The value of insurance is then,

\[
W(\alpha) = \begin{cases} 
\theta \hat{\alpha}_S + (1 - \theta)(\hat{\alpha}_L + p\hat{\alpha}_D) & p \leq p^* \\
(\hat{\alpha}_L + p\hat{\alpha}_D) - \theta(c + c_A) & p > p^*
\end{cases}
\]

The cost for the insurer from offering a contract \( \alpha \) is:

\[
K(\alpha) = \begin{cases} 
\hat{\alpha}_S & p \leq p^* \\
(\hat{\alpha}_L + p\hat{\alpha}_D) & p > p^*
\end{cases}
\]

\[
W(\alpha) = \begin{cases} 
\hat{\alpha}_S + (1 - \theta)(c + c_A) + (1 - \theta)\hat{\alpha}_D(p - p^*) & p \leq p^* \\
\hat{\alpha}_S + (1 - \theta)(c + c_A) + \hat{\alpha}_D(p - p^*) & p > p^*
\end{cases}
\]

The cost for the insurer from offering a contract \( \alpha \) is:

\[
K(\alpha) = \begin{cases} 
\hat{\alpha}_S & p \leq p^* \\
\hat{\alpha}_S + (c + c_A) + \hat{\alpha}_D(p - p^*) & p > p^*
\end{cases}
\]

We can see that the joint surplus is independent of \( \hat{\alpha}_S \). In fact, the solution is the same as in the baseline model setting \( \hat{\alpha}_S = 0 \).

**Lemma 8.** Paying for settlement is never optimal, i.e., \( \hat{\alpha}_S = 0 \).

**Proof.** It is easy to see that \( \hat{\alpha}_L = c_A \) in the optimal contract. Consider \( \hat{\alpha}_S > 0 \) and \( \hat{\alpha}_D \) that induce some threshold \( p^* = \frac{\hat{\alpha}_S + c}{\hat{\alpha}_D} \). Consider a new contract, \( \hat{\alpha}'_S = 0 \) and \( \hat{\alpha}'_D < \hat{\alpha}_D \) such that \( p^* = \frac{c}{\hat{\alpha}_D} \). Notice that \( W(\alpha) - K(\alpha) \) is decreasing in \( \hat{\alpha}_D \) for \( p \leq p^* \) and independent of \( \hat{\alpha}_D \) for \( p > p^* \). Moreover, \( W(\alpha) - K(\alpha) \) is independent of \( \hat{\alpha}_S \). Therefore, the solution conditional on any particular \( p^* \) is the contract with the lower \( \hat{\alpha}_D \). \( \square \)
B.3 Risk Aversion

In this appendix we consider the case where the agent is risk averse. We show that many key insights from our main model are preserved.

Risk aversion introduces several elements that are absent in the baseline case. First, insurance affects an agent’s litigation payoffs through two channels: (1) it increases the expected value of the lottery the agent faces when going to litigation; and (2) it reduces the risk of going to litigation. Under risk neutrality, the reduction of risk did not play a role in the agent’s payoff. Second, the level of wealth of the agent becomes relevant. In the case of risk neutrality, we assume the agent’s wealth is at least $d$, so the agent can always pay for damages, but other than that the level of wealth is irrelevant for the decision of buying insurance. Under risk aversion, the agent’s wealth may determine the agent’s level of risk aversion, which affects the equilibrium transfer under bargaining. In addition, there is no separability between the cost of insurance for the agent and the settlement payoff. So even in the absence of wealth effects (e.g., CARA utility), the price of insurance may alter the bargaining core. Third, the settlement fee paid by the agent, as well as the willingness to pay for insurance, do not generally have closed-form solutions. As a result, for many parts of the main analysis the model under risk aversion is not analytically tractable.

Consider a risk averse agent with initial level of wealth $w$ covered by an insurance policy $\alpha = (\alpha_L, \alpha_D)$, bought at some price $Q$, and with preferences over lotteries represented by an increasing and concave Bernoulli utility function $u(\cdot)$. If the third party and the agent go to litigation, the expected payoff of the agent is

$$u(CE(p, \alpha, Q)) \equiv pu(w - c_A + \alpha_L - d + \alpha_D - Q) + (1-p)u(w - c_A + \alpha_L - Q),$$

where $CE(p, \alpha, Q)$ denotes the certainty equivalent of the risky litigation outcome under insurance policy $\alpha$ bought at price $Q$. Notice that $\alpha_L$ increases the expected value of the lottery, $\alpha_D$ both increases the value and the variance of litigation. In fact, when $\alpha_D = d$, there is no risk associated with going to litigation. Also, notice that the price of the insurance $Q$ has a non-linear effect on the certainty equivalent.

Suppose the third party has a credible litigation threat, i.e. $pd \geq c$. Parties are mutually better off if they can agree on settlement terms and avoid litigation. Under a settlement, the agent just pays a transfer to the third party, and neither party incurs litigation costs. A feasible settlement agreement is a transfer $T$ from the agent to the third party such that $pd - c \leq T$ and $u(CE(p, \alpha, Q)) \leq u(w - Q - T)$. Equivalently, the bargaining core corresponds to transfers $T$ such that

$$T_{\min}(p) \equiv pd - c \leq T \leq w - Q - CE(p, \alpha, Q) \equiv T_{\max}(p, \alpha, Q).$$
Without insurance parties always settle because \( u(CE(p, 0, 0)) \leq u(w - pd - c_A) \) is equivalent to \( pd + c_A \leq w - CE(p, 0, 0) \), so the bargaining core is not empty.

Consider an insurance contract \( \alpha \) sold at price \( Q \). We compute the settlement fee as solution to the maximization of the Nash-product:

\[
T^\alpha(p, Q) \in \arg \max_T \left( u(w - Q - T) - u(CE(p, \alpha, Q)) \right) \theta (T - (pd - c))^{1-\theta}
\]

subject to \( T_{\text{min}}(p) \leq T \leq T_{\text{max}}(p, \alpha, Q) \). \hspace{1cm} (19)

An interior solution for problem (19) satisfies

\[
\left( \frac{\theta}{1-\theta} \right) u'(w - Q - T^\alpha) = \frac{u(w - Q - T^\alpha) - u(CE(p, \alpha, Q))}{T^\alpha - (pd - c)}.
\]

Conditional on \( p \), the agent’s willingness to pay for insurance policy \( \alpha \) sold at price \( Q \) is then \( T^0(p, 0) - T^\alpha(p, Q) \), provided that \( T_{\text{max}}(p, \alpha, Q) \geq T_{\text{min}}(p) \), i.e. parties settle. When \( T_{\text{max}}(p, \alpha, Q) < T_{\text{min}}(p) \), insurance induces litigation and the agent’s willingness to pay is \( CE(p, \alpha, Q) + w - T^0(p, 0) \).

Under risk neutrality, we show that: (1) \( W(p, \alpha) \) is strictly increasing in \( p \), (2) \( W(p, \alpha) \) is supermodular in \( p \) and \( \alpha \), and (3) the insurer’s profit is strictly negative for any type \( p \) that enters litigation. These three conditions are crucial for our results on the nature of equilibrium under perfect competition with asymmetric information (Proposition 4 and Theorems 1 and 2).

Figure 13 shows the willingness to pay under risk neutrality and two specifications of risk aversion: CARA utility \( u(x) = -\exp(-\sigma x) \) and CRRA utility \( u(x) = \frac{x^{1-\eta}}{1-\eta} \). As long as the agent is not too risk averse, the features of the willingness to pay in the linear case are preserved. In addition, the insurer’s profit is negative for types that litigate. Risk aversion does alter the agent’s willingness to litigate. Conditional on the same insurance policy \( \alpha \), a risk averse agent is less willing to litigate compared to a risk neutral agent, because going to litigation is risky. In the figures, \( p^* \) rises relative to the case of risk neutrality.
Risk aversion introduces two primary complications into the model. First, from equation (20), we see that the price and the terms of insurance both affect the settlement transfer (whereas with risk neutrality, the transfer is not affected by the price of insurance). Second, the price $Q$ affects the decision to settle in a possibly non-monotonic way: recall that settlement obtains with risk neutrality, the transfer is not affected by the price of insurance). Second, the price $Q$ affects the decision to settle in a possibly non-monotonic way: recall that settlement obtains with risk neutrality, the transfer is not affected by the price of insurance. However, we can show that the optimal monopoly insurance policy under complete information is qualitatively very similar under risk aversion. The maximum willingness to pay of an agent of type $p$ that settles is $T^0(p, 0) - T_{\text{min}}(p)$. The monopolist then charges price $Q^{CI} = T^0(p, 0) - T_{\text{min}}(p)$ and chooses $\alpha^{CI}_L$ and $\alpha^{CI}_D$ to set $T_{\text{max}}(p, \alpha^{CI}, Q^{CI}) = T_{\text{min}}(p)$. It follows that the equilibrium transfer under bargaining is $T^{\alpha}(p, Q^{CI})$, because $T_{\text{max}}(p, \alpha^{CI}, Q^{CI}) = T_{\text{min}}(p)$ and the agent is willing to pay $Q^{CI}$. Because $T^{\alpha}(p, Q) = T_{\text{max}}(p, \alpha, Q) = T_{\text{min}}(p)$, the optimal contract under monopoly makes the agent indifferent between settlement and litigation, and the price extracts the entire surplus from the agent. Under competition, the same contract would obtain as an equilibrium, and sell for a price of zero. These characteristics mirror the primary insights in Proposition 2.

For the general class of risk averse preferences, the outcome of bargaining may depend upon wealth $w$ and the price of insurance $Q$. Our model is then not analytically tractable for analysis beyond the complete information case. And while we can simulate outcomes for certain classes of utility functions, this introduces a taxonomy of possible cases to consider.

Figure 13: The figure shows the willingness to pay and the cost for the insurer for different utility specifications. The figure on the left correspond to a risk-neutral agent (baseline case). The indifference point between settlement and litigation is $p^* = 0.5$. The figure in the middle corresponds to a CARA utility with parameter $\sigma = 0.1$ and the figure on the right corresponds to CRRA utility with parameter $\eta = 0.25$. Under risk aversion $p^*$ shifts to the right compared to the case of risk neutrality. The simulations consider the following parameters: $c = c_A = 1$, $d = 5$, $\theta = 0.8$, $w = 7$, and contract $(\alpha_L, \alpha_D) = (1, 2)$.

---

21 Under full insurance, $\alpha_{FI} = (\alpha_L, \alpha_D) = (c_A, d)$, we have $CE(p, \alpha_{FI}, Q) = w - Q$ so $T_{\text{max}}(p, \alpha_{FI}, Q) = 0 < T_{\text{min}}(p)$. Under policy $\alpha$, $T_{\text{max}}(p, 0, Q^{CI}) = w - T^0(p) + T_{\text{min}} - CE(p, 0, Q^{CI}) > T_{\text{min}}$, because the agent always settle without insurance, i.e., $CE(p, 0, 0) < w - T^0(p)$, which implies $CE(p, 0, Q^{CI}) < w - T^0(p)$. Then, by continuity, we can find $\alpha^{CI}$. 

9
(e.g., increasing risk aversion, decreasing risk aversion, etc.). Analyzing these cases for a wide array of distributions of $p$ is beyond the scope of this paper.

The main purpose of this appendix is to show that our findings are not specific to risk neutrality, but extend to risk averse agents. To illustrate this further, we next analyze a widely used class of preferences, represented by mean-variance utility. These preferences have the feature that the certainty equivalent is linear in $Q$. After discussing some basic observations on liability insurance under mean-variance utility, we provide evidence from simulations of the optimal monopoly contract under symmetric information, to show that our insights from the main model continue to hold when the agent is not too risk averse.

### B.3.1 Mean-variance preferences

Consider mean-variance preferences, represented by

$$U(X) = E(X) - \frac{\sigma Var(X)}{2}.$$  

An agent with these preferences evaluates lottery $X$ according to its mean and variance. Under insurance policy $(\alpha_L, \alpha_D)$, the certainty equivalent under litigation is

$$CE(p, \alpha) = w - (c_A - \alpha_L) - p(d - \alpha_D) - \frac{\sigma p(1-p)(d - \alpha_D)^2}{2}.$$  

The only difference with the risk neutral case is the last term

$$RP(p, \alpha_D) \equiv \frac{\sigma p(1-p)(d - \alpha_D)^2}{2},$$

which corresponds to the agent’s risk-premium. The bargaining surplus is

$$S_B = c + c_A - \alpha_L - p\alpha_D + RP(p, \alpha_D),$$

which is positive for low $p$, but may be negative for high $p$. Indeed, $S_B = 0$ for a cutoff value of $p^*$ that satisfies

$$p^* = \frac{c + c_A - \alpha_L}{\alpha_D} + \frac{RP(p^*, \alpha_D)}{\alpha_D} > \frac{c + c_A - \alpha_L}{\alpha_D} = p_{RN}^*.$$  

If $p > p^*$, then litigation ensues. When $RP(p^*, \alpha_D) > 0$, the threshold $p^*$ is strictly higher than $p_{RN}^*$, the threshold for litigation under risk neutrality. Risk aversion makes the agent more inclined to settle.

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22See for example Grant and Polak (2013)
The settlement fee is given by \( T = pd - c + (1 - \theta)S_B \), so the agent’s payoff when settlement occurs is \( w - T \). Thus, the willingness to pay for insurance, is

\[
W(p, \alpha) = \begin{cases} 
(1 - \theta)[\alpha_L + p\alpha_D + \Delta_{RP}(p, \alpha_D)] & \text{if } p \leq p^* \\
\alpha_L + p\alpha_D - \theta(c + c_A + RP(p, 0)) + \Delta_{RP}(p, \alpha_D) & \text{if } p > p^* 
\end{cases}
\]

(21)

where \( \Delta_{RP}(p, \alpha_D) \equiv RP(p, 0) - RP(p, \alpha_D) \) is the difference between the risk-premium without and with insurance.

The difference between risk aversion and risk neutrality is clear from this expression. First, compared to a risk neutrality, a risk averse agent of type \( p \) is willing to pay \( \Delta_{RP}(p, \alpha_D) \) extra because insurance reduces risk. When the agent settles, the risk reduction puts the agent in a better bargaining position, so the agent captures an extra \((1 - \theta)\Delta_{RP}(p, \alpha_D)\) of the bargaining surplus. Second, when \( p \) is taking risks when going to litigation. Insurance reduces this risk, so the agent is willing to pay \( \Delta_{RP}(p, \alpha_D) \) for the reduction in the risk premium. However, the opportunity cost of litigation is \( \theta c + c_A + RP(p, 0) \), which correspond to what agent would have captured as bargaining rents had the agent settled without insurance.

Differentiating the willingness to pay with respect to \( p \) we obtain

\[
\frac{\partial W}{\partial p}(p, \alpha) = \begin{cases} 
(1 - \theta)\alpha_D \left[ 1 + \frac{\sigma(1 - 2p)(2d - \alpha_D)}{2} \right] & \text{if } p \leq p^* \\
\alpha_D + \frac{\sigma(1 - 2p)}{2}(\alpha_D(2d - \alpha_D) - \theta d^2) & \text{if } p > p^* 
\end{cases}
\]

(22)

Next, consider the cross-partial with respect to \( \alpha_D \).

\[
\frac{\partial^2 W}{\partial p \partial \alpha_D}(p, \alpha) = \begin{cases} 
(1 - \theta) [1 + \sigma(1 - 2p)(d - \alpha_D)] & \text{if } p \leq p^* \\
1 + \sigma(1 - 2p)(d - \alpha_D) & \text{if } p > p^* 
\end{cases}
\]

(23)

It is obvious that for sufficiently low \( \sigma \) we have that \( \frac{\partial W}{\partial p} > 0 \) and \( \frac{\partial^2 W}{\partial p \partial \alpha_D} \geq 0 \) for all \( p \). That is, the willingness to pay is strictly increasing in \( p \) and supermodular in \( p \) and \( \alpha \) when the agent’s risk aversion is low. It is also straightforward to show that \( W(p, \alpha) < K(p, \alpha) \) for sufficiently low \( \sigma \), so the insurer incur losses by inducing litigation. These results mirror our findings from Figure 13 that key characteristics of willingness to pay are preserved under sufficiently low levels of risk aversion.
Simulation of Optimal Contract with Risk Aversion

We have already shown that results under complete information (e.g., Proposition 2 from subsection 3.1) and results for perfect competition with asymmetric information (e.g. Proposition 4, Theorem 1 from subsection 3.3) hold when $\sigma$ is low. The following set of simulations consider the symmetric information setting of subsection 3.2.

For these simulations we consider the following parameters:

$$c = c_A = 1, \ d = 5, \ \theta = 0.8, \ w = 7.$$ 

We also consider several distributions which reflect cases considered in Example 1 from subsection 3.2.

<table>
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<th>$\alpha_D$</th>
<th>$p^*$</th>
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<tr>
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<table>
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<th>$F(p) = 1 - (1 - p)^{1.5}$</th>
<th>$F(p) = p$</th>
<th>$F(p) = \sqrt{p}$</th>
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</table>

Table 1: Optimal monopoly contract under symmetric information for mean-variance preferences for different distribution of types. The first row in each table is the optimal contract for the risk-neutral case.

Results from these simulations are in Table 1. Results from subsection 3.2 tend to hold when $\sigma$ is relatively low. For $\sigma \leq 0.1$, we see that the optimal $\alpha_L$ equals $c_A$. Hence, the general finding from Proposition 1 that $\alpha_L < c_A$ is weakly dominated by $\alpha_L = c_A$ (with or without uncertainty about $p$) holds for low $\sigma$.

When $\sigma \leq 0.1$, the conditions for the optimal $p^*$ also reflect results from Lemmas 2 and 3 and Corollary 3 (as highlighted by Example 1). If $F(p) = 1 - (1 - p)^{\alpha}$, with $\alpha > 1$, then the optimal $p^* < 1$. If $F(p) = p^{\alpha}$, then the optimal $p^* = 1$. Both sets of results are the same as under risk neutrality.
For higher levels of risk aversion, damages insurance plays a more dominant role. We see that when $\sigma = 3$, it is optimal for all $F(p)$ to set $\alpha_L = 0$ and rely exclusively on damages insurance. When agents are strongly risk-averse, it pays far more to use $\alpha_D$ to raise willingness to pay, because it reduces the risk premium.

Our main conclusion from the results in this appendix is that, although solving the model for general risk aversion preferences is not tractable, our results hold when the agent’s risk aversion is not too high.

### B.4 Bargaining under Incomplete Information

Instead of Nash bargaining under complete information, we consider an agent that is privately informed about $p$, the probability that the court finds the agent liable, and a third party that is uninformed about $p$. The third party makes a take-it-or-leave-it offer to the agent and if the offer is rejected the parties engage in costly litigation.

The third party needs to receive at least $E[p]d - c$ as a settlement compensation, otherwise it is individually rational for the third party to litigate. As a benchmark case, we study the case of no insurance. An agent of type $p$ pays in expectation $pd + c_A$ from going to litigation so any settlement offer $S$ such that $S \leq pd + c_A$ will be accepted by these agents. Denote by $\hat{p}_{NI}(S) \equiv \frac{S-c_A}{d}$ the agent that is indifferent between accepting offer $S$ and going to litigation.

Notice that in this framework, high-risk types will accept a settlement offer while and low-risk types will litigate. In the baseline case of bargaining under complete information it is always optimal to settle without insurance. In contrast, with incomplete information and depending on the settlement offer, low-risk types prefer to litigate than to settle.

Anticipating the decision of the agent, the third party will make a settlement offer $S$ to maximize its expected payoff

$$S_{NI}^* \in \arg \max_{S \in [E[p]d-c, \infty)} \int_{\hat{p}_{NI}(S)}^{\hat{p}_{NI}(S)} [pd - c]dF(p) + S[1 - F(\hat{p}_{NI}(S))].$$

Taking the first order condition we find that the optimal threshold satisfies

$$\frac{1 - F(\hat{p}_{NI})}{f(\hat{p}_{NI})} = \frac{c + c_A}{d}.$$ (24)

With an increasing hazard rate, this condition is necessary and sufficient. The optimal settlement offer is then $S_{NI}^* = p_{NI}^*d + c_A$. Agents of type $p < p_{NI}^*$ litigate while agents of type $p \geq p_{NI}^*$ settle.
Consider now the case of an agent that is covered by the liability insurance policy \((\alpha_L, \alpha_D)\). The expected payment of an agent of type \(p\) covered by this insurance policy from rejecting an offer and going to litigation is

\[
\pi_L^A(\alpha) = (c_A - \alpha_L) + p(d - \alpha_D).
\]

A settlement offer \(S\) is, then, accepted if and only if \(S \leq \pi_L^A(\alpha)\).

For the sake of exposition, consider a liability insurance contract that partially cover damages, i.e., \(\alpha_D < d\). An agent of type \(p\) accepts the settlement fee \(S\) if and only if

\[
p \geq \hat{p}_a(S) \equiv \frac{S - (c_A - \alpha_L)}{d - \alpha_D}.
\]

The settlement offer \(S\) depends on the insurance policy \(\alpha\) bought by the agent. The third party’s optimal settlement offer conditional on a liability insurance policy \(\alpha\) is the solution to

\[
S^*(\alpha) \in \arg\max_{S \in [E[p]d - c, \infty)} \int_{0}^{\hat{p}_a(S)} [pd - c]dF(p) + S[1 - F(\hat{p}_a(S))]
\]

Taking FOC and writing the problem in terms of \(p^*_a(S) \equiv \hat{p}(S^*)\) we get:

\[
\left(\frac{1 - F(p^*_a)}{f(p^*_a)}\right) = \frac{c + c_A - (\alpha_L + p^*_a\alpha_D)}{d - \alpha_D}.
\]

Notice that, in contrast to the case of no insurance, the FOC may not be sufficient even with an increasing hazard rate. For the purpose of exposition, assume for now that \(p^*_a\) is interior. Let \(G(x) = \frac{1 - F(x)}{f(x)}\) and assume that \(G(\cdot)\) is decreasing. Then, \(p^*_N < p^*_a\) if \(G(p^*_N) \geq G(p^*_a)\), which is true if and only if

\[
p^*_a \geq \frac{c + c_A}{d} - \frac{\alpha_L}{\alpha_D}.
\]

This shows that liability insurance contract may reduce or increase the number of types that litigate. This is in contrast to the case of bargaining under complete information, where insurance could only increase the amount of litigation. Another difference is that every agent that settle it does by paying the same settlement fee. The willingness to pay of an agent for insurance depends on the value of not buying insurance. Hence, there are four potential cases: an agent settles without insurance but litigates with insurance; an agent settles with and without insurance; an agent litigates without insurance but settles with insurance; an agent litigates with and without insurance.

For given contract \((\alpha_L, \alpha_D)\), we define the willingness to pay of an agent of type \(p\) by

\[23\]When \(\alpha_D = d\), the value of going to litigation is constant and equal to \(c_A - \alpha_L\). The complete solution of this case is available upon request.
Notice that the willingness to pay depends on the equilibrium settlement offered by the third party. Hence, the optimal liability insurance policy, then is chosen by solving

$$\max_{(\alpha_L, \alpha_D)} \int_0^{\alpha_L} [W(p, \alpha|S^*(\alpha)) - (p\alpha_D + \alpha_L)]dF(p) + \int_{\alpha_L}^{1} W(p, \alpha|S^*(\alpha))dF(p)$$

In contrast with the case of Nash-bargaining under complete information, the optimal monopoly contract, even when the agent is ex-ante informed about $p$, does not have an analytically tractable solution. For this reason, we provide some intuition for the two-type case.

**Two-type case**

Consider the two-type case as defined in Definition 2 and allow for arbitrary values $0 \leq p_L < p_H \leq 1$. Assume the agent is protected by the liability insurance policy $\alpha = (\alpha_L, \alpha_D)$. The third party considers only two possible settlement offers:

- $S^L = (c_A - \alpha_L) + p^L(d - \alpha_D)$
- $S^H = (c_A - \alpha_L) + p^H(d - \alpha_D)$

By offering $S^L$ the low-risk type is indifferent between settlement and litigation and the high-risk agent is strictly better off by accepting the offer. Settlement offer $S^H$ leaves the high-risk type indifferent between accepting the offer or litigation but low-risk type rejects the offer and litigate. The third party’s outside option is $E[p]d - c$ because it can always make a ‘bad faith’ settlement offer that forces both types to litigate. Hence, we have three cases:

1. The third party makes offer $S^L$, both types of agents settle, and the third party’s payoff is

$$\pi_{TP}(S^L) = (c_A - \alpha_L) + p^L(d - \alpha_D)$$

2. The third party makes offer $S^H$, high-risk types settle but low-risk type litigate, and the third party’s payoff is

$$\pi_{TP}(S^H) = \lambda[(c_A - \alpha_L) + p^H(d - \alpha_D)] + (1 - \lambda)[p^Ld - c]$$

3. The third party forces litigation by making a bad faith offer ($S = +\infty$) and third party in this case is

$$\pi_{TP}(S^\infty) = (\lambda p^H + (1 - \lambda)p^L)d - c$$
We can show that
\[
\pi_{TP}(S^H) = \pi_{TP}(S^\infty) + \lambda(c + c_A - \alpha_L - p^H \alpha_D) \equiv Y \\
\pi_{TP}(S^H) = \pi_{TP}(S^L) + \lambda(p^H - p^L)(d - \alpha_D) - (1 - \lambda)(c + c_A - \alpha_L - p^L \alpha_D) \equiv Z \\
\pi_{TP}(S^L) = \pi_{TP}(S^\infty) + Y - Z
\]

The optimal offer, then is determined by \(Y, Z, \) and \(Y - Z\).

Consider first the case of no insurance by setting \(\alpha_L = \alpha_D = 0\). It is easy to see that, in this case, we have\( Y = \lambda(c + c_A) \) and \(Z = \lambda(p^H - p^L)d - (1 - \lambda)(c + c_A)\).

Obviously, \(Y > 0\) so an offer that always leads to litigation \((S^\infty)\) cannot be chosen by the third party. Also, it can be easily shown that \(Z < 0\) \(\Rightarrow Y > Z\) and, therefore, the optimal offer is either \(S^H\) or \(S^L\). In fact, without insurance, the optimal offer is \(S^L\) if
\[
(p^H - p^L) \left( \frac{\lambda}{1 - \lambda} \right) < \frac{c_A + c}{d}
\]
and \(S^H\) if this condition does not hold. Notice that this condition is the discrete case analogous to the first order condition \((24)\).

To characterize the optimal offer with insurance, we need to determine as a function of \(\alpha = (\alpha_L, \alpha_D)\) the values of \(Y(\alpha), Z(\alpha), \) and \(W(\alpha) \equiv Y(\alpha) - Z(\alpha)\).

We can show that
\[
Y(\alpha) = 0 \iff \alpha_L = c + c_A - p^H \alpha_D \\
Z(\alpha) = 0 \iff \alpha_L = c + c_A - \left( \frac{\lambda}{1 - \lambda} \right) (p^H - p^L)d + \left( \frac{\lambda p^H - p^L}{1 - \lambda} \right) \alpha_D \\
W(\alpha) = 0 \iff \alpha_L = c + c_A - \lambda(p^H - p^L)d - p^L \alpha_D
\]

The slopes of these linear functions are ordered: \(\frac{\lambda p^H - p^L}{1 - \lambda} > -p^L > -p^H\), as well as the intercepts. Also, notice that \(Y(\alpha) = 0\) and \(W(\alpha) = 0\) intersect at \(\alpha_D = \lambda d\) and also \(Z(\alpha) = 0\) and \(W(\alpha) = 0\) intersect at the same point. Define \(\omega = (\frac{\lambda}{1 - \lambda}) (p^H - p^L)d\).

Figure 14 shows regions where \(S^H \succeq S^L \) \((Y > 0)\), \(S^H \succeq S^\infty \) \((Z > 0)\), and \(S^L \succeq S^\infty \) \((W > 0)\), for a given contract \(\alpha\). This picture is drawn for a particular set of parameters, such that \(c + c_A > \omega\). However, the parameters \(\lambda, p^H, p^L\) and \(d\), determine both the value of \(\omega\) and
Figure 14: Regions to determine the preference of the third party among different settlement offers for a given insurance policy $\alpha = (\alpha_L, \alpha_D)$

the slope of the curve where $Z(\alpha) = 0$. Given these regions, we can now find the optimal settlement offers, which are illustrated in Figure 15.

From Figure 15 it is easy to see the case of no insurance. When $(\alpha_L, \alpha_D) = (0, 0)$ either $c + c_A > \omega$ in which case the optimal settlement offer is $S_L$ or $c + c_A < \omega$, in which case the optimal settlement offer is $S_H$. In general, the settlement offer will depend on the liability contract $\alpha$. Notice that $\alpha_L \leq c_A$. When $\alpha_D = \lambda d$, the intersection of the three regions occurs at $\alpha_L = c + c_A - p^H \lambda d$. When $\alpha_D = d$, the maximum value of $\alpha_L$ for $W(\alpha) = 0$ (i.e., intersection of the regions where $S^\infty$ and $S_L$ at $\alpha_D = d$) is above $c_A$ if and only if $\lambda < \frac{c - p^L d}{p^H - p^L}$. In that case, the offers considered by the third party would only be $S_H$ and $S_L$. If that is not the case, the third-party would consider an offer $S^\infty$.

Given that we know what the optimal settlement offer is for a given insurance contract $\alpha$, let’s now determine the optimal contract offered by a monopolist under symmetric information.

First, consider the case where $c + c_A > \omega$. In this case, without insurance, the optimal settlement offer is $S_L$. Hence, every agent gets the same outside option from not buying insurance, which is to pay $S_L$ as a settlement fee.

Denote by $S^*(\alpha)$ settlement fee offered by the third party, illustrated in Figure 15. The willingness to pay for insurance is:

- zero if $S^*(\alpha) = S_L$
Figure 15: Regions to determine the optimal settlement offer for a given insurance policy $\alpha = (\alpha_L, \alpha_D)$

- $S^L - S^H$ for type $p^H$ and zero for type $p^L$ if $S^*(\alpha) = S^H$
- $S^L - (pd + c_A)$ for type $p$ if $S^*(\alpha) = S^\infty$

Because the agent is uninformed about its type at the moment of buying insurance, the expected willingness to pay is zero when $S^*(\alpha) = S^L$ and negative otherwise. Thus, the insurer cannot profit in this case, and must offer a contract such that $S^*(\alpha) = S^L$. Any one of these contracts must be sold at price zero.

Second, consider the case $c + c_A < \omega$. In this case, without insurance the optimal settlement offer is $S^H$ and only high-risk types settle. There is scope to make profits by offering a policy contract such that $S^*(\alpha) = S^L$. Any other policy contract would generate an expected willingness to pay of zero.

The willingness to pay for insurance contract $\alpha$ such that $S^*(\alpha) = S^L$ is zero for the low-risk types and equal to $S^H - S^L$ for high-risk types. Therefore, the expected willingness to pay for insurance is

$$\lambda(p^H - p^L)(d - \alpha_D).$$

With this insurance contract, there is no litigation in equilibrium. Hence, the effect of insurance in this case is to reduce the amount of litigation, in contrast with the baseline case where the opposite occurred. Because everyone settles, there is no cost incurred by the insurer. Hence, it would like to offer the smallest largest possible $\alpha_D$ such that the pair $(\alpha_L, \alpha_D)$ is in the region where $S^*(\alpha) = S^L$. 
Notice that the region where $S^*(\alpha)$ is non-empty for the case $c + c_A < \omega$ if and only if the intersection of the three regions at $\alpha_D = \lambda d$ occurs at some point $\alpha_L > 0$. Then, we require that $c + c_A - p^H \lambda d > 0$. Hence, the two conditions that are necessary for insurance to have value are

$$p^H \lambda < \frac{c + c_A}{d} < \left(\frac{\lambda}{1 - \lambda}\right)(p^H - p^L).$$

Notice that these two conditions imply that the slope of the line where $Z(\alpha) = 0$ is positive, that is, $\lambda p^H > p^L$. The lowest possible $\alpha_D$ that meets the condition $S^*(\alpha) = S^L$ is when $\alpha_L = 0$ and $Z(0, \alpha_D) = 0$. Hence,

$$\alpha_D^* = \frac{\lambda(p^H - p^L)d - (1 - \lambda)(c + c_A)}{\lambda p^H - p^L}.$$

We can summarize the results in the following proposition:

**Proposition 7.** Consider the monopoly problem under symmetric information for the two type case. When $c + c_A > \left(\frac{\lambda}{1 - \lambda}\right)(p^H - p^L)d$, insurance does not have positive value because without insurance the optimal settlement offer is $S^L$ and all types settle. However, when this inequality does not hold, insurance may have positive value for the high-type agent who is able to settle for a lower settlement fee. For this to be true, additionally to $c + c_A \leq \left(\frac{\lambda}{1 - \lambda}\right)(p^H - p^L)d$ we require $p^H \lambda < \frac{c + c_A}{d}$. If these two conditions hold, the optimal liability insurance contract is

$$\alpha^*_L = 0, \quad \alpha_D^* = \frac{\lambda(p^H - p^L)d - (1 - \lambda)(c + c_A)}{\lambda p^H - p^L}.$$  

That is, the optimal liability contract partially covers damages and does not cover litigation costs. A monopolist offers this contract at price $\lambda(p^H - p^L)(d - \alpha_D^*)$ and a perfectly competitive market offers it for free.

Notice the difference between the optimal liability insurance contract when the negotiation is under incomplete information versus when it is under complete information. First, insurance reduces litigation in equilibrium when the negotiation is under complete information. Second, insurance is valuable only for some distribution of types and not always as it was the case with complete information bargain. Third, the optimal contract does not cover litigation costs and only partially covers damages.

The analysis of a competitive equilibrium under adverse selection is simple. First of all, offers must be such that only settlement offer $S^L$ is induced in equilibrium. An insurance policy $\alpha'$ that induces the third party to offer $S^H$ does not have any value for the agents and therefore must be sold at a price of zero. No insurer finds this deviation profitable. Hence, consider offers $\alpha \in \{\alpha : S^*(\alpha) = S^L\}$. If this set is empty, equilibrium does not exist. If this set is
not empty, the most profitable contract is

$$\alpha^* = \left( \alpha^*_L = 0, \alpha^*_D = \frac{\lambda(p^H - p^L)d - (1 - \lambda)(c + c_A)}{\lambda p^H - p^L} \right).$$

Hence, a pooling equilibrium at this contract exists. However, there is multiplicity of equilibrium because any contract $\alpha \in \{ \alpha : S^*(\alpha) = S^L \}$ in addition to $\alpha^*$ that is sold at price of zero it would also be part of an equilibrium.

B.5 Control over the Settlement Decision

In this extension we consider the optimal assignment of control over the settlement process. In our main model, we assumed that in general the agent decides whether to settle or litigate and negotiates the settlement, which is motivated by the features of actual liability insurance contracts that we observe in some industries, such as in patent litigation. In this framework the agent benefits from the ability to negotiate a better settlement with the third party, but the option to litigate gives rise to an ex post moral hazard problem. Instead, the agent and insurer may in some settings prefer an insurance contract whereby the insurer negotiates the settlement and controls the decision whether to settle or litigate, to avoid the problem of ex post moral hazard.

To study this problem, analogously to our main model, suppose that the insurer contracts with the agent, then observes $p$ and negotiates a settlement with the third party, under the threat of litigation. The insurer offers a contract $\hat{\alpha}_D \in [0, d]$ to cover the possible damages that the agent may have to pay if found liable, as in the main body of the paper. Since the insurer controls the litigation process, it pays the litigation cost $c_A$; alternatively, this can also be modeled analogously as the agent paying the litigation cost, and the insurance contract covering (some part of) the litigation cost. We assume, as in the literature on litigation insurance (Meurer (1992)), that the insurer must negotiate “in good faith,” a restriction which in practice is interpreted to mean that the insurer must negotiate a settlement which maximizes I and A’s joint payoff. Equivalently, this can also be seen as a requirement that the insurer must leave the agent no-worse-off than if it had not bought insurance. Under both of these interpretations, since $\hat{\alpha}_D$ is a transfer between the agent and the insurer, the parties are indifferent over all $\hat{\alpha}_D$. For generality, we also allow for the possibility that the insurer is better than the agent at negotiating a settlement: suppose the insurer has a bargaining power $\theta_I$, rather than $\theta$.

First, notice that this model of settlement is in fact analogous to our baseline model with no insurance: one party (in this case the insurer) negotiates a settlement to maximize I and A’s joint payoff, which is equivalent to a model without insurance where the agent negotiates a
settlement to maximize its own payoff, though possibly with different bargaining power.

In this extension, the agent’s payoff without insurance is

$$\bar{V} = -c_A - pd + \theta(c + c_A).$$

The agent’s payoff with insurance (where the insurer bargains) is

$$V = -c_A - pd + \theta_I(c + c_A).$$

So the agent and insurer’s net joint surplus from insurance (relative to having no insurance) is

$$W = (\theta_I - \theta)(c_A + c).$$

It is clear that such insurance cannot be profitable if $\theta > \theta_I$, so we will focus on the case where $\theta_I \geq \theta$. Also, notice that this surplus is independent of $p$: all types value this kind of insurance contract by the same amount. With a monopolist insurer, the optimal price of this insurance is $W$, whereas with competition it is 0. In both settings the bargaining surplus is always positive, so there is never any litigation in equilibrium. Moreover, because this surplus is independent of $p$, the joint surplus from such insurance is the same across different market and information structures. Whether $p$ is the agent’s private information or not at the time of contracting with the insurer is in fact irrelevant in this case—both parties anticipate that at the time of bargaining, I knows $p$ and bargains to maximize A and I’s joint payoff (which is analogous to our baseline model where A bargains without insurance). A receives no information rents, since the net joint surplus from this insurance contract is independent of $p$.

For each market structure and information structure, we can now compare the insurer’s overall profit in our main model against its profit from selling insurer-controlled insurance. We mainly focus on the cases where setting $p^* = 1$ is optimal, although analogous comparisons and intuitions emerge in all cases, where $p^* < 1$ may be optimal.

**Monopoly under symmetric information**

To begin, consider a monopoly setting with symmetric information. In the case where $p^* = 1$ is optimal, from Proposition 3 the insurer’s profit is:

$$P_M(1) = E_p(W(p, 1)) = \int_{\frac{1}{2}}^{1} (1 - \theta)(c_A + cp)dF(p).$$
We compare this against the insurer’s profit in this extension:

\[ \bar{P}_M \equiv \int_{\frac{1}{\theta_I}}^{1} (\theta_I - \theta)(c_A + c)dF(p). \]

The profit from agent-controlled insurance can be re-written as

\[ P_M(1) = \int_{\frac{1}{\theta_I}}^{1} (1 - \theta)(c_A + c)dF(p) - \int_{\frac{1}{\theta_I}}^{1} (1 - \theta)c(1 - p)dF(p). \]

So we have

\[ \bar{P}_M \geq P_M(1) \iff \int_{\frac{1}{\theta_I}}^{1} (1 - \theta)c(1 - p)dF(p) \geq \int_{\frac{1}{\theta_I}}^{1} (1 - \theta_I)(c_A + c)dF(p) \]

Notice that for \( \theta_I \) sufficiently high (e.g. \( \theta_I = 1 \)), the right-hand side is 0 and the left-hand side is positive (independent of \( \theta_I \)), so insurer-controlled insurance is optimal. On the other hand, for \( \theta_I \) low enough (e.g. \( \theta_I = \theta \)), we have \( c(1 - p) < c + c_A \), so the inequality is reversed, hence agent-controlled insurance is optimal. There exists a unique threshold \( \tilde{\theta}_I \) given by the expression

\[ \int_{\frac{1}{\theta_I}}^{1} (1 - \theta)c(1 - p)dF(p) = \int_{\frac{1}{\theta_I}}^{1} (1 - \tilde{\theta}_I)(c_A + c)dF(p), \]

such that for \( \theta_I > \tilde{\theta}_I \), insurer-controlled contracts are optimal, whereas for \( \theta_I \leq \tilde{\theta}_I \), agent-controlled contracts are optimal. Moreover, when the agent and insurer are equally good at bargaining, i.e. \( \theta = \theta_I \), agent-controlled insurance contracts are better. Hence our results from the main model are fairly robust, and the contract we characterized continues to be optimal when we allow insurer-controlled contracts to be offered.

**Competition under symmetric information**

Now consider a competitive market where insurers and agents are symmetrically uninformed. To see whether agent-controlled or insurer-controlled insurance will be sustained as an equilibrium, we must compare the insurer and agent’s net joint surplus from each type of contract. In the case where \( p^* = 1 \) is optimal, from Proposition 3 the insurer and agent’s joint surplus is

\[ JSC(1) \equiv E_p(W(p, 1)) = \int_{\frac{1}{\theta_I}}^{1} (1 - \theta)(c_A + cp)dF(p). \]

With an insurer-controlled insurance contract, the insurer and agent’s joint surplus is

\[ W_C = \bar{P}_M \equiv \int_{\frac{1}{\theta_I}}^{1} (\theta_I - \theta)(c_A + c)dF(p). \]
Hence we have the exact same comparison as with a monopoly under symmetric information: for $\theta_I > \tilde{\theta}_I$, insurer-controlled contracts are offered in equilibrium, whereas for $\theta_I \leq \tilde{\theta}_I$, agent-controlled contracts are offered in equilibrium. When the agent and insurer have equal (or similar enough) bargaining power, our equilibrium results from the main model continue to hold.

**Monopoly under private information**

Next, consider the monopoly setting with private information. In the case where $p^* = 1$ is optimal, from Theorem 3, the insurer offers a menu of two contracts: $(c_A, 0)$ sold at price $(1 - \theta)c_A$, for types $p \leq \bar{p}$, and $(c_A, c)$ sold at price $(1 - \theta)(c_A + c\bar{p})$, for types $p > \bar{p}$. The insurer’s total revenue here is

$$R_M(1) \equiv \int_{\frac{\theta_I}{3}}^{\bar{p}} (1 - \theta)c_A dF(p) + \int_{\bar{p}}^{1} (1 - \theta)(c_A + c\bar{p}) dF(p)$$

We compare this against the insurer’s profit in this extension:

$$\tilde{P}_M = \int_{\frac{\theta_I}{3}}^{1} (\theta_I - \theta)(c_A + c) dF(p).$$

So we have

$$\tilde{P}_M \geq R_M(1) \iff \int_{\frac{\theta_I}{3}}^{\bar{p}} (\theta_I - \theta)c dF(p) + \int_{\bar{p}}^{1} (\theta_I - \theta)c(1 - \bar{p}) dF(p) \geq \int_{\frac{\theta_I}{3}}^{\bar{p}} (1 - \theta_I)c_A dF(p) + \int_{\bar{p}}^{1} (1 - \theta_I)(c_A + c\bar{p}) dF(p)$$

As before, for $\theta_I$ sufficiently high (e.g. $\theta_I = 1$), the right-hand side is 0 and the left-hand side is positive, so insurer-controlled insurance is optimal. On the other hand, for $\theta_I$ low enough (e.g. $\theta_I = \theta$), the left-hand side is 0 while the right-hand side is positive, so the inequality is reversed, hence agent-controlled insurance is optimal. There exists a threshold $\tilde{\theta}_I$ given by the expression

$$\int_{\frac{\theta_I}{3}}^{\bar{p}} (\theta_I - \theta)c dF(p) + \int_{\bar{p}}^{1} (\theta_I - \theta)c(1 - \bar{p}) dF(p) \geq \int_{\frac{\theta_I}{3}}^{\bar{p}} (1 - \tilde{\theta}_I)c_A dF(p) + \int_{\bar{p}}^{1} (1 - \tilde{\theta}_I)(c_A + c\bar{p}) dF(p)$$

such that for $\theta_I > \tilde{\theta}_I$, insurer-controlled contracts are optimal, whereas for $\theta_I \leq \tilde{\theta}_I$, agent-controlled contracts are optimal. As in the the setting with symmetric information, the results from our main model are quite robust and continue to hold, absent any major differences in bargaining power.
Competition under private information

Now consider a competitive market where the agent is privately informed about its type. To see whether agent-controlled or insurer-controlled insurance will be sustained as an equilibrium, we must again compare the insurer and agent’s net joint surplus from each type of contract. From Theorems 1 and 2, the only possible agent-controlled equilibrium contract is a pooling contract with $p^* = 1$. When such an equilibrium exists, the insurer and agent’s joint surplus is

$$JS_C(1) \equiv E_p(W(p, 1)) = \int_{\frac{1}{2}}^{1} (1 - \theta)(c_A + cp)dF(p).$$

With an insurer-controlled insurance contract, the insurer and agent’s joint surplus in this case is

$$W_C = P_M \equiv \int_{\frac{1}{2}}^{1} (\theta_I - \theta)(c_A + c)dF(p).$$

Both of these are identical to the case where agents are uninformed, and thus our conclusions coincide: for $\theta_I > \tilde{\theta}_I$, insurer-controlled contracts are offered in equilibrium, whereas for $\theta_I \leq \tilde{\theta}_I$, agent-controlled contracts are offered in equilibrium. When the agent and insurer have equal (or similar enough) bargaining power, our equilibrium results from the main model continue to hold.

B.5.1 Alternative models of control

There are other possible ways to model the optimal allocation of control over the decision whether to settle or litigate. In particular, we can also consider a setting where the insurer decides whether to litigate or not and bargains over a settlement fee with the third party, but is not subject to the “good faith” requirement that we considered above. In such a case, the insurer negotiates a settlement to maximize its own payoff, rather than its joint payoff with the agent. Suppose at $t = 1$ the agent privately learns $p$, the insurer offers a contract, $\hat{\alpha}_D$, at $t = 2$ the third party sues, at $t = 3$ the insurer observes $p$ and bargains over a settlement with the third party, under complete information, under the threat of litigation at $t = 4$. If the parties settle, the insurer pays the settlement fee; if the parties litigate, the insurer bears the litigation cost $c_A$.

In this case, the insurer’s payoff from going to litigation is $-p\hat{\alpha}_D - c_A$. For the third party, the payoff from going to litigation is $pd - c$. Hence the joint surplus from litigation is

$$p(d - \hat{\alpha}_D) - (c + c_A).$$

If instead the insurer and the third party settled, their joint surplus would be zero. Hence
settlement maximizes surplus when

\[ p(d - \hat{\alpha}_D) - (c + c_A) \leq 0 \iff p \leq \frac{c + c_A}{d - \hat{\alpha}_D} \]

In this case, the Nash bargaining transfer the insurer pays the third party is:

\[ T =pd - c + (1 - \theta)[c + c_A - p(d - \hat{\alpha}_D)] \]

The insurer’s payoff after the bargaining negotiation is:

\[ \pi_I(p) = \begin{cases} -\{(pd - c + (1 - \theta)[p\hat{\alpha}_D + c_A - (pd - c)]\} & p \leq \frac{c + c_A}{d - \hat{\alpha}_D} \\ -(p\hat{\alpha}_D + c_A) & p > \frac{c + c_A}{d - \hat{\alpha}_D} \end{cases} \]

Define \( \gamma(p, \hat{\alpha}_D) \equiv p\hat{\alpha}_D + c_A \). Then the insurer’s the cost of providing insurance \( \alpha \) to type \( p \) can be written as

\[ K(p, \alpha) = \begin{cases} \theta(pd - c) + (1 - \theta)\gamma(p, \alpha) & pd - c \leq \gamma(p, \alpha) \\ \gamma(p, \alpha) & pd - c > \gamma(p, \alpha) \end{cases} \]

Notice that \( \gamma(p, \alpha) = pd - c \) iff \( p = p^* = \frac{c + c_A}{d - \hat{\alpha}_D} \), and the insurer’s cost is positive for all types, in contrast to the case in which the agent controls the negotiation.

Consider now the agent’s willingness to pay. Without insurance, an agent of type \( p \) settled and paid \( pd - c + (1 - \theta)(c + c_A) \). When the insurer controls the lawsuit, because the agent does not bargain, the agent’s payoff when the insurer settles is 0. When there is litigation, instead, the agent pays \( p(d - \hat{\alpha}_D) \). Therefore, the willingness to pay for insurance \( \hat{\alpha}_D \) for an agent of type \( p \) is

\[ W(p, \hat{\alpha}_D) = \begin{cases} pd - c + (1 - \theta)(c + c_A) & p \leq p^* = \frac{c + c_A}{d - \hat{\alpha}_D} \\ p\hat{\alpha}_D - c + (1 - \theta)(c + c_A) & p > p^* = \frac{c + c_A}{d - \hat{\alpha}_D} \end{cases} \]

Then we have:

\[ W(p, \alpha) - K(p, \alpha) = \begin{cases} (1 - \theta)[p(d - \alpha)] & if \ p \leq p^* = \frac{c + c_A}{d - \hat{\alpha}_D} \\ -\theta(c + c_A) & if \ p > p^* = \frac{c + c_A}{d - \hat{\alpha}_D} \end{cases} \]

Let \( \Delta \equiv d - \alpha \). Then we can write the difference \( W - K \) as function of \( p^* \) and \( \Delta \):

\[ W(p, \alpha) - K(p, \alpha) = \begin{cases} (1 - \theta)(c + c_A) + (1 - \theta)(p - p^*)\Delta & if \ p \leq p^* \\ -\theta(c + c_A) & if \ p > p^* \end{cases} \]
Compare this with the benchmark case when the agent bargains:

\[
(benchmark) \ W(p, \alpha) - K(p, \alpha) = \begin{cases} 
(1 - \theta)(c + c_A) + (1 - \theta)(p - p^*)\hat{\alpha}_D & \text{if } p \leq p^* \\
-\theta(c + c_A) & \text{if } p > p^* 
\end{cases}
\]

The problem is identical except here we have \(\Delta\) instead of \(\hat{\alpha}_D\). Recall that in the benchmark case \(\hat{\alpha}_D = \frac{c}{p^*}\). Now we have \(\Delta = \frac{c + c_A}{p^*}\). Therefore the problem in terms of \(p^*\) can be written as:

\[
W(p, p^*) - K(p, p^*) = \begin{cases} 
(1 - \theta) \left[(c + c_A) \frac{p}{p^*}\right] & \text{if } p \leq p^* \\
-\theta(c + c_A) & \text{if } p > p^* 
\end{cases}
\]

Notice that the net surplus from an insurer-controlled contract \(\alpha\) that induces some cutoff type \(p^*\) is lower than from an analogous agent-controlled contract \(\tilde{\alpha}\) that induces the same cutoff type \(p^*\), for each type \(p\). This equivalence in terms of \(p^*\), between an insurer-controlled contract \(\alpha\) and an agent-controlled contract \(\tilde{\alpha}\), requires that \(\tilde{\alpha} = d - \alpha - \frac{c_A}{c}\alpha\). For types \(p < p^*\), the net surplus from an insurer-controlled contract is in fact strictly lower than from an agent-controlled one. Hence an insurer-controlled contract cannot be optimal under complete, symmetric, or perfect information, as long as an equivalent agent-controlled contract with \(\tilde{\alpha}\) is feasible, and our main results regarding agent-controlled insurance continue to hold. Figure 16 represents this graphically.

Figure 16: Net surplus from an agent-controlled contract with \(p^*\) (in blue), compared to an insurer-controlled contract with \(p^*\) (in red)