Matching and Revenue-Risk Sharing Contract with Heterogeneous Risk Averse Supplier and Retailer

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August 6, 2018

Abstract

The following analysis covers a decentralized matching model with a bilateral supply chain including risk averse suppliers and retailers. We characterize the stable matchings with endogenous production plan and revenue sharing contract. Depending on the trade-off between the expected revenue and demand uncertainty, both positive assortative matching (PAM) and negative assortative matching (NAM) can occur at a stable matching equilibrium. These results shed light on the role of the endogenous production plan in coordination supply chain allocation and inventory management.

Keywords — assortative matching, supply chain management, risk sharing, stable matching

1 Introduction

Risk averse agents in bilateral matching markets form their coordination supply chains according to their preferences on agents other the side. And the matched pairs in the coordination supply chain must necessarily agree upon the desired sharing contract and a production plan. For example, ahead of the holiday season, the cosmetics supply chain signs a supply contract and faces

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the challenge of balancing its stocks. In a two-sided market, each retailer and supplier is able to sign a contract with their favorable partners and various types of contracts. If the retailer and supplier sign a buyback contract, the retailer will order the proper amount of product with approximately a 30% discount off of retail prices. If products are unpopular in the market, the extra stock will be returned to the supplier. Under this buy-back contract, cosmetics manufacturers commonly face an excessive stock issue, thus many brands reduce the total production to avoid the risk of overstocking since the old batch of cosmetics will quickly deteriorate. In the kind of this newsvendor problem, the risk averse supplier has a better chance to produce an insufficient amount, which means that the product might become out of stock more quickly and the supply chain may lose potential profit. To refrain from low production from a high risk averse supplier, some retail companies sign an “exclusive products” contract with manufacturers to produce goods that can only be sold in their stores. In this way, the supplier takes less of the share of profit and retailers are prohibited from returning excessive products.

The uncertainty of the market profoundly increases the importance of risk averseness of each of the agents in the market. Due to the fact that cosmetics production takes 4-6 weeks of leading time and that customized products may take even longer, so it is a crucial issue for the supply chain to sign an optimal contract with an ideal partner.

Inspired by the cosmetics supply chain question, we developed a matching model in the endogenous production plan scenario to answer questions: When risk averse retailers and suppliers need to team up to become a supply chain ahead of a sale season, which company is the best to work with? How much product should be made? And how can they share the revenue in a risky industry? By anticipating the risk from an uncertain demand, agents encounter multiple dimensions of strategic choices in the balance between alleviating riskiness and achieving higher profit. The coordination of supply chains has been studied in a significant body of literature, where upper-stream suppliers and down-stream retailers are capable of coordinating to maximize the expected profit in a risky business environment. Also, due to the utilization of the Internet, the information available to supply chain partners in the market becomes more transparent, so that the searching cost to find suppliers and retailers has been heavily reduced and agents are more able to match with various
partners. Taking these facts into account, this paper is focuses on the frictionless matching in the coordination supply chain. Therefore, the intention of this study is to introduce an approach to suppliers’ and retailers’ strategies, and to identify three dimensions of a strategy set: optimal coordinating contracts, matching patterns and production choices.

This paper discusses the match between heterogeneous risk averse agents on both sides of the supply chain. Considering the riskiness of the business environment, a less risk averse agent can either match with an opposing type of agent to be allocated a larger proportion of uncertain profit and riskiness, or match with an identical agent to achieve a more challenging project which is associated with higher expected profit and higher variance. Similarly for the high risk averse agent, this model offers low risk averse agents two channels for maximizing their expected return: to match with a low risk averse agent but to take less profit and less risk, or team up with an identical agent to choose a safer production plan with an equally split contract. Although the heterogeneity of suppliers and retailers can be observed in various aspects, this model focuses on how risk aversion types influence matching and coordination contracts in supply chains.

In this model, each product’s demand is uncertain at the time of making the decision, so that the bilateral supply chain is required to choose the amount of production they commit to produce before the demand is revealed. We use the mean-variance approach in available production choice set, so that the supply chain endogenously decides an optimal positively-correlated mean-variance combination. Naturally, a more profitable production plan is associated with a higher uncertainty of demand.

In economics theory, assortative matching was introduced by Becker (1973) and it can applied to determine the matching strategy. This set of matching strategy is the kind of model used to describe the decision to match with the same(positive) or opposite(negative) type. In a coordinated supply chain management, we suggest assortative matching as a proper approach to analyze the sorting model of bilateral supply chains. Assortative matching is mutually acceptable stable equilibrium by both agents, therefore, the equilibrium creates a stable matching outcome.

In sum, this model determines a stable matching strategy and risk-revenue sharing contract
under a risky business environment. As we characterized the endogenous production selection in the range of positive correlated mean and variance, the model reveals that under risky business conditions, both positive and negative assortative matching can occur in equilibrium. More specifically the “easy”\footnote{The easiness here is in reference to the low demand uncertainty associated with a higher marginal expected return.} business environment causes a stronger centralized effect that leads to negative assortative matching; the “challenging”\footnote{Challenging here indicates the marginal increase of expected return being higher when the project has relatively high uncertainty of demand.} environment gives rise to positive assortative matching due to the decentralized effect. Under the setting of both retailers and suppliers being heterogeneous with respect to their risk aversion type, the model provides the solution of: (i), matching patterns; (ii), the optimal risk-profit sharing contract between each distinct supply chain; (iii), optimal production plan.

2 Literature

The previous literature is scant in the field of matching in coordinating supply chains, and the existing literature on it is focused on discussing the matching in unbalanced markets, i.e. countable suppliers and $N$ retailers case. Azevedo and Leshno\footnote{Azevedo and Leshno (2016) discuss this matching market with finite heterogeneous ability suppliers and continuous types of retailers.} discuss this matching market with finite heterogeneous ability suppliers and continuous types of retailers. We was inspired by the unbalanced matching in supply chain coordination, but we focus on extending both sides of the supply chain to $N$. Therefore, this article studies a different class of matching model, where both sides of agents can be continuous types and agents are heterogeneous in respect to risk aversion.

This study closely follows two lines of literature: risk aversion in the coordination supply chain and the bilateral matching model. The extant supply chain management literature is lacking a model to illustrate the process of sorting for risk averse agents. By modeling both contract and production plan in the matching process, this paper illustrates a new angle of supply chain management.

Most existing studies on coordinating supply profoundly address the advantages and limitations
of revenue sharing contracts, and make comparison to the non-coordinating ones see (Cachon and Lariviere, 2005). Based on Pasternack (2008), the buyback contract allows the coordinating supply chain for profit sharing. Also the risk averse agents in the supply chain have drawn more attention. Gan et al. (2003) firstly illustrated how risk aversion would alter the classical coordination supply chain with Pareto-optimal solutions. Chen et al. (2013) study the stable matching of one risk averse retailer and a group of competing suppliers. They allow the model includes more than one supplier and their model indicates the least risk averse agents bear all risk, and the contract determines the inventory for maximum expected payoff. They also emphasize the importance of stability, which guarantees the coordinate supply chain is maximizing both agents’ individual and entire supply chain’s profits simultaneously without deviation.

The methodology literature stream we have been following is the stream of assortative matching with a risk sharing model. Started with Becker (1973), the assortative matching in the bilateral groups model was proved to be negative in equilibrium, but the result was blamed due to the inconsistency of empirical evidence. The previous literature, (Chiappori and Reny, 2016) mainly discussed the scenario where agents, both retailer and supplier, take the project as a given. In doing so, they can access channels to maximize their expected return by project choice. However, it is more realistic that the agents are free to choose their strategic production and sign the optimal revenue sharing contract to maximize their expected profit in a coordinated supply chain. The classical assortative matching model predicts the negative assortative matching (See Chiappori and Reny (2016); Legros and Newman (2000); Schulhofer-Wohl (2006)). A less risk-averse agent acts as a good seller of insurance while a more risk-averse agent is a good buyer of it. Moreover, a stable match must reflect such seller-buyer nature of risk sharing. More specifically, Li et al. (2013) proves that when couples can control the variance of their project and share according to the linear sharing rule the positive assortative matching can be optimal. Yu Wang et al. (2014) also presented a model of possible PAM and applied it on the informal insurance market.
3 The model

Consider a market with balanced groups of $N$ suppliers and $N$ retailers. Suppliers and retailers need to team up as bilateral supply chains to produce and sell product during the sale season. Both suppliers and retailers are risk averse. Suppliers and retailers possess CARA utility function over money outcomes, $w$:

$$u(w, r) = -e^{-rw},$$

where $r \in [\underline{r}, \overline{r}]$ is an Arrow-Pratt degree of risk aversion coefficient.

Agents are heterogeneous with respect to their risk aversion coefficients. Agent $i$’s risk aversion coefficient is denoted by $r_i$ and is publicly known. The supply chain $(i, j)$ choose a production plan $(\mu_{ij}, \sigma_{ij}^2)$. $\mu$ represents how profitable the product is and $\sigma^2$ indicates the risk caused by demand uncertainty. The product is differentiated and the supply chain has the market power to affect the price and the unit profit accordingly.

$P_{ij}$ represents the unit profit function of products $ij$, that is

**Assumption 1.**

$$P_{ij} = \alpha - \frac{\alpha}{k} Q_{ij} + \epsilon_{ij}$$

The unit profit depicted by the quantity $Q_{ij}$ produced which chosen by supply chain $(i, j)$. The total market size is $\alpha$, and the consumers’ sensitivity of price captured by $\beta$. We also assume the ratio of market size and price sensitivity as $k$. The production uncertainty represented by $\epsilon_{ij} \sim N(0, \sigma_{ij}^2)$.

The coordination supply chain jointly determine the price by maximizing the expectation of profit function $\pi_{ij}$:

$$E[\pi_{ij}] = E[P_{ij}Q_{ij}]$$

The optimal price for supply chain $(i, j)$ is $\frac{k}{2}$, and the profit function for production $(i, j)$. $\pi_{ij}$ follows $N(\frac{\alpha k}{4}, \frac{k^2}{4} \sigma_{ij}^2)$.

In sum, each unique supply chain $(i, j)$ chooses a production plan $\Pi_{ij}$ which is a random variable.
distributed normally with mean \( \frac{\alpha_{ij} k}{4} \) and variance \( \frac{k^2 \sigma_{ij}^2}{4} \). We make the following assumption about the frontier of the feasible production plans:

**Assumption 2.** A production plan \((\alpha, \sigma^2)\) is feasible if and only if:

\[
\alpha \leq (\sigma^2)^\gamma
\]  

(3)

The parameter \( \gamma \) denotes how costly it is to increase expected profit by increasing the demand variance. By imposing an increasing function of expectation of \( \pi \) with \( 0 < \gamma < 1 \), we capture the idea that higher expected profit is associated with higher uncertainty in demand. Concavity implies decreasing marginal expected returns to risk tolerance.

First, suppliers and retailers are matched into supply chains, and then the supply chain that is formed with supplier \( i \) and retailer \( j \) commit to the production plan \((\alpha_{ij}, \sigma_{ij}^2)\) and a revenue sharing contract. During the sale season, the supplier produces according to the production plan they agreed on and the retailer sells it according to the contract. At the end of sale season, the shocks are realized and supplier and retailer split the revenue according to their contract.

### 3.1 Equilibrium concept

A matching is a one-to-one function \( f : N \rightarrow N \) mapping each supplier to a retailer.

**Matching structure** A matching function \( f \) is positive (negative) assortative\(^3\) if

\[
r_i \geq r_j
\]  

(4)

implies

\[
r_{f(i)} \geq (\leq) r_{f(j)}
\]  

(5)

\(^3\)Suppose a supply chain matching market contains 100 suppliers and 100 retailers with heterogeneous risk aversion type. From the lowest Arrow-Pratt degree of risk averse (the least risk averse) \( i = 1 \) to the highest Arrow-Pratt degree of risk averse (the most risk averse) \( i = 100 \) we mark suppliers as \( \{S_1, S_2, \ldots, S_{100}\} \) retailers as \( \{R_1, R_2, \ldots, R_{100}\} \). Negative assortative matching is the matching pattern in which agents are paired as \((S_1, R_{100}), (S_2, R_{99}), (S_3, R_{98}), \ldots, (S_{100}, R_1)\), which is the most risk averse retailer willing to match with least risk averse supplier in the market. In contrast, if positive assortative matching is the one where the most risk averse retailers is matched with the most risk averse supplier, and vice versa. More specifically, under PAM the supply chain will pair as \((S_1, R_1), (S_2, R_2), (S_3, R_3), \ldots, (S_{100}, R_{100})\).
**Contracts** Fix a supply chain \((i, j)\) and a production plan \(\pi_{ij} \sim N\left(\frac{\alpha_i k}{\beta}, \frac{\beta^2 \sigma_j^2}{\beta}ight)\). A contract \([m_i(\cdot), m_j(\cdot)]\) is a pair of functions such that when realized profit is \(y\), the supplier’s payoff is \(m_i(\cdot)\) and retailer’s payoff is \(m_j(\cdot)\).

Let \((u_i, u_j)\) stand for a payoff pair for the supplier and the retailer. \((u_i, u_j)\) is feasible if there exist a production plan \((\mu, \sigma)\) and a contract \((m_i(\pi), m_j(\pi))\) with

\[
m_i(\pi) + m_j(\pi) \leq \pi
\]  

for all \(y\), and

\[
u_i \leq E[m_i(\pi)], u_j \leq E[m_j(\pi)]
\]

An equilibrium consists of a payoff allocation \(\{(u_i, u_j)\}_{i,j} \in N\) and a matching function \(f\) such that

1. For all \(i, j\) if \(f(i) = j\), then \((u_i, u_j)\) is feasible for \((i, j)\);

2. There exists no \((i, j)\) and a payoff pair \((\tilde{u}_i, \tilde{u}_j)\) such that:

   (a) \((\tilde{u}_i, \tilde{u}_j)\) is feasible for \((i, j)\), and

   (b) \(\tilde{u}_i > u_i, \tilde{u}_j > u_j\).

4 Feasible payoffs

In this section we characterize the set of feasible payoffs for an arbitrary supply chain \((i, j)\). We separate this in two steps: first, we characterize the the achievable payoffs, taking as given the production plan. Then based on this, we characterize Pareto frontier of possible payoffs by varying the production plan.
4.1 Optimal contract

Stability requires that the contract of a supply chain generates a Pareto optimal payoff vector, given the production plan \( \pi_{ij} \sim (\alpha_{ij}, \sigma_{ij}^2) \). Therefore any equilibrium contract must solve:

\[
\max_{m_{ij}} \{Eu_i[\pi_{ij} - m_{ij}(\pi_{ij})]\} + \lambda\{Eu_j[m_{ij}(\pi_{ij})]\}
\]  

(8)

By varying \( \lambda \) over \( \mathbb{R}_{++} \), the solution to this problem spans the Pareto frontier of possible payoff for the matched pair.

Proposition 1 characterizes the solution of this problem for arbitrary \( \lambda \)

**Proposition 1.** Fix \( \lambda > 0 \), contract that solves (7) is linear with respect to total revenue \( \pi_{ij} \) and is given by:

\[
\pi_{ij} - m_{ij}(\pi_{ij}) = \frac{r_j \pi_{ij} - \log \lambda}{r_i + r_j}
\]  

(9)

\[
m_{ij}(\pi_{ij}) = \frac{r_i \pi_{ij} + \log \lambda}{r_i + r_j}
\]  

(10)

*Proof see appendix.*

Let \( CE_i(m_i(\pi_{ij})) \) and \( CE_j(m_j(\pi_{ij})) \) represents the certainty equivalent of the lottery induced by contracts \( m_i(\pi_{ij}) \) and \( m_j(\pi_{ij}) \) for agent \( i \) and \( j \) respectively. We define the compound Arrow-Pratt coefficient of risk aversion for supplier \( j \) and retailer \( i \) as \( R_{ij} = \frac{r_ir_j}{r_i + r_j} \).

**Lemma 4.1.** If \( (m_i, m_j) \) solves (8), then

\[
CE_i(m_i(\pi_{ij})) + CE_j(m_j(y\pi_{ij})) = \alpha_{ij}(\sigma_{ij}^2)\frac{k}{4} - R_{ij}\frac{k^2}{8}\sigma_{ij}^2
\]  

(11)

*Proof in appendix.*

Let \( CE_{ij}(\alpha, \sigma) \) represents the total certainty equivalent for the team where they choose across at production plan \((\mu, \sigma^2)\), that is,

\[
CE_{ij} = \alpha_{ij}(\sigma_{ij}^2)\frac{k}{4} - R_{ij}\frac{k^2}{8}\sigma_{ij}^2
\]  

(12)
Note that the total certainty equivalent is independent of \( \lambda \). \( \lambda \) only affect how supply chain splitting the revenue and not affect the business strategy as the reputation of firms. Under our setting, the reputation \( \lambda \) represents the irrelevant transfer between the supplier and retailer during the cooperation. Moreover, the total certainty equivalent is frictionlessly transferable with in the supply chain. This transferability allows us to apply Becker (1973) approach directly when payoff are expressed in certainty equivalent terms.

It suffice to modify the definition of feasibility as follow: A pair of certainty equivalents \((CE_i, CE_j)\) is feasible if and only if

\[
CE_i + CE_j \leq \max_{\alpha, \sigma^2} CE_{ij}(\alpha, \sigma^2)
\]

subject to \( \alpha \leq (\sigma^2)\gamma \).

The solution to (12) is straightforward and it is given by:

\[
\hat{\sigma}_{ij}^2 = \left( \frac{k}{2\gamma} R_{ij} \right)^{\frac{1}{\gamma-1}}
\]

and

\[
\hat{\alpha}_{ij} = \left( \frac{k}{2\gamma} R_{ij} \right)^{\frac{\gamma}{\gamma-1}}
\]

Observe that \( \sigma_{ij}^2 \) is decreasing in \( R_{ij} \). It is intuitive that the more risk averse the supply chain is, the less risky production plan is optimal. From here, let \( \overline{CE}_{ij} \) represents the total certainty equivalent value of (13).

5 Coordinating supply chain with endogenous project selection

For each pair \((i, j)\), the Pareto frontier pairs \((CE_i, CE_j)\) satisfy \( CE_i + CE_j = \overline{CE}_{ij} \). Therefore, the sorting properties of equilibrium matching are determined by the modularity properties of \( \overline{CE}_{ij} \). Lemma 5 records this.
Lemma 5.1. According to Becker (1973), all equilibrium matchings are negative (positive) assortative if the team’s certainty equivalent $CE_{ij}$ is submodular (supermodular) in $(r_i, r_j) \frac{\partial^2 CE}{\partial r_i \partial r_j} \leq 0 (\geq 0)$.

Consider a simple example of two suppliers and two retailers in matching market, each group (suppliers and retailers) has one high risk averse agent and one low risk averse agent, i.e. each type includes agents such that $i \in (H, L)$ where $r_H > r_L$. Assume $CE_{ij}$ is the total certainty equivalent of the supply chain formed with agents type $i, j$ where $i, j \in [H, L]$. In $\Delta CE_i = CE_{iL} - CE_{iH}$, $\Delta CE_i$ describes how much total certainty equivalent varies by providing agent $i$ a higher risk averse agent as his/her new partner. Note that if this decrease is larger for $i = H$ than $i = L$, equilibrium matchings are NAM, conversely, if it is larger for $i = L$ the equilibrium matchings are PAM.

It is well known that when the production plan is exogenous, the equilibrium matching is NAM and the total certainty equivalent indicates submodularity in the agents’ risk averse coefficient with $\Delta CE_H > \Delta CE_L$. The high risk averse agent has a higher cost to match with another high risk averse agents than does a low risk averse agent. In terms of the increase of social surplus, the total certainty equivalent increase from replacing one high risk averse agent in the pair $(H, H)$ with the low risk averse agent to form a new pair $(H, L)$ is higher than replacing one high risk averse agent in a pair $(H, L)$ with a low risk averse agent to form a new pair $(L, L)$.

By allowing the endogenous production plan, the matched pair can pick the production plan that maximizes the total certainty equivalent on the production plan constraint function $\alpha \leq (\sigma^2)^\gamma$, and $r_i, r_j$ are not always substituted. Both $\Delta CE_H < (>) \Delta CE_L$ are possible and the result depends on $\gamma$.

To understand this, it is convenient to analysis the indifference curve of a supply chain:

$$\alpha_{ij}(\sigma^2_j(R_{ij})) = R_{ij} \frac{k}{2} \sigma^2_j(R_{ij}) + \frac{4}{k} CE_{ij}.$$

We can break down the motion of replacing each agent’s $L$ type partner with an $H$ type into two parts:

- The indifference curve implement the rotation motion to express the risk preference. This motion is well studied by exogenous case, we record it as $\Delta CE^ex_i$. 

11
On the other hand, when the production plan is endogenous, the indifference curve also comply the movement on the frontier of \(\alpha(\sigma^2)\), we note it as \(\Delta CE_{en}^i\).

\(\Delta CE_i\) can be decomposed into two steps as we discussed above:

\[
\Delta CE_i = \frac{k}{4} \left\{ \left[ (\alpha_iL - \frac{\sigma_{iL}^2 k}{2} R_iL) - (\alpha_iL - \frac{\sigma_{iL}^2 k}{2} R_iH) \right] - \right. \\
\left. \left[ (\alpha_{iH} - \frac{\sigma_{iH}^2 k}{2} R_{iH}) - (\alpha_{iL} - \frac{\sigma_{iL}^2 k}{2} R_{iH}) \right] \right\}
\]

Certainty equivalent loss in exogenous production plan \(\Delta CE_{ex}^i\)

Certainty equivalent adjustment by endogenous production plan adjustment \(\Delta CE_{en}^i\)

(16)

By decomposing the change in the certainty equivalent as (15), \(\Delta CE_{ex}^i\) and \(\Delta CE_{en}^i\) clarify how the endogeneity of production plan may lead to super modularity. In Figure 1, we subtract the endogenous production adjustment from the total certainty equivalent and demonstrate how \(\Delta CE_{ex}^i\) changes if only the exogenous production plan is permitted. The total certainty equivalent decrease is always more drastic for \(H\) type agents because of \(\Delta CE_{ex}^H > \Delta CE_{ex}^L\).

However, the high risk averse supply chain \((H, H)\) will have a larger certainty equivalent adjustment when they are able to choose production plan on the frontier, reflected by \(\Delta CE_{en}^H > \Delta CE_{en}^L\). Specifically, the more risk averse the agent is, the more certainty equivalent loss will be caused by matching with a higher risk averse partner. Simultaneously though, when the more risk averse pair is able to choose the production plan, they are capable of getting a higher certainty equivalent than a lower risk averse pair by choosing a low risk production plan.

In the supply chain matching market, high risk averse agents are balancing between matching with \(H\) or with \(L\). By matching with \(H\), the supply chain \((H, H)\) would acquire a low uncertainty production plan with low profit; by matching with \(L\), the supply chain \((H, L)\) would acquire a medium uncertainty production plan with medium profit.

The low risk averse agents also facing the trade off between matching with \(H\) and \(L\). Likewise, \((H, L)\) pair would adopt a medium profit production plan, and when the matching is negative, \(L\) can take larger share of revenue by providing informal insurance to the \(H\). In contrast, \((L, L)\) is
able to take a high profit high uncertainty production plan with an equally share payment contract.

The modularity of total certainty equivalent is determined by $\gamma$ and the condition is suggested by Proposition 2:

**Proposition 2.** When $\gamma > \frac{1}{2}$ all equilibrium matchings are positive assortative (PAM), and when $\gamma < \frac{1}{2}$ all equilibrium matchings are negative assortative matching (NAM).

*Proof see appendix.*

Figure 2 and 3 illustrate that the concavity of the production plan constraint $\alpha(\sigma^2)$ is the
Figure 2: Endogenous production plan with $\gamma > \frac{1}{2}$
Figure 3: Endogenous production plan with $\gamma < \frac{1}{2}$
crucial ingredient to conclude the stable matching pattern in the endogenous production plan. When $\theta(\zeta^2)$ becomes less concave, which is when $\gamma > \frac{1}{2}$, we have $\Delta CE_L > \Delta CE_H$, and therefore PAM is stable. On the other hand, when $\theta(\zeta^2)$ becomes more concave, which is when $\gamma > \frac{1}{2}$, $\Delta CE_L < \Delta CE_H$ and NAM becomes stable. Also Figure 1 demonstrate that when the production plan is exogenous and fix for the agents, the only possible equilibrium is NAM, due to the concavity of $\alpha(\sigma^2)$ and the result has been proved and widely discussed from previous literature.

We can now conclude that when the available set $\alpha(\sigma^2)$ becomes more concave, matching with a heterogeneous type of agent generates a higher certainty equivalent for the entire industry. On the contrary, when $\alpha(\sigma^2)$ is less concave, PAM is the stable matching equilibrium.

6 Discussion

This paper suggest an improved matching mechanism theory in the coordination supply chain analysis to predict a more accurate matching pattern. Suggested by Ackerberg and Botticini (2002), the production plan raised by endogenous matched partners can be the indicator of agents’ risk taking types. By observing the endogenous production plan, assortative matching theory is able to reveal the risk sharing behavior and the degree of risk aversion between the partners. Empirically, the agents’ degree of risk aversion considered to be difficult to observe and measure. Therefore, if the theory has flaws initially, the predicted agents’ degree of risk aversion can be questionable. In Example 1, we compose a comparison between random matching and assortative matching equilibrium to address the importance of accounting for the endogenous production plan.

Example

In Figure 4, we compare the optimal production plan suggested by this model. Recall that the cutoff of PAM and NAM in this model is $\gamma = 0.5$. If the environment changes slightly by $\nu$, the matching pattern can be altered drastically.

According to (Babcock et al., 1993), individual’s Arrow-Pratt risk aversion coefficient lays between $[0,1]$, therefore I randomly draw the risk averse heterogeneous retailers and suppliers in
Figure 4: Optimal $\sigma^2$ choice of random matching vs endogenous matching under NAM where $\gamma = 0.3$

such interval\footnote{Due to the widespread of risk coefficient cause the difficulty of scaling and emphasize the comparison, we picked the risk aversive coefficient $r \in [0.4, 0.6]$}. Figure 4 exhibits that how the optimal production plan would vary by slightly changing the business environment $\gamma$. When $\gamma = 0.5 + \nu$, PAM is the stable matching and the production plan selection is much more dispersed than the random matching; when $\gamma = 0.5 - \nu$, NAM is stable and the production plan is highly concentrated compared to the random matching.

Recall the exogenous production plan model that only predicts NAM as the unique stable equilibrium. If the empirical evidence denotes the evenly distributed production plan, the risk sharing pattern suggested by exogenous production plan model will not be accurately revealed and the risk aversion coefficient must be erroneous. In particular, the range of agents’ degree of risk aversion can be fallaciously amplified and the risk aversion levels of agents can be underestimated.

7 Conclusion

My model introduces the distinct scope to examine risk aversion agents in coordination supply chain by extending the strategy space to three dimensions: matching pattern, product selection, and revenue sharing contract. This approach permits more general assumption with respect to risk aversion problem in supply chain management: we allowed $N \times N$ risk averse suppliers and retailers

\footnote{Due to the concavity of $\mu(\sigma)$ function, the low risk project ought to be chosen by more supply chains. However compared to the NAM, production choice under PAM is very evenly spread}
in the matching market and also provide a stable matching equilibrium.
References


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Appendices

Proof of Proposition 1

Proof.

\[
\max_{m_{ij}} \{Eu_i[\pi_{ij} - m_{ij}(\pi_{ij})]\} + \lambda \{Eu_j[m_{ij}(\pi_{ij})]\}
\]

Take F.O.C

\[
u'_i(\pi_{ij} - m_{ij}(\pi_{ij})) = \lambda u'_j(m_{ij}(\pi_{ij}))
\]

Take Log on both side.

\[
r_i[\pi_{ij} - m_{ij}(\pi_{ij})] = r_j m_{ij}(\pi_{ij}) - \log \lambda
\]

Subtract \(m_{ij}(\pi_{ij})\):

\[
m_{ij}(\pi_{ij}) = \frac{r_i \pi_{ij} + \log \lambda}{r_i + r_j}
\]

Therefore we can easily address that: For supplier \(i\), the optimal contract is:

\[
\pi_{ij} - m_{ij}(\pi_{ij}) = \frac{r_j \pi_{ij} - \log \lambda}{r_i + r_j}
\]

For retailer \(j\), the optimal contract is:

\[
m_{ij}(\pi_{ij}) = \frac{r_i \pi_{ij} + \log \lambda}{r_i + r_j}
\]

Again, the total payoff generated from the supply chain is \(\pi_{ij}\)

\[\square\]

Proof of Lemma 4.1

Proof.

\[
\max_{m_{ij}} \{Eu_i[\pi_{ij} - m_{ij}(\pi_{ij})]\} + \lambda \{Eu_j[m_{ij}(\pi_{ij})]\}
\]
Take F.O.C

\[ u'_i(\pi_{ij} - m_{ij}(\pi_{ij})) = \lambda u'_j(m_{ij}(\pi_{ij})) \]

Take Log on both side.

\[ r_i[\pi_{ij} - m_{ij}(\pi_{ij})] = r_jm_{ij}(\pi_{ij}) - \log \lambda \]

Subtract \( m_{ij}(\pi_{ij}) \):

\[ m_{ij}(\pi_{ij}) = \frac{r_i\pi_{ij} + \log \lambda}{r_i + r_j} \]

Therefore the optimal contract is linear with respect to final profit \( \pi_{ij} \). For agent \( i \) the final payoff is \( \pi_{ij} - m_{ij}(\pi_{ij}) \) which gives us:

\[ \pi_{ij} - m_{ij}(\pi_{ij}) = \frac{r_j\pi_{ij} - \log \lambda}{r_i + r_j} \]

Also agent \( j \) achieve \( m_{ij}(\pi_{ij}) \) for final payoff. The Pareto optimal expected utility for entire supply chain (1) can be reorganized as:

\[ Eu_i(\frac{r_j\pi_{ij} - \log \lambda}{r_i + r_j}) + Eu_j(\frac{r_i\pi_{ij} + \log \lambda}{r_i + r_j}) \]

By the definition of Certainty equivalent and normal distribution we can derive CE for each agent:

\[ u_i(CE_i) = Eu_i[\pi_{ij} - m_{ij}(\pi_{ij})] = Eu_i(\frac{r_j\pi_{ij} + \log \lambda}{r_i + r_j}) \]

and

\[ u_j(CE_j) = Eu_j[m_{ij}(\pi_{ij})] = Eu_j(\frac{r_i\pi_{ij} + \log \lambda}{r_i + r_j}) \]

We derive the certainty equivalent for each agent:

\[ CE_i = \frac{r_i}{r_i + r_j} \frac{k}{4} \alpha_{ij} - \frac{r_j}{2} \frac{r_i}{r_i + r_j} k^2 \frac{\sigma_{ij}^2}{4} + \frac{\log \lambda}{r_i + r_j} \]
and

\[ CE_j = \frac{r_j}{r_i + r_j} \frac{k}{4} \alpha_{ij} - \frac{r_j}{r_i + r_j} \left( \frac{r_j}{2} \right)^2 \sigma_{ij}^2 - \log \frac{\lambda}{r_i + r_j} \]

Hence the Certainty equivalent of total supply chain can be added for both supplier and retailer pair \((i, j)\). Also because endogenous product choice \(\alpha_{ij}\) is a function of \(\sigma_{ij}^2\).

\[ CE_{ij} = \frac{k}{4} \alpha_{ij}(\sigma_{ij}^2) - R_{ij} \frac{k^2}{8} \sigma_{ij}^2 \]

Where \(R_{ij} = \frac{r_ir_j}{r_i + r_j}\).

\[ \square \]

**Proof of Proposition 2**

**Proof.** From Lemma 3.1

\[ CE_{ij} = \frac{k}{4} \alpha_{ij}(\sigma_{ij}^2) - R_{ij} \frac{k^2}{8} \sigma_{ij}^2 \]

And assumption 1

\[ \alpha \leq (\sigma^2)^\gamma, 0 < \gamma < 1 \]

We can re-express the total certainty equivalent in equilibrium as:

\[ CE_{ij} = \frac{k}{4} (\sigma_{ij}^2)^\gamma - R_{ij} \frac{k^2}{8} \sigma_{ij}^2 \]

Take F.O.C with respect to \(\sigma^2\)

\[ \frac{k}{4} (\sigma_{ij}^2)^{2(\gamma-1)} - \frac{R_{ij}k^2}{8} = 0 \]

Therefore

\[ \sigma_{ij}^2 = \left( \frac{k}{2\gamma} R_{ij} \right)^{\frac{1}{\gamma-1}} \]

Plug the optimal \(\sigma_{ij}\) back to \(CE\)

\[ CE_{ij} = \left( \frac{R}{2\gamma} \right)^{\frac{1}{\gamma-1}} - \frac{R_j}{2} \left( \frac{R}{2\gamma} \right)^{\frac{1}{\gamma-1}} \]

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\[ CE_{ij} = R_{ij}^{\frac{\gamma}{1 - \gamma}} \left[ \left( \frac{k}{2\gamma} \right)^{\frac{\gamma}{1 - \gamma}} - \frac{1}{2} \left( \frac{k}{2\gamma} \right)^{\frac{1}{1 - \gamma}} \right] \]

\[ CE_{ij} = \frac{r_i r_j}{r_i + r_j} \frac{\gamma}{1 - \gamma} \left[ \left( \frac{k}{2\gamma} \right)^{\frac{\gamma}{1 - \gamma}} - \frac{1}{2} \left( \frac{k}{2\gamma} \right)^{\frac{1}{1 - \gamma}} \right] \]

First we take cross partial derivative with respect to \( r_i \)

\[ \frac{\partial CE}{\partial r_i} = \left( \frac{\gamma}{1 - \gamma} \right) \left( \frac{1}{r_i} + \frac{1}{r_j} \right)^{\gamma - 1} \left( - \frac{1}{r_i^2} \right) \left[ \left( \frac{k}{2\gamma} \right)^{\frac{\gamma}{1 - \gamma}} - \frac{1}{2} \left( \frac{k}{2\gamma} \right)^{\frac{1}{1 - \gamma}} \right] \]

Then we take cross partial derivative with respect to \( r_j \)

\[ \frac{\partial^2 CE}{\partial r_i \partial r_j} = \left( \frac{\gamma}{1 - \gamma} \right) \left( \frac{1}{1 - \gamma} - 1 \right) \left( \frac{1}{r_i} + \frac{1}{r_j} \right)^{\gamma - 2} \left( - \frac{1}{r_i^2} \right) \left( - \frac{1}{r_j^2} \right) \left[ \left( \frac{k}{2\gamma} \right)^{\frac{\gamma}{1 - \gamma}} - \frac{1}{2} \left( \frac{k}{2\gamma} \right)^{\frac{1}{1 - \gamma}} \right] \]

Since \( 0 < \gamma < 1 \), if \( k > \gamma \)

\[ \gamma > \frac{1}{2} \frac{\partial^2 CE}{\partial r_i \partial r_j} > 0 \]

PAM is optimal. And

\[ \gamma < \frac{1}{2} \frac{\partial^2 CE}{\partial r_i \partial r_j} < 0 \]

NAM is optimal.

If \( k < \gamma \), the result is reversed. However, when \( k < \gamma \), \( CE_{ij} < 0 \) and the supply chain will choose the \( \sigma^2 = 0 \) and quit the market. Therefore we will only consider \( k > \gamma \).