The Market Game with Production and Arbitrary Returns to Scale: Coordination and Price Stickiness

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The Market Game

- The Shapley-Shubik market game has been studied extensively in the general equilibrium literature, but has seen very few applications in either macroeconomics or industrial organization
- A major reason for this is likely the fact that the model has been studied primarily in the context of exchange economies
    - The problem with exchange environments is that because allocations are not Pareto optimal, it leads to the question as to why markets don't reopen, or why agents don't arrange for things like wash trades (per Peck and Shell's "Liquid markets and competition" paper) which lead to optimal outcomes
- What would seem to be missing is the production side of the economy
    - But even here, with standard convexity assumptions, as in the Dubey-Shubik (1977, *J. Math. Econ.*) model, the optimality issue begs the question of enforcement of antitrust policies to restore the perfectly competitive efficiency result

The Market Game

- This observation suggests that a critical aspect of the market game that has been missing is the phenomenon of increasing returns to scale in production
  - The role of IRTS in creating imperfect competition is well-known in the industrial organization literature
  - Furthermore, even casual observation of industry structure in any modern technologically sophisticated economy will catelog major industrial sectors in which large monopolies and oligopolies occur: energy, communication, information collection and distribution, transportation, and agriculture are among those featuring significant concentration in the U.S. economy
  - Indeed, with the exception of increasing returns due to specialization, there is every reason to believe that increasing returns to scale, scope or network density will give rise to large firms with significant market power
  - In this economic environment, then, we find a clear justification for considering the imperfectly competitive framework of the Shapley-Shubik model

The Market Game

- In a previous paper, we have shown how to extend the market game with production to incorporate increasing returns to scale
- The existence results available in this framework are considerably weaker than those one obtains with convex technologies, because firms facing increasing returns can make losses, and, as a result, can't populate the economy in arbitrary numbers
- One of the more interesting things to emerge from this extension, however, is a simple but powerful equilibration result based on market share adjustements between the different coordination equilibria that arise in these models

Cooordination Equilibria

- Contemporary New Keynesian macroeconomic models encapsulate Keynes' original observation on how sticky wages could lead to prolonged slumps via models of coordination failures
- These models have the key feature that because prices must be chosen before they can be sticky, NK models necessarily posit imperfect competition
  - These models typically invoke the Dixit-Stiglitz model of monopolistic competition (which involve increasing returns to specialization)
  - Price stickiness (which is generally not endogenous in the DS model) is generated by the imposition of so-called "menu costs" for price adjustment
  - Other models invoke some aspect of menu costs to impose staggered price changes (after Calvo's original model)
- Taken together, these models can exhibit multiple coordination equilibria with less than full employment of resources
- As much of the current (continuing) debate over Keynesian macro demonstrates, these models have been useful in explaining the aftermath of the 2008 financial crisis and resulting recession

Coordination Equilibria

- Despite this success, there are important unresolved questions that neither standard NK models, nor the alternative RBC model seem to be able to answer
- Specifically
  - How much do relative prices actually move around? Is the data consistent with the idea that menu costs can be large?
  - Given empirical evidence (even going back to the Great Depression) that economies seem to eventually recover from even very large downturns, is this consistent with the NK hypotheses?

Coordination Equilibria

- Evidence of price movements
- Klenow and Malin's recent work (*Microeconomic Evidence on Price-Setting, NBER WP, 2010*) lays out a series of stylized facts on price changes
    - *Prices change at least once a year on average*
    - *Sales and product turnover are important to micro price flexibility*
    - *There is substantial heterogeneity in the frequency of price changes across goods*
    - *Relative price changes are transitory*
    - *Price changes are typically not synchronized across the business cycle*

Coordination Equilibria

- The KM stylized facts suggest
    - Prices aren't actually that sticky; firms seem to be able to make adjustments when they wish, suggesting that menu costs aren't large
    - Firms use temporary price adjustments to adjust inventory and change production actions
- Combined with the fact that economies seem to recover from downturns, given enough time, and the KM observation that prices aren't synchronized across business cycles, the empirical evidence suggests that NK models' reliance on price stickiness may not be justified

Coordination Equilibria

- Our contribution
  - The Shapley-Shubik market game gives rise naturally to Pareto rankable coordination equilibria (see Peck, Shell and Spear)
  - We show that in the model with production and small numbers of firms, there is an equilibration mechanism which allows prices to remain fixed across different Nash equilibria, without any need to invoke menu costs or staggered adjustment processes.
  - We also show the equilibriation mechanism can operate to move the economy toward full employment of resources only if there are firms operating increasing returns to scale technologies in the model
  - Finally, we will also show that this equilibration mechanism also strongly suggests a role for monetary policies in accommodating this equilibration

The Model

- There are two types of agents
  - $M < \infty$ standard consumers endowed with primary factors of production $e_i \in \mathbb{R}^N_+$ only, who sell these factors to firms that produce outputs of consumer goods that enter in the preferences of consumers.
    - This is the so-called *Hechsler-Ohlin* assumption on the model structure
  - Preferences of standard consumers are as in Peck, Shell, and Spear [*J. Math Econ*, 21, 1992], defined over consumption goods vectors $x_i \in \mathbb{R}^\ell_+$.
  - There are $K_j < \infty$ producers of finitely many types $j = 1, ..., \ell$ who produce outputs of consumption good j using a production technology specified by a production function $q_{k_j} = f_j\left(\varphi_{k_j}\right)$ where $\varphi_{k_j} \in \mathbb{R}^N_+$ is the vector of inputs for producer $k = 1, ..., K_j$ in production sector $j = 1, ..., \ell$. We let $\sum_j K_j = J$ be the total number of firms.
    - We assume that consumers are exogenously endowed with ownership shares of each firm. Specifically, we let $\theta_i^{k_j}$ = consumer i's ownership share of firm k in sector j.
    - Shareholder unanimity issue

The Model

- Timing convention on firm actions relative to consumers.
  - Firms face a hard time-to-produce constraint which requires that they procure inputs and engage in production activities before they can actually deliver output and receive payment for their production activities.
  - So, we divide the market game into two subperiods.
    - In the first, producers must procure inputs for production, based on their expectations of prices they will receive for produced output.
    - Once production decisions are made and output is produced, consumer households receive the value of their offers of primary factors as income.
    - They then decide what to bid on the trading posts for produced goods.
    - As noted previously, we assume that producers act to maximize profits.

The Model: Producer Actions

- For the time being, we let p_j be the price of output good j, and r_n be the price of input good n, and $r^T = [r_1, ..., r_N]$ the vector of input prices. Profit for firm k_j is then

$$\pi_{k_j} = p_j f_{k_j}\left(\varphi^{k_j}\right) - r \cdot \varphi^{k_j}$$

- Prices for inputs are determined on the trading posts for inputs. Producer k_j's bid on trading post $n = 1, ..., N$ is denoted $w_{k_j}^n$, and we let $w_{k_j}^T = \left[w_{k_j}^1, ..., w_{k_j}^N\right] \in \mathbb{R}_+^N$ denoted the producer's vector of bids for inputs. The aggregate bid across all producers for input good n is

$$W^n = \sum_{j=1}^{\ell} \sum_{k_j}^{K_j} w_{k_j}^n.$$

- As is standard, we let the aggregate bid on trading post n except for that of producer k_j be denoted $W_{-k_j}^n$.

The Model: Producer Actions

- The price of input good n is then defined as

$$r^n = \frac{W^n}{E^n}$$

where $E^n = \sum_{i=1}^{M} e_i^n$.

- Each producer's allocation of input goods is given by the producer's own bid for the input divided by the price

$$\varphi_{k_j}^n = \frac{w_{k_j}^n}{r^n} = w_{k_j}^n \frac{E^n}{W^n}.$$

This, of course, is just the standard market game rule that allocates a share of the aggregate offer to an agent in the same proportion as that agent's bid on the trading post is to the aggregate bid.

- Producers earn unit of account revenues from the sale of their outputs on the trading posts for output goods.

The Model: Producer Actions

- With $q^j_{k_j} = f_j\left(\varphi_{k_j}\right)$, $j = 1, ..., \ell$, a firm in output sector j will offer all of its output on the trading post, so we can define the aggregate offer of good j as

$$Q^j = \sum_{k_j=1}^{K_j} q^j_{k_j}.$$

- As before, we let $Q^j_{-k_j}$ be the total offer of good j less that of producer k_j. Given the output price p_j for good j, producer k_j can spend $p_j q^j_{k_j}$ units of account on the purchase of input goods. Hence, producer k_j faces a budget constraint for bids on inputs of the form

$$\sum_{n=1}^{N} w^n_{k_j} \leq p_j q^j_{k_j}.$$

  - Note that from the allocation rule, if we substitute for $w^n_{k_j}$ in the expression above, we obtain the statement that profits can't be negative.

The Model: Consumer Actions

- Consumers bid on trading posts for output goods of the firms in the second subperiod of the game.
  - As noted above, consumers will offer their full endowment for sale on the trading posts for inputs, and will thus, given the timing structure of the game, receive income from the sale of endowments of

$$r \cdot e_i = \sum_{n=1}^{N} r^n e_i^n = \sum_{n=1}^{N} \frac{W^n}{E^n} e_i^n$$

    where the aggregate bids and offers on the input markets are determined by the producers.
  - In addition to their income from the sale of inputs, consumers also receive the shares of the producer firms profits they are endowed with, so that consumer i's total income is

$$y_i = \sum_{n=1}^{N} \frac{W^n}{E^n} e_i^n + \sum_{j=1}^{\ell} \sum_{k_j=1}^{K_j} \theta_i^{k_j} \pi_{k_j}.$$

The Model: Consumer Actions

- Given the general shareholder unanimity issue, we assume here that households simply take the value of their endowments as given.
- Given these assumptions, we let consumer i's bid on trading post j be denoted b_i^j and define the aggregate bid on trading post j as

$$B^j = \sum_{i=1}^{M} b_i^j.$$

  - As above, we let the aggregate bid on trading post j except for that of consumer i be denoted B_{-i}^j. The price of output good j is then defined as the ratio of the total bids for the good to the total offer of the good

$$p_j = \frac{B^j}{Q^j}.$$

  - Consumer i's allocation of consumption good j is then given by the usual market game allocation rule

$$x_i^j = \frac{b_i^j}{p_j} = b_i^j \frac{Q^j}{B^j}$$

The Model: Consumer Actions

- Consumer i's bids for output goods in terms of units of account are then constrained by the budget relation

$$
\begin{aligned}
\sum_{j=1}^{\ell} b_i^j &\leq y_i = r \cdot e_i + \sum_{j=1}^{\ell} \sum_{k_j=1}^{K_j} \theta_i^{k_j} \pi_{k_j} \\
&= \sum_{n=1}^{N} r^n e_i^n + \sum_{j=1}^{\ell} \sum_{k_j=1}^{K_j} \theta_i^{k_j} \pi_{k_j} \\
&= \sum_{n=1}^{N} \frac{W^n}{E^n} e_i^n + \sum_{j=1}^{\ell} \sum_{k_j=1}^{K_j} \theta_i^{k_j} \pi_{k_j}.
\end{aligned}
$$

Strategic Interactions: Producers

- A producer's best response to other producers' actions is determined by the solution to the optimization problem

$$\max_{b_{k_j}} \frac{B^j}{Q^j} f_{k_j}\left(\varphi^{k_j}\right) - \sum_{n=1}^{N} \frac{W^n}{E^n} \varphi_n^{k_j}$$

subject to

$$\sum_{n=1}^{N} w_{k_j}^n \leq p_j q_{k_j}$$

$$\varphi_n^{k_j} = w_{k_j}^n \frac{E^n}{W^n}$$

$$q_{k_j} = f_j\left(\varphi_{k_j}\right).$$

Producer First-order Conditions

- On the producer side, we note first that the second constraint in the producer's optimization is simply the requirement that profits be non-negative. We assume in the analysis below that this constraint doesn't bind, though it is necessary in principle, for example, with increasing returns to scale technologies.
- Making substitutions from the other constraints into the producer's optimization, we obtain an unconstrained optimization

$$\max_{b_{k_j}} \frac{B^j}{Q^j} f_{k_j}\left(\left[w^1_{k_j}\frac{E^1}{W^1}, \ldots, w^N_{k_j}\frac{E^N}{W^N}\right]\right) - \sum_{n=1}^{N} w^n_{k_j}.$$

- Taking first-order conditions yields

$$\frac{B^j}{Q^j}\frac{\partial f_{k_j}}{\partial \varphi^n_{k_j}}\left[\frac{E^n}{W^n} - \frac{w^n_{k_j}E^n}{\left(W^n\right)^2}\right] - \frac{B^j f_{k_j}}{\left(Q^j\right)^2}\frac{\partial f_{k_j}}{\partial \varphi^n_{k_j}}\left[\frac{E^n}{W^n} - \frac{w^n_{k_j}E^n}{\left(W^n\right)^2}\right] - 1 = 0$$

Producer First-order Conditions

- Simplifying the first-order-condition yields

$$\frac{B^j}{Q^j}\frac{\partial f_{k_j}}{\partial \varphi^n_{k_j}}\frac{E^n W^n_{-k_j}}{\left(W^n\right)^2}\frac{Q^j_{-k_j}}{Q^j}-1=0$$

or

$$\frac{p_j}{r^n}\frac{\partial f_{k_j}}{\partial \varphi^n_{k_j}}\frac{W^n_{-k_j}}{W^n}\frac{Q^j_{-k_j}}{Q^j}-1=0.$$

- Note that if the market is such that we have a very large number of producers, the two ratios on the right-hand side of the first term will be almost one, and the expression here boils down to the statement that the value of the marginal product of the n^{th} input should equal it's price.

Strategic Interactions:Consumers

- A consumer's best responses are determined by the solution to the optimization problem

$$\max_{b_i} u_i [x_i]$$

subject to

$$\begin{aligned} x_i^j &= b_i^j \frac{Q^j}{B^j} \text{ for } j = 1, \ldots, \ell \\ \sum_{j=1}^{\ell} b_i^j &\leq \sum_{n=1}^{N} \frac{W^n}{E^n} \omega_i^n + \sum_{j=1}^{\ell} \sum_{k_j=1}^{K_j} \theta_i^{k_j} \pi_{k_j} \\ \pi_{k_j} &= \frac{B^j}{Q^j} f_{k_j}\left(\varphi^{k_j}\right) - r \cdot \varphi^{k_j}. \end{aligned}$$

Consumer First-order Conditions

- Consumers will take the bids on the input markets as given, and hence will also take the firms' outputs as given, although they will take account of the effect their own bids on the product markets has on the prices in the profits of each firm. The first-order conditions for this optimization problem are

$$\frac{\partial u_i}{\partial x_i^j}\left[\frac{Q^j}{B^j}-\frac{b_i^j Q^j}{\left[B^j\right]^2}\right]+\lambda\left[\frac{\sum_{k_j=1}^{K_j}\theta_i^{k_j} f_{k_j}}{Q^j}-1\right]=0$$

- Simplifying yields

$$\frac{\partial u_i}{\partial x_i^j}\left[\frac{Q^j}{B^j}\frac{B_{-i}^j}{B^j}\right]+\lambda\left[\frac{\sum_{k_j=1}^{K_j}\theta_i^{k_j} f_{k_j}}{Q^j}-1\right]=0.$$

Consumer First-order Conditions

- We do not need to consider the effect of a change in household i's bid on input prices because of the envelope theorem as applied to firms' profit maximizations. This follows from the fact that we can view household i's income as its share of aggregate profit net of it's own sales of primary factors. If firms are not maximizing this, then some firm is not maximizing profit, and hence the assertion follows. Finally, note that the household first-order condition is also consistent with what we get in the competitive limit, since the right-hand term will go to $-\lambda$ as the number of firms and consumers gets large, while the left-hand side goes to the marginal utility divided by the price.

Existence of Producer Best-Responses

- Under the condition that

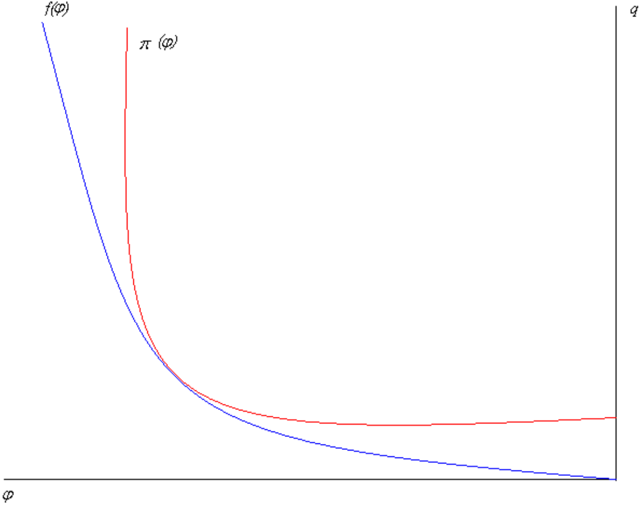
$$\lim_{\varphi \to \infty} \frac{1}{f(\varphi)} D_{\varphi} f < \infty$$

  profits become negative as φ gets large, and hence, a bounded optimum to the profit maximization problem exists
- **Remark:** It is possible that if RTS grow faster than exponential (e.g. recombinant growth processes, see Weitzman, *QJE*, 1998), even the strategic market game framework may not rule out pathological infinities of things like profits.
- **Remark:** With IRTS (and unlike the case of DRS), firms can make losses at low levels of output

▸ Proof

Existence of Producer Best-Responses

- Hence, we obtain the following picture



Existence of Nash Equilibrium

- We can show existence of equilibrium generally via an application of Kakutani's fixed-point theorem, once we find conditions that guarantee that all agents' optimization problems are continuous. Assuming this for the moment, we would then define a mapping

$$\beta = \begin{bmatrix} b_1 \\ \vdots \\ b_M \\ w_1 \\ \vdots \\ w_J \end{bmatrix} \in \ \Delta^{\ell M + NJ - 1}$$

  where $\Delta^{\ell M + NJ - 1}$ is the simplex of each household's ℓ-vector of bids b_i for output goods, and w_j is the n-vector of bids for input goods for producer j.

Existence of Nash Equilibrium

- Next let $\tilde{\beta}(\beta)$ be the vector of best-responses to β and define a mapping (as in the Peck-Shell-Spear paper)

$$\zeta : \Delta_{\varepsilon}^{\ell M+NJ-1} \rightarrow \Delta_{\varepsilon}^{\ell M+NJ-1}$$

by

$$\zeta(\beta) = \frac{1}{\imath \cdot \tilde{\beta}(\beta)} \tilde{\beta}(\beta).$$

- Given the assumption that all objective functions are continuous, this mapping is upper-hemi-continuous via the maximum principle. Quasi-concavity of utility and production functions implies that the maximizing correspondence is also convex-valued. The finiteness of aggregate endowments (together with the sell-all assumption on endowments) implies that the best responses are also bounded, and hence the mapping ζ is well-defined. Via the Kakutani fixed-point theorem, then, there exists a fixed point of the mapping, which is clearly the Nash equilibrium.

Existence of Nash Equilibrium

- The assumption above about continuity of objective functions is not trivial, since we need to solve the "0/0" problem whereby (for example) a monopoly firm can make arbitrarily small bids for inputs and end up obtaining all of the input resources, even though in the limit (as the bids go to zero), the firm gets nothing.
- We can't fix this the way one does in an exchange environment by requiring bids to be non-zero and using Inada conditions, since with an arbitrary number of IRTS firms, we need to allow for the possibility that some firms will find shutting down optimal.

Existence of Nash Equilibrium

- The simplest way to ensure that all firms have continuous profit functions, then, is to assume that each sector includes some DRS firms who always find it profitable to produce, and hence enforce positive bids for input resources.
- While this assumption does end up allowing us to prove a general existence result, the result itself is quite weak since it says nothing about whether or not IRTS firms are active in equilibrium, and how many can be active.
- However, since our results on coordination equilibrium and price-stickiness will end up requiring the presence of both IRTS and DRS firms, we content ourselves with the weak existence result.

Coordination and Price Stickiness

- As we noted above, the market game exhibits indeterminacy in terms of the quantities of goods household put up for offer.
- In the pure exchange version of the model, Peck, Shell and Spear characterize this as market thickness, and show that the Nash equilibrium allocations corresponding to different degrees of market thickness have real effects
- In the production market game, this indeterminacy shows up in the extent of households' offerings on the markets for inputs.
    - While we have assumed so far that households offer all of their endowments of input goods, this assumption is arbitrary and can be replaced with an arbitrary offer of less than the full aggregate endowment

Coordination and Price Stickinesst

- While the indeterminacy may not be apparent in our specification of the model, it can be seen via the following argument
  - Consider a sequence of economies in which households have positive endowments of all goods, make positive offers of all goods, and have positive marginal utility for all goods (including primary factors)
  - Assume that in the limits, endowments of produced goods (and offers of these goods by households) go to zero, while the utility functions of households converge (in the topology of $\mathbb{C}^r$ uniform convergence) to utilities in which the marginal utility of primary factors is zero everywhere
  - Everywhere along this sequence, it will be the case that households can either make bids or offers, but not both; this is as in the pure exchange case
  - This indeterminacy shows up as an equality of the first-order conditions with respect to bids and offers
  - Since the FOCs are continuous, then, in the limit we obtain the same indeterminacy with respects to bids and offers that obtains in the pure exchange case

Coordination and Price Stickiness

- In the macro literature, equilibria in which there is slack in the input markets is referred to as a coordination (or coordination failure) equilibrium
- In the market game with production, there is a remarkable emergent mechanism for equilibration across coordination equilibria in the model, based on individual firm's market shares, with all prices for outputs and inputs left fixed
- This equilibration mechanism is apparent from the first-order conditions for firm profit maximization

$$\frac{B^j}{Q^j}\frac{\partial f_{k_j}}{\partial \varphi^n_{k_j}}\frac{E^n W^n_{-k_j}}{(W^n)^2}\frac{Q^j_{-k_j}}{Q^j} - 1 = 0$$

or, writing the ratios of aggregate bids to offers as prices,

$$\frac{p_j}{r^n}\frac{\partial f_{k_j}}{\partial \varphi^n_{k_j}}\frac{W^n_{-k_j}}{W^n}\frac{Q^j_{-k_j}}{Q^j} - 1 = 0.$$

Coordination and Price Stickiness

- Writing this first-order condition in standard vector form relating the production gradient for firm k_j to the vector of input prices gives

$$p^j \left(\frac{Q^j_{-k_j}}{Q^j} \right) D_\phi f_{k_j} - \hat{W} \hat{W}^{-1}_{-k_j} r = 0$$

- The key thing to observe here is that, holding p^j and r fixed, given degrees of freedom to vary the output shares and input bill shares for each firm, we can effectively span the full range of each firm's production gradient.
- This ability to mimic price effects via market shares is constrained by the fact that
  - Market shares must sum to one
  - With prices for outputs constant, product market adjustments are limited to movements along each consumer's income expansion path; hence, for this analysis, we will assume that there are many households so that demand in the product markets is competitive

Coordination and Price Stickiness

- Since market shares enter the firm's first-oder conditions as one minus market share, the adding up requirement means that we have $(J-1)N$ free input bill share parameters, and $\sum_{j=1}^{\ell}(K_j-1)$ free product market share parameters.
- The fact that with prices fixed, adjustments on the product market are restricted to income effects means that adjustments across coordination equilibria with prices fixed can occur only for relaxations of the total input use constraint in response to some external effect on aggregate demand

Coordination and Price Stickiness

- These two restrictions pose some interesting questions
  - Given that we would generally need ℓ output price variables and N input price variables in conjunction with consumer choices and firm input allocations to find equilibrium, how does the number of firms affect the equilibirum calculation when prices are fixed?
  - Can adjustments in firm market shares in response to exogenous shocks to demand maintain the economy in Nash equilibrium? If so, does the model impose any restrictions for equilibration to work?
  - Given the restriction to generating equilibration via income effects on the product market, we will clearly need to make adjustments in households' bids, as well as firms bids on the input markets in order to keep prices fixed. What does this require in terms of adjustment in the units of account we are using to denominate prices? What implications might this have for monetary policy adjustments in a real world environment?

Coordination and Price Stickiness

- To address the first question, we need to write down the equilibrium conditions we wish to maintain when total resources vary.
- These are the first-order conditions

$$\begin{aligned} Du_i(x_i) - \lambda_i p &= 0 \\ p \cdot x_i &= r \cdot e_i + \sum_{j=1}^{\ell} \sum_{k_j} \theta_i^{k_j} \pi_{k_j} \end{aligned}$$

for households; and

$$\begin{aligned} p^j \left(\frac{Q^j_{-k_j}}{Q^j} \right) D_\phi f_{k_j} - \hat{W} \hat{W}^{-1}_{-k_j} r &= 0 \\ \phi^{k_j} = \hat{w}_{k_j} \hat{W}^{-1} E &= \left(I - \hat{W}_{-k_j} \hat{W}^{-1} \right) E \end{aligned}$$

for firms

Coordination and Price Stickiness

- The allocation rules for the market game imply (given the prices) that the product and factor markets clear, so the only remaining requirements for equilibrium are the price constancy rules

$$\begin{aligned} p - \hat{B}\hat{Q}^{-1}\iota &= 0 \\ r - \hat{W}\hat{E}^{-1}\iota &= 0 \end{aligned}$$

where $\iota^T - [1, \ldots, 1]$ is the sum vector, and $\hat{}$ over an aggregate variable denotes making the vector into a diagonal matrix with the embedded vector down the main diagonal. In this expression, the aggregate bids on the product and input markets are taken as variables, since they can be varied independently of market shares.

Coordination and Price Stickiness

- This yields a system of $M(\ell+1)+JN+\ell+N$ equilibrium conditions, in $M(\ell+1)+(J-1)N+\sum_{j=1}^{\ell}(K_j-1)+\ell+N$ variables. So, to have at least as many variables as we do equations, then, we need

$$(J-1)N+\sum_{j=1}^{\ell}(K_j-1)\geq JN$$

or

$$\sum_{j=1}^{\ell}(K_j-1)\geq N$$

Coordination and Price Stickiness

- If we let F be the average number of firms per output sector, the inequality above becomes

$$(F - 1)\,\ell \geq N$$

which yields

$$F \geq 1 + \frac{N}{\ell}$$

- To get a rough estimate of this number, we use data compiled by Sébastien Miroudot, Rainer Lanz and Alexandros Ragoussis ("Trade in Intermediate Goods and Services", OECD Trade Policy Working Paper No. 93).

Coordination and Price Stickiness

- From their table 5

		Number of SITC Commodities Lines Classified by Use			
Industry	Total	Intermediate	Consumption	Capital	Other
Agriculture and Fishing	193	112	79	2	0
Mining and quarrying	75	75	0	0	0
Food products	299	113	186	0	0
Textiles and wearing apparel	375	205	169	0	1
Wood publishing and printing	152	117	35	0	0
Refined petroleum	17	15	0	1	1
Chemical products	483	446	37	0	0
Rubber and plastic products	70	58	12	0	0
Metal products	373	323	21	28	1
Mechanical products	395	108	28	252	7
Office machinery and computers	30	4	1	25	0
Radio, TV and communication equipment	70	33	6	31	0
Medical, precision and optical instruments, watches and clocks	130	42	26	62	0
Motor vehicles	33	16	2	14	1
Other transport equipment	56	14	14	26	2
Other manufactured goods	281	170	80	30	1
Electricity, gas and water	3	3	0	0	0
	3035	1854	696	471	
			N/L	1.588689	

Coordination and Price Stickiness

- Using the sum of consumption and capital outputs to estimate ℓ, with the intermediate goods representing N, we obtain $\frac{N}{\ell} = 1.6$, so that we would need $F > 3$.
- The answer to the second question involves two steps.
  - We need to show first that firms will in fact have incentives to respond to changes in market shares in ways that move the economy back toward equilibrium after a shock; this turns out to require some assumptions
  - We then need to show what amounts to a full-rank condition for applying the implicit function theorem in order to formally show the result

Coordination and Price Stickiness

- To show conditions under which firms will have incentives to respond to market share changes, consider a simple one-sector version of the model with a single output produced using a single input, and suppose that all firms in this economy produce using a strictly diminishing returns technology
- Suppose we are at a slack coordination equilibrium, and the demand for the output good increases

A Coordination and Price Stickiness Result

- What happens if some firm responds to the demand shock by increasing its own output, other firms holding output constant?
  - The relevant first-order condition to answer this question is
$$p\frac{Q_{-k}}{Q}f'(\phi) - r\frac{W}{W_{-k}} = 0$$
or
$$f'(\phi) = \frac{r}{p}\frac{W}{W_{-k}}\frac{Q}{Q_{-k}}.$$
  - If the firm increases output, its market share of output goes up, so one minus market share goes down. Similarly, its share of the wage bill will go up, so one minus that goes down. Putting both of these parameters on the right-hand-side of the FOC, the right-hand-side goes up. To maintain optimality, then, the firm would want to adjust output to increase its marginal product of the input. For a firm with DRS technology, however, this requires reducing output. Hence, this firm would not wish to respond to the demand shock.
  - The opposite, however, is true of a firm operating an increasing returns technology.
  - Finally, this result generalizes easily to the case of multiple inputs

Coordination and Price Stickiness

- In a mixed RTS economy, with some firms operating IRTS technologies and some DRS or CRS technologies, the IRTS firms will respond to the shock by increasing output.
- In this case, though, something interesting happens
  - Consider a DRS firm. This firm will find its share of output falling as the IRTS firms increasing production. It will also see its share of the wage bill falling.
  - In this case, the right-hand-side of the FOC above will be smaller, so the firm's optimal response will be to make its marginal product smaller, which it does by increasing output
- Hence, we have a scenario in which it is possible to think of a leader-follower equilibration, which has IRTS firms moving first in response to changes in aggregate demand, followed by adjustments among the non-IRTS firms.

Coordination and Price Stickiness

- It remains, then, to show that the implicit function theorem (or, more generally, a transversality result) will apply in the neighborhood of the Nash equilibrium for an economy under slack
- The Jacobian matrix has $M(\ell+1)+JN+\ell+N$ rows, and $M(\ell+1)+\sum_{j=1}^{\ell}(K_j-1)+(J-1)N+\ell+N$ columns. Calculating the Jacobian yields a matrix of the form

$$JACOBIAN = \begin{bmatrix} \mathbf{H}_1 & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & * & \mathbf{0} \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \cdots & \mathbf{H}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & * & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{G}_{k_1} & \cdots & \mathbf{0} & * & * \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{G}_{k_\ell} & * & * \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \hat{Q}^{-1} & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \hat{E}^{-1} \end{bmatrix}$$

Coordination and Price Stickiness

In the Jacobian,

$$\mathbf{H}_i = \left[\begin{array}{cc} D^2 u_i & -p \\ -p^T & 0 \end{array} \right] \text{ for } i = 1, ..., M$$

(i.e. the derivatives of the consumer FOCs with respect to x_i, and λ_i);

$$\mathbf{G}_{k_j} = \left[\begin{array}{cc} p^j Df_{k_j} & -p^j \frac{Q^j_{-k_j}}{Q^j} \left[D^2 f_{k_j} - \frac{1}{Q^j} DfDf^T \right] \hat{E} - \hat{r} \end{array} \right]$$

(i.e. the derivatives of firm FOCs with respect to $\frac{Q^j_{-k_j}}{Q^j}$, and $\frac{W^n_{-k_j}}{W^n}$; here $\hat{r} = diag\ r$ and $\hat{E} = diag\ E$), and $*$ denote derivatives with respect to B and W that we don't need to worry about.

Coordination and Price Stickiness

Since the Jacobian matrix can be reduced to a square matrix in block upper-triangular form, it is easily verified that the matrix has full row rank, once we note that in the sub-matrices $\mathbf{G}_{k_j}$ the derivatives with respect to $\frac{Q^j_{-k_j}}{Q^j}$ ensure that even if some firm's technology exhibits constant returns to scale (in which case the second derivative matrix would be singular in the relative price direction), there will be a non-zero perturbation possible in the direction of the relative price vector.

A Coordination and Price Stickiness Result

- Hence, we have the following

Theorem

When total input resources are variable, there exist there are solutions to the equilibrium equations in variables consisting of household consumptions and Lagrange multipliers, and firm input wage bill shares, and firm output market shares, in a neighborhood of any given input constrained Nash equilibrium for the economy.

Discussion

- This result clearly implies that firms can maintain Nash equilibrium in the market game via output adjustments alone, without any need for changes in relative prices
  - This, in turn, suggests that even very small costs to changing prices will lead firms to limit such changes
- The need for adjustments in the units of account normalization to maintain nominal prices suggests a role for monetary policy in providing the degrees of freedom for making these adjustment
  - This also suggests that some of the price adjustments delineated in the Klenow-Malin stylized facts reflect adjustments in nominal prices needed to maintain stability of relative prices, but this is obviously open to further study

Thank you!

Existence of Interior Producer Best-Responses

- We show that as long as output prices are strictly positive, any producer's profit function is strictly quasi-concave. (For this analysis, we supress consideration of the specific production sector). On the revenue side, let

$$h_k = D_\varphi (pq_k) = D_\varphi f(\varphi) \left[\frac{dpq_k}{dq_k} \right].$$

Now

$$\frac{dpq_k}{dq_k} = \frac{B}{Q}\left[1 - \frac{q_k}{Q}\right] = \frac{BQ_{-k}}{Q^2} \geq 0$$

- We now assume that the production function is homogeneous of degree $\delta \geq 0$, so that by Euler's theorem, $\varphi \cdot D_\varphi f = \delta f(\varphi)$. Now, consider

$$\begin{aligned} \varphi \cdot h_k &= \varphi \cdot D_\varphi f(\varphi) \left[\frac{dpq_k}{dq_k} \right] \\ &= \delta q_k \frac{BQ_{-k}}{Q^2} \geq 0 \end{aligned}$$

and asymptotically zero.

Existence of Interior Producer Best-Responses

- Next, differentiating again with respect to φ, we find(φ)

$$\frac{d^2 pq_k}{dq_k^2} = \frac{d}{dq_k}\left[\frac{BQ_{-k}}{Q^2}\right] = -2\frac{BQ_{-k}}{Q^3} < 0$$

- It now follows that

$$D_\varphi h_k = \frac{BQ_{-k}}{Q^2} D_\varphi^2 f - 2 D_\varphi f D_\varphi f^T \frac{BQ_{-k}}{Q^3} = \frac{BQ_{-k}}{Q^2}\left[D_\varphi^2 f - \frac{2}{Q} D_\varphi f D_\varphi f^T\right]$$

since $Q = \sum f_k\left(\varphi^k\right)$. Now, Via Euler's theorem

$$\varphi \cdot D_\varphi f = \delta f\left(\varphi\right)$$
$$\Rightarrow D_\varphi f + D_\varphi^2 f \varphi = \delta D_\varphi f$$
$$\Rightarrow D_\varphi^2 f \varphi D_\varphi f^T = \left(\delta - 1\right) D_\varphi f D_\varphi f^T$$

Existence of Interior Producer Best-Responses

- Substituting above, we have

$$\begin{aligned} D_\varphi^2 f - \frac{2}{Q} D_\varphi f D_\varphi f^T &= D_\varphi^2 f - \frac{2}{Q(1-\delta)} D_\varphi^2 f \varphi D_\varphi f^T \\ &= D_\varphi^2 f \left[I - \frac{2}{Q(1-\delta)} \varphi D_\varphi f^T \right]. \end{aligned}$$

- Now, assuming that the production function is strictly quasi-concave, along any direction ρ orthogonal to $D_\varphi f$, we have

$$\rho^T D_\varphi h_k \rho = \rho^T D_\varphi^2 f \rho < 0$$

and hence, the firm's revenue function will be quasi-concave.

Existence of Interior Producer Best-Responses

- Now, consider the cost side, with

$$C(\varphi) = r \cdot \varphi.$$

Differentiating yields

$$D_{\varphi} C = r^{T} + \varphi^{T} D_{\varphi} r.$$

- Since

$$r = E^{-1} W$$

$$D_{\varphi} r = E^{-1} D_{\varphi} w.$$

Existence of Interior Producer Best-Responses

- With

$$\begin{aligned} w &= \hat{W}\hat{E}^{-1}\varphi \\ \Rightarrow D_\varphi w &= \hat{W}\hat{E}^{-1} + \hat{\varphi}\hat{E}^{-1}D_\varphi w \\ \Rightarrow D_\varphi w &= \left[I - \hat{\varphi}\hat{E}^{-1}\right]^{-1}\hat{W}\hat{E}^{-1} \\ &= \hat{E}\hat{E}_{-k}^{-1}\hat{W}\hat{E}^{-1} \\ &= \hat{E}_{-k}^{-1}\hat{W} \end{aligned}$$

  so that

$$D_\varphi r = \hat{W}\hat{E}_{-k}^{-1}\hat{E}^{-1}.$$

- Hence,

$$\begin{aligned} D_\varphi C &= W^{T\hat{E}-1} + \varphi^T\hat{W}\hat{E}^{-1}\left[\hat{E}_{-k}\right]^{-1} \\ &= W^{T\hat{E}-1}\left[I + \hat{\varphi}\hat{E}_{-k}^{-1}\right] \gg 0. \end{aligned}$$

Existence of Interior Producer Best-Responses

- Differentiating again with respect to φ yields:

$$\begin{aligned} D_{\varphi}^{2}C &= \hat{E}^{-1}\left[I+\hat{\varphi}\hat{E}_{-k}^{-1}\right]D_{\varphi}w+\hat{W}\hat{E}_{-k}^{-1}\hat{E}^{-1} \\ &= \hat{E}^{-1}\left[I+\hat{\varphi}\hat{E}_{-k}^{-1}\right]\hat{E}_{-k}^{-1}\hat{W}+\hat{W}\hat{E}_{-k}^{-1\hat{E}-1} \\ &= \left[2I+\hat{\varphi}\hat{E}_{-k}^{-1}\right]\hat{W}\hat{E}_{-k}^{-1\hat{E}-1} \end{aligned}$$

- Since this matrix is positive definite, it follows that the cost function is strictly convex. Unfortunately, this is the most we can say if we are dealing with IRTS firms, since the sum of a quasi-concave and concave function need not be quasi-concave. In particular, we would need to explicitly check the second-order conditions to determine whether a solution to the first-order conditions was in fact a maximum or minimum.

Existence of Interior Producer Best-Responses

- Now, if we impose the mild assumption that

$$\lim_{\varphi \to \infty} \frac{1}{f(\varphi)} D_\varphi f < \infty$$

this will guarantee that as φ gets large, profit becomes negative since

$$D_\varphi \pi = \frac{BQ_{-k}}{Q^2} D_\varphi f - \varphi^T \hat{E}^{-1} D_\varphi w - \hat{E}^{-1} W.$$

Under the restriction above, $D_\varphi \pi$ will be asymptotically negative. The restriction itself bounds the production function below something exponential, and is sufficient, though not necessary. In this case, then, we are guaranteed that the firm will seek an interior profit maximum. Note, however, that for small φ, $D_\varphi f$ approaches zero, and it can easily occur that $D_\varphi \pi$ becomes negative, indicating that the firm's best response in this case is to shut down.

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