# Bidding for Teams


Benoit Julien[*] UNSW Australia

John Kennes[†] Aarhus University

Moritz Ritter[‡] Temple University


February 7, 2015


**Abstract**

We construct a directed search model of team production. Teams have an optimal size as workers' productivity may positively depend on the presence of other workers, generating local increasing returns to scale. We demonstrate that increasing the complementarity between workers within a team increases dispersion in wages. This suggests that the proliferation of human resource management practices that emphasize team production may have contributed to the increase in income inequality in recent decades. Additionally, we demonstrate that efficiency in competing auctions model may require a reserve price in excess of the seller's valuation if there are positive externalities among buyers.



Keywords: Competing Auctions, Directed Search, Inequality, Team Production, Wage Dispersion

JEL classification: D44, D83, E24, J31



[*]Email: benoit.julien@unsw.edu.au

[†]Email: jkennes@econ.au.dk

[‡]Email: moritz.ritter@temple.edu

# 1 Introduction

Over the past three decades, organizational change within firms has lead to a proliferation of modern human resource management practices, such as self-managed teams and performance pay (e.g. Bloom and Van Reenen, 2010). Concurrent with these changes to the internal organization of firms, income inequality both across (skill premium) and within (residual inequality) groups of similar workers rose markedly, especially among workers in the top half of the income distribution (e.g. Autor et al., 2008).[1] In this paper, we ask whether the changes to the internal organization of firms, embodied by an increased prevalence of team work may have contributed to the observed increase in inequality.[2]

In teamwork the productivity of one team member depends on the presence of other team members, i.e. there are complementarities between workers. Therefore, as the organization of firms evolves from one in which individual workers produce independently, to one in which the presence of other workers is crucial for a worker's productivity, the dispersion of workers' productivity and wages may increase. To study this question, we develop a framework in which teams produce a homogenous good with identical S-shaped production technologies, using labor as the only input. The S-shaped technology captures that production on the firm level is organized in teams and that for small enough teams the addition of new members may increase the average productivity of all members (for example, because of increased specialization or complementary skills). However, once a team becomes sufficiently large, the gains from further specialization are outweighed by the increased coordination necessary and the average product begins to decrease.

Teams recruit workers in a frictional labor market in which unmatched workers can only apply to one team and cannot coordinate with each other on which team to apply to. We follow the directed search literature in that we allow teams to post a trading mechanism before workers decide which team to apply to.[3] We consider two types of mechanisms. First, we begin by assuming

[1]Autor et al. (2005) report that "male 90/50 (upper-tail) residual wage inequality rose during both halves of the sample: by 4.4 log points from 1973 to 1989 and by 4.0 log points from 1989 to 2005."

[2]Lemieux et al. (2009) find that the incentive side of modern human resource management, the growing incidence of performance-pay, can help to explain a significant amount of the the growth in wage dispersion, especially on the top of the earnings distribution.

[3]See Peters (1984) for an early contribution and Montgomery (1991) and Burdett et al. (2001) for further development of that approach.

that teams run simple auctions: all workers on a team submit their wage demands and the team selects workers based on the bids. Due to the presence of increasing returns for small teams, the resulting equilibrium is not constrained efficient. Small teams attract an inefficiently small number of applicants since workers are not fully rewarded for their strategic complementarity. Efficiency requires that each worker receive her marginal product, which is not feasible in the presence of increasing returns since any wage demand is limited by the worker's average product.

We then consider competing auctions in which teams compete against each other for applicants by posting an amenity. The resulting competition among teams leads to a constrained efficient allocation, but the posted amenity is not zero for teams with increasing returns. This contrasts to the existing literature on competing auctions (Peters and Severinov, 1997, Julien et al. 2000, Albrecht et al. 2012, 2014), which finds that a reserve price of zero implements the constrained efficient allocation. However, typically only two agents trade with each other in these models, which is akin to assuming constant returns. Here we show that deviating from the assumption of constant returns breaks the zero reserve price result.[4]

We then extend the static model into a dynamic framework and show computationally that the model can qualitatively deliver an increase in residual wage inequality as the complementarity between workers in a team increases. Workers who are employed on ideal-sized teams (producing at the maximum of the average product curve) extract all surplus through the auction and hence receive high wages. Workers who are employed on teams with fewer than the average product maximizing number of workers accept a low wage today because of the option value of growing into an ideal-sized team in the future. Previously, under constant returns to scale, this option value was not present and wages on these teams were higher and the wage dispersion was lower.

While search models often do not generate much wage dispersion (e.g. Hornstein et al., 2011), the auction model with on-the-job search has proven successful in generating residual wage disparity (Julien et al., 2006). In our model, workers do not search on the job, but rather partially filled teams continue attracting new workers until the optimal team size is reached. Therefore, teams have a high reservation value in the auction stage, but workers are able to extract much of the

[4] Lester et al. (2015) demonstrate that the zero reserve price result also critically hinges on the meeting technology between buyers and sellers.

surplus. If the number of workers is smaller or equal to the number of spots on the team, the workers extract all surplus, while the opposite is true if there are more workers than spots. This leads to a dispersion in wages, even if the workers' productivity is independent of the team size; the coefficient of variation of wages with constant return teams is 0.79. Changes to the production technology change the distribution of surplus, increasing wages on ideal-sized teams at the expense of wages on smaller teams, therefore increasing wage dispersion. With critical mass teams (no output is produced unless the team is fully staffed) the coefficient of variation of wages is 1.3.

Some previous work has investigated directed search models with multi-worker firms; these models use wage posting rather than the competing auctions mechanism we study.[5] The model closest to ours is Tan (2012), who studies a directed search model in which firms have an optimal size, giving rise to local increasing returns. In that model, firms can have an optimal size of either one or two workers and she finds that firms which seek to hire two workers post larger wages if the local increasing returns are sufficiently strong. While we share the idea of local increasing returns in production, our focus on the auction equilibrium leads us to a natural exploration of the efficiency properties of our environment and her focus is the wage-size differential across firms, rather than income inequality. Shi (2002) also considers a directed search model in which a firm can hire a second worker. However, in that model the source of local increasing returns is that firms with two workers can charge higher prices in a frictional goods market, giving rise to a positive size-wage differential even if both workers are equally productive.[6]

Our paper is also related to the literature studying the sale of multiple items in a directed search framework. In Burdett et al. (2001), the market outcome depends on the number of units for sale at each seller, buyers' valuations are independent of the number of buyers at a seller's location. Geromichalos (2012) studies a directed search model in which sellers of goods post general trading mechanisms in which the price and quantity sold depend on the number of buyers. His focus is on describing the matching technology; while his setup allows for a positive consumption externality

[5]In addition to the papers discussed here, several other authors have developed search models with "large firms." These models typically feature a decreasing returns to scale production technology and are used for quantitative analysis. See, for example, Hawkins (2013) and Kaas and Kircher (2013) for models with a competitive search environment and Helpman et al. (2010) and Cosar et al. (2014) for models with a random search environment.

[6]Lester (2010) studies a model in which firms can hire either one or two workers, but focuses on the case in which each worker produces the same output and firms differ in the cost of creating vacancies.

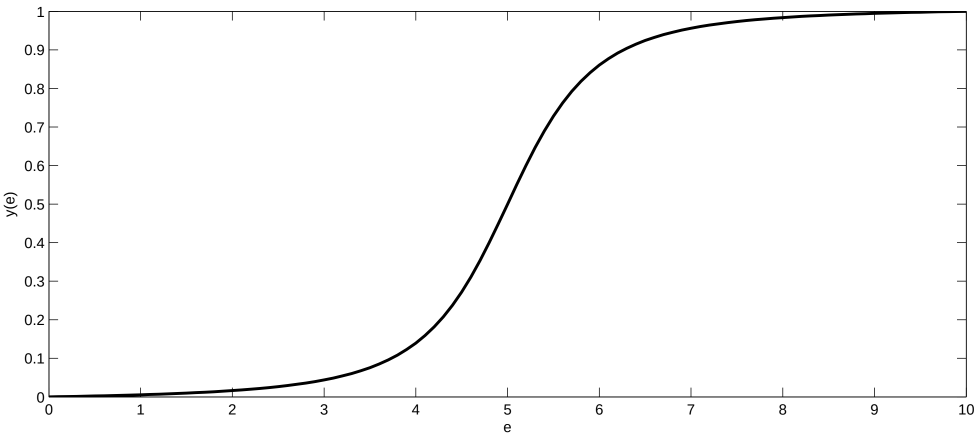



Figure 1: Team Production Technology

between buyers (corresponding to increasing returns), he does not explore that possibility in his analysis. In a related paper (Julien et al., 2014), we explicitly focus on the case of a good with a positive consumption externality and show that the competing auction equilibrium is efficient and sellers post a negative reserve price.

The remainder of the paper is consists of two parts. In section 2, we develop a model of team production and study the efficiency of its directed search equilibrium. In section 3, we extend the static model into a dynamic framework and explore its implications for wage dispersion.

## 2 Static Model

There is large, exogenous number of teams, denoted $n$. Teams differ by the number of incumbent workers; let $n(\tilde{e})$ to denote the number of teams at the start of the period with $\tilde{e}$ incumbent workers. A team produces $y(e)$ units of output, where $e$ denotes the number of workers employed by that team after the matching stage (described below). We assume that smaller teams may be subject to increasing average productivity and large teams are subject to diminishing average productivity.[7] Figure 1 depicts a function $y\,(e)$ with this property. Let $e^*$ denote the maximum output per agent on a team, i.e. $y(e^*)/e^* \geq y(e)/e$, $\forall\, e$. Note that while the marginal product is diminishing for some

[7]This technology is consistent with a conventional U-shaped average cost curve for the production of output by a group where the costs are measured by the number of members in the group.

$e < e^*$, only if $e > e^*$ does the average product exceed the marginal product. Going forward, we call teams with $e < e^*$ as having increasing returns and teams with $e > e^*$ as having decreasing returns. Also, there is a maximum team size, denoted $\bar{e}$. Any workers in excess of $\bar{e}$ have a non-positive marginal product, i.e. $y(\bar{e}) \geq y(e), \forall\, e > \bar{e}$.

The total number of workers in this market is determined by free entry and exit. Therefore, any workers who enters this economy at the start of the period must earn an expected return of $W > 0$, the cost of entry.

The problem of determining team membership and pricing in this economy is modeled as a three stage game. In the first stage, the teams post vacancies and choose a wage determination mechanism. We begin by assuming that teams post a simple auction without reserve price and then explore competing auctions. In the second stage, workers enter and decide which team to apply to. Workers can only apply to one team and and cannot coordinate with each other. As common in the literature, we focus on symmetric mixed strategies, that is workers apply to identical teams with equal probabilities. In the third stage, workers auction their labor services to the highest bidder (or, in the case of wage posting, the firm selects at random from the number of applicants) and production occurs.

### 2.1 Efficient Allocation

We begin our analysis by describing the constrained efficient allocation. Consider a planner who takes as given the problem of coordination and maximizes the value of a team subject to the alternative payoffs of each agent. The social planners can add new workers at a cost of $W$ and create exits and earn a return of $W$. Therefore, the planner's problem for a team that has $e$ incumbent workers is to choose queue lengths $q(e)$ and exits $x(e)$ to solve

$$Z(e) = \max_{q,x\geq 0} \left\{ \sum_{z=0}^{\infty} y\left(e - x + z\right) Pr(z \mid q(e)) - Wq(e) + Wx(e), 0 \right\}, \tag{1}$$

where

$$Pr(z \mid q(e)) = \frac{\left(q\left(e\right)\right)^{z}}{z!} e^{-q(e)},$$

is the number of arriving bidders as a function of the expected queue length.[8]

If the optimal queue length $q^*(e)$ is positive, the solution of the social planning problem is given by the following first-order condition,

$$W = \sum_{z=0}^{\infty} \left[ y\left(e+z+1\right) - y\left(e+z\right) \right] \frac{\left(q^*\left(e\right)\right)^z}{z!} e^{-q(e^*)} \tag{2}$$

and the number of exits is equal to zero. If the optimal queue length is zero, the number of exits is determined by the condition that the marginal productivity is equal to the outside value of a worker:

$$W = y\left(e - x^*(e)\right) - y\left(e - x^*(e) - 1\right). \tag{3}$$

## 2.2 Simple Auction

First, to fix ideas, assume that all teams post simple auctions. In that auction game, the incumbent workers and the unemployed workers who selected that specific team attempt to sell their labor services to the team by submitting their wage demand. The team (buyer) selects the lowest bidders to fill the vacancies on the team. In the case of more lowest bid applicants then vacancies the team randomly selects among those applicants.

In this game, the equilibrium wage-bid function of each worker is given by:

$$w(e) = \begin{cases} \triangle_e & if \quad e \geq e^* \\ \frac{y(e)}{e} & if \quad e < e^* \end{cases} \tag{4}$$

where $e$ is the number of bidders and $\Delta_e = y(e) - y(e-1)$ denotes the marginal productivity of the $e$th worker. To see that this is an equilibrium bidding function first consider the case of diminishing returns, i.e. the case $e \geq e^*$. In that case, if all other workers demand their marginal product, an individual worker finds it optimal to also bid the marginal product. Any bid in excess of the marginal product will be declined but a bid at the marginal product will be accepted and a smaller

[8] Recall that we restrict our attention to the symmetric equilibrium in which identical workers randomize among identical teams. Assuming a large market, the probability that a team is visited by $z$ new workers is then Poisson distributed with parameter $q(e)$, see Julien et al. (2000).

bid does not increase the acceptance probability. Note that this implies that the wage will be zero if $e > \bar{e}$.

Now consider the case of increasing returns, i.e. $e \leq e^*$. A bid by each agent for their marginal contribution is not feasible given the output of the team. The maximum a team can pay each worker is her average product. Hence, for each bidder, it is a best strategy to bid $y(e)/e$, if the others are bidding $y(e)/e$. Bidding higher (i.e. asking for a higher share of output) than $y(e)/e$ is not feasible, because total output is $y(e)$, and bidding lower than $y(e)/e$ does not change the probability of acceptance, which is one if the bidder chooses $y(e)/e$. Note that in the case of diminishing average product, $e > e^*$, a wage bid of the average product is feasible but not optimal. If all $e$ workers demanded their average product, the team would hire fewer than $e$ workers and make a positive profit. Each worker could then increase her probability of being hired by bidding less. With decreasing returns the maximum wage bid that leads to the team hiring the worker with probability one is her marginal product.

Let $q(e)$ denote the expected queue length of applicants to a team with $e$ incumbent members. This queue length is the outcome of free entry of agents subject to the assumption that these agents play common mixed strategies over which seller to visit in this market. Let $\hat{W}(e)$ be the expected utility of an incumbent member on a club with $e$ incumbent agents. If the queue length for any particular club type is $q(e)$, the expected payoff of this incumbent agent is given by

$$\hat{W}(e, q(e)) = \sum_{z=0}^{\infty} w(e+z)\, Pr(z \mid q(e)) \tag{5}$$

where

$$Pr(z \mid q(e)) = \frac{(q(e))^z}{z!} e^{-q(e)}$$

is the number of arriving bidders as a function of the expected queue length. An incumbent worker on a team with $e$ incumbents can choose to stay, $\sigma(e) = 0$, or exit $\sigma(e) = 1$. Therefore, the incumbent solves

$$\sigma^*(e \mid W) = \arg\max_{\sigma(e)} \left\{ (1 - \sigma(e))\hat{W}(e, W) + \sigma(e) W \right\} \tag{6}$$

If any agent chooses to remain on the team, we must have $\hat{W}(e) \geq W$ since agent can always quit.

Similarly, if an agents enters this economy, their expected payoff of choosing a team with $e$ incumbent workers is given by

$$W\left(e,q\left(e\right)\right)=\sum_{z=0}^{\infty}w\left(e+z+1\right)Pr(z\mid q\left(e\right)), \tag{7}$$

which is the wage paid if the worker and $z$ additional new workers apply to a team with $e$ incumbent workers times the probability that $z$ additional workers also chose this team. If the queue length at a team is positive, the free entry of agents gives $W=W(e,q\left(e\right))$.

**Proposition 2.1.** *If incumbent membership is greater than $e^{*}$, the social planning solution is equivalent to the decentralized equilibrium with simple auctions. Otherwise the social planner prefers a longer queue of buyers ($q^{*}\left(e\right)\geq q\left(e\right)$).*

*Proof.* Let $t\left(e\right)$ denote the difference between the right hand side of the social planner's first-order condition, (2), and the right hand side of the free entry condition, (11), evaluated at $q^{*}(e)$, $W=W(e,q^{*}\left(e\right))$. This gives

$$\begin{aligned} t(e) &= \sum_{z=0}^{\infty}\triangle_{e}Pr(z\mid q^{*}(e))-\sum_{z=0}^{\infty}w\left(e+z+1\right)Pr(z\mid q^{*}(e)) \\ &= \sum_{z=0}^{e^{*}-e}\left[\triangle_{e+z}-\frac{y\left(e+z\right)}{e+z}\right]\frac{\left(q^{*}\left(e\right)\right)^{z}}{z!}e^{-\left(q^{*}(e)\right)} \end{aligned} \tag{8}$$

If the team production function $y(e)$ is not subject to increasing returns, then there exists a solution to the simple auction model that is identical to the social planner's solution. However, if there is a region of local increasing returns to scale, where $\triangle_{e}>\frac{y(e)}{e}$, then $t\left(e\right)$ is positive if the queue length is equal to $q^{*}\left(e\right)$. The smaller individual payoffs in this region means that the overall queue length is shorter in the decentralized economy. □

In the case of diminishing returns to scale, each bidder extracts their marginal contribution. This is an obvious requirement for efficient decisions within an organization, because the bidders are then rewarded their marginal returns to the value of a match.

With increasing returns, the worker does not receive his full contribution and hence there will be inefficiently little entry. In essence, the worker has a positive externality on all other workers in his team that she is not fully rewarded for. The function $t(e)$ can thusly be interpreted as the subsidy on worker entry that can be used to implement the social planner's solution.[9] If the social efficient queue length is positive, then the incumbents should not quit. Therefore, the same subsidy must also be awarded to incumbents in order to ensure that they do not quit. Consequently, the implementation of the subsidy is complicated by the fact that similar transfers must also be awarded to incumbents.

## 2.3 Competing auctions equilibrium

Now consider the possibility that teams can advertise amenities before workers make their search decision.[10] An interpretation of such an amenity could be the provision of health care benefits, gym memberships or firm provided lunches. Let the amenity be denoted $d$. Therefore, workers may submit bids against the team's output and the amenity, i.e. from the viewpoint of the worker "output function" is now given by $y(e) + d(e)$. Given this amenity, the wage-bid function of an agent is given by:

$$w(e,d) = \begin{cases} \triangle_e & if \quad e \geq e^* \\ \frac{y(e)+d}{e} & if \quad e < e^* \end{cases} \tag{9}$$

where $\Delta_e = y(e) - y(e-1)$ denotes the marginal productivity of the $e$th worker. Note that the amenity cause higher bids (greater demands for surplus) only if $e < e^*$. Otherwise, competitive bidding returns the entire cost of the amenity back to the team. Consequently, the amenity has no effect on the team's profits if the team is subject to diminishing returns to scale for all possible employment levels.

Given this wage-bid function, the expected wage of applying to a team with $e$ incumbents and

[9] This is related to the result in Mortensen (2009) that in an island matching model with random matching a simple auction does not solve the coordination problem; to implement the constrained efficient allocation, a tax on island entry is needed. Here a subsidy (negative "Mortensen tax") on worker entry is needed since the planner's coordination incentive is stronger than the individual agents' coordination incentive under a simple auction.

[10] Alternatively, teams can also promise all applicants a payment for applying to the team. Note that a minimum wage promise that workers could not underbid would not lead to an efficient outcome since teams would restrict hiring such that the marginal product of the last worker is equal to the minimum wage.

posted amenity $d$ as function of the queue length is given by:

$$W(e,d,q) = w(e+1,d)e^{-q(e,d)} + w(e+2,d)q(e,d)\,e^{-q(e,d)} + w(e+3,d)\frac{(q(e,d))^2}{2}e^{-q(e,d)} + w(e+4,d)\frac{(q(e,d))^3}{3!}e^{-q(e,d)} + ... \tag{10}$$

Since workers are free to enter or exit and have an outside option $W$, the queue length have to satisfy

$$W(e,d,q) \le W,\ \ q(e) \ge 0,\ \ q(e)\,(W(e,d,q) - W) = 0 \tag{11}$$

**Lemma 1.** *The reaction function of workers to the amenity is well behaved,*
*i.e.* $q'(d,e) \equiv \frac{\partial q'(d,e)}{\partial d} > 0$.

*Proof.* Totally differentiate the function $W = W(e,d,q)$ as follows:

$$\begin{aligned} 0 =& \frac{1}{e+1}e^{-q(e,d)} - w(e+1,d)q'(e,d)\,e^{-q(e,d)} \\ &+ \frac{1}{e+2}q(e,d)\,e^{-q(e,d)} - w(e+2,d)q'(e,d)\left[e^{-q(e,d)} + q(e,d)\,e^{-q(e,d)}\right] \\ &+ \frac{1}{e+3}\frac{q(e,d)^2}{2}e^{-q(e,d)} - w(e+3,d)q'(e,d)\left[q(e,d)\,e^{-q(e,d)} + \frac{q(e,d)^2}{2}e^{-q(e,d)}\right] + ... \end{aligned}$$

Collecting terms, it follows straightforward that $q'(e,d) > 0$. □

The function $q(e,d)$ is increasing in the amenity $d$, because the payoff of the worker is made higher in every realization of applicants when the team has an increasing average product, $e < e^*$ and the payoff is independent of the realization of the number of applicants when these realizations are in the region of decreasing average product, $e > e^*$.

Given this well behaved reaction function between worker queues and team amenities, we can now describe the seller's problem.

$$S(e,d) = \max_{q,d}\left\{\sum_{z=0}^{\infty}(y(e+z) - w(e+z,d))\,Pr\,(z \mid q(d,e))\right\}$$

subject to the worker's optimal search decision, (11). Substituting out $w(e,d)$ using the constraint gives:

$$S(e,d) = \max_{d} \left\{ \sum_{z=0}^{\infty} \left(y\left(e+z\right)\right) Pr(z \mid q\left(d,e\right)) - Wq\left(d,e\right) \right\} \tag{12}$$

The first order condition of the seller's problem is given by

$$\left[ \sum_{z=0}^{\infty} \left[y\left(e+z+1\right) - y\left(e+z\right)\right] \frac{\left(q^{**}\left(e\right)\right)^{z}}{z!} e^{-q^{**}(e)} \right] q'(d,e) = Wq'(d,e).$$

Thus

$$\sum_{z=0}^{\infty} \triangle_{e+z} \frac{\left(q^{*}\left(e\right)\right)^{z}}{z!} e^{-q^{*}(e)} = W \tag{13}$$

We can prove the following proposition.

**Proposition 2.2.** *The competing auction equilibrium is constrained efficient.*

*Proof.* Follows from the team's and planner's first-order condition, (13) and (2). □

Obviously, an incumbent will also choose not to quit if the queue length is positive in equilibrium. Therefore, this decentralized equilibrium also implements efficient quitting if the queue length is positive. Moreover, if the queue length is zero, the quitting decision is also constrained efficient, because the equilibrium amenity is zero, and thus the private decision to quit is equivalent to that given by the social planner.

# 3 Dynamic model

The dynamic model is a repeated version of the static model with free entry of workers and a fixed number of teams.[11] The workers are long lived, risk neutral and have a common discount factor of $\beta$. Each unmatched worker has an opportunity cost of $W$. A team with $e$ members continues to produce $y\left(e\right)$ units of output, but now each team is subject to a constant and random rate of exit, $\delta > 0$, such that all employed workers become unemployed. The exiting team is replaced by a new team with zero incumbent workers. To simplify exposition, we assume that a team may employ

[11] Alternatively, we can also assume that there is also a fixed number of agents and solve for the expected return to unmatched agent. See Kennes and Knowles (2012), for example.

one or two workers, i.e. $y(e) = y(2)$, $\forall\, e > 2$, but the model can be extended to allow for more than two workers.

Let $\Lambda(e)$ denotes the joint value of a team after unemployed workers approached the team and are bidding for employment.

$$\Lambda(e) = \begin{cases} \beta(1-\delta)J(0) & if \quad e=0 \\ y(1) + \beta\left[\delta W + (1-\delta)\, J(1)\right] & if \quad e=1 \\ y(2) + \beta\left[\delta 2W + (1-\delta)\, J(2)\right] & if \quad e=2 \\ y(2) + \beta\left[\delta 2W + (1-\delta)\, J(2)\right] + (e-2)\beta W & if \quad e>2 \end{cases} \tag{14}$$

where $J(e)$ denotes expected joint value of a team with $e$ incumbent workers at the beginning of the period, prior to unemployed workers making their search decision:

$$J(e) = \sum_{z=0}^{\infty} Pr\left(z \mid q(e)\right) \Lambda(e+z)\,. \tag{15}$$

The joint value of a team is the current team output plus the continuation value of the team including the opportunities for rematching of the workers not hired and laid off if the team is subject to the exit shock. If the team does not exit, the following outcomes are dependent on how many applicants the team has in the current period. If $e \leq 2$, then all applicants are hired in the current period and retained in the next period. If $e > 2$, then only 2 workers are hired in the current period and retained in the next period.[12]

Using this, the bidding function of each agent is given by

$$S(e) = \begin{cases} \Lambda(e) - \Lambda(e-1) & if \quad e > \hat{e} \\ \frac{\Lambda(e)-\Lambda(0)}{e} & if \quad e \leq \hat{e} \end{cases} \tag{16}$$

where $S(e)$ denotes the bidder's share of joint value. The employment level $\hat{e}$ is the level at which the joint value function switches from increasing to decreasing returns, making it feasible for teams

[12] If $y(2)/2$ is sufficiently smaller than $y(1)$, i.e. if the technology experiences strongly diminishing marginal returns, it is possible that one worker would decide to leave at the beginning of the next period and search for a new team. We rule this out by assumption.

to pay workers their marginal contribution. With size teams who can employ either one or two workers, the interesting case is $\hat{e} = 2$; for the first worker, average and marginal surplus contribution coincide and workers in excess of 2 add only their continuation value $\beta W$.

The expected value of approaching a team with $e$ workers and a queue length $q(e)$, $W(e, q(e))$ is given by:

$$W\left(0, q\left(0\right)\right) = e^{-q(0)}S(1) + q(0)e^{-q(0)}S(2) + \left(1 - e^{-q(0)} - q(0)e^{-q(0)}\right)\beta W \tag{17}$$

$$W\left(1, q\left(1\right)\right) = e^{-q(1)}S(2) + \left(1 - e^{-q(1)}\right)\beta W \tag{18}$$

Similarly, the expected payoff of an incumbent worker employed by a team with $e$ incumbents is given by:

$$\hat{W}\left(1, q\left(1\right)\right) = e^{-q(1)}S(1) + q(1)e^{-q(1)}S(2) + \left(1 - e^{-q(1)} - q(1)e^{-q(1)}\right)\beta W \tag{19}$$

$$\hat{W}\left(2, q\left(2\right)\right) = S(2) \tag{20}$$

Here we used the fact that no worker will approach a team that already employs two workers, since the value of applying would be $\beta W$, which is less than the cost of entering, $W$. If the queue length at a team is positive, the free entry of agents gives $W = W(e, q\left(e\right))$.[13] A worker on a team with $e$ incumbents can choose to stay or quit but as noted in footnote 12, we rule out the case that $y(e)$ has such strong decreasing returns.

## 3.1 Distribution of Team Sizes

Given the queue lengths $q(e)$ from above, we can solve for the distribution of team sizes. Let $n\left(e\right)$ denote the number of teams with $e$ incumbent workers at the start of the period and let $N(e+z)$ denote the number of teams with $e$ incumbent workers and $z$ applicants. Any team that has $N\left(e\right)$ current workers exits at the end of the period with probability $\delta$ and all teams with $e + z \geq 3$ will

[13]Note that in the case of an increasing average product, if there exists a positive equilibrium queue length, there may also exists another queue length that also solves equation (17). In this case, we can always focus on the longest queue length, which is equivalent to selecting the directed search equilibrium which is most favorable to workers.

be a size 2 team in the following period. Therefore,

$$n(e) = \begin{cases} (1-\delta)N(e) & e = 1 \\ (1-\delta)\sum_{s=2}^{\infty} N(s) & e \geq 2 \end{cases}.$$

The number of teams without members at the start of the period is given by $n(0) = \delta(1 - N(0)) + N(0)$, which is the number of teams with workers that were dissolved and the unfilled teams from last period.

Within the period, the team can increase (or not) in size by recruiting. Thus, the number of teams with $e$ workers in the current period is given by

$$N(e) = \sum_{i=0}^{e} n(i)\, Pr(e - i \mid q(i)) \tag{21}$$

where $Pr(e - i \mid q(i))$ is the probability that a team with $i$ incumbent workers receives $e - i$ applicants and hence has $e$ workers after the matching stage. Given, the values of $q(e)$, the equations for $n(e)$ and $N(e)$ form a block recursive system of equations, which implies that the equations are linearly independent and thus this system of equations has a unique solution.

### 3.2 Wages and Inequality

The wage paid to the worker in the current period depends on the current share $S(e)$ and the expected share in the future, which depends on the expected number of hires in the next period. Let $w(e)$ denote the wage paid to a worker on a team with $e$ workers in the production stage. Then $S(e)$ is given by:

$$S(e) = w(e) + \beta(1-\delta)\hat{W}(e') + \beta\delta W, \tag{22}$$

where $e' = \min\{e, 2\}$. Using equation (16) and assuming that $\hat{e} = 2$, we obtain the following wage schedule:

$$w(1) = y(1) + \beta(1-\delta)\left(J(1) - J(0) - \hat{W}(1)\right) \tag{23}$$

$$w(2) = (1 - \beta(1-\delta))\left(\frac{y(2)}{2} + \beta(1-\delta)\frac{\Lambda(2) - J(0)}{2}\right) \tag{24}$$

$$w(e) = \beta(1-\delta)\left(W - \frac{\Lambda(2) - J(0)}{2}\right), \; e > 2 \tag{25}$$

From (23) through (25), it becomes apparent that the wage contract can be interpreted as the team selling itself to its workers. To see that, rearrange (23), which gives

$$w(1) + \beta(1-\delta)\hat{W}(1) = \beta(1-\delta)\left(J(1) - J(0)\right).$$

That is, the payoff of the worker on a size one team, today's wage plus the continuation value of being employed by a size one team at the beginning of the next period, is the value of the team with one employer less the value of the team with zero employers. Similarly, for size two teams we get

$$2\frac{w(2)}{(1 - \beta(1-\delta))} = y(2) + \beta(1-\delta)\left(\Lambda(2) - J(0)\right). \tag{26}$$

That is, the net present value of the wages paid on a size 2 team is equal today's output plus the discounted value of the joint team less the value of a team without employees.

Since the team sells itself to its workers, the wage payment today does not need to be positive. In particular, in the case of more than two workers on the team, when competition among workers in the bidding process allows the team to extract the full surplus, the wage paid will be negative since $\hat{W} = S(2) > W$. The relationship between $w(1)$ and $w(2)$ depends on the primitives of the economy, the production technology, the discount factor and the team break-up rate.

As discussed in the introduction, the U.S. and other developed economies have experienced two seemingly unrelated trends over the past three decades, an increase in inequality and the proliferation of modern human resource management practises, such as self-managed teams and performance pay (Bloom and Van Reenen, 2010). We now computationally analyze the relationship between the production technology and the wage function and investigate whether our model can qualitatively deliver an increase in wage inequality as the complementarity between workers in a team increases.

#### 3.2.1 Numerical Analysis

Consider the following production technology:

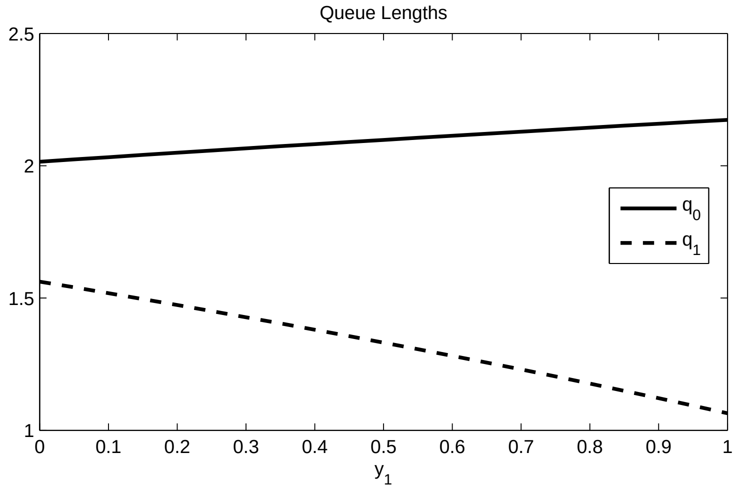
$$y(e) = \begin{cases} 2 & if \quad e = 2 \\ y_1 & if \quad e \neq 2, \end{cases} \tag{27}$$

with $y_1 \in [0, 1]$. Here $y_1$ captures the degree of complementarity between workers on a team. If $y_1 = 1$, the average product is independent of the team size and there are no complementarities. We think of this as the production structure before the organizational change towards teamwork. On the other hand, if $y_1 = 0$, the first worker is completely unproductive without the second worker present. This 'critical mass' team is an extreme example of complementarity in production.[14] We set $\delta = 0.25$ and $\beta = 0.9$ and choose $W$ such that the unemployment rate is 10%. We begin by varying the degree of complementarity by decreasing $y_1$ from 1, i.e. constant returns in production, to $y_1 = 0$, i.e. a critical mass team where no output is produced without two workers present.

Figure 2 shows how the queue lengths respond to changes in the complementarity between the two workers in the team. The queue length at a team without workers is slightly increasing in $y_1$, while the queue length at a team with one worker already present is more strongly decreasing. If $y_1 = 0$, a team only produces output if there are two workers present, hence applying to a team with already one worker eliminates the chance of being alone on a team and not being productive. However, for the free-entry condition to hold, there has to be a sufficiently high probability that the team ends up with more than two workers and as a result the queue length at teams with one worker already employed is relatively long. As $y_1$ increases, the relative value of a size two to size one team decreases and so $q(1)$ falls. For teams without workers the queue length changes little as $y_1$ increases because the decrease in the value of being employed by a size two team is offset by the

[14] Alternatively, we could set $y_1 = 1$ and vary $y_2 \geq 2$. We choose to vary $y_1$ since it is naturally bounded by zero and one.

Figure 2: Equilibrium Queue Length



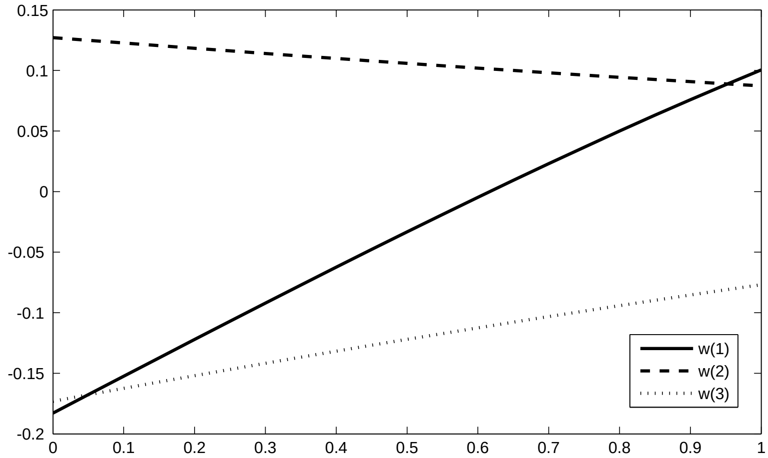

increase in value of being employed by a size one team.

Figure 3 shows the wages as a function of $y_1$. The level of wages is low relative to the output workers produce, even if there are exactly two workers on a team and the worker are able to attract all the surplus. This is a result of the assumption that teams, which fail to hire workers, can rematch in the following period. In the following period, they face a good chance of being matched with more than two workers and extract all surplus in the auction game. Hence the reservation value of the team, $J(0)$, is significantly larger than zero.

The wage on teams with two workers increases slightly, as the complementarity between workers increases (i.e. $y_1$ goes down). This is because as the value of a team without workers ($J(0)$) decreases as $y_1$ goes down, which increases the surplus the workers receive. The increase in $w(2)$ in turn increases the payment made by workers on teams who have more than two workers in the bidding stage who will be at a size two team in the following period. Being on a size two team is more attractive and the bid to buy that team increases and $w(3)$ becomes more negative. Not surprisingly, as the complementarity between workers increases, the wage on teams with one worker decreases markedly. Form equation (23), this is for two reasons, first the output of that worker falls. Second, the value of being a worker on a size one team at the beginning of the following period, $\hat{W}(1)$, increases because of the increase in $w(2)$.

Significant wage dispersion arises through the dynamic rematching, akin to on-the-job search.

Figure 3: Wages



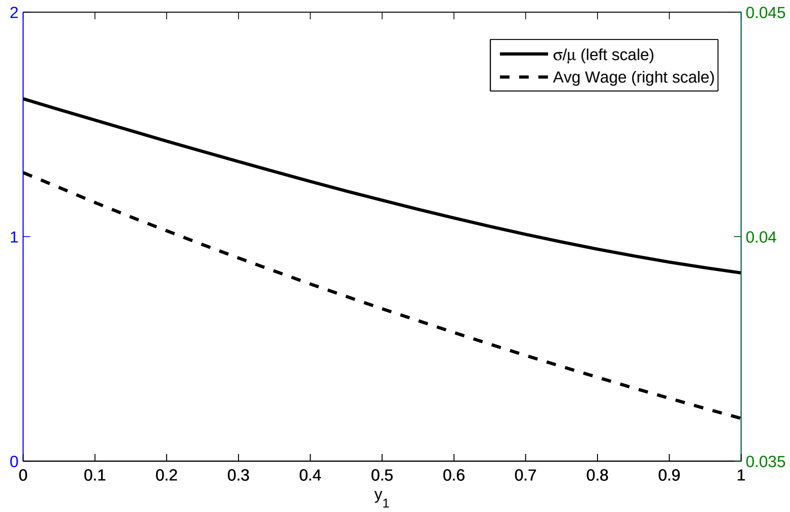

Teams with only one worker will likely attract at least one more worker in the following period. With constant returns, this does not affect the productivity of the existing worker and the wage on teams with one and two workers is almost identical.[15] However, once there are strong complementarities the value of being on team with one worker present at the beginning of the following period is more valuable and the worker is willing to accept a lower wage (possibly even a negative wage) in order to purchase the team. As Figure 4 shows, wage dispersion increases markedly as complementarities are getting stronger ($y_1$ decreases); the coefficient of variation increases by more than 50% from 0.79 to 1.3.

As common in the search literature, we do not have an explicit notion of a firm; a firm may consist of a single team or be a collection of teams. Thus, we do not take an explicit stand on whether organizational change increases dispersion of wages across or within firms.[16] The Fifth European Working Conditions Survey (EWCS2010) reports that the prevalence of teamwork varies both across industries and firm sizes. It is therefore likely that both across and within firm inequality are affected, as difference in organizational forms across firms increase through different rates of

[15]It is slightly higher on teams with one worker because of the chance of having three or more workers in the following period, giving all future surplus to the team.

[16]Dispersion in wages across firms appears to be the main contributor to the increase in inequality (e.g. Lazear and Shaw, 2009), but using matched employer-employee data, some authors find that at least some of the increase in inequality can be attributed to the increases in within firm dispersion. Using data from 1963 to 1986, Davis and Haltiwanger (1991) find that for non-production workers about half of wage inequality growth occurs within firms. More recently, using data from 1992 to 2007, Barth et al. (2014) find that, depending on the sample restrictions, changes in the within firm variance account for one fifth to one third of the overall increase in the variance of wages.

Figure 4: Wage Dispersion




organizational change.

In addition to residual inequality, our model may also be used to shed light on the rise of the skill premium. Most workers are employed on size two teams – larger and smaller team typically only exist for one period – the average wage is increasing, even though output (labor productivity) is falling. Going from constant returns to critical mass teams increases the average wage by 8.8% and the productivity adjusted wage by 15.2%. To the extent that team production is more common among high-skilled workers, this may help to explain the increase in the skill premium that occurred concurrent with the increase in residual inequality, see Figure 4.[17] An increase in teamwork then generates an increase in the premium of the worker group using teams, and an increase in the dispersion among those workers, consistent with the increase in the skill premium and the increase in the 90/50 residual dispersion (Autor et al., 2008).

## 3.3 Efficiency

Since the teams sell themselves to the workers, workers are the residual claimants, any externality is getting internalized by the workers. However, workers discount the future, so the efficiency will depend on how strongly workers discount. We computationally analyze the role of the discount

[17]The EWCS2010 finds that higher the level of educational attainment are associated with higher the frequency of teamwork.

factor $\beta$ in the efficiency of simple auction equilibrium.

Consider a simple critical mass team where two workers are needed to fulfil a task. The output of the team with a period is given by

$$y(e) = \begin{cases} 2 \quad if \quad e = 2 \\ 0 \quad if \quad e \neq 2 \end{cases} \tag{28}$$

As before, we set $\delta = 0.25$ and we choose $W$ such that the unemployment rate is 10%. We vary the discount factor $\beta$ to study its effect on the inefficiency of the simple auction mechanism. The resulting values as a function of the discount factor are reported in Table 1.

Table 1: Changes in asset values as a function of discounting

| | [1] | [2] | [3] |
|---|---|---|---|
| $\beta$ | $\Lambda(1) - \Lambda(0)$ | $\frac{\Lambda(2)-\Lambda(1)}{2}$ | [1]/[2] |
| 0 | 0 | 1.0000 | 0 |
| 0.2000 | 0.0566 | 1.0738 | 0.0527 |
| 0.4000 | 0.1323 | 1.1546 | 0.1146 |
| 0.6000 | 0.2450 | 1.2405 | 0.1975 |
| 0.8000 | 0.4560 | 1.3259 | 0.3439 |
| 0.8500 | 0.5493 | 1.3460 | 0.4081 |
| 0.9000 | 0.6837 | 1.3658 | 0.5006 |
| 0.9500 | 0.9219 | 1.3886 | 0.6639 |
| 0.9750 | 1.0880 | 1.4014 | 0.7764 |
| 0.9850 | 1.2030 | 1.4102 | 0.8531 |
| 0.9900 | 1.2673 | 1.4151 | 0.8956 |
| 0.9950 | 1.3412 | 1.4208 | 0.9440 |
| 0.9980 | 1.3916 | 1.4250 | 0.9766 |
| 0.9990 | 1.4094 | 1.4264 | 0.9881 |
| 0.9999 | 1.4259 | 1.4276 | 0.9988 |

Since the team seeks a critical mass of two workers, the value of a team with no members is subject to increasing returns to scale since a team with only one worker produces no output. Thus,

$$\frac{\Lambda(2) - \Lambda(1)}{2} \geq \Lambda(1) - \Lambda(0) \tag{29}$$

In the static model, which is analogous to assuming that $\beta = 0$, the value of the club is given by $\Lambda(e) = y(e)$. If the discount factor increases, then $\Lambda(e)$ also increases to reflect the possible returns to future matching rounds. Table 1 illustrates the key limit result: If the discount factor approaches one, the average asset value of a critical mass team approaches a constant.

$$\lim_{\beta \to 1} \frac{(\Lambda(2) - \Lambda(1))/2}{\Lambda(1) - \Lambda(0)} = 1$$

Therefore, opportunities for rematching function to linearize the net payoffs of a club relative to the number of members. The corollary to this result is that the subsidy on worker entry ("Mortensen tax") relative to the value of the team will fall to zero.

A closely related experiment is to increase the frequency of offer rounds holding the discount factor constant (De Fraja and Sakovics (2001)). In this case, we scale the value of output each period by the time available for playing chess given the rate of discounting and we scale the probability of a match shock inside the period to reflect the shorter period length.

**Proposition 3.1.** *The simple auction equilibrium converges to the constrained efficient allocation if the length of time between offer rounds falls to zero.*

*Proof.* If the time between offer rounds falls to zero, then agents effectively do not discount future matching opportunities and thus $(\Lambda(2) - \Lambda(1))/2 \cong \Lambda(1) - \Lambda(0)$. The asset value of a club is also bounded by $\Lambda(2) \leq y(1)/\beta$, which is the present value of a club that is filled in each period. □

The intuition is that a single buyer can effectively purchase the team and act as a private goods seller in the next period. This reduces the importance of establishing a critical mass of buyers in any particular period and thus the model functions more closely as a private goods model. Therefore, if we increase the opportunities for repeated matching, the competing auction environment will converge to the simple auction environment.

# 4 Conclusion

We have developed a directed search model of team production. Teams are characterized by complementarities between workers, giving rise to local increasing returns. We show that the static directed search equilibrium is constrained efficient but the reserve price is not zero. We then computationally demonstrate that changes to the organization of production that emphasize team production are increasing wage dispersion. This is consistent with the concurrent increase in residual wage inequality, especially on the top of the earnings distribution, and the proliferation of teamwork in modern human resource management.

# References


[1] Albrecht, James, Pieter Gautier and Susan Vroman (2012): "A Note on Peters and Severinov, 'Competition Among Sellers Who Offer Auctions Instead of Prices'." *Journal of Economic Theory*, Vol. 147, pp. 389-392.

[2] Albrecht, James, Pieter Gautier and Susan Vroman (2014): "Efficient Entry in Competing Auctions." *American Economic Review*, Vol. 104 (10), pp. 3288-3396.

[3] Autor, David H., Lawrence F. Katz and Melissa S. Kearney (2008): "Trends in U.S. Wage Inequality: Revising the Revisionists." *Review of Economics and Statistics*, Vol. 90(2), pp. 300–323.

[4] Barth, Erling, Alex Bryson, James C. Davis and Richard Freeman (2014): "It's Where You Work: Increases in Earnings Dispersion across Establishments and Individuals in the U.S.." NBER Working Paper No. 20447.

[5] Bloom N, Van Reenen J, Ashenfelter O, Card D. Human resource management and productivity. Handbook of Labor Economics (2010) 4B(Elsevier, Amsterdam) 16971767

[6] Burdett, Ken, Shouyong Shi and Randall Wright (2001): "Pricing and matching with frictions." *Journal of Political Economy* Vol. 109(5), pp. 1060-1085.

[7] Cosar, Kerem, Nezih Guner and James Tybout (2014): "Firm Dynamics, Job Turnover, and Wage Distributions in an Open Economy." *American Economic Review,* forthcoming.

[8] Davis, Steve J. and John Haltiwanger (1991): "Wage Dispersion between and within US Manufacturing Plants, 1963 - 1986." *Brooking Papers on Economic Activity,* pp. 115–180.

[9] De Fraja, Gianni and József Sákovics (2001): "Walras Retrouvé: Decentralized Trading Mechanisms and the Competitive Price." *Journal of Political Economy*, Vol. 109(4), pp. 842-863.

[10] Eurofound (2012): "Fifth European Working Conditions Survey." Publications Office of the European Union, Luxembourg.

[11] Geromichalos, Athanasios (2012): "Directed Search and Optimal Production." *Journal of Economic Theory*, Vol. 147(6), pp. 2303-2331.

[12] Hawkins, William (2013): "Competitive Search, Efficiency, and Multi-worker Firms." *International Economic Review* Vol. 54(1), pp. 219-251.

[13] Helpman, Elhanan, Oleg Itskhoki and Stephen J. Redding (2010): "*Inequality and Unemployment in a Global Economy.*" *Econometrica*, Vol. 78(4), pp. 1239-1283.

[14] Julien, Benoit, John Kennes and Ian King (2000): "Bidding for Labor." *Review of Economic Dynamics*, Vol. 3(4), pp. 619-649.

[15] Julien, Benoit, John Kennes and Ian King (2006): "Residual Wage Disparity And Coordination Unemployment." *International Economic Review*, Vol. 47(3), pp. 961-989.

[16] Julien, Benoit, John Kennes and Moritz Ritter (2014): "Efficient club formation in a frictional environment." *Economics Letters*, Vol. 125(3), pp. 372-375.

[17] Kaas, Leo and Philip Kircher (2014): "Efficient Firm Dynamics in a Frictional Labor Market." Working Paper.

[18] Kennes, John and John Knowles (2012): "Do Marital Prospects Dissuade Unmarried Fertility?" Working Paper.

[19] McAfee, Preston (1993): "Mechanism Design by Competing Sellers." *Econometrica*, Vol 61, pp. 1281-1312.

[20] Montgomery, James D. (1991): "Equilibrium wage dispersion and interindustry wage differentials." *The Quarterly Journal of Economics* Vol. 106(1), pp. 163-179.

[21] Mortensen, Dale (2009): "Island Matching." *Journal of Economic Theory*, Vol 144, pp. 2336-2353.

[22] Lazear, Edward and Kathryn Shaw, eds. (2009): "The Structure of Wages: An International Comparison." The University of Chicago Press.

[23] Lemieux, Thomas W., Bentley MacLeod and Daniel Parent (2009): *The Quarterly Journal of Economics* Vol. 124(1), pp. 1-49.

[24] Lester, Benjamin (2010): "Directed Search with Multi-Vacancy Firms." *Journal of Economic Theory*, Vol. 145(6), pp. 2108-2132.

[25] Lester, Benjamin, Ludo Visschers and Ronald Wolthoff (2015): "Meeting Technologies and Optimal Trading Mechanisms in Competitive Search Markets." *Journal of Economic Theory*, Vol. 155, pp. 1-15.

[26] Peters, Michael (1984): "Bertrand equilibrium with capacity constraints and restricted mobility." *Econometrica,* Vol. 52 pp. 1117-1128.

[27] Peters, Michael and Sergei Severinov (1997): "Competition among sellers who offer auctions instead of prices." *Journal of Economic Theory*, Vol. 75, pp. 141-179.

[28] Shi, Shouyong (2002): 'Product market and the size-wage differential." *International Economic Review*, Vol. 43(1), pp. 21-54.

[29] Tan, Serene (2012): "Directed Search and Firm Size." *International Economic Review*, Vol. 53(1), pp. 95-113.