A Model of Decentralized Search inside Organizations

Zhouzheng Li
Department of Economics, Ohio State University

February 15, 2015

Abstract

This paper analyzes a dynamic model where experts sequentially search potential projects and make recommendations to the decision maker. The decision maker, usually a generalist, knows little about the details of projects, has to pick one project to launch for the future after hearing reports from the experts. The experts (i.e. division managers) search repeatedly and sequentially send costless report to the decision maker. Both experts as well as the decision maker shares the return of the implemented project, but each expert gets extra fixed benefit if his project is implemented. Our model reflects two features of hierarchical decision making inside an organization especially in R&D companies. One is the dynamic search and the other is the information transmission between the decision maker and division managers, however less research is done previously as we are aware of. Attracted by the extra benefit, two experts tends to exaggerate the projects in their report. However, the exaggeration will harm the decision maker's payoff. As experts take turns to report, we show that always letting one expert report is the best among all orders of reporting. When incorporate effort in search, the decision maker's situation is different. Even though both experts tend to exaggerate the quality of the project, they also compete to search harder which would cancel out the former side-effect. We show that letting two experts to search projects could sometimes be better than just having one expert to search. We also figure out the optimal order of reporting among all possible orders.

JEL Classification Numbers: D23, D74, D83

Keywords: Multiple experts; Search; Information transmission; Competition

1 Introduction

Inside many organizations, such as R&D firms, and research organizations, the generating of new ideas, hiring new senior employees, and developing new products are vital to the success of the organization. For example, Google has many divisions spending a significant part of their working time developing different new ideas among search engine, advertisement, apps, self-driving cars; Economics departments send professors from different fields to search and interview candidates on job markets every year in AEA meeting. All of those examples involve the activity of decentralized search inside an organization. Usually, when a project is found, the manager of the division make presentations to the head of the company about the profitability (quality) of the project. Since the decision maker (DM) of an organization is not a specialist in every field, he faces challenges of picking and implementing a successful project only by their communication during the presentations. The information are asymmetric between experts and the decision maker, i.e. the quality of potential projects, and the search activities are hard to observe and monitor (Aghion and Tirole 1994, Holmstrom 1989, Holmstrom and Milgrom 1994), which brings many issues in theory and practice. The decentralized search behavior is very common inside an organization and has several distinctive features compared with search in market. First, the structure is hierarchical, as senior management (decision maker) selects and implements projects that experts propose. The second feature is that experts share the return of implemented projects without bearing the risk of obtaining nothing if they fail to innovate (Levinthal and March 1981, 1993). While in competitive market, the searchers faces the risk of failure such as contingent search among headhunting companies. Another feature is the scarcity of resource inside an organization (such as limited strategic and financial resources, limited managerial attention), and it is impossible to implement every proposed projects (Rotemberg and Saloner 1994, Lovas and Ghoshal 2000, Foss 2003, Ocasio 1997, Stein 1997). So the selection among projects brings competition between experts. The stylized structure of search behavior is intensively studied in many management literatures, and most of them focused on contracting and designing reward system (Lazear and Rosen 1981, Lambert et al. 1993, Prendergast 1999). They modeled the information of projects as observable or partially observable by the DM. In this paper, we assume the information is not observable by the DM, but the DM can infer some information through communication with experts. We ask these questions, how experts communicate with the DM? Will competition between experts helps the DM to obtain a better project?

Briefly speaking, the search and decision making process follows a certain time line when a firm is planning to launch a new project. The experts develop and search new ideas for potential projects and make recommendations to the DM. The DM who can not observe the quality of project has to make an "accept" or "reject" decision after listening to their presentations. There is a pre-assigned order for experts' presentations, i.e. a random expert could present in the first period, then if his project gets rejected, the other expert could present his project in the following period. The order could be any pre-assigned sequence and different sequence may affect behaviors of both experts and the DM. In each period, an expert conducts search and make presentation. If an acceptance decision is made, that project will be implemented; otherwise, another period starts in which another expert conducts search and makes presentation. This process continues until one project is finally selected. Delay cost incurs in each period which could be interpreted as the loss due to a delay in making an investment. So there will not be endlessly search. As in the same company, both experts and the DM prefer high quality project being launched sine they both share the profit. However, the expert will be given an extra benefit if his project gets adopted. In the cheap talk literature, we also call the extra benefit as the expert's bias and the bias is commonly known. Since only one project will be selected, experts competes with each other and tends to exaggerate the quality of the project in order to capture the extra benefit. We want to ask how much the information will be transmitted to the DM in the presence of the biased experts? We also model experts with intensive search ability, i.e. each expert could make high effort or low effort to search, which changes the distribution of the quality of the obtained project. With high effort to search, the expert is more likely to obtain a project, but there is cost of high effort. Low effort will change nothing and there is no cost in search. So we may think that experts also compete in search which may provide higher quality project. One question is that if experts can exert effort in search, will this elicit a good project? As we mentioned above, the order of presentation could be any sequence. The DM has ability to pre-assign the presentation order to the experts. We will show the presentation order will change the experts incentive of communication and search and finally affect the quality of project selected. From the mechanism design perspective, without considering the money transfer, if the DM can make binding commitment assigning the orders, then what will be the optimal presentation order? One may argue that with money transfer, to let the winning expert subside the loser will cure the incentive problem. However, the mechanism without money transfer is realistic and more interesting in our model, because [literature, justification]

In classic search problem, there is one individual makes a draw each period from exogenous and known distribution which is independently and identically distributed across time. After every draw, the DM has to decide whether to accept the search result and the realization of the payoff or to reject and continue to search. The DM faces tradeoff between search and stopping. Higher payoff will eventually arrive with further search, but the there is cost of delay across periods. There are a lot of surveys on search in the framework of optimal stopping time (Lippman and McCall 1976, Mortensen 1986, Rogerson. et al. 2005). Our model has two elements, search and communication. In classic search model, the communication is not needed if there is only one expert in search. When the DM delegate search to two experts or more, the experts face competition. Since the information and the search is not observable, we study the communication as the only channel for the DM to infer the quality of projects. In organizational search, multiple experts search projects, and competition turns out to be the key feature. Competitions lie in search and communication. While competition in communication brings loss on information, competition in search motivates experts to exert effort. On the other side, compared with previous models of organizational search, we introduce the element of communication, so we connect the search model to the cheap talk models (Crawford and Sobel 1982, Battaglini 2002, Chakraborty and Harbaugh 2010, Dessein 2002, Li, Rantakari, and Yang 2014). Since our model studies decentralized search, the literatures on collective search model are related to our paper. In those papers, search can be conducted by committees in which the implement of project needs agreement among all experts, and many are modeled through voting (Albresht, J. A. Anderson, and S. Vroman 2010, Chan and Suen 2012, Compte and Jehiel 2010, Moldovanu and Shi 2013). None of them studies the hierarchy and the communication inside an organization, but in real life, both ways of search exist.

In the paper, we focus on a stationary equilibrium which is widely used in search literatures. We prove the existence and characterize the equilibrium in different meeting procedures. Staring from the single expert model, we show that since the DM only consult one expert, the expert always capture the extra benefit, so their interests are totally aligned and the expert can be treated as the DM. With two experts, we discussed effortless search and intensive search. Start from effortless search where experts does not have an option on how to search, both experts make random draw from the same set of values with certain successful rate. They prefer high quality project, but the biases provide experts incentive to exaggerate. If two projects' quality are close but not very low, then each expert tends to send high message trying to get his own project implemented. With multiple experts, the presentation order matters. We focus sequentially reporting order indicated by p, a probability that the same expert stays presenting another project in the next round. Some specific orders could be observed from the reality: an alternating rule, in which each expert takes turns to propose projects; a Markov rule, a coin is flipped to decide whether the same expert should stay in the next period. In equilibrium, the DM makes "yes" or "no" decision according to the experts' report, and the experts has a cutoff strategy, in which the cutoff equals the benefit that he reports high message and the value of continuing search. This sequential presentation orders and private benefit create competition between experts in communication. Compared to the single expert model, capturing the extra benefit is never a sure option to the experts. Only if the expert's project is accepted, he can obtain the extra benefit. The tradeoff for the experts is not only between the marginal benefit of search and cost of delay, but also the extra benefit. This effect will make the cutoff lower and experts will be more hurry to conclude the search and capture the extra benefit ahead of the opponent. The DM always accept the project when the experts report high value about the project because given the cutoff quality, waiting only incurs extra cost other than obtaining higher expected quality. Generally speaking, the single expert model could be treated as a special case of the two expert model. The extra benefit might go to the other expert in any sequential rules, so continuing to search increase the risk of losing the extra benefit. In order to compensate the loss, the experts are more hurry and less picky on projects. For two-expert model, we also demonstrate examples of an alternating rule and a Markov rule (with transition probability all 1/2). The result shows that the continuation value of search determines the cutoff in different reporting orders. In the Markov rule, each expert still has half chance to make proposal and grab the extra benefit in the following period, while in the alternating rule, the other expert has 100% chance. Therefore, the continuing to search is less risky in the Markov rule, so both experts become more cautious on investigating the quality of the project instead of bluffing to grab the bonus. The search duration depends on the cutoff of acceptance. Since the project is a random draw from a knowing distribution, the higher acceptance cutoff, the longer it takes to find. Obviously, it takes longer time to find acceptable project in the Markov rule than in the alternating rule. Longer search duration has higher delay cost, however, we show that the delay effect is less than the benefit of higher quality, so the Markov rule generate higher expected payoff for the DM. If search is effortless, the DM always has loss because of communication. If we increase the probability of staying in the next period, both that expert and DM are better off. Therefore, it is always the best to let single expert search.

After investigating the baseline model, we will discuss the model endogenous search behavior. The search behavior is vital to the search efficiency: with probability s, an expert find a project from a fixed distribution on the support $\Omega = [0, 1]$ which is i.i.d across periods. Otherwise, he finds nothing. In the baseline model, we can focus our attention on competition in communication and study how the conflict of interest of experts affects the DM's decision. While studying the model of endogenous search, we will show how competition plays like a double-edged blade to the DM. In our model, we will show the order of reporting really matters. Two experts sequentially reports their projects to the DM, and in the first period, they will flip a coin to decide who is the first. In reality, there are several forms of orders, we study cases that there are sequentially assigned orders for experts to recommend, and we compare two specific cases: an alternating rule and a Markov rule with $p = \frac{1}{2}$. As a benchmark case, we start from analyzing single expert case.

The search is intensive in many examples of decentralize search inside an organization. The experts exert effort searching for potential project, then he has a larger chance to be successful or develop a high quality idea, but exerting effort has cost. There exists the equilibrium that both experts exert high effort in search. As the cost of search is not very high, the benefit of search is higher than the cost of effort. Since both experts prefer high quality project as well, they would compete on searching as well as capturing private benefit. Under those conditions, the competition in search alleviate efficiency loss in communication. Even though the loss in communication still exist, the expected quality rises because of competition leads to higher quality level. It is possible that the DM can be better off by letting multiple experts search instead of one. More generally, we investigate the effect of the transition probability and the optimal design of the meeting procedure. An interest result is that, when the expert has larger probability to stay in the next period, the higher acceptance cutoff there will be, but it is less likely that the expert exerts high effort. As the DM's payoff is increasing in project quality, the optimal transition probability is set on the boundary when high effort equilibrium exist.

We also considered other meeting procedure, the simultaneous rule. When the DM meets two experts at the same time, we show that the equilibrium is a partition equilibrium . The first cutoff (acceptance cutoff) is determined by the continuation value of search and other cutoffs comes from the comparison of messages from both experts. Since the number of partitions is finite, fully revealing equilibrium does not exist. Unlike the classic cheap talk equilibrium, the relation between the DM' payoff and the number of partitions is non-monotonic. The first partition determines whether to compare both candidates or to continue to search. Since the partition length is increasing from the second partition, with more partitions, the DM can reveal more information about the projects' quality, but the first partition cutoff is lower, which means that the expected quality he picks from is lower. We show the optimal number of partitions and compare it with random rule as well.

In order to be tractable, we made some restrictions. One restriction is "non-recall" of the projects which means the rejected projects will never have chance to meet again. We also restrict our analysis to the pure strategy and stationary equilibrium. These restrictions are common in search literatures we introduced above.

In the next section, we are going to formally present the model and provide some preliminary analysis. In section 3, we study the equilibrium when competition is in communication. In section 4, we study two-dimensional competition case and propose the optimal design of meeting procedure. In section 5, we show some of the results from simultaneous communication. In section 6, we show some of the results of asymmetric experts. The last section is our conclusion.

2 Model

A decision maker (DM) is planning to launch a new project in two potential areas. There are two experts $i = 1, 2$ in each department responsible for collecting information and searching for qualified projects. The return of the project is θ_i distributed according to F_i on support $\Omega_i \subset [0, 1]$ which is independently and identically distributed across period. The realization of θ_i can only be observed by the expert i. Time is discrete and all players discount the future at the same rate $\delta \in (0, 1)$. In each period, the DM consults one expert according to a pre-determined order. We model the order as a symmetric Markov process with a transition matrix $\begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix}$. We call p the persistency, which is the probability of consulting the same expert in the next period. $1-p$ is the probability of consulting the other expert. Many rules we could observe in reality are included in this Markov rule. For example, $p=0$ defines an alternating rule, in which the DM deterministically consults the other expert in the next period; $p=1$ means the DM always consults the same expert; and $p=1/2$ defines a random rule in which the DM flips a coin to decide who is the next. In the initial period, the expert is randomly selected.

At the beginning of the period, an option θ_i arrives for the expert from a given distribution $F(\theta)$ on the support Ω_i and the value θ_i is realized to the expert. The the expert sends message m_i about the project to the DM. After learning the message, the DM decides which project to adopt, or to continue to the next period. Let $d_t \in \{1,2\}$ be the DM's acceptance decision in period $t = 1, 2, ...$

Given the project d adopted at period t, the DM's payoff is $U_p(t,d) = \delta^t \theta_d$. δ is the discount to the future value. The delay cost would not be very small, so that the DM would not end up with always ignoring one expert. The DM faces to tradeoff between the return of the project and the delay cost accumulating in each period. The agent i's payoff is given as follows, given period t and choice d,

$$U_i = \begin{cases} \delta^t \theta_j & \text{if } d \neq i \\ \delta^t(\theta_j + b_i) & \text{if } d = i \end{cases} \quad t = 0, 1, 2, ...$$

where $b_i \in [0,1)$ is agent i's private benefit (bias) if his project gets adopted. Actually, as we will analyze that the period is determined by the acceptance cutoff $\hat{\theta}$ in equilibrium, duration t is a function of $\hat{\theta}$. From the payoff functions of the DM and experts, there is some alignment of interest, i.e. they all care about the return of the project and they prefer to have the higher quality project to be implemented. However, interest conflict exists between experts. The expert whose project being implemented will obtain an extra benefit b_i called bias. Since only one expert can collect the private benefit, they compete to get their own project implemented. If the returns of both project are close, both experts are eagerly to capture the private benefit. We will show that the bias affect how expert report the information of their projects to the DM. For convenience, we make restriction of non-recall; i.e. once a project is discarded, it is never to be considered. Each player makes decision to maximize his own expected payoff.

Let M_{it} to be the set of message at period t, $M_t = \Pi_{i=1}^2 M_{it}$, and let $\mathbf{M}^t = \Pi_{s=1}^t M_t$ strategy for the DM is $d_t : \mathbf{M}^t \rightarrow \{yes, no\}$. The strategy for the expert is the communication rule $\mu_{it} : \Pi_{s=1}^t \Omega_i \rightarrow M_{it}$. And the belief for the DM is a function $g_t : \mathbf{M}^t \rightarrow \Omega$.

3 Baseline Model

We focus on stationary Bayesian equilibria that employ cutoff strategies.

Lemma 1 *In equilibrium, the expert has at most two meaningful messages.*

Proof. Suppose there are three messages, l, m, and h for the expert i, with $E_g(\theta_i|l) < E_g(\theta_i|m) < E_g(\theta_i|h)$. Consider the pure strategy equilibrium, the DM only decides to accept or reject. Since the DM's payoff is $E_g(\theta|msg)$, it could only be the case that the DM accept the project by hearing h, rejects by hearing l and m; or accept the project by hearing h and m, rejects by hearing l. In any of those cases, for example the former one, the message l, and m leads to the same outcome that expert i's project gets rejected. We could always combine those consecutive messages to one message. Therefore the expert has at most two meaningful messages. ■

The expert message could be in any forms, but that are equivalent to the two-message equilibrium in outcome. The expert sends message as "high" and "low" conditioned on the realization of the project he find. Specifically, expert i reports "high" if $\theta_i \geq \hat{\theta}_i$; otherwise sends "low", where $\hat{\theta}_i$ is the cutoff for expert i. The DM accept the project immediately after hearing the "high" message, and reject the project by hearing the "low" message and then pick one expert to consult according to a pre-determined order p in the next period. Think about the DM first, the DM has two actions, "accept" and "reject". Any messages that could induce the DM's acceptance are equivalent, so we just refer them as "high" message, and vice versa.

Let v_i^a and v_i^n denote, respectively, the value of not being consulted (inactive) and being consulted (active) at the beginning of the period. The values for each expert are determined by two Bellman equations:

$$\begin{cases} v_i^a = max_{\bar{\theta}_i} \int_{\bar{\theta}_i}^1 (\theta_i + b_i) dF(\theta_i) + F(\bar{\theta}_i)\delta(pv_i^a + (1-p)v_i^n) \\ v_i^n = \int_{\bar{\theta}_j}^1 \theta_j dF(\theta_j) + F(\bar{\theta}_j)\delta(pv_i^n + (1-p)v_i^a) \end{cases} \quad i = 1, 2, i \neq j \tag{1}$$

Where we use $\hat{\theta}$ for the equilibrium cutoff, and $\bar{\theta}$ for any cutoff available. In the first equation, when the DM is consulting the other expert, the value of being inactive has two parts except the delay cost each period, the expected return of the other project if the other project is implemented, and the continuation value of the next period. The second equation is similar to the first except that the expert has chance to obtain the private benefit. The expert chooses the cutoff to maximize the value.

The first order condition of the second equation is, [1]

$$\hat{\theta}_i + b_i = \delta(pv_i^a + (1-p)v_i^n) \tag{2}$$

The above FOC is the indifference condition. The left hand side is the current return if the DM accept the expert i's project, and the right hand side is the value of continuing searching.

The system of equations can be rewritten as,

$$\begin{cases} v_i^n &= \int_{\hat{\theta}_j}^1 \theta_j dF(\theta_j) + \delta(pv_i^n + (1-p)v_i^a)F(\hat{\theta}_j) \\ v_i^a &= \int_{\hat{\theta}_i}^1 (\theta_i + b_i)dF(\theta_i) + \delta(pv_i^a + (1-p)v_i^n)F(\hat{\theta}_i) \end{cases} \quad i = 1, 2, i \neq j \tag{3}$$

The value of v_i^n and v_i^a can be solved recursively as functions of $\hat{\theta}$. Then, using the indifference condition, the cutoff can be solved.

For the DM, let v_i^p be the value that expert i is active.

$$v_i^p = \int_{\hat{\theta}_i}^1 \theta_i dF(\theta_i) + \delta(pv_i^p + (1-p)v_j^p)F(\hat{\theta}_i) \tag{4}$$

The DM's value is similar to the active expert's value except the bias b_i. In equilibrium, the DM decides to accept the project i if expert i sends message "high", or reject if hearing "low" message. Obviously, babbling is one equilibrium of the model. If experts always send "high" message, then the DM always ignores experts' recommendations. We are interested in the meaningful equilibrium where the DM could extract some information about the projects.

Proposition 1 *There exist stationary Bayesian equilibrium if $b_2(1-\delta p)/\delta < E(\theta)$. And the equilibrium strategy for the experts is characterized by an acceptance cutoff $\hat{\theta}_i$, if $\theta \geq \hat{\theta}_i$, the expert*

[1] This is one-shot deviation check. fix other periods value under optimal cutoff, to see that cutoff is the best response in this period.

reports "high" message to the DM, otherwise reports "low"; and the DM accepts the project if the message is "high", and rejects at "low" message;

From Proposition 1, the existence of cutoff equilibrium requires that the bias of experts is not too high. And in the following proposition, we show the condition for the uniqueness in symmetric equilibrium.

Proposition 2 *There exist a unique symmetric stationary Bayesian equilibrium if $b(1-\delta p)/\delta < E(\theta)$ and $\delta < \frac{1}{f+1}$. And the equilibrium strategy for the experts is characterized by an acceptance cutoff $\hat{\theta}$, if $\theta \geq \hat{\theta}$, the expert reports "high" message to the DM, otherwise reports "low"; and the DM accepts the project if the message is "high", and rejects at "low" message;*

From the existence of the meaningful equilibrium, the bias should not be too large, or else the bias provides too much incentive for the experts to lie thus the DM would not believe any message from them. In the equilibrium, both experts message has some effect of strategic complement. Just think about one expert increase the cutoff, then the inactive period value to the second expert increases which would encourage the second expert to rise his cutoff. Due to the complementarity of strategy, if both expert are very confident in the future, it is possible that they would "coordinate" to keep the current cutoff or to be more picky. Therefore, multiple equilibrium might appear. How they coordinate with multiple equilibria might be interesting, however, in this paper, we exclude multiplicity by restricting the discount factor in order to emphasis the main message of this paper. We focus our attention on the symmetric equilibrium, and there could be asymmetric equilibrium when experts are symmetric. We left the asymmetric equilibrium analysis at the end of the paper.

In the baseline model we focus on the competition of communication, both experts obtains a project from a known distribution. Both experts are identical, so the search behavior looks as if one expert search again and again. However, the incentives are different from that of the single expert search. With two experts search, one expert would not have chance to capture the extra benefit when he is not in active period. That could lead the expert to exaggerate the quality of the project in order to capture the benefit. Thus, the cutoff of reporting "high" ($\hat{\theta}$) would be lower. Due to the discount for the future, in equilibrium the DM would accept the project when the expert report the project as "high" quality project.

3.1 The DM's problem and single expert problem

As a benchmark, we start from analyzing the decision maker's problem. If the decision maker could search project himself, this becomes the classic optimal stopping time problem. If the DM delegate search to one expert, the expert will definitely obtain the extra benefit since the expert is the only source searching projects. From the perspective of the expert, since his project will always be implemented, the extra benefit doesn't basically affect the expert's incentive in report. The interests of the DM and the expert are aligned, and their tradeoff is the cost of delay so the acceptance cutoff of single expert case is the ideal cutoff of the DM's.

Firstly, we focus on the DM's problem. If the DM search for projects, assume the stopping cutoff is $\hat{\theta}_p$, then his value can be written recursively as the following form,

$$v^p = \int_{\hat{\theta}_p}^{1} \theta dF(\theta) + \delta v^p F(\hat{\theta}_p) \tag{5}$$

Clearly, given a cutoff $\hat{\theta}_p$, the expected quality of the project is $\int_{\hat{\theta}_p}^{1} \theta dF(\theta)$. The probability of continuing searching is $F(\hat{\theta}_p)$ and the next-period value is discounted by δ. The alternative form of the value is,

$$v^p = \frac{1}{1 - \delta F(\hat{\theta}_p)} \int_{\hat{\theta}_p}^{1} \theta dF(\theta) \tag{6}$$

There is trade off between the return of the project and the search time. Setting higher stopping cutoff would find higher quality project on average but finding such project takes longer time and the future value is discounted. The first order condition of the value function is,

$$\frac{dv^p}{d\hat{\theta}_p}|_{\hat{\theta}_p=\hat{\theta}_p^*} = \frac{-\hat{\theta}_p^* f(\hat{\theta}_p^*)(1 - \delta F(\hat{\theta}_p^*)) + \delta \int_{\hat{\theta}_p}^{1} \theta dF(\theta) \cdot f(\hat{\theta}_p^*)}{(1 - \delta F(\hat{\theta}_p))^2} = 0 \tag{7}$$

The above equation can be simplified as,

$$\hat{\theta}_p^* = \delta v^p \tag{8}$$

Thus, the optimal stopping cutoff $\hat{\theta}^*$ makes the DM indifferent between ending search and continuing search.

$$\hat{\theta}_p^*(1 - \delta F(\hat{\theta}_p^*)) = \delta \int_{\hat{\theta}_p^*}^{1} \theta dF(\theta) \tag{9}$$

Simplify,

$$\hat{\theta}_p^* = \frac{\delta}{1-\delta F(\hat{\theta}_p^*)} \int_{\hat{\theta}_p^*}^1 \theta dF(\theta) \tag{10}$$

If the DM let a single expert to search, actually the single expert's behavior is the same as in the decision making problem. Actually, the single expert's value function is similar except the bias term,

$$v = \int_{\hat{\theta}}^1 (\theta + b) dF(\theta) + \delta v F(\hat{\theta}) \tag{11}$$

alternatively,

$$v = \frac{1}{1-\delta F(\hat{\theta})} \int_{\hat{\theta}}^1 (\theta + b) dF(\theta) \tag{12}$$

The only difference between (6) and (12) is that the expert has a constant extra benefit b than the DM. This does not affect the expert's incentive on choosing cutoff of recommendation. In equilibrium, the cutoff is determined by the indifference condition of the expert,

$$\hat{\theta} + b = \delta v \tag{13}$$

Thus,

$$\hat{\theta} + b = \frac{\delta}{1-\delta F(\hat{\theta})} \int_{\hat{\theta}}^1 (\theta + b) dF(\theta) = \frac{\delta}{1-\delta F(\hat{\theta})} \int_{\hat{\theta}}^1 (\theta) dF(\theta) + \frac{\delta(1-F(\hat{\theta}))}{1-\delta F(\hat{\theta})} b \tag{14}$$

$$\hat{\theta} = \frac{\delta}{1-\delta F(\hat{\theta})} \int_{\hat{\theta}}^1 (\theta) dF(\theta) + \frac{\delta - 1}{1-\delta F(\hat{\theta})} b \tag{15}$$

which is similar to eq (8) except the bias term. Because of the discount, the expert's interests is distorted from the DM's. Therefore, the equilibrium cutoff of single expert case is distorted away from the ideal cutoff of the DM's. The following proposition shows that there is a downward distortion of expert's cutoff.

Proposition 3 *In the decision maker's problem, there exist unique optimal stopping cutoff $\hat{\theta}_p^*$. When $\theta \leq \hat{\theta}_p^*$, the DM's expected payoff is monotonically increasing. If the DM let a single expert to search, then the experts' equilibrium cutoff $\hat{\theta} < \hat{\theta}_p^*$.*

The first part of the proposition shows a classic stopping time problem for the DM. In the proposition, we show that there is a unique peak of the the DM's payoff. And what good for the model is that there is monotonicity of the DM's payoff when the cutoff is smaller than the optimal cutoff. The monotonicity gives a lot of convenience in analysis. In the second part of the proposition, it shows that the single expert's interest is not perfectly aligned with the DM. Although the expert could always secure the extra benefit b, but when to get the extra benefit also matters. Due to the discount to the future value, the expert has extra incentive and tries to conclude the search earlier, so the expert would be less picky than the DM should be.

3.2 Two experts

Will the competition between two experts benefit the DM? The answer is no. When two experts search project for the DM, we show that the competition worsens communication between experts and the DM. The expert's payoff is determined by the profit of the selected project and the extra benefit if his project is chosen. However, the DM only cares about the quality of the project. We show that the extra benefit gives the expert incentive to manipulate the message sent to the DM. For example, when there are two experts take turns to report new projects to the DM, each expert will end up exaggerating the quality of his project, in the way of saying "my project quality is high, which is 8 out of 10" even when the actual quality is not as high. In equilibrium, the DM accept the project reported as "high", because that it the best the DM could do. It is very interesting that the competition between two experts actually harm the DM's payoff. When the competition is more tough, experts tends to exaggerate more. The toughness of the competition is different in different orders of recommendation. In order to capture the toughness, we denote the reporting order (rule) characterized by p (we call it persistence), the probability that the expert remains to report in the next period, we show that the difference between values in active period and the inactive period is higher with smaller p. The difference is larger, each expert is more willing to talk big in order to capture the extra benefit when he is reporting.

Let's first symmetric experts with $b_1 = b_2 = b$ and suppose the cutoff of sending "high" message is $\hat{\theta}$. The experts' values are determined by the following Bellman equations,

$$\begin{cases} v^a & = \int_{\hat{\theta}}^1 (\theta + b) dF(\theta) + \delta(pv^a + (1-p)v^n)F(\hat{\theta}) \\ v^n & = \int_{\hat{\theta}}^1 \theta dF(\theta) + \delta(pv^n + (1-p)v^a)F(\hat{\theta}) \end{cases} \tag{16}$$

The difference (Δ) of the continuation value between active and inactive period is,

$$\Delta_{a-n} = \frac{b(1-F(\hat{\theta}))}{1-\delta(2p-1)F(\hat{\theta})} \tag{17}$$

Since the expert might capture the extra benefit in active period, the difference is always positive. Notice that the numerator of the above equation is positive, and the denominator is positive. The difference is increasing in b and one could imagine that given any reporting rule, if the difference is larger, then the active period is more attractive to the expert.

The average value of both active and inactive period is,

$$\bar{v} = \frac{1}{2}(v^a + v^n) = \frac{1}{2(1-\delta F(\hat{\theta}))} \int_{\hat{\theta}}^1 (2\theta + b) dF(\theta) \tag{18}$$

Notice that the average value does not contain p, so p only allocate total fixed value to active periods and inactive periods. When p is smaller, each expert has less chance to report in the next period, given projects in hand, it makes them more likely to report as high quality project. However, from the DM's perspective, the DM likes to make the chance for the same expert to report higher as to lower experts' incentive to mis-report. The following proposition indicates the p should be 1.

Proposition 4 *Among all sequentially reporting orders characterized by p, the equilibrium cutoff quality is increasing in p, and the optimal order for the DM is $p = 1$, letting one expert always report.*

Proof. By (28)(29), and $v^a > v^n$, so $pv^a + (1-p)v^n$ is increasing in p. Since v^a, v^n are positive and continuous in $\hat{\theta}$, by Lemma (6) in appendix, the cutoff quality $\hat{\theta}$ is increasing in p. It is easy to verify that $\hat{\theta} \leq \hat{\theta}_S \leq \hat{\theta}_p^*$. By Proposition 3 (monotonicity), $v^p(\hat{\theta}) \leq v^p(\hat{\theta}_S) \leq v^p(\hat{\theta}_p^*)$ for all $p < 1$. By Proposition 3, the DM's payoff is increasing if . Therefore, the optimal order for the DM achieves at $p = 1$. ■

Usually we might think competition between experts would help the DM to obtain higher quality project, but this result seems a little bit counter intuitive at the first glance. Actually, it is quite clear to understand. The main source that harms the competition is experts' biases. In the stand of an expert, when it is his turn to recommend the project, he faces tradeoff between exaggeration and being honest. Given the other expert and the DM's strategies, the exaggeration could increase the possibility that his project being accepted thus capturing the extra benefit, but the quality of his project might be low; while being honest, he would give the chance to his opponent, but a better project might be found. With one expert searching, that expert would always capture the extra benefit, and the only thing matters is when to capture it. However, with two experts search, the possibility of losing the extra benefit is larger than that of single expert search, which provide experts incentive to persuade the DM harder by manipulating the messages, thus the communication becomes more noisy. The DM's expected payoff was harmed by the noisy communication. This phenomena could always be observed in politics where two or more parties have similar power to speak, and both parties tend to have more incentive to manipulate what they speak for their own good. In those scenarios, the greater good is always harmed due to the competition between politicians.

The following corollary discusses how other parameters affect the communication.

Corollary 1 *When two experts search,*

1. *if experts' bias is higher, experts tend to exaggerate more;*
2. *if the discount rate δ is smaller, experts tend to exaggerate more.*

As expert's biases are higher, it rises the difference of values between active periods and inactive periods. The active periods seems more attractive to them. So, experts tends to exaggerate the quality of his project hoping to capture the extra benefit. The discount rate δ reflect their patience. If the experts are not patient (discount the future value largely), then the current project and the extra benefit in current period are more attractive than the project obtained in the future, so experts exaggerate in order to get project accepted.

3.3 Some focal rules of reporting

In many real life examples, many groups compete intensively for influence where the general audience know little about true information. Between political parties, political power struggle is very common. They acquires balanced chance to speak. In the real life, we always observe simple rules fair to each party, such as the alternating rule, where each party take turns to speak; a super symmetric rule, called random rule, in which each party flips a coin to decide who to speak in each period. The question is, does the DM really benefit from multi-party competition? The answer is clearly not, because when parties pursuing their own interest, no matter how small interest that would be, both parties tends to speak loud but send less meaningful information. In this section, we focus on the balanced focal rules, and demonstrate how those competition harms the information and which balanced rule is better. It seems that competition could somehow balance their power that might do better to the society. However, it is the self-interest that worsen the communication. Both parties tends to speak loud but less meaningful information is sent to the public. We shall assume $F(\theta) \backsim U[0,1]$.

Definition 1 *An* ***alternating rule*** *is a reporting order with persistence* $p = 0$*. A* ***random rule*** *is a reporting order with* $p = 1/2$*.*

Alternating rule:

In the alternating rule, the expert's cutoff quality is $\hat{\theta}_A$, and the expert's values are determined by the following Bellman equations,

$$\begin{cases} v^n & = (1-\hat{\theta}_A)\frac{1+\hat{\theta}_A}{2} + \delta\hat{\theta}_A v^a \\ v^a & = (1-\hat{\theta}_A)(\frac{1+\hat{\theta}_A}{2} + b) + \delta\hat{\theta}_A v^n \end{cases} \tag{19}$$

solve v^n, we have the indifference condition,

$$\hat{\theta}_A + b = \delta\frac{1-\hat{\theta}_A^2}{2(1-\delta\hat{\theta}_A)} + \frac{\delta^2\hat{\theta}_A b}{1-(\delta\hat{\theta}_A)^2} \tag{20}$$

where the LHS is the payoff of sending "high" message that equals the payoff of continuing to search as the RHS. The RHS contains the discounted expected return, and expected extra benefit. The experts' expected payoff from the quality of the project are the same as the DM's.

Markov rule:

In Markov rule with $p = 1/2$, the Bellman equations are,

$$\begin{cases} v^n &= (1-\hat{\theta}_M)\frac{1+\hat{\theta}_M}{2} + \frac{1}{2}\delta\hat{\theta}_M(v^a + v^n) \\ v^a &= (1-\hat{\theta}_M)(\frac{1+\hat{\theta}_M}{2} + b) + \frac{1}{2}\delta\hat{\theta}_M(v^a + v^n) \end{cases} \tag{21}$$

and the corresponding indifference condition is,

$$\hat{\theta}_M + b = \delta\frac{1-\hat{\theta}_M^2}{2(1-\delta\hat{\theta}_M)} + \delta\frac{b(1-\hat{\theta}_M)}{2(1-\delta\hat{\theta}_M)} \tag{22}$$

Compared with the alternating rule, the indifference condition is similar except the expected extra benefit term. Too see it clearly, let $\delta = 1$, the benefit term in the random rule is $\frac{b}{2}$, since the chance of reporting project is the same. However, in the alternating rule, the expected extra benefit is less, since in the next period this expert won't have chance, and in the next next period, the chance really depends on whether his opponents project is accepted, which largely reduce the expectation.

Corollary 2 *Compared with the alternating rule, the cutoff quality is higher in Markov rule.*

This corollary is a direct result from Proposition 4. In the alternating rule, the expert have a lower cutoff, in other words, less picky to select the project since he doesn't have chance to report and capture the benefit in the next period. Intuitively, the more likely the expert has chance to bring out his project, the more picky the expert is. When the expert is sure to stay in the next period ($p = 1$), there is the maximum of the acceptance threshold. $p = 1$ returns to the single expert model. The figure 3.3 shows the relation of cutoffs in reporting order. When $0 < p < 1$, the $\hat{\theta}$ lies between $\hat{\theta}_A$ and $\hat{\theta}_p^*$.

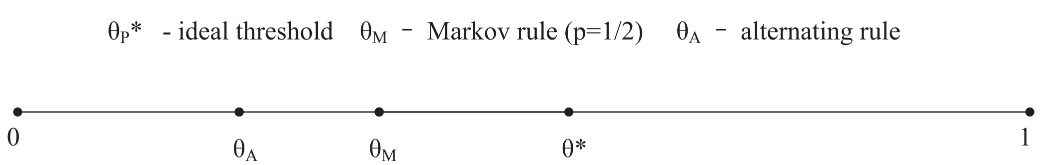

Figure 1: The relation of acceptance thresholds in different rules

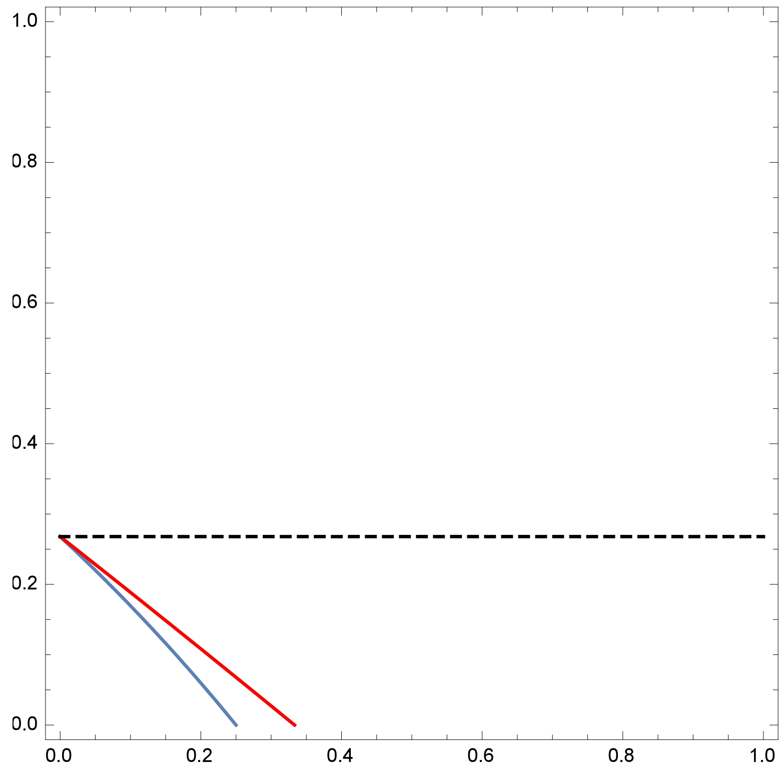

Figure 2: Comparison of the focal rules with $\delta = 0.5$.

The fig 2 demonstrates the DM's optimal cutoff, equilibrium cutoffs in focal rules. x axis is the bias and y axis is $\hat{\theta}$. The dotted line is the DM's optimal cutoff which is irrelevant to b. The red line is the cutoff in the alternating rule which is higher than the blue line, the cutoff in the super random rule. With larger bias, the information becomes more noisy since their deviations to the DM's optimal cutoff become larger. The distance between cutoffs in two rules become larger as well.

Since competition worsen the communication, the DM would be worse off when there is existing political power struggle between parties. In this perspective, single party should let the people be better off, however, people cannot allocate the power of parties easily.

In reality, we could always observe scenarios that the DM has ability to allocate the power and multiple experts' phenomena exists especially inside firms. For example, firms like Google, it has many project based groups: one division may research future cars and one division may develop Android system. Unlike the politic parties, those groups in companies do not need to be balanced. Whether those groups in a company could survive really depends on their performance. In the baseline model, we lack of modeling the behavior of search which could potentially support the argument on competition. In those companies, the search activity is endogenous, and costly. The innovation of the projects inside firms always engages costly and uncertain search activities (March and Simon 1958, Cyert and March 1963, Nelson and Winter 1982). Actually, no one could be sure of successfully innovating. But the probability of successfully innovating depends on how much effort the expert puts in. With more effort in search, there is less probability of failing to innovate.

4 Endogenous Search

In this section, we endogenize the search behavior, while making effort in search can improve the probability of obtaining a project. We define s as the successful rate (the probability of obtaining a project). The search cost is e,a function of s.

Assumption 1 *The search cost function $e(s)$ is continuous, increasing, convex, and satisfies condition on $[0,1]$: $lim_{s\to 1}e(s) < 1$, $lim_{s\to 1}e'(s) = 1$ and $e(0) = 0$, $e'(0) = 0$.*

The above assumption on effort e means the cost is higher if you need higher successful rate to find a project. On the other hand, one can understand this by its inverse function $s = e^{-1}(E)$ which means that if you put more effort in search, then the probability of finding a project is higher. The assumption on search cost is a standard assumption of cost function. We could also say that the search cost reflect how intensively the expert searches.

As we observe that the intensive search has impacts on the successful rate, so it affects the experts' strategies. This phenomenon can be observed within R&D firms and in academia, the researchers devote time and labor to read papers and to try new ideas, then they are more successful to develop new projects. However, the search activities are hard to observe and monitor, and the DM only hears reports from experts. By introducing effort in search, we could imagine that there exist competitions in both search and communication. While the competition in communication worsens the communication, the competition in search could provide incentive to search harder and alleviate the side effect in the communication. We will show that in such scenario, miscommunication does always exist, and experts make significant amount of effort in search. In the single expert case, since the single expert will surely obtain the extra benefit, he may not have enough incentive to search hard and we could refer this expert as "honest but lazy" expert. With two experts competition, the experts are "dishonesty but diligent". Is the "honest but lazy" case better or "dishonesty but diligent" case? That really depends how much the intensive search offsets the effect of mis-communication. In many environment, the DM would like to involve both experts to search.

We still following the concept of stationary Bayesian equilibrium and only add the effort level to the expert. The effort cannot be observed by the DM, so the DM's strategy is the function of the message. The equilibrium structure are basically the same as the baseline model, except that the experts optimize the search behavior. We will show that under some conditions, the effort is high enough to offset the mis-communication, and the DM is better off by consulting two experts. Compared with the baseline model, we will show that there exist some order with persistence $p < 1$ that makes the DM better off among all possible orders.

4.1 Equilibrium Characterization

When there is two experts, there could be improvement of the cutoff quality which also benefits the DM. We can write the Bellman equations for the experts $i, j = 1, 2,\ j \neq i$

$$\begin{cases} v_i^a & = max_{\bar{\theta}_i, s_i} s_i \int_{\bar{\theta}_i}^1 (\theta + b_i) dF(\theta) - e(s_i) + \delta(pv_i^a + (1-p)v_i^n)(1 - s_i + s_i F(\bar{\theta}_i)) \\ v_i^n & = s_j \int_{\bar{\theta}_j}^1 \theta dF(\theta) + \delta(pv_i^n + (1-p)v_i^a)(1 - s_j + s_j F(\bar{\theta}_j)) \end{cases} \tag{23}$$

$$\begin{cases} \hat{\theta}_i + b_i & = \delta(pv_i^a + (1-p)v_i^n) \\ \frac{de}{ds}|_{s_i = s_i^*} & = \int_{\hat{\theta}_i}^1 (\theta + b_i) dF(\theta) - \delta(1 - F(\hat{\theta}_i))(pv^a + (1-p)v^n) = \int_{\hat{\theta}_i}^1 (\theta - \hat{\theta}_i) dF(\theta) \end{cases} \tag{24}$$

So the simplified Bellman equation is,

$$\begin{cases} v_i^a & = s_i \int_{\hat{\theta}_i}^1 (\theta + b_i) dF(\theta) - e(s_i) + \delta(pv_i^a + (1-p)v_i^n)(1 - s_i + s_i F(\hat{\theta}_i)) \\ v_i^n & = s_j \int_{\hat{\theta}_j}^1 \theta dF(\theta) + \delta(pv_i^n + (1-p)v_i^a)(1 - s_j + s_j F(\hat{\theta}_j)) \end{cases} \tag{25}$$

For symmetric experts, $b_i = b_j$, and we consider symmetric equilibrium where $\hat{\theta}_i = \hat{\theta}_j$. By the first order condition on search cost, it must satisfy $s_i = s_j$.

$$\begin{cases} \hat{\theta} + b & = \delta(pv^a + (1-p)v^n) \\ \frac{de}{ds}|_{s=s^*} & = \int_{\hat{\theta}}^1 (\theta + b) dF(\theta) - \delta(1 - F(\hat{\theta}))(pv^a + (1-p)v^n) = \int_{\hat{\theta}}^1 (\theta - \hat{\theta}) dF(\theta) \end{cases} \tag{26}$$

So the simplified Bellman equation is,

$$\begin{cases} v^a & = s\int_{\hat{\theta}}^1 (\theta + b) dF(\theta) - e(s) + \delta(pv^a + (1-p)v^n)(1 - s + sF(\hat{\theta})) \\ v^n & = s\int_{\hat{\theta}}^1 \theta dF(\theta) + \delta(pv^n + (1-p)v^a)(1 - s + sF(\hat{\theta})) \end{cases} \tag{27}$$

The difference (Δ) of the continuation value between active and inactive period is,

$$\Delta_{a-n} = \frac{sb(1 - F(\hat{\theta})) - e(s)}{1 - \delta(2p-1)(1 - s + sF(\hat{\theta}))} \tag{28}$$

The average value of both active and inactive period is,

$$\bar{v} = \frac{1}{2}(v^a + v^n) = \frac{s\int_{\hat{\theta}}^1 (2\theta + b) dF(\theta) - e(s)}{2(1 - \delta(1 - s + sF(\hat{\theta})))} \tag{29}$$

Let's first look at the indifference condition on $\hat{\theta}$, and it has similar form as the baseline model. Let $\phi(\hat{\theta}) = \delta(pv^a + (1-p)v^n) - b$, and the explicit form is as below,

$$\begin{aligned} \phi(x) &= \delta(\bar{v} + \frac{2p-1}{2}\Delta_{a-n}) - b \\ &= \delta\{\frac{s\int_x^1 (2m + b) dF(m) - e(s)}{2(1 - \delta(1 - s + sF(x)))} + \frac{2p-1}{2}\frac{sb(1 - F(x)) - e(s)}{1 - \delta(2p-1)(1 - s + sF(x))}\} - b \\ &= \delta\{\frac{s\int_x^1 m dF(m)}{1 - \delta(1 - s + sF(x))} \\ &+ (sb(1 - F(x)) - e(s))\frac{p - \delta(2p-1)(1 - s + sF(x))}{(1 - \delta(2p-1)(1 - s + sF(x)))(1 - \delta(1 - s + sF(x)))}\} - b \end{aligned} \tag{30}$$

The function $\phi(\hat{\theta})$ is the experts' continuation value minus the bias which could be illustrated as normalized continuation value. The normalized value function is continuous and differentiable, and in order to simplify our analysis, we assume $\phi'(x) < 1$.

Assumption 2 *$\phi(x, s)$ satisfies $\frac{\partial \phi}{\partial x} < 1$, and $\frac{\partial \phi}{\partial s} > 0$, $\phi(0, 1) > 0$.*

This assumption put implicity restriction on δ, b, and the distribution function. In that assumption, $\frac{\partial \phi}{\partial x} < 1$ is similar to what we required for the uniqueness in Proposition 2. The function

$\phi(x)$ contains v^a which is determined by the successful rate, bias and the cost of search. Ceteris paribus the assumption $\frac{\partial \phi}{\partial s} > 0$ requires that the marginal cost of search is never too large so that experts could benefit from search. The last assumption is similar to the boundary condition as Proposition 2. Later we will show the parameter region satisfies the condition.

Proposition 5 *The DM consults two expert with reporting order characterized by persistence p, if assumptions 1 and 2 hold, there exist a unique symmetric stationary Bayesian equilibrium characterized by $(s^*, \hat{\theta})$.*

The existence condition basically requires that there is enough incentive to search. In the assumptions, we make restrictions that the search cost should not be too large compared with the expectation of the project. The first part of eq (26) determines whether expert to search. The boundary condition $\phi(0, 1) > 0$ means that given the maximum effort, it still encourages expert to search instead of sending message of nonsense. Because in that case, the value of "search" is higher than the value of "accept immediately". When $s = 0$, then experts cannot find any project, thus they don't need to search, so they set $\hat{\theta} = 0$ sending no informational message. The second part of eq (26) determines how much effort experts should choose. The marginal benefit gained by the extra successful rate is lower with higher $\hat{\theta}$. However, the marginal search cost is increasing. With higher cutoff, experts make less effort.

The explicit form of the equilibrium solution is impossible to solve. But the uniqueness of the equilibrium is very helpful in our analysis. The cutoff is bounded $\hat{\theta} \in [\underline{\theta}, \overline{\theta}] \in (0, 1)$. It is easy to see that $\underline{\theta} > 0$, because $\hat{\theta} = 0$ means the message contains no information. Also, the upper bound of cutoff should be less than 1. Consider the indifference condition on cutoff, the continuation value is always less than $1 + b$. If the cutoff were 1, then the indifference condition never holds. Because The assumptions are sufficient conditions, which means in reality, a larger parameter space could support the existence and uniqueness of such equilibrium.

Corollary 3 *With endogenous search,*

1. *if experts' bias is higher, experts tend to exaggerate more; but they put more effort to search.*

2. *If $sb(1 - F(\bar{\theta})) \geq e(s)$, with higher persistence p, experts tend to exaggerate less; but they put less effort to search; Else if $sb(1 - F(\underline{\theta})) \leq e(s)$, with higher persistence p, experts tend to exaggerate more; but they put more effort to search.*

The corollary states that there are two types of experts' behavior, "honest but lazy" and "dishonest but diligent". The honest expert doesn't exaggerate too much but make little effort to search, but the expert who works hard tends to exaggerate his project too much. In the standing point of the DM, before starting the searching game, there is a very practical question for her: should she hire two expert to search project or should she just hire one expert? The above result tells us that there is tradeoff between information loss and the search efficiency for the designer. The first part of the corollary shows how the experts' characteristics affect the communication and the search behavior. With larger bias b, the communication is worse which is aligned with the baseline model, but it incentivize experts to search harder. Since experts biases are observable by the DM, this result gives some implications on what types of experts should be hired when a firm is about to search a potential project. The recruiter should understand that if there is no extra benefit that both experts compete for, there is no incentive for them to work hard, although competition could worsen communication.

The second part of the corollary gives impression that given experts already hired by the firm, would the DM affect the experts' behavior by manipulating the reporting order characterized by the persistence p? The answer is yes. There are basically two cases characterized by difference of the search benefit and its cost. Usually, if the search cost is smaller than the search benefit, namely $sb \geq e(s)$, experts' value in active periods is higher than their value in inactive periods. Otherwise, the inactive periods' value is smaller, in which expert "free-rides" each other. In this case, experts wants to get their own project implemented. So if the expert has more chance to report in the next period, then he would not hurry to send "high" message which is aligned with the baseline model. However, the honesty in communication harms the search efficiency. In the second case when search cost is higher than the benefit, the inactive period value is higher than that in active period. This looks like a free-riding phenomena. No expert wants to be in active period to search, but both of them want the other expert to conduct search to bear the cost. This case really exists in equilibrium because experts make decisions on the value $pv^a + (1-p)v^n$. It means that the chance of free-riding the other experts could compensate the loss in searching. In this case, the incentive has an opposite direction, and with higher persistence p, experts would be harder to say "high" message, and search with less effort.

4.2 Free riding effect

In the presence of search cost, there naturally be two scenarios, one is when the search cost is not high $sb(1-F(\bar{\theta})) \geq e(s)$, then experts have higher continuation value in active periods; the other case is that if the search cost is too high $sb(1-F(\underline{\theta})) \leq e(s)$, then experts have higher continuation value when he is inactive. For the former case, there is strong incentive for the experts to compete in communication; while the search cost is high, the experts is worse off in active periods, so the experts tends to be more picky on projects in order to let the other expert to search. In this sense, experts rise the cutoff in order to "free-ride" the other. This problem arises from the sequential reporting rule and the cost of search. As we could imagine, this problem exists even if there is no biased experts. However, the "free-ride" effect is different between cases when there is only one expert to search and when there are two experts to search. In the single expert search case, there is essentially no "free-ride" effect, but in the two experts search case, expert always "free-rides" each other. As we observed, the "free-ride" effect would rise the cutoff. When the experts are biased, incentive of exaggeration would lower the cutoff. Since the "free-ride" effect and the distortion of communication are not linear, it is needed to understand whether the "free-ride" effect is the same when there is unbiased experts and are biased experts. For convenience, we focus on uniform distribution, and assume $e(s) = s^2$. Let $G(x) = \int_x^1 \theta DF(\theta)$.

Lemma 2 *When experts are unbiased $b = 0$, let $\hat{\theta}_0^{multi}$ be the equilibrium cutoff when two experts search, $\hat{\theta}_0^{single}$ be the equilibrium cutoff when single expert search, then $\hat{\theta}_0^{multi} > \hat{\theta}_0^{single}$ and experts pays more effort in single-expert search case.*

For biased experts case, the continuation value contains b. Let v_0 be the values of unbiased experts case. The first order condition with biased experts could be simplified as,

$$\begin{cases} \hat{\theta} &= \delta(pv_0^a + (1-p)v_0^n) + \Delta \\ s &= \frac{(1-\hat{\theta}^2)}{4} \end{cases} \tag{31}$$

where,

$$\Delta = \frac{\delta sb(1-\hat{\theta})}{2}\left(\frac{1}{1-\delta(1-s+s\hat{\theta})} + \frac{1}{1-\delta(2p-1)(1-s+s\hat{\theta})}\right) - b < 0 \tag{32}$$

Observe that Δ is increasing in s and δ. So, $\Delta < \frac{b(1-\hat{\theta})}{2}(\frac{1}{1-\hat{\theta}} + \frac{1}{1-(2p-1)\hat{\theta}}) - b < b - b < 0$.

The residual Δ depicts the difference of continuation value between unbiased and biased experts cases. The comparative statics of Δ indicates how the distortion in communication interact with the "free-ride" effect.

Lemma 3 Δ *is increasing in* p.

The proof is obvious. The equilibrium cutoff is determined by the continuation value and we refer R_0^M, R_b^M, R_0^S and R_b^S to the RHSs of the indifference condition. By Lemma **??**, the equilibrium cutoff could be compared by the RHSs. $\Delta_0 = R_b^S - R_0^S$, and $\Delta_p = R_b^M - R_0^M$. Since $\Delta_0 = \Delta(p = 1) > \Delta_p = \Delta(p)$, $R_b^S - R_0^S > R_b^M - R_0^M$, that is $R_b^S - R_b^M > R_0^S - R_0^M$. As we discussed, when there is bias, the equilibrium cutoff in both single expert case and multi-expert case are comparatively lower. By comparing the RHSs, we further understand that the difference between those cutoffs are higher when experts are biased. It implies that the "free-ride" effect is relatively larger when experts are biased.

With an option of making effort in search, the competition encourages experts to search harder. As we can observe the incentive to search hard comes from experts' bias. Higher bias leads to lower cutoff in equilibrium, however high effort in search would generate a potential project in shorter time. The competition between experts has those two opposite effects that cancel out with each other. In the Corollary, as the reporting order probability p increases, experts are less likely to make high effort in search. This is because experts expect high chance of reporting to the DM, and the threat from the other expert is exogenously reduced by the reporting order. The delay cost is relatively high, search helps experts to find projects faster. However, if search cost is relatively high, experts would not afford to search.

4.3 Focal rules

We continue to discuss the two orders: the alternating rule and the random rule. In these examples, we will focus on the parameter of bias b and investigate what the equilibrium like with different b. We further assume that $e(s) = \frac{s^k}{c}$

The DM's payoff is:

$$v^p = \frac{s(1-\hat{\theta}^2)}{2(1-\delta(1-s+s\hat{\theta}))} \tag{33}$$

The alternating rule (p=0):

We summarize the Bellman equation and condition for high effort equilibrium,

$$\begin{cases} v^a = s(1-\hat{\theta}_A)(\frac{1+\hat{\theta}_A}{2}+b) - \frac{s^k}{c} + \delta(1-s+s\hat{\theta}_A)v^n \\ v^n = s(1-\hat{\theta}_A)\frac{1+\hat{\theta}_A}{2} + \delta(1-s+s\hat{\theta}_A)v^a \end{cases} \tag{34}$$

The experts' indifference condition for the cutoff is: $\hat{\theta}_A + b = \delta v^n$. And the first order condition on the effort is, $\frac{k}{c}s^{k-1} = \frac{(1-\hat{\theta}_A)^2}{2}$.

The Markov rule (p=1/2):

We summarize the Bellman equation and condition for high effort equilibrium,

$$\begin{cases} v^a = s(1-\hat{\theta}_M)(\frac{1+\hat{\theta}_M}{2}+b) - \frac{s^2}{2} + \delta(1-s+s\hat{\theta}_M)(\frac{1}{2}v^a + \frac{1}{2}v^n) \\ v^n = s(1-\hat{\theta}_M)\frac{1+\hat{\theta}_M}{2} + \delta(1-s+s\hat{\theta}_M)(\frac{1}{2}v^n + \frac{1}{2}v^a) \end{cases} \tag{35}$$

The experts' indifference condition is: $\hat{\theta}_M + b = \frac{\delta}{2}(v^a + v^n)$. And the first order condition on the effort is, $\frac{k}{c}s^{k-1} = \frac{(1-\hat{\theta}_M)^2}{2}$.

Example $\delta = 0.85$, $b = 0.35$, $k = 2.5$, $c = 2.5$.

The figure 3 compares the equilibrium of the alternating rule and the random rule. In the figure, x axis is the cutoff $\hat{theta}$, and y is the successful rate s. The two upward sloping solid curves are the indifference condition of cutoffs (red curve is one in the alternating rule, and the other is that in the random rule). The red curve and the blue curve intersect where the search cost equals the extra benefit. After that point, the value $v^a \leq v^n$ and if the equilibrium is in that area, then experts would rather stay in inactive period to "free-ride" his opponent. The downward sloping solid curve is the first order condition on the effort. It is clear that the cutoff is higher in the random rule however, the successful rate is higher in the alternating rule. The example shows that experts cannot have two virtues "loyalty" and "diligence" at the same time. With more equal chance to speak, experts tends to be more honest but lazy, and vice versa. We also drew the DM's indifference curves as dotted curves. The DM's expected payoff is is larger to the east. In the alternating rule the DM's payoff is 0.475 which is lower than 0.478 in the random rule, and the DM is better off under the random rule. In this environment, the communication effect dominant the

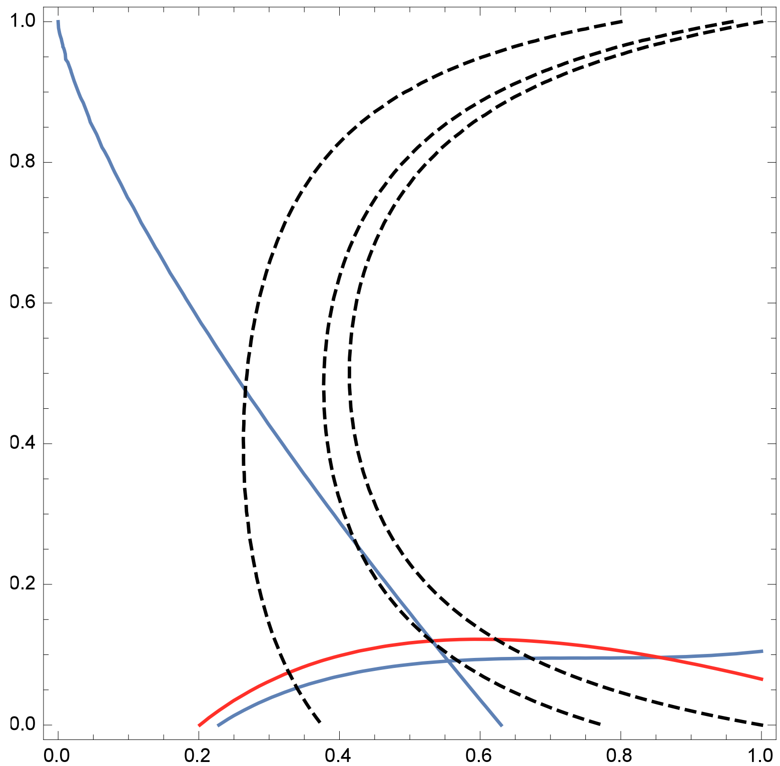

Figure 3: The equilibrium in alternating rule and the random rule.

searching effect. For the DM, the random rule would be better.

4.4 Optimal Order Design

A natural question is that are there any other intermediate meeting procedure that improves the acceptance threshold and further maximize the DM's payoff? As indicated in examples 1, 2 and 3, if high effort equilibrium exists, the existence condition gives the upper bound of the maximum cutoff $\hat{\theta}^*$. It provide a way to solve the optimal p^* and the optimal $p^* < 1$. Otherwise, if only the low effort equilibrium exists, then $p^* = 1$. As in the examples, the equilibrium exhibits discontinuity at $\hat{\theta}^*$, and it switches from the high effort equilibrium to low effort equilibrium. For low effort equilibrium, $p = 1$ determines the highest cutoff, so we also have to compare either p^* or $p = 1$ gives the DM higher payoff.

The DM's payoff in general form is,

$$v^p = s \int_{\hat{\theta}}^{1} \theta dF(\theta) + \delta v^p (1 - s + sF(\hat{\theta})) \tag{36}$$

which is simplified as,

$$v^p = \frac{s}{1 - \delta(1 - s + sF(\hat{\theta}))} \int_{\hat{\theta}}^{1} \theta dF(\theta) \tag{37}$$

Lemma 4 *The DM's value function is increasing in* s*. In the decision maker's problem, there exist unique optimal stopping cutoff* $\hat{\theta}_p^*$*. When* $\theta \leq \hat{\theta}_p^*$*, the DM's expected payoff is monotonically increasing. If the DM let experts to search, then the experts' equilibrium cutoff* $\hat{\theta} < \hat{\theta}_p^*$*.*

Lemma 5 *The DM's indifference curves* $s = \mu(\hat{\theta}, v^p)$ *are convex functions.*

Proof. Consider any v^p, the indifference curve is

$$s = \frac{v^p(1-\delta)}{\int_{\hat{\theta}}^{1} (\theta - \delta v^p) dF(\theta)} \tag{38}$$

The derivative of s to $\hat{\theta}$ is,

$$s' = \frac{v^p(1-\delta)}{(\int_{\hat{\theta}}^{1} (\theta - \delta v^p) dF(\theta))^2} (\hat{\theta} - \delta v^p) f(\hat{\theta}) \tag{39}$$

Actually, $s' < 0$ by Lemma 4, because $\hat{\theta} < \hat{\theta}_p^*$ and v^p function is monotonic increasing at left side of $\hat{\theta}_p^*$. ■

There are two factors in DM's payoff, the successful rate and the cutoff quality. With two experts, the equilibrium cutoff is below the optimal cutoff for the corresponding successful rate. Keep other parameters constant, larger persistence p would create larger continuation value, and leads to higher cutoff, thus create lager payoff for the DM. On the other hand, experts has less incentive to make effort to search which reduces the successful rate s, since there is search cost.

In fig 3, we observe that the equilibrium cutoff move along the curve (effort curve) where marginal search benefit equals marginal search cost. In that figure, we also draw the indifference curve for the DM. Which equilibrium makes the DM better off depends on the location of the cutoffs. The effort curve $s = \omega(\hat{\theta})$ is decreasing and the DM's indifference curves are convex functions $s = \mu(\hat{\theta}, v^p)$. Intuitively, if the equilibrium cutoff is on the point of tangency of curves ω and μ, then the corresponding p is the optimal reporting rule. Otherwise, we obtain corner solutions, either $p = 0$ or $p = 1$.

Claim 4 *The optimal persistence* p^* *to the DM has the following characteristics, 1. If the optimal* *rule is interior* $0 < p^* < 1$*, then there must exist an equilibrium* $(s^*, \hat{\theta})$ *satisfies* $\frac{d\omega}{d\hat{\theta}} = \frac{d\mu}{d\hat{\theta}}$*. 2. Otherwise, the optimal rule is a corner solution, either* $p = 0$ *or* $p = 1$*.*

From the condition of the effort level, we have proved that in equilibrium, larger cutoff corresponds to lower successful rate. And in the DM's expected payoff function, the DM prefers larger successful rate and higher cutoff. If the optimal rule $0 < p^* < 1$ exists, the corresponding equilibrium $(s^*, \hat{\theta})$ must also solve the maximization problem with constraint of effort level. However, such pair of s and $\hat{\theta}$ might not exist in some environment. If there is no such pairs, then the optimal p^* should be a corner solution either $p^* = 1$ or $p^* = 0$. In the example of last section, numerically, we found that for many parameters, the optimal persistence $p^* = 1$. From the examples we found that when s is very small, then the effect of s dominates the effect of $\hat{\theta}$; and when s is large, the effect of $\hat{\theta}$ dominates s. In many examples, the equilibrium successful rate turns out to be high, but the cutoff $\hat{\theta}$ is very low. So an increase in the cutoff would increase the DM's payoff largely and the communication effect turn out to be greater than the searching effect. Below is a figure describing the equilibrium with any p.

Example $\delta = 0.85$, $b = 0.35$, $k = 2.5$, $c = 2.5$.

The green, red, blue curves are respectively $p = 1$, $p = 1/2$ and $p = 1$. And for any $0 < p < 1$, the curve lies between green and blue curves. Their intersects with the deep blue (downward sloping) curve are the equilibrium. We also draw the tangent condition for optimal interior p^*. As we could observe, the tangent point lies outside the equilibrium area, and the corner solution is obtained $p^* = 1$. Since the conditions for equilibrium exhibits high non-linearity, it is impossible to compare different equilibrium directly. In the special case with $F \backsim U[0, 1]$ and power function form search cost, the competition in search doesn't provide enough incentive to overcome the communication effect.

5 Asymmetric experts in endogenized search

In this section, we discuss experts with different biases $0 < b_1 < b_2 < 1$. And we focus on two reporting orders: the alternating rule and the random rule. In the random rule, the probability of expert 1 report is τ. With asymmetric biases, we could explore more properties on the experts. We focus on a general random rule characterized by τ.

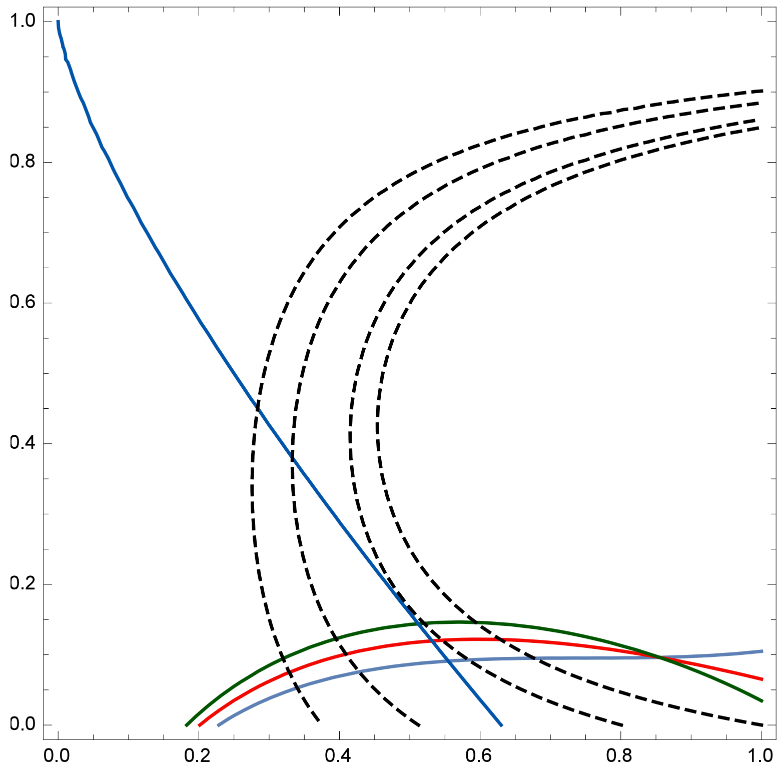

Figure 4: Optimal p

We construct the equilibrium that all agents are playing high effort with corresponding cutoff. Similarly, the continuation values of each agent are, for the expert 1,

$$\begin{cases} v_1^a = s_1(1-\hat{\theta}_1)(\dfrac{1+\hat{\theta}_1}{2}+b_1) - \dfrac{s_1^2}{2} + \delta(1-s_1+s_1\hat{\theta}_1)(\tau v_1^a + (1-\tau)v_1^n) \\ v_1^n = s_2(1-\hat{\theta}_2)(\dfrac{1+\hat{\theta}_2}{2}) + \delta(1-s_2+s_2\hat{\theta}_2)(\tau v_1^a + (1-\tau)v_1^n) \end{cases} \tag{40}$$

For the expert 2,

$$\begin{cases} v_2^a = s_2(1-\hat{\theta}_2)(\dfrac{1+\hat{\theta}_2}{2}+b_2) - \dfrac{s_2^2}{2} + \delta(1-s_2+s_2\hat{\theta}_2)((1-\tau)v_2^a + \tau v_2^n) \\ v_2^n = s_1(1-\hat{\theta}_1)(\dfrac{1+\hat{\theta}_1}{2}) + \delta(1-s_1+s_1\hat{\theta}_1)((1-\tau)v_2^a + \tau v_2^n) \end{cases} \tag{41}$$

The first order conditions for the equilibrium of expert 1 is,

$$\begin{cases} \hat{\theta}_1 + b_1 & = \delta(\tau v_1^a + (1-\tau)v_1^n) \\ s_1^* & = (1-\hat{\theta}_1)(\frac{1+\hat{\theta}_1}{2}+b_1) - \delta(1-\hat{\theta}_1)(\tau v_1^a + (1-\tau)v_1^n) \end{cases} \tag{42}$$

and for expert 2 is,

$$\begin{cases} \hat{\theta}_2 + b_2 & = \delta((1-\tau)v_2^a + \tau v_2^n) \\ s_2^* & = (1-\hat{\theta}_2)(\frac{1+\hat{\theta}_2}{2} + b_2) - \delta(1-\hat{\theta}_2)((1-\tau)v_2^a + \tau v_2^n) \end{cases} \tag{43}$$

For two asymmetric experts, we could define two experts distance of exaggeration by $d = \hat{\theta}_1 - \hat{\theta}_2$.

Corollary 5 *In random rule characterized by τ, the equilibrium has the following features,*

1. *if expert exaggerate more, then he searches harder.*

2. *if $\frac{b_2}{b_1} \geq \frac{1-\delta\tau}{1-\delta(1-\tau)}$, then expert 2 exaggerate more than expert 1.*

3. *the distance between two experts is increasing in b_2, decrease in b_1 and if $b_1 + b_2 \geq 1/4$, then d is increasing in τ.*

This corollary shows that given a random reporting rule to experts, the expert with smaller bias would report with a higher cutoff. This is not only determined by the bias, but also the probability of reporting. Even if expert 1 has less bias, but his chance to report is low, he may exaggerate more than expert 2. Moreover, the experts who exaggerate less tends to make less effort to search which is aligned with previous results. The 3rd result is about the distance of cutoffs. If we could increase the probability τ, the distance of two cutoffs would be larger. In fact, τ is the probability that expert 1 reporting, if τ is higher, then he has a higher continuation value, so he has less incentive to exaggerate. On the other hand, $1-\tau$ is the probability that expert 2 reporting. If τ is higher, then expert 2 expect lower continuation value so he tends to exaggerate more. When expert has smaller bias, then compared to the continuation value, capturing the extra benefit in current period seems less attractive, so expert's cutoff is higher. This explains why the distance is decreasing in b_1 and increasing in b_2.

6 Conclusion

Decentralized search is very common inside organizations especially R&D firms and academic organizations. The decision maker who is not specialist in every filed, has very limited ability to access the information of potential projects. The experts, specialists, are responsible for develop ideas and make proposals to the DM. If the DM accepts one proposal, he commit resources to implement the project; otherwise, another projects will be suggested and the DM faces a decision making problem again. The process of search and proposing continues until final project is accepted. The DM shares the return of the future project, prefers high quality projects, but he has delay cost in each period due to time cost and arranging meetings with experts. The experts whose interests are partially aligned with the organization, prefers high quality project but compete for private benefit if his own project wins. Many management papers investigate how to design the reward system to motivate experts search. Compared to those papers, we design from the meeting procedure without using monetary transfer. Given the inability of observing the information of the projects and monitoring the search activities of the experts, we show that the DM still have some ability to control the quality of the projects. We find that controlling the meeting procedure with the experts can actually affect their incentives in search and communication. When the expert has chance to make proposal in the next period, the expert will be more honesty and be more picky. The random rule, in which the expert has 1/2 to meet the DM in the next period, turns out to be better than the alternating rule, since the same expert does not have chance to meet the DM in the next period. Due to competition, both experts has incentive to exaggerate, in other words, acceptance threshold is lower. On the other hand, if the search is intensive, the competition provide experts incentives to search intensively. Therefore, the competition is possible to make the DM better off. It explains why organizational search always involves many employees from competition perspective. We also studied simultaneous communication in extension part. In equilibrium, the DM is able to reveal more precise information of both projects, but he must allow mixing more low quality project to compare. It turns out that the simultaneous communication rule is worse than the random rule, so we give up this design.

Throughout all the results, we assume that the rejected project never comes again. This assumption might be strong, and in future work it is possible to loose some of the assumptions. Another strong assumption is the independence of projects. To be more realistic, the projects might have correlations. For example, some ideas come from cooperation of employees and knowledge sharing exists in organizations. By loosing the assumption independence, it allows us to broadly study cooperation and competition in organizational search. On the modeling of the intensive search, the search could be modeled in many ways as we observes in reality. An alternative way of modeling is that exerting effort in search would obtain a project that stochastically dominate that without effort, but that would not change the essential result.

7 Appendix

Proof of Proposition 1. In any period, the cutoff $\hat{\theta}$ is chosen when the active expert is indifferent between reporting "high" and "low". Thus, the indifference condition for the expert ($i = 1, 2$)is,

$$\hat{\theta}_i + b_i = \delta(pv_i^a + (1-p)v_i^n) \tag{44}$$

Let $g_i(\hat{\theta}_i, \hat{\theta}_j) = \delta(pv_i^a + (1-p)v_i^n) - b_i$. For the system of equations $i = 1, 2$.

$$\begin{cases} \hat{\theta}_i &= g_i(\hat{\theta}_i, \hat{\theta}_j) \\ \hat{\theta}_j &= g_j(\hat{\theta}_i, \hat{\theta}_j) \end{cases} \quad i = 1, 2, i \neq j \tag{45}$$

The function $g_i(x)$ is continuous in $[0, 1]$. It is easy to see $g_i < 1$. Because $v_i^n < v_i^a < 1$, then $g_i(\hat{\theta}_i, \hat{\theta}_j) < \delta(1 + b) - b < 1$. Moreover, $g_i(\hat{\theta}_i, \hat{\theta}_j) > \delta v_i^n - b_i > \delta E(\theta) - b_i$. If $\delta E(\theta) - b_2 > 0$, then $g_i > 0$, $\forall i = 1, 2$. By Brouwer fixed-point theorem, the there exist fixed point $(\hat{\theta}_i, \hat{\theta}_j)$.

Assume $\hat{\theta}_i$ is the equilibrium cutoff, we just need to verify it is an equilibrium. The DM's belief is truncated distribution of F, i.e. $g_h(\theta) = f(\theta|\theta \geq \hat{\theta})$, $g_l(\theta) = f(\theta|\theta < \hat{\theta})$. If the DM hears the "high" message, in equilibrium, the value of implementing the project should be greater than that of continuing searching. Actually,

$$E_{g_h}(\theta_i|\theta_i \geq \hat{\theta}_i) > \delta v_j^p \tag{46}$$

Too see this, by eq (4), let $G(\hat{\theta}_i) = \int_{\hat{\theta}_i}^1 \theta dF(\theta)$,

$$v_i^p = \frac{(1 - \delta F(\hat{\theta}_j)p)G(\hat{\theta}_i) + \delta F(\hat{\theta}_i)(1-p)G(\hat{\theta}_j)}{1 - \delta p(F(\hat{\theta}_i) + F(\hat{\theta}_j)) + \delta^2 F(\hat{\theta}_i)F(\hat{\theta}_j)(2p-1)} \tag{47}$$

On the other hand, since the belief is truncated distribution, $E_{g_h}(\theta|\theta \geq \hat{\theta}_i) = \int_{\hat{\theta}_i}^1 \frac{\theta}{1-F(\hat{\theta}_i)} dF(\theta) = \frac{G(\hat{\theta}_i)}{1-F(\hat{\theta}_i)}$.

$$
\begin{aligned}
&E_{g_h}(\theta|\theta \geq \hat{\theta}_i) - v_j^p = \frac{G(\hat{\theta}_i)}{1-F(\hat{\theta}_i)} - \delta v_j^p \\
&= \frac{G(\hat{\theta}_i)}{1-F(\hat{\theta}_i)} - \frac{(1-\delta F(\hat{\theta}_i)p)G(\hat{\theta}_j) + \delta F(\hat{\theta}_j)(1-p)G(\hat{\theta}_i)}{1-\delta p(F(\hat{\theta}_i)+F(\hat{\theta}_j)) + \delta^2 F(\hat{\theta}_i)F(\hat{\theta}_j)(2p-1)} \\
&= \frac{1}{(1-F(\hat{\theta}_i))(1-\delta p(F(\hat{\theta}_i)+F(\hat{\theta}_j)) + \delta^2 F(\hat{\theta}_i)F(\hat{\theta}_j)(2p-1))} \\
&\times [G(\hat{\theta}_i)(1-\delta(F(\hat{\theta}_i)+F(\hat{\theta}_j)) - \delta^2 F(\hat{\theta}_j)(1-p(1+F(\hat{\theta}_i)))) - \delta(1-F(\hat{\theta}_i))G(\hat{\theta}_j)(1-\delta p F(\hat{\theta}_i))] > 0
\end{aligned} \tag{48}
$$

Similarly, we could prove $E_{g_h}(\theta|\theta \geq \hat{\theta}_i) < v_j^p$. Therefore the existence is proved. ■

Proof of Proposition 2. In any period, the cutoff $\hat{\theta}$ is chosen when the active expert is indifferent between reporting "high" and "low". Thus, the indifference condition for the expert is,

$$
\hat{\theta} + b = \delta(pv^a + (1-p)v^n) \tag{49}
$$

By eqs (28)(29),

$$
\begin{aligned}
\hat{\theta} + b &= \delta(pv^a + (1-p)v^n) \\
&= \delta(p(\bar{v} + \frac{\Delta_{a-n}}{2}) + (1-p)(\bar{v} - \frac{\Delta_{a-n}}{2})) \\
&= \delta(\bar{v} + \frac{2p-1}{2}\Delta_{a-n}) \\
&= \frac{\delta}{2(1-\delta F(\hat{\theta}))}\int_{\hat{\theta}}^1 (2\theta + b)dF(\theta) + \delta\frac{2p-1}{2}\frac{b(1-F(\hat{\theta}))}{1-\delta(2p-1)F(\hat{\theta})} \\
&= \frac{\delta}{1-\delta F(\hat{\theta})}\int_{\hat{\theta}}^1 \theta dF(\theta) + \delta\frac{b(1-F(\hat{\theta}))(p-\delta(2p-1)F(\hat{\theta}))}{(1-\delta(2p-1)F(\hat{\theta}))(1-\delta F(\hat{\theta}))}
\end{aligned} \tag{50}
$$

Let $g(\hat{\theta}) = \delta(pv^a + (1-p)v^n) - b$,

$$
g(x) = \frac{\delta}{1-\delta F(x)}\int_x^1 s dF(s) + \delta\frac{b(1-F(x))(p-\delta(2p-1)F(x))}{(1-\delta(2p-1)F(x))(1-\delta F(x))} - b \tag{51}
$$

Existence: The function $g(x) - x$ is continuous in $[0,1]$, $g(1) = -b-1 < 0$ and $g(0) = \delta E(x) - b(1-\delta p)$. By the intermediate value theorem, if $\delta E(x) > b(1-\delta p)$, there exist $\hat{\theta} \in [0,1]$ such that $g(\hat{\theta}) - \hat{\theta} = 0$.

Assume $\hat{\theta}$ is the equilibrium cutoff, we just need to verify it is an equilibrium. The DM's belief is truncated distribution of F, i.e. $g_h(\theta) = f(\theta|\theta \geq \hat{\theta})$, $g_l(\theta) = f(\theta|\theta < \hat{\theta})$. If the DM hears the "high" message, in equilibrium, the value of implementing the project should be greater than that of continuing searching. Actually,

$$E_{g_h}(\theta_i|\theta_i \geq \hat{\theta}_i) > \delta v_j^p \tag{52}$$

Too see this, by eq (4),

$$v^p = \frac{1}{1-\delta F(\hat{\theta})} \int_{\hat{\theta}}^1 \theta dF(\theta) \tag{53}$$

On the other hand, since the belief is truncated distribution, $E_{g_h}(\theta|\theta \geq \hat{\theta}) = \int_{\hat{\theta}}^1 \frac{\theta}{1-F(\hat{\theta})} dF(\theta)$. Since $\delta \in (0,1)$, $\frac{1}{1-F(\hat{\theta})} \int_{\hat{\theta}}^1 \theta dF(\theta) \geq \frac{1}{1-\delta F(\hat{\theta})} \int_{\hat{\theta}}^1 \theta dF(\theta)$. Similarly, we could prove $E_{g_h}(\theta|\theta \geq \hat{\theta}) < v^p$. Thus, eq (52) holds and equilibrium exists.

Uniqueness: Since $g(x)$ is continuous on $[0,1]$, by mean value theorem, for any nonempty set $[x,y] \subset [0,1]$, there exists $x^* \in [x,y]$ such that,

$$g(y) - g(x) = g'(x^*)(y-x) \tag{54}$$

Since $g(0) > 0$ and $g(1) < 0$, if $g'(x) < 1$, then the fixed point is unique.

Suppose the upper bound of $f(x)$ is $\bar{f}$, and it is obvious that $\bar{f} \geq 1$ since the length of support is 1.

$$g'(x) = \delta\{f(x)\frac{-x(1-\delta F(x)) + \delta \int_x^1 s dF(s)}{(1-\delta F(x))^2} + bH(x)\} \tag{55}$$

In which $H(x)$ is, (collect 1st and last terms, and 2nd and 3rd terms at the second line)

$$\begin{aligned} H(x) = & \frac{(-f(x)(p-\delta(2p-1)F(x)) + (1-F(x))(-\delta(2p-1)f(x)))(1-\delta(2p-1)F(x))(1-\delta F(x))}{(1-\delta F(x))^2(1-\delta(2p-1)F(x))^2} \\ & - \frac{(1-F(x))(p-\delta(2p-1)F(x))(-\delta(2p-1)f(x)(1-\delta F(x)) + (1-\delta(2p-1)F(x))(-\delta f(x)))}{(1-\delta F(x))^2(1-\delta(2p-1)F(x))^2} \\ & = \frac{\delta(p-1)(2p-1)f(x)(1-F(x)))}{(1-\delta F(x))(1-\delta(2p-1)F(x))^2} + \frac{(\delta-1)f(x)(p-\delta(2p-1)F(x))}{(1-\delta F(x))^2(1-\delta(2p-1)F(x))} \end{aligned} \tag{56}$$

If $p > 1/2$, $H(x) < 0$. Since the first term is negative. For the second term, we only need $p - \delta(2p-1)F(x)) > 0$. Actually this inequality holds automatically. In other words, we need

$\delta < \frac{p}{(2p-1)F(x)}$. Since $min\{\frac{p}{(2p-1)F(x)}\} = \frac{p}{2p-1} = 1$, δ naturally satisfies.

$$\begin{aligned} g'(x) &= \delta\{f(x)\frac{-x(1-\delta F(x)) + \delta\int_x^1 sdF(s)}{(1-\delta F(x))^2} + bH(x) \\ &= \delta f(x)\{\frac{-x}{1-\delta F(x)} + \frac{\delta}{(1-\delta F(x))}\cdot\frac{\int_x^1 sdF(s)}{1-F(x)}\cdot\frac{1-F(x)}{1-\delta F(x)} + b\cdot H(x) \\ &< \delta f(x)\frac{\delta}{1-\delta} \\ &< \delta^2\bar{f}\frac{1}{1-\delta} \end{aligned} \tag{57}$$

If $p \leq 1/2$, the second term is negative and the first term is positive that matters.

$$\begin{aligned} &\frac{\delta(p-1)(2p-1)f(x)(1-F(x)))}{(1-\delta F(x))(1-\delta(2p-1)F(x))^2} \leq \frac{\delta(1-p)(1-2p)f(x)}{(1+(1-2p)F(x))^2} \\ &\leq \delta(1-p)(1-2p)f(x) \end{aligned} \tag{58}$$

$$\begin{aligned} g'(x) &= \delta\{f(x)\frac{-x(1-\delta F(x)) + \delta\int_x^1 sdF(s)}{(1-\delta F(x))^2} + bH(x) \\ &\leq \delta f(x)\{\frac{-x}{1-\delta F(x)} + \frac{\delta}{(1-\delta F(x))}\cdot\frac{\int_x^1 sdF(s)}{1-F(x)}\cdot\frac{1-F(x)}{1-\delta F(x)} + b\cdot H(x) \\ &< \delta f(x)\{\frac{\delta}{1-\delta} + \delta(1-p)(1-2p)\} \\ &< \delta^2\bar{f}\{\frac{1}{1-\delta} + 1\} \\ &= \delta^2\bar{f}\frac{2-\delta}{1-\delta} \end{aligned} \tag{59}$$

In the second line of eq (57,59), if condition $\Theta = \{\delta | \frac{\delta^2}{1-\delta}\bar{f} < 1 \;\&\&\; \delta^2\bar{f}\frac{2-\delta}{1-\delta} < 1\}$ holds, then the equilibrium is unique. In eq (57), since $\delta^2\bar{f}\frac{1}{1-\delta} \leq \delta\bar{f}\frac{1}{1-\delta}$, the more sufficient condition for uniqueness is $\delta < \frac{1}{\bar{f}+1}$. And In eq (59), we could check if $\delta < \frac{1}{\bar{f}+1}$, $g'(x) < 1$. Thus the uniqueness is proved. ■

Proof of Proposition 3. The optimal stopping cutoff for the DM's problem is determined by eq (10).

Let

$$g(\hat{\theta}) = \frac{\delta}{1-\delta F(\hat{\theta}_p^*)}\int_{\hat{\theta}_p^*}^1 \theta dF(\theta) \tag{60}$$

It is easy to see that $g(x)$ is continuous in $[0,1]$, $g(0) = E(\theta) > 0$ and $g(1) = 0 < 1$, so there exist $\hat{\theta}_p^*$ satisfies eq (10) due to the intermediate value theorem. For such function, similar to the proof in Proposition 1,

$$\begin{aligned} g'(x) &= \delta f(x)\frac{-x(1-\delta F(x)) + \delta\int_x^1 sdF(s)}{(1-\delta F(x))^2} \\ &= \delta f(x)\left(\frac{-x}{1-\delta F(x)} + \frac{\delta}{(1-\delta F(x))}\cdot\frac{\int_x^1 sdF(s)}{1-F(x)}\cdot\frac{1-F(x)}{1-\delta F(x)}\right) \\ &< f(x)\frac{\delta}{1-\delta} \\ &< \bar{f}\frac{\frac{1}{1+2\bar{f}}}{1-\frac{1}{1+2\bar{f}}} \\ &= \frac{1}{2} < 1 \end{aligned} \tag{61}$$

Therefore, the optimal cutoff is unique. Notice that $v^p = g(x)$, the optimal cutoff satisfies eq (7), which is $g'(\hat{\theta}_p^*) = 0$. Suppose v^p is not monotonically increasing, then there must be some $\tilde{\theta} \neq \hat{\theta}_p^*$ such that $g(\tilde{\theta}) < 0$, by the continuity of $g'(x)$ in $[0,1]$ and $g'(0) > 0$, there must be some point $\theta^{**} \in [0,\tilde{\theta}]$ such that $g'(\theta^{**}) = 0$. So θ^{**} is another optimal cutoff which is against the uniqueness.

The case of letting single expert to search is the special case $p = 1$ of the baseline model. Therefore the equilibrium cutoff is unique. Compare eqs (15) and (10), the RHS of eq (10) is larger than the RHS of eq (15). It is easy to check that when $\hat{\theta} = 0$, both RHSs are positive, and we know the cutoffs are unique. By Lemma 6, their fixed points satisfy $\hat{\theta} < \hat{\theta}_p^*$. ■

Proof of Corollary 1. To show bias b decreases the equilibrium cutoff $\hat{\theta}$, consider the indifference condition for the expert,

$$\hat{\theta} + b = \delta(pv^a + (1-p)v^n) \tag{62}$$

Recall eq (63), let $g(\hat{\theta}) = \delta(pv^a + (1-p)v^n) - b$

$$g(x) = \frac{\delta}{1-\delta F(x)}\int_x^1 sdF(s) + \delta\frac{b(1-F(x))(p-\delta(2p-1)F(x))}{(1-\delta(2p-1)F(x))(1-\delta F(x))} - b \tag{63}$$

In $g(x)$ only the 2nd and 3rd term are determined by b, so let

$$
\begin{aligned}
h(x) &= \delta \frac{b(1-F(x))(p-\delta(2p-1)F(x))}{(1-\delta(2p-1)F(x))(1-\delta F(x))} - b \\
&= b\frac{\delta(1-F(x))(p-\delta(2p-1)F(x)) - (1-\delta(2p-1)F(x))(1-\delta F(x))}{(1-\delta(2p-1)F(x))(1-\delta F(x))} \\
&< b(1-F(x))\frac{\delta(p-\delta(2p-1)F(x)) - (1-\delta(2p-1)F(x))}{(1-\delta(2p-1)F(x))} \\
&< \frac{b(1-F(x))}{(1-\delta(2p-1)F(x))}(p-\delta(2p-1)F(x)) - (1-\delta(2p-1)F(x)) \\
&< \frac{b(1-F(x))}{(1-\delta(2p-1)F(x))}(p-1) \\
&< 0
\end{aligned}
\tag{64}
$$

In the above inequalities, we used the result that $\delta < 1$ and $p \leq 1$.

2. Similar to the proof in part 1, it is easy to see that the function $g(x, \delta)$ is increasing in δ. Thus, smaller discount rate leads to lower equilibrium cutoff. ■

Proof of Proposition 5. Consider the first order condition in eq (26) and we need to prove the two curves intersect at unique point. Let's consider the second part first. Firstly, $\int_{\hat{\theta}}^{1}(\theta - \hat{\theta})dF(\theta)$ is decreasing in $\hat{\theta}$ and its RHS is increasing in s, so by assumption of convexity of $e(s)$, $\hat{\theta}$ is decreasing in s. If $s = 0$, we could solve $\hat{\theta} = 0$ and if $s = 1$, $\hat{\theta} = 0$.

The first part of the condition is basically the same as the condition in Proposition 2. Take the first order derivative on both sides, and collect terms,

$$
\frac{d\hat{\theta}}{ds} = \frac{\frac{\partial \phi}{\partial s}}{1 - \frac{\partial \phi}{\partial \hat{\theta}}}
\tag{65}
$$

By assumption 2, $\hat{\theta}$ is increasing in s.

Check the boundary of the curve. If $s = 0$, then $\phi(x, 0) = 0$, so we could solve $\hat{\theta} = 0$. If $s = 1$, it is similar to the baseline model except the search cost $e(s)$. We need to prove the solution $\hat{\theta} \geq 0$. Consider $\hat{\theta} = \phi(\hat{\theta}, 1)$. It is obvious that ϕ is continuous, and $\phi(1, 1) < 0$. Since $\phi(0, 1) > 0$, by the intermediate value theorem, there exist a $\hat{\theta} \in [0, 1]$. Since $\frac{d\phi(x)}{dx} < 1$, the solution is unique.

In summary, the first curve $L_1 : \hat{\theta}(s)$ is increasing in s, with boundary conditions $\hat{\theta}(0) = 0$ and $\hat{\theta}(1) \geq 0$; and the second curve $L_2 : \hat{\theta}(s)$ is decreasing in s, with boundary conditions $\hat{\theta}(0) = 1$ and $\hat{\theta}(1) \leq 0$. Therefore, the two curves determines unique solution $(s^*, \hat{\theta})$. ■

Proof of Corollary 3. For the first part, what we need to check that the term contains b in $\phi(x)$ has negative sign. If $v^a \geq v^n$. By the Bellman equations,

$$v^a < s\int_{\hat{\theta}}^{1}(\theta+b)dF(\theta) - e(s) + \delta v^a(1-s+sF(\hat{\theta})) \tag{66}$$

$$\begin{aligned}\phi(x) &< \delta v^a - b \\ &< M + \frac{\delta s(1-F(x))b}{1-\delta(1-s+sF(x))} - b \\ &= M + \frac{\delta - 1}{1-\delta(1-s+sF(x))} < 0\end{aligned} \tag{67}$$

where M is the term that contains no b. Therefore, $\phi(x)$ is decreasing in b. On the other hand, if $v^a < v^n$. However, v^n contains no b, so the only term contains b in $\phi(x) < \delta v^n - b$ is negative. In summary, $\phi(x)$ is decreasing in b. The first part is proved.

For the second part, recall eq (26). $\frac{de}{ds}$ is increasing in s due to convexity. The RHS of the equation is decreasing in $\hat{\theta}$. Since $\hat{\theta} + b = \delta(pv^a + (1-p)v^n)$, we have

$$\begin{aligned}RHS &= \int_{\hat{\theta}}^{1}(\theta+b)dF(\theta) - (1-F(\hat{\theta}))(\hat{\theta}+b) \\ &= \int_{\hat{\theta}}^{1}\theta dF(\theta) - (1-F(\hat{\theta}))\hat{\theta}\end{aligned} \tag{68}$$

Take derivative with respect to $\hat{\theta}$, we have

$$\begin{aligned}\frac{dRHS}{d\hat{\theta}} &= -\hat{\theta}f(\hat{\theta}) - 1 + F(\hat{\theta}) + \hat{\theta}f(\hat{\theta}) \\ &= F(\hat{\theta}) - 1 < 0\end{aligned} \tag{69}$$

Therefore, the effort level is decreasing in $\hat{\theta}$. For the first part of the proposition, we only need to prove that $\hat{\theta}$ is increasing in p. If $sb(1-F(\bar{\theta})) \geq e(s)$, then $v^a \geq v^n$, so $pv^a + (1-p)v^n$ is increasing in p. By Lemma 6, $\hat{\theta}$ is increasing in p. Else if $sb(1-F(\underline{\theta})) \leq e(s)$, then $v^a < v^n$, $pv^a + (1-p)v^n$ is decreasing in p. By Lemma 6, $\hat{\theta}$ is decreasing in p.

■

Proof of Lemma 2. By the continuation value functions eq (29,28), we have,

$$v^a = \frac{1}{2}[\frac{2sG(\hat{\theta}) - s^2}{1-\delta(1-s+sF(\hat{\theta}))} - \frac{s^2}{1-\delta(2p-1)(1-s+sF(\hat{\theta}))}] \tag{70}$$

$$v^n = \frac{1}{2}[\frac{2sG(\hat{\theta}) - s^2}{1-\delta(1-s+sF(\hat{\theta}))} + \frac{s^2}{1-\delta(2p-1)(1-s+sF(\hat{\theta}))}] \tag{71}$$

And the first order conditions are,

$$\begin{cases} \hat{\theta} & = \delta(pv^a + (1-p)v^n) \\ s & = \frac{(1-\hat{\theta}^2)}{4} \end{cases} \tag{72}$$

The continuation values are functions of p, and $v^a(p) > v^a(0)$ and $v^n(p) > v^a(0)$. Since $v^a(p) < v^n(p)$, in eq (27), we replace the RHS v^n to v^a, then the value of $\tilde{v}^a$ after replaced is smaller than v^a, and $\tilde{v}^a$ equals $v^a(0)$. Thus, $v^a(p) > v^a(0)$. And it is obvious that $v^n(p) > v^a(0)$. Therefore, $\hat{\theta} = \delta(pv^a(p) + (1-p)v^n(p)) > \delta v^a(p)$. By Lemma 6, $\hat{\theta}_0^{multi} > \hat{\theta}_0^{single}$.

The second part of the FOD describes the relation between s and $\hat{\theta}$ are negative. So $s_{multi} < s_{single}$. ■

Proof of Lemma 4. It is easy to show that v^p is increasing in s. The indifference condition for $\hat{\theta}_p^*$ is $\hat{\theta}_p^* = \delta v^p = g(\hat{\theta}_p^*)$. This indifference condition is a special case of ϕ with $b = 0$ and $e = 0$. By assumption 2, there exist unique $\hat{\theta}_p^*$. And similar to the proof in Proposition 3, we could prove the monotonicity of the value function at $\theta \le \hat{\theta}_p^*$. Further, by Lemma 6, since the value $\phi \le g$, the expert's equilibrium cutoff $\hat{\theta} < \hat{\theta}_p^*$. ■

Proof of Corollary 5. For the first part, we could substitute $\hat{\theta}_1 + b_1 = \delta(\tau v_1^a + (1-\tau)v_1^n)$ into $s_1^* = (1-\hat{\theta}_1)(\frac{1+\hat{\theta}_1}{2} + b_1) - \delta(1-\hat{\theta}_1)(\tau v_1^a + (1-\tau)v_1^n)$, and similar to expert 2, we have

$$s_i^* = \frac{(1-\hat{\theta}_i)^2}{2} \tag{73}$$

for all $i = 1, 2$. Clearly, s_i^* is decreasing in $\hat{\theta}_i$. Moreover, if their cutoff is the same, then s_i^* also equals.

For the second part, let's consider expert 1 first. Substitute $\hat{\theta}_1 + b_1 = \delta(\tau v_1^a + (1-\tau)v_1^n)$ into v_1^a and v_1^n, then plug them back to the indifference condition,

$$\begin{aligned} \hat{\theta}_1 = \delta[&\tau s_1(1-\hat{\theta}_1)\frac{1+\hat{\theta}_1}{2} - \frac{\tau s_1^2}{2} + \tau(1-s_1+s_1\hat{\theta}_1)\hat{\theta}_1 \\ &+ (1-\tau)s_2(1-\hat{\theta}_2)\frac{1+\hat{\theta}_2}{2} + (1-\tau)(1-s_2+s_2\hat{\theta}_2)\hat{\theta}_1] \\ &+ (\delta\tau - 1)b_1 \end{aligned} \tag{74}$$

and similarly,

$$\hat{\theta}_2 = \delta[(1-\tau)s_2(1-\hat{\theta}_2)\frac{1+\hat{\theta}_2}{2} - \frac{(1-\tau)s_2^2}{2} + (1-\tau)(1-s_2+s_2\hat{\theta}_2)\hat{\theta}_2 \\ + \tau s_1(1-\hat{\theta}_1)\frac{1+\hat{\theta}_1}{2}) + \tau(1-s_1+s_1\hat{\theta}_1)\hat{\theta}_2] \\ + (\delta\tau - 1)b_1 \tag{75}$$

Let $W_i = 1 - s_i + s_i\hat{\theta}_i$ Thus,

$$(1-\tau W_1 - (1-\tau)W_2)(\hat{\theta}_1 - \hat{\theta}_2) = \frac{\delta}{2}(1-\tau(s_1^2+s_2^2)) + (1-\delta(1-\tau))b_2 - (1-\delta\tau)b_1 \tag{76}$$

In the LHS, since $W_i \leq 1$, any convex combination of W_i is also less than 1. In the RHS, $s_i \leq \frac{1}{2}$, so $1-\tau(s_1^2+s_2^2)) > 0$. If $\frac{b_2}{b_1} \geq \frac{1-\delta\tau}{1-\delta(1-\tau)}$, then $\hat{\theta}_1 > \hat{\theta}_2$.

For the third part, notice that $W_1 > W_2$. To see this, $W_1 - W_2 = s_2 - s_1 + s_1\hat{\theta}_1 - s_2\hat{\theta}_2 > (s_2 - s_1)(1-\hat{\theta}_2) > 0$. Take total derivative,

$$(1-\tau W_1 - (1-\tau)W_2)\frac{\partial d}{\partial \tau} = (W_1 - W_1)d - \frac{\delta}{2}(s_1^2+s_2^2) + \delta(b_2+b_1) \tag{77}$$

where $\frac{1}{2}(s_1^2+s_2^2) \leq \frac{1}{4}$. If $b_1 + b_2 \geq 1/4$, then d increases in τ. And it is easy to see d increase in b_2 and decrease in b_1. ■

Lemma 6 *f and g are continuous function on R, for any $x \in R$, $f > g$ and $f(0), g(0) \geq 0$. If there exist unique x^*, y^* such that $x^* = f(x^*)$ and $y^* = g(y^*)$, then $x^* > y^*$.*

Proof. Let $\bar{x}^* = min(x^*, y^*)$, so $x \leq \bar{x}^*$, $f(x) > g(x) \geq x$. When $x = \bar{x}^*$, $f(\bar{x}^*) > g(\bar{x}^*) = \bar{x}^*$. By uniqueness of $y^* = g(y^*)$, we have $\bar{x}^* = y^* = min(x^*, y^*)$. Thus, $x^* > y^*$. ■

The above lemma 6 can be scaled from R to $[0,1]$. Actually, the explicit expression of the threshold quality in alternating rule is tedious, but by indifference condition, we find that the threshold quality is determined by the next-period value. We will show that for the Markov rule, the next-period value is higher than that under alternating rule, thus we could use this to prove propositions.

Mathematica code for focal rules: see the simulation file "simulation.nb".

References

Aghion, P., and J. Tirole. "The management of innovation," *The Quarterly Journal of Economics* 109(4), 1994, 1185-1209.

Albrecht, J., A. Anderson, and S. Vroman "Search by Committee," *Journal of Economic Theory* 145(4), 2010, 1386-1407.

Chan, J. and W. Suen "Does Simple Majority Rule Produce Hasty Decisions?," *mimeo 2012*

Compte, O. and P. Jehiel. "Bargaining and Majority Rules: A Collective Search Perspective," *Journal of Political Economy* 118, 2010, 189C221.

Battaglini, M. "Multiple Referrals and Multidimensional Cheap Talk," *Econometrica* 70(4), 2002, 1379-1401.

Chakraborty, A. and R. Harbaugh. "Persuasion by Cheap Talk," *American Economic Review* 100 (5), 2010, 2361-82.

Crawford, V. and J. Sobel. "Strategic Information Transmission," *Econometrica* 50(6), 1982, 1431-1451.

Cyert, R. and J. March. "A Behavioral Theory of the Firm," *Prentice Hall, Englewood Cliffs, NJ* 1963.

Dessein, W. "Authority and Communication in Organizations," *Review of Economic Studies* 69, 2002, 811-838.

Foss, N. "Selective intervention and internal hybrids: Interpreting and learning from the rise and decline of the Oticon spaghetti organization," *Organization Science* 14(3), 2003, 331-349.

Holmstrom, B. "Agency costs and innovation," *Journal of Economic Behavior & Organization* 12(3), 1989, 305-327.

Holmstrom, B., and P. Milgrom "The firm as an incentive system," *American Economic Review* 84(4), 1994, 972-991.

Lambert, R., D. Larcker, and K. Weigelt "The structure of organizational incentives, " *Administrative Science Quarterly* 38(3), 1993, 438-461.

Lazear, E., and S. Rosen. "Rank-order tournaments as optimum labor contracts, " *Journal of Political Economy* 89(5), 1981, 841-864.

Levinthal, D, and G. March. "A model of adaptive organizational search, " *Journal of Economic Behavior & Organization* 2(4), 1981, 307-333.

Levinthal, D, and G. March. "The myopia of learning, " *Strategic Management Journal* 14(special issue), 1993, 95-112.

Lippman, S and J. McCall. "The economics of job search: A survey" *Economic Inquiry* 14, 1976, 155-189.

Li, Z., H. Rantakari, and H. Yang. "Competitive Cheap Talk," *working paper* 2014.

Lovas, B., and S. Ghoshal. "Strategy as guided evolution," *Strategic Management Journal* 21(9), 2000, 875-896.

March, J. and H. Simon. "Organizations," *Wiley, New York* 1958.

Moldovanu, B. and X. Shi. "Specialization and partisanship in committee search," *Theoretical Economics* 8, 2013, 751C774.

Mortensen, D. "Job search and labor market analysis," *Handbook of Labor Economics* North-Holland, Amsterdam, 1986, 849-920.

Nelson, R. and G. Winter. "An Evolutionary Theory of Economic Change," *Harvard University Press* 1982.

Ocasio, W. "Towards an attention-based view of the firm," *Strategic Management Journal* 18(summer special issue), 1997, 187-206.

Prendergast, C. "The provision of incentives in firms," *Journal of Economic Literature* 37(1), 1999, 7-63.

Rogerson, R., R. Shimer, and R. Wright, “Search-theoretic models of the labor market: A survey,” *Journal of Economic Literature* XLIII, 2005, 959-988.

Rotemberg, J., and G. Saloner, “Benefits of narrow business strategies,” *American Economic Review* 85(5), 1994, 1330-1349.

Stein, J., “Interal capital markets and the competition for corporate resources,” *The Journal of Finance* 52(1), 1997, 111-133.