Opting Out of Buyer-Seller Networks

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Abstract

Buyer-seller networks where price is determined in a centralized framework (specifically by an ascending-bid auction) are important in many economic examples such as certain real estate markets and even in radio spectrum sharing. However, it may be that some sellers are better off not participating in the market. We consider what happens if sellers can make a take it or leave it offer to one of their linked buyers before the centralized price is determined and thus such a seller can choose not to participate in the auction. We give conditions on the graph and buyers' valuations under which the buyer and seller will both agree to such a take it or leave it offer. Specifically, the buyer-seller pair will choose to opt out of the auction if (i) the seller acts as a bridge in the network where if he is removed then the initial graph splits and if (ii) the probability of a buyer having a high valuation is large enough or if the expected price from the auction over the initial graph is low perhaps due to there being a large number of sellers.

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1 Introduction

We consider buyer-seller networks where price is determined in a centralized framework, specifically by an ascending-bid auction. However, the sellers also have the opportunity to not participate in the auction, but to instead make an offer to one of their linked buyers. There are many examples where buyer-seller networks use auctions to determine prices. Perhaps the best known example would be real estate. Many countries use auctions to determine prices in real estate markets and even in the U.S. there are online real estate sites that sell by auction; see Maher (1989), Lusht (1996), and Thanos and White (2014). Real estate sales can be interpreted as taking place on a network since the seller's property has a particular location and each buyer also has a certain location (or locations) where he wishes to purchase property; these locations create buyer-seller links. However, it may be the case that a seller would prefer to make an offer to a particular buyer directly rather than go through the auction. An alternative example is that of art auctions where each seller has a particular painting he is selling and buyers want to purchase paintings by particular artists; such preferences can create buyer-seller links. Another example is that of secondary spectrum sales where a primary spectrum user holds a license to use a specific spectrum band in a given area. Unlicensed secondary users would like to access the spectrum by purchasing idle spectrum channels from primary users whose license covers their location. These locations create a buyer-seller network where a link indicates a secondary user (or buyer) is located in a primary user's (or seller's) licensed area; see Zhang and Zhou (2014). The electrical engineering literature often suggests auctions as the best mechanism for such sales[1]; see Zhang, et al. (2013) and Chun and La (2013). However, it may be that the primary license holder would prefer to opt out of the auction and make an offer directly to a certain buyer.

Specifically, we consider buyer-seller networks with an ascending bid auction where each seller is able to exit the auction and make a take it or leave it offer to a buyer. Thus, before the auction occurs the seller can choose a buyer that he is linked to and offer to sell the good to this buyer at a price of his choosing. The buyer can accept this price or the buyer can choose to return to the auction that will take place without the seller; once the seller chooses to make an offer he has opted out of the auction and cannot return. We give conditions under which the buyer and seller will both agree to such a take it or leave it offer and thus will choose to opt out of the auction. Specifically, the seller is able to make such an offer if (i) he acts as a bridge in the network where if he is removed then the initial graph splits and if (ii) the probability of a buyer having a high valuation is large enough or if the expected price from the auction over the initial graph is low perhaps because of there being a large number of sellers. We consider both the case of an allocatively complete initial graph (in proposition 1) and the case of an allocatively incomplete initial graph (in proposition 8).

The paper most closely related to the current one is Kranton and Minehart (2001). They examine a buyer-seller network where goods are sold in an ascending bid auction. They show that for a given link pattern the equilibrium prices are pairwise stable in that no linked buyer and seller can renegotiate and obtain a better

[1] Alternatively, Zhang and Zhou (2014) consider a mechanism for sharing channels where primary users set quotas based on secondary user's locations in order to maximize profits.

deal. Corominas-Bosch (2004) also consider buyer-seller networks in a centralized market where prices are determined through bilateral bargaining. Note that in our model players opt out before the auction takes place which is different from the renegotiation that can take place in Kranton and Minehart (2001) which occurs ex-post. Another difference is that in our game after the seller makes the take it or leave it offer he cannot return to the auction. To show that the offer is credible the seller exits the auction and after he makes his offer there is not time for him to return to the auction and the auction proceeds without him.

There are many interpretations for why the seller does not return to the auction even if his take it or leave it offer is rejected. For instance, consider the housing market example where the game is not repeated. Here there may be some sellers who value the good above what they could get in the ascending bid auction. If these sellers decide to make a take it or leave it offer, then if the offer is rejected, they will not return to the auction as they do not expect to get a high enough payoff from the auction. Essentially these sellers are choosing to exit the auction perhaps because there are many buyers and sellers in the market and the expected auction price is thus low. However, if sellers have more control over the price, then they are willing to sell the house. Alternatively, consider a market with repeated sales[2] such as radio spectrum. Here the seller may choose not to return to the auction to gain credibility as a tough negotiator in the future. If he does not return to the auction, then if the game is repeated the buyers will know he will not return to the auction and will be more willing to accept his current offer.

[2]Fainmesser (2012) considers repeated buyer-seller games played on a network where the threat of a loss of repeated interactions can facilitate cooperation.

There is a large literature on buyer-seller networks. Some papers focus on co-operative approaches to buyer-seller bargaining such as seller cooperatives or Nash bargaining; see Wang and Watts (2006) and Bayati et al. (2015), respectively. While others consider de-centralized bargaining with bilateral opportunities; see Abreu and Manea (2012), de Fontenay and Gans (2014), Condorelli and Galeotti (2012), and Hatfield et al. (2013). Alternatively, Elliott (2014) allows buyers and sellers to invest in relationships and shows that over-investment results when players wish to create outside options while Board and Pycia (2014) considers buyer-seller networks where the buyer has an outside option. We add to this literature by focusing on what happens if sellers can credibly exit the market.

The current paper is related to exchange networks in the sociology literature. There are many papers looking at the relationship between network position and power in exchange networks; see Markovsky, Willer, and Patton (1988), Cook and Yamagishi (1992), Lucas et al. (2001), and Skvoretz and Willer (1993). Here one agent may have power over another if he controls the others resources. In the economics literature, Manea (2011) considers a player's strength in infinite horizon buyer-seller networks with random matching. In the current paper, we find that a seller who acts as a bridge has power over some buyers and may be able to entice such buyers to opt out of the auction.

2 Model

Let $M = \{1, 2, ..., i, ...m\}$ represent the set of sellers and $N = \{1, ..., j, ..., n\}$ the set of buyers in a region. Assume $m \leq n$. Without loss of generality, assume that each seller has one unit of the good to sell.

If buyer j observes seller i's prices we say that j and i are linked. We let g represent the set of links between sellers and buyers with G representing the set of all possible such graphs. We use notation $ij \in g$ to represent a link between i and j. We let $g - i$ represent the graph that would be obtained when i and all of i's links are removed from graph g. Let $|g|$ represent the number of linked sellers in g. We define $c \subset g$ to be a component of g if for every linked $i \in c$ and for every linked $j \in c$ there exists a path $\{ik, k\ell, ..., jj\}$ such that each link in the path is in c. Let $|c|$ represent the number of linked sellers in c. We say that $i \in M$ is a bridge if $g - i$ has more components than does g.

We assume that i pays a cost k_{ij} for maintaining each of his links $ij \in g$ and j pays a cost $\widetilde{k}_{ij}$ for maintaining each of his links $ij \in g$.

Buyer j would like to purchase at most one unit where v_j represents the value that j would receive from using the unit. Specifically, it is assumed that each v_j is a random variable independently and identically distributed on $[\underline{v}, \overline{v}]$ with continuous distribution F, where $\underline{v} \geq 0$. We let $v_{(\ell)}$ represent the ℓth highest order statistic of the n values and let $f_{v_{(\ell)}}$ represent its density function. We represent this buyer by (ℓ). Thus $f_{v_{(\ell)}}(v_\ell) = \frac{n!}{(\ell-1)!(n-\ell)!}[F(v_\ell)]^{n-\ell}[1 - F(v_\ell)]^{\ell-1}f(v_\ell)$. We let $Pr[v = v_{(\ell)}] = \frac{(n-1)!}{(\ell-1)!(n-\ell)!}[F(v_\ell)]^{n-\ell}[1 - F(v_\ell)]^{\ell-1}$ which represents the probability that v is the ℓth highest order statistic.

We consider two different methods of price generation. The first is prices which are set by an ascending bid auction. The price that seller i receives in the auction is represented by p_i^a. However, it is possible that seller i would prefer not to participate in the auction. In this case, seller i picks a buyer j such that $ij \in g$ and offers price p_{ij} to that buyer. Buyer j can either agree to pay price p_{ij} for the good, or can refuse to pay the price and can choose to participate in the auction that takes place without seller i. Buyer j must be linked to another seller besides i in order to participate in the auction that takes place without i. Once seller i has decided not to participate in the auction, then this decision is final. He may participate in a later auction, but not in this one. An explanation could be that the auction takes place quickly and once i has opted out there is not time for him to opt back in.

Seller i receives utility u_i where

$$u_i = \begin{cases} p - \sum_{j:ij\in g} k_{ij} & \text{i sells his unit at price } p \\ -\sum_{j:ij\in g} k_{ij} & \text{otherwise.} \end{cases}$$

Buyer j receives payoff $\widetilde{u}_j$ where

$$\widetilde{u}_j = \begin{cases} v_j - p - \sum_{i:ij\in g} \tilde{k}_{ij} & j\text{buys a good at price } p, \\ -\sum_{i:ij\in g} \widetilde{k}_{ij} & \text{otherwise.} \end{cases}$$

Next we define an *allocatively complete* network. This definition is from Kranton and Minehart (2001). Network g is allocatively complete if and only if for every $B \subseteq N$ of size m, there exists a feasible allocation such that every $j \in B$ obtains a good.

3 Results

We start with a proposition which shows that a seller may gain from not participating in the auction, but from instead just offering a price to a particular buyer that he is linked to for the good.

For simplicity we set the costs of maintaining links $k_{ij} = \widetilde{k}_{ij} = 0$ for all i and j with the understanding that adding in such costs will only strengthen our results in the sense that it will increase the cost of staying in the auction substantially and thus make opting out more likely for both agents.

Assumption A1: Let g be allocatively complete and consist of a single component and let $g-i$ consist of at least two allocatively complete components for some $i \in M$. Choose a $j \in N$ such that $ij \in g$ and such that j is not guaranteed a good in the auction over $g-i$ even if $v_j \geq v_{|m-1|}$. Let $c \in g-i$ be the component of $g-i$ that j is a member of.

Under assumption A1 seller i acts as a bridge in graph g. Proposition 1 gives conditions under which seller i can use his position as a bridge to entice buyer j to opt out of the auction.

Proposition 1. *Let assumption A1 be true for some $i \in M$ and $j \in N$. Seller i and buyer j will choose to opt out of the auction and exchange the good at price p_{ij} if for $|c| > 0$, $\frac{E[v_{(m+1)}]}{Pr(v_j \geq p_{ij})} \leq p_{ij} \leq min\{v_j(1 - Pr(v_j \geq v_{(|c|)})) + E[v_{(|c|+1)}|v_j \geq v_{(|c|)}], v_j\}$ and if for $|c| = 0$, $\frac{E[v_{(m+1)}]}{Pr(v_j \geq p_{ij})} \leq p_{ij} \leq v_j$.*

Note that the condition $\frac{E[v_{(m+1)}]}{Pr(v_j \geq p_{ij})} \leq p_{ij}$ guarantees that seller i will be better off exchanging the good at price p_{ij} given F while $p_{ij} \leq min\{v_j(1 - Pr(v_j \geq v_{(|c|)})) +$

$E[v_{(|c|+1)}|v_j \geq v_{(|c|)}], v_j\}$ guarantees that j is better off participating in the exchange given v_j.

Proof. First we show that such a j exists that meets assumption A1. Note that it is not possible for a component of $g-i$ to consist of only sellers and not buyers as g was a single component and $g-i$ simply removes one seller from g and by definition sellers are not connected to each other; thus, all components of $g-i$ consist of either only buyers or of both buyers and sellers. As $g-i$ consists of at least two components, there must be at least one component that has less than $m-1$ buyers; call this component c. As c has less than $m-1$ buyers it is not possible for $m-1$ sellers to purchase from c. Thus, there must exist a $j \in c$ such that $ij \in g$ and such that j is not guaranteed a good even if $v_j \geq v_{(|m-1|)}$.

Next we consider the case where $|c| > 0$ and show that buyer j will choose to opt out of the auction if the conditions of Proposition 1 are met. If j decides to opt out of the auction, then he will receive a payoff of $v_j - p_{ij}$. If j decides to participate in the auction over graph $g-i$, then by assumption c is allocatively complete and so the items in component c will sell to the buyers in c with the top $|c|$ values at price $v_{(|c|+1)}$. Thus j will win an item only if $v_j \geq v_{(|c|)}$. So j's expected payoff from participation in the auction is $Pr(v_j \geq v_{(|c|)})(v_j) - E[v_{(|c|+1)}|v_j \geq v_{(|c|)}]$. So j prefers to opt out of the auction if $v_j - p_{ij} \geq Pr(v_j \geq v_{(|c|)})(v_j) - E[v_{(|c|+1)}|v_j \geqslant v_{(|c|)}] \geq 0$ or if $p_{ij} \leq min\{v_j(1 - Pr(v_j \geq v_{(|c|)})) + E[v_{(|c|+1)}|v_j \geq v_{(|c|)}], v_j\}$.

Now we consider the case where $|c| > 0$ and show that seller i will choose to opt out of the auction if our conditions are met. Seller i will prefer to opt out if his expected payoff from opting out is greater than his expected payoff from participating in the auction over g. If i participates in the auction, then he expects for the good to be sold at the price $v_{(m+1)}$. Since g is allocatively complete, the buyers with the top m values for the good will all win an item and the items will be sold at price $v_{(m+1)}$; thus, i expects to receive $E[v_{(m+1)}]$ from participating in the auction. If seller i decides to opt out of the auction, then his expected payoff is $p_{ij} \cdot Pr(j \text{ accepts } p_{ij})$. Since $p_{ij} \leq v_j(1 - Pr(v_j \geq v_{(|c|)}) + E[v_{(|c|+1)}|v_j \geq v_{(|c|)}]$, we know that j accepts p_{ij} if $v_j \geq p_{ij}$. Thus, i's expected payoff from opting out is $p_{ij} \cdot Pr(v_j \geq p_{ij})$. So if $p_{ij} \cdot Pr(v_j \geq p_{ij}) \geq E[v_{(m+1)}]$, then i will choose to opt out of the auction.

Next we consider the case where $|c| = 0$. If $|c| = 0$, then there are no sellers in the component that j is a member of in network $g - i$. Thus, if i makes an offer of p_{ij} to j, then if j rejects the offer he will not receive a good and will end up with a payoff of 0. Thus j only rejects the offer if $p_{ij} > v_j$ and i will only make the offer if his expected payoff from the offer, $p_{ij} \cdot Pr(v_j \geq p_{ij})$, is greater than his expected payoff from participating in the auction over g, $E[v_{(m+1)}]$. □

In the following corollary we assume the assumptions of Proposition 1 hold true and allow the seller to choose the price which maximizes his expected profits to offer to buyer j.

Corollary 2. *Seller i will choose $p_{ij} = max_p p \cdot Pr(p \leq min\{v_j(1 - Pr(v_j \geq v_{(|c|)})) + E[v_{(|c|+1)}|v_j \geq v_{(|c|)}], v_j\})$ if $|c| > 0$ and $p_{ij} = max_p p \cdot Pr(p \leq v_j)$ if $|c| = 0$.*

Next we illustrate the proposition with an example.

Example 3. Let $m = 3$, $n = 4$, and $g = \{11, 12, 22, 23, 33, 34\}$. Let $v_i \in \{1, 2, 9, 10\}$ each with equal probability. In an ascending bid auction over g the buyers with the top three valuations each win a good at price $v_{(4)}$ where $E[v_{(4)}] = (.6838)(1) + (.2539)(2) + (.0586)(9) + (.003906)(10) = 1.75806$; note that (.6838) is the probability that at least one of the v_i's equals 1. Next we consider the graph without seller 2 and all of his ties which we call $g-2$. Seller 2 can offer price p_{22} to buyer 2. Buyer 2 will accept this price if doing so makes him better off than he is from participating in the auction over $g-2$. If $v_2 = 10$, then buyer 2 expects to pay price $\frac{1}{4}(1+2+9+\frac{1}{2}10) = 4.25$ in the auction over $g-2$. To see this note that the component of $g-2$ that buyer 2 is a member of is $c = \{11, 12\}$. Thus, buyer 2 wins the item if buyer 1 has $v_1 < 10$ and expects to pay price v_1. While if $v_1 = 10$, then buyer 2 expects to win the item with probability $\frac{1}{2}$ and pay price $v_1 = 10$. Thus, buyer 2 expects to win the item with probability $1 - (\frac{1}{2})(\frac{1}{4}) = .875$. Here buyer 2's expected payoff from entering the auction is: $(.875)(10) - 4.25 = 4.5$. If $v_2 = 9$, then buyer 2's expected payoff from the auction is $(\frac{1}{2} + \frac{1}{4}(\frac{1}{2}))(9) - (\frac{1}{4})(1+2+9(\frac{1}{2})) = (.625)9 - 1.875 = 3.75$. If $v_2 = 2$, then $E[\widetilde{u}_2] = (\frac{1}{4} + \frac{1}{4}(\frac{1}{2}))(2) - (\frac{1}{4})(1 + 2(\frac{1}{2})) = .25$. If buyer $v_2 = 1$, then $E[\widetilde{u}_2] = 0$. Therefore, if $p_{22} \leq 5.25$ and if buyer 2 has valuation $v_2 = 9$ or $v_2 = 10$, then if 2 opts out of the auction he will receive a payoff of $v_2 - p_{22} \geq 4.5$ if $v_2 = 10$ and a payoff of $v_2 - p_{22} \geq 3.75$ if $v_2 = 9$. Thus both types of high value buyers will choose to accept p_{22}. Seller 2 will receive a payoff greater than his payoff from the auction over g if $Pr(buyer\ 2\ accepts) \cdot p_{22} \geq 1.75806$. Given that $p_{22} \leq 5.25$, buyer 2 will accept if he has one of the top two valuations and this occurs with probability $\frac{1}{2}$. Thus if $p_{22} \geq 3.5172$, then seller 2 prefers to opt out of the auction. So for any $3.5172 \leq p_{22} \leq 5.25$, both seller 2 and the top value buyers are better off trading the good at price p_{22}. Note that since the probability of buyer 2 accepting is equal to $\frac{1}{2}$ for all $3.5172 \leq p_{22} \leq 5.25$, seller 2's expected payoff will be maximized at the largest price in this range. Seller 2 cannot gain from raising the price and only selling to a buyer with $v_2 = 10$ as the most this buyer would pay is $p = 10 - 4.5 = 5.5$, but the probability of acceptance (or of the buyer being this type) would fall to $\frac{1}{4}$; thus, seller 2's expected payoff from such an offer would be lower than if he set $p_{22} = 5.25$.

Next we allow a seller to make price offers to multiple buyers that he is tied to. Specifically, seller i can simultaneously offer price p^i to all buyers j such that $ij \in g$. Each buyer j can either agree to pay price p^i for the good or can refuse to pay the price and then can participate in the auction that takes place without seller i. If multiple buyers agree to pay price p^i then seller i picks one at random to obtain the good. The remaining buyers can then participate in the auction that takes place without seller i. We assume that buyers behave somewhat myopically in that they do not take into account another buyer accepting the offer when they make their own decisions.

Remark 4. We assume that if seller i's offer is rejected, then i does not return to the auction and participate in it. It could be that there is a chance that the game will be repeated in the future and a chance this repetition will be repeated; thus, there is a chance the game could be repeated an infinite number of times. If i returns to the auction this period, then in all future periods the buyers will not believe that i will opt out of the auction if his offers are rejected. Thus, the buyers have no incentive to accept a price that is below what they would receive in the auction with i. In this case there is no mutual gain from opting out and the full auction over g will always be played in the future. Thus if i values the future enough, then he will have incentive today to not re-enter the auction if all his offers are rejected. Note that buyer j always has incentive to return to the auction if he is unable to purchase the good from i, as he cannot gain in the future from abstaining from the auction.

Example 5. This is a continuation of Example 4 where now seller 2 can make simultaneous offers of p^2 to both buyers 2 and 3. Note that if buyer 2 has one of the top two values, then he will agree to opt out of the auction as before or as long as $p^2 \leq 5.25$ since any price lower than 5.25 will guarantee that he receives a payoff greater than what he would get if he remains in the auction without seller 2. Similarly if buyer 3 has one of the top two values he will agree to opt out if $p^2 \leq 5.25$. We know from example 4, that seller 2 will agree to opt out of the auction if $Pr(buyer\ 2\ or\ 3\ accepts\ offer\ p^2) \cdot p^2 \geq 1.75806$. Buyer 2 or 3 will accept if he has one of the top two values. Thus the probability that buyer 2 or 3 accepts is equal to the probability that one or both of these buyers has $v_j = 9$ or $v_j = 10$. This probability equals $1 - \frac{1}{4} = \frac{3}{4}$. Thus seller 2 opts out if $\frac{3}{4} \cdot p^2 \geq 1.75806$ or if $p^2 \geq 2.34408$. So now for any $2.34408 \leq p_{22} \leq 5.25$, both seller 2 and the top two buyers are better off opting out of the auction. Allowing the seller to make more offers has increased the range of prices which support opting out of the auction. As seller 2 can now make offers to more buyers the probability of his offer being rejected is lower and he does not need to charge as high of a price to opt out.

Next we generalize this example in the proposition below.

Proposition 6. *Let assumption A1 be true for seller i and for all $j \in \{j1, j2, ..., j\ell\}$. Seller i and buyers $j1, ..., j\ell$ will choose to opt out of the auction and exchange the* *good at price p if* $\frac{E[v_{(m+1)}]}{Pr(v_{j1} \geq p \cup v_{j2} \geq p \cup ... \cup v_{j\ell} \geq p)} \leq p$ *and if p satisfies all of the right hand side inequality constraints listed in Proposition 1.*

Comparing Propositions 1 and 6 we see that the range of prices that allow for opting out has increased. Note that even though all buyers $j1, ..., j\ell$ choose to opt out only one of them will end up with the good.

Proof. We show that if p satisfies the inequalities given in Proposition 6, then i and $j1, j2, ..., j\ell$ will all choose to opt out of the auction. Since the right hand side inequality constraints from Proposition 1 are met, we know from the proof of Proposition 1 that all $j \in \{j1, ..., j\ell\}$ will prefer to opt out of the auction. Next we show that seller i will also prefer to opt out. Seller i prefers to opt out if his expected payoff from opting out is greater than his expected payoff from participating in the auction over g. Recall that the good is sold at price $v_{(m+1)}$ in the auction over g. Thus i prefers to opt out if $p \cdot Pr(\text{at least one of } j1, ..., j\ell \text{ accepts } p) \geq E[v_{(m+1)}]$. Since the assumptions of Proposition 1 are met for all $j1, .., j\ell$ we know that any $j \in j1, ..., j\ell$ will accept the offer p if $v_j \geq p$. Thus the probability that at least one of $j1, ..., j\ell$ accepts p equals $Pr(v_{j1} \geq p \cup v_{j2} \geq p \cup ... \cup v_{j\ell} \geq p)$ and so i will choose to opt out if $\frac{E[v_{(m+1)}]}{Pr(v_{j1} \geq p \cup v_{j2} \geq p \cup ... \cup v_{j\ell} \geq p)} \leq p$. □

Corollary 7. *If the conditions of Proposition 1 are met so that each i and $j \in \{j1, ..., j\ell\}$ pair choose to opt out of the auction, then the conditions of Proposition 6 will also be met and i and $j1, ..., j\ell$ will also choose to opt out of the auction collectively.*

Proof. First we show that if i and $j1, j2, ..., j\ell$ meet the conditions of Proposition

1 sufficient for i and $j \in \{j1, ..., j\ell\}$ to opt out of the auction, then there exists a p that meets all of the inequalities of Proposition 1. To see this note that the left hand side inequalities are $\frac{E[v_{(m+1)}]}{Pr(v_j \geq p_{ij})} \leq p_{ij}$ for $j \in \{j1, ..., j\ell\}$. As we have assumed that each v_j is identically distributed we know that $Pr(v_j \geq p_{ij})$ is the same for all $j \in \{j1, ..., j\ell\}$. Thus $\frac{E[v_{(m+1)}]}{Pr(v_j \geq p_{ij})}$ is the same for all j. So choosing a $p = \frac{E[v_{(m+1)}]}{Pr(v_j \geq p_{ij})}$ will meet all of the inequalities of Proposition 1.

Next we show that the conditions of Proposition 6 are met. As $Pr(v_{j1} \geq p \cup v_{j2} \geq p \cup ... \cup v_{j\ell} \geq p) \geq Pr(v_j \geq p)$ it must be that $\frac{E[v_{(m+1)}]}{Pr(v_{j1} \geq p \cup v_{j2} \geq p \cup ... \cup v_{j\ell} \geq p)} \leq \frac{E[v_{(m+1)}]}{Pr(v_j \geq p)} \leq p$ and Proposition 6 holds true. □

Next we consider the case where the graph g is not allocatively complete and where one seller receives less than what he would expect to receive from an allocatively complete graph. Such a seller has even more to gain from exiting the auction.

Next we define an *allocatively complete seller*. Let $N_i \subseteq N$ be the set of buyers who are linked to seller $i \in M$. Then i is an *allocatively complete seller* if for all $B \subseteq N$ of size m such that $B \cap N_i \neq \oslash$ there exists a feasible allocation such that every $j \in B$ obtains a good.

Assumption A2: Let g not be allocatively complete and let g consist of a single component. Assume $g - i$ consists of at least two allocatively complete components for some $i \in M$ such that i is an allocatively complete seller. Choose a $j \in N$ such that $ij \in g$ and such that j is not guaranteed a good in the auction over $g - i$ even if $v_j \geq v_{|m-1|}$. Let $c \in g - i$ be the component of $g - i$ that j is a member of.

Assumption A2 requires seller i to be a bridge who is an allocatively complete seller. Thus i's removal splits the graph but i is not contributing to the allocatvie incompleteness of graph g in the sense that buyers linked to i are always guaranteed a good if they have a top valuation.

Proposition 8. *Let assumption A2 be true for some $i \in M$ and $j \in N$. Seller i and buyer j will choose to opt out of the auction and exchange the good at price p_{ij} if for $|c| > 0$, $A \leq p_{ij} \leq min\{v_j(1 - Pr(v_j \geq v_{(|c|)})) + E[v_{(|c|+1)}|v_j \geq v_{(|c|)}], v_j\}$ and if for $|c| = 0$, $A \leq p_{ij} \leq v_j$. For some $A \leq \frac{E[v_{(m+1)}]}{Pr(v_j \geq p_{ij})}$.*

Proof. The assumptions of Proposition 9 differ from those of Proposition 1 in that g is no longer allocatively complete, but i is an allocatively complete seller. Note that there have been no changes to the assumptions on buyer j. Thus by Proposition 1, buyer j will choose to opt out of the auction and exchange the good at price p_{ij} if for $|c| > 0$, $p_{ij} \leq min\{v_j(1 - Pr(v_j \geq v_{(|c|)})) + E[v_{(|c|+1)}|v_j \geq v_{(|c|)}], v_j\}$ and if for $|c| = 0$, $p_{ij} \leq v_j$.

Next we show that seller i will also choose to opt out of the auction. First we show that i's expected payoff from participating in the auction is $A' \leq E[v_{(m+1)}]$. Since i is an allocatively complete seller we know that if the top m value buyers include at least one buyer in N_i, then all buyers will obtain the good at price $v_{(m+1)}$. However, as g is not allocatively complete there exists a subset of buyers of size m such that not all of them can obtain the good. Let these buyers have the top m values. As i is allocatively complete it must be that none of these buyers are linked to i. Thus i will not sell his good to one of these top value buyers. Instead i will sell his good to a buyer with value v_{m+1} or lower and thus he must sell the good to this buyer at a price $p < v_{(m+1)}$. Thus, i will either sell the good at $v_{(m+1)}$ or at a lower price and so the expected price that i receives from participating in the auction is $A' \leq E[v_{(m+1)}]$.

If i decides to opt out of the auction, his expected payoff is $p_{ij} \cdot Pr(j \text{ accepts } p_{ij})$. As in Proposition 1, j accepts p_{ij} if $v_j \geq p_{ij}$. Thus, i's expected payoff from opting out is $p_{ij} \cdot Pr(v_j \geq p_{ij})$. Therefore, i opts out of the auction if $p_{ij} \cdot Pr(v_j \geq p_{ij}) \leq A'$ or if $p_{ij} \leq \frac{A'}{Pr(v_j \geq p_{ij})} \equiv A$ where $A \leq \frac{E[v_{(m+1)}]}{Pr(v_j \geq p_{ij})}$. □

Next we give an example which illustrates Proposition 8.

Example 9. Let $m = 2$, $n = 4$ and $g = \{11, 12, 13, 23, 24\}$. Let $v = \{1, 2, 9, 10\}$ and let each occur with equal probability. First notice that seller 2 is an allocatively complete seller since if buyers 3 or 4 have at least one of the top two valuations, then all the top value buyers can receive the good. However, if buyers 1 or 2 have the top two valuations, then they cannot both receive the good; thus g is not allocatively complete. Next we consider the sellers expected payoffs if both sellers participate in an ascending bid auction. Notice that if $v_3 = v_{(4)}$ or if $v_4 = v_{(4)}$, then $p_2^a = v_{(4)}$. In all other cases $p_2^a = v_{(3)}$. While $p_1^a = v_{(2)}$ if either $v_3 = v_{(4)}$ and $v_4 = v_{(3)}$ or if $v_3 = v_{(3)}$ and $v_4 = v_{(4)}$. In all other cases $p_1^a = v_{(3)}$. Thus, $E[u_1] = \frac{1}{8}E[v_{(2)}] + \frac{7}{8}E[v_{(3)}]$ and $E[u_2] = \frac{1}{2}E[v_{(4)}] + \frac{1}{2}E[v_{(3)}]$. Since seller 2 expects to receive $v_{(4)}$ half of the time this seller may benefit from exiting the auction and selling directly to buyer 4 who is not linked to any other seller and so cannot remain in the auction if seller 2 opts out. If seller 2 offers to sell directly to buyer 4 at price p_{24}, then his expected payoff is $E[u_2] = p_{24} \cdot Pr[v_4 \geq p_{24}]$. If seller 2 participates in the auction, he expects to receive $E[u_2] = \frac{1}{2}E[v_{(4)}] + \frac{1}{2}E[v_{(3)}] = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 2 = 1.5$. If instead seller 2 sells directly to buyer 4 at price $p_{24} = 9$, then he expects this price to be accepted half of the time and $E[u_2] = \frac{1}{2} \cdot 9 = 4.5 > 1.5$. Buyer 4 is not liked to any other seller except seller 2; thus by Proposition 8 he accepts p_{24} as long as $p_{24} \leq v_4$. In the language of

Proposition 8, if $A = \frac{E[u_2]}{Pr(v_4 \geq 9)} = \frac{1.5}{.5} = 3 \leq p_{24} \leq 9$ then both seller 2 and any buyer 4 with $v_4 \geq 9$ will choose to opt out of the auction.

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