Flexible Top Trading Cycles and Chains Mechanism: Maintaining Diversity in Erasmus Student Exchange

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Abstract

In Europe, every year more than 200,000 college students study in different countries thanks to the Erasmus Student Exchange program. The program aims to improve the integration all around the Europe. Although this exchange program is successful in many dimensions, it has two major drawbacks. First of all, the number of exchange students imported and exported by countries are not balanced. Unbalanced exchange causes financial burden for countries that are importing more exchange students then their exports. Secondly, we do not observe diversity among the exchange students. That is, certain countries are exchanging students mainly between each other. To solve these two problems observed in the current practice, we propose a version of Top Trading Cycles and Chains (TTCC) mechanism. We show that our mechanism solves these observed problems while satisfying constrained efficiency, strategy-proofness and fairness. To our knowledge, this is the first application of TTCC mechanism to the many to one market.

1 Introduction

The Erasmus student exchange program is a student exchange program between colleges of the 33 member countries.[1] Close to 3 million students have participated to the program since 1987. The number of students benefiting from the program is increasing each year; in 2011, more than 231,408 students attended a college in another member country as an exchange student. The number of member colleges is more than 4,000.[2] The Erasmus program aims to improve the quality of higher education and strengthen its European dimension. Considering the growing numbers of exchange students, one can think that the program is fulfilling its purpose. In this paper, we first show that the student exchange program works inefficiently and the outcome of the current procedure does not fulfill the diversity objective. Then, we propose a centralized mechanism which can easily increase the efficiency of the student exchange program while maintaining other desirable features.

Students from member colleges can participate in the Erasmus student exchange program. Each college from the member countries has the right to apply to be a member institute. All membership applications of universities are sent to the European Commission. Once a university is awarded with membership, it needs to sign bilateral agreements with the other member institutions. In particular, the student exchanges are done between the member universities that have signed a bilateral contract with each other. The bilateral agreement includes information about the number of students that will be exchanged between the two universities in a given period.

The selection process of the exchange students is mostly done as follows. The maximum number of students that can be exported to a partner university is determined based on the bilateral agreement with that partner and the number of students who have been exported since the agreement was signed. The students submit their list of preferences over the partner universities to their home university.

[1] The member countries are EU Member States, Iceland, Liechtenstein, Norway, Switzerland and Turkey.

[2] The numbers are collected from tables provided on web page of European Commission. See the http://ec.europa.eu/education/erasmus/statistics_en.htm

	2004/05	2005/06	2006/07	2007/08	2008/09	2009/10	2010/11	Total
UK	9052	9,264	9,273	8,452	8,636	8,770	8,927	62,374
Sweden	3,928	4,532	4,827	5,403	5,793	6,060	6,348	36,891
Denmark	2,087	2,684	2,958	3,292	3,625	3,934	4,262	22,842
Poland	-6,058	-6,911	-7,489	-7,744	-7,256	-6,079	-4,640	-46,177
Germany	-5,154	-5,959	-6,006	-5,752	-5,685	-6,102	-6,059	-40,717
Turkey	-843	-2,024	-3,117	-4,475	-4,560	-5,117	-5,209	-25,345

Table 1: Balance of Member Countries

Each university ranks its own students based on predetermined criteria, e.g., GPA and seniority. Based on the ranking, a serial dictatorship mechanism is applied to place students at the available slots. Finally, the list of the students who received the slots of the partner universities is sent to the partners. The partner universities most likely accept all the students on the list.

We can consider the whole assignment procedure as n independent assignment procedures where n is the number of member institutions. The reason is each college has available seats in the other colleges that it has bilateral agreement and these seats can be used only by that college.

An exchange student pays her tuition to her own college, not to the one importing her. This may lead to financial issues when a college imports more than it exports. For instance, when a college exports 10 students and imports 20 students, it receives tuition for only 10 students and spends money for 20 students. To prevent this financial issue, colleges may be precautionary and set the maximum number of students to be exchanged at a low level. When we consider the balance of countries we observe that some countries are running huge positive balances over the years and while others are running huge negative balances (Table 1). Between 2004 and 2011, the UK imported 62,374 students more than it exported; while Poland exported 46,177 students more than it imported.[3]

We can interpret the numbers in Table 1 as follows: Between 2004-2010 more

[3]The numbers are collected from tables provided on web page of European Commussion. See the http://ec.europa.eu/education/erasmus/statistics_en.htm

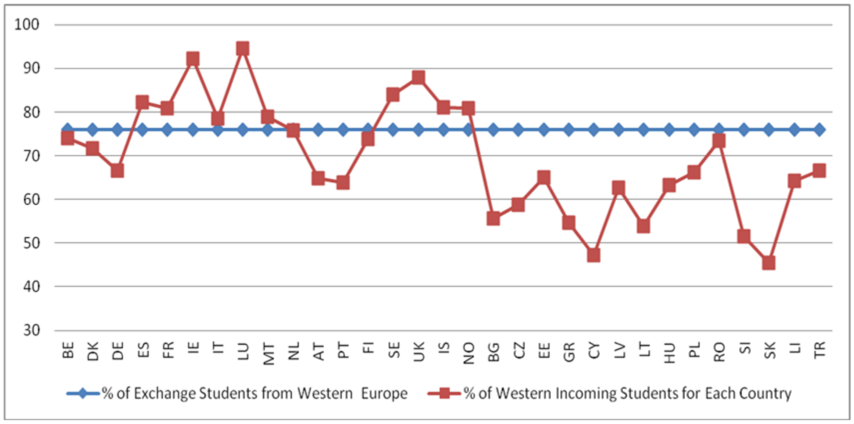


Figure 1: Comparision of the Percentages Incoming Western European Exchange Students

than 60,000 students studied at colleges in UK without paying tuition. Given the average tuition fee is around \$13,000[4], the financial cost of Erasmus program to the British colleges in 2011 is more than \$ 110,000,000. The financial burden of having positive imbalance has been considered as the main reason for colleges in UK to draw up agreements with colleges in Central and Eastern European countries.[5] A study by The Higher Education Funding Council for England in 2004, also showing that colleges are seeking to reduce this imbalance of incoming and outgoing students, and the associated financial burden it generates.[6]

One can think that the main reason for the observed imbalances between the number of incoming and outgoing students is the integration of more countries, mainly the Eastern European countries, to the program. Therefore, imbalances can be minimized by drawing up the agreements between Eastern and Western European countries. However, we illustrate that this will not solve the problem.

In Figure 1, we illustrate the fact that the incoming students of the western Europe countries are mostly from the western Europe (2010-2011). On the other

[4]See http://www.theguardian.com/education/2011/apr/18/tuition-fees-universities-maximum-charge.

[5]See http://www.timeshighereducation.co.uk/164221.article.

[6]http://www.international.ac.uk/media/1438322/E-04-22.pdf

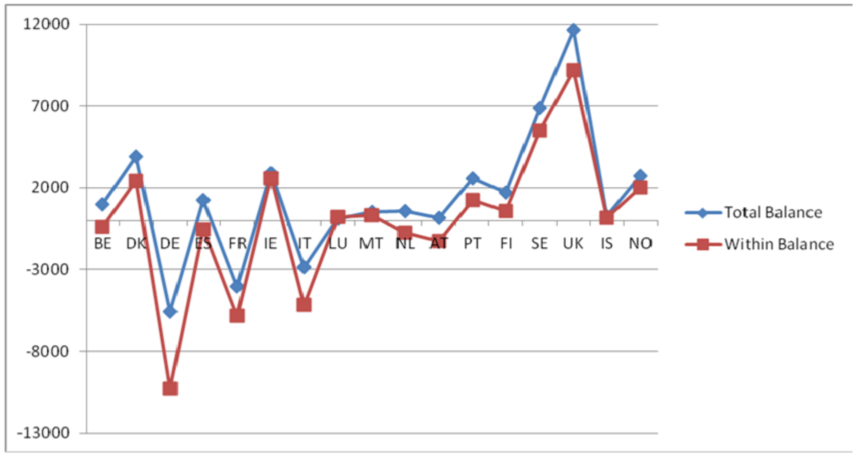


Figure 2: Difference Between Incoming and Outgoing Students

hand, when we consider the percentage of western European students in the incoming sets of the eastern Europe countries below the percentage of the ratio of the western students in all the exchange students. For instance, 92% and 45% of incoming sets of Ireland and Slovakia are from western Europe, respectively (75% of the exchange students are from Western Europe). Instead of looking for country by country if we compare the Western and Eastern Europe we see that 78% of the incoming sets of Western Europe is from Western Europe whereas 61% of the incoming sets of Eastern Europe is from Western Europe.

In Figure 2, we calculate the difference between the incoming and outgoing students when we only consider an exchange market composed of only the western European countries. This figure shows that the countries who suffer from having a high positive balance (i.e. UK, Sweden and Ireland) would have the same problem when we have a Exchange market composed of only the western European countries. In other words, the cause of positive balance of western European countries is not the eastern ones.

Figure 1 shows that colleges in Western Europe are exchanging students between each other. The same observation is also true for Eastern Europe colleges. These numbers indicate that the set of exchange students for most of the colleges are not diverse. This is not consistent with the aim of the Erasmus program and European Union since both give high importance to diversity and integration between all countries and nations. The main reason not to maintain diversity is that colleges from Western Europe are not sign bilateral contract with colleges from Eastern Europe due to the fear of having imbalances.

In this paper we aim to come up with a new design and mechanism for Erasmus program in order to maintain a reasonable balance between imports and exports, and diversity in the set of exported students by each college. The huge imbalance between countries and the lack of diversity in the set of exchange students can be easily solved if a version of Top Trading Cycles and Chains (TTCC) [Roth et al., 2004] mechanism is adopted to this market. The reason for focusing on TTCC is that it is a very flexible mechanism and we can incorporate balancedness and diversity constraints within the mechanism. Adoption of the TTCC mechanism requires the market to be centralized. For instance, when we apply TTCC mechanism the number of incoming students for UK cannot exceed its low number of outgoing students. The students from other Western European countries may prefer to go the other countries which have high negative imbalance such as the Eastern European countries. Moreover, we can require each college to get a minimum number of exchange students from all countries and maintain desired levels of diversity. Applying TTCC mechanism may also increase the total number of students participating to the Erasmus program. Because, colleges would not fear from the financial burden of the imbalances and they may sign bilateral agreements with more partners. In particular, we show that our proposed TTCC mechanism guarantees the diversity in the exchange students and not allow colleges to have imbalances more than permitted levels. Moreover, it select an efficient outcome and makes it dominant strategy for students to report their true preferences.

The mechanism that we introduce is a generalization of TTCC mechanism introduced by Roth et al. [2004] for kidney exchange problem which is a one to one matching problem. It is also related to the Top Trading Cycles (TTC) mechanism which has been introduced for house allocation problem [Shapley and Scarf, 1974, Abdulkadiroğlu and Sönmez, 1999, Pápai, 2000]. TTC mechanism has been also studied in the many to one matching problems such as school choice problem introduced by Abdulkadiroğlu and Sönmez [2003]. Our mechanism differs from the mechanisms studied in these paper since it incorporates more feasibility constraints. Moreover, to our knowledge, this is the first application of TTCC mechanism to the many to one matching markets. Ehlers et al. [2014] include diversity constraints to their models. However, in their model the minimum quota constraint cannot be satisfied by a strategy-proof mechanism. In the matching literature, student exchange programs are first studied by Dur and Ünver [2014]. Dur and Ünver [2014] propose a variant of TTC mechanism for the student exchange programs. Their TTC mechanism selects an outcome in which the number of imports and the number of exports are the same for each college. Differently, we allow some level of imbalance between the number of imports and exports for each college.

2 Model

An **Erasmus student exchange problem** consists of

- a set of **colleges** $C = \{c_1, ..., c_m\}$,
- a set of **students** $S = \underset{c \in C}{\cup} S_c$ where S_c is the set of students who are studying in college c,
- an **import quota** matrix $q = (q_{c,c'})_{c,c' \in C}$ where $q_{c,c'}$ is the maximum number of students who will be "imported" by college c from college c',
- an **aggregate quota** vector $Q = (Q_c)_{c \in C}$ where Q_c is the maximum number of students who will be "imported" by college c,
- a **imbalance-tolerance** vector $b = (b_c)_{c \in C}$ where b_c is the maximum tolerated difference between the number of students who will be imported and exported by college c,

- a list of college internal rankings $\succ = (\succ_c)_{c \in C}$ where $\succ_c$ is the **internal priority order** of students in S_c based on some exogenous rule,
- a list of **student preferences** $P = (P_s)_{s \in S}$ where P_s is the preference relation of student s over colleges including the remaining unmatched option.

For students, we assume there is an outside option, referring to remaining unmatched within the exchange program, and is denoted by $c_\emptyset$. When a student is unmatched it means that student remains in her current school. We take $Q_{c_\emptyset} = |S|$ and $q_{c_\emptyset, c} = |S_c|$ for all $c \in C$. Let R_s denote the associated at least as good as relation of student s.

Without loss of generality we assume that $b_c \geq 0$ for all $c, c' \in C$. This assumption is consistent with the preferences of colleges observed in the real word. Exporting more students than imports is not something that can be tolerated. In particular, it is something desired by colleges. Note that when $\sum_{c' \in C} q_{c,c'} \leq Q_c$ then the aggregate quota becomes redundant. Therefore, we take $\sum_{c' \in C} q_{c,c'} \geq Q_c$. In the rest of the paper we fix C, S, q, Q, b and $\succ$ and denote a problem with P.

A *matching* is a function $\mu : S \to C \cup c_\emptyset$ such that $|\mu^{-1}(c) \cap S_{c'}| \leq q_{c,c'}$, $|\mu^{-1}(c)| \leq Q_c$ and $|\mu^{-1}(c)| - |\left(\bigcup_{c' \in C \setminus \{c\}} \mu^{-1}(c')\right) \cap S_c| \leq b_c$. One can consider the last three restrictions as the feasibility conditions. Let $\mathcal{M}$ be the set of possible matchings.

A matching μ is *fair* if there does not exist a student pair (s, s') such that (1) $\{s, s'\} \subseteq S_c$ (2) $s \succ_c s'$ and (3) $\mu(s') P_s \mu(s)$. In words, under a fair matching, whenever a student envies another student from her home college, then she has lower internal priority than the student she envies.

Student s *strictly prefers* matching μ to matching μ' if he strictly prefers $\mu(s)$ to $\mu'(s)$, $\mu(s) P_s \mu'(s)$. A matching μ Pareto dominates matching μ' if there does not exist a student i strictly preferring μ' to μ and at least one student strictly prefers μ to μ'. A matching μ is *Pareto efficient* if it cannot be Pareto dominated by another matching $\mu' \in \mathcal{M}$.

A *mechanism* ψ is a procedure which selects a matching for each problem P. Let $\psi(P)$ be the matching selected by ψ and $\psi_s(P)$ be the assignment of student s.

A mechanism ψ is *fair* (*Pareto efficient*) if for all problems it selects a fair (Pareto efficient) matching.

A mechanism ψ is *strategy-proof* if it is (weakly) dominant strategy for all students to tell their preferences truthfully. Formally, a mechanism φ is strategy-proof if for every preference profile P and P'_s $\psi_s(P) R_s \psi_s(P'_s, P_{-s})$ for all student $s \in S$. Here, P_{-s} represents the true preference profile of students except s.

3 Current Practice

As mentioned in the Introduction, the colleges prefer to have a tolerable imbalance between the number of students imported and exported. Once the bilateral agreements are signed between the colleges, the colleges cannot control the difference between their imports and exports. In order not to have an intolerable imbalance, colleges set their bilateral quotas lower than the actual one. Let $\tilde{q}_{c,c'} \leq q_{c,c'}$ and $\tilde{q}_{c',c} \leq q_{c',c}$ be the quotas agreed on between all $c, c' \in C$. Each college $c \in C$ places its students to the colleges that they sign bilateral agreements by using serial dictatorship mechanism where the number of available seats in college c' is $\tilde{q}_{c,c'}$. The set of students who will be exported by college c is determined in the following steps:

Serial Dictatorship Mechanism:

Step 1: Student with the highest priority in $\succ_c$ is assigned to his most popular college, c', with available seats for students from college c , $\tilde{q}_{c',c} > 0$. We remove that student and reduce the number of available seats in college c' for students from c by one.

In general,

Step k: Student with the highest priority in $\succ_c$ among the remaining ones is assigned to his most popular college with available remaining seats for students from college c. We remove that student and reduce the number of available seats in college c' for students from c by one.

The mechanism terminates when all students are assigned.

This mechanism is run by all colleges independently and all exchange students are determined.

Note that there will be no priority violation and students cannot benefit from misreporting their true preferences. These follow from the properties of the serial dictatorship mechanism. Moreover, given the quotas set in the bilateral agreements we cannot find another matching Pareto dominating the outcome of current practice. However, since the colleges set their bilateral quotas lower than the actual ones not to have intolerable imbalances, the outcome of the current practice may not be efficient under the true bilateral quotas. In the following two sections, we propose two mechanisms under which colleges do not have intolerable imbalance independent of the bilateral and aggregate quotas they report. Hence, there will be no harm to report their true quotas. When the colleges report their true quotas, the number exchanges will increase.

4 Generalized Top Trading Cycles and Chains Mechanism

In this section, we propose a mechanism that satisfies fairness, Pareto efficiency and strategy-proofness. In particular, we propose a version of the Top Trading Cycles and Chains (TTCC) mechanism [Roth et al., 2004]. TTCC mechanism is introduced for kidney exchange problem. Different from the kidney exchange, the student exchange is a many to one matching problem in which colleges can be matched to more than one student. Another difference from the kidney exchange is that, in the student exchange problem there are more feasibility constraints, i.e., bilateral quota, aggregate quota, and the tolerable balance. Due to these differences, the exact version of TTCC mechanism introduced by Roth et al. [2004] cannot be applied to the student exchange problem.

The kidney exchange problem can be considered as a special case of the student exchange problem where each college's aggregate quota is one and maximum tolerable balance of each college is one. Hence, we generalize the TTCC mechanism in order to make it suitable for student exchange problem and we name it as Generalized Top Trading Cycles and Chains (gTTCC) mechanism. Before giving the description of the mechanism, we define two notions that we use. A *cycle* is a list of colleges and students $(c_1, s_1, c_2, s_2, ..., c_k, s_k)$ where c_n points to s_n and s_n points to c_{n+1} (c_{k+1}=c_1). A *chain* is a list of colleges and students $(c_1, s_1, c_2, s_2, ..., c_k)$ where c_n points to s_n for all $n \in \{1, ..., k-1\}$ and s_n points to c_{n+1}. Here, college c_1 is the *tail* and c_k is the *head* of the chain.

For a given student exchange problem, the gTTCC mechanism determines its outcome as follows.

gTTCC Mechanism for Student Exchange

Let $\pi(c)$ be the random number assigned to college $c \in C$. We define three different counters:

1. Let $a_{c,c'}$ track the number of available positions in c left for students from c' in the fixed portion of the matching.
2. Let A_c track the number of available positions in c left for all students.
3. Let d_c track the difference between the number of students imported by c and the number of exported students from c in the fixed portion of the matching.

Initially set $a_{c,c'} = q_{c,c'}$, $A_c = Q_c$, and $d_c = 0$ for all $c, c' \in C$.

Step 1: Define $\tilde{C}_c = \{c' \in C \cup c_\emptyset | a_{c',c} > 0,\ A_{c'} > 0,\ d_{c'} < b_{c'}\}$. Each college $c \in C$ points to the student $s \in S_c$ who has the highest internal priority among the remaining ones. For each college $c \in C$ students with the highest internal priority in S_c points to her favorite college in $\tilde{C}_c$. The null college $c_\emptyset$ points to the students pointing to it.

Proceed to Step 2 if there is no cycle. Otherwise, locate each cycle, and assign each student to the college that she points to. The assigned students are removed. For each college $c \in C$ in a cycle pointed by a student $s \in S_{c'}$ reduce A_c and $a_{c,c'}$ by 1. Return to Step 1.[7]

[7]Note that if a student s points to $c_\emptyset$ then there will be a cycle composed of s and $c_\emptyset$.

Step 2: If there are no students left, we are done. If not, then we have chains and all chains are ending with colleges whose all students have been already removed. Select the chain whose head is the college with the lowest π. Assign students in that chain to the colleges that they are pointing to. If a student from c' is assigned to c then reduce $a_{c,c'}$, A_c by one. Let c_h and c_t be the head and tail of the selected chain, respectively. Reduce d_{c_t} and increase d_{c_h} by one. The assigned students are removed. Return to Step 1.

The mechanism terminates when all students are assigned.

By restricting the colleges that can be pointed by students in Step 1, the gTTCC mechanism selects a feasible matching. Also note that, each college $c \in C$ will have balance no more than b_c independent of the quotas they report. Hence, the colleges have no reason to set lower quotas not to have intolerable imbalance. We illustrate the mechanism in the following example.

Example. There are four colleges and each college has two students: $C = \{A, B, C, D\}$, and $S_A = \{a_1, a_2\}$, $S_B = \{b_1, b_2\}$, $S_C = \{c_1, c_2\}$, and $S_D = \{d_1, d_2\}$. Quotas are given as:

	$q_{\cdot,A}$	$q_{\cdot,B}$	$q_{\cdot,C}$	$q_{\cdot,D}$	Q	b
A	$-$	1	1	$-$	2	0
B	2	$-$	0	1	2	0
C	1	1	$-$	1	2	1
D	1	1	1	$-$	2	1

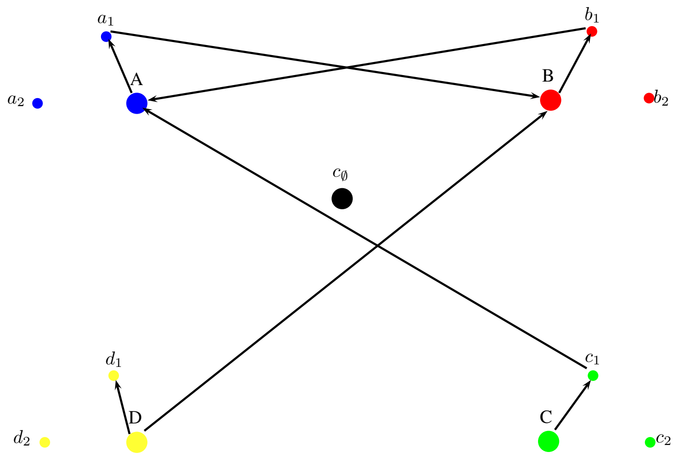
Each student with the lower number has higher priority than the one with higher number at his home college, i.e., $1 \succ_a 2$. The preferences of the students are given as:

P_{a_1}	P_{a_2}	P_{b_1}	P_{b_2}	P_{c_1}	P_{c_2}	P_{d_1}	P_{d_2}
B	B	A	D	A	A	B	B
C	D	D	A	B	B	C	C
$c_\emptyset$	$c_\emptyset$	C	C	$c_\emptyset$	D	$c_\emptyset$	$c_\emptyset$
D	C	$c_\emptyset$	$c_\emptyset$	D	$c_\emptyset$	A	A

Suppose that $\pi(a) < \pi(c) < \pi(b) < \pi(d)$. Now we are ready to illustrate how the gTTCC mechanism works. We first set the counters as below:

	$a_{\cdot,A}$	$a_{\cdot,B}$	$a_{\cdot,C}$	$a_{\cdot,D}$	A	d
A	0	1	1	1	2	0
B	2	0	0	1	2	0
C	1	1	0	1	2	0
D	1	1	1	0	2	0

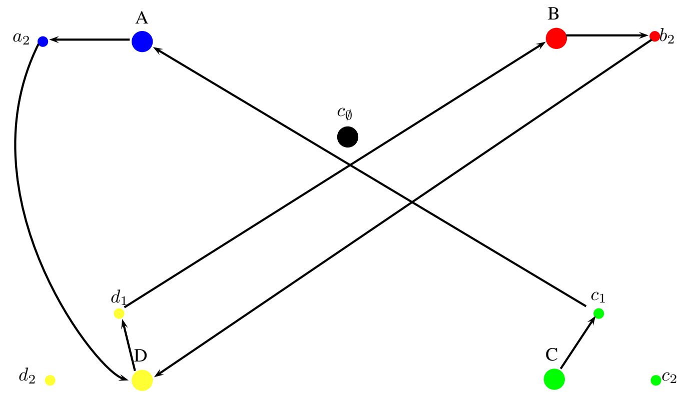
Round 1: Based on the quota profiles we calculate the set of colleges that can be pointed by students as follows: $\tilde{C}_A = \{B, C, D\}$, $\tilde{C}_B = \{A, C, D\}$, $\tilde{C}_C = \{A, D\}$, and $\tilde{C}_D = \{A, B, C\}$. Given the available sets for students, in this round each student pointed by her home college can point to her top choice.



In this round, we have a unique cycle. We assign a_1 to B, and b_1 to A. We update the counters:

	$a_{\cdot,A}$	$a_{\cdot,B}$	$a_{\cdot,C}$	$a_{\cdot,D}$	A	d
A	0	0	1	1	1	0
B	1	0	0	1	1	0
C	1	1	0	1	2	0
D	1	1	1	0	2	0

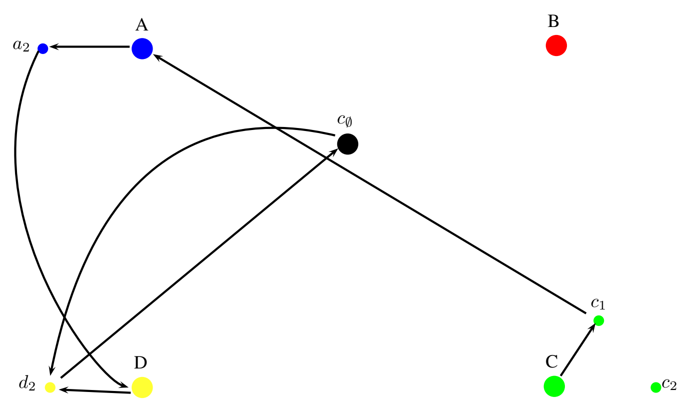
Round 2: We update the set of colleges that can be pointed by students:$\tilde{C}_A = \{C, D\}$, $\tilde{C}_B = \{A, C, D\}$, $\tilde{C}_C = \{A, D\}$, and $\tilde{C}_D = \{A, B, C\}$. Note that since B is not in $\tilde{C}_A$ anymore, a_2 cannot point to B although B is her top choice and it has available seat.



We have a unique cycle in which d_1 points to B and b_2 points to D. We assign d_1 to B and b_2 to D. We update the counters:

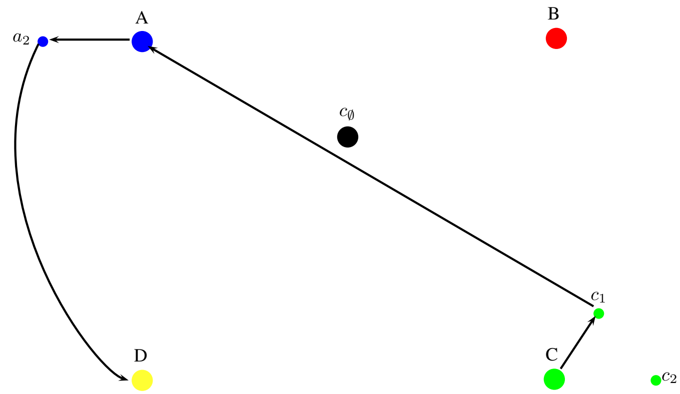
	$a_{.,A}$	$a_{.,B}$	$a_{.,C}$	$a_{.,D}$	A	d
A	0	0	1	1	1	0
B	1	0	0	0	0	0
C	1	1	0	1	2	0
D	1	0	1	0	1	0

Round 3: We update the set of colleges that can be pointed by students:$\tilde{C}_A = \{C, D\}$, $\tilde{C}_B = \{A, C\}$, $\tilde{C}_C = \{A, D\}$, and $\tilde{C}_D = \{A, C\}$.



We have a unique cycle between d_2 and the null college, $c_\emptyset$. We assign d_2 to null college. Since this cycle does not include any college from C, we do not need to update the counters.

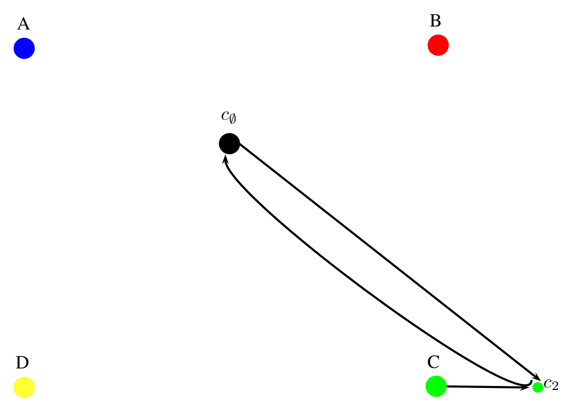
Round 4: The sets of colleges that can be pointed by students are the same as the ones in Round 3.



We do not have any cycles. We have two chains in this round. Since $\pi(A) < \pi(C)$, $C \longrightarrow c_1 \longrightarrow A \longrightarrow a_2 \longrightarrow D$ is selected. Students in this chain is assigned to the colleges that they point to. We update the counters:

	$a_{.,A}$	$a_{.,B}$	$a_{.,C}$	$a_{.,D}$	A	d
A	0	0	0	1	0	0
B	1	0	0	0	0	0
C	1	1	0	1	2	-1
D	0	0	1	0	0	1

Round 5: We update the sets of colleges that can be pointed by students: $\tilde{C}_A = \{C\}$, $\tilde{C}_B = \{A, C\}$, $\tilde{C}_C = \emptyset$, and $\tilde{C}_D = \{C\}$. Since $\tilde{C}_C = \emptyset$, the only remaining agent $c_2$cannot point to any college in C.



We have a unique cycle in which c_2 points to $c_\emptyset$. We assign her to $c_\emptyset$.

Since all students are assigned the mechanism terminates. The outcome selected by gTTCC is $\mu = \begin{pmatrix} a_1 & a_2 & b_1 & b_2 & c_1 & c_2 & d_1 & d_2 \\ B & D & A & D & A & c_\emptyset & B & c_\emptyset \end{pmatrix}$. ■

In kidney exchange problem, TTCC is Pareto efficient and strategy-proof. Since the kidney exchange is a one-to-one problem our fairness notion is redundant in that problem. In Theorem 1, we show that the gTTCC preserves the appealing properties of the TTCC in student exchange problem, moreover it is also fair.

Theorem 1 *The gTTCC mechanism is fair, Pareto efficient and strategy-proof.*

Proof. Denote the outcome of gTTCC mechanism for a given problem P by μ.

gTTCC is fair: Suppose μ is not fair. Then, there exists s, s' and c such that $s \succ_c s'$ and $\mu(s') P_s \mu(s)$. Since s points to $\mu(s)$, which is a less preferred than $\mu(s')$, $\mu(s')$ should have been removed from $\tilde{C}_c$ before s is pointed by c. By the definition of the mechanism, s' is removed after s. Hence, $\mu(s')$ cannot be in $\tilde{C}_c$ when s' starts pointing a college. This is a contradiction.

gTTCC is Pareto efficient: In each round of gTTCC, each student $s \in S_c$ points to her most preferred school among the ones in $\tilde{C}_c$. A college c' is not in $\tilde{C}_c$ since at least one of the feasibility conditions is binding. Therefore, in order to make s better off at least one student will be worse off or at least one of the feasibility conditions will be violated. In either case, there does not exist another matching $\mu' \in \mathcal{M}$ Pareto dominating μ.

gTTCC is Strategy proof: To be added. ■

Unlike the kidney exchange problem minimal chain selection rule does not satisfy strategy-proofness. In this mechanism although we remove the chains selected in a given round the final outcome is still Pareto efficient.

5 Modeling for Diversity: Introducing Minimum Quota

In this section, we introduce an additional constraint to our model. In particular, we add a minimum quota requirement. We represent the minimum quota by $m_{c,c'}$ and it is the minimum number of students from c' required to be assigned to c. Minimum quota restriction can be considered as a tool to guarantee diversity in the students exchanges. As shown in Figure 1, the student exchanges are done between certain countries, i.e., the Western European countries are exchanging students mostly between each other. The minimum quota requirement will help to increase the diversity in the set of exchange students imported by each country.

Unlike the other requirements, minimum quota requirement cannot be satisfied in all problems. In particular, for some problems there may not exist a matching in which the minimum quota requirement is satisfied for each college. In order to guarantee the existence of a matching in which the minimum quota requirement is satisfied, we need some assumptions. We list our assumptions below:

1. $\sum_{c' \in C \setminus \{c\}} m_{c',c} \leq |S_c|$: Each college should have enough students to satisfy the minimum quota in all other schools.

2. $\sum_{c' \in C \setminus \{c\}} m_{c,c'} \leq Q_c$: Each college should have enough seats for the minimum number of students to be assigned

3. $m_{c,c'} \leq q_{c,c'}$: Minimum number of students need to be assigned cannot be more than the maximum number of students can be assigned

4. $\sum_{c' \in C \setminus \{c\}} m_{c,c'} \leq b_c$: It is possible that college c cannot export any student and in that case the assignment of minimum students should not violate the tolerance.

5. If $m_{c,c'} > 0$ then all students in $S_{c'}$ prefers c to $\emptyset$: This is required in order not to force a student to be assigned to a school that he does not consider acceptable.

When all the assumptions listed above hold, the following mechanism will select an outcome in which the minimum quota restriction is satisfied.

gTTCC with Minimum Quota

Step 0: Let $\pi(c)$ be the random number assigned to college $c \in C$. We define six different counters and an identifier. Let $a_{c,c'}$ track the number of available positions in c left for students from c' in the fixed portion of the matching. Let A_c track the number of available positions in c left for all students. Let d_c track the difference between the number of students imported by c and the number of exported students from c in the fixed portion of the matching. Let $g_{c,c'}$ track the minimum number of students from c' needed to be assigned to c to satisfy the minimum quota requirement. Let h_c track the number of remaining students from c. Let $k_{c,c'}$ be the identifier such that students from college c can point to c' if $k_{c,c'} = 0$, and students from college c cannot point to c' if $k_{c,c'} = 1$. Initially set $a_{c,c'} = q_{c,c'}$, $A_c = Q_c$, $d_c = 0$, $g_{c,c'} = m_{c,c'}$ and $h_c = |S_c|$ for each college $c, c' \in C$. Set $k_{c,c'} = 0$ for all $c, c' \in C$.

Step 1: If $h_c = \sum\limits_{c' \in C \setminus \{c\}} g_{c',c}$ then assign each remaining students in S_c to the colleges with $g_{c',c}$ one by one starting from the student with the highest internal priority and based on their preferences. Update the counters based on the assignments. If $A_c = \sum\limits_{c' \in C \setminus \{c\}} g_{c,c'}$, then set $k_{c',c} = 1$ for all colleges c' such that $g_{c,c'} = 0$. If $h_c = 0$ and $b_c - d_c = \sum\limits_{c' \in C \setminus \{c\}} g_{c,c'}$ then set $k_{c',c} = 1$ for all colleges c' such that $g_{c,c'} = 0$.

Step 2: Define $\tilde{C}_c = \{c' \in C \cup c_\emptyset | a_{c',c} > 0,\ A_{c'} > 0,\ d_c < b_c,\ k_{c,c'} = 0\}$. Each college $c \in C$ points to the student $s \in S_c$ who has the highest internal priority among the remaining students. For each college $c \in C$ students with the highest internal priority in S_c points to her favorite college in $\tilde{C}_c$. The null college $c_\emptyset$ points to the students pointing to it.

Proceed to Step 3 if there is no cycle. Otherwise locate each cycle, and assign each student to the college that she points to. The assigned students are removed. For each college $c \in C$ in a cycle pointed by a student $s \in S_{c'}$ reduce A_c, $a_{c,c'}$, $g_{c,c'}$ and h_c by 1 . Return to Step 1.[8]

Step 3: If there are no students left, we are done. If not, then all chains are ending with colleges whose all students have been already removed. Select the chain

[8]Note that if a student s points to $c_\emptyset$ then there will be a cycle composed of s and $c_\emptyset$.

whose head is the college with the lowest π. Assign students in that chain to the colleges that they are pointing to. If a student from c' is assigned to c then reduce $a_{c,c'}$, A_c, $g_{c,c'}$ and $h_{c'}$ by one. Let c_h and c_t be the head and tail of the selected chain, respectively. Reduce d_{c_t} and increase d_{c_h} by one. The assigned students are removed. Return to Step 1.

The mechanism terminates when all students are assigned.

From the definition of the mechanism, it is clear that the outcome of gTTCC with minimum quota is a matching, i.e., satisfies the feasibility constraints, and the number of students from c assigned to c' is not less than $m_{c',c}$.

The main difference between gTTCC mechanism introduced in the previous section and the mechanism introduced here is the inclusion of the control step, Step 1. In particular, in Step 1 we check whether with the remaining students the minimum quota restriction will be satisfied. For instance, if the number of the remaining students from college c is equal to the remaining quota requirement then all these students must be assigned to one of the colleges with minimum quota greater than 0. Similarly, if the number of remaining seats in a college is equal to the minimum quota reserved then it can only accept students in order to fulfill its minimum quota requirement and all the other students cannot be assigned to that college.

Before showing the appealing properties of gTTCC mechanism with minimum quota, we introduce a notion:

We say that a matching μ is *constrained efficient* if there does not exist any other matching $\tilde{\mu}$ such that $|\tilde{\mu}^{-1}(c) \cap S_{c'}| \geq m_{c,c'}$ for all $c, c' \in C$.

Theorem 2 *gTTCC mechanism with minimum quota is fair, constrained Pareto efficient and strategy-proof.*

Proof.

We define a round of gTTCC as the interval between two consecutive step 1.

Strategy-proofness: Consider a student $s \in S_c$ who becomes active in round k. He cannot change the assignments done before round k. Moreover, if the number of students not assigned in c is more than the minimum quota reserved for c, then s

will not be assigned to a school in order to satisfy minimum quota requirement. If a college c' is not in $\tilde{C}_c$ is round k then it will not be in $\tilde{C}_c$ no matter what s submits.

Suppose s points to his most preferred school in $\tilde{C}_c$ in round k. If he is assigned then no need to manipulate.

If he is not assigned then there is either a cycle or chain not including him. Student s cannot change this cycle or chain by submitting a different preference profile.

If a college becomes unavailable in the next round that college belongs to either a chain or cycle in round k. Since s cannot change the cycles or chain selected in round k by misreporting, he cannot be assigned to the colleges become unavailable in the next round no matter what he submits. Moreover, if s can form a cycle or chain in round k by reporting another preference profile then he can form the same cycle in the following rounds when he submits his true preference.

Hence, we can consider s as a passive student in round k and consider round $k+1$ as the first round he becomes active and redo the same analysis.

Fairness: As we have shown in Theorem 1, if $s \succ_c s'$ then student s can take the assignment of s' before he is removed. Hence, s will not envy the assignment of s' under the outcome of gTTCC.

Constrained Efficiency: We can follow the steps in the proof of Theorem 1 for Pareto efficiency. There will be difference when minimum quota requirement binds. That will be the reason why the outcome is constrained efficient and not Pareto efficient.

■

6 Conclusion

To be added.

References

Atila Abdulkadiroğlu and Tayfun Sönmez. House allocation with existing tenants. *Journal of Economic Theory*, 88:233–260, 1999.

Atila Abdulkadiroğlu and Tayfun Sönmez. School choice: A mechanism design approach. *American Economic Review*, 93:729–747, 2003.

Umut Dur and M. Utku Ünver. Two-sided matching via balanced exchange: Tuition and worker exchanges. Working paper, 2014.

Lars Ehlers, Isa Hafalir, Bumin Yenmez, and Muhammed Yildirim. School choice with controlled choice constraints: Hard bounds vs. soft bounds. *Journal of Economic Theory*, 153:648–683, 2014.

Szilvia Pápai. Strategyproof assignment by hierarchical exchange. *Econometrica*, 68: 1403–1433, 2000.

Alvin E. Roth, Tayfun Sönmez, and M. Utku Ünver. Kidney exchange. *Quarterly Journal of Economics*, 119:457–488, 2004.

Lloyd Shapley and Herbert Scarf. On cores and indivisibility. *Journal of Mathematical Economics*, 1:23–37, 1974.