Background Risk and Insurance Take Up under Limited Liability
(Preliminary and Incomplete)

T. Randolph Beard and Gilad Sorek*

March 3, 2018

Abstract

We study the effect of a non-insurable background risk on insurance take-up choices over insurable risks, made by risk averse agents under limited liability laws. This economic environment applies, for example, to the consumer’s decision to purchase medical insurance in the face of non-insurable income risk under limited liability provided by the bankruptcy laws. We show that a wealth deteriorating background risk that is not bounded by limited liability decreases insurance take up, whereas a wealth deteriorating background risk that is bounded by limited liability does not affect insurance take up. Further, a mean preserving background risk that is not bounded by limited liability decreases insurance take up by prudent consumers, whereas a mean preserving background risk that is bounded by limited liability increases insurance take up.

JEL Classification: I 38
Key-words: Insurance take up, Bankruptcy, Background Risk.

---

*Economics Department, Auburn University, Auburn Alabama. Emails: beardtr@auburn.edu, gms0014@auburn.edu.
1 Introduction

Developed economies use various legal rules intended to guarantee minimum levels of wealth to consumers\(^1\), and to limit punishment in cases of insufficient wealth to compensate for damages. The most prominent example of such limited liability mechanisms is consumer bankruptcy law.

Sinn (1982) has shown that limited liability decreases the demand for insurance. Keeton and Kwerel (1984) derived similar results for a more detailed specification of drivers’ demands for liability insurance. Gollibier, et al. (1997) showed that limited liability induces (risk averse) firms to increase their exposure to risky investment. Sorek and Benjamin (2016) studied the implications of limited liability to health care markets, arising from consumers’ ability to avoid paying medical bills under the protection of bankruptcy laws.

The present work elaborates on the previous studies on the implications of limited liability for a consumer’s willingness to hedge against an insurable risk, by introducing an additional, non-insurable, background risk.

Eeckhoudt et al. (1996) showed that when liability is not limited, consumer risk aversion implies that wealth deteriorating background risk and mean preserving increase both work to increase the absolute risk aversion. That is consumers willing to hedge against the insurable risk is increasing.

The present analysis shows that, under limited liability, a wealth deteriorating background risk that is not bounded by limited liability decreases insurance take up, whereas a wealth deteriorating background risk that is bounded by limited liability does not affect insurance take up. Furthermore, we find that a mean preserving background risk that is not bounded by limited liability decreases insurance take up by prudent consumers, whereas a mean increase in preserving background risk that is bounded by the limited liability increases insurance take up.

Our study is closely related to the work by Fei and Schlesinger (2008) on the effect of a state-dependent background risk on the demand for insurance. In their work the size of a zero mean background risk can vary in different insurable-loss states. They show that a prudent individual will buy either more insurance or less insurance than with no background risk, depending on the relative size of the background risk in the loss states vis-a-vis the no-loss states.

In the present analysis the effective background risk also depends on the realization of the insurable-loss state, but in a non-symmetric manner: the limited-liability trunks only the downside background risk, and it is more effective under the bad realization of the insurable risk. We show that a wealth deteriorating background risk that is not bounded by limited liability decreases insurance take up, whereas a wealth deteriorating background risk that is bounded by limited liability does not affect insurance take up. Further, a mean preserving background risk that is not bounded by limited liability decreases insurance take up by prudent consumers, whereas a mean preserving background risk that is bounded by limited liability increases insurance take up.

The theoretical topic under study has a natural implication for the contemporary debate the surrounds the American healthcare policy. Prior to the implementation of the Affordable Care Act

---

\(^1\)Through welfare programs that are designed to secure a statutory minimal level of material wealth.
(ACA) in 2010, the large number of uninsured people in the United States was a focus of academic study and policy concern (Gruber 2008). The ACA provided health insurance coverage for about half of these 45 million uninsured American adults. Now, however, steps to eliminate the individual mandates and related mechanisms in the ACA, coupled with all-out efforts to repeal the entire law, again focuses attention on the insurance take up decision.

Recent studies have provided both theoretical and empirical evidence on the importance of personal bankruptcy laws in the insurance take up decision (See for example Mahoney 2015; Sorek and Benjamin 2016). In particular, bankruptcy itself provides an informal (and incomplete) form of insurance against sufficiently large medical bills However, research suggests that the protection given by bankruptcy decreases with the consumer’s wealth level, or level of attachable assets. This, in turn, implies that bankruptcy serves as a substitute for medical insurance primarily for lower income families.

The present paper can be interpreted as an examination of the effects of uninsurable income (or wealth) risk on consumer decision to purchase medical insurance when bankruptcy protection is available\(^2\).

The remainder of the paper is organized as follows: Section 2 presents the modeled economic environment; Section 3 studies the effect of background risk on insurance take up choices under limited liability; Section 4 concludes this article.

2 The Model

Agent’s utility from wealth is \( u(\cdot) \), where \( u'(\cdot) > 0 \) and \( u''(\cdot) < 0 \), so agents are assumed to be risk averse. Initial wealth, denoted \( w \), is subject to a medical risk and an income risk. The medical risk is discrete, imposing medical expense \( m > 0 \) with probability \( \pi \in (0,1) \) (and no medical expense with probability \( 1 - \pi \)). The stochastic medical expense can be insured for an actuarially fair premium \( p = \pi m \). The downward risk is bounded by a limited liability policy, such as consumers bankruptcy, under which net wealth cannot fall below the level \( B \). Finally assume that wealth is subject also to a discrete non-insurable risk which hits consumers with probability \( \rho \) and magnitude \( L \). The appendix shows that under limited liability and background risk the expected utility is not necessarily concave, and thus we cannot follow the approach applied by Eeckhoudt, Gollier and Schlesinger (1996), with more general distributions of the background risk, based on Nachman’s (1982) results. Instead, we are elaborating on the framework that was studied by Sorek and Benjamin (2016) on the medical insurance take-up decisions which depend on consumers wealth. Whereas Sorek and Benjamin (2016) study insurance purchase decisions on the extensive margins, their modeling approach to the insurable risk and the background risk are very similar to one employed by Fei and Schlesinger (2008) in their study on the individual consumer’s intensive demand for insurance in the face of a state-dependent background risk. The exact nature of the modeled background risk is clarified in the following section.

\(^2\)Where the income risk is determined by employment security and other measures of macroeconomic stability (e.g. stock market uncertainty measure).
3 Insurance take up

Under the above specifications, we study consumers’ insurance take up choices. Sorek and Benjamin (2016) show that when there is no background risk there exists initial wealth level \( \tilde{w} \) such that \( \tilde{w} - m < B \) (i.e. for which limited liability is binding), above (below) which everyone (no one) buys insurance. This is because the limited liability provided partial insurance (by limiting potential losses) which is decreasing with initial wealth level.

In what follow we will study the effect of a non-uninsurable background risk on consumers’ willingness to the actuarially fair medical insurance, starting with a wealth deteriorating background risk, and then moving to a mean preserving spread.

3.1 Wealth deteriorating background risk

3.1.1 Discrete background risk

Consider a discrete wealth deteriorating background risk, which decreases initial wealth by \( L \) with probability \( \rho \). We focus first on the case where background risk alone can not cause bankruptcy. We consider this risk "unbounded" as it is not affected by the limited liability policy. In this case, consumers’ expected utility with and without medical insurance, denoted \( E_I(u) \) and \( E_{UI}(u) \) respectively, are given by

\[
E_I(u) = \rho u(w - p - L) + (1 - \rho) u(w - p)
\]

\[
E_{UI}(u) = [(1 - \rho) \pi + \rho \pi] u(B) + (1 - \pi)(1 - \rho) u(w) + \rho (1 - \pi) u(w - L)
\]

Hence consumers’ utility gain from buying insurance, denoted \( \Delta E(u) = E_I(u) - E_{UI}(u) \), can be written as

\[
\Delta E(u) = u(w - p) - \pi u(B) - (1 - \pi) u(w) - \rho[u(w - p) - u(w - L - p)] + (1 - \pi)[u(w) - u(w - L)]
\]

Proposition 1 The unbounded deteriorating background risk works to decrease insurance take up.

Proof. The sum of the first three addends in (2) is the gain from having medical insurance when there is no background risk. The expression in the brackets in (2) is positive and increasing with \( L \) (\( \forall L > 0 \)). Hence, the gains from insurance take up are decreasing with the probability for wealth deteriorating risk \( \rho \) and its magnitude \( L \).

Suppose now that the wealth deteriorating background risk is large enough to cause bankruptcy by itself, that is \( L < w - B \). We consider this background risk as a bounded one (that is bounded
by the limited liability policy). In this case consumers’ expected utility with and without insurance are given by

\[ E_I(u) = \rho u(B) + (1 - \rho) u(w - p) \]  
\[ E_{UI}(u) = [(1 - \rho) \pi + \rho (1 - \pi) + \rho \pi] u(B) + (1 - \pi)(1 - \rho) u(w) \]

therefore, the gain from buying insurance can be written as

\[ \Delta E(u) = (1 - \rho) [u(w - p) - \pi u(B) - (1 - \pi) u(w)] \]  

**Proposition 2** The bounded wealth deteriorating background risk does not affect insurance take up.

**Proof.** Inspection of (4) reveals the expression in the brackets is the utility gain from insurance when there is no background risk at all. 

For the case \( m < w - L \) the background risk increases the incentive for buying insurance, hence over all insurance take up will decrease.

### 3.2 Mean preserving background risk

Next we consider a symmetric, mean preserving, background risk that increases or decreases income by \( L \) with equal probability - \( \frac{L}{2} \). We start again with that case of unbounded risk, i.e. \( w - L > B \), for which consumers’ expected utility with and without insurance are given by

\[ E_I(u) = \frac{\rho}{2} u(w - p - L) + (1 - \rho) u(w - p) + \frac{\rho}{2} u(w - p + L) \]
\[ E_{UI}(u) = \pi u(B) + (1 - \pi) \left[ (1 - \rho) u(w) + \frac{\rho}{2} u(w + L) + \frac{\rho}{2} u(w - L) \right] \]

and the utility gain from insurance can be presented as

\[ \Delta E(u) = (1 - \rho) [u(w - p) - (1 - \pi) u(w) - \pi u(B)] + \frac{\rho}{2} [u(w - p - L) - (1 - \pi) u(w - L) - \pi u(B)] + \frac{\rho}{2} [u(w - p + L) - (1 - \pi) u(w + L) - \pi u(B)] \]

Note that the term in the first brackets in (6) is the utility from buying insurance when there is no background risk. Then, the terms on the second and third brackets of (6) are the utility gains from buying insurance under limited liability, for initial wealth levels \( w - L \) and \( w + L \), respectively.

The first derivative of these expressions (in each brackets) with respect to initial wealth is positive:

\[ \frac{\partial \Delta E(u)}{\partial w} = u'(w - p) - (1 - \pi) u'(w) > 0, \forall \pi \in (0, 1). \]

However, the second derivative is negative
for prudent consumers, i.e. for $u''(\bullet) > 0 : \frac{\partial^2 u}{\partial w^2} = u''(w - p) - (1 - \pi) u''(w)$. In this case, for the marginal insured consumer under no background risk, the loss from having insurance under the negative background shock is higher than the gain from having insurance under a positive background shock. The latter analysis provides the following proposition.

**Proposition 3** For prudent consumers, unbounded mean preserving background risk works to decrease insurance take up.

**Proof.** Proof is provided in the analysis of equation (6) above.

Finally, consider a bounded mean-preserving risk, i.e. suppose $w - L < B$. in this case a negative background risk by itself leads to consumer bankruptcy. Moreover we assume first that here that $w + L - m > B$. That is, a positive background shock prevent the possibility of getting bankrupt. Under these assumptions, consumers’ expected utility with and without insurance is given by

$$E_I(u) = \frac{\rho}{2} u(B) + (1 - \rho) u(w - p) + \frac{\rho}{2} u(w - p + L)$$  \hspace{1cm} (7)

$$E_{UI}(u) = \pi \left[ (1 - \frac{\rho}{2}) u(B) + \frac{\rho}{2} (w + L - m) \right] + (1 - \pi) \left[ (1 - \rho) u(w) + \frac{\rho}{2} u(w + L) + \frac{\rho}{2} u(B) \right]$$  \hspace{1cm} (8)

and utility gain from buying insurance can be presented as

$$\Delta E(u) = (1 - \rho) [u(w - p) - (1 - \pi) u(w) - \pi u(B)] + \frac{\rho}{2} [u(w + L - p) - \pi u(w + L - m) - (1 - \pi) u(w + L)]$$  \hspace{1cm} (9)

**Proposition 4** The bounded mean preserving background risk works to increase insurance take up.

**Proof.** The expression in the first brackets of (8) is the utility gain from buying insurance when there is no background risk. The term in the second brackets in (8) is the gain from buying an actuarially fair insurance under unlimited liability, and thus is positive for any risk averse consumer. Hence the overall gain from buying insurance is increasing with the introduction of the bounded mean-preserving background risk.
\[ E_{UI} (w) = \frac{\rho}{2} u(B) + (1 - \rho) \left[ \pi u(w - m) + (1 - \pi) u(w) \right] + \frac{\rho}{2} \left[ \pi u(w + L - m) + (1 - \pi) u(w + L) \right] \]

and the utility gain from insurance is:

\[
\begin{align*}
\Delta E (u) &= (1 - \rho) \left[ u(w - p) - \pi u(w - m) - (1 - \pi) u(w) \right] + \\
&\quad + \frac{\rho}{2} \left[ u(w + L - p) - \pi u(w + L - m) - (1 - \pi) u(w + L) \right] + \\
&\quad + \frac{\rho}{2} \left[ u(w - p - L) - u(B) \right]
\end{align*}
\]

The sign of the above expression is ambiguous as the first two addends are positive and the last one is negative. For sufficiently low risk aversion the sum is negative.

4 Generalized risks (to be completed)

Consider all possible medical risks - for any combination of medical risk that are

Consider now a generalized wealth deteriorating background risk is subject to a general c.d.f form \( F(L) \). Then, integrating Propositions (1)-(2) yields the following Corollary

**Corollary 1** Wealth deteriorating risk weakly decreases insurance take up.

**Proof.** The c.d.f of the background risk \( F(L) \) can be decomposed into discrete risks, \( f(L) \) for which either proposition (1) or (2) hold. Hence, the fatter the tails of the distribution the larger is the relative negative effect ■

Consider now a general mean-preserving and symmetric c.d.f for the background risk \( F(L) \), with . Propositions (3)-(4) provide the following corollary for the general distribution

**Corollary 2** A mean preserving increase in background risk can either increase or decrease insurance take up.

**Proof.** Any symmetric distribution can be presented as a combination of discrete binary distributions, for each of which either proposition (3) or (4) applies. ■

5 Conclusion

TO BE ADDED
References


Appendix

Suppose that uninsured income shock, denoted \( \varepsilon \), follows a cdf \( F(\varepsilon) \) with the finite support \( 2L > 0 \), such that the limited liability policy is binding. In this case expected utility, for the general cdf and for the uniform case (on the right hand side) are given by:

\[
E(u) = \int_{-(w-B)}^{L} u(w+\varepsilon)f(\varepsilon) + F(-w+B)u(B) = \frac{1}{4L} \left[ u^2(w+L) - u^2(B) + 2(-w+B+L)u(B) \right] \tag{10}
\]

The first and second derivatives of (1A) with respect to \( w \) are:

\[
\frac{d}{dw} \frac{du}{dy} \int_{g_1(y)}^{g_2(y)} f(x,y) dx = \int_{g_1(y)}^{g_2(y)} \frac{df}{dx} f(x,y) dx + g_2'(y) f(g_2(y), y) - g_1'(y) f(g_1(y), y). \tag{3A}
\]

\( ^3 \)Equations (2A)-(3A) are derived by the Leibniz rule:
\[
\frac{dE(u)}{dw} = \int_{-(w-B)}^{L} u'(w + \varepsilon)f(\varepsilon)\,d\varepsilon = \frac{1}{2L}[u(w + L) - u(B)] 
\] 
(2A)

\[
\frac{d^2 E(u)}{dw} = \int_{-(w-B)}^{L} u''(w + \varepsilon)f(\varepsilon)\,d\varepsilon + f(B)u'(B) = \frac{u'(w + L)}{2L} > 0 
\] 
(3A)

Equation (3A) demonstrates the possible convexity of the expected utility function under limited liability and background risk which prevents us applying the approach used by Schlesinger et al. (1996), that is based on Nachman’s (1982) results.