Hold-up and the Timing of Ex Post Renegotiation*

Jafar Olimov†

January 1, 2015

Abstract

I study how contracting parties choose the timing of ex post renegotiation, and whether the ability to schedule the ex post renegotiation can help to resolve the hold-up problem. I show that depending on the renegotiation protocol and parameter restrictions, the assignment of the right to decide when to renegotiate can attenuate or completely resolve the hold-up problem.

JEL codes: D23, K12, L14

Keywords: renegotiation timing, hold-up, bargaining

A typical case of a relationship-specific (hereafter, specific) investment involves a lengthy construction or an installation stage. For instance, in the classic GM-Fisher Body example, Fisher Body undertook specific investment by constructing a plant of a certain capacity with car body dyes and presses used exclusively in the production of GM cars (Klein, Crawford and Alchian (1978)). In another often

*I would like to thank Brian Roe, James Peck, Yaron Azrieli, Huanxing Yang, Howard Marvel, John Kagel, Noah Dormady and seminar participants of the OSU theory lunch group for help and many suggestions, which helped to improve the paper. The paper is based on the first chapter of my dissertation. I acknowledge research support by the National Institute of Food and Agriculture (grant #2008-25400-18701).

†Research Center SHARQ, Dushanbe, Tajikistan; Email: olimov@gmail.com

cited work on specific investment, Joskow (1987) considers the construction of coal-fired electricity-generating power plants. In both these cases, the capacity choice of production facilities was irreversible once the construction stage had begun, and the construction stage took at least a year, if not longer. A more recent example of a project with specific investment and a lengthy construction stage is the Keystone oil pipeline (phase 1) project from Alberta to Illinois. Similar to the previous two examples, the capacity of the oil pipeline and other specifications of the project could not be altered at the construction stage due to Canadian and US federal regulations, while the construction stage itself took more than two years.

The incomplete contracts literature suggests that because contracting parties can not specify all contingencies in a contract, once the uncertainty about the non-investor's outside option is realized, the investor (she) and the non-investor (he) engage in ex post renegotiation. Further, the incomplete contracts literature assumes that the ex post renegotiation occurs at some moment after the investor makes an irreversible investment decision without making a distinction whether the renegotiation takes place before, during or after the construction stage. The standard result in the literature is that the investor anticipates this ex post renegotiation and, unless she is a residual claimant with certainty, chooses an ex ante sub-optimal investment (plant capacity) level.

In reality, however, when drafting an initial incomplete contract, contracting parties include special clauses, which regulate when the ex post renegotiation can take place. These clauses contain time-specific provisions, provisions related to changes in market conditions, and/or provisions related to the progression of a relationship.[1] The clauses with time-specific provisions are called "scheduled reviews" and include specific dates or time periods, when contracting parties can review contract terms.[2] The presence of such clauses in long-term contracts implies that after the realization of uncertainty, the investor and the non-investor

[1] For a discussion of various types of renegotiation provisions in natural gas contracts, see, for example, Crocker and Masten (1991).

[2] For description of scheduled reviews see, for example, Cellich (1999).

typically have to wait for a specific time period to engage in ex post renegotiation. Depending on the provisions, the ex post renegotiation can take place prior to the construction stage, at different points within the construction stage, or well after the completion of the construction stage.

In this paper I study the effect of timing of the ex post renegotiation on the resolution of the hold-up problem. I show that in general it is possible to set the timing of ex post renegotiation to encourage the investor to make an ex ante socially optimal investment decision in equilibrium. I also study the investor's and the non-investor's equilibrium timing choices. I show that depending on the renegotiation protocol and parameter restrictions, the assignment of the right to decide when to renegotiate can result in the ex ante socially optimal investment. This way I am able to show that contracting parties can use renegotiation timing clauses in incomplete contracts to attenuate or, in certain circumstances, to completely resolve the hold-up problem.

The observation that contracting parties schedule the ex post renegotiation can be traced back to Macneil (1978) in his discussion of the US contract law. Macneil (1978) observes that if contracting parties cannot decide on some parameters of a contract, they often sign an "agreement to agree" and schedule the renegotiation at a later time period (p. 870). If the cost of renegotiation varies across time, then the renegotiation timing choice can have a direct impact on the renegotiation outcome and the investor's ex ante investment decision. From an economist's point of view, two important questions arise. The first question is whether it is possible to schedule the ex post renegotiation to achieve social optimality, and the second question is how to organize the process of determination of the renegotiation timing, so that the investor makes a socially optimal investment decision. As I show in the paper, it is possible to achieve the social optimality of equilibrium investment by appropriately setting the timing of ex post renegotiation. The answer to whether contracting parties set the renegotiation timing to achieve social optimality depends on assumptions about model parameters such as the distribution function of the non-investor's outside option, the shape of the investor's production cost function and the non-investor's valuation function. Depending on parameter restrictions, we can achieve social optimality by allocating the right to choose the renegotiation timing and the right to make the first offer.

To differentiate among time periods, when ex post renegotiation can take place, I introduce the production delay cost. I assume that with each period of the renegotiation delay, the investor's production cost goes up. In addition, I assume that as the relationship progresses, this production delay cost increases. To illustrate this idea, suppose that as in the Joskow's case, an investor commits to construct a coal-fired electricity generating plant of a certain capacity. Suppose that once the construction has begun, the investor's capacity choice is irreversible. Further, the construction of the plant takes two periods. In the first period the investor purchases the land for the construction site, and in the second period the investor constructs the plant. Next, suppose that the investor's partner can renegotiate with the investor either in the first or in the second period. If the delay due to renegotiation takes place in the first period, the investor faces no waiting cost, since the cost of maintaining a piece of land is negligible. If the delay due to renegotiation takes place in the second period, when the plant has already been constructed, the investor has to pay a positive cost of maintaining the plant while the parties renegotiate. The production delay cost captures the increase in the investor's production cost from maintaining the plant, should the renegotiation delay happen in the second period.

To model the ex post renegotiation, I use an alternating-offers bargaining game, where parties bargain over a division of the trade surplus and only the non-investor's has an outside option. In the presence of a non-zero production delay cost, with each period of delay the investor's production cost goes up, while the trade surplus proportionally falls. Consequently, an alternating-offers bargaining game with a production delay cost and one-sided outside option reduces to the game, where with each period of delay the trade surplus shrinks at a higher rate than the outside option. As a result, the renegotiation timing decision amounts to the choice of a depreciation rate for the trade surplus. As I show in the paper, if the investor is the first mover in the bargaining game, then an increase in the depreciation rate of the trade surplus strengthens her bargaining position as the first mover, which in turn encourages the investor to raise her investment. Then, I show that if the investor has the right to make the first offer and has the right to set the renegotiation timing, the investor chooses to renegotiate, when the trade surplus completely depreciates with delay, and necessarily makes an ex ante socially optimal investment decision.

The investor can invest optimally in equilibrium, even if the renegotiation period with a complete depreciation of the trade surplus is not in her choice set, or if she doesn't have the right to make the first offer. I obtain this result, because the choice of a renegotiation period with a higher depreciation rate has two opposing effects. On the one hand, it increases the payoff of the party making the first offer and equally lowers the payoff of the party making the second offer. On the other hand, it lowers the probability of the best bargaining outcome and the payoff in the worst bargaining outcome for the first mover, and equally increases the probability and the payoff for the second mover. A party deciding when to renegotiate sets the renegotiation timing to balance these opposing effects.

The chosen renegotiation period determines the depreciation rate of the trade surplus, which in turn determines the investor's ex ante equilibrium investment decision. Whether the choice of the renegotiation period results in the social optimality of the investor's equilibrium investment decision depends on parameters of the model, the identity of the party making the timing decision, and the identity of the party making the first offer. However, if we appropriately assign the right to make the first offer and the right to choose the renegotiation period, the investor's incentive to raise investment can be sufficient to attenuate or to completely resolve the hold-up problem, even if the investor is not a residual claimant with certainty.

From the modeling perspective, this paper is most closely related to works by

Chiu (1998) and DeMeza and Lockwood (1998). As in these related works, I use Rubinstein's alternating-offers bargaining game to model the ex post renegotiation (Osborne and Rubinstein (1990) and Binmore et. al. (1989)). In contrast to Chiu (1998) and DeMeza and Lockwood (1998), I do not rely upon ownership rights to resolve the hold-up problem. [3] Instead, I introduce the production delay cost in the bargaining game and exploit the adjustment in the distribution of bargaining outcomes arising from changes in investment.

There is a vast body of literature on the social optimality of specific investment. Grossman and Hart (1986) suggest a solution to the hold-up problem, in which the investor owns the means of production. In their setup the hold-up problem is resolved, because the investor gains residual control rights through allocation of ownership rights. Aghion, Dewatripont, and Rey (1994) propose a solution where contracting parties become residual claimants with certainty by means of a renegotiable contract specifying a "financial hostage" for the buyer and a breach remedy for the seller.[4] Nöldeke and Schmidt (1995) employ an option contract involving a third-party verifiability of the delivery decision, while Edlin and Reichelstein (1996) introduce a contract specifying breach remedies or expectation damages contingent on specific performance.

Several studies employ dynamic solutions to the hold-up problem. Che and Sákovics (2004) find that the allocation of bargaining rights loses its importance in a dynamic setting. If contracting parties are willing to participate in a relationship and they are sufficiently patient, the investment dynamics alone are sufficient to induce optimal specific investment. However, this result holds only in the infinite-time setting. In the finite-time setting, the investor under-invests. Guriev and

[3] Chiu (1998) and DeMeza and Lockwood (1998) resolve the hold-up problem by adjusting ownership rights to make the non-investor's outside option bind with probability 1.

[4] As I show in this paper, if parties can set the timing of ex post renegotiation, the investor's position as a residual claimant is sufficient but is not necessary for the resolution of the hold-up problem. Although the assignment of the right to choose the renegotiation timing to the investor can be viewed as a form of a financial hostage paid by the non-investor, the investor can invest optimally even if she is not a residual claimant, or if she does not have the right to set the renegotiation timing.

Kvasov (2005) propose a solution to the hold-up problem in the finite-time setting. The authors use the Nash folk theorem for finitely repeated games and find that a fixed-term contract of a sufficient length can induce the investor to undertake a socially optimal specific investment.

In the next section I present the model and equilibrium strategies in the alternating-offers bargaining game with the production delay cost. In Section 2, I solve for an equilibrium investment decision, and in Proposition 2, I present conditions when equilibrium investment is ex ante social optimal, sub-optimal, or exceeds the socially optimal level. In Section 3, I study the investor's and the non-investor's equilibrium renegotiation timing choices for the case, when the investor is the first mover. In Section 4, I present a numerical example, where contracting parties can choose from a menu of two renegotiation periods. In Section 5, I discuss results, and in the last section I conclude. In the appendix, I present several extensions including the one, where the non-investor is the first mover.

1 Model

In this section I present the static perfect information model of one-sided specific investment and the production delay cost. Two risk-neutral parties (an investor (I) and a non-investor (N)) engage in trade over a good whose production involves specific investment. At date $t = 0$ the parties sign an agreement, which specifies the time $t \in \{2, .., T\}$ of the ex post renegotiation. Further, at time $t = 0$ the investor makes the investment decision σ, $\sigma \in [0, \Sigma]$, at the convex investment cost $c(\sigma)$. Investment σ raises the investor's linear production cost of the tradable good, $s(\sigma) = s\sigma$, and the non-investor's linear valuation of the tradable good, $b(\sigma) = b\sigma$, where $b > s$. [5] Since the investor's investment choice is irreversible, I assume that after σ is determined at $t = 0$, neither the investment σ nor the investment cost $c(\sigma)$ can change in any subsequent period of time.

[5] I discuss the case with non-linear production cost and valuation functions in the appendix.

At time $t = 1$, the parties learn the realization of the non-investor's outside option, $\underline{v} \in [L, V]$, $0 \leq L < V$. To capture the assumption that the investment is completely sunk, I assume that the investor's outside option is always zero. After the arrival of $\underline{v}$, the parties proceed to the renegotiation stage in the scheduled period $t \in \{2, .., T\}$.

In the renegotiation stage, the non-investor can accept the outside option $\underline{v}$ or obtain a share of the trade surplus $w(\sigma) = b(\sigma) - s(\sigma) = (b - s)\sigma$, which is a difference between the non-investor's valuation $b(\sigma)$ and the investor's production cost $s(\sigma)$. If there is a one period renegotiation delay, then depending on $t \in \{2, .., T\}$, the investor's production cost goes up by $\alpha_t s(\sigma)$ and the remaining trade surplus and the outside option shrink by a factor of $\delta \in (0, 1)$. Hence, with one period of delay, the non-investor can accept the depreciated outside option $\delta\underline{v}$ or bargain over a division of the trade surplus of size $\delta(w(\sigma) - \alpha_t s(\sigma))$.

Since $\alpha_t s(\sigma)$ and $w(\sigma)$ are linear functions of σ, I can rewrite the expression $\delta(w(\sigma) - \alpha_t s(\sigma))$ as $(\delta - \rho_t)w(\sigma)$, where $\rho_t = \alpha_t \delta \frac{s}{b-s}$ captures the production delay cost. [6] I assume that for any $t \in \{1, .., T\}$, we have $\rho_t \leq \rho_{t+1}$, $\rho_t \in [0, \delta]$, or equivalently, $a_t \leq a_{t+1}$, $a_t \in [0, \frac{b-s}{s}]$. In other words, I assume that the production delay cost parameter ρ_t is non-decreasing in time and is bounded above by the common discount factor $\delta \in (0, 1)$.

If the investor and the non-investor agree on the division of the trade surplus, the investor completes the investment project, produces the tradable good of quantity $q = 1$, delivers it to the non-investor, and splits the trade surplus with the non-investor according to the negotiated division. I present the timing of the model in the figure below.

[6] With linear trade surplus, the production delay cost parameter ρ_t does not depend on σ. If we assume that the trade surplus is concave, then the production delay cost parameter $\rho_t(\sigma) = \alpha_t \delta \frac{s(\sigma)}{w(\sigma)}$ depends on σ. The results of the model hold for the case with concave trade surplus, although the exposition becomes more complicated. I present an extension of the model with concave trade surplus in the appendix.

$$\underset{t=0}{\begin{bmatrix}\text{Investor sets } \sigma; \\ \text{reneg. period} \\ t \in \{2, .., T\} \\ \text{is chosen;}\end{bmatrix}} \rightarrow \underset{t=1}{\begin{bmatrix}\text{Outside} \\ \text{option} \\ \underline{v} \\ \text{arrives}\end{bmatrix}} \rightarrow \underset{t \in \{2,..,T\}}{\begin{bmatrix}\text{Ex post} \\ \text{renego-} \\ \text{tiation} \\ \text{with } \rho_t\end{bmatrix}} \rightarrow \underset{t=T+1}{\begin{bmatrix} . \\ \text{Trade} \\ \text{is} \\ \text{finalized}\end{bmatrix}}$$

Figure 1. Timing of the model.

I assume that the trade surplus $w(\sigma)$ always exceeds the cost of investment $c(\sigma)$ (i.e. $w(\sigma) > c(\sigma)$), so that it is always socially optimal to invest. In addition, since $b > s$, the trade is always efficient. Lastly, I assume that all parameters of the model are fully observable and verifiable, and the timing of the ex post renegotiation is contractible.

1.1 Ex Post Renegotiation

In the ex post renegotiation stage the parties engage in the alternating-offers bargaining game with a common discount factor $\delta \in (0, 1)$ and the production delay cost parameter $\rho_t \in [0, \delta]$. Following Rubinstein (1982), I use the concept of a subgame perfect Nash equilibrium (SPNE). In this section, I derive SPNE strategies for the case when $\rho_t \in [0, \delta)$. I consider the case with $\rho_t = \delta$ in Section 2.

The bargaining game in this section coincides with the alternating-offers bargaining game, where the investor moves first, the outside option depreciates at rate δ, and the trade surplus depreciates at rate $\delta' = \delta - \rho_t$, $\delta' \leq \delta$.[7] The procedure of the bargaining game is standard. Prior to the investor's offer, the parties learn the non-investor's outside option $\underline{v} \in [L, V]$, where $0 \leq L < \frac{(\delta - \rho_t) w(\sigma)}{1 + \delta - \rho_t} < V < w(\sigma)$.[8] After learning the non-investor's outside option, the investor proposes a division

[7] See Osborne and Rubinstein (1990) and Binmore et. al. (1989) for an analysis of the game with $\delta' = \delta$.

[8] I ignore the case when $w(\sigma) < V$, since the presence of the case does no affect the resolution of the hold-up problem in the model. To see this note that when $w(\sigma) < V$, the non-investor takes the outside option and terminates the relationship. Although the non-investor obtains no positive payoff in this case, the investor is still a residual claimant of the specific investment with a socially optimal incentive to invest.

$x = (x_I, x_N)$ to the non-investor, where x_I is the investor's share of $w(\sigma)$ and x_N is the non-investor's share of $w(\sigma)$.[9] If the non-investor doesn't accept the investor's proposal, the non-investor can either take the outside option $\underline{v}$ or propose a new division $y = (y_I, y_N)$, where y_I is the investor's share of $w(\sigma)$ and y_N is the non-investor's share of $w(\sigma)$. In the latter case, the trade surplus $w(\sigma)$ depreciates at rate $\delta - \rho_t$ and the outside option depreciates at rate δ. The game continues until either the investor or the non-investor accepts an offer or until the non-investor takes the outside option. I present SPNE strategies of the bargaining game in Proposition 1.

Proposition 1. *Let $\rho_t \in [0, \delta)$, then there are three cases:*

Case 1. If $\underline{v} \in [L, \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t})$, the investor's unique SPNE strategy is to offer a division $x = (\frac{w(\sigma)}{1+\delta-\rho_t}, \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t})$ and to accept any division y with $y_I \geq \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}$. The non-investor's unique SPNE strategy is to offer a division $y = (\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}, \frac{w(\sigma)}{1+\delta-\rho_t})$, to accept any division x with $x_N \geq \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}$, and never take outside option $\underline{v}$.

Case 2. If $\underline{v} \in [\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}, \frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2})$, the investor's unique SPNE strategy is to offer a division $x = (\frac{w(\sigma)}{1+\delta-\rho_t} - \frac{\rho_t \underline{v}}{1-(\delta-\rho_t)^2}, \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} + \frac{\rho_t \underline{v}}{1-(\delta-\rho_t)^2})$ and to accept any division y with $y_I \geq \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} - \frac{(\delta-\rho_t)\rho_t \underline{v}}{1-(\delta-\rho_t)^2}$. The non-investor's unique SPNE strategy is to offer a division $y = (\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} - \frac{(\delta-\rho_t)\rho_t \underline{v}}{1-(\delta-\rho_t)^2}, \frac{w(\sigma)}{1+\delta-\rho_t} + \frac{(\delta-\rho_t)\rho_t \underline{v}}{1-(\delta-\rho_t)^2})$, to accept any division x with $x_N \geq \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} + \frac{\rho_t \underline{v}}{1-(\delta-\rho_t)^2}$, and never take outside option $\underline{v}$.

Case 3. If $\underline{v} \in [\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}, V]$, the investor's unique SPNE strategy is to offer a division $x = (w(\sigma) - \underline{v}, \underline{v})$ and to accept any division y with $y_I \geq (\delta - \rho_t)(w(\sigma) - \underline{v})$. The non-investor's unique SPNE strategy is to offer a division $y = ((\delta - \rho_t)(w(\sigma) - \underline{v}), w(\sigma) - (\delta - \rho_t)(w(\sigma) - \underline{v}))$, to accept any division x with $x_N \geq \underline{v}$, and take outside option $\underline{v}$ if $x_N < \underline{v}$.

Proof: see appendix.

By Proposition 1, there is no renegotiation delay, and the non-investor imme-

[9] I assume that the investor is the first to propose a division. I consider the case where the non-investor is the first mover in the appendix.

diately accepts the investor's offer $x = (x_I, x_N)$ for any value of the non-investor's outside option $\underline{v}$. In Case 1, $\underline{v}$ is non-binding (less than the non-investor's payoff in a bargaining game without his outside option), the non-investor's threat of taking $\underline{v}$ is not credible, and the parties bargain as if no outside option exists. In particular, when $\underline{v} < \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}$, where $\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}$ is the non-investor's payoff in the game without an outside option, the non-investor immediately accepts the investor's offer $x = (\frac{w(\sigma)}{1+\delta-\rho_t}, \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t})$. Since the investor is the first mover and the non-investor is the second mover, the investor obtains a larger share of $w(\sigma)$ while the non-investor obtains a smaller share of $w(\sigma)$. This is a standard outcome in the alternating-offers bargaining game, where the trade surplus depreciates at rate $\delta - \rho_t$ and the investor moves first (see Rubinstein (1982)).

In Case 2, the investor's outside option $\underline{v}$ is binding (exceeds $\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}$) but is close enough to being non-binding (bounded above by $\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}$). As a result, the non-investor is able to extract an additional payoff above $\underline{v}$. To see this, note that the investor's offer can be rearranged in the following way: $x = (\frac{w(\sigma)}{1+\delta-\rho_t} - \frac{\rho_t\underline{v}}{1-(\delta-\rho_t)^2}, \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} + \frac{\rho_t\underline{v}}{1-(\delta-\rho_t)^2}) = (\frac{w(\sigma)-\underline{v}}{1+\delta-\rho_t} + \frac{(1-\delta)\underline{v}}{1-(\delta-\rho_t)^2}, \underline{v} + \frac{(\delta-\rho_t)(w(\sigma)-\underline{v})}{1+\delta-\rho_t} - \frac{(1-\delta)\underline{v}}{1-(\delta-\rho_t)^2})$, and the non-investor's equilibrium payoff $x_N = \underline{v} + \frac{(\delta-\rho_t)(w(\sigma)-\underline{v})}{1+\delta-\rho_t} - \frac{(1-\delta)\underline{v}}{1-(\delta-\rho_t)^2}$ strictly exceeds $\underline{v}$ for $\underline{v} \in [\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}, \frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2})$.

To see why the non-investor's payoff in Case 2 exceeds $\underline{v}$, consider the non-investor's rearranged payoff $\underline{v} + \frac{(\delta-\rho_t)(w(\sigma)-\underline{v})}{1+\delta-\rho_t} - \frac{(1-\delta)\underline{v}}{1-(\delta-\rho_t)^2}$. Since the non-investor's outside option is binding, in equilibrium the non-investor obtains at least $\underline{v}$ (the first term in the rearranged payoff). As a result, the parties bargain over the difference between the trade surplus and the outside option, $w(\sigma) - \underline{v}$. Given that the trade surplus depreciates at rate $\delta - \rho_t$ and the investor is the first mover, the equilibrium division is $(\frac{w(\sigma)-\underline{v}}{1+\delta-\rho_t}, \frac{(\delta-\rho_t)(w(\sigma)-\underline{v})}{1+\delta-\rho_t})$, where the non-investor obtains $\frac{(\delta-\rho_t)(w(\sigma)-\underline{v})}{1+\delta-\rho_t}$ (the second term in the rearranged payoff). Because the non-investor is the second mover and is willing to continue bargaining rather than to take his outside option immediately, the investor extracts some payoff proportional to the share of the outside option lost with one period of delay $\frac{(1-\delta)\underline{v}}{1-(\delta-\rho_t)^2}$ (the third term in the rearranged payoff).

If the outside option doesn't not depreciate ($\delta = 1$), then the investor cannot extract the payoff proportional to the share of the outside option lost with one period of delay, and the third term in the rearranged payoff is zero. As a result, when $\delta = 1$, the non-investor's payoff is a sum of his outside option and a share of the difference between the trade surplus and the outside option obtained in a bargaining game, where the trade surplus depreciates at rate $1 - \rho_t$, $\underline{v} + \frac{(1-\rho_t)(w(\sigma)-\underline{v})}{2-\rho_t}$.

If the non-investor's outside option is sufficiently high, so that $\underline{v}$ exceeds the non-investor's equilibrium payoff in Case 2, then the non-investor prefers to take the outside option immediately rather than to bargain over the difference between the trade surplus and the outside option. In Case 3, when $\underline{v} \geq \underline{v} + \frac{(\delta-\rho_t)(w(\sigma)-\underline{v})}{1+\delta-\rho_t} - \frac{(1-\delta)\underline{v}}{1-(\delta-\rho_t)^2}$ or, $\underline{v} \geq \frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}$, the investor offers to the investor an equivalent of his outside option $\underline{v}$, and the non-investor immediately accepts the offer.

Whether the players employ strategies in Case 2 depends on the size of the non-investor's outside option. If the outside option is very high, then for the non-investor the loss from rejecting the outside option (in terms of the lost value with one period of delay) exceeds the gain from continuing bargaining over the difference between the trade surplus and the outside option, and the non-investor obtains an equivalent of his outside option. If the outside option is binding but is sufficiently small, the non-investor prefers to bargain over the difference between the trade surplus and his outside option rather than to take an equivalent of his outside option immediately. The investor foresees this outcome and compensates the non-investor with an extra payoff above $\underline{v}$, so that there is no inefficient delay. To see this point, let's consider the case when the outside option doesn't not depreciate ($\delta = 1$) and the non-investor does not face any loss from postponing the acceptance of $\underline{v}$. Then, the non-investor prefers to continue bargaining over a positive difference between the trade surplus and his outside option for any binding outside option below $w(\sigma)$, and the strategies in Case 2 are SPNE for any $\underline{v} \in [\frac{(1-\rho_t)w(\sigma)}{2-\rho_t}, w(\sigma))$.

Lastly, observe that for $\rho_t > 0$, the interval $[\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}, \frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2})$ is non-empty, so that Case 2 always exists. However, when $\rho_t = 0$, the interval in Case 2 no longer exists, and Proposition 1 reduces to the outside option principle of Binmore et. al. (1989) (Cases 1 and 3 in Proposition 1).

1.2 Expected Payoffs

To obtain investor's and the non-investor's expected payoffs, I introduce the distribution function of the non-investor's outside option $F_{\underline{V}}(.)$. I define the non-investor's outside option as a random variable $\underline{V}$ with the distribution function $F_{\underline{V}}(.)$, continuous density function $f_{\underline{V}}(.) = F'_{\underline{V}}(.)$, support $[L, V]$, and a realized value $\underline{v}$. Since investment σ is relationship-specific, σ does not affect the distribution function of the non-investor's outside option $F_{\underline{V}}(.)$.

By Proposition 1, I obtain the following distribution of expected equilibrium payoffs. With probability $\lambda_1(\sigma, \rho_t) = F_{\underline{V}}(L \leq \underline{V} \leq \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t})$, the parties' expected payoffs are $(\frac{w(\sigma)}{1+\delta-\rho_t}, \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t})$. With probability $\lambda_2(\sigma, \rho_t) = F_{\underline{V}}(\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} \leq \underline{V} \leq \frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2})$, the parties' expected payoffs are

$$\begin{aligned}&(\tfrac{w(\sigma)}{1+\delta-\rho_t} - \tfrac{\rho_t}{1-(\delta-\rho_t)^2}E[\underline{V}|\underline{V} \in [\tfrac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}, \tfrac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}]]),\\ &\tfrac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} + \tfrac{\rho_t}{1-(\delta-\rho_t)^2}E[\underline{V}|\underline{V} \in [\tfrac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}, \tfrac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}]]).\end{aligned}$$

And with probability $\lambda_3(\sigma, \rho_t) = F_{\underline{V}}(\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2} \leq \underline{V} \leq V)$, the parties' expected payoffs are

$$\begin{aligned}&(w(\sigma) - E[\underline{V}|\underline{V} \in [\tfrac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}, V],\\ &E[\underline{V}|\underline{V} \in [\tfrac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}, V]).\end{aligned}$$

The probabilities $\langle\lambda_1(\sigma, \rho_t), \lambda_2(\sigma, \rho_t), \lambda_3(\sigma, \rho_t)\rangle$, where $\sum_{i=1}^{3}\lambda_i(\sigma, \rho_t) = 1$, depend on the cdf of the outside option $F_{\underline{V}}(.)$, discount rate δ, production delay cost parameter ρ_t and the trade surplus $w(\sigma)$. It is easy to see that an increase in investment raises the probability $\lambda_1(\sigma, \rho_t) = F_{\underline{V}}(L \leq \underline{V} \leq \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t})$, $\frac{\partial\lambda_1(\sigma,\rho_t)}{\partial\sigma} > 0$, while an increase in ρ_t lowers the probability $\lambda_1(\sigma, \rho_t)$, $\frac{\partial\lambda_1(\sigma,\rho_t)}{\partial\rho_t} < 0$.

Lastly, I impose the concavity condition on the cdf $F_{\underline{V}}(.)$ and functions $w(\sigma)$ and $c(\sigma)$. This condition together with the linearity of $w(\sigma)$ and the convexity of $c(\sigma)$ guarantees that the investor's objective function is globally concave with respect to σ:

$$\frac{\rho_t(\delta-\rho_t)w(\sigma)}{(1-\delta+\rho_t)(1+\delta-\rho_t)^2}\frac{\partial\lambda_1^2(\sigma,\rho_t)}{\partial\sigma\partial\sigma}+\frac{(\delta-\rho_t)w'(\sigma)}{1+\delta-\rho_t}\left(\frac{\rho_t\frac{\partial\lambda_1(\sigma,\rho_t)}{\partial\sigma}}{1-(\delta-\rho_t)^2}+\lambda_3(\sigma,\rho_t)\right)< \; c''(\sigma), \forall\sigma.$$

The right-hand side of the condition is positive, because $c''(\sigma) > 0$. The sign of the left-hand side is indeterminate, because the sign of $\frac{\partial\lambda_1^2(\sigma,\rho_t)}{\partial\sigma\partial\sigma}$ is indeterminate and the magnitude of the terms is unknown. I do not impose any other restrictions on the distribution function $F_{\underline{V}}(.)$ beyond those implied by the concavity condition.

2 Equilibrium Specific Investment

The investor's equilibrium investment choice maximizes her expected payoff from the relationship with the non-investor. The investor's expected payoff, EU_I, is the difference between the expected negotiation payoff and the cost of specific investment $c(\sigma)$.

$$\begin{aligned}EU_I(\sigma,\rho_t) &= -c(\sigma)+\lambda_1(\sigma,\rho_t)\frac{w(\sigma)}{1+\delta-\rho_t}+\\ &\lambda_2(\sigma,\rho_t)\left(\frac{w(\sigma)}{1+\delta-\rho_t}-\frac{\rho_t}{1-(\delta-\rho_t)^2}E[\underline{V}|\underline{V}\in[\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t},\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}]]\right)+\\ &\lambda_3(\sigma,\rho_t)\left(w(\sigma)-E[\underline{V}|\underline{V}\in[\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2},V]]\right)\end{aligned}$$

Given that EU_I is globally concave by the concavity condition, the equilibrium specific investment $\sigma_t^* = \sigma^*(\rho_t) = \underset{\sigma\geq 0}{argmax} EU_I$ is a unique maximizer of EU_I. Hence, the equilibrium specific investments σ_t^* is completely characterized by the following first-order maximization condition (see appendix for derivation):

$$(1)\; w'(\sigma)+\left[\frac{\partial\lambda_1(\sigma,\rho_t)}{\partial\sigma}\frac{\rho_t}{1-(\delta-\rho_t)^2}\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}-(1-\lambda_3(\sigma,\rho_t))\frac{(\delta-\rho_t)w'(\sigma)}{1+\delta-\rho_t}\right]=c'(\sigma).$$

The first-order optimization condition equalizes the investor's expected marginal cost and benefit. The equilibrium investment σ_t^* is ex ante socially optimal, if it satisfies the social optimality condition $w'(\sigma_t^*) = c'(\sigma_t^*)$. This is possible to achieve, if the terms in square brackets are equal to zero. Hence, the necessary and sufficient condition for the ex ante social optimality of equilibrium investment can be expressed in the following way:

$$(2)\ \frac{\partial\lambda_1(\sigma_t^*,\rho_t)}{\partial\sigma}\frac{\rho_t}{1-(\delta-\rho_t)^2}\frac{(\delta-\rho_t)w(\sigma_t^*)}{1+\delta-\rho_t} = (1-\lambda_3(\sigma_t^*,\rho_t))\frac{(\delta-\rho_t)w'(\sigma_t^*)}{1+\delta-\rho_t}.$$

The left-hand side of condition (2) is the investor's incentive to increase investment. This incentive appears, because an increase in investment raises the probability that the non-investor's outside option is non-binding, $\frac{\partial\lambda_1(\sigma,\rho_t)}{\partial\sigma} > 0$, in which case the investor is able to retain the difference between her payoffs when the non-investor's outside option is binding $(\frac{w(\sigma)}{1+\delta-\rho_t} - \frac{\rho_t}{1-(\delta-\rho_t)^2}E[\underline{V}|\underline{V} \in [\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}, \frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}])$ and non-binding $(\frac{w(\sigma)}{1+\delta-\rho_t})$. The size of this difference at the margin is $\frac{\rho_t}{1-(\delta-\rho_t)^2}E[\underline{V}|\underline{V} = \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}] = \frac{\rho_t}{1-(\delta-\rho_t)^2}\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}$. As a result, the total magnitude of the incentive to increase investment amounts to $\frac{\partial\lambda_1(\sigma_t,\rho_t)}{\partial\sigma}\frac{\rho_t}{1-(\delta-\rho_t)^2}\frac{(\delta-\rho_t)w(\sigma_t)}{1+\delta-\rho_t}$.

The right-hand side represents the investor's incentive to lower investment, because with probability $1-\lambda_3(\sigma,\rho_t)$ the investor is not a residual claimant and, therefore, gives up the $\frac{\delta-\rho_t}{1+\delta-\rho_t}$ share of the increase in the trade surplus, $w'(\sigma)$, to the non-investor. In expectation, the size of this disincentive is equal to $(1-\lambda_3(\sigma_t,\rho_t))\frac{(\delta-\rho_t)w'(\sigma_t)}{1+\delta-\rho_t}$.

When these two opposing incentives are equal, the investor's equilibrium investment is ex ante socially optimal. However, when the left-hand side of condition (2) exceeds the right-hand side, the investor ex ante over-invests in equilibrium, and when the left-hand side of condition (2) is below the right-hand side, the investor ex ante under-invests in equilibrium.

Let's consider the investor's equilibrium investment when $\rho_t = 0$ and $\rho_t = \delta$. When $\rho_t = 0$, the investor's expected payoff reduces to $EU_I(\sigma,0) = \lambda_1(\sigma,0)\frac{w(\sigma)}{1+\delta} + (1-\lambda_1(\sigma,0))(w(\sigma) - E[\underline{V}|\underline{V} \in [\frac{\delta w(\sigma)}{1+\delta}, V]]) - c(\sigma)$. The maximization of $EU_I(\sigma,0)$ with respect to σ gives the following equilibrium condition: $w'(\sigma) - \lambda_1(\sigma,0)\frac{\delta w'(\sigma)}{1+\delta} = c'(\sigma)$. Note than no equilibrium investment σ_t^* can simultaneously satisfy this equilibrium condition and the social optimality condition, $w'(\sigma) = c'(\sigma)$, for any $\lambda_1(\sigma,0) > 0$, which implies that when $\rho_t = 0$, the investor ex ante under-invests in equilibrium.

When $\rho_t = \delta$, the investor is a residual claimant with certainty, and her expected payoff is $EU_i(\sigma, \delta) = w(\sigma) - E[\underline{V}] - c(\sigma)$. Further, the investor's equilibrium investment $\sigma_t^* = \arg\max EU_i(\sigma, \delta)$ coincides with the solution to the social optimality condition, $w'(\sigma) = c'(\sigma)$. Consequently, when $\rho_t = \delta$, the investor's equilibrium investment is ex ante socially optimal. We obtain this result, because when $\rho_t = \delta$, the trade surplus completely depreciates with one period of delay, and the investor as the first mover offers to the non-investor an equivalent of his outside option. The non-investor immediately accepts the investor's offer, because if he continues bargaining, his payoff from splitting the trade surplus with the investor is at most zero. As a result, the non-investor accepts an equivalent of his outside option, the investor becomes a residual claimant with certainty and invests optimally.

Unless $\rho_t = \delta$, the right-hand side of condition (2) is non-zero, and the disincentive to invest is always present. The relevant question is whether there is a sufficiently strong investment incentive to compensate for this disincentive when $\rho_t < \delta$. The answer is clear for the case with $\rho_t = 0$: the incentive to invest is always below the socially optimal level. However, as condition (2) shows, the answer is not clear for the case with $\rho_t \in (0, \delta)$. Depending on parameter values, the equilibrium investment level can be socially optimal, sub-optimal, or exceed the socially optimal level. I solve condition (2) for the socially optimal $\widehat{\rho}(\sigma_t^*)$ and define the range of parameters leading to different investment outcomes in Proposition 2.

Proposition 2.

Let $\alpha(\sigma_t^*) = \frac{\partial \lambda_1(\sigma_t^*, \rho_t)}{\partial \sigma} \frac{1}{1-\lambda_3(\sigma_t^*, \rho_t)} \frac{w(\sigma_t^*)}{w'(\sigma_t^*)}$, $\widehat{\rho}(\sigma_t^*) = \frac{\sqrt{\alpha(\sigma_t^*)^2 - 4\alpha(\sigma_t^*)\delta + 4} - (\alpha(\sigma_t^*) - 2\delta)}{2}$, *and* $\widehat{\rho}(\sigma_t^*) \in (0, \delta)$.

a. For any t, such that $\rho_t = 0$ *or condition (2) is a negative inequality, the equilibrium investment* σ_t^* *is ex ante socially sub-optimal.*

b. For any t, such that $\rho_t = \delta$ *or* $\rho_t = \widehat{\rho}(\sigma_t^*)$*, the equilibrium investment* σ_t^* *is* *ex ante socially optimal.*

c. For any t, such that condition (2) is a positive inequality, the equilibrium investment σ_t^* *exceeds the ex ante socially optimal level.*

Note 1: The value $\widehat{\rho}(\sigma_t^*)$ is strictly positive, since the square root term in $\widehat{\rho}(\sigma_t^*)$, $\sqrt{\alpha(\sigma_t^*)^2 - 4\alpha(\sigma_t^*)\delta + 4}$, strictly exceeds $\alpha(\sigma_t^*) - 2\delta$ for any $\delta < 1$. For this reason we can also discard the second solution with the negative square root term.

Note 2: The assumption that ρ_t is increasing in time does not imply that the assignment of the ex post renegotiation in all time periods after t results in the ex ante over-investment and the assignment of the ex post renegotiation in all time periods prior to period t results in the ex ante under-investment. I obtain this result, because by the implicit function theorem, the sign of the partial derivative of the ex ante equilibrium investment with respect to ρ_t is indeterminate, $\frac{d\sigma_t^*}{d\rho_t} \lesseqgtr 0$.

3 Equilibrium Timing of Renegotiation

The investor's equilibrium investment is socially optimal, if a social planner can specify and enforce the renegotiation timing leading to the socially optimal investment. The role of a social planner in this context is usually given to a court. More specifically, when the investor and the non-investor sign an initial incomplete contract at time $t = 0$, the court can contractually specify the ex post renegotiation timing leading to the socially optimal specific investment. However, as Posner (1999) argues, courts in general have very limited capability to assess and process circumstances in which contracting parties operate. As a result, it is reasonable to assume that no court can calculate and enforce the renegotiation timing leading to the socially optimal investment.

According to Macneil (1978), often parties themselves specify the timing of ex post renegotiation, and include it as a clause in a contract. In this context, the court's role reduces to a more realistic task of enforcement of the renegotiation timing clause. In this section I study unilateral timing choices of the investor and the non-investor under the assumption that the investor, as before, is the first mover in the bargaining game. I discuss timing choices under the assumption that the non-investor is the first mover in the appendix.

Let's consider a scenario, where the investor unilaterally decides when to renegotiate. The investor's problem in period $t = 0$ amounts to finding a solution $(\sigma^*_{t_I}, t^*_I)$, which maximizes the investor's expected utility function EU_I. The investor's equilibrium investment choice is characterized by condition (1). The investor's equilibrium timing choice is characterized by boundary restrictions and a solution to condition (3).

$$(3)\ (1-\lambda_3(\sigma,\rho_t))\frac{w(\sigma)}{(1+\delta-\rho_t)^2} = -\frac{\partial\lambda_1(\sigma,\rho_t)}{\partial\rho_t}\frac{\rho_t}{1-(\delta-\rho_t)^2}\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}+ \lambda_2(\sigma,\rho_t)\frac{1-\delta^2+\rho_t^2}{(1-(\delta-\rho_t)^2)^2}E[\underline{V}|\underline{V}\in[\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t},\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}]])$$

I obtain condition (3) by optimizing the investor's expected payoff EU_I with respect to ρ_t. The left-hand side of condition (3) captures the investor's marginal benefit from acting as the first mover. The investor obtains this marginal benefit, because when the outside option is non-binding, the investor's share of the trade surplus $\frac{w(\sigma)}{1+\delta-\rho_t}$ increases in ρ_t. This happens because with higher ρ_t and smaller $\delta-\rho_t$, the trade surplus loses more value, when it is the non-investor's turn to offer a division of the trade surplus. As a result, when the investor offers an initial division, the non-investor is willing to accept a lower share of the trade surplus.

The right-hand side of condition (3) captures the investor's marginal cost from an increase in ρ_t. Since $\frac{\partial\lambda_1(\sigma,\rho_t)}{\partial\rho_t} < 0$, the right-hand side is always positive. The first marginal cost term captures the investor's marginal cost from the drop in the probability $\lambda_1(\sigma,\rho_t)$ and an associated payoff reduction by $\frac{\rho_t}{1-(\delta-\rho_t)^2}\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}$. The second marginal cost term captures the marginal cost from an increase in $\frac{\rho_t}{1-(\delta-\rho_t)^2}$ occurring with probability $\lambda_2(\sigma,\rho_t)$.

Condition (3) does not imply that the investor's net marginal benefit from an increase in ρ_t is strictly positive. It is straightforward to show that the second marginal cost term is strictly less than the marginal benefit term. However, the size of the first marginal cost term cannot be specified, since the size of $\frac{\partial\lambda_1(\sigma,\rho_t)}{\partial\rho_t}$

depends on the assumption about the distribution function of the outside option. As a result, it is impossible to define all ρ_t's, when the marginal cost side of condition (3) exceeds the marginal benefit side without additional assumptions.

Further, without assuming that the investor's expected payoff EU_I is concave with respect to ρ_t, it is impossible to argue that a solution to condition (3) maximizes EU_I. However, without making any concavity assumptions or restricting the range of parameters, for any $t_I^* \in [2, ...T]$ with $\rho_{t_I^*} \in [0, \delta]$, the investor always chooses t_I^* with $\rho_{t_I^*} = \delta$, and the investor's equilibrium investment is necessarily socially optimal. I summarize this result in Proposition 3.

Proposition 3.

For $\rho_t \in [0, \delta]$, the investor's equilibrium timing choice is t_I^* such that $\rho_{t_I^*} = \delta$, and the investor's equilibrium investment $\sigma^*_{t_I^*}$ is ex ante socially optimal.

Proof:

The investor's expected payoff $EU_I(\sigma, \rho_t)$ achieves the global maximum at $\rho_t = \delta$. To see this, note that the non-investor never accepts any investor's offer below his outside option. As a result, for any investment σ, $\sigma \in [0, \Sigma]$, the investor payoff can never exceed $w(\sigma) - E[\underline{V}] - c(\sigma) = EU_I(\sigma, \rho_t = \delta)$. Further, by Proposition 2, the investor's equilibrium investment $\sigma_{\rho_t = \delta}$ is socially optimal. ■

Proposition 3 suggests that the investor always prefers to renegotiate, when $\rho_t = \delta$ and the trade surplus completely depreciates with delay. When $\rho_t = \delta$ or $\delta - \rho_t = 0$, the investor's first-mover advantage achieves its maximum, and the investor is able to appropriate all the trade surplus above the non-investor's outside option. This happens, because when $\rho_t = \delta$, the non-investor's payoff, if he rejects the investor's offer and makes an offer with one period of delay, is zero. This gives the investor an opportunity to extract all the trade surplus above the investor's outside option for any level of specific investment. By Proposition 2, this makes the investor a residual claimant for all contingencies and necessarily makes the investor's equilibrium investment ex ante socially optimal.

In general, the range of renegotiation periods maybe constrained and not include some period s with $\rho_s = \delta$ as an option. In this case the investor's equilibrium timing choice is driven by boundary restrictions and condition (3). Nevertheless, without making any concavity assumptions, I conclude that if $\rho_t \in [0, \delta)$ the investor always prefers to renegotiate when $\rho_t \neq 0$. To see this, note that when $\rho_t = 0$, condition (3) reduces to the strictly positive inequality $\frac{w(\sigma)}{(1+\delta)^2} > 0$ for any $\sigma \in (0, \Sigma]$. As a result, the investor always has an incentive to choose to renegotiate in some period t_I^* with $\rho_{t_I^*} > 0$. Further, by Proposition 2, if $\rho_t < \delta$, then the investor's equilibrium investment can be socially optimal, sub-optimal, or exceed the socially optimal level. I state these results in Proposition 4.

Proposition 4.

For $\rho_t \in [0, \delta)$, the investor always chooses t_I^* such that $\rho_{t_I^*} > 0$, and the investor's equilibrium investment $\sigma_{t_I^*}^*$ can be ex ante socially optimal, sub-optimal, or exceed the social optimality level.

By Proposition 4, if the set of renegotiation periods is constrained and does not include a period with $\rho_t = \delta$, the investor's equilibrium investment can be socially optimal, sub-optimal, or exceed the socially optimal level. A more precise conclusion requires a set of assumptions about the shape of the expected payoff function. Nevertheless, without making any additional assumptions, Propositions 3 and 4 suggest that if the investor unilaterally decides when to renegotiate and is the first mover in the bargaining game, then her equilibrium investment does not necessarily result in under-investment. Moreover, if the investor is allowed to choose any ex post renegotiation period, then her equilibrium investment is socially optimal.

In the rest of the section I consider a scenario where the non-investor makes the renegotiation timing decision. The non-investor's equilibrium timing decision $t_N^* \in [2, ...T]$ with $\rho_{t_N^*} \in [0, \delta]$ maximizes his expected payoff $EU_N(\sigma_{\rho_t}, \rho_t)$ with respect to ρ_t conditional on the investor's equilibrium investment σ_{ρ_t}. The non-investor's expected payoff $EU_N(\sigma_{\rho_t}, \rho_t)$ is as follows:

$$EU_N(\sigma_{\rho_t}, \rho_t) = \lambda_1(\sigma_{\rho_t}, \rho_t)\frac{(\delta-\rho_t)w(\sigma_{\rho_t})}{1+\delta-\rho_t} +$$
$$\lambda_2(\sigma_{\rho_t}, \rho_t)(\frac{(\delta-\rho_t)w(\sigma_{\rho_t})}{1+\delta-\rho_t} + \frac{\rho_t}{1-(\delta-\rho_t)^2}E[\underline{V}|\underline{V} \in [\frac{(\delta-\rho_t)w(\sigma_{\rho_t})}{1+\delta-\rho_t}, \frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma_{\rho_t})}{1-\rho_t-(\delta-\rho_t)^2}]]) +$$
$$\lambda_3(\sigma_{\rho_t}, \rho_t)E[\underline{V}|\underline{V} \in [\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma_{\rho_t})}{1-\rho_t-(\delta-\rho_t)^2}, V]])$$

The optimization of $EU_N(\sigma_{\rho_t}, \rho_t)$ with respect to ρ_t gives condition (4):

$$(4)\ \lambda_2(\sigma, \rho_t)\frac{1-\delta^2+\rho_t^2}{(1-(\delta-\rho_t)^2)^2}E[\underline{V}|\underline{V} \in [\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}, \frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}]])$$
$$-\frac{\partial\lambda_1(\sigma,\rho_t)}{\partial\rho_t}\frac{\rho_t}{1-(\delta-\rho_t)^2}\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} = (1-\lambda_3(\sigma, \rho_t))\frac{w(\sigma)}{(1+\delta-\rho_t)^2}.$$

The interpretation of condition (4) is similar to the interpretation of condition (3). The left-hand side of condition (4), or the non-investor's marginal benefit side, is a sum of two terms, where the first term captures the marginal benefit from an increase in $\frac{\rho_t}{1-(\delta-\rho_t)^2}$ occurring with probability $\lambda_2(\sigma, \rho_t)$, and the second term captures the marginal benefit from an increase in payoff by $\frac{\rho_t}{1-(\delta-\rho_t)^2}\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}$ when the probability $\lambda_1(\sigma, \rho_t)$ falls in response to an increase in ρ_t. The right-hand side of condition (4), or the non-investor's marginal cost side, contains a single marginal cost term: the drop in the non-investor's marginal payoff given an increase in the investor's bargaining power as the first mover.

The non-investor's expected payoff $EU_N(\sigma_{\rho_t}, \rho_t)$ has a global minimum at $\rho_t = \delta$, where it takes the value of $E[V]$. As a result, the non-investor never chooses to renegotiate in any period s with $\rho_s = \delta$. Whether the non-investor prefers to renegotiate when $\rho_t = 0$ or when $0 < \rho_t < \delta$ depends on the shape of $EU_N(\sigma_{\rho_t}, \rho_t)$ and parameter restrictions. I summarize investment outcomes for this case in Proposition 5.

Proposition 5.

For $\rho_t \in [0, \delta]$, the non-investor chooses t_N^* such that $\rho_{t_N^*} < \delta$. If $\rho_{t_N^*} = 0$, then the investor ex ante under-invests in equilibrium. If $\rho_{t_N^*} > 0$, then the investor's equilibrium investment $\sigma_{t_N^*}^*$ can be ex ante socially optimal, sub-optimal, or exceed the socially optimal level.

Proof:

To see that the non-investor never chooses to renegotiate when $\rho_t = \delta$, note that $EU_N(\sigma_{\rho_t}, \rho_t = \delta) = E[V]$, which is the non-investor's expected outside payoff. Hence, any $\rho_t < \delta$ can weakly increase the non-investor's expected payoff.

Note that if $\rho_{t_N^*} = 0$, then the non-investor's expected payoff achieves a local maximum. To see this, observe that condition (4) is strictly negative at $\rho_{t_N^*} = 0$ for any investment σ, $\sigma \in [0, \Sigma]$. Further, by Case (a) of Proposition 2, if $\rho_{t_N^*} = 0$, the investor under-invests in equilibrium.

Other possible candidates for local maxima are ρ_t's satisfying condition (4). Whether these candidate points are global maxima, local maxima, or not maxima depends on the shape of $EU_N(\sigma_{\rho_t}, \rho_t)$. However, if the global maximum is achieved at $\rho_t \in (0, \delta)$, then by Proposition 2, the investor's equilibrium investment can be socially optimal, sub-optimal, or exceed the socially optimal level. ■

Proposition 5 suggests that the non-investor timing decision either leads to under-investment or to a range of outcomes, where ex ante social optimality is one of the outcomes. Similar to Proposition 4, it is impossible to make a more precise prediction about the investor's equilibrium investment without imposing additional assumptions on the shape of the non-investor's expected payoff function. However, while an allocation of the right to make an unconstrained timing decision to the investor necessarily leads to the social optimal investment, an allocation of the same right to the non-investor may result in under-investment.

4 Numerical Example

In this section I demonstrate results of previous sections in a numerical example. Following the illustration in the introduction, suppose that an investor constructs a plant of capacity σ. The plant capacity level is irreversible, and the plant has no value outside the relationship with the non-investor . The trade surplus from constructing the plant and trading with the non-investor is $w(\sigma) = 0.5\sigma$, while the cost of investment is $c(\sigma) = \frac{\sigma^2}{10}$. The ex post renegotiation follows an alternating-offers bargaining game, where the common discount factor is $\delta = 0.9$, the investor proposes first, and only the non-investor has an outside option.

In period $t = 0$ the investor chooses between two renegotiation periods: if the renegotiation begins in period $t = 2$, the investor faces no production delay cost ($\rho_{t=2} = 0$), and if the renegotiation begins in period $t = 3$, the investor loses $\rho_{t=3} w(\sigma)$, $\rho_{t=3} = 0.5$, with each round of renegotiation. In addition, in period $t = 0$ the investor decides on the capacity level σ. Next, in period $t = 1$ the non-investor's outside option arrives. With probability $\lambda(\sigma, \rho_t) = prob(\underline{V} \leq \frac{(\delta - \rho_t) w(\sigma)}{1+\delta-\rho_t})$ the non-investor's outside option $\underline{V}$ is non-binding, with probability $1 - \lambda(\sigma, \rho_t)$ the non-investor's outside option is binding.

$$\underset{t=0}{\begin{bmatrix} \text{Investor} \\ \text{chooses } \sigma \text{ and} \\ \text{reneg. period} \\ t \in \{2,3\} \end{bmatrix}} \rightarrow \underset{t=1}{\begin{bmatrix} \underline{V} \\ \text{arrives} \end{bmatrix}} \rightarrow \underset{t=2}{\begin{bmatrix} \text{Ex post} \\ \text{reneg. with} \\ \rho_{t=2} = 0 \end{bmatrix}} \text{ or } \underset{t=3}{\begin{bmatrix} \text{Ex post} \\ \text{reneg. with} \\ \rho_{t=3} = 0.5 \end{bmatrix}} \rightarrow \underset{t=4}{\begin{bmatrix} \text{Trade is} \\ \text{finalized} \end{bmatrix}}$$

Figure 2. Timing of the example

The ex ante socially optimal capacity level $\sigma^* = 2.5$ maximizes the difference between the trade surplus and the investment cost, $w'(\sigma) = c'(\sigma)$. The investor's equilibrium capacity level depends on the choice of the ex post renegotiation period $t \in \{2, 3\}$. Suppose that the ex post renegotiation takes place in period $t = 2$ with $\rho_{t=2} = 0$. Then, with probability $\lambda(\sigma, \rho_{t=2}) = prob(\underline{V} \leq \frac{\delta w(\sigma)}{1+\delta})$ the investor obtains $\frac{w(\sigma)}{1+\delta}$, while the non-investor obtains $\frac{\delta w(\sigma)}{1+\delta}$ (Case 1 of Proposition 1). With probability $1 - \lambda(\sigma, \rho_{t=2})$, the non-investor obtains his expected outside option $E[\underline{V} | \underline{V} \geq \frac{\delta w(\sigma)}{1+\delta}]$, and the investor obtains $w(\sigma) - E[\underline{V} | \underline{V} \geq \frac{\delta w(\sigma)}{1+\delta}]$ (Case 3 of Proposition 1). The investor's equilibrium capacity level $\sigma^*_{t=2}$ satisfies the following condition: $w'(\sigma) - \lambda(\sigma, \rho_{t=2}) \frac{\delta w'(\sigma)}{1+\delta} = c'(\sigma)$. It is easy to see that the investor's capacity choice is sub-optimal for any $\lambda(\sigma, \rho_{t=2}) > 0$, because with probability $\lambda(\sigma, \rho_{t=2})$ the investor is not a residual claimant and gives up the $\frac{\delta}{1+\delta}$ share of the increase in the trade surplus. The magnitude of the disincentive is captured by the negative term, $-\lambda(\sigma, \rho_{t=2})\frac{\delta w'(\sigma)}{1+\delta}$. To see this, suppose that $\lambda(\sigma^*_{t=2}, \rho_{t=2}) = 0.5$. Then, the investor's equilibrium capacity level is $\sigma^*_{t=2} = 1.9 < 2.5 = \sigma^*$.

Next, suppose that the ex post renegotiation takes place in period $t = 3$ with $\rho_{t=3} = 0.5$. By Proposition 1, with probability $\lambda(\sigma, \rho_{t=3})$ the investor obtains $\frac{w(\sigma)}{1+\delta-\rho_{t=3}}$, while the non-investor obtains $\frac{(\delta-\rho_{t=3})w(\sigma)}{1+\delta-\rho_{t=3}}$ (Case 1 of Proposition 1). If the outside option is binding, then with probability $1 - \lambda(\sigma, \rho_{t=3})$, the investor obtains $\frac{w(\sigma)}{1+\delta-\rho_{t=3}} - \frac{\rho_{t=3}}{1-(\delta-\rho_{t=3})^2}E[\underline{V}|\underline{V} \in [\frac{(\delta-\rho_{t=3})w(\sigma)}{1+\delta-\rho_{t=3}}, V]]$, while the non-investor obtains $\frac{(\delta-\rho_{t=3})w(\sigma)}{1+(\delta-\rho_{t=3})} + \frac{\rho_{t=3}}{1-(\delta-\rho_{t=3})^2}E[\underline{V}|\underline{V} \in [\frac{(\delta-\rho_{t=3})w(\sigma)}{1+\delta-\rho_{t=3}}, V]]$ (Case 2 of Proposition 1).[10] Note that the investor's payoff falls exactly by $\frac{\rho_{t=3}}{1-(\delta-\rho_{t=3})^2}E[\underline{V}|\underline{V} \in [\frac{(\delta-\rho_{t=3})w(\sigma)}{1+\delta-\rho_{t=3}}, V]]$ as the non-investor's outside option becomes binding.

The investor's equilibrium capacity level $\sigma^*_{t=3}$ satisfies the following condition: $w'(\sigma) + [\frac{\partial\lambda(\sigma,\rho_{t=3})}{\partial\sigma}\frac{\rho_{t=3}}{1-(\delta-\rho_{t=3})^2}\frac{(\delta-\rho_{t=3})w(\sigma)}{1+\delta-\rho_{t=3}} - \frac{(\delta-\rho_{t=3})w'(\sigma)}{1+\delta-\rho_{t=3}}] = c'(\sigma)$ (see derivation of condition (1)). If the increase in the probability of a non-binding outside option at the equilibrium capacity level, $\frac{\partial\lambda(\sigma,\rho_{t=3})}{\partial\sigma}|_{\sigma^*_{t=3}}$, is 0.672, then the equilibrium capacity level is $\sigma^*_{t=3} = 2.5 = \sigma^*$. Hence, if the ex post renegotiation takes place in period $t = 3$ with $\rho_{t=3} = 0.5$, the investor's equilibrium capacity level is ex ante socially optimal.

Next, let's see whether the investor prefers to renegotiate in period $t = 2$ or in period $t = 3$. Assume that probabilities of non-binding outside options in two cases are $\lambda(\sigma^*_{t=2}, \rho_{t=2}) = prob(\underline{V} \leq 0.45) = 0.5$ and $\lambda(\sigma^*_{t=3}, \rho_{t=3}) = prob(\underline{V} \leq 0.36) = 0.45$, the upper bound of the outside option is $V = 0.7$, and expected conditional outside options are $E[\underline{V}|\underline{V} \in [\frac{\delta w(\sigma^*_{t=2})}{1+\delta}, V]] = E[\underline{V}|\underline{V} \in [0.45, 0.7]] = \frac{0.7+0.45}{2} = 0.58$ and $E[\underline{V}|\underline{V} \in [\frac{(\delta-\rho_{t=3})w(\sigma^*_{t=3})}{1+\delta-\rho_{t=3}}, V]] = E[\underline{V}|\underline{V} \in [0.36, 0.7]] = \frac{0.7+0.36}{2} = 0.53$.

If the ex post renegotiation takes place in period $t = 2$, the investor expected payoff is $EU_I(\sigma^*_{t=2}, \rho_{t=2}) = 0.074$. Whereas, if the ex post renegotiation takes place in period $t = 3$, the investor expected payoff is $EU_I(\sigma^*_{t=3}, \rho_{t=3}) = 0.094$. Since the investor's expected payoff is higher in the latter case, the investor's decision to

[10] For simplicity, in this example I assume that the upper bound of the outside option V is less than $\frac{(\delta-\rho_{t=3})(1-\delta+\rho_{t=3})w(\sigma)}{1-\rho_{t=3}-(\delta-\rho_{t=3})^2}$. As a result, we have $\lambda(\sigma, \rho_t) = \lambda_1(\sigma, \rho_t)$, $1 - \lambda(\sigma, \rho_t) = \lambda_2(\sigma, \rho_t)$, and $\lambda_3(\sigma, \rho_t) = 0$.

renegotiate at $t = 3$ is an equilibrium, and the investor's equilibrium capacity choice is socially optimal.

Lastly, let's consider the non-investor's timing decision. If the ex post renegotiation takes place at $t = 2$, the non-investor's expected payoff is $EU_N(\sigma^*_{t=2}, \rho_{t=2}) = w(\sigma^*_{t=2}) - EU_I(\sigma^*_{t=2}, \rho_{t=2}) = 0.876$. If the ex post renegotiation takes place at $t = 3$, the non-investor's expected payoff is $EU_N(\sigma^*_{t=3}, \rho_{t=3}) = w(\sigma^*_{t=3}) - EU_I(\sigma^*_{t=3}, \rho_{t=3}) = 1.171$. As a result, given the freedom to choose the renegotiation timing, the non-investor also prefers to renegotiate at $t = 3$, in which case the investor makes the socially optimal investment decision.

This example shows that in equilibrium the investor invests optimally and also decides to renegotiate, when there is a positive cost of production delay. The positive cost of production delay improves the investor's bargaining position as the first mover and endows the investor with a sufficiently strong incentive to raise specific investment and achieve social optimality, even though the investor is not a residual claimant with certainty. Of course, if the investor had a choice to renegotiate in some period s with $\rho_{t=s} = \delta$, then by Proposition 3 the investor would prefer to renegotiate in period s, become a residual claimant with certainty, and necessarily make a socially optimal capacity choice.

In this example, the non-investor also prefers to renegotiate, when there is a positive cost of production delay, even though the presence of this cost weakens his bargaining position as the second mover. We obtain this outcome, because the effect of the production delay cost on the investor's incentive to raise investment is sufficiently high to compensate the non-investor for his loss from the weaker bargaining position.

5 Discussion

The analysis in previous sections suggests that if the cost of ex post renegotiation is not the same across periods, then the use of timing clauses regulating the ex post renegotiation can serve as an instrument with a potential to resolve the hold-up problem. In particular, parties can use these timing clauses alone or in conjunction with other instruments such as liquidation damages, financial hostages, or contract duration clauses to eliminate or to reduce the incentive to under-invest in environments with specific investment and incomplete contracts.

The effect of the production delay cost on the trade surplus is the key assumption underlying the effectiveness of timing clauses in resolution of the hold-up problem. If the production delay cost does not affect the trade surplus, then the use of renegotiation timing clauses in incomplete contracts is irrelevant, because the primitives of the renegotiation procedure remain the same in all periods after the realization of outside uncertainty. In this case, the timing of the ex post renegotiation should be driven only by the realization of external uncertainty and not depend on schedules. However, empirical evidence suggests that the timing of ex post renegotiation is guided by complex contractual provisions, which also include time schedules.

Proposition 3 suggests that if the investor is the first mover in the bargaining game, then she always prefers to renegotiate, when the production delay cost is sufficiently high, so that the trade surplus completely depreciates with one period of delay. This result appears counter-intuitive, because the investor chooses to renegotiate, when the loss from a potential renegotiation delay or the renegotiation break-up achieves its maximum. However, a higher depreciation rate also strengthens the investor's bargaining position as the first mover. In particular, with the trade surplus depreciating at a higher rate, the non-investor's threat to reject a smaller share of the trade surplus loses credibility, and the investor is able to obtain a larger share for herself. With a completely depreciating trade surplus, the investor is able to extract all the trade surplus above the non-investor's outside option and become a residual claimant with certainty. This, in turn, results in the social optimality of the investor's equilibrium investment decision.

The investor's equilibrium investment can be ex ante socially optimal, if the renegotiation period with a complete depreciation of the trade surplus is not in the choice set, or if the investor is not the first mover. In the example, the parties cannot choose to renegotiate when the trade surplus completely depreciates, and the investor invests optimally in equilibrium regardless of who determines the renegotiation timing. In this respect, the example illustrates a case, where the allocation of the right to choose the renegotiation timing is irrelevant in resolving the hold-up problem. Further, in the appendix I present conditions for the social optimality of the investor's equilibrium investment, when the investor is not the first mover.

However, I show that in general the resolution of the hold-up problem in the presence of a varying cost of ex post renegotiation rests on three elements: the choice set of possible renegotiation periods, the identity of a party deciding when the renegotiation offer can be made, and the identity of a party making the renegotiation offer. In particular, as the previous discussion suggests, if the choice set of renegotiation periods is unconstrained, then the assignment to the investor of the right to make the first renegotiation offer and the right to determine when to make this offer results in the ex ante socially optimal specific investment. If the choice set of renegotiation periods is constrained, then depending on the set of available renegotiation periods, an allocation of one or both of these rights to the investor or to the non-investor can result in the socially optimal equilibrium investment.

An important feature of the model is that although the production delay cost increases over time, the parties never incur this cost, and there is no efficiency loss from renegotiating later rather than sooner. This is a consequence of the perfect information assumption in the bargaining game. Since parties have complete information in the bargaining game, they immediately reach an agreement, and there is no renegotiation delay no matter when the ex post renegotiation is scheduled. In this respect, an increase in the production delay cost serves to support the first mover's credible threat to extract a larger share of the trade surplus rather than as the cost, which the parties necessarily pay. Consequently, if we relax the perfect information assumption and introduce information asymmetry, then in equilibrium parties may renegotiate with delay. Since the production delay cost increases in time, the presence of the efficiency loss from renegotiating later may change some results in this paper.

6 Conclusion

In the paper I study how renegotiation timing clauses in incomplete contracts can be used to resolve the hold-up problem. I show that it is possible specify the timing of ex post renegotiation to motivate the investor to make a socially optimal specific investment in equilibrium, even if the investor is not a residual claimant with certainty.

The resolution of the hold-up problem in this paper depends on three components: the set of periods, when ex post renegotiation can take place, the identity of the party making the timing decision, and the identity of the party making the first renegotiation offer. I show that depending on parameters restrictions, the assignment of the right to set the renegotiation timing and the right to make the first offer to the investor or the non-investor can lead to the socially optimal equilibrium investment.

The underlying assumption in the model is that the cost of ex post renegotiation varies across periods. In particular, I assume that as time progresses, the cost of production delay increases and the trade surplus, which the parties bargain over, depreciates at a higher rate with the renegotiation delay. I assume that parties renegotiate according to the rules of the alternating-offers bargaining game, and I derive equilibrium strategies in the game, where the trade surplus and the outside option depreciate at different rates.

7 References

1. Aghion, Philippe, Mathias Dewatripont, and Patrick Rey, "Renegotiation Design with Unverifiable Information," *Econometrica*, 62(1994), 257-282.

2. Binmore, Kenneth, Avner Shaked, and John Sutton, "An Outside Option Experiment," *Quarterly Journal of Economics*, 104 (1989), 753-770;

3. Cellich, C. (1999) "Contract Renegotiations: The Neglected Phase of The Process," *International Trade Forum*, 24: 2, pp. 11-15.

4. Che, Yeon-Koo, and József Sákovics, "A Dynamic Theory of Holdup," *Econometrica*, 72 (2004), 1063-1103.

5. Chiu, Y. Stephen, "Noncooperative Bargaining, Hostages, and Optimal Asset Ownership," *American Economic Review*, 88 (1998), 882-901.

6. Crocker K. and S. Masten, (1991), "Pretia ex Machina? Prices and Process in Long-Term Contracts," *Journal of Law and Economics*, 34:1, 69-99.

7. De Meza, David and Lockwood, Ben, "Does Asset Ownership Always Motivate Managers? Outside Options and the Property Rights Theory of the Firm," *Quarterly Journal of Economics*, 113(1998), 361-86.

8. Edlin, Aaron, and Stefan Reichelstein, "Holdups, Standard Breach Remedies, and Optimal Investment," *American Economic Review*, 86 (1996), 478-501.

9. Grossman, Sanford, and Oliver Hart, "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Investment," *Journal of Political Economy*, 94 (1986), 691-719.

10. Guriev, Sergei, and Dmitriy Kvasov, "Contracting on Time," *American Economic Review*, 95 (2005), 1369-1385.

11. Joskow, Paul, "Contract Duration and Relationship-Specific Investments: Empirical Evidence from Coal Markets," *American Economic Review,* 77 (1987), 1968-1985;

12. Klein, Benjamin, Robert Crawford, and Armen Alchian, "Vertical Integration, Appropriable Rents, and the Competitive Contracting Process," *Journal of Law and Economics*, 21(1978), 297-326.

13. Macneil, Ian, "Contracts: Adjustment of Long-Term Economic Relations under Classical, Neoclassical, and Relational Contract Law," *Northwestern University Law Review,* 72 (1978), 854-905.

14. Nöldeke, Georg and Klaus Schmidt, "Option Contracts and Renegotiations: A Solution to the Hold-up Problem," *RAND Journal of Economics*, 26 (1995), 163 - 179.

15. Osborne, Martin, and Ariel Rubinstein, *Bargaining and Markets,* (San Diego: Academic Press, Inc., 1990).

16. Posner, Eric, "Theory of Contract Law under Conditions of Radical Judicial Error," *Northwestern University Law Review,* 94 (1999), 749-775.

17. Rubinstein, Ariel, "Perfect Equilibrium in a Bargaining Model," *Econometrica*, 50 (1982), 97-109.

8 Appendix

8.1 Proof of Proposition 1

To prove Proposition 1, I follow the approach of Osborne and Rubinstein (1990). I define the complete, transitive and reflexive preference ordering $\succeq_{i\in\{I,N\}}$ over the set $(X \times P) \cup \{D\}$. The set X is defined as the set of all agreements $X =$

$\{(x_I, x_N) \in R^2 : x_I + x_N = w(\sigma)$ and $x_{i\in\{I,N\}} \geq 0\}$. The set $P = \{0, 1, 2, 3...\}$ defines periods when players can make offers, and D is the disagreement payoff.

Next, note that preferences underlying the utility function $U_i(x_i, p) = \delta^p(x_i - \alpha_t s(\sigma)p)$, $\delta \in (0, 1)$, $i \in \{I, N\}$, satisfy the following six assumptions for any $(x, p) \in X \times P$ and $U_i(D) = -\infty$.

Assumption 1: The disagreement payoff D is the worst outcome: for all $(x, p) \in X \times P$, we have $(x, p) \succeq_{i\in\{I,N\}} D$.

Assumption 2: Parties prefer larger shares of $w(\sigma)$: $\forall\ x, y \in X$ and $p \in P$, $(x, p) \succ_{i\in\{I,N\}} (y, p)$ if and only if $x > y$.

Assumption 3: Parties prefer to agree on the bargaining outcome sooner than later: $\forall\ x, y \in X$ and $p, l \in P$, we have $(x, l) \succeq_{i\in\{I,N\}} (y, p)$ if $l < p$, and $(x, l) \succ_{i\in\{I,N\}} (y, p)$ if $l < p$ and $x_{i\in\{I,N\}} > 0$.

Assumption 4: Preference ordering is continuous: $\forall n$, if $(x_n, p) \succeq_{i\in\{I,N\}} (y_n, p)$, then $(x, p) \succeq_{i\in\{I,N\}} (y, p)$, where $lim_{n\to\infty} x_n = x$ and $lim_{n\to\infty} y_n = y$ and $x_n, y_n, x, y \in X$ and $p \in P$.

Assumption 5: Preferences are stationary: $\forall\ x, y \in X$ and $p \in P$, we have $(x, p) \succ_{i\in\{I,N\}} (y, p + 1)$ if and only if $(x, 0) \succ_{i\in\{I,N\}} (y, 1)$.

Assumption 6: The cost of delay increases in x_i, $i \in \{I, N\}$: the difference $x_i - \delta(x_i - \alpha_t s(\sigma))$ is an increasing function of x_i, $i \in \{I, N\}$.

By Theorem 3.4 in Osborne and Rubinstein (1990), the bargaining game with players' preferences represented by the utility function $U_i(x_i, p) = \delta^p(x_i - \alpha_t s(\sigma)p)$, $\delta \in (0, 1)$, has a unique SPNE. Further, the pair of SPNE strategies is the solution $\{\overline{x}, \overline{y}\} = \{(\overline{x}_I, \overline{x}_N), (\overline{y}_I, \overline{y}_N)\}$ to the system: $\overline{y}_I = v_I(\overline{x}_I, 1)$ and $\overline{x}_N = v_N(\overline{y}_N, 1)$. Here, $\overline{x}_I$ is I's share when I makes an offer, $\overline{x}_N$ is N's share when I makes an offer, $\overline{y}_I$ is I's share when N makes an offer, $\overline{y}_N$ is N's share when N makes an offer, $v_I(\overline{x}_I, 1)$ is I's share when I makes an offer with one period of delay, and $v_N(\overline{y}_N, 1)$ is N's share when N makes an offer with one period of delay.

Given the utility function $U(w(\sigma), p) = \delta^p(w(\sigma) - \alpha_t s(\sigma)p)$, with one period of delay the parties bargain over $\delta(w(\sigma) - \alpha_t s(\sigma))$. Using the normalization in

Section 1, we obtain $\delta(w(\sigma) - \alpha_t s(\sigma)) = (\delta - \rho_t)w(\sigma)$, where $\rho_t = \alpha_t \delta \frac{s}{b-s}$.

Let $\underline{v} < \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}$ as in Case 1, and consider the investor's offer of the form $\overline{x}(w(\sigma)) = (\alpha w(\sigma), (1-\alpha)w(\sigma))$ and the non-investor's offer of the form $\overline{y}(w(\sigma)) = ((1-\beta)w(\sigma), \beta w(\sigma))$, where α is the investor's share of $w(\sigma)$ when the investor makes an offer and β is the non-investor's share of $w(\sigma)$ when the non-investor makes an offer. The SPNE conditions require that $(1-\beta)w(\sigma) = \alpha(\delta-\rho_t)w(\sigma)$ and $(1-\alpha)w(\sigma) = \beta(\delta-\rho_t)w(\sigma)$. The solution to these two equations is $\alpha = \frac{1}{1+\delta-\rho_t}$ and $\beta = \frac{\delta-\rho_t}{1+\delta-\rho_t}$. By plugging in α and β into $\overline{x}(w(\sigma))$ and $\overline{y}(w(\sigma))$ we obtain a pair of subgame perfect strategies $\overline{x} = (\frac{w(\sigma)}{1+\delta-\rho_t}, \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t})$ and $\overline{y} = (\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}, \frac{w(\sigma)}{1+\delta-\rho_t})$. Further, since $\underline{v} < \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}$, the non-investor never takes $\underline{v}$. For uniqueness of these strategies, see pages 46-48 in Osborne and Rubinstein (1990).

Next, let $\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} \leq \underline{v} < \frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}$ as in Case 2. Since $\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} \leq \underline{v}$, the investor has to offer at least $\underline{v}$ to the non-investor. Hence, consider the investor's offer of the form $\overline{x}(w(\sigma)) = (\alpha(w(\sigma)-\underline{v}), \underline{v} + (1-\alpha)(w(\sigma)-\underline{v}))$ and the non-investor's offer of the form $\overline{y}(w(\sigma)) = ((1-\beta)(w(\sigma)-\underline{v}), \underline{v} + \beta(w(\sigma)-\underline{v}))$, where α is the investor's share of the trade surplus above the non-investor's outside option and β is the non-investor's share of the trade surplus above the non-investor's outside option. Given that with one period of delay the parties bargain over $(\delta-\rho_t)(w(\sigma)-\underline{v})$ and $\delta\underline{v}$, the SPNE conditions require that $\underline{v} + (1-\alpha)(w(\sigma)-\underline{v}) = \delta\underline{v} + (\delta-\rho_t)\beta(w(\sigma)-\underline{v})$ and $(1-\beta)(w(\sigma)-\underline{v}) = (\delta-\rho_t)\alpha(w(\sigma)-\underline{v})$. Solving for α and β, we obtain $\alpha = \frac{1}{1+\delta-\rho_t} + \frac{\underline{v}(1-\delta)}{(w(\sigma)-\underline{v})(1-(\delta-\rho_t)^2)}$ and $\beta = \frac{1}{1+\delta-\rho_t} - \frac{\underline{v}(\delta-\rho_t)(1-\delta)}{(w(\sigma)-\underline{v})(1-(\delta-\rho_t)^2)}$. By plugging in α and β into $\overline{x}$ and $\overline{y}$ we obtain a pair of subgame perfect strategies $\overline{x} = (\frac{w(\sigma)}{1+\delta-\rho_t} - \frac{\rho_t\underline{v}}{1-(\delta-\rho_t)^2}, \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} + \frac{\rho_t\underline{v}}{1-(\delta-\rho_t)^2})$ and $\overline{y} = (\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} - \frac{(\delta-\rho_t)\rho_t\underline{v}}{1-(\delta-\rho_t)^2}, \frac{w(\sigma)}{1+\delta-\rho_t} + \frac{(\delta-\rho_t)\rho_t\underline{v}}{1-(\delta-\rho_t)^2})$. Note that the non-investor prefers to accept the investor's offer $\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} + \frac{\rho_t\underline{v}}{1-(\delta-\rho_t)^2}$ than to take $\underline{v}$, since $\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} + \frac{\rho_t\underline{v}}{1-(\delta-\rho_t)^2} > \underline{v}$ if an only if $\underline{v} < \frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}$. I prove the uniqueness of these strategies at the end of the proof.

Lastly, let $\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2} \leq \underline{v}$ as in Case 3. Then, the pair of strategies $x = (w(\sigma)-\underline{v}, \underline{v})$ and $y = ((\delta-\rho_t)(w(\sigma)-\underline{v}), w(\sigma) - (\delta-\rho_t)(w(\sigma)-\underline{v}))$ is

SPNE. To see this, note that if the non-investor rejects the investor's offer $x = (w(\sigma) - \underline{v}, \underline{v})$, the highest offer the non-investor can get is $w(\sigma) - (\delta - \rho_t)(w(\sigma) - \underline{v})$ with one period of delay. Since $(\delta - \rho_t)[w(\sigma) - (\delta - \rho_t)(w(\sigma) - \underline{v})] < \underline{v}$, whenever $\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2} \leq \underline{v}$, the non-investor does not have an incentive to reject the investor's offer $x = (w(\sigma) - \underline{v}, \underline{v})$. To see that the investor's strategy is subgame perfect, consider the investor's incentive to reject the non-investor's offer $y = ((\delta - \rho_t)(w(\sigma) - \underline{v}), w(\sigma) - (\delta - \rho_t)(w(\sigma) - \underline{v}))$. In case of a rejection, the investor at most can obtain $w(\sigma) - \underline{v}$ with one period of delay, or $(\delta - \rho_t)(w(\sigma) - \underline{v})$. Hence, the investor's threat of rejection of any offer $y_I < (\delta - \rho_t)(w(\sigma) - \underline{v})$ is credible and the investor's strategy is subgame perfect. For uniqueness of these strategies, see pages 57-58 in Osborne and Rubinstein (1990).

Finally, I prove the uniqueness of SPNE strategies in Case 2. Define M_s as the supremum and m_s as the infimum of the investor's payoffs over all SPNE payoffs in a game where the investor proposes first. Similarly, define $M_b + \underline{v}$ as the supremum and $m_b + \underline{v}$ as the infimum of the non-investor's payoffs over all SPNE payoffs in a game where the non-investor proposes first. Next, I show that the following inequalities must hold:

(1) $m_b + \underline{v} \geq w(\sigma) - (\delta - \rho_t)M_s$

(2) $M_s \leq w(\sigma) - ((\delta - \rho_t)m_b + \delta\underline{v})$

(3) $m_s \geq w(\sigma) - ((\delta - \rho_t)M_b + \delta\underline{v})$

(4) $M_b + \underline{v} \leq w(\sigma) - (\delta - \rho_t)m_s$

If the investor rejects the non-investor's offer, then the highest payoff the investor can obtain in the present value terms is $(\delta - \rho_t)M_s$. This implies that the non-investor can obtain at least $w(\sigma) - (\delta - \rho_t)M_s$, which is the right-hand side of inequality (1). Since $m_b + \underline{v}$ is the infimum of the non-investor's SPNE payoffs, inequality (1) should hold.

If the non-investor rejects the investor's offer, then the smallest payoff the non-investor can obtain in the present value terms is $(\delta - \rho_t)m_b + \delta\underline{v}$. As a result, at most the investor can obtain is $w(\sigma) - ((\delta - \rho_t)m_b + \delta\underline{v})$, and inequality (2) should hold.

Similarly, if the non-investor rejects the investor's offer, then the highest payoff the non-investor can obtain in the present value term is $(\delta - \rho_t)M_b + \delta\underline{v}$. As a result, the investor can at least obtain $w(\sigma) - ((\delta - \rho_t)M_b + \delta\underline{v})$ implying that inequality (3) should hold.

If the investor rejects the non-investor's offer, then the lowest payoff the investor can obtain in the present value terms is $(\delta - \rho_t)m_s$. This implies that the non-investor can obtain at most $w(\sigma) - (\delta - \rho_t)m_s$, which is the right-hand side of inequality (4). Since $M_b + \underline{v}$ is the supremum of the non-investor's SPNE payoffs, inequality (4) should hold.

Since the investor's strategy $x = (\frac{w(\sigma)}{1+\delta-\rho_t} - \frac{\rho_t\underline{v}}{1-(\delta-\rho_t)^2}, \frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} + \frac{\rho_t\underline{v}}{1-(\delta-\rho_t)^2})$ and the non-investor's strategy $y = (\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} - \frac{(\delta-\rho_t)\rho_t\underline{v}}{1-(\delta-\rho_t)^2}, \frac{w(\sigma)}{1+\delta-\rho_t} + \frac{(\delta-\rho_t)\rho_t\underline{v}}{1-(\delta-\rho_t)^2})$ are subgame perfect, we must have that when $\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} < \underline{v} < \frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}$, then $m_s \leq \frac{w(\sigma)}{1+\delta-\rho_t} - \frac{\rho_t\underline{v}}{1-(\delta-\rho_t)^2} \leq M_s$ and $m_b + \underline{v} \leq \frac{w(\sigma)}{1+\delta-\rho_t} + \frac{(\delta-\rho_t)\rho_t\underline{v}}{1-(\delta-\rho_t)^2} \leq M_b + \underline{v}$.

Next, I claim that if $\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} < \underline{v} < \frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}$, then $m_s = \frac{w(\sigma)}{1+\delta-\rho_t} - \frac{\rho_t\underline{v}}{1-(\delta-\rho_t)^2} = M_s$ and $m_b + \underline{v} = \frac{w(\sigma)}{1+\delta-\rho_t} + \frac{(\delta-\rho_t)\rho_t\underline{v}}{1-(\delta-\rho_t)^2} = M_b + \underline{v}$. To see this, note that inequalities (2) and (1) together imply that $w(\sigma) - M_s \geq (\delta - \rho_t)m_b + \delta\underline{v} \geq (\delta - \rho_t)(w(\sigma) - (\delta - \rho_t)M_s) + \rho_t\underline{v}$ or that $\frac{w(\sigma)}{1+\delta-\rho_t} - \frac{\rho_t\underline{v}}{1-(\delta-\rho_t)^2} \geq M_s$. The resulting inequality together with the constraint $\frac{w(\sigma)}{1+\delta-\rho_t} - \frac{\rho_t\underline{v}}{1-(\delta-\rho_t)^2} \leq M_s$ implies that $M_s = \frac{w(\sigma)}{1+\delta-\rho_t} - \frac{\rho_t\underline{v}}{1-(\delta-\rho_t)^2}$. If we plug $M_s = \frac{w(\sigma)}{1+\delta-\rho_t} - \frac{\rho_t\underline{v}}{1-(\delta-\rho_t)^2}$ into inequality (1), we obtain $m_b \geq \frac{w(\sigma)-\underline{v}}{1+\delta-\rho_t} - \frac{(\delta-\rho_t)(1-\delta+\rho_t)\underline{v}}{1-(\delta-\rho_t)^2}$. This inequality together with the constraint $m_b + \underline{v} \leq \frac{w(\sigma)}{1+\delta-\rho_t} + \frac{(\delta-\rho_t)\rho_t\underline{v}}{1-(\delta-\rho_t)^2}$ implies that $m_b + \underline{v} = \frac{w(\sigma)}{1+\delta-\rho_t} + \frac{(\delta-\rho_t)\rho_t\underline{v}}{1-(\delta-\rho_t)^2}$.

Inequalities (3) and (4) together imply that $m_s \geq w(\sigma) - ((\delta - \rho_t)M_b + \delta\underline{v}) \geq w(\sigma) - (\delta - \rho_t)(w(\sigma) - (\delta - \rho_t)m_s - \underline{v}) - \delta\underline{v}$ or that $m_s \geq \frac{w(\sigma)}{1+\delta-\rho_t} - \frac{\rho_t\underline{v}}{1-(\delta-\rho_t)^2}$. This inequality together with the constraint $m_s \leq \frac{w(\sigma)}{1+\delta-\rho_t} - \frac{\rho_t\underline{v}}{1-(\delta-\rho_t)^2}$ implies that $m_s = \frac{w(\sigma)}{1+\delta-\rho_t} - \frac{\rho_t\underline{v}}{1-(\delta-\rho_t)^2}$. If we plug $m_s = \frac{w(\sigma)}{1+\delta-\rho_t} - \frac{\rho_t\underline{v}}{1-(\delta-\rho_t)^2}$ into inequality (4), we obtain $M_b + \underline{v} \leq \frac{w(\sigma)}{1+\delta-\rho_t} + \frac{(\delta-\rho_t)\rho_t\underline{v}}{1-(\delta-\rho_t)^2}$. This inequality together with the constraint $\frac{w(\sigma)}{1+\delta-\rho_t} + \frac{(\delta-\rho_t)\rho_t\underline{v}}{1-(\delta-\rho_t)^2} \leq M_b + \underline{v}$ implies that $M_b + \underline{v} = \frac{w(\sigma)}{1+\delta-\rho_t} + \frac{(\delta-\rho_t)\rho_t\underline{v}}{1-(\delta-\rho_t)^2}$. ■

8.2 Derivation of the investor's first-order condition

The derivative of the investor's objective function with respect to σ yields the following equation:

$$\text{FOC: } \frac{\lambda_1(\sigma)w'(\sigma)}{1+\delta-\rho_t} + \frac{\lambda_1'(\sigma)w(\sigma)}{1+\delta-\rho_t} + \lambda_3(\sigma)\Big(w'(\sigma) - \frac{\partial E[\underline{V}|\underline{V}\in(\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2},V)]}{\partial\sigma}\Big)+$$
$$\lambda_3'(\sigma)\Big(w(\sigma) - E[\underline{V}|\underline{V}\in(\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}, V)]\Big) - c'(\sigma)+$$
$$\lambda_2(\sigma)\Big(\frac{w'(\sigma)}{1+\delta-\rho_t} - \frac{\rho_t \underline{v}}{1-(\delta-\rho_t)^2}\frac{\partial E[\underline{V}|\underline{V}\in(\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t},\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2})]}{\partial\sigma}\Big)+$$
$$\lambda_2'(\sigma)\Big(\frac{w(\sigma)}{1+\delta-\rho_t} - \frac{\rho_t \underline{v}}{1-(\delta-\rho_t)^2}E[\underline{V}|\underline{V}\in(\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}, \frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2})]\Big) = 0.$$

In the next step I expand the terms $\frac{\partial E[\underline{V}|\underline{V}\in(\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2},V)]}{\partial\sigma}$ and $\frac{\partial E[\underline{V}|\underline{V}\in(\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t},\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2})]}{\partial\sigma}$.

$$\frac{\partial E[\underline{V}|\underline{V}\in(\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2},V)]}{\partial\sigma} = \frac{\partial}{\partial\sigma}\frac{\int_{\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}}^{V}\underline{V}dF_{\underline{V}}(\underline{V})}{F_{\underline{V}}(V)-F_{\underline{V}}(\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2})} = \frac{\partial}{\partial\sigma}\frac{\int_{\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}}^{V}\underline{V}dF_{\underline{V}}(\underline{V})}{\lambda_3(\sigma)},$$

which by Leibniz integration rule is equal to

$$\frac{-\frac{(\delta-\rho_t)(1-\delta+\rho_t)w'(\sigma)}{1-\rho_t-(\delta-\rho_t)^2})\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}f_{\underline{V}}(\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2})}{\lambda_3(\sigma)} - \frac{\lambda_3'(\sigma)E[\underline{V}|\underline{V}\in(\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2},V)]}{\lambda_3(\sigma)}.$$

$$\frac{\partial E[\underline{V}|\underline{V}\in(\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t},\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2})]}{\partial\sigma} = \frac{\partial}{\partial\sigma}\frac{\int_{\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}}^{\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}}\underline{V}dF_{\underline{V}}(\underline{V})}{F_{\underline{V}}(\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2})-F_{\underline{V}}(\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t})} =$$

$$\frac{\partial}{\partial\sigma}\frac{\int_{\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}}^{\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}}\underline{V}dF_{\underline{V}}(\underline{V})}{\lambda_2(\sigma)},$$

which by Leibniz integration rule is equal to

$$\frac{w'(\sigma)w(\sigma)\{(\frac{(\delta-\rho_t)(1-\delta+\rho_t)}{1-\rho_t-(\delta-\rho_t)^2})^2 f_{\underline{V}}(\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2})-(\frac{(\delta-\rho_t)}{1+\delta-\rho_t})^2 f_{\underline{V}}(\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t})\}}{\lambda_2(\sigma)} -$$
$$\frac{\lambda_2'(\sigma)E[\underline{V}|\underline{V}\in(\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t},\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2})]}{\lambda_2(\sigma)}.$$

Next, I plug in the resulting expressions into FOC and take derivatives of $\lambda_1(\sigma)$, $\lambda_2(\sigma)$, and $\lambda_3(\sigma)$. By noting that $\lambda_1'(\sigma) = f(\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t})\frac{(\delta-\rho_t)w'(\sigma)}{1+\delta-\rho_t}$, $\lambda_2'(\sigma) = f(\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2})\frac{(\delta-\rho_t)(1-\delta+\rho_t)w'(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}$
$-f(\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t})\frac{(\delta-\rho_t)w'(\sigma)}{1+\delta-\rho_t}$, and $\lambda_3'(\sigma) = -f(\frac{(\delta-\rho_t)(1-\delta+\rho_t)w(\sigma)}{1-\rho_t-(\delta-\rho_t)^2})\frac{(\delta-\rho_t)(1-\delta+\rho_t)w'(\sigma)}{1-\rho_t-(\delta-\rho_t)^2}$, I obtain the investor's first-order condition.

8.3 Equilibrium Investment When the Non-Investor Proposes First

If the non-investor proposes first, then by Proposition 1, the investor accepts the non-investor offer y. As a result, the distribution of expected payoffs is as follows. With probability $\mu_1(\sigma,\rho_t) = F_{\underline{V}}(L \leq \underline{V} \leq \frac{w(\sigma)}{1+\delta-\rho_t})$, the parties' expected payoffs are $(\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t}, \frac{w(\sigma)}{1+\delta-\rho_t})$. With probability $\mu_2(\sigma,\rho_t) = F_{\underline{V}}(\frac{w(\sigma)}{1+\delta-\rho_t} \leq \underline{V} \leq \frac{(1-\delta+\rho_t)w(\sigma)}{1-\delta(\delta-\rho_t)})$, the parties' expected payoffs are

$$(\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} - \frac{(\delta-\rho_t)\rho_t}{1-(\delta-\rho_t)^2}E[\underline{V}|\underline{V} \in [\frac{w(\sigma)}{1+\delta-\rho_t}, \frac{(1-\delta+\rho_t)w(\sigma)}{1-\delta(\delta-\rho_t)}]],$$
$$\frac{w(\sigma)}{1+\delta-\rho_t} + \frac{(\delta-\rho_t)\rho_t}{1-(\delta-\rho_t)^2}E[\underline{V}|\underline{V} \in [\frac{w(\sigma)}{1+\delta-\rho_t}, \frac{(1-\delta+\rho_t)w(\sigma)}{1-\delta(\delta-\rho_t)}]]).$$

And with probability $\mu_3(\sigma,\rho_t) = F_{\underline{V}}(\frac{(1-\delta+\rho_t)w(\sigma)}{1-\delta(\delta-\rho_t)} \leq \underline{V} \leq V)$, the parties' expected payoffs are

$$((\delta-\rho_t)(w(\sigma) - E[\underline{V}|\underline{V} \in [\frac{(1-\delta+\rho_t)w(\sigma)}{1-\delta(\delta-\rho_t)}, V]),$$
$$w(\sigma) - (\delta-\rho_t)(w(\sigma) - E[\underline{V}|\underline{V} \in [\frac{(1-\delta+\rho_t)w(\sigma)}{1-\delta(\delta-\rho_t)}, V]])).$$

The investor's expected payoff is $EU_I'(\sigma,\rho_t)$.

$$EU_I'(\sigma,\rho_t) = -c(\sigma) + \mu_1(\sigma,\rho_t)\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} +$$
$$\mu_2(\sigma,\rho_t)(\frac{(\delta-\rho_t)w(\sigma)}{1+\delta-\rho_t} - \frac{(\delta-\rho_t)\rho_t}{1-(\delta-\rho_t)^2}E[\underline{V}|\underline{V} \in [\frac{w(\sigma)}{1+\delta-\rho_t}, \frac{(1-\delta+\rho_t)w(\sigma)}{1-\delta(\delta-\rho_t)}]]) +$$
$$\mu_3(\sigma,\rho_t)((\delta-\rho_t)(w(\sigma) - E[\underline{V}|\underline{V} \in [\frac{(1-\delta+\rho_t)w(\sigma)}{1-\delta(\delta-\rho_t)}, V]]))$$

The non-investor's expected payoff is $EU_N'(\sigma,\rho_t)$.

$$EU_N'(\sigma,\rho_t) = -c(\sigma) + \mu_1(\sigma,\rho_t)\frac{w(\sigma)}{1+\delta-\rho_t} +$$
$$\mu_2(\sigma,\rho_t)(\frac{w(\sigma)}{1+\delta-\rho_t} + \frac{(\delta-\rho_t)\rho_t}{1-(\delta-\rho_t)^2}E[\underline{V}|\underline{V} \in [\frac{w(\sigma)}{1+\delta-\rho_t}, \frac{(1-\delta+\rho_t)w(\sigma)}{1-\delta(\delta-\rho_t)}]]) +$$
$$\mu_3(\sigma,\rho_t)(w(\sigma) - (\delta-\rho_t)(w(\sigma) - E[\underline{V}|\underline{V} \in [\frac{(1-\delta+\rho_t)w(\sigma)}{1-\delta(\delta-\rho_t)}, V]])).$$

The investor's equilibrium investment satisfies the following condition:

$$w'(\sigma) + [\frac{\partial\mu_1(\sigma,\rho_t)}{\partial\sigma}\frac{(\delta-\rho_t)\rho_t}{1-(\delta-\rho_t)^2}\frac{w(\sigma)}{1+\delta-\rho_t} - \frac{1-(\delta-\rho_t)^2\mu_3(\sigma,\rho_t)}{1+\delta-\rho_t}w'(\sigma)] = c'(\sigma).$$

Since the socially optimal investment is characterized by $w'(\sigma) = c'(\sigma)$, any equilibrium investment $\sigma^*_{\rho_t}$ satisfying $\frac{\partial\mu_1(\sigma,\rho_t)}{\partial\sigma}\frac{(\delta-\rho_t)\rho_t}{1-(\delta-\rho_t)^2}\frac{w(\sigma)}{1+\delta-\rho_t} = \frac{1-(\delta-\rho_t)^2\mu_3(\sigma,\rho_t)}{1+\delta-\rho_t}w'(\sigma)$ is socially optimal. Depending on parameter restrictions, it is possible to define values $\rho_t \in [0,\delta]$, at which equilibrium investment $\sigma^*_{\rho_t}$ is socially optimal.

Consider the non-investor's equilibrium choice of ρ_t. Note that when $\rho_t = \delta$, the investor's expected payoff is $EU_I'(\sigma, \rho_t = \delta) = -c(\sigma)$, in which case the investor's equilibrium investment choice is $\sigma^*_{\rho_{t=\delta}} = 0$. Hence, the non-investor equilibrium timing choice is such that $\rho_t < \delta$. For the same reason, the investor never chooses $\rho_t = \delta$.

Next, consider the case when $\rho_t = 0$. The slope at the investor's choice of ρ_t is characterized by the following condition:

$$\mu_2(\sigma, \rho_t)\frac{\delta(1-(\delta-\rho_t)^2)-2\rho_t}{(1-(\delta-\rho_t)^2)^2}E[\underline{V}|\underline{V} \in [\frac{w(\sigma)}{1+\delta-\rho_t}, \frac{(1-\delta+\rho_t)w(\sigma)}{1-\delta(\delta-\rho_t)}]])+$$
$$\frac{\partial\mu_1(\sigma,\rho_t)}{\partial\rho_t}\frac{(\delta-\rho_t)\rho_t}{1-(\delta-\rho_t)^2}\frac{w(\sigma)}{1+\delta-\rho_t} = [\mu_3(\sigma, \rho_t)w(\sigma) + (1-\mu_3(\sigma, \rho_t))\frac{w(\sigma)}{(1+\delta-\rho_t)^2}].$$

When $\rho_t = 0$, the condition above becomes a negative inequality: $0 > (1 - \mu_1(\sigma, \rho_t = 0))w(\sigma) + \mu_1(\sigma, \rho_t = 0))\frac{w(\sigma)}{(1+\delta)^2}$. This means that $\rho_t = 0$ is a local maximum. Whether $\rho_t = 0$ is a global maximum depends on the shape of the expected payoff $EU_I'(\sigma, \rho_t)$. Nevertheless, we can see that if $\rho_t = 0$ is a global maximum, the investor's ex ante under-invests in equilibrium. However, if the investor's equilibrium choice of ρ_t is above 0, then the equilibrium investment can be socially optimal, sub-optimal or exceed the socially optimal level.

Next, consider the case when $\rho_t = 0$ is the non-investor's choice of ρ_t. The slope at the non-investor's choice of ρ_t is characterized by the following condition:

$$\mu_2(\sigma, \rho_t)\frac{\delta(1-(\delta-\rho_t)^2)-2\rho_t}{(1-(\delta-\rho_t)^2)^2}E[\underline{V}|\underline{V} \in [\frac{w(\sigma)}{1+\delta-\rho_t}, \frac{(1-\delta+\rho_t)w(\sigma)}{1-\delta(\delta-\rho_t)}]])+$$
$$[\mu_3(\sigma, \rho_t)w(\sigma) + (1-\mu_3(\sigma, \rho_t))\frac{w(\sigma)}{(1+\delta-\rho_t)^2}] = \frac{\partial\mu_1(\sigma,\rho_t)}{\partial\rho_t}\frac{(\delta-\rho_t)\rho_t}{1-(\delta-\rho_t)^2}\frac{w(\sigma)}{1+\delta-\rho_t}.$$

At $\rho_t = 0$, the condition above becomes a positive inequality: $(1 - \mu_1(\sigma, \rho_t = 0))w(\sigma) + \mu_1(\sigma, \rho_t = 0))\frac{w(\sigma)}{(1+\delta)^2} > 0$. This means that the non-investor always has an incentive to increase ρ_t from 0. As a result, the non-investor's equilibrium choice of ρ_t belongs to an open interval $(0, \delta)$, and the associated equilibrium investment can be socially optimal, sub-optimal or exceed the socially optimal level. I summarize these results in the next proposition.

Proposition A

a. For $\rho_t \in [0, \delta]$, the investor chooses t_I^* such that $\rho_{t_I^*} < \delta$. If $\rho_{t_I^*} = 0$, then the investor ex ante under-invests in equilibrium. If $\rho_{t_I^*} > 0$, then the investor's *equilibrium investment* $\sigma^*_{t^*_I}$ *can be ex ante socially optimal, sub-optimal, or exceed the social optimality level.*

b. For $\rho_t \in [0, \delta]$*, the non-investor chooses* t^*_N *such that* $0 < \rho_{t^*_N} < \delta$*, and the investor's equilibrium investment* $\sigma^*_{t^*_N}$ *can be ex ante socially optimal, sub-optimal, or exceed the social optimality level.*

8.4 Model with Non-Linear Production and Valuation Functions and the Investor Proposing First

Let $b(\sigma)$ be a nonlinear valuation function, and $s(\sigma)$ be a nonlinear production function. Assume that $b'(\sigma) > 0$ and $w'(\sigma) = b'(\sigma) - s'(\sigma) > 0$. Then, parties bargain over a division of the trade surplus $w(\sigma) = b(\sigma) - s(\sigma)$, given that the non-investor can accept his outside option $\underline{v}$. With one period of delay the parties bargain over a division of $\delta(w(\sigma) - \alpha_t s(\sigma))$, given that the non-investor can accept the depreciated outside option $\delta\underline{v}$.

As before, we can rewrite the depreciated trade surplus $\delta(w(\sigma) - \alpha_t s(\sigma))$ as $(\delta - \rho_t(\sigma))w(\sigma)$, where the production delay cost parameter $\rho_t(\sigma) = \alpha_t \delta \frac{s(\sigma)}{b(\sigma) - s(\sigma)}$ depends on investment σ. As previously, I assume that for any $t \in \{1, .., T\}$, $\rho_t(\sigma) \leq \rho_{t+1}(\sigma)$, $\rho_t(\sigma) \in [0, \delta]$, and $\delta \in (0, 1)$.

Since the primitives of the bargaining game are unchanged, we can use equilibrium strategies in Proposition 1. Let's consider the case, when the investor makes the timing decision. Since $\rho_t(\sigma)$ depends on σ, the investor maximizes her expected utility function $EU_I(\sigma)$ with respect to the single parameter σ.

$$EU_I(\sigma) = -c(\sigma) + \lambda_1(\sigma, \rho_t(\sigma))\frac{w(\sigma)}{1+\delta-\rho_t(\sigma)} +$$
$$\lambda_2(\sigma, \rho_t(\sigma))\left(\frac{w(\sigma)}{1+\delta-\rho_t(\sigma)} - \frac{\rho_t(\sigma)}{1-(\delta-\rho_t(\sigma))^2}E[\underline{V}|\underline{V} \in [\frac{(\delta-\rho_t(\sigma))w(\sigma)}{1+\delta-\rho_t(\sigma)}, \frac{(\delta-\rho_t(\sigma))(1-\delta+\rho_t(\sigma))w(\sigma)}{1-\rho_t(\sigma)-(\delta-\rho_t(\sigma))^2}]]\right) +$$
$$\lambda_3(\sigma, \rho_t(\sigma))(w(\sigma) - E[\underline{V}|\underline{V} \in [\frac{(\delta-\rho_t(\sigma))(1-\delta+\rho_t(\sigma))w(\sigma)}{1-\rho_t(\sigma)-(\delta-\rho_t(\sigma))^2}, V]])$$

Under the assumption that $EU_I(\sigma)$ is globally concave, the investment $\sigma^*_t = \underset{\sigma \geq 0}{argmax} EU_I$ defines both the equilibrium investment and the renegotiation timing choice. The investment σ^*_t is characterized by the following first-order condition:

$$w'(\sigma) + [\frac{\partial\lambda_1(\sigma,\rho_t(\sigma))}{\partial\sigma}\frac{\rho_t(\sigma)}{1-(\delta-\rho_t(\sigma))^2}\frac{(\delta-\rho_t(\sigma))w(\sigma)}{1+\delta-\rho_t(\sigma)}$$
$$-\lambda_2(\sigma,\rho_t(\sigma))\frac{1-\delta^2+\rho_t^2(\sigma)}{(1-(\delta-\rho_t(\sigma))^2)^2}\frac{\partial\rho_t(\sigma)}{\partial\sigma}E[\underline{V}|\underline{V}\in[\frac{(\delta-\rho_t(\sigma))w(\sigma)}{1+\delta-\rho_t(\sigma)},\frac{(\delta-\rho_t(\sigma))(1-\delta+\rho_t(\sigma))w(\sigma)}{1-\rho_t(\sigma)-(\delta-\rho_t(\sigma))^2}]]$$
$$-(1-\lambda_3(\sigma,\rho_t(\sigma)))(\frac{(\delta-\rho_t(\sigma))w'(\sigma)}{1+\delta-\rho_t(\sigma)}-\frac{w(\sigma)}{(1+\delta-\rho_t(\sigma))^2}\frac{\partial\rho_t(\sigma)}{\partial\sigma})]=c'(\sigma).$$

As before, the equilibrium investment σ_t^* satisfying the first-order condition is socially optimal, if the sum of terms in square brackets is zero. Whether the sum in square brackets is zero depends on the size and the sign of $\frac{\partial\rho_t(\sigma)}{\partial\sigma}=\alpha_t\delta[\frac{s'(\sigma)}{w(\sigma)}-\frac{s(\sigma)w'(\sigma)}{w^2(\sigma)}]$. The sign of $\frac{\partial\rho_t(\sigma)}{\partial\sigma}$ depends on the sign of $s'(\sigma)$. If $s'(\sigma)<0$, then $\frac{\partial\rho_t(\sigma)}{\partial\sigma}<0$ and $\frac{\partial\lambda_1(\sigma,\rho_t(\sigma))}{\partial\sigma}>0$. If $s'(\sigma)>0$, then $\frac{\partial\rho_t(\sigma)}{\partial\sigma}$ and $\frac{\partial\lambda_1(\sigma,\rho_t(\sigma))}{\partial\sigma}$ could be positive or negative. Hence, any further analysis of the model requires making assumptions about functional forms of the production cost function, valuation function, and the distribution function of the outside option.

Next, consider the case when the non-investor sets the timing of the ex post renegotiation. Then, the setup of the model is identical to the model with linear production and valuations functions. If the non-investor sets $\rho_t(\sigma)$, the investor chooses investment σ while treating $\rho_t(\sigma)$ as a constant. As a result, predictions of the model in this case should coincide with predictions of the model with linear production and valuation functions, the non-investor making the timing decision, and the investor acting as the first mover.