Optimal Markdown and Priority Pricing with Demand Uncertainty*

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Abstract

Markdown and priority pricing schemes facilitate complex pricing schedules for sellers and enable strategic buyers to purchase products at desirable prices. In a pre-announced markdown mechanism, price of a product declines over time according to a specific schedule until it sells out, inducing high-value customers to purchase earlier and at higher prices. This paper considers an environment with uncertain demand and discusses the existence and characterization of a model that determines producer’s optimal pricing and allocation rule as a preannounced and multistep markdown schedule. The mechanism focuses on the operational implications of allotting scarce resources, when at the same time, customers are heterogeneous in their valuations and sensitivities towards availability of product. The proposed mechanism suggests that a carefully designed multistep markdown pricing could achieve optimal revenue when selling a single unit. However, when the monopolist has multiple units, he should modify implementation of markdown pricing by either hiding number of available products at each stage or selling them via contingent contracts and upfront payments. Further we discuss the inefficient outcome of “commodity burning”, where monopolist may consider disposing of a portion of supply. Finally, we illustrate that seller’s optimal scheme includes offering supplementary insurance to the risk averse customers.

Keywords: Markdowns, Priority Pricing, Mechanism Design

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1 Introduction

Price skimming is a marketing strategy where seller sets an initially high price for the product and lowers it over time. Thus, it enables a manufacturer, who has marketing power, to extract a greater proportion of consumer surplus. Customers with higher valuations are incentivized to purchase at earlier stages, and these higher prices insure them against the product selling out. Stokey (1979, 1988) was one of the first to use such a model to investigate the emergence of higher quality products in a learning-by-doing framework.

Recent technological improvements in electronic commerce have enabled retailers to exercise more innovative and complicated pricing schemes. One such unique format is automated and decreasing scheduled prices, called “Automatic Markdown E-commerce”. For example “Pricetack.com”, an e-commerce website, enables sellers to offer their products for sale using a novel falling price schedule. Manufacturers specify a schedule of falling prices with short time intervals between the markdowns. Buyers may purchase the product at the initial price or wait for more discounts and a better deal in the future. However, buyers risk losing the product to other potential customers if the item sells out before it falls to the targeted low price. Figure 1 provides an example, a cell-phone jammer offered with a falling price schedule, on the “Pricetack.com” website. The initial price of the product was $320, on October 4, 2011, until when the price was marked down by $20. The price would further drop to $280 on October 14 and $240 on November 3 if it did not sell out by then. The same mechanism is used widely and at a fast growing rate by other websites such as “Pricefalls.com”. Figure 2 shows how price of an item drops according to a preannounced schedule until it sells out or the supplier ends the listing. Filene’s Basement (Figure 3), a retailer, also employed markdown system in its Boston store until September 2007 when it underwent construction. Similarly, “Next to New,” a retail store located in Austin, TX, implements markdown pricing for its products that include household furnishing items and clothing. Price items are reduced every 30 days at a preannounced rate (usually 25%) until they sell out. Further examples have been practiced by other retailers, such as “plunging prices” implemented by Sam’s club\(^1\), Dutch flower auction known as Aalsmeer flower auction (Figure 4), and fashion markets.

Priority services implement a similar mechanism to discriminate between customers. Supplier provides a menu of options with different prices and service orders, where customers should pay higher for better service ranking. Afterwards, when uncertainties are realized, supply becomes rationed according to the service ranks of the contracts. We see a strong link between these models and automated markdowns. The higher customers pay, the earlier they obtain the product or the higher quality of service they receive. Seminal works by Wilson (1989), Chao and Wilson (1987) and Oren, Smith and Wilson (1985) investigate markets with priority pricing schedules, such as the electricity markets. Wilson (1989) considers the

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\(^1\)The full example is available in the study by Elmaghraby et al. (2008).
effect of uncertainty in supply and excludes randomness in demand. Chao and Wilson (1987) formulate efficient rationing in the presence of demand uncertainty, while Oren, Smith and Wilson (1985) take capacity fees as a part of total tariff and illustrate how a specified capacity with known date of delivery should be priced. However, optimal pricing and rationing is still not obvious in the pricing context, especially when supply is scarce and demand is uncertain, and in the presence of customers who strategically choose the time of purchase.

Although preannounced and priority pricing schemes segment customers based on their valuations, they may lead to the “strategic waiting” phenomenon. Here, people who are willing to pay high price for a merchandise might still postpone their purchase to get the product at clearance price rather than buying the item earlier. However, if the pricing scheme can somehow create incentives for those with higher valuations so that they purchase products at higher prices, retailer would be able to extract more from the market by performing a second degree price discrimination. However, knowing customer preferences and using non-linear prices designed for different qualities of product, suppliers may capture a large proportion of surplus by distinguishing between types of consumers.

In this paper, we study the pricing scheme in a model where demand is uncertain. We employ Mechanism Design, which is a standard tool to solve profit maximizing problems, in order to design supplier’s optimal resource allocation. The goal of Mechanism Design is to achieve profit maximizing solutions where customers are incentivized to attain purchasing behaviors aligned with the objectives of the designer. For instance, Vickrey (1961) Clarke (1971) Groves (1973) mechanism, known as the VCG mechanism, designs payment schedules or money transfers in order to implement second degree price discrimination. We employ models of priority and markdown pricing schemes and explore the possibility of implementing optimal mechanism via revenue maximizing pricing schedules. In addition, we allow the seller to have commitment over pricing schedule and argue that dynamic pricing should be abandoned in such environments. The present study sheds light on the following questions by combining previous streams of the literature:

- Is it possible to obtain optimal revenue via markdown pricing scheme? How should sellers implement such mechanisms?

- Is it optimal for seller to commit to a specific pricing path? If so, should he dispose of a fraction of the unsold products?

- How do the answers to the above questions differ when buyers are risk averse? How about the case when supplier has more than one item to sell?

Previous works have partly addressed these questions. However, a comprehensive study is missing in the literature to explain the growing and widespread use of multiple-step markdown

\footnote{In second degree price discrimination, sellers do not directly observe types of customers, and therefore, are not able to fully extract consumer surplus.}
pricing especially by online retailers. Diverse streams of past studies find it optimal to implement a sealed bid auction in the beginning period and ignore the dynamic setting. Other studies solve for the best pricing strategies when seller has no commitment over prices in the future. Therefore, it is appealing to understand how and why mechanisms with committed pricing schedules are becoming more popular. These issues together with the need for a deeper analysis of pricing multiple units have been the motivations for the present study.

This paper considers environments where there are uncertainties in demand and formulates optimal pricing schemes for a monopolist. It shows that uncertainty enables producer to further squeeze customers by performing multiple-step markdown pricing. The current study adds to the literature in three ways and characterizes optimal pricing schemes when seller has commitment power against finite number of customers with uncertain demand structure. Firstly, we consider a seller with single unit of a unique product and demonstrate that, absent risk aversion and income effects, optimal pricing mechanism could be implemented in the form of multiple-step markdown pricing. We further study the case of sellers with multiple units of supply and find that offering products via markdown pricing is not optimal in general. However, if seller employs one of the following two operational techniques, he can benefit from the advantages of running a markdown pricing (e.g. low complexity and easy to implement) and still attain optimal profit. First is to hide the number of items available and the quantity sold at every single step except at the beginning period, and second is to run a priority pricing auction in the beginning, distribute the goods and charge customers based on an optimal allocation and pricing rule. Moreover, we study the case of risk averse customers and find that offering insurance options to customers is optimal for the retailer and also pareto improving. In other words, when customers are sensitive to the quality of service or availability of product, monopolist and customers all benefit from having the retailer, or a third party, supplement contracts with insurance. Therefore, retailer can attain higher profits from market by enforcing a preannounced pricing scheme with upfront payments and supplementary insurance options. On the other side of market, customers will be better off hedging themselves against risks of unavailability of product. Table 1 provides a summary of related research in pricing against strategic customers compared with our study.

In addition, we illustrate how markdown or priority pricing schedules generate inefficiencies. We argue that a retailer may intentionally dispose of a portion of its products rather than putting them up for sale. In other words, decision of a supplier shall include “commodity burning” if the realized demand for higher prices falls below a particular limit. The purpose of commodity burning is to discourage higher value customers to wait until the last minute for clearance sales or further discounts. Thus, although the decision to throw away a fraction of products is inefficient in a social-welfare-maximization point of view, it should be used by retailers to prevent the strategic waiting phenomenon in order to increase profit.

Our mechanism is a type of second degree price discrimination. It is different from first
degree price discrimination and the seller can not extract the whole surplus since he does not know the true valuations of customers. The equilibrium prices show that each customer (except the lowest type who pays her own valuation) pays lower than her valuation\(^3\). The differential amount - the difference between valuation and payment - is called "rent" and it increases as type increases, i.e., higher types receive higher rents. The reason is that high value customers have private information and the mechanism should give them greater rent to provide incentive to reveal true valuations.

We find it a reasonable assumption to consider commitment to a specific pricing path for retailers with multiple number of products who consider profitability of future sales as well. One should note that some retailers commit to a specific announcement and stick to it even if it turns out not to be optimal. Retailers such as “pricefalls.com” may find it profitable to change the price path of an item after they observe its sale at an initial price, but they may also find it too costly to do so. Similar online retailers who run parallel auctions and proactively seek more products to sell on their websites, will threaten the credibility of pricing schedules for their other products if they forgo the predetermined pricing and start selling in a sequentially rational way. Furthermore, pricing with no commitment has other significant disadvantage for sellers. As Coase (1972) argues, when a seller has no commitment power, multistep pricing leads to zero profit - due to the strategic behavior of the customers and that the seller competes with himself over different periods.

The theoretical model consists of a profit maximizing monopolist who provides a product with a corresponding price schedule. On the other side, each customer maximizes her expected utility given available options. All future prices are determined at an initial date and before the realization of uncertainties. We show that a retailer benefits from the uncertain world and extracts more from customers by exercising priority or markdown schemes. In Section 3, we start with a model where the monopolist has a single item for sale and solve for optimal mechanisms including preannounced markdowns and priority pricing. We postpone the discussion of multiunit sale until Section 3.2 where the supplier has a fixed and commonly known stock of product. We formulate the solution and provide the optimal pricing schedule in both single-unit and multiunit cases. Furthermore, we compare priority pricing with preannounced markdowns and argue that it is not optimal to sell sequentially when multiple units are for sale.

The organization of this paper is as follows: Section 2 discusses the relevant literature. Section 3 provides a model based on priority services and markdown pricing in the presence of uncertain demand for a single unit of product. It characterizes the revenue maximizing allocation and pricing scheme, and pursues with the case where seller has multiple units to sell. Section 4 investigates optimal pricing and allocation to risk averse customers and characterizes an equilibrium with upfront payments and insurance schemes. Finally, Section

\(^3\)For clarity purposes, we assume that seller is male and customers are female and use the corresponding pronouns throughout the paper.
5 concludes, provides operational management implications and outlines questions and future lines of research.

2 Literature Review

Given that the strategic waiting of higher value customers is considered a loss to retailers, the literature provides operational policy advice such as suggesting that suppliers offer markdowns, limited availability, advance discounts, or higher future prices, to dissuade high value customers to postpone their purchase. Su and Zhang (2008) analyze a form of retailers’ price discrimination where they may inform customers that future purchasing opportunities would be unappealing. The signaling could be performed with either future high prices or limited availability. Surasvadi et al. (2014) illustrate the group-buy pricing mechanism for customers who arrive according to a poisson process. The pricing allows the seller to sell multiple units at a regular price and also a discounted price that becomes activated when a pre-specified number of reservations are made. Segal (2003) considers a supplier with multiple units of a commodity who is willing to sell them through an auction to customers with single unit demand. He finds that, when valuations are private information, an optimal pricing schedule should charge the customers based on the other customers’ bids. Lazzati and Van Essen (2014) solve for the nearly optimal mechanism when seller is unaware of the distribution of customers, Su (2007) studies capacity rationing, Aviv and Pazgal (2008) consider preannounced pricing and committed capacity planning, and Yin et al. (2009) compare the profitability of two inventory display formats: display all and display one. The latter study focuses on a setting in which monopolist announces a fixed price path (a premium and end-of-period price) and manipulates customer expectations on the availability via an appropriate display format. Although the framework is very limited and allows for only two types of customers and only two price steps, the findings have intriguing operational implications and provide support for the superiority of display one over the display all format. Lazear (1986) also addresses a similar question and investigates price dynamics when there are fixed number, T, of sale periods: just two regular and clearance prices when \( T = 2 \). \( N \) customers with homogenous but unknown-to-seller valuations arrive in each period and seller chooses a strictly falling price schedule to attain the highest possible revenue. However, this analysis ignores the fact the customers’ heterogeneity except in the dimension that some customers may be serious buyers and others may be just shoppers walking around in the store. Furthermore, seller has no choice on the number of sale periods in this framework. However, it was one of the novel ideas at the time and a later study by Pashigian and Bowen (1991) investigate the Lazear (1986) model empirically. Other relevant studies are Rao and Peterson (1998) and Van Mieghem (2000) who solve for the optimal pricing of priority ser-

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4A comprehensive reference on these topics could be found in Aviv et al. (2009).
vices in a queueing context, and Anderson and Dana (2009) who state the conditions under which supplier chooses not to offer advance buy discounts.

Other types of price discrimination could be performed by last minute and opaque selling strategies, as discussed by Jerath et al. (2009). The Priceline.com or Hotwire.com websites, for instance, offer hotel deals with approximate locations and last minute flight deals. Recent studies have also focused on the impact of the intertemporal behavior of customers on producers’ profits and the appropriate mechanisms that suppliers could employ for revenue management. Other forms of uncommon price discrimination is the case studied by Deneckere and McAfee (1996): one of the observed strategies of manufacturers has been intentional damaging a portion of their goods in order to price discriminate. Firms may intentionally produce their goods at lower qualities with almost the same marginal cost to achieve higher profits.

We build our model on the previous streams of literature who investigate price discrimination when supply is scarce relative to demand. Harris and Raviv (1981) focus on an environment with uncertain demand and show that when supply is abundant, fixed pricing is optimal. In the case of scarce supply, they illustrate how a Vickery mechanism should be used by a producer. Another study by Liu and Van-Ryzin (2008) points out that in the case of preannounced prices, optimal strategy for a monopolist could be to deliberately understock a product and cause shortage in order to create incentive for higher value customers to buy earlier and at higher prices. A related study by Elmaghraby et al. (2008) extends those findings to the optimal mechanisms in the presence of multiunit demand under both complete and incomplete information. They find that under known demand structure and scarce supply, two step markdown schedule is optimal. Moreover, they show that under certain conditions - private and incomplete information - three step markdowns might be implemented by a monopolist. However, they claim that seller will never find it optimal to employ more than three steps. The recent paper by Osadchiy and Vulcano (2010) build on Elmaghraby et al. (2008) and argue that when customers are heterogeneous in their valuations and arrival time, a binding reservations mechanism could be superior to markdown pricing when it is costly for customers to wait for the product. However, they compare their model with a simple two step markdown pricing absent uncertainty in the demand structure. In a related study, Krishna (2002) provides optimal auctions and revenue equivalence between different sets of prices in a multiperiod setup. Customers are independently and identically drawn from a known distribution in his framework. He shows that multiple unit auctions share two common features: revenue equivalence and prices following a martingale process, i.e., prices in the following period are expected to be the same as current price and knowledge of past prices will never help predict future price levels.

The last set of relevant research include the work by McAfee and Vincent (1997), Skreta (2006), Horner and Samuelson (2011), and McAfee and Vincent (1993) who solve the pricing problem of a monopolist unable of committing to future prices. Skreta (2006) shows that
a seller with full bargaining power may post prices in each period. She makes a prominent contribution to the dynamic pricing schemes since other studies’ methodologies do not apply to the frameworks in which seller behaves sequentially. The reason is that the revelation principle is invalid in such cases and it makes the analysis too complicated to solve for an equilibrium without utilizing this principle. McAfee and Vincent (1997) investigate a closely related problem where seller chooses a sequentially rational mechanism against finitely many buyers. They find that seller implements a sequence of first price or second price auctions. Time horizon is infinite, and therefore, seller lacks the ability to effectively enforce a reserve price. In another relevant study by Horner and Samuelson (2011) seller sets prices in discrete steps for finite number of customers. Because of the existence of a deadline to sell the items, seller benefits from having some bargaining power, and therefore, the outcome differs from that by Coase (1972), known as the Coase conjecture. Under certain conditions, the conjecture shows that monopolist will compete with itself over different periods of sales and could not earn positive profit. The reason is simply because retailer has no commitment over future prices. In addition, the research by McAfee and Vincent (1993), studies the declining price auctions of identical items that are sold sequentially. They study a falling price setting and find that, for the existence of pure strategy bidding functions in equilibrium, it is necessary to assume that utility functions exhibit non-decreasing absolute risk aversion. Otherwise, there is a positive probability that outcomes be inefficient. The study differs from ours in that we focus on the optimality of declining prices (maximum revenue to seller) while they question the efficiency of mechanism (maximum social welfare).

3 Model

The monopolist is risk neutral and has a commonly known fixed supply, $S$, which is equal to one unit in this Section. The seller has already paid all the costs to produce, i.e., sunk costs, and we assume there are no expenses regarding storage. Also, the object has no value for the seller if unsold and may be disposed of at the end of the sales season. On the other side, customers are heterogeneous in their valuations for the product (for instance, a celebrity might value a fashion clothing more than others). Furthermore, we assume that the prices are announced before the demand is realized. Customers’ valuations are private information and therefore, unobservable to others in the market. There are finite, $M$, number of customers with valuations $\{v_1, v_2, ..., v_M\}$ drawn independently from an identical and commonly known distribution with cumulative density function $F(.)$ (where $f(.)$ denotes the probability density function)\(^5\). Furthermore, possible valuations are equally distanced from each other over the

\(^5\)In reality, only some of the whole population are aware of, for example, pricetack.com and check their website. A seller should implement markdown pricing to target this group. However, it does not mean that people who are not aware of the website do not need the product or are casual buyers. It just means that seller need not worry about them and he should have a good approximation of the distribution of the people
normalized interval between zero and one, i.e., \( \{\theta_1 = \frac{1}{N}, \theta_2 = \frac{2}{N}, \ldots, \theta_N = \frac{N}{N}\} \). For simplicity, we consider a distribution where each possible valuation is equally likely to occur with probability \( \frac{1}{N} \).

Conditional on consuming the product, utility of a consumer with valuation \( v_i \) is represented by:

\[
U(v_i, p) = v_i - p,^6
\]

where \( p \) denotes the price and \( u(.) \) is the utility function. One way of selling the product is the fixed price scheme. In fixed price sales, seller posts a price \( p \) and customers with valuations greater than or equal to that price level purchase the product. However, there is a better alternative for the seller in which he takes into account the uncertainties in demand and implements a discriminating mechanism between different types of customers to attain higher revenue. But how should he determine the price path? And is it possible to sell the item via markdowns and achieve the highest possible revenue, in expected terms, at the same time?

One could consider three methods of price discrimination between different types of customers. One method is to implement a falling price schedule as performed by “Pricefalls.com” and “Pricetack.com”. These websites announce a future price path for each product with exact markdowns. On the demand side, people observe price schedule and pick up the stage to step in and purchase the product. As illustrated by Figure 1, a retailer has specified five markdowns to sell a cell phone jammer at the Pricetack.com website. Customers pick their favorite price and wait for the time when the price drops to that specific level. A strategic buyer may wait for lower prices, but she also faces the risk of unavailability if she decides to postpone her purchase.

The second method to sell the product, as described by Horner and Samuelson (2011), is to implement sequentially rational sales in which price is updated in every single step in a dynamic environment. Therefore, seller will have no commitment on the plunging prices and one may even observe a jump in prices since no announcements for future prices are credible.

Finally, we consider the option that seller implements a sealed bid auction in the first period. Customers announce their bids via sealed envelopes, or with providing their credit card numbers, and seller allocates the product to the highest bid with prices determined by the mechanism. As we will discuss, the latter mechanism achieves optimal profit, the maximum surplus a seller could extract via a pricing scheme, but it suffers from being too complicated for a common customer to practice. One restriction to implement such a mechanism is to have all customers bid in the beginning of the sale and seller allocate the who are checking his website. It is therefore crucial that seller considers the distribution of his audience rather than the total population, if different.

\(^6\)The utility function is assumed to be linear in our framework until section 4, where we relax this assumption and address the case with risk averse customers with concave utility function \( U(v_i, p) = u(v_i - p) \).
good in a static framework. However, customers are usually more convenient with observing dynamic prices with gradual changes rather than deciding about their desirable bid in a complicated mechanism. Furthermore, a retailer may be able to attract more customers with simpler and easier to understand pricing methods. In summary, we compare the following three mechanisms and discuss managerial implications and optimality of each scheme:

A) Priority pricing and allocation based on ranking order
B) Multiple step pricing with a preannounced and committed path
C) Priority pricing with upfront payments and supplementary insurance

People are served according to priority pricing and allocation rules in model A. Supplier implements a mechanism by allowing a representative customer \(i\) to choose among a set of priority orders, \(r_i \in [r_T, \ldots, r_{N-1}, r_N]\), where \(1 \leq T \leq N\), and \(r_T\) is the lowest ranking order that seller may allocate product. The first or top ranking order, \(r = r_N\), is served first with the highest probability of receiving the product, and the lowest ranking order, \(r = r_T\), is served last with the least probability.\(^8\) Each priority level is paired with a corresponding price and the menu of contracts is denoted by \(<r_i, p(r)>\), where \(r\) is the set of choices of all customers including customer \(i\). In other words, pricing may depend not only on what ranking order an individual picks, but also on the choices of all others who are willing to purchase the product. We will show later in this section that there is an optimal mechanism in which pricing can depend only on self-choice, and therefore, seller can avoid the complexity and just post a price to each ranking order. The second plan, \(B\), is similar to the priority pricing \(A\) in the way that it contains a set of prices implemented as markdown pricing. Rather than offering a menu of prices and ranking orders, seller starts with an initial price \(p_N\), lowers it to \(p_{N-1}\) in the second period and continues decreasing the price on a preannounced path until it reaches \(p_T\). However, the difference with plan \(A\) is in that the sale is not implemented in a single period. It starts with an initially high price, continues plunging according to a committed schedule and terminates either at price \(p_T\) or when the item sells out before price reaches that level. Our “Pricrack.com” example, Figure 1, is a perfect illustration of plan \(B\). The retailer has specified five markdowns to sell a cell phone jammer. People observe the price path and make their mind about when to step in and purchase the product. We will show that when there is single unit for sale, there exist both types of \(A\) and \(B\) plans, which are equivalent in attaining optimal revenue. However,

\(^7\)Monopolist offers different choices to discriminate between different types of customers, and since there are \(N\) types in the market, maximum number of choices in the menu should be \(N\) which occurs when \(T = 1\), i.e., offering more options than the number of types is redundant.

\(^8\)One may assign a number to each ranking order to indicate who is served first, second and so on, i.e., \(r_N = 1, r_{N-1} = 2, \ldots, r_T = N - T + 1, \ldots, r_1 = N\). In other words, we can map the ranking orders to a number between \(1\) and \(N\) in a decreasing order so that it implies that the highest ranking order, \(r_N\), is served first, second highest, \(r_{N-1}\), second and so on.
their difference is significant when there are multiple units for sale and multi-step pricing
with a preannounced path may not be optimal. Section 3.2 discusses our findings in more
detail. Finally, plan C implements a mechanism similar to plan A with the difference that
seller offers supplementary insurance with a menu of priority options. Seller bears more risk
and pays all or a portion of customers’ money back if not served. The scheme is discussed
in detail in Section 4.

3.1 Optimal Pricing

Thus far, the monopolist’s problem to solve for a mechanism with optimal revenue seems
very complicated. Customers are strategic and there are too many ways to implement price
discrimination. However, the complication could be simplified by employing the revelation
principle:

**Proposition 1.** Revelation Principle: for any scheme in our incomplete information game,
there exists a payoff equivalent scheme with an equilibrium which is both direct and truthtelling,
i.e., players report their types truthfully.

Using the revelation principle, we can limit our search to the set of mechanisms in which
customers report their types truthfully. Therefore, we consider cases in which customer i has
a valuation \( v_i \in \{ \theta_1 = \frac{1}{N}, \theta_2 = \frac{2}{N}, ..., \theta_N = \frac{N}{N} \} \), and given truthfullness of other customer,
she should not have incentive to report any valuation except the true one, i.e., \( \hat{v}_i = v_i \), where
\( \hat{v}_i \) denotes the report of customer i for her type.

Since there are N possible valuations for each customer, when a representative customer
i reports the highest valuation \( \hat{v}_i = \theta_N \), it is as if she has chosen the highest ranking, \( r_N \),
among all possible ranking orders, i.e., \( r(\theta_N) = r_N \). Similarly if someone declares \( \hat{v}_i = \theta_{N-1} \),
it is as if she has chosen the second highest rank, i.e., \( r(\theta_{N-1}) = r_{N-1} \), and the person who
announces that her type is the lowest, \( \hat{v}_i = \theta_1 \), means that she is ranked the last in the ranking
order, i.e., \( r(\theta_1) = r_1 \). Therefore, from now on, we assume that each person i with valuation
\( v_i \) declares his type by choosing a ranking order, \( \hat{r}_i \). The choice will be truthtelling in the
new messaging space if \( \hat{r}_i = r(v_i) \in \{ 1, 2, ..., N \} \), where \( r(.) \) is the function defined above,
which maps valuations to the ranking orders. The function is one to one and decreasing if we
consider \( \{ r_1 = N, r_2 = N - 1, ..., r_N = 1 \} \), i.e., the higher the valuation, the lower the ranking
order. This function allows us to use the nice intuitive features of pricing based on ranking
orders while at the same time, it keeps the mechanism direct, i.e., as if customers message
their types directly. For the purpose of convenience, we denote \( r(\theta_l) \) with \( r_l \), \( \forall l \in \{ 1, ..., N \} \)
throughout this paper.

We denote the allocation rule for customer i, in an optimal mechanism, by \( g_i(\hat{r}_i, \hat{r}_{-i}) \),
which is a function of her choice, \( \hat{r}_i \), and the choices of all other people in the market, \( \hat{r}_{-i} \)

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9A proof of the theorem can be found in Myerson (1979).
which is a vector that contains ranking orders chosen by all people other than customer \( i \). A customer who chooses the rank \( \hat{r}_i \), will be served according to the allocation rule \( g_i(\hat{r}_i, \hat{r}_{-i}) \) and should transfer \( t_i(\hat{r}_i, \hat{r}_{-i}) \) which depends on the declarations of all customers. Therefore, given report \( \hat{r} = (\hat{r}_1, ..., \hat{r}_M) \) by all customers, seller allocates supply to them according to the allocation rule \( g(\hat{r}) = [g_1(\hat{r}), ..., g_M(\hat{r})] \), and transfer rule \( t(\hat{r}) = [t_1(\hat{r}), ..., t_M(\hat{r})] \).

According to the revelation principle, we are interested in the truth-telling schemes in which players report their rankings according to their true types, \( \hat{r}_i = r(v_i) \). Therefore, we denote truth-telling reports by \( r \) and distinguish any other report by \( \hat{r} \) for clarity purposes.

Therefore, assuming that all other players are ranked truthfully, individual \( i \) takes expectation over other consumers’ choices, \( r_{-i} \), and obtains expected probability of receiving the product, \( q_i(\hat{r}_i) \), and expected payment, \( p_i(\hat{r}_i) \), for service order \( \hat{r}_i \) as:

\[
q_i(\hat{r}_i) = E_{-i}[g_i(\hat{r}_i, r_{-i})] \quad (1)
\]
\[
p_i(\hat{r}_i) = E_{-i}[t_i(\hat{r}_i, r_{-i})] \quad (2)
\]

where \( E_{-i} \) denotes expectation taken over all possible states of demand for all customers except \( i \), given that everybody else is truthful.

If customer \( i \) with valuation \( v_i \) is willing to participate in the game, she will expect to receive the product with probability \( q_i(\hat{r}_i) \) at the expected price \( p_i(\hat{r}_i) \) if she chooses the ranking order \( \hat{r}_i \). If she decides to take the contract with ranking order \( \hat{r}_i \), her expected utility will be:

\[
U_i(\hat{r}_i; v_i) = E_{-i}[u(g_i(\hat{r}_i, r_{-i})v_i - t_i(\hat{r}_i, r_{-i}))] \quad (3)
\]

which will reduce to the following expression if customers are represented by quasilinear utility functions:

\[
U_i(\hat{r}_i; v_i) = E_{-i}[g_i(\hat{r}_i, r_{-i})v_i - t_i(\hat{r}_i, r_{-i})] = q_i(\hat{r}_i)v_i - p_i(\hat{r}_i) \quad (4)
\]

Therefore, due to the incomplete information environment, customers face an allocation rule with corresponding prices, and report their ranking orders (or their valuations, equivalently) based on their privately known information. We assume that if a customer decides to stay out of market, she will gain zero utility. Therefore, we normalize outside option’s utility to zero, and interpret it as not taking any contract. Therefore, we can formulate the customer’s maximization problem as:

\[
\max \{0, \max_{\hat{r}_i} \{U_i(\hat{r}_i; v_i)\} \} \quad (5)
\]

We can see that the expected utility function, if non-zero, satisfies the condition that agents’ marginal rate of substitution is increasing in type, i.e. \( \frac{\partial}{\partial v_i} \left( \frac{\partial U_i(\hat{r}_i; v_i)}{\partial q_i} \right) > 0 \). In short, the assumption is a sufficiency condition to insure that any monotonic allocation \( q(.) \) is implementable. Therefore, designer can search over the monotonic allocation mechanisms
for an optimal solution. This mechanism should offer a better deal to higher value agents. Otherwise, in any mechanism that higher types face punishment for reporting true values, they have incentives to misreport themselves as lower types.

The timing is as follows: First, supplier designs a mechanism to maximize his profit from allocating supply to the heterogenous customers. For any set of choices by the customers (ranking order \( r \), or equivalently, valuations), a contract specifies payments for the bidders. Then, customers choose their ranking orders from seller’s menu. And finally, seller serves customers and charges them according to the allocation rule.

The monopolist designs a mechanism to incentivize higher value customers to purchase earlier at higher price. Two distinct structures could be assumed for pricing: one with static implementation, where customers bid in advance and choose a ranking order with a corresponding price rule where delivery is contingent on the realized demand (Plan \( A \)), and another with multiple markdowns where price plunges down until it reaches a certain price floor or it sells out (Plan \( B \)). For the case of linear utilities, we show that for any optimal mechanism \( A \), there exists an equivalent scheme, \( B \), which is implemented dynamically and according to a preannounced price path.

In the simple case of a two period sale, for instance, monopolist decides to sell at \( P_H \) in period one, and if any product left, at \( P_L \) in period 2. A consumer buys at the first period if utility of purchasing at the higher price is greater than buying cheaper but with higher risk of unavailability. In other words, customer purchases the product earlier if the following IC\(^{10}\) condition holds: 

\[ q_1 v_i - P_H \geq q_2 v_i - P_L \]

where \( q_i \) is the probability of obtaining the good in period \( i \).

We start by solving for the optimal mechanism. We show that the monopolist can obtain maximum attainable profit by running mechanism \( A \) and that there exists an equivalent \( B \) scheme (5 markdowns in the example by Figure 1). When offered a menu of priority services, we show that customers who choose higher prices, are served with more certainty or with superior priority in equilibrium. Buyers signal about their types by choosing a service ranking between 1 and \( N \), where each ranking order corresponds to a purchasing time or markdown if the product is not sold out by then. We consider the earlier steps as higher priority orders with more certainty in obtaining the product. One should note that we assume that a smaller service ranking, \( r \), corresponds to higher probability of being served. We show that the monopolist serves the customers based on their service orders, i.e., the people who have chosen higher payments are prioritized over the people who have committed to pay less\(^{11}\). Also, as we will discuss, retailer may not be willing to serve all customers.

\(^{10}\)Incentive Compatibility or Self Selection.

\(^{11}\)As illustrated by Myerson (1981), Mas-Colell, Whinston and Green (1995) and Tirole (1988) the assumption is valid under two conditions when customers are drawn independently from an identical distribution. First, the utility function be concave and strictly increasing, and second, the hazard rate of the distribution function be increasing in type/valuation. The former is satisfied by assuming that the customers are risk neutral, and the latter is true for a large class of distributions, including normal, uniform, etc.
In other words, the seller might be willing to serve the customers who declare types greater than $\theta_T$, which we call as the cutoff type. Thus, optimal number of steps could be lower than $N$ in equilibrium.

Recall that a truthtelling strategy means that the best strategy for a customer is to reveal her true type among all possible types, $v_i \in \{\theta_1, \ldots, \theta_N\}$, given that everybody else is truthful. Therefore, the following Incentive Compatibility conditions should hold for the truthful customer $i$ with any possible type $v_i = \theta_l$ (where $r_l = r(\theta_l)$ is the corresponding truthtelling ranking choice):

$$E_{-i}[g_i(r_l, r_{-i})\theta_l - t_i(r_l, r_{-i})] \geq E_{-i}[g_i(\hat{r}_i, r_{-i})\theta_l - t_i(\hat{r}_i, r_{-i})] \quad \forall l, \hat{r}_i \in \{1, \ldots, N\}$$

or equivalently,

$$q_i(r_l)\theta_l - p_i(r_l) \geq q_i(\hat{r}_i)\theta_l - p_i(\hat{r}_i) \quad \forall l, \hat{r}_i \in \{1, \ldots, N\}$$

Furthermore, seller can not force buyers to participate in the pricing scheme. Therefore, in addition to the incentive compatibility conditions, the mechanism should also provide the participants with at least their outside utility (Individual Rationality or IR condition). The outside utility is called “outside option” or “reserve utility” and is normalized to zero:

$$E_{-i}[g_i(r_l, r_{-i})\theta_l - t_i(r_l, r_{-i})] \geq 0 \quad IR \ Condition$$

or equivalently,

$$q_i(r_l)\theta_l - p_i(r_l) \geq 0$$

Before solving the monopolist’s revenue maximization problem, one should note that the solution is symmetric across the individuals. The reason is that all valuations are independently and identically distributed. Therefore, if a set of reports $r'$ can be obtained by rearranging $r$, then any individual who reports $x$ in the first set, $r$ will be treated with the same allocation and transfer rule as any individual with the same report in the set $r'$. To express the symmetry in terms of equations:

$$g_i(r_i = x, r_{-i}) = g_j(r'_j = x, r'_{-j}), \quad t_i(r_i = x, r_{-i}) = t_j(r'_j = x, r'_{-j})$$

The symmetry of the solution implies that expected payment and allocation to different individuals with the same ranking should be the same, too. Therefore, index $i$ can be dropped from $q_i(.)$ and $p_i(.)$ in the IC and IR conditions (Equation 7 and Equation 9, respectively) and we denote them by $q(.)$ and $p(.)$ from now on.

Now, we can state the monopolist’s problem in terms of solving for the optimal choice among the set of truthtelling allocation rules and transfers $< g(r), t(r) >$ - the choice that maximizes his expected revenue. In mathematical terms, we can express the merchant’s
problem as:
\[
\text{Max } ME_r E_{-i}[t_i(r_l, r_{-i})] = ME_r p(r_l) = \frac{M}{N} \sum_{k=1}^{N} p(r_k)
\]  
\[
\text{maximized over } <g(r), t(r)>
\]
\[
\text{subject to IC and IR conditions}
\]

where \(E_r\) is the expectation over the uncertain ranking/valuation of the customer (from the point of view of the seller), and \(E_{-i}\) is the expectation from the point of view of customer \(i\) with valuation \(v_i = \theta_i\) who considers reservation prices of other customers as unknown.

The solution technique includes two steps. First we ignore a number of constraints and solve the maximization problem. Then, having solved the maximization problem and after finding the allocation rule, we show that the ignored constraints are satisfied. The constraints to put aside are individual rationality (IR) constraints of all types except the lowest participating type \(\theta_T\), and all but one incentive compatibility (IC) condition of every customer. The IC and IR constraints to ignore are:

\[
E_{-i}[g_i(r_l, r_{-i})\theta_l - t_i(r_l, r_{-i})] \geq 0 \forall l \neq T
\]

\[
E_{-i}[g_i(r_l, r_{-i})\theta_l - t_i(r_l, r_{-i})] \geq E_{-i}[g_i(\hat{r}_i, r_{-i})\theta_l - t_i(\hat{r}_i, r_{-i})]
\]

\[
\forall l, \hat{r}_i \in \{1, ..., N\} \text{ such that } \hat{r}_i \neq r_{l-1}
\]

which means that, for each customer with valuation \(\theta_l\), it is sufficient to only consider the condition that reporting \(r_{l-1}\) and mimicking the closest smaller type, \(\theta_{l-1}\), is not profitable for her. Using all the simplifying assumptions, one could solve for the optimal set of contracts and obtain the following priority pricing scheme:

**Proposition 2.** Optimal scheme is a priority pricing rule with a cut-off rank \(T\), where priority prices are \(\{p_T^*, p_{T+1}^*, ..., p_N^*\}\). The prices are paid upfront and customers participate in a lottery-type sale in which their payments may not result in the delivery of product. Any customer with valuation \(\theta_l \geq \theta_T\) chooses the option \(<r_i, p_l^*>\) from the menu and receives the product at price \(p_l^*\) if no higher value customer is present in the market. Optimal allocation rule, \(g_i(r_i, r_{-i})\), and cut-off rank \(T\) are defined by:

\[
g_i(\hat{r}_i, \hat{r}_{-i}) = \begin{cases} 0 & \text{if } \exists j \text{ such that } \hat{r}_j > \hat{r}_i \\ 1 & \text{if } \nexists j \text{ such that } \hat{r}_j \geq \hat{r}_i \\ \frac{1}{K} & \text{if } \exists K \text{ number of } j \text{ such that } \hat{r}_j = \hat{r}_i \text{ and } \hat{r}_i = \max_k \{\hat{r}_k\} \end{cases}
\]

where \(i, j, k \in \{1, ..., M\}\)
and $T = \text{ArgMin}_l \{\theta_l - (N - l)\Delta \theta_l\}$ such that $\{\theta_l - (N - l)\Delta \theta_l \geq 0\}$ such that

and prices are obtained by:

$$p^*_l = \theta_l q(r_l) + \sum_{j=T}^{l-1} (\theta_j - \theta_{j+1}) q(r_j) \quad \text{given} \quad q(r_l) = E_{-i}[g_i(r_l, r_{-i})]$$  \hspace{1cm} (12)

In other words, a customer who chooses the option $<r, p(r)>$, will be served if no other customer chooses a higher priority (which is matched with a higher price). It is interesting to study some special cases for our equilibrium:

- If you let population be too big, i.e. $M \rightarrow \infty$, then posted price will be optimal. In other words, with big populations, seller is almost confident that there is somebody with the highest possible valuation, and therefore, sets a single price (known as posted price) equal to the highest possible valuation $p = 1$.

- If you let $N \rightarrow \infty$, customers’ distribution approaches a continuous distribution and virtual valuation of a customer with type $\theta_l$ can be calculated as $\psi(\theta_l) = 2\theta_l - 1$. The $\psi(.)$ function is the same as virtual valuation obtained by Myerson (1981), Wilson (1989), and Mas-Colell et al. (1995) for continuous distribution functions, i.e. $\theta_l - \frac{1-F(\theta_l)}{f(\theta_l)}$, which will be equal to $2\theta_l - 1$ for our uniform distribution.

Proposition 3. Priority pricing could be implemented by offering a menu of options with rankings and payments conditional on receiving the product. The set of prices are $\{\tilde{p}^*_1, \tilde{p}^*_2, ..., \tilde{p}^*_N\}$ defined by:

$$\tilde{p}^*_l = \frac{p^*_l}{q^*_l}$$  \hspace{1cm} (13)

where $p^*_l$ and $q^*_l$ are equilibrium priority price and probability of being served for customers who choose ranking order $r_l$ specified according to Proposition 2.

This proposition and the following proposition (that provides an equivalent mechanism to priority pricing) explain how online retailers, “pricetack.com” and “pricefalls.com” for instance, find it optimal to implement multiple-step markdown pricing with payments conditional on the delivery of product.

Proposition 4. When customers are risk neutral, for any priority pricing $A$ with a menu of options $<r_l, \tilde{p}^*_l>$ with contingent payments, there exists an equivalent markdown scheme, $B$. In mechanism $B$, prices fall according to the same path, $<\tilde{p}^*_l>$, and yield to the same total revenue.
The last proposition is of essential import since it guarantees that there exist an easily implementable markdown pricing scheme and provides the optimal set of prices. The scheme provides the seller with optimal revenue, and at the same time, has the nice feature of a easy to understand schedule with decreasing multiple-step prices rather than a complicated pricing and allocation rule. Seller starts with an initially high price, decreases it over time until the price reaches the price floor or it sells out throughout the process.

3.2 Multiunit Markdowns

We have characterized markdown pricing as optimal mechanism to sell a product. However, an interesting operational question is that if we can prescribe the same mechanism under other circumstances when seller has more than one unit of product. In the following propositions, first we characterize the optimal pricing and allocation rule and then present the equivalent markdown scheme to implement that optimal scheme.

Proposition 5. When the seller has a total of $S < M$ units, optimal allocation rule is an upfront payment or priority pricing where allocation rule is defined by:

\[
    g_i(\hat{r}_i, \hat{r}_{-i}) = \begin{cases} 
    0 & \text{if } C_{\hat{r}_i}(\hat{r}) \geq S \\
    \max\{1, \frac{S-C_{\hat{r}_i}(\hat{r})}{K}\} & \text{if } C_{\hat{r}_i}(\hat{r}) < S \text{ and } \exists K \text{ number of } j \text{ such that } \hat{r}_j = \hat{r}_i \end{cases}
\]

where $C_{\hat{r}_i}(\hat{r})$ is the number of people who choose a ranking order strictly higher than $\hat{r}_i$, $i, j \in \{1, \ldots, M\}$, and $\hat{r}_i, \hat{r}_j \in \{1, \ldots, N\}$. Furthermore, prices are the same as in the case with only one unit presented by Equation 12.

However, since markdown pricing reveals information to customers at every stage, it is not optimal to implement it in a dynamic setting. At every single period, few items may be sold, and depending on the number of items left, incentives of customers will be altered in the following period. Therefore, if seller sticks to a preannounced price schedule and reveals information regarding the number of items left, the scheme will not be optimal for most possible distributions of demand. However, there are alternatives via which seller can obtain optimal profit. Monopolist should either hide any information regarding the quantity sold and quantity left or modify markdown scheme and implement a priority pricing schedule instead. These arguments let us state the following proposition:

Proposition 6. Seller with multiple units of product should implement one of the specific forms of markdown pricing to achieve optimal revenue. First is to hide number of available supply during the whole process of plunging prices. And second is to offer priority services in which customers choose the price level in advance and pay conditional on receiving product.
Our analysis shows that when a seller puts up multiple units for sale, he should sell them via either priority pricing scheme or hiding information regarding the availability. The first mechanism is implemented in the beginning period and before the realization of uncertainties. Higher ranked customers receive the product with greater probabilities but at higher prices. And the second method hides the number of remaining products at each stage so that customers’ incentives are not distorted. This way, customers do not get information regarding number of units sold in previous periods and quantity remaining at each stage.

4 Extension: Risk Averse Customers

In this section, we assume that customers are risk averse and denote their utility by a strictly increasing and concave function, \( u(x) = -e^{-\alpha x} \), with constant absolute risk aversion (CARA). Retailer is risk neutral and cares only about the expected revenue. Utility of choosing \( \hat{r}_i \) among the available menu of ranking orders is defined by equation 3. Similar to the solution technique in the previous section, we use the revelation principle and search over the set of truth-telling equilibria. The maximization problem, after ignoring the unnecessary IC and IR constraints, will be the optimal choice of allocations and transfers, \( <g(r), t(r)> \), to solve the following problem:

\[
\begin{align*}
\text{Max } & ME_{\hat{r}_l}E_{-i}[t_i(r_l, r_{-i})] \\
E_{-i}[u(g_i(r_l, r_{-i})\theta_l - t_i(r_l, r_{-i}))] & \geq E_{-i}[u(g_i(\hat{r}_i, r_{-i})\theta_l - t_i(\hat{r}_i, r_{-i}))] \quad (IC) \\
E_{-i}[u(g_i(r_l, r_{-i})\theta_l - t_i(r_l, r_{-i}))] & \geq 0 \quad (IR) \\
\forall l, \hat{r}_i \in \{1, ..., N\} \text{ such that } \hat{r}_i = r_{l-1}
\end{align*}
\]

The following proposition provides the optimal allocation rule in the new framework where customers are risk averse:

**Proposition 7.** When customers are risk averse with CARA utility function, \( u(x) = -e^{-\alpha x} \), producer adds insurance options to the markdown scheme. Retailer offers a menu of priority rankings and upfront payments \( <r_l, \bar{p}_l> \), where each ranking \( r_l \) is supplemented with \( \theta_l = r^{-1}(r_l) \) units of insurance \( I_{r_l} \):

\[
I_{r_l}(C_l(r); S) = \begin{cases} 
1 & \text{if } S \leq C_l(r) \\
1 & \text{if } S > C_l(r) \text{ and } g_i(r_l, r_{-i}) = 0 \\
0 & \text{if otherwise}
\end{cases}
\]

where \( C_l(r) \) is the number of people who are served before \( r_l \). Optimal allocation rule is the same characterized by equation 14 and the set of priority price, \( \{\bar{p}_l\} \), is obtained by
iterative solution of the following equalities with $\tilde{p}_T = \theta_T$:

$$\tilde{p}_{l+1} = \theta_{l+1} - u^{-1}(E_{-i}[u(g(r_l, r_{-i})\Delta\theta_l + \theta_l - \tilde{p}_l)])$$

(17)

As suggested by equation 16, one unit of insurance $I_{r_l}$ provides one unit of payment transfer in case demand is too high to serve a customer with ranking order $r_l$. In other words, the insurance pays to the customer if realized number of customers with higher ranking orders, denoted by $C_l(r)$, become greater than total supply $S$.

Therefore, customers bear no risk by transferring the uncertainties to the monopolist. The intuition behind the optimal equilibrium is that producer is risk neutral and customers are risk-averse. Thus, the optimal equilibrium includes contracts where seller compensates customers’ uncertainties with insurance, and in return, gets paid higher in expected terms.

As implied by Proposition 7 and Proposition 2, monopolist chooses market coverage by designing a contract where no customer with valuation lower than $\theta_T$ is willing to participate. By choosing the lowest service order, he decides what portion of market to serve. An appealing operational implication is for situations where demand by the participating customers falls below the supply level. The exceeding amount will be thrown away or unsold, and customers with lower valuations will not be served by any kind of clearance sales - what we call as “commodity burning”. Such experiments are seen, for instance, in airlines’ ticket sales. United Airlines, for example, would fly with fully occupied coach seats and empty business class seats if it turns out that the demand for the latter is low. Or as another instance, Filene’s basement was willing to give the remaining items to charity after a specific date if unsold until then (Figure 5). In such examples, seller forgoes profit from last minute sales. By committing to a preannounced pricing schedule, supplier extracts more from high value customers who are incentivized to purchase the product at higher prices.

Proposition 7 states that upfront payment schedule combined with insurance yields to higher profit relative to pricing schemes with no insurance. By insuring customers against losses, seller can extract more ex-ante surplus from the market. It is because risk averse buyers pay higher in expected terms if asked to pay in advance for a certain quality of service relative to the contracts where they should pay after the uncertainties are realized. Therefore, an operational advice for producers is to bear more risk and offer insurance schemes in order to maximize profits when customers are sensitive towards disruptions or availability of product. Similar mechanisms could be called “cashback” mechanisms, as in Ho et al. (2013), in which seller pays back some of the amount in the form of cashback so that he can effectively perform second degree price discrimination among the customers.

One should note that total upfront payment, $\tilde{p}_{l+1}$, includes both insurance premium and the price for the delivery of the product. Therefore, we can decompose total payment into two segments: insurance premium $\tilde{p}_{ri}$ per unit insured and payment for the priority service $p(r_l)$:
\[ \bar{p}_i = p(r_i) + \theta_i \bar{p}_{r_i} \]

Insurance \( I_{r_i} \) pays off when the product is not allocated to the customer, i.e., \( g(r_i, r_{-i}) = 0 \) or \( I_{r_i} = 1 \). In addition, expected payoff of one unit of the insurance is equal to its premium for an actuarially fair insurance:

\[ \bar{p}_{r_i} = E_{r_{-i}} I_{r_i} (C_i(r); S) \]  \hspace{1cm} (18)

Therefore, payment for the priority service can be stated as:

\[ p(r_i) = \bar{p}_i - \theta_i \bar{p}_{r_i} \]  \hspace{1cm} (19)

In all, total upfront price can be seen as two separate payments: payment for the priority service and insurance against the risk of unavailability. Seller’s optimal strategy is to offer contingent contracts with upfront payments and supplementary insurance in order to maximize his expected revenue. At the same time, risk averse customers participate in a lottery-type mechanism where they may not receive the product if demand becomes too high.

5 Concluding Remarks

We study the revenue maximizing pricing schedule of a monopolist in the presence of uncertain demand and against heterogeneous customers who may differ in their valuations for the product. This paper considers a structure of uncertain demand and case where supply may contain single or multiple units of product. In addition, it considers the scenarios where seller commits to a future price path. In so doing, we contribute to the literature in the following ways.

First, we are able to illustrate that the optimal policy is to price discriminate via performing multistep markdown pricing or priority schemes, as performed by the “pricetack.com” and “pricefalls.com” websites. The pricing schedule is preannounced in the way that the price path is introduced in the beginning of the horizon. On the other hand, priority scheme could be carried out via an auction with simultaneous bids when supplier prefers to collect sealed bids and allocate the products in a single period. Both falling price schedule and priority options incentivize higher value customers to purchase at higher prices, and therefore, enable the monopolist to extract more surplus from market. This is because each customer must decide either to buy the product at a higher price or at a lower price but with a certain risk. In the “pricetack.com” case, customer may wait to purchase item at a discounted rate but will face the risk of the product selling out during the sales season. We show that when there is a single unit of supply, dynamic sales with a preannounced price path performs as well as the optimal mechanism in which seller offers menu of ranking-price options. However,
when there are multiple units of supply, we find that it is not optimal to implement a scheme with plunging prices unless seller hides information regarding the quantity sold and quantity remaining at each period. The lower revenue of dynamic pricing, when supply information is revealed, is due to the revelation of information to customers. They update their strategies at every single sale period after they observe the number of quantities sold and the amount left. Fortunately, there are two ways seller can avoid this and achieve optimal revenue. First is to hide number of available stock of goods at every single period except the beginning supply which is a common knowledge. And second is to implement a specific form of modified priority pricing in which customers choose their desired price in the beginning and are charged conditional on receiving the product. To implement this, seller may receive each customer’s choice and credit card information in the beginning, allocate the goods according to their ranking orders and charge each one with the corresponding price if allocated the product. Finally, monopolist’s allocation rule is different from the efficient scheme in that “commodity burning” might happen in equilibrium. That is, if high value customers are too few to purchase the product at the very early stages, monopolist may dispose of the goods rather than putting them up for sale at very low prices.

In addition, we show that when customers are sensitive to the availability of product or quality of service, represented by risk averse utility functions, there exists another pricing strategy that dominates the proposed multistep or priority pricing schedule. In this case, monopolist offers contingent contracts, complemented by a menu of options with upfront payment and supplementary insurance, and serves customers conditional on the realization of demand. A customer pays in advance for a product and risks loss of her money if she does not receive the product. On the other hand, seller offers her some reimbursement to insure her against possible loss.

Future research should take into account more detailed allocation and pricing schemes that are made feasible by recent technological innovations, e.g., markdown pricing schemes with unknown price floor. Another direction is the introduction of different forms of knowledge about the distribution of customers. One way is to consider unknown aggregate population, or different types of customers (informed vs. uninformed about the rest of the demand) which could significantly change the optimal pricing scheme. Furthermore, future studies need to take into account the possibility that seller knows the distribution of potential demand, but is unaware of the presence of customers in the market. For instance, seller may learn about the distribution of customers’ valuations by experience, but he might not know with certainty which customers need the product during the sale period.
References


Appendix: Proofs of the Propositions

. Proof of Proposition 2

Given the linearity of utility functions and the symmetry assumption that customers are drawn independently from an identical distribution, we can denote the expected payment of choosing ranking \( r \) by \( p(r) = p_i(r) \forall i \), which is the same across all individuals. It simplifies our analysis since there will be no heterogeneity among the customers, and hence, one can take expectations of the equations without worrying about differences across individuals. Therefore, the maximization problem and the constraints reduce to the following:

\[
\begin{align*}
\text{Max } & ME_{r,l}p(r_l) = \frac{M}{N} \sum_{k=1}^{N} p(r_k) \\
\text{maximized over } & <g(r), \{p(r_k)\}> \\
\text{subject to } & q(r_l)\theta_l - p(r_l) \geq q(\hat{r}_i)\theta_l - p(\hat{r}_i) \quad (IC) \\
& \forall l, \hat{r}_i \in \{1, ..., N\} \text{ such that } \hat{r}_i = r_{l-1} \\
& \text{and } q(r_l)\theta_l - p(r_l) \geq 0 \forall l \in \{1, ..., N\} \quad (IR)
\end{align*}
\]

Note that prices are total expected payments and not prices contingent on delivery. Before solving the maximization problem, one can see that if the participation constraint for a customer with valuation \( \theta_T \) holds, then all the IR constraints for the others can be ignored. It is easy to show that if the IR constraint for \( \theta_l \) and the incentive compatibility condition for \( \theta_{l+1} \) hold, i.e.,

\[q(r_{l+1})\theta_{l+1} - p(r_{l+1}) \geq q(r_l)\theta_{l+1} - p(r_l) \text{ and } q(r_l)\theta_l - p(r_l) \geq 0\]

then, since \( \theta_{l+1} > \theta_l \), the IR condition holds for \( \theta_{l+1} \) automatically, i.e., \( q(r_{l+1})\theta_{l+1} - p(r_{l+1}) \geq 0 \). Therefore, it is sufficient to consider only the individual rationality constraint of the lowest type, \( \theta_T \). Given the IR constraint for \( \theta_T \), the optimal price \( p^*_T \) will be:

\[p^*_T = q(r_T)\theta_T\]  

One can see that the other incentive compatibility conditions should hold with equality to maximize revenue. Therefore, price \( p(r_l) \) is obtained by iteratively plugging the price \( p(r_{l-1}) \) in the IC condition of customer with valuation \( v_i = \theta_l \):
\[ p(r_l) = \theta_l q(r_l) + \sum_{j=T}^{l-1} (\theta_j - \theta_{j+1}) q(r_j) \] (22)

The next step includes plugging optimal prices in equation 20 and solving the maximization problem:

\[ \text{Max } ME_{r_l} [\theta_l q(r_l) + \sum_{j=T}^{l-1} (\theta_j - \theta_{j+1}) q(r_j)] \] 

maximized over \(<g(r), T>\)

The expectation term could be written as the sum of all possibilities as:

\[ \text{Max } M \frac{1}{N} \sum_{l=T}^{N} [\theta_l q(r_l) + \sum_{j=T}^{l-1} (\theta_j - \theta_{j+1}) q(r_j)] \] (23)

which is equal to:

\[ \text{Max } M \frac{1}{N} \sum_{l=T}^{N-1} [\theta_l q(r_l) + \sum_{j=T}^{l-1} (\theta_j - \theta_{j+1}) q(r_j)] \] (24)

where \(\Delta_{\theta_l} = \theta_{l+1} - \theta_l\). The equation above implies that if valuations be equally distanced from each other, then \(\Delta_{\theta_l}\) will be a constant term. Therefore, the coefficient of \(q(r_l)\) - defined as \(\psi(\theta_l) = \theta_l - (N - l) \Delta_{\theta_l}\) - will be increasing in \(l\). Since the profit function is linear in the \(q(r_l)\) terms, then it is optimal to make \(q(r_N)\) as big as possible. Given, \(q(r_N)\), then it is optimal to make \(q(r_{N-1})\) as big as possible. The allocation rule continues in the same way until the seller stops before allocating anything to type \(T - 1\) for whom the coefficient of \(q(r_{T-1})\) becomes negative. Therefore, the allocation rule includes a priority scheme denoted by equation 11 where \(T\) is obtained from solving:

\[ T = \text{ArgMin}_l \{\theta_l - (N - l) \Delta_{\theta_l}\} \text{ such that } \{\theta_l - (N - l) \Delta_{\theta_l} \geq 0\} \] (26)

. Proof of Proposition 3

The proof is very straight forward by stating that customers are risk neutral and only care about expected utilities. Utility functions are quasilinear, and therefore, one should redo the proof for Proposition 2 by using the new utility functions after rewriting them in terms of contingent prices,
\[ p_l^*: \]
\[ q^*(r_l) (\theta_l - p_l^*) = q^*(r_l) \theta_l - p_l^* \]

**Proof of Proposition 4**

For any option, \(<r_l, p_l^*>\), in plan A, customer with valuation \(\theta_l\) shall receive the product with probability \(q_l^*\) and will pay \(\tilde{p}_l^*\) contingent on delivery. Therefore, expected utility will be:

\[ q_l^* (\theta_l - p_l^*) \]

Alternatively, the monopolist could implement a markdown scheme, B, in which price of the product declines according to the pricing scheme \(<\tilde{p}_N^*, \tilde{p}_{N-1}^*, ..., \tilde{p}_T^*>\). In this case, prices decline every period according to a preannounced schedule. If a customer wants to wait until a later period to purchase the item, she will expect to pay lower but will also face higher risk of unavailability. Initial price will be \(\tilde{p}_N^*\), and given that any customer with type \(\theta_{N-1}\) purchases the product in period 2 at price \(\tilde{p}_{N-1}^*\), it attracts the highest value customers, \(\theta_N\), to purchase the product at the initial price. This is because all the IC and IR conditions of the customers are the same as those in the mechanism with priority pricing.

If no customer has the valuation \(\theta_N\) and price declines to \(\theta_{N-1}\) in the following period, customers will update their beliefs with the information that there is no person with valuation \(\theta_N\). Therefore, one could use Bayesian update to find the new distribution function, \(f_n(.\)) for any remaining possible valuation \(l \in \{1, 2, ..., N - 1\}\):

\[ f_n(\theta_l) = \frac{f(\theta_l)}{F(\theta_{N-1})} \]

Given the new distribution function and that any customer with valuation \(\theta_l\) waits for price \(\tilde{p}_l^*\) to purchase the product, all individuals including the customer with valuation \(\theta_{N-1}\) will update her beliefs, and the incentive compatibility inequalities will change accordingly. The conditions will change only in that the new set of \(q_l^*\) will be obtained by using the new distribution function \(f_n(.)\). One should note that since customers' utilities are quasilinear, all the IC conditions stay the same. Therefore, customer \(\theta_{N-1}\)'s behavior will be the same and she will choose to purchase the product at period 2, if available. Similarly, if there were no sales in the first two periods, beliefs will be updated correspondingly, IC conditions will stay unchanged in the same way, and customer with valuation \(\theta_{N-2}\) will still choose to buy the product at \(\tilde{p}_{N-2}^*\). The argument continues in the same way until we show that individuals with valuation \(\theta_T\) choose to purchase at a price equal to their maximum willingness to pay, \(\tilde{p}_T^* = \theta_T\).
We complete the proof by stating that since mechanism A is optimal and mechanism B leads to equivalent strategies and prices, then, markdown pricing leads to the optimal profit as well.

Proof of Proposition 5

The monopolist’s maximization problem is the same as the problem in Proposition 2 with identical incentive compatibility and individual rationality constraints. However, the difference is that the number of available units for sale is more than one. Therefore, the seller allocates more than one unit rather than only one unit to the bidders. Similarly, because the maximization problem is a linear equation with bigger coefficients on the expected probability of delivery to the higher ranked customers, the monopolist adopts a similar priority scheme. He starts with allocating the units to the highest order customers, $r_N$, and charges them $p_N$. Then, the monopolist allocates the leftover to the customers with the next highest ranking order, $r_{N-1}$ and charges them $p^*_{N-1}$. The seller continues delivering the products to the customers based on their orders until it completely sells out. A tie-braking rule is also adopted. When number of individuals demanding the product at current price level becomes greater than number of units left, customers are served with equal chances. We complete the proof by stating that the price equations remain the same since incentive compatibility and individual rationality constraints remain the same in the new truth-telling mechanism.

Proof of Proposition 6

Proposition 5 provides the optimal pricing and allocation rule for a seller with multiple units. To implement the same pricing and allocation rules, designer of the markdown pricing should make sure that the incentives of the customers are not distorted throughout the process of markdowns. And the only way to implement such a mechanism is that no additional information be revealed during the markdowns compared with the optimal rules. Therefore, either all trading and commitments should occur before the realization of uncertainties or the information regarding the number of remaining supplies be hidden by the seller.

Proof of Proposition 7

The seller is risk neutral and designs a contract to maximize total expected profit. On the other hand, customers are risk averse. For a risk averse buyer, an allocation that delivers a certain average utility is superior to any original allocation that delivers an uncertain utility with the same expected payoff. Therefore, using the specific CARA property of the utility function, it is straightforward to show that optimal allocation requires that the risk neutral monopolist bears the risk in equilibrium.
The mechanism allows the retailer to implement any set of money transfers. Therefore, the seller who is willing to bear all or a portion of customers' risks, will consider offering payments to the customers in the form of insurance contracts. We show that the ability to make insurance transfers enables the seller to extract the maximum possible surplus from the demand side.

One unit of insurance, $I_{r_1}$, costs $\tilde{p}_{r_1}$. It provides one unit of payment for a customer with ranking order $r_1$ when demand by higher order customers is too high to serve him. If customer $i$ with type $\theta_l$ supplements her choice, $<r_1,t_i(r_1,r_{-i})>$, with $x_i$ units of the corresponding insurance, $<I_{r_1},\tilde{p}_{r_1}>$, her expected utility will be:

$$U(r_1,x_i;\theta_l) = E_{-i}[u(g_i(r_1,r_{-i})\theta_l - t_i(r_1,r_{-i}) + I_{r_1}x_i - x_i\tilde{p}_{r_1})]$$ (27)

Full insurance of the customer requires that:

$$I_{r_1} = 1 - g_i(r_1,r_{-i}), \quad t_i(r_1,r_{-i}) = p(r_1), \quad x_i = \theta_l$$ (28)

which means that the optimal insurance should provide $\theta_l$ units of insurance to a customer with valuation $\theta_l$, and should pay him when he does not receive the product. Furthermore, the transfer function $t_i(r_1,r_{-i})$ should be independent of the choices of other customers. Therefore, it can be denoted by $p(r_1)$, a function that depends only on the customer’s own choice. Therefore, the utility function will become:

$$U(r_1,x_i = \theta_l;\theta_l) = u(\theta_l - p(r_1) - \theta_l\tilde{p}_{r_1})$$ (29)

We follow the same steps as in the proof for the Proposition 2. We combining priority price with insurance premium and denote total payment for choice $l$ by $\tilde{p}_l = p(r_1) + \theta_l\tilde{p}_{r_1}$. Considering that the lowest participating customer, the cut-off type with valuation $\theta_T$, obtains zero expected utility in equilibrium, one can obtain $\tilde{p}_T = \theta_T$. Incentive compatibility conditions hold with equality and give us equilibrium prices. Payment by type $\theta_{l+1}$ can be obtained by repetitive iterations of the IC conditions (starting from the IC condition of the customer with type $\theta_T$):

$$u(\theta_{l+1} - \tilde{p}_{l+1}) = E_{-i}[u(g(r_1,r_{-i})\theta_{l+1} + (1 - g(r_1,r_{-i}))\theta_l - \tilde{p}_l)]$$ (30)

The equation above has a unique solution where $\tilde{p}_{l+1} > \tilde{p}_l$. It is because the utility function is strictly increasing and we have:

$$g(r_1,r_{-i})\theta_{l+1} + (1 - g(r_1,r_{-i}))\theta_l < \theta_{l+1}$$
which means that the customer pays an upfront amount, receives an insurance and becomes indifferent between receiving and not receiving the product. Rewriting Equation 30, one can obtain:

\[ \tilde{p}_{t+1} = \theta_{t+1} - u^{-1}(E_{-i}[u(g(r_l, r_{-i})\Delta \theta_l + \theta_i - \tilde{p}_i)]) \]

and the iterative solution of each price in terms of the lower price gives us:

\[ \tilde{p}_{t+1} = \theta_{t+1} - u^{-1}(E_{-i}[u(g(r_l, r_{-i})\Delta \theta_l + u^{-1}(E_{-i}[u(g(r_{l-1}, r_{-i})\Delta \theta_{l-1} + ...)])])) \] (31)

Assuming that the valuations are equally distanced, i.e., \( \Delta \theta_l = \Delta \forall l \), prices can be used to rewrite the maximization problem as:

\[ \text{Max } M \frac{1}{N} \sum_{t=T}^{N} \{ \theta_{t+1} - u^{-1}(E_{-i}[u(g(r_l, r_{-i})\Delta \theta_l + u^{-1}(E_{-i}[u(g(r_{l-1}, r_{-i})\Delta \theta_{l-1} + ...)])]] \} \] (32)

maximized over \( < g(r), T > \)

According to the new maximization problem, it is sufficient to define the allocation rule and cut-off rank to solve for the optimal scheme. Customers have strictly increasing and concave utility functions, and therefore, inverse utility function \( u^{-1}(.) \) is also strictly increasing. Therefore, it becomes easy to see that solving the maximization problem includes making \( g(r_N, r_{-i}) \) as large as possible. Then, given \( g(r_N, r_{-i}) \), one should make \( g(r_{N-1}, r_{-i}) \) as big as possible and continue until the cut-off rank gets served by the allocation rule, \( g(r_T, r_{-i}) \). Therefore, the optimal allocation rule will be the same as in the case with risk neutral customers (equation 14), where the seller implements priority pricing with higher ranked customers served first. To obtain the cut-off rank, one should note that it is not possible to provide a close form solution and it should be obtained by computing and comparing equation 32 for different values of \( T \).
Figure 1: Cell Phone Jammer Sold by a Falling Price Schedule on “Pricetack.com”

Figure 2: Markdown Pricing on “Pricefalls.com”
Figure 3: Filene’s Basement

Figure 4: Aalsmeer Flower Auction
<table>
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<tr>
<th>Reference</th>
<th>Uncertainty</th>
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<th>Commitment(^b)</th>
<th>Aggregate Demand</th>
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<td>Liu and Van Ryzin (2008)(^c)</td>
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<td>Present Paper</td>
<td>Demand</td>
<td>Yes</td>
<td>Incomplete</td>
<td>Yes</td>
<td>Known and Finite</td>
</tr>
</tbody>
</table>

\(^a\)Under incomplete information, customers have privately known valuations drawn independently from a known distribution.
\(^b\)The seller has commitment on pricing schedule.
\(^c\)Seller has two periods to sell the product and has no power over determining the number of pricing steps in this paper. Seller’s tools are deliberate rationing and prices in two period.
\(^d\)Customers have waiting costs in this framework which could be heterogeneous.
\(^e\)Customers have waiting costs in this framework, and they could be heterogeneous in terms of having different degrees of impatience.

Table 1: Summary of related research on dynamic pricing with strategic customers