Nonlinear Pricing with Local Network Effects

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Abstract

I present a model of second degree price discrimination of a local network good where consumers are located in a social network, and their private valuations are endogenous and depends on the participation of their connections. A monopolistic seller decides the lowest contract available for purchase which determines participation through a threshold game of complements played among the buyers. Relative to a benchmark with exogenous types, participation is likely to be larger with local network externalities since a participating consumer generates positive externalities to other participating consumers through direct and indirect social relationships. The importance of these externalities depends on identifiable properties of the underlying network structure. Under certain network structures, the monopolist is forced to maintain full, or close to full, participation as not offering a contract to low-demand types can lead potential high-value consumers to leave the market. A situation where the monopolist decides not to produce at all, despite production being socially desirable, may occur.

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1 Introduction

For certain products, our purchasing decisions are influenced by the decisions of our social relations. For instance, the phone contract you choose to subscribe to depends on how many in you social network you can reach, and whether you should join a certain file-sharing service may depend on whether your friends are on the same service. In other words, buyers generate positive externalities to other buyers they have relations with.

This paper addresses the strategy of a monopolist seller of a such "local network good". When buyers generate externalities to other buyers, and these externalities are partly extracted by the seller, how are profits maximized? The specific situation considered here is a monopolistic firm who offers a menu of vertically differentiated contracts. Because externalities are generated between connected consumers, the set of profit maximizing contracts ultimately depends on the network structure. Key properties of the network structure determining the set of contracts are identified in this paper.

Goods with local network externalities range from fashion to social networking sites to telephony. The subset of products considered in the present paper are products where it is feasible to imperfectly discriminate across consumers by offering a menu of products of different quantity or quality as first analyzed by Mussa and Rosen (1978) and Maskin and Riley (1984). The key addition to the standard model is that the valuation of a consumer (its privately observed "type") is endogenously determined by the participation of its connections. Thus, when a consumer experiences an increase in the number of friends buying a compatible product, then this consumer will have a higher benefit of a given quantity (or quality) and is inclined to adopt a contract with higher quantity whenever contracts are incentive compatible.

Examples: (i) Mobile operators typically offer a set of plans with a given amount of SMS, talk time and data included in the monthly fee. To some extent, the optimal plan for a consumer depends on the number of individuals in its social network using a compatible service. For "friends and family plans" the benefit is limited to the number of friends being customers at the same mobile operator. (ii) In addition to online storage, users of cloud services like Dropbox and Google Drive may benefit from sharing files with individuals using the same service. The more individuals your account is connected to, the higher is the benefit of an extra byte of storage, and you may be more likely to go for a contract with more storage. (iii) Until recently a premium account for USD

9.99/month was needed for accessing group video calls on Skype.[1] Those with many Skype contacts would therefore be more inclined to accept the premium offer.

In these markets, participation of some consumers affect the benefit of other participating consumers. Therefore, the seller is prone to offer contracts to seemingly unprofitable and scarcely connected consumers. The purchasing decision of one consumer depends on the decisions taken by those he is connected to. In effect, the consumers are playing a so-called *threshold game of complements*, a game where a consumer buys only if at least a given number of his friends buys the same product. At a given threshold participation in this game crucially depends on the network structure. Thus, the strength of local externalities are determined by *how* consumers are linked. For instance the decision of one consumer's purchasing decision may depend on his friend buying, who again buys only if another friend buys. Thus, the decision of your friends' friends' (...) friends' friends may be detrimental for your own purchasing decision. In that case local externalities are strong, and the seller may want to sell to as many as possible. If decisions of indirect connections are less important for your decision, then local externalities are weaker, and the seller may find it optimal to offer a set of products which only highly connected (high demand) consumers would be willing to buy.

For network structures where scarcely connected consumers are detrimental for the participation for all other consumers, the monopolist has no choice but to sell to everyone, or almost everyone. In that situation the monopolist may suffer losses from scarcely connected consumers that exceed profits from the more connected ones. Due to asymmetric information, not all externalities can be extracted by the monopolist, and the monopolist may decide not to produce at all, despite production being socially desirable: a complete market failure.

The model also provides a rationale for why free-of-charge contracts sometimes are offered in markets with local network externalities. In addition to being directly unprofitable, a free contract increases information rents given to high-demand customers. However, due to the externalities generated, the value to other consumers increase, which also benefits the seller.

This work relates to previous research on price discrimination with network effects, pricing in social networks, and games on networks.

[1] https://web.archive.org/web/20140330011125/http://www.skype.com/en/premium/ On April 28, 2014, group video calls became free of charge

To the best of my knowledge Candogan et al. (2012) and Bloch and Quérou (2013) are the only ones to study price discrimination on social networks. Under first-degree (perfect) price discrimination both find that for linear-quadratic payoff functions the optimal quantity sold is proportional to the Bonacich (1987) centrality.[2] Furthermore, they find that with constant marginal costs, the unit price charged is the same for all nodes in a network, even if perfect price discrimination is feasible. Bloch and Quérou (2013) identify two opposing effects with respect to centrality and pricing: central nodes have a higher willingness to pay, but their consumption also generate most consumption externalities to other nodes which suggest that central nodes should be subsidized. With constant marginal costs, these two effects cancel each other out. However, if costs are quadratic, the former effect dominates, and more central nodes are charged higher prices.

The present paper differs from the above in two central aspects. First, I study a model in which perfect price discrimination is not feasible, and discrimination is of second-degree with a monopolist designing a set of incentive compatible contracts. Second, in my model consumers' benefit from adjacent consumption is a function of the *binary* choice of whether neighbors purchase or not, while Candogan et al. (2012) and Bloch and Quérou (2013) consider a setting where network benefits depends positively on neighbors' consumption levels.

Second-degree price discrimination (nonlinear pricing) with global (as opposed to local) network effects have previously been studied by Sundararajan (2004a) and Csorba (2008). These papers differ as the former assumes network value and intrinsic value being separable, whereas the latter does not. While Csorba (2008) concludes that network effect always increase consumption, the results from Sundararajan (2007) depends on how an increase in type affects the two types of values. On a more general level these papers, as well as the present one, deal with contracting with externalities which most notably is treated in Segal (1999). To the best of my knowledge, the present paper is the first to model nonlinear pricing with local network effects.

Non-discriminatory monopoly pricing in social networks is covered by among others Sääskilahti (2007), Campbell (2013), Shin (2014), Carroni and Righi (2014) and Ajorlou et al. (2014), in which Sääskilahti (2007) and Campbell (2013) consider static models, while the three latter papers study dynamic models. Campbell (2013), Carroni and

[2]The results from Bloch and Quérou (2013) and Candogan et al. (2012) do to a large extent follow from results in Ballester et al. (2006), Corbo et al. (2007) and Bramoullé et al. (2014) who all analyze games of strategic substitutes and complements with linear-quadratic utility.

Righi (2014) and Ajorlou et al. (2014) differ from the others as their models have no consumption externalities, but information about a product's existence spreads through the social network by word-of-mouth. Duopoly models with local network effects are considered in Banerji and Dutta (2009) and Fjeldstad et al. (2010).

This paper also relates to work on games on networks. When payoffs depend on actions taken by one's connections, consumers' actions are strategic complements. Adoption games are studied in among others Jackson and Yariv (2007) and Sundararajan (2007). In the present paper, a simple approach is modeled in which consumers are playing a *threshold game of complements* in a similar setup as the one presented in Jackson (2008). I show that under a maximal and pareto-dominant Nash equilibrium, there are identifiable properties of the network structure which determines the set of contracts offered by the monopolist. Threshold games of complements are special cases of games of strategic complements where the increase in payoffs by taking a higher action increase in the actions of other players. Games of strategic complements are closely related to games of strategic substitutes in which best-response actions are decreasing in the action taken by others (public good games). Several variations of such games are treated in among others Galeotti et al. (2010) and Jackson and Zenou (2014).

The rest of the paper is structured as follows: Part 2 sets up the main ingredients of the model, and part 3 provides a benchmark model where the monopolist contracts with exogenous types. The main model where types depend on actions taken in the network is presented in part 4. Part 5 briefly analyzes extensions to the model. An application with actual network data is provided in part 6. Part 7 concludes.

2 Model preliminaries

There is a finite set of consumers, $I = \{1, 2, \ldots, n\}$. The connections between the consumers are represented by a network described by an $n \times n$ adjacency matrix $\boldsymbol{G}$. An element g_{ij} of $\boldsymbol{G}$ describes the connection between nodes i and j. The network is assumed to be undirected and unweighted, i.e. $g_{ij} \in \{0, 1\}$ for all $i, j \in I$, where $g_{ij} = g_{ji} = 1$ implies that consumers i and j are connected, while i and j are not connected whenever $g_{ij} = g_{ji} = 0$. By convention $g_{ii} = 0$ for all $i \in I$. The neighborhood of consumer i, $N_i(\boldsymbol{G})$, is the set of all agents who are connected to i: $j \in N_i(\boldsymbol{G}) \Leftrightarrow g_{ij} = 1$.

The *network degree* of agent i, $\tilde{d}_i(\boldsymbol{G})$, is the number of i's connections:[3]

[3]This is usually referred to as simply the "degree". It will be clear shortly why a distinction between

$$\tilde{d}_i(\boldsymbol{G}) = \sum_{j=1}^{n} g_{ij}$$

The set of network degrees in network $\boldsymbol{G}$ is $\{0, 1, \ldots, \tilde{m}\}$ where $\tilde{m} \leq n-1$. Without loss of generality consumers with no connections are disregarded. Hence, the set of network degrees is limited to $\tilde{D} = \{1, 2, \ldots, \tilde{m}\}$.

Each consumer i takes an action $b_i \in \{0, 1\}$ where $b_i = 1$ implies that i buys the product.[4] The set of actions taken by all consumers is represented by a vector $\boldsymbol{b} = (b_1, b_2, \ldots, b_n)$. The *degree* of agent i, $d_i(\boldsymbol{b}; \boldsymbol{G})$, is defined as the number of buyers in the neighborhood of i:

$$d_i(\boldsymbol{b}; \boldsymbol{G}) = \sum_{j=1}^{n} g_{ij} b_j.$$

Hence, this paper departs somewhat from the standard vocabulary in the network literature: What I call *degree* is in fact the *buyer degree* (the type of degree consumers care about), while *network degree* is what traditionally is referred to as *degree* in the literature.

Each consumer's degree (number of *buyers* in the neighborhood) determines its *type*, θ, and the type is assumed to increase in a consumer's degree:

$$\theta(d_i + 1) > \theta(d_i).$$

Hence, the type of a given consumer is a function of the actions taken in ones neighborhood. Two consumers with the same degree are of the same type, and the type is therefore independent of the consumer's identity. For the remainder of the paper the type of any consumer with d buyers in its neighborhood is referred to as θ_d.

The probability of a randomly selected agent having degree d (equivalently, having type θ_d) is defined as:

$$\beta(d) = \Pr(\theta = \theta_d | \boldsymbol{b}, \boldsymbol{G}),$$

which is distributed according to a probability density function $\phi(\theta)$ with the associated cumulative density function $\Phi(\theta)$. Since each consumer's type is determined by the

"network degree" and "degree" is made.

[4]The quantity (or quality), q, of the product bought may differ between individuals. We set $b_i = 1$ if $q_i > 0$.

actions taken by one's neighbors, the degree distribution, $\phi(\theta)$, depends on the actions taken in the network. Following the above abuse of notation, the share of consumers having degree d are referred to as β_d for the remainder of the paper.

The net utility of a consumer with degree d has the following functional form:

$$U_i(q, P, d) = \begin{cases} \theta_d v(q) - P - f & \text{if } b_i = 1 \\ 0 & \text{if } b_i = 0, \end{cases}$$

where q is the quantity (or quality) of the good obtained, P is the price paid for a given quantity, and $v(q)$ is concave and increasing in q: $v' > 0, v'' < 0$. The parameter $f \geq 0$ is a transaction cost imposed on the consumer by obtaining the product. This can be interpreted as a cost associated with learning how to use the product,[5] or as the value of an outside option assumed to be the same for all types (e.g. sharing files over e-mail instead of using a cloud service). Since network benefits from obtaining the social good depend only on the discrete choice taken by one's neighbors and not the quantities consumed, the monopolist could offer a virtually costless quantity $\epsilon \to 0$ to ensure participation among low-demand consumers. The parameter f ensures that such behavior is costly for the seller as consumers must be compensated for the transaction cost.

Since a consumer's type depends positively on the number of buyers in one's neighborhood, consumption exhibits direct positive externalities, i.e. the marginal benefit from q is increasing in neighbors' actions. For $d > d'$:

$$\frac{\partial U(q, P, d)}{\partial q} > \frac{\partial U(q, P, d')}{\partial q}.$$

Actions are also *strategic complements*, i.e. a consumer's payoff by taking a higher action is increasing in neighbors' actions. For $\theta > \theta'$:[6]

$$\theta v(q) > \theta' v(q).$$

[5] A similar assumption is made in Sundararajan (2007).

[6] The general definition stated in e.g. Jackson and Zenou (2014): For all i, $a_i \geq a_i'$ and $a_{-i} \geq a_{-i}'$:

$$u_i(a_i, a_{-i}, \boldsymbol{G}) - u_i(a_i', a_{-i}, \boldsymbol{G}) \geq u_i(a_i, a_{-i}', \boldsymbol{G}) - u_i(a_i', a_{-i}', \boldsymbol{G}).$$

In the present paper actions are binary, $b_i \in \{0, 1\}$, thus the above inequality translates to $\theta v(q) - 0 \geq \theta' v(q) - 0$.

Finally, the assumptions on θ and the utility function ensures that the Spence-Mirrlees single-crossing condition is satisfied:

$$\frac{\partial}{\partial\theta}[-\frac{\partial U/\partial q}{\partial U/\partial P}] = v'(q) > 0.$$

3 Benchmark model: Exogenous types

As a benchmark this section provides a standard textbook model where types are exogenous, and the monopolist offer a set of incentive compatible contract resulting in a separating equilibrium. For sake of comparison in the subsequent parts, I assume in this section that the distribution of types is equivalent to the *network degree* distribution in the social network, and types are therefore exogenously given and determined by their exogenously given network degree, $\tilde{d}_i$. For instance, in this benchmark a node with two links is type 2, with gross utility $\tilde{\theta}_2 v(q) - f$, regardless of whether the connections have obtained the good or not.[7] The model in this section is a version of the one provided in Bolton and Dewatripont (2005), where the only extension is the transaction cost $f \geq 0$ imposed on the participating consumers, as well as the type distribution being determined by the network degree distribution in an exogenously given network $\boldsymbol{G}$.

Assumption 1. *The monopolist has full information of the structure of the network* $\boldsymbol{G}$*. The identity (location) of each individual on the network is unknown to the monopolist.*

The assumption is equivalent to each individual's type being private information, but the distribution of types being known to the monopolist. Hence, the monopolist cannot perfectly discriminate over the network. However, it can offer a set of contracts the consumers can select from. Degrees are distributed over a probability density function $\phi(\tilde{\theta})$ with the associated cumulative density function $\Phi(\tilde{\theta})$. The *Mills ratio*, $M(\tilde{\theta}_d)$, is the reciprocal of the *hazard ratio* interpreted as the share of nodes with degrees exceeding $\tilde{d}$ relative to the share of nodes with degree $\tilde{d}$:

$$M(\tilde{\theta}_d) = \frac{1 - \Phi(\tilde{\theta}_d)}{\phi(\tilde{\theta}_d)} = \frac{1 - \sum_{i=1}^{d} \tilde{\beta}_i}{\tilde{\beta}_d}.$$

From the Mills ratio, the following assumption is imposed.

[7] Parameters that are exogenous in the benchmark model which are endogenized in the main model are denoted with a tilde.

Assumption 2.

$$\frac{\Delta}{\Delta \tilde{d}}\left[\tilde{\theta}_d - M(\tilde{\theta}_d)\Delta\tilde{\theta}_d\right] > 0$$

for all $\tilde{d}$, where $\Delta\tilde{\theta}_d = \tilde{\theta}_{d+1} - \tilde{\theta}_d$.

Assumption 2 is slightly less restrictive than assuming a decreasing Mills ratio (equivalently increasing hazard ratio). The assumption ensures equilibrium quantities are increasing in types so that the maximization problem subject to local incentive compatibility constraints is sufficient, and is normally satisfied for most unimodular distributions. This simplifying assumption ensures that "bunching" (local pooling) of types following the ironing procedure by Myerson (1981) is not implemented.

The monopolist faces the cost function $c(q) = cq$, and solves the following problem subject to local constraints:[8]

$$\max_{P_d, q_d, k} \sum_{d=k}^{\tilde{m}} [P_d - cq_d]\tilde{\beta}_d.$$

subject to:

$$\tilde{\theta}_k v(q_k) - P_k - f \geq 0 \quad \text{(PC)}$$

$$\tilde{\theta}_d v(q_d) - P_d \geq \tilde{\theta}_d v(q_{d-1}) - P_{d-1} \text{ for all } \tilde{d} > k \quad \text{(IC)}$$

$$q_m \geq q_{m-1} \geq \cdots \geq q_k \quad \text{(QQ)}$$

Following Assumption 2 the constraint (QQ) is automatically satisfied. The parameter k denotes the lowest type offered a contract. Hence, if $k = 1$, we have full participation. If $k \geq 2$ then nodes of degree $k-1$ and lower are not offered a contract satisfying their participation constraints. I return to the choice of k shortly. First, for a given k the first order conditions are:

$$\theta_d v'(q_d^{SB}) = \frac{c}{1 - M(\tilde{\theta}_d)\dfrac{\Delta\tilde{\theta}_d}{\tilde{\theta}_d}} \text{ for } k \leq \tilde{d} \leq \tilde{m}. \quad (1)$$

[8]The cost function $c(q)$ could alternatively be interpreted as a variable transaction cost to consumers as in Sundararajan (2004b), which may be more natural in a setting for information goods where marginal costs are zero.

For $\tilde{d} = \tilde{m}$, $M(\tilde{\theta}_m) = 0$, thus q_m is the only efficient offered quantity as marginal benefit equals marginal cost. The monopolist distorts the quantity downwards for all types less than m in order to profit more from higher types. From equation (1) it is clear that quantities are increasing in types following Assumption 2. The set of profit maximizing prices are found from (PC) and (IC) in the maximization problem:

$$P_k = \tilde{\theta}_k v(q_k^{SB}) - f \tag{2}$$

$$P_d = \tilde{\theta}_d v(q_d^{SB}) - f - \sum_{i=k}^{d-1} \Delta\tilde{\theta}_i v(q_i) \text{ for } \tilde{d} \geq k+1, \tag{3}$$

where $\sum_{i=k}^{d-1} \Delta\tilde{\theta}_i v(q_i)$ is the information rent given to types greater than $\tilde{\theta}_k$. For notational ease, I will throughout the rest of the paper refer to q_d^{SB} as q_d defined by equation (1), unless otherwise stated.

3.1 The lowest contract

There are two types of costs associated with participating consumers. The first are direct types of costs: production costs cq_d and the transaction cost f. The second is the indirect cost of the information rent that must be offered to all other consumers with degree larger than k. For $f = 0$, the lowest participating type, k, is given by the lowest type where the inequality

$$\tilde{\theta}_k - M(\tilde{\theta}_k)\Delta\tilde{\theta}_k > 0$$

is satisfied. There is no real and positive value of q if the above inequality is violated, following the first-order-condition in (1).

Generally, with the above inequality satisfied, the profits from the lowest participating type can be written as:

$$\tilde{\beta}_k[\tilde{\theta}_k v(q_k) - cq_k - f] - [1 - \Phi(k)]\Delta\tilde{\theta}_k v(q_k).$$

The last part in the above expression is the information rent given to degrees $\tilde{d} \geq k+1$, which otherwise would not be given if degree k nodes where not served. It is therefore profitable to serve nodes of degree k whenever the gross consumer surplus less information rents are positive, i.e, whenever the inequality

$$f \leq \tilde{\theta}_k v(q_k) - cq_k - \frac{1 - \Phi(k)}{\tilde{\beta}_k}\Delta\tilde{\theta}_k v(q_k)$$

holds. Note that $(1-\Phi(k))/\tilde{\beta}_k$ is the Mills ratio, $M(\tilde{\theta})$. Inserting the expression for the profit maximizing quantity q_k from equation (1), the expression can be simplified to:

$$f \leq cq_k[\frac{v(q_k)}{q_k v'(q_k)} - 1] =: f_c(k). \tag{4}$$

Proposition 1. *When types are exogenous, the lowest participating type, k, is given by the lowest integer k in which the inequality $f \leq f_c(k)$ holds, provided that q_k has a real and positive solution:* $1 - M(\tilde{\theta}_k)\Delta\tilde{\theta}_k/\tilde{\theta}_k > 0$.

Hence, if the lowest k which satisfies $f \leq f_c(k)$ is greater than 1, the monopolist implements a shutdown policy of degrees $1 \leq \tilde{d} \leq k-1$.

With k defined as in Proposition 1, the profit function is given as follows:

$$\pi^* = \sum_{d=k^*}^{m} \tilde{\beta}_d[(\frac{1}{El_v(q_d)} - 1)cq_d - f] \tag{5}$$

where $El_v(q_d) = q_d v'(q_d)/v(q_d)$ is the utility elasticity w.r.t. quantity consumed.[9]

4 Main model: Endogenous types

The degree, d, of each consumer depends on the number of participating neighbors, and is for that reason endogenously determined as described in part 2. In turn, types, θ, and the degree distribution in the network is endogenously determined.

Since the benefit, and in effect the decision to participate, depends on the participation of those the consumers are connected to, there are several equilibria. As will be clear shortly, the agents are playing a *threshold game of complements*, depending on the set of contracts available. This is a special type of coordination game in which player i chooses action $b_i = 1$ (buy) only if at least a given threshold of i's neighbors take the same action.

Definition. *In a threshold game of complements with threshold k in network $\boldsymbol{G}$, $\mathcal{T}(k;\boldsymbol{G})$, and payoff functions $U_i(b_i, d_i(\boldsymbol{b};\boldsymbol{G}))$ payoffs are such that:*

$$U_i(1, d_i(\boldsymbol{b};\boldsymbol{G})) \geq U_i(0, d_i(\boldsymbol{b};\boldsymbol{G})) \text{ if and only if } d_i(\boldsymbol{b};\boldsymbol{G}) \geq k.$$

[9] Equivalent to output elasticity of capital/labor for a firm.

(a) Minimal equilibrium **(b)** Maximal equilibrium

Figure 1: Minimal and maximal Nash equilibria in a threshold game of complements with threshold 1.

A pure strategy Nash equilbrium in the game $\mathcal{T}(k; \boldsymbol{G})$ is a profile of strategies $\boldsymbol{b} = (b_1, b_2, \ldots, b_n)$ such that:

$$b_i = 1 \text{ if } d_i(\boldsymbol{b}; \boldsymbol{G}) \geq k, \text{ and}$$

$$b_i = 0 \text{ if } d_i(\boldsymbol{b}; \boldsymbol{G}) < k.$$

A threshold game of complements in networks typically have multiple equilibria. Figure 1 illustrates that even a simple network with two connected nodes, there are two equilibria with zero and full participation, respectively. In order to limit the number of model outcomes, the following assumption is imposed:

Assumption 3. *The realized Nash equilibrium in a threshold game of complements, $\mathcal{T}(k; \boldsymbol{G})$, is the maximal equilibrium. I.e. the Nash equilibrium with the highest level of participation.*

While it may be appear natural that consumers are able to coordinate to the Pareto-dominant outcome, the maximal equilibrium assumption is a strong one for one-shot games, in particular for thresholds of 2 and higher. In Section 5.1 I show that by adding dynamics and some fairly simple assumptions on consumers' expectations, the maximal Nash equilibrium is implementable in finite time. Thus, the optimal contracts described in this section can be interpreted as the optimal long-run contracts for any period following the time that the maximal Nash equilibrium is reached.

Identifying the maximal equilibrium is a straight-forward exercise: Having all nodes take the high action as a prior, $b_i = 1$ for all i, nodes having degrees below k reduce their actions to $b_i = 0$. This process is repeated until no further nodes decrease their actions. Except for the minimal equilibrium given by $b_i = 0$ for all i, identifying all other Nash equilibria is a tedious combinatorial exercise. The equilbria do not only depend on the prior distribution of actions, but also the order in which actions are taken.[10]

[10]There are 2^n possible prior distributions of $\boldsymbol{b}$, and $n!$ different orders actions can take for the first round of elimination.

The maximal Nash equilibrium for the game $\mathcal{T}(k; \boldsymbol{G})$ depends on the structure of the network $\boldsymbol{G}$. A necessary condition for participation for a threshold of $k \geq 2$ is that the network contains a *cycle*, a sequence of links connecting a sequence of nodes that starts and ends at the same node. The general requirement for non-zero participation is the existence of what I define as a k-cycle, C_k.

Definition. *C_k is a collection of nodes who take action $b_i = 1$ in the maximal Nash equilibrium in a threshold game complements with threshold k.*

C_k is therefore the set of nodes that survives sequential elimination in a threshold game of complements, starting out with full participation. Figure 2 illustrates nodes in C_2 and C_3. Starting with full participation, the three nodes outside C_2 drop out at a threshold game of complements with threshold 2, while the rest of the nodes still have at least two adjacent nodes participating, and are therefore in a C_2 per definition. One of the nodes outside C_2 has a network degree of 2. However, one of its adjacent nodes are of degree 1, leaving the node with only one adjacent buyer. Thus, this node drops out at threshold 2.

Note that the degree-2 node located in the middle of the C_2 is *not* in a cycle, despite being in a C_2. However, having two separate *paths* leading to nodes in a cycle "protects" this node from sequential elimination of nodes of degree 1.[11] At threshold 3, all nodes except for the group of four to the left drop out. Therefore, having many links is not sufficient for participation. In C_3 three of the nodes have exactly three links, while the top right node in C_2 have 5 links, but does not participate at threshold 3 since less than three (zero) of its neighbors do. Note that the set of nodes in C_3 also are contained in C_2 and so on: $C_m \subseteq C_{m-1} \subseteq \cdots \subseteq C_1$.

Since the type of each node now is determined endogenously, Assumption 2 from the benchmark case is modified:

Assumption 4. *For any maximal Nash equilibrium in a threshold game of complements, $\mathcal{T}(k; G)$, with types $\theta_d = \theta(d)$, the following inequality holds for all d.*

$$\frac{\Delta}{\Delta d}\Big[\theta_d - M(\theta_d)\Delta\theta_d\Big] > 0$$

[11] A *path* is a sequence of links connecting a sequence of nodes in which each node appears at most once in the sequence.

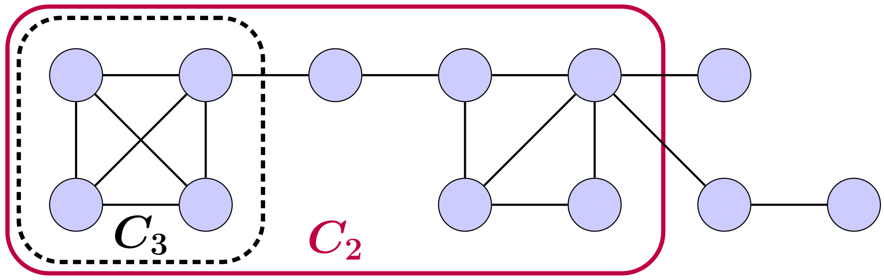


Figure 2: Nodes contained in the solid box are in C_2, nodes in the dashed box are in C_3. Nodes that are in a C_3 are also in C_2. All nodes in the network are in C_1.

As in Section 3 the above assumption ensures that local incentive compatibility is sufficient for obtaining a solution to the monopolist's maximization problem, i.e., bunching of some types is not implemented in equilibrium.

A profit maximizing monopolist solves the following problem:

$$\max_{P_d, q_d, k} \sum_{d=k}^{m} [P_d - cq_d]\beta_d$$

s.t.

$$\theta_k v(q_k) - P_k - f \geq 0 \tag{PC}$$

$$\theta_d v(q_d) - P_d \geq \theta_d v(q_{d-1}) - P_{d-1} \text{ for all } d > k \tag{IC}$$

$$q_m \geq q_{m-1} \geq \cdots \geq q_k \tag{QQ}$$

$$d_i = d_i(k, \boldsymbol{G}) = d_i(\mathcal{T}(k; \boldsymbol{G})) \Rightarrow \theta_d = \theta_d(k),\ \beta_d = \beta_d(k) \tag{KK}$$

As before, k is the node with the lowest degree that is offered a contract satisfying individual rationality. The solution to the above maximization problem at a given k is equivalent to the first order conditions in the benchmark with exogenous types, and is omitted for preservation of space. However, if full participation is not implemented, the distribution of types may differ between the two cases. With $M(\theta_d) \neq M(\tilde{\theta}_d)$ for some d, the optimal contract yields a different quantity and price to a given type in the two cases.

The main difference from the benchmark with exogenous types is that the monopolist must take into account the maximal Nash equilibrium from the threshold game of complements, $\mathcal{T}(k; \boldsymbol{G})$. The condition (KK) emphasizes that the types, θ_d, and the degree distribution, β_d, depend on the lowest participating type, k, through the realized equilibrium of the threshold game of complements.

Negative prices are allowed for. This may be unrealistic as consumers could take the cash without using the product, unless there is a commitment mechanism to consume (i.e. not incurring the transaction cost f, which might be interpreted as switching from the outside option). Hence, the "cash takers" would not generate any externalities to consumers in their neighborhood. The case with prices being bounded to non-negative values is considered in Part 5 in an extension to the model.

Before proceeding to identification of the set of profit maximizing contracts, we have the following trivial, albeit not unimportant, result:

Proposition 2. *If the optimal contract for exogenous types has full participation, $k^* = 1$, then the optimal contract with endogenous types is also with full participation, $k^* = 1$.*

Proof. With the assumption on the maximal Nash equilibrium being imposed in $\mathcal{T}(1; \boldsymbol{G})$, network degree equals degree for all individuals: $\tilde{d}_i = d_i$ for all i. Hence, the payoff for all consumers are equal in the two cases, and the maximization problems for the monopolist are identical in the two cases. □

In order to properly explore the workings of price discrimination in social networks, the rest of the paper focuses on the case where full participation is *not* optimal for exogenous types.

4.1 The lowest contract

A shutdown is implemented whenever the monopolist set $k > 1$, i.e., consumers of degree 1 are not offered a contract.[12] If a shutdown is implemented, the set of participating nodes is defined by those who take the action $b_i = 1$ in the maximal equilibrium of $\mathcal{T}(k; \boldsymbol{G})$. Participation of a given node depends on its position in the network $\boldsymbol{G}$, summarized by the following proposition:

Proposition 3. *If the lowest participating type is θ_k, then only nodes in C_k participate.*

Proof. The lowest contract offered is (P_k, q_k), defined by the equality $\theta_k v(q_k) - P_k - f = 0$. Thus, for $\# \in \{1, \dots, k-1\}$ a consumer of degree $k - \# < k$, i.e., type $\theta_{k-\#} < \theta_k$, has its participation constraint violated at the lowest contract offered: $\theta_{k-\#} v(q_k) - P_k - f < 0$.

[12] Nodes of network degree 1 is assumed to exist in the network $\boldsymbol{G}$. Generally, a shutdown is implemented whenever $k > \min \tilde{D}$.

By definition, in a maximal equilibrium of $\mathcal{T}(k; \boldsymbol{G})$ all participating nodes are in C_k and have degrees k or higher. □

Network degrees, $\tilde{d}_i$, and *degrees*, d_i may differ substantially when full participation is not implemented. All nodes not in C_k have their degrees reduced to zero by a shutdown. Hence, potential high-degree consumers may drop out of the market. An example is the central node of a *star* component who has a high degree, and in turn a high benefit of participating if his neighbors, all of degree 1, participate.

In addition, nodes who are in a C_k, and therefore choose to participate, have their degree reduced if they are connected to nodes outside a C_k relative to a case where degrees of $k-1$ and higher participate. In short, the degree of a node i, and in return its type, θ_i, may be reduced if a shutdown is implemented:

$$\frac{\Delta d_i}{\Delta k} \leq 0.$$

4.1.1 Properties of location and network structure

The magnitude of a shutdown on degrees ultimately depends on the network structure of $\boldsymbol{G}$. In order to analyze the effects further, I classify three key attributes to the location of a node i. Three key subsets of $i \in G$, denoted X_k, Y_k, and Z_k are defined as follows:

1. Nodes that are in a C_k and are only adjacent to nodes in a C_k:

$$X_k = \{i \in \boldsymbol{G} : i \in C_k, (\forall j \in N_i, j \in C_k)\}.$$

2. Nodes that are in a C_k and are adjacent to at least one individual not in a C_k:

$$Y_k = \{i \in \boldsymbol{G} : i \in C_k, (\exists j \in N_i, j \notin C_k)\}.$$

   This set can be divided into further subsets $Y_{k\#}$ where $\# \in \{1, 2, \cdots, l\}$ is the number of i's neighbors who is not in a C_k.

3. Nodes that are not in a C_k:

$$Z_k = \{i \in \boldsymbol{G} : i \notin C_k\}.$$

For $i \in X_k$ a shutdown of nodes of degree $k-1$ does not affect participation or the gross benefit for a given quantity, thus the degree distribution within this set of consumers remains the same after a shutdown. For $i \in Y_k$ a shutdown reduces the degree of each i by at least one unit, thus the willingness to pay for a given quantity is reduced in this segment. For $i \in Z_k$ the degree is less than k for all nodes, thus none in this segment participate.

Define any variable ω_d associated with degree d its associated value $\omega_{d|k}$ when k is the lowest participating degree. The share of nodes who have degree d and are in X_k, Y_k or Z_k are denoted $\beta_d^{X_k}, \beta_d^{Y_k}, \beta_d^{Z_k}$, respectively. For simplicitly I assume the utility elasticity w.r.t. q to be constant.[13] The difference in profits by increasing the lowest contract from k to $k+1$ is then:

$$\begin{aligned}\pi_{k+1} - \pi_k = (\frac{1}{E} - 1)c \Bigg[& - \beta_{k|k}[q_{k|k} - \hat{f}] \\ & + \sum_{d=k+1}^{m} \beta_d^{X_{k+1}} [q_{d|k+1} - q_{d|k}] \\ +[\sum_{d=k+1}^{m} \beta_{d|k+1}^{Y_{k+1}} q_{d|k+1} - & \sum_{d=k+1}^{m} \beta_{d|k}^{Y_{k+1}} q_{d|k}] \\ & -[\sum_{d=k+1}^{m} \beta_{d|k}^{Z_{k+1}} q_{d|k} - \hat{f}] \Bigg].\end{aligned} \tag{6}$$

In the above equation $\hat{f} = f/(1/E - 1)c$. If profits increase by increasing the lowest contract, the first difference equation (6) is positive. Since the step function $\pi^*(k)$ is concave for $k \in \mathbb{N}$ when evaluated at profit maximizing pairs of $(q_d(k), P_d(k))$, the profit maximizing level of participation, k^* is given by the solution of the following problem:

$$k^* = \arg\max_k k \text{ s.t. } \pi_{k+1} - \pi_k \leq 0. \tag{7}$$

That is, k^* is the highest k in which the first difference in (6) is non-positive, meaning it is not optimal to increase the lowest contract further. At k^* we can solve for the direct profits obtained from the lowest contract:

[13] I.e. a function of the form $v(q) = \sigma q^E$ where $0 < E < 1$ and σ a positive constant.

$$\begin{aligned}\beta_{k^*}[(\frac{1}{E}-1)cq_{k^*}-f] &= \pi_{k^*}-\pi_{k^*+1} \\ +(\frac{1}{E}-1)c\Bigg[\sum_{d=k^*+1}^{m}\beta_d^{X_{k^*+1}}[q_{d|k^*+1}-q_{d|k^*}] \\ +[\sum_{d=k^*+1}^{m}\beta_{d|k^*+1}^{Y_{k^*+1}}q_{d|k^*+1}-\sum_{d=k^*+1}^{m}\beta_{d|k^*}^{Y_{k^*+1}}q_{d|k^*}] \\ -[\sum_{d=k^*+1}^{m}\beta_{d|k^*}^{Z_{k^*+1}}q_{d|k^*}-\hat{f}]\Bigg]\end{aligned} \tag{8}$$

Direct profits from lowest type is a non-negative term, $\pi_{k^*}-\pi_{k^*+1}$, in addition to three terms which are interpreted as the local network externalities accrued to monopolist from participation of consumers of degree k^*. If the lowest contract is increased to k^*+1, these three terms represent changes in profits from nodes located in X_{k+1}, Y_{k+1}, and Z_{k+1}, respectively.

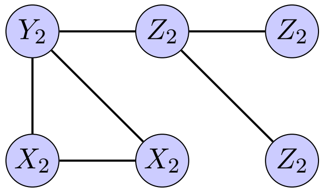
The change in profits from the consumer segment in X_{k+1} is ambiguous, and the change is due to changes in the degree distribution, which affects the optimal quantity offered to each d by equation (1).[14] From the consumer segment in Y_{k+1}, where all nodes have their degree decreased, it is clear that among nodes with degree k or higher, the degree distribution with k stochastically dominates the distribution without k, i.e.,

$$\sum_{i=k}^{d}\beta_{d|k+1}^{Y_{k+1}} > \sum_{i=k}^{d}\beta_{d|k}^{Y_{k+1}} \text{ for all } d<m.$$

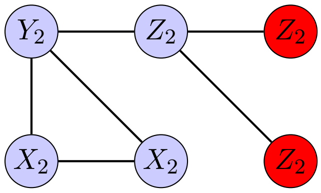
Hence, the consumers in this segment all have a lower benefit from consumption when removing the contract to degree k nodes, yielding lower profits to the monopolist. In the final segment of consumers outside C_k all profits are lost, despite having a potential large share of consumers with high willingness to pay without a shutdown.

Thus, it is indeed possible that the monopolist derives direct negative profits from the lowest participating type, k^*. If this is the case, the profits lost from other consumers by increasing the lowest contract by a unit must be greater than the foregone losses of consumers of degree k^*.

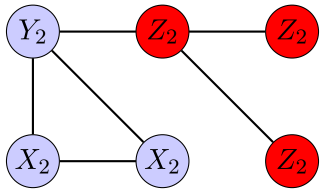
[14]Numerical results suggest that this effect is positive since the Mills ratio shifts down from a shutdown in various generated networks.



(a) Full participation



(b) Shutdown with exogenous types



(c) Shutdown with endogenous types

Figure 3: Shutdown of consumers with degree 1.

Example

A simple example illustrating the effect of a shutdown of degree 1 nodes in a network, i.e., going from $k = 1$ to $k = 2$, is illustrated in Figure 3. Three of the nodes are in Z_2. At $k = 1$ two of these nodes have degree 1, and one node has degree 3. Of the remaining nodes who are in a C_2 (equivalent to a cycle in this example), the degree-3 node is in Y_2 since it is connected to one node outside the cycle. The two remaining nodes are of degree 2 and are only connected to nodes in the cycle and therefore belong to subset X_2.

With full participation, the degree distribution is $\boldsymbol{\beta} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, with $\sum \boldsymbol{\beta} = 1$. Suppose, $(\frac{1}{E} - 1)cq_1 - f < 0$, i.e. serving nodes of $d = 1$ yields negative direct profits. When types are exogenous a shutdown only leads unprofitable nodes of network degree 1 to drop out, and profits are increased by $\frac{1}{3}|(\frac{1}{E} - 1)cq_1 - f|$. However, when types depend on participation in a node's neighborhood, we have that, in addition to the two degree-1 nodes, the degree-3 node in Z_2 drops out, and the degree-3 node in Y_2 has its degree reduced by one unit, making this node less profitable to the monopolist. Therefore, with a shutdown of nodes with $d = 1$ the remaining degree distribution is $\boldsymbol{\beta} = (0, \frac{1}{2}, 0)$.

4.1.2 Continuous approximation

The discrete nature of the problem makes comparative statics difficult since a shutdown of a given type is not a continuous choice. Without loss of generality, the benchmark with exogenous types can be stated with a continuum types. For types depending on participation in a node's neighborhood, and thus indirectly by the lowest participating degree, k, a continuous *approximation* of the effects of a shutdown is presented. The continuous approximation of the model captures the two main effects described in the preceding section: The lowest participating type affects the total size of participating consumers (the nodes in Z_k), and skews degree distribution leftwards as some nodes have their type decreased (from the nodes in Y_k).

Exogenous types

With a continuum of types $\theta \in [\theta_L, \theta_H]$ distributed over some pdf $\phi(\theta)$, profits can be written as (see e.g. Bolton and Dewatripont (2005)):

$$\int_{\theta_L+k}^{\theta_H} \Big[\big[\theta v[q(\theta)] - cq(\theta)\big]\phi(\theta) - v[q(\theta)](1-\Phi(\theta)) - f \Big] d\theta.$$

For $k = 0$, we have full participation. The profit maximizing level of $q(\theta)$ is given by the first-order condition:

$$v'[q(\theta)] = \frac{c}{\theta - M(\theta)},$$

where $M(\theta)$ is the Mills ratio:

$$M(\theta) = \frac{1-\Phi(\theta)}{\phi(\theta)}.$$

By assumption $\theta - M(\theta)$ is strictly increasing in θ, i.e., $M'(\theta) < 1$. Thus, the profits can be written as:

$$\int_{\theta_L+k}^{\theta_H} \Big[v[q(\theta)](\theta - M(\theta)) - cq(\theta) - f \Big] \phi(\theta) d\theta.$$

The value function is derived by inserting $c/v'[q(\theta)]$ for $\theta - M(\theta)$ from the profit maximizing level of $q(\theta)$.

$$\pi(\theta, k) = \int_{\theta_L+k}^{\theta_H} \Big[cq(\theta)[\frac{1}{El(q(\theta))} - 1] - f \Big] \phi(\theta) d\theta \tag{9}$$

Here $El(q(\theta)) = \frac{v(q(\theta))}{q(\theta)v'(q(\theta))}$. In this case it is straight-forward to calculate the profit maximizing level of k:

$$\frac{d\pi(\theta,k)}{dk} = 0$$

$$\Leftrightarrow cq(\theta_L + k^*)[\frac{1}{El(q(\theta))} - 1] - f = 0,$$

or $k^* = 0$ if $cq(\theta_L)[1/El(q(\theta)) - 1] - f > 0$.

Endogenous types

To get an approximation of monopoly profits as a function of the lowest participating degree, equation (9) is modified as follows:

$$\pi(\theta,k) = (1-z(k))\int_{\theta_L+k}^{\theta_H}\Big[cq(\theta,k)[\frac{1}{El(q(\theta,k))}-1]-f\Big]\phi(\theta+y(k))d\theta \qquad (10)$$

Here, $z(0), y(0) = 0$, $y'(k), z'(k) > 0$ and $z(\theta_H - \theta_L) = 1$. The function $z(k)$ describes that increasing the lowest participating type marginally decreases the market size as some of the nodes not in a C_k. The other effect, $y(k)$, describes the fact that nodes located in C_k, but adjacent to nodes outside a C_k, have their degree decreased, and the degree distribution shifts left for that reason.[15] Assuming $El(q(\theta,k)) = E$ (constant), the profit maximizing choice of k is identified by the first-order-condition:

$$\begin{aligned}
&\text{Traditional effect:} && -(1-z(k))(cq(\theta_L+k,k)[\tfrac{1}{E}-1]-f)\phi(\theta_L+k+y(k)) \\
&\text{Market size reduced:} && -z'(k)\int_{\theta_L+k}^{\theta_H}\Big[cq(\theta,k)[\frac{1}{E}-1]-f\Big]\phi(\theta+y(k))d\theta \\
&\text{Distribution effect:} && +(1-z(k))\int_{\theta_L+k}^{\theta_H}\Bigg[\frac{\partial q(\theta,k)}{\partial k}c[\frac{1}{E}-1] \\
& && \qquad +\big(cq(\theta,k)[\tfrac{1}{E}-1]-f\big)\phi'(.)y'(k)\Bigg]d\theta = 0
\end{aligned}$$

The traditional effect by an increase in k is positive for $cq(\theta_L + k^*)[\frac{1}{E} - 1] - f < 0$, and therefore equivalent to that of exogenous types. As long as total monopolist profits is positive, the effect from a reduction in market size is negative. The final effect on distribution is divided in two parts. Due to a shift in the type distribution, the Mills ratio may be affected. The sign of $\partial q/\partial k$ is ambiguous, but numerical results suggest that the sign is positive for among others Poisson random networks . The second part is the effect on profits from each type and has a negative net effect: Assuming a unimodular distribution profits increase for low types (where $\phi(.)$ is increasing), and negative for high types (where $\phi(.)$ is decreasing). Profits per type is increasing in θ since $\partial q/\partial\theta > 0$, thus the overall effect is negative.

Re-arranging the first order condition, the optimal level of k is given by:

[15]Hence, this approximation assumes that all types decrease by the same order by an increase in k.

$$cq(\theta_L + k, k)[\frac{1}{E} - 1] - f =$$

$$\frac{1}{(1 - z(k))\phi(\theta_L + k + y(k))}\Bigg(- z'(k)\int_{\theta_L+k}^{\theta_H}\Big[cq(\theta, k)[\frac{1}{E} - 1] - f\Big]\phi(\theta + y(k))d\theta \tag{11}$$

$$+(1 - z(k))\int_{\theta_L+k}^{\theta_H}\left[\frac{\partial q(\theta, k)}{\partial k}c[\frac{1}{E} - 1] + \big(cq(\theta, k)[\frac{1}{E} - 1] - f\big)\phi'(.)y'(k)\right]d\theta\Bigg) < 0$$

Proposition 4. *If the continuous case has an interior solution, the monopolist derives negative direct profits from the lowest participating type,* $\theta_L + k$.

Corollary 1. *The lowest participating type,* $\theta_L + k$*, is lower with endogenous types than exogenous types.*

The intuition is as equivalent as with discrete types: Participating nodes of a given degree yields a positive externality as nodes Z_k would not participate otherwise, as well as nodes in Y_k maintain a higher degree, translating to increased profits to the monopolist.

4.2 Complete market failure

Since participation of a given node yields positive externalities to those in its neighborhood, the monopolist may choose to serve unprofitable nodes in order to profit from the increased profits in the neighborhood of those nodes. Moreover, following Proposition 3, the monopolist is unable not to serve nodes of degree k if no nodes are in C_{k+1}. In that case the monopolist faces an "all-or-nothing" trade-off between serving a large share of unprofitable customers and not to produce at all.

Hence, a complete market failure may occur. That is, despite production being socially desirable the monopolist may choose not to produce at all if there are no nodes in C_{k+1} and losses from the unprofitable nodes exceed the profits from the profitable nodes.

Complete market failure occurs when production would yield a positive social surplus but negative profits. This may occur since the consumer's surplus, in this case the information rents, are not accrued to the monopolist. The social surplus is given by total benefits to consumers less the production costs, cq, and the transaction cost per consumer, f:

$$W = \sum_{d=k}^{m} \beta_d(\theta_d v(q_d) - cq_d - f)$$

Some of the social surplus is distributed to the consumers through information rents, $r_d(q_d) = \sum_{i=k}^{d-1}(\theta_{d+1} - \theta_d)v(q_i)$, for all degrees larger than k. Thus, the monopoly profits are given by:

$$\pi = \sum_{d=k}^{m} \beta_d[\theta_d v(q_d) - cq_d - f - r_d(q_d)].$$

The condition is summarized under the following proposition.

Proposition 5. *Suppose there exists a $k = \bar{k}$ such that there exists nodes in C_k, while there does not exist nodes in C_{k+1} in the network $\boldsymbol{G}$. Then there will be no production despite production being socially desirable if the following inequalities hold for all $k \leq \bar{k}$:*

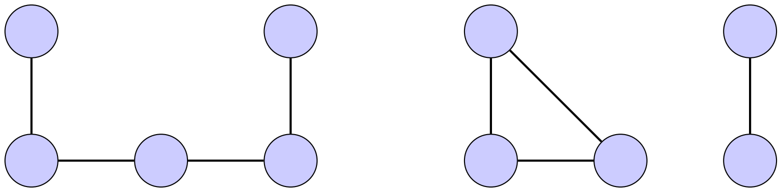
$$\frac{\sum_{d=k}^{m} \beta_d[\theta_d v(q_d) - cq_d - r_d(q_d)]}{\sum_{d=k}^{m} \beta_d} \leq f < \frac{\sum_{d=k}^{m} \beta_d[\theta_d v(q_d) - cq_d]}{\sum_{d=k}^{m} \beta_d}. \tag{12}$$

Within a certain interval of the transaction cost, f, total information rents exceed the potential monopoly losses from a positive production level. The intuition behind the result is as follows: Since no C_{k+1} is contained in the network: $C_{k+1} \notin \boldsymbol{G}$, a shutdown of nodes with degree k would lead to a complete shutdown of *all* nodes. If the losses of the unprofitable nodes exceed the profits from the rest of the nodes, a profit-maximizing monopolist would therefore choose not to produce at all.

As first illustrated by Akerlof (1970), information asymmetries can lead to complete market failure when types are not separable. With separation and market power a positive production level can be restored, but as illustrated here, the presence of network externalities can shut down the market completely, despite no coordination failure and seperability of types in equilibrium.

Example

The above result is easiest illustrated in a so-called path network, the simplest network without cycles (therefore no C_2). In this type of network, full participation or no participation (shutdown) are the only possibilities for the monopolist as only serving nodes of degree 2 ultimately leads to zero participation, which can be easily shown through backward induction with initial drop-out of the peripheral nodes.



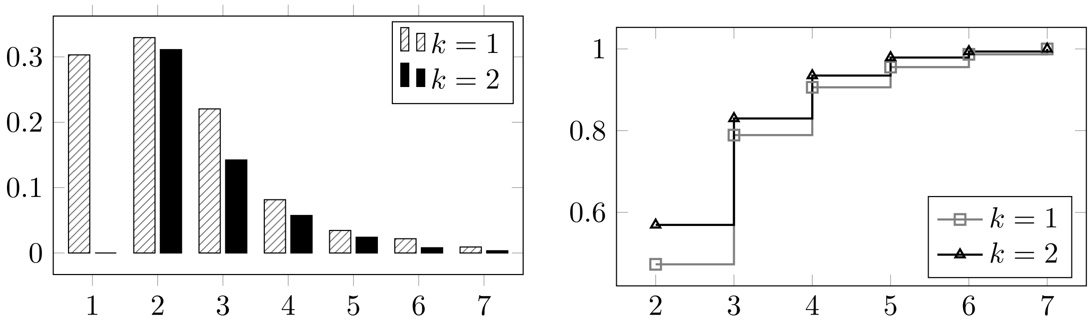
(a) **(b)**

Figure 4: Two networks with equal degree distribution but different topology.

Figure 4 (a) illustrates a path network of 5 nodes. Under full participation there are two nodes of degree 1, and three nodes of degree 2. Thus, the distribution of degrees under full participation is $(\beta_1, \beta_2) = (2/5, 3/5)$. Suppose further that the the payoff function takes the form $v(q) = \sqrt{q}$, and parameters as follows: $c = 0.5, \theta_1 = 2, \theta_2 = 3$. It can be shown that under full participation the profit maximizing price-quantity pairs are $(P_1, q_1) = (1 - f, 0.25)$ and $(P_2, q_2) = (8.5 - f, 9)$. Hence, the monopolist earns profits $\frac{1}{5}(0.875 - f)$ and $\frac{1}{5}(4 - f)$ from each node of degree 1 and 2, respectively, yielding total profits of $2.75 - f$.

From equation (4) it can be derived that nodes of degree 1 yield negative profits for $f > 0.125$. However, since this network is without cycles, the monopolist serves all nodes as long as total profits is positive, i.e., for all $f < 2.75$. However, an information rent of 0.5 is given to all nodes of degree 2, i.e., for 3/5 of the consumers yielding a social surplus of $3 - f$. Hence, for $2.75 < f < 3$ we experience a complete market failure where monopoly production would yield a positive (although not maximized) social surplus, but with negative monopoly profits. Under perfect price discrimination (i.e. welfare maximum), production is optimal for $f < 3.5$, and profits from degree 1 nodes are positive for $f < 2$.

As a comparison, the network in Figure 4 (b) represents a network with the exact same degree distribution. However, here all the degree 2 nodes are in a cycle, and are not connected to degree 1 nodes. Thus, no externalities, direct or indirect, go from degree 1 to degree 2 consumers, and vice versa. A profit maximizing monopolist would choose to only serve nodes of degree 2 if $f > 0.125$, and total shutdown occurs at $f > 3.75$ which coincides with the critical value of f for a positive social surplus. Note that under the assumption of a maximal Nash equilibrium, offering a menu of contacts in the network in Figure 4 (b) yields the exact same outcome as under exogenous types.



(a) Distribution of degrees

(b) $k = 1$ stochastic dominance.

Figure 5: Panel (a): Degree distribution with lowest participating degree of 1 and 2, respectively from random Poisson network with $n = 1000$ and $p = 2/n$ (degree 0 nodes omitted). Panel (b): Cumulative degree distribution conditional on $d \geq 2$. Distribution without shutdown stochastically dominates distribution with shutdown.

4.3 Numerical analysis: Poisson random graphs

A social network can take a vast range of topologies. Due to its simplicity and several convenient properties, Poisson random graphs are widely analyzed in the social network literature, and serve as a nice benchmark. In the random networks analyzed here, two nodes connect randomly with probability p. The degree distribution is therefore binomial, in which degrees are approximately Poisson distributed with expectation np for large n.

In the present model, Poisson random graphs are convenient due to its statistical properties. First, Mills ratios are strictly decreasing, which ensures that local incentive constraints are sufficient for designing a set of optimal contracts. Second, these types of networks experience sharp phase transitions for the existence of cycles at $pn > 1$ (Erdős and Rényi, 1960), which also implies the existence of nodes in C_2. I provide evidence from simulations that similar phase transitions also exist for the emergence of C_k for $k \geq 3$.

In this section, I first investigate the effects on participation and distribution of degrees at different level of participation. Second, I briefly look at the implications for the monopolist's profits under specific functional forms and parameter values.

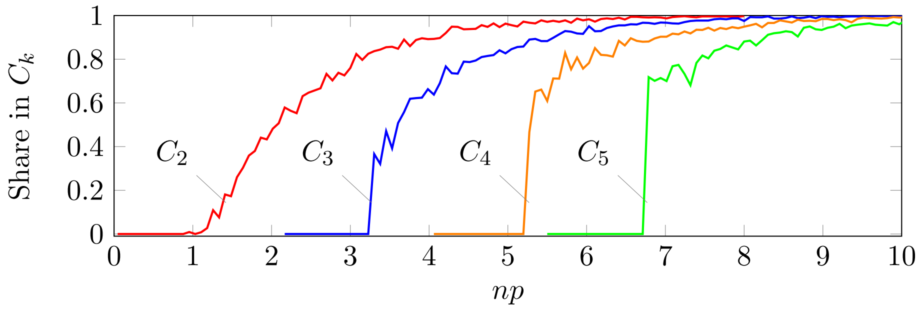


Figure 6: Share of nodes being in $C_k, k \in \{2, 3, 4, 5\}$, for given pn in Poisson random graph, $n = 1000$ in simulations.

4.3.1 Statistical properties

When increasing the lowest participating degree from $k = 1$ to $k = 2$, there are two effects: First, nodes outside of C_2 choose not to buy under a threshold game of complements, i.e., the market becomes smaller. Second, some participating nodes will be of lower degrees since they are connected to nodes outside of C_2. Degree distributions of a random graph with $np = 2$ for $k = 1$ and $k = 2$ are illustrated in Figure 5.[16] At $k = 1$ we have full participation, thus the distribution sums to 1. Panel (a) illustrates the effects on the population size. By construction, no nodes of degree 1 participate at $k = 2$, but for all $d \geq 2$ there are fewer individuals of each degree compared to $k = 1$. Thus, there are potentially fewer profitable customers left in the market. Panel (b) emphasizes the effect a shutdown has on the distribution degrees. Conditional on $d \geq 2$ the distribution with $k = 1$ stochastically dominates the distribution with a shutdown. Hence, there are not only fewer profitable customers, but a given consumer of $d \geq 2$ is expected to be less profitable.

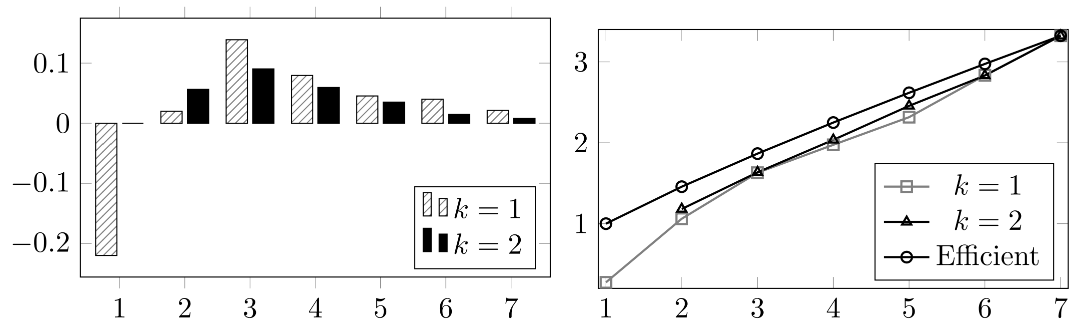
Another aspect of interest is the share of nodes located in C_k for different levels of k. Erdős and Rényi (1960) proved that for $n \to \infty$ there exists a cycle (therefore also C_2) with certainty for $pn > 1$. Figure 6 shows the share of nodes located in C_k for $k \in \{2, 3, 4, 5\}$ for simulations with $n = 1000$, and $np \in [0, 10]$. The theorem of Erdős and Rényi is evident as nodes in C_2 appears once np exceeds 1. Interestingly, similar phase transitions seem to exists for higher k's. E.g. at a threshold around $np = 3.3$ nodes in C_3 appears, and the phase transitions for higher k's appear to be more extreme with respect to the share located in C_k once the critical threshold is reached. In the

[16] When full participation is not implemented, degrees are no longer Poisson distributed.

previous example with $np = 2$, around 55% of the nodes are in C_2. Despite around 43% of the participating nodes being of degrees $d \geq 3$ for $k = 2$, none are in C_3. Thus for a random graph with $np = 2$, only two regimes, $k \in \{1, 2\}$, is feasible for a positive level of participation.

4.3.2 Profits and quantities

How do the effects on particiaption and degree distribution by a shutdown affect the monopolist's profits? On one side, potential losses from low degree consumers are foregone. On the other side, the monopolist cannot benefit from the externalities they generate. For the same network as depicted in Figure 5 (a), Figure 7 (a) illustrates the profits generated from each contract offered for $k \in \{1, 2\}$ for a given functional form and parameter values described in the figure caption.



(a) Profits per type

(b) Quantity offered to each type

Figure 7: Parameters: $v(q) = \sqrt{q}, \theta(d) = 1 + \sqrt{d}, c = 1, f = 1$. (a) Profits, measured as gross consumer surplus less costs and information rent, per contract offered. (b) Quantity increasing in type.

When increasing the lowest participating contract, the monopolist foregoes the direct negative profits from serving degree 1 consumers. On the other hand, less profits are generated from the rest of the population. This loss is interpreted as the share of externalities derived to the monopolist from participation of degree 1 consumers.

However, a shutdown affects the distribution of degrees, which has an impact on the design of the offered contracts. In this specific numerical example, the Mills ratio shifts down following a shutdown, leading to an increase in quantity offered to each participating type. Intuitively, there are relatively fewer consumers of high degrees. Thus, the monopolist finds it optimal to increase the information rents given to consumers of lower degrees, i.e., by offering higher quantities to low-demand customers, as illustrated in Figure 7 (b). In this example, profits derived from degree 2 consumers increase after a shutdown, despite there being fewer degree 2 consumers in the population. The change in the monopolist's optimal behavior therefore partially offset the negative effects by a shutdown on participation and distribution of types.

5 Extensions

5.1 Implementation of the maximal equilibrium when consumers fail to coordinate

One can easily argue that the maximal equilibrium of a threshold game of complements is unlikely to be realized in a one-shot game. By adding dynamics and adaptive expectations with respect to a consumers' future degree, I show that the maximal equilibrium of any k can be reached under a simple mechanism. Although implementation of the maximal equilibrium is attainable, the process of attaining maximal equilibrium can be costly to the monopolist.

Denote the shortest path between two nodes i and j in the same component of $\boldsymbol{G}$ as its *geodesic* $\Gamma_{ij}(\boldsymbol{G})$. The following result follows:

Lemma 1. *In a dynamic threshold game of complements of threshold $k = 1$ in discrete time and adaptive expectations, $E_i[d_{it}] = d_{it-1}$, the maximal Nash equilibrium is reached at latest at time $\bar{t} = \max_{ij} \Gamma_{ij}(\boldsymbol{G})$, $t \in \{0, 1, 2, \cdots, \}$, if*
(i) There is at least one initial adopter in each separate component in $\boldsymbol{G}$, and
(ii) The initial adopter(s) buy(s) in the first two initial periods: $t = 0, 1$.

Proof. Suppose the network $\boldsymbol{G}$ is connected. The proof can be generalized to a network with separate components as each component without loss of generality can be described as an independent network. Denote the action taken by individual i at time t as b_{it}, and fix an arbitrary node i with action $b_{i0} = b_{i1} = 1$. At $t = 1$, then $b_{j1} = 1$ for all $j \in N_i$, and $t = 2$, $b_{l2} = 1$ for all $l \in N_j$ for all $j \in N_i$. In $t = 2$, i voluntarily takes action $b_{2i} = 1$ following as best response to actions b_{j1} for $j \in N_i$. Iterating, for later periods no players will reduce their actions, and a node will take action $b = 1$ at time t if there is at least one neighbor taking the same action in $t - 1$. Thus, the time t in which a node initially adopts corresponds to the shortest path length from the initial adopter

i. Full adoption is then implemented at the time corresponding to the largest geodesic from i, $t = \max_j \Gamma_{ij}(\boldsymbol{G})$, and the maximum time feasible to implement full adoption is $\max_{ij} \Gamma_{ij}(\boldsymbol{G})$. □

The lemma implies that the maximal Nash equilibrium is attainable for any k:

Proposition 6. *In a dynamic threshold game of complements of any threshold k in discrete time and adaptive expectations, $E_i[d_{it}] = d_{it-1}$, the maximal Nash equilibrium is then reached in finite time if the monopolist increases k from an initial threshold $k = 1$ with full participation.*

Proof. Given the initial full participation, the game coincides with the algorithm to identify the maximal Nash equilibrium in games of strategic complements. See e.g. Jackson and Zenou (2014). □

Lemma 1 and Proposition 6 imply that in a dynamic contracting setting, maximal coordination among consumers is implementable, even when we start out with zero participation. The key is that the monopolist start out with a set of contracts in which the lowest contract is individual rational for consumers with at least one buyer among their connections. In each separate component in the network at least one consumer is "seeded",[17] which leads to full participation in finite time (Lemma 1). When full participation is implemented at $k = 1$, the desired lowest contract $k^* \geq 1$ can be implemented at the maximal equilibrium by increasing the lowest contract (Proposition 6).

A potential problem is that some consumers will have their intratemporal participation constraint (and incentive contraint) violated as some of their peers unadopt and thus are stuck with a "too high" contract in some period t_c before being able to downgrade or leave the market in $t_c + 1$, i.e., $\theta_{d-\#} v(q_d) - P_d < 0$ where # is the number of unadopted friends in the preceding period. Foreseeing that the monopolist may increase the lowest contract, consumers must have their present value net utility satisfied before adopting a higher contract. The monopolist may have to commit to maintain the lowest contract at $k = 1$ for a certain amount of time, and must with certainty give the degree-1 consumers some surplus during the time before the maximal equilibrium is reached to insure them against the risk of having a one-period negative surplus.

[17]Since the monopolist by assumption cannot identify the individuals in the network, we must assume that the monopolist publicly offers one first-come-first-serve contract of quantity q_1 at a negative price $P \leq -2f$ which lasts for two periods.

Assume a discount factor $\delta \in [0,1]$,[18] and denote s_i the time i initially adopted, and $\tau_i - 1$ the time in which the last change in adoption in i's neighborhood occurred. In order to ensure full participation when the consumers expect an increase in k the following participation constraint must hold for all i:

$$E_i[\sum_{t=s_i}^{\tau_i} \delta^{\tau_i - s_i}(\theta_{idt}v(q_{dt}) - P_{dt} - f)] \geq 0,$$

$$d_{it} \geq d_t \text{ for } k = 1, \text{ and}$$

$$d_{it} \leq d_t \text{ for } k \geq 2.$$

As a consequence of adaptive expectations, the present value participation constraint implies possibly suboptimal intratemporal contract choices for both the consumers and the monopolist. In the adoption phase, a consumer may adopt a too low contract due to the lag following adaptive expectations. In the unadopting phase, the opposite may occur, i.e., that consumer has a too high contract for a limited amount of time.

I conclude this section by stating that under less favorable assumptions on the one-shot threshold game, the maximal equilibrium is attainable by adding dynamics and adaptive expectations. The long run contracts under the maximal equilibrium with k as the lowest participating degree is equivalent to the contracts studied in Section 4. As the maximal equilibrium is not reached immediately, present value profits are lower compared to contracts with maximal equilibrium reached initially. Thus, coordination failure in one-shot games are more costly to the monopolist, consumers and welfare relative to immediate successful coordination.

5.2 Non-negative prices

The assumption of allowing for negative prices may be unrealistic since consumers could take the money without adopting the product (and therefore escaping the transaction cost f). Subsequently they would not pass on the externality to potential consumers in one's neighborhood. By imposing a minimum price $P = 0$, consumers no longer have the incentive to take the money and run. Moreover, non-negative prices fits better with the empirical facts of markets analyzed in this paper. The model has shown that offering a product at a loss (including giving away for free) can be an optimal strategy due to positive externality low-demand customers may generate to others. Parts of the

[18] With $\delta = 0$ consumers are myopic, and participation constraint satisfied in period $t = s_i$ is sufficient.

externality can be extracted by the monopolist through higher price-quantity pairs to other consumers.

A potential contract offering a quantity q_k^0 for free is such that the consumer's surplus of the lowest participating type, k, is zero, i.e.

$$\theta_k v(q_k^0) = f. \tag{13}$$

The condition $P_d \geq 0$ is binding whenever q_d^{SB} from equation (1) implies a negative price. Hence, the quantity offered to nodes of degree d is therefore

$$q_d = \max\{q_k^0, q_d^{SB}\}.$$

Note that $P_d = 0$ may imply a bunching at the bottom where consumers of degrees larger than k are offered the contract $(P_k, q_k) = (0, q_k^0)$. Monopoly profits when $P_d = 0$ and binding are lower than when negative prices are allowed for due to the increased information rent that must be given to nodes of higher degrees.

Lemma 2. *At a given k profits are lower if the non-negativity constraint on prices binds.*

Proof. Given k and the maximal equilibrium in $\mathcal{T}(k, \boldsymbol{G})$, the value function, $\pi^*(\boldsymbol{q}^*, \boldsymbol{P}^*; k)$, is the solution to the maximized Lagrangian,

$$\mathcal{L}(\boldsymbol{q}, \boldsymbol{P}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \sum_{d=k}^{m} \Big([P_d - cq_d]\beta_d + \lambda_{d \geq k+1}[\theta_d v(q_d) - \theta_d v(q_{d-1}) - P_d + P_{d-1}] - \mu_d[P_d - l] \Big) \\ + \lambda_k[\theta_k v(q_k) - P_k - f],$$

where prices are constrained s.t. $P_d \geq l$ for all $d \geq k$ ($l = 0$ by assumption). From the Kuhn-Tucker complementary slackness condition we have that $\mu_d > 0$ if $P_d = l$, and $\mu_d = 0$ otherwise. From the envelope theorem:

$$\frac{\partial \pi^*}{\partial l} = -\sum_{d=k}^{m} \mu_d \leq 0.$$

If $P_k = l$, then $\mu_k > 0$, and it follows that $\partial \pi^* / \partial l < 0$. □

The intuition behind the above result is clear: When prices cannot be negative, and the constraint binds, higher quantities must be given to low types in order to satisfy individual rationality. This translates into higher information rents, which reduces monopoly profits. The result also suggests that if a shutdown of low-degree types is feasible, it is more likely to occur since a larger share of the externalities is transferred to consumers through information rents instead of monopoly profits. However, if a shut-down of consumers of degree k is not feasible, lower monopolist profits increase the set of parameters in which a complete market failure occurs:

Proposition 7. *If there exists a k such that there are nodes in C_k, but no nodes in C_{k+1} in the network $\boldsymbol{G}$, and the non-negativity constraint binds, then the set of f in which complete market failure occurs is increased.*

Proof. The interval of f in which a complete market failure occurs is given by inequality (12). First, note that for q_d where $P_d > 0$, quantities are not affected due to quasi-linear net utility. From Lemma 2 we know that the left-hand-side of the inequality is decreased whenever at least one price binds, i.e., the loss from increased information rents is larger than the increased profits from larger quantity-price pairs. The right-hand-side of the inequality is increased provided that $\theta_d v'(q_k^0) \geq c$, i.e. quantities for at least one type is increased towards the socially desirable quantity. □

5.3 Heterogeneous outside options

The parameter f, interpreted as a transaction cost or as the value of the outside option, is assumed to be constant across all types. When this parameter is interpreted as a transaction cost (e.g. a switching cost like learning cost or hassle of adoption), there is no rationale for this parameter to systematically vary over types. However, interpreted as an outside option it can be argued that f is type dependent.

In that case, it seems natural that the value of the outside option is increasing in a consumer's *network degree* which is exogenously given by a consumer's location in the network. If the market we are studying is a cloud service with file sharing, the outside option may be the benefit of sending and receiving files over e-mail. The value of this outside option can increase in the number of neighbors you have, independent of whether your neighbors are buyers or not.

Hence, the set of outside options is $\boldsymbol{f} = \{f_1, f_2, \ldots, f_m\}$, with $\Delta f_{\tilde{d}}/\Delta \tilde{d} > 0$. Since the outside option is independent of actions taken, heterogeneity does not affect incentive constraints. However, participation constraints may be affected, as will be illustrated shortly.

Define $h = \max_i(\tilde{d}_i | d_i = k)$, i.e. the highest *network degree* of the participating nodes with the lowest *degree*, k, where $h \geq k$. The highest value of the outside option of a participating consumer with degree k is then f_h, where $h = 1$ with full participation, i.e., $k = 1$.

An individual with degree k and network degree h has its participating constraint binding:

$$P_k = \theta_k v(q_k) - f_h.$$

Hence, consumers of degree k and network degree $\tilde{d} < h$ will obtain a surplus of $f_h - f_{\tilde{d}}$ by participating. It may be feasible that the monopolist does not offer contracts to the nodes with the highest network degrees. However, these nodes may have an important location in the network and thus generate large externalities.

A consumer of degree $d > k$ and network degree $\tilde{d}$ will then have the following net utility from participation:

$$U(d; \tilde{d}, k) = -(f_{\tilde{d}} - f_h) + \sum_{i=k}^{d-1} (\theta_{i+1} - \theta_i) v(q_i).$$

As before, the last element is the information rent to all nodes of degrees $d > k$. Hence, it is possible that consumers of degrees larger than k do not participate if their outside option is sufficiently valuable.

Proposition 8. *Consider the value of outside options increasing in network degrees:* $\Delta f_{\tilde{d}} / \Delta \tilde{d} > 0$. *If there exist at least one node of degree k and network degree h who receives zero surplus, then a node of degree d and network degree $\tilde{d}$ will not participate if the difference in outside options $f_{\tilde{d}} - f_h$ exceeds the information rent, i.e., if the following inequality holds:*

$$f_{\tilde{d}} - f_h > \sum_{i=k}^{d-1} (\theta_{i+1} - \theta_i) v(q_i).$$

It is not immediately clear whether this may pose problem for the monopolist at high or low levels of k. At $k = 1$, then $h = 1$, and the difference $f_{\tilde{d}} - f_h$ is likely to be large for some individuals, but the information rent is also larger for high levels of participation. For larger k, the information rent is lower, but f_h is larger. Thus, without having specific information on functional forms and network structure, no clear conclusions can be made.

The possibility of non-participation of high-degree types does not necessarily pose major problems to the monopolist. First, degrees and network degrees are positively correlated and information rents are increasing in degrees. Second, if full participation is not imposed, then some of the nodes with the lowest degree may have quite high network degrees, and therefore high f_h. Third, if $P \geq 0$ binds, information rents are higher than when negative prices are allowed for. If $P > 0$ and non-participation poses a problem, the monopolist can reduce the price to all consumers by some level ψ such that $\psi \geq \sum_{i=k}^{d-1} \Delta\theta_i v(q_i) + f_h - f_{\tilde{d}}$ for all consumers of degrees $d \geq k$.

An equilibrium where all consumers obtain a positive surplus is a possibility under heterogeneous outside options. In that case, the monopolist profits are reduced compared to the case where $f_{\tilde{d}} = f$ for all $\tilde{d}$. If the inequality in Proposition 8 applies for at least one individual, the set of parameters in which a complete market failure can occur is increased.

5.4 Changes in the network structure

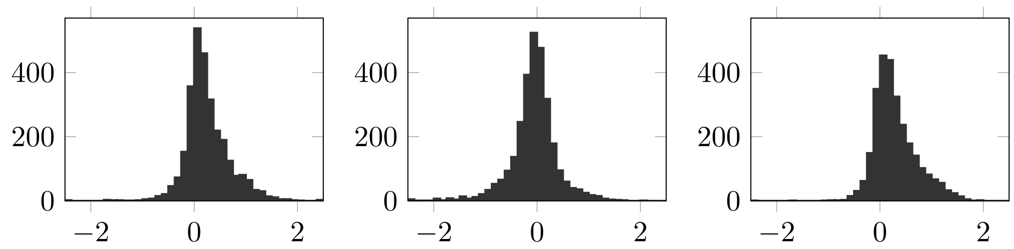
The network structure determines the extent consumers of different degrees generate externalities to each other. This section aims to identify how certain alterations of a given network affects the lowest optimal level of participation, k^*. As the variations of possible networks and alterations are so vast, general analytical results are difficult to obtain. For that reason I conduct several simulation exercises with randomly generated networks of size $n = 40$ with a link forming at random between two individuals with probability $p = 2/n = 0.05$.[19] These are the "unmodified" networks, and profits for a given functional form, sets of parameters and k are denoted π_k^U.

Modified networks were then generated in which alterations of the unmodified networks were made in order to explore the effects from weighted relationships (links of different weights), self-links (valuation independent of neighbors' actions), and clustering (higher frequency of three individuals all being connected to each other). In all these modified networks, the expected sum of degrees where held constant.

A constant elasticity functional form, $v(q) = q^\alpha$, was assumed and type parameter of an individual of degree d given by $\theta(d) = 1 + \sqrt{d}$. For each generated network, α and c where drawn from a uniform distribution over $[0, 1]$, and f was then set to solve the equation $\pi_{k=1}^U(f) = \pi_{k=2}^U(f)$. I.e., I only consider situations in which a monopolist is indifferent between serving degree-1 consumers or not.[20] The goal of this simulation

[19]Since nodes without links are omitted, the expected size of then network is $E(n) = 40(1 - Binomial(0, 40, p = 0.05)) \approx 34.86$.

[20]For each case, 4000 networks and a random set of parameters where generated. Simulation outcomes



(a) Weighted links.
Mean 0.25, $t = 29.9$

(b) Self-links.
Mean -0.06, $t = -7.2$

(c) Clustering.
Mean 0.29, $t = 38$

Figure 8: Realizations of simulated differences in log-profits of modified networks, $\pi^M_{k=2} - \pi^M_{k=1}$, $M \in \{Weight, Self, Cluster\}$, with parameters such that profits in unmodified networks were equal at $k = 1$ and $k = 2$: $\pi^U_{k=2} - \pi^U_{k=1} = 0$. Mean difference in log-profits and t-values of mean givens in captions of the above subfigures.

exercise is then to estimate the expected difference in profits of the contracts $k = 1, 2$ in the "modified" networks: $\pi^M_{k=2} - \pi^M_{k=1}$. If this difference is positive (negative) and statistically significant, I conclude that a certain alteration to the network makes is more (less) likely that the lowest contract is increased (decreased). A limitation of this approach is that the conclusions only are applicable to the specific network and parameter generating processes the simulations are based on. Summaries of the simulation results are provided in Figure 8.

5.4.1 Weighted links

Arguably, we value some relationships higher than others. In this section, I modify the network to allow for different weighting of links between individuals in the sense that a consumer's value of a given quantity depends not only on *how many* neighbors who participate, but may also depend on *who* of his neighbors who participate.

In order to work the extension into the existing model with discrete types, I apply I mean-preserving spread approach. For each existing link, $g_{ij} = g_{ji} = 1 \in G$, from the standard model, let each $g_{ij} = g_{ji}$ take values $\{0, 1, 2\}$ with probabilities $(r/2, 1 - r, r/2)$ where $r \in [0, 1]$. Thus, with probability r, a link is either severed or "doubled" with equal probability, or remains at $g_{ij} = g_{ji} = 1$ with probability $1 - r$. Note that the network is still undirected. Thus, an individuals' network degree, $\tilde{d}_i(\boldsymbol{G}) = \sum_{j=1}^n g_{ij}$,

in which a solution of f were not found or profits were negative: $\pi^U_{k=1} = \pi^U_{k=2} < 0$, were omitted. These accounts for an excess of 1/4 of the outcomes.

remains the same on average. The question of interest is then whether the optimal k is likely to be affected when some relationships are stronger than others.

There are two opposing effects: That some links have double weight means that the nodes at the end of these links are less dependent on other direct and indirect relationships to have their degree being at least k so that their participation constraints are satisfied. Thus, this effect goes in the direction of increasing the lowest contract, k. On the other hand, as some links now are severed, nodes who previously was in a C_k may now be out. For instance, a node who previously was located in a cycle, may no be out if the cycle is broken. In return, nodes connected to these nodes will have their degrees reduced if the lowest contract is increased. This effect goes in the direction of decreasing the lowest contract.

2867 simulations where an existing link in the unmodified network got weight $g_{ij} = g_{ji} = 2$ (doubled) with probability .25 and severed, $g_{ij} = g_{ji} = 0$, with the same probability were conducted. Differences in log-profits in the two regimes for the weighted networks, $\pi^W_{k=2} - \pi^W_{k=1}$, are shown in Figure 8a. As illustrated, there are several observations with both positive and negative values. Relative to a network with unweighted links, the direction of the optimal lowest contract can be both positive and negative. However, the mean is positive and equal to 0.25 with a corresponding t-value of 29.9 under zero mean null hypothesis. The result suggests that in expectation, the optimal k is higher, at least not lower, with a weighted network compared to an unweighted one under this specific network generating process. The intuition for the result is as follows: Weighted links increases the correlation of valuation between connected links. Thus, for the nodes who have links with weight 2 are now less dependent on the participation of lower value types through direct and indirect connections. Thus, externalities from low-demand types to high-demand types are weakened, which incentivizes the seller to increase k.

5.4.2 Self links: Network independent types

Two individuals with the same amount of participating neighbors may still value a product differently. In the same manner as in the extension with weighted links I apply a mean-preserving spread approach to types, where the type, θ of a given node i with degree d is given by $\theta_{id} = \theta(d + \hat{v})$, where $\hat{v}$ is a stochastic variable that take values $\{-1, 0, 1\}$ with probabilites $(\rho/2, 1-\rho, \rho/2)$, respectively.

2910 simulations with $\rho = 0.25$ where conducted. Differences in log-profits under the two regimes with self links, $\pi^S_{k=2} - \pi^S_{k=1}$, are illustrated in Figure 8b. As in the weighted links case there is large variation. However, the mean difference in log-profits is -0.06 with t-value of -7.2. This suggests that with mean preserving self links, the optimal contract is in expectation lower than under a network without self links in this specific network generating process.

Why do we find the opposite result from the weighted links case? One explanation may be that weighted links is similar to adding self links on a pair of connected nodes. Thus, relative to the unweighted network, there will be an increased correlation in the valuation between connected nodes. Thus, if pairs or nodes either become weaker or stronger, the stronger pairs will be less affected by increasing the lowest contract. With self links there is no increased correlation in types among connected consumers, and a negative effect similar to the effect of breaking up cycles dominates.

5.4.3 Clustering

There is strong empirical evidence that two individuals are more likely to be connected if they share a connection. One common measure of such connections in a network is *clustering* in which in some form counts the number of *triads*, i.e., instances in which a group of three nodes have links to each other. The most basic measure is *overall clustering coefficient* which measures the number of triads relative to "stars" of three nodes, where one node is connected to two other nodes without the other two being connected:

$$Cl^{overall}(\boldsymbol{G}) = \frac{\sum_{i;j\neq i;k\neq j;k\neq i} g_{ij}g_{ik}g_{jk}}{\sum_{i;j\neq i;k\neq j;k\neq i} g_{ij}g_{ik}}$$

Other measures of clusering exist, such as the *average clustering measure* (see e.g. Jackson (2008)), and the preferred measure depends on the phenomenon one wish to study. For the current exercise, any commonly used clustering measure will increase. Thus, the clustering measure is not of concern for the comparative statics.

Clustering of a given random network was increased in the following manner. For each random network, then for each "star" (i is connected to j and k without j and k being connected), j and k where connected with probability .25, increasing the number of triads in each network. For each link added, a randomly selected link in the network was removed, keeping number of links constant.

Results from 2776 simulations are illustrated in Figure 8c. The mean log-difference of $\pi^C_{k=2} - \pi^C_{k=2}$ is 0.29, and significantly positive with a t-value of 38. The results are strikingly similar to those of weighted links. A reason for this might be that, just as in the case of weighted links, correlation in degrees between connected links increase relative to the unmodified network. A larger share of nodes are likely to be in a C_2 (all nodes in a triad is in a cycle and therefore also a C_2), and a random node in a C_2 will also be less likely to be connected to a node outside C_2.

6 Empirical application: Selling mobile plans in Indian villages

The following example is based on social network data from Banerjee et al. (2013) initially used to analyze participation in microfinance loans in several villages in the Indian state of Karnataka. Based on questionnaires in 75 villages, 12 adjacency matrices were constructed for each village describing various types of social relations across villagers.[21] The analysis here is based on the so-called "friends" networks.[22] The first column in Table 1 provides descriptive statistics of the degree distribution across all of the 75 villages (degree 0 nodes omitted).[23]

Consider a mobile phone operator offering a menu of vertically differentiated mobile plans to the villagers, and the marginal utility of a minute of talk time is increasing in the number of friends subscribing to a mobile plan. Thus, under incentive compatible contracts, individuals with more friends subscribe to a plan with more talk time.

Suppose the phone operator knows the network structure and has estimated demand functions for each degree. The operator must then consider its choice of k, i.e., the lowest degree being willing to subscribe to a plan.

With full participation 14784 consumers subscribe to a plan. The most sold contract (4423 or 29.2%) is to consumers who only uses the phone to call one friend, and it

[21]Some examples of relations are friendships, visits, advice, borrowing/lending money/kerosene, and temple/church/mosque companions.

[22]Friendships are undirected and unweighted and are based on mentioning or being mentioned to answering: "Name the 4 non-relatives whom you speak to the most." If i mentions j, j mentions i, or both, then i and j are defined as friends.

[23]The social network data were collected from a random sample of villagers, yielding a sample of approximately 46% of all households per village (Banerjee et al., 2013).

	$k = 1$	$k = 2$	$k = 3$
Mean degree	2.64	3.18	3.93
Mode	1	2	3
Median	2	3	3
Standard deviation	1.82	1.65	1.45
Skewness (3rd moment)	2.76	3.38	3.51
Minimum (= k)	1	2	3
Maximum	39	34	22
Sum of degrees	39082	28054	4544
No. of buyers	14784	8828	1156
Share of total population	100%	59.7%	7.8%
Buyers with $d = k$	4323	3851	603
Buyers with $d \geq k + 1$ and $\notin C_{k+1}$	1633	3821	553
- % of buyers with $d \geq k + 1$	15.6%	76.8%	100%
Mean conditional on $d \geq k + 1$	3.32	4.09	4.95

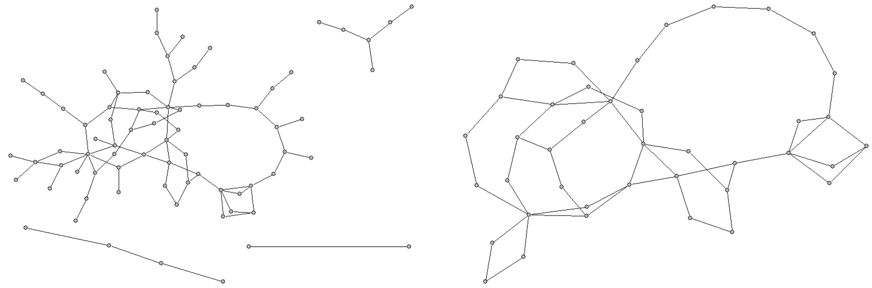
Table 1: Descriptive statistics and realizations of maximal equilibria in threshold games of complements in 75 villages in Karnataka, India. Data from "friend networks" in Banerjee et al. (2013). 2211 nodes without links are omitted.

is therefore not unimaginable that the monopolist incurs losses from these individuals through low prices and high information rents to more highly connected consumers.

If the operator should choose to impose a shutdown of degree-1 consumers, it must also take into the negative impact a shutdown has on profits through decreased network benefits. As the second-to-last element in the first column in Table 1 reveals, not offering a contract to degree 1 nodes leads to an additional exclusion of 15.6% of the total population having degrees of at least 2 under full participation, but not with the exclusion of degree 1 consumers (i.e., they are not in C_2). Figure 9 illustrates the set of participating nodes under the two regimes in one specific village.

Shutting down an additional type, i.e., not offering a contract to degree-2 nodes appear more costly in terms of participation and degree distribution. Now 76.8% of consumers having degrees of 3 or more when $k = 2$ are excluded from the market (nodes not in C_3). In fact, in 37 of the 75 villages no one subscribe to the plan. Of all the 14784 villagers, no one is in a C_4, thus $k = 3$ is the highest lowest contract which is feasible with a positive level of participation.

In addition to a large effect on participation by excluding low-degree consumers, the table captures how the degree distribution is affected. Not surprisingly, the maximum



(a) 75 participating nodes with $k = 1$ **(b)** 40 participating nodes with $k = 2$

Figure 9: Friend network from "Village 10" in Karnataka from Banerjee et al. (2013). "Name the 4 non-relatives whom you speak to the most." There are no nodes in C_3.

degree decreases in k, and the mean degree is substantially lower than the mean without shutdown conditional on degrees strictly higher than k. Generally the skewness increases in k, suggesting that the degree distributions becomes more "left heavy", with a larger share of consumers having lower degrees.[24]

7 Conclusion

How low should the monopolist go? This paper analyzed the set of profit maximizing contracts for a monopolist selling vertically differentiated products where consumers benefit from individuals in their social network purchasing one of these products. These local network externalities are likely to cause the lowest participating type to be lower compared to a benchmark with demand being independent of one's neighbors' actions. The extent scarcely connected (low demand) consumers generate externalities to the rest of the network depends on the importance of indirect connections, which follows from identifiable properties of the network structure. Thus, when designing the menu of available contracts, the monopolist does not only care about the distribution of degrees, but also how the distribution is affected by removing the lowest contract.

In order to analyze how a networks' degree distribution is affected by removing the lowest contract, key properties of the network structure which determines the strength of local network externalities are identified. Of special importance is the share of nodes participating in a threshold game of complements, and within this group of nodes the

[24]Decreasing standard errors may also explain increased skewness.

share having their degree reduced by increasing the game's threshold.

From these key properties, the analysis revealed that the monopolist may be forced to maintain a high level of participation, since removing the lowest contract would lead all consumers to leave the market as less externalities are generated. In relation to this, there exists a condition for a complete market failure to occur: a situation where monopoly production yields a positive social surplus, but negative private profits. Even though network externalities incentivize the monopolist to sell to more consumers, the same effects may lead to the opposite extreme in which no products are sold.

References

Ajorlou, A., Jadbabaie, A., and Kakhbod, A. (2014). Dynamic pricing in social networks: The word of mouth effect. SSRN Working paper.

Akerlof, G. A. (1970). The market for "lemons": Quality uncertainty and the market mechanism. *The Quarterly Journal of Economics*, 84(3):488–500.

Ballester, C., Calvó-Armengol, A., and Zenou, Y. (2006). Who's who in networks. wanted: The key player. *Econometrica*, 74(5):1403–1417.

Banerjee, A., Chandrasekhar, A. G., Duflo, E., and Jackson, M. O. (2013). The diffusion of microfinance. *Science*, 341(6144):363–370.

Banerji, A. and Dutta, B. (2009). Local network externalities and market segmentation. *International Journal of Industrial Organization*, 27(5):605–614.

Bloch, F. and Quérou, N. (2013). Pricing in social networks. *Games and Economic Behavior*, 80(0):243–261.

Bolton, P. and Dewatripont, M. (2005). *Contract Theory.* The MIT Press, Cambridge, MA.

Bonacich, P. (1987). Power and centrality: A family of measures. *American Journal of Sociology*, 92(5):1170–1182.

Bramoullé, Y., Kranton, R., and D'Amours, M. (2014). Strategic interaction and networks. *American Economic Review*, 104(3):898–930.

Campbell, A. (2013). Word-of-mouth communication and percolation in social networks. *The American Economic Review*, 103(6):2466–2498.

Candogan, O., Bimpikis, K., and Ozdaglar, A. (2012). Optimal pricing in networks with externalities. *Operations Research*, 60(4):883–905.

Carroni, E. and Righi, S. (2014). Pricing in social networks under limited information. Working paper.

Corbo, J., Calvó-Armengol, A., and Parkes, D. (2007). The importance of network topology in local contribution games. In Deng, X. and Graham, F., editors, *Internet and Network Economics*, volume 4858 of *Lecture Notes in Computer Science*, pages 388–395. Springer Berlin Heidelberg.

Csorba, G. (2008). Screening contracts in the presence of positive network effects. *International Journal of Industrial Organization*, 26(1):213–226.

Erdős, P. and Rényi, A. (1960). On the evolution of random graphs. *Publications of the Mathematical Institute of the Hungarian Academy of Sciences*, 5:17–61.

Fjeldstad, Ø., Moen, E. R., and Riis, C. (2010). Competition with local network externalities. SSRN Working paper.

Galeotti, A., Goyal, S., Jackson, M. O., Vega-Redondo, F., and Yariv, L. (2010). Network games. *The Review of Economic Studies*, 77(1):218–244.

Jackson, M. O. (2008). *Social and Economic Networks.* Princeton University Press, Princeton, NJ.

Jackson, M. O. and Yariv, L. (2007). Diffusion of behavior and equilibrium properties in network games. *The American Economic Review*, pages 92–98.

Jackson, M. O. and Zenou, Y. (2014). Games on networks. In Young, P. and Zamir, S., editors, *Handbook of Game Theory*, volume 4. North Holland.

Maskin, E. and Riley, J. (1984). Monopoly with incomplete information. *The RAND Journal of Economics*, 15(2):171–196.

Mussa, M. and Rosen, S. (1978). Monopoly and product quality. *Journal of Economic Theory*, 18(2):301–317.

Myerson, R. B. (1981). Optimal auction design. *Mathematics of Operations Research*, 6(1):58–73.

Sääskilahti, P. (2007). Monopoly pricing of social goods. MPRA Paper 3526. University Library of Munich.

Segal, I. (1999). Contracting with externalities. *The Quarterly Journal of Economics*, 114(2):337–388.

Shin, E. (2014). Monopoly pricing and diffusion of (social) network goods. Working paper.

Sundararajan, A. (2004a). Nonlinear pricing and type-dependent network effects. *Economics Letters*, 83(1):107–113.

Sundararajan, A. (2004b). Nonlinear pricing of information goods. *Management Science*, 50(12):1660–1673.

Sundararajan, A. (2007). Local network effects and complex network structure. *The B.E. Journal of Theoretical Economics*, 7(1):1–35.