On the Role of Reputation Mechanism in Firm’s Technology Choices

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Abstract

This paper studies the problem of transferring tacit technologies between business partners. In the absence of a pre-existing language to describe the tacit technology, firms resort to simple reputation mechanisms to facilitate technology transfers. Under the optimal mechanism, firms of similar natures make different technology choices and such difference persists even in the long-run. The framework is used to study the trade-off between experimenting with new technologies and exploiting the existing technologies. It also sheds light on when vertical integration solves the above stated incentive problem.

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1 Introduction

Economists have documented large and persistent technology differences between businesses of similar natures (see Syverson (2011) for a detailed survey). Yet after a decade of extensive research, we are still far from a complete understanding of what derives this ubiquitous observation. This paper takes a step forward by studying technology transfers between firms in vertical business relationships. The central question it aims to address is how the incentive problems in technology transfer shape the firms’ technology choices.

The paper starts from the premise that new technologies arise when firms in vertical relationships specialize in different parts of the production procedure. However, many of these technologies are tacit, as they cannot be transferred by any form of disembodied communication. To assimilate the technologies discovered by their partners, firms resort to embodied communication (henceforth “experimentation”) that is costly and uncertain, e.g., sending technicians and key personnel to the other firms, running joint pilot projects, etc. However, this is a challenging task when incentives are not perfectly aligned across the firms. Say, for concreteness, that the more knowledgeable firm can put the resource for experimentation to private use. Then when it claims that the environment is conducive to experimentation, the less knowledgeable firm cannot tell if it is indeed the case or if its counterparty is acting for its own benefit. Similarly, when experimentation fails to produce a breakthrough, the less knowledgeable firm does not know if it is due to bad luck or because its counterparty has not experimented at all.

In the presence of repeated interactions, this tension is solved by a simple reputation mechanism. In the aforementioned setting, the mechanism assigns the more knowledgeable firm a trustworthiness score, which is downgraded if the firm suggests
experimentation but fails to produce a breakthrough, and is upgraded if the latter suspends experimentation on the basis that the environment is inconducive. A high trustworthiness score implies a large surplus share in the event of successful knowledge transfer, whereas a low score raises the chance of relationship severance.

The reputation mechanism generates unique footprints on the process of technology transfer and establishes a linkage between the firm’s technology choice and its experience with technology transfer. In this way, it provides a rationale for firm-level technology differences that persist even in the long run. Such persistence stems from two sources. First, the uncertainty inherent in experimentation leads to relationship severance between some firms but not others. Second, notice that trustworthiness is rewarded by an increased share of future surplus. If this means a policy that favors the new technologies developed by the existing partners, then any difference between the initial partners gets propagated, not diminished over time.

The mechanism also sheds light on a number of related issues on technology adoption. First, it predicts that when facing the trade-off between experimenting with a new technology and exploiting an existing technology, a resource constrained firm is prone to experimentation when its partner is trustworthy and reverts to exploitation as the partner’s reputation deteriorates. In this way, it provides a foundation for the disruption cost in technology adoption (Arrow (1962), Holmes et al. (2011)). Second, the mechanism speaks to when vertical integration solves the incentive problem in technology transfer. By projecting the incentive cost onto the key dimensions of the model, I derive comparative static results on how the benefit of vertical integration changes with the underlying environment.

The results are consistent with a number of stylized facts. For example, the reputation mechanism is observed in the Japanese auto industry (Aoki (1988)), where it is common for big firms like Toyota to subcontract part of the design and production
to small suppliers. To assimilate the technologies that the suppliers acquire through first-hand experience, the big firms rank the suppliers by their cooperativeness in knowledge transfer and prioritize them accordingly in future businesses. The relationship between reputation and technology choice is also found in the oil drilling industry, where there is a significant productivity gain from enriching the producer’s design with the driller’s geological expertise. In this setting, Kellogg (2011) documents a positive correlation between productivity and relationship stability. Finally, the results on vertical integration are supported by studies in industry organization that production chains facilitate the transfer of intangible inputs such as tacit knowledge (Hortacsu and Syverson (2007), Atalay et al. (2012)).

This paper is related to three strands of the literature. First, it is built upon the extensive empirical literature on tacit technologies. See, besides the aforementioned references, Boldrin and Levine (2008) for detailed historical accounts, Acemoglu and Zilibotti (2001) and Bertrand and Schoar (2003) for skill mismatch and managerial talent, respectively, Irwin and Klenow (1994) and Levitt et al. (2012) for learning-by-doing, and Keller (2002), Burstein and Monge-Naranjo (2009), Keller and Yeaple (2012) for evidence from trade. By studying the incentive problem caused by such tacitness, I derive its impact on technology choices and organizational structures. For related studies, see Acemoglu et al. (2007) for theoretical and empirical analyses on why firms with advanced technologies adopt decentralized organization structures, and Acemoglu et al. (2007) for a theoretical model on how contract incompleteness and legal institutions affect technology choices.

This paper is also closely related to the literature on incomplete contract. See Hart and Moore (1988, 1990) for the implication of renegotiation on asset allocation, Segal (1999) and Nabil et al. (2006) for the impact of language complexity on the optimal contractual form, Tirole (2009) for the consequence of unawareness, and Ellison and
for the lack of contractual language in long-term relationships.

Technically, the current analysis borrows heavily from the methodology on continuous time contracting. See, among others, Sannikov (2008) and Hörner and Samuelson (2009).

The rest of the paper is organized as follows: Section 2.1 describes the baseline model; Section 2.2 formulates the problem and streamlines key steps of the proof; Section 3 states the main results and discusses their real-world implications; Section 4 extends the baseline model to study the impact of contractual restrictions, new vs. old technologies and vertical integration; Section 5 concludes. All proofs are relegated to the Appendix.

2 The Model

2.1 Set Up

Time is discrete and is indexed by $t = 0, \Delta, 2\Delta, ...$, where $\Delta$ is a small positive number. There is a principal (she) and an agent (he). Both are neutral and share a common interest rate $e^{-r\Delta}$. In each period, the principal can spend a cost $\kappa \Delta$ to obtain a unit of capital, whereas the agent has free access to one unit of labor.

There is a productive arm $X^0$ and infinitely many unproductive arms $X^1, X^2, ...$. Each arm is available independently with probability $p$ in each period. When receiving a unit of capital and a unit of labor (or “being pulled”), the productive arm is activated with probability $q\Delta$—in which case it generates a positive output, whereas the unproductive arms always yield a private benefit $\pi \Delta$ to the agent. Denote by $S$ the present value of the net surplus that is generated by the productive arm, and
let $rS > \pi - \kappa$ such that it is efficient to pull to the productive arm whenever it is available.

At the beginning of $t = 0$, all arms are covered by identical looking boxes, and only the agent knows if the productive arm is available and where it is located. The box containing the productive arm is opened once and for all when the arm is activated for the first time, while boxes that contain the unproductive arms can never be opened.

Before we proceed, it is useful to reinterpret the model in a concrete setting. Take, for example, the case of Toyota (Aoki 1988) which subcontracts a substantial fraction of the design and production to small suppliers. Imagine that a subcontractor invents a design that enhances Toyota’s productivity, but the subtlety of the design makes it a poor subject of direct communication. To solve the problem, Toyota sends technicians to the subcontractor with the aim to figure out the design through joint experimentation. However, the company is aware that the blueprint shown by the subcontractor may not be genuine. Indeed, it could be the designs of the gadgets that benefit the subcontractor only. If the subcontractor tricks Toyota to work on the gadgets, then it is essentially using Toyota as a cheap source of labor.

In this setting, the key ingredients of the model mean the following:

- The productive arm represents the opportunity to experiment with the new design. In reality, the key personnel who understand this design may not always be available to help Toyota. This is why the productive arm is assumed to be present with probability $p$ in each period. Note that this assumption is non-restrictive since $p$ can take any value between zero and one.

- The unproductive arms represent the designs that benefit the subcontractor only. By adding infinitely many such arms to the model, I make the subcontractor’s participation crucial because there is no chance that Toyota can
experiment with these many arms on her own.

- The boxes are used to model the lack of pre-existing languages to describe the new design. Put it differently, Toyota cannot tell from reading the blueprint or from the subcontractor’s description whether this is the design she wants.

- The box-opening event occurs when Toyota learns the essence of the design through experimentation.

- The expected future surplus $S$ includes, but is not limited to the productivity gain from this particular design. In general, it represents the value of continuing the relationship with this subcontractor.

To maximize her ex-ante payoff, the principal proposes a relational contract to the agent at $t = 0$. Time elapses as follows. In period $t$, the available arms $\mathcal{X}_t \subseteq \{X^0, X^1, X^2, \ldots\}$ are realized. The agent observes $\mathcal{X}_t$ and sends a message $m_{a,t} \in \mathcal{M}_a = \{H, L\}$ to the principal indicating whether the productive arm is available (“$H$” means available and “$L$” means otherwise). Given the agent’s message, the principal decides whether to contribute capital, $d_t \in \mathcal{D} = \{Y, N\}$. In the former case ($d_t = Y$), she also sends a message $m_{p,t} \in \mathcal{M}_p = \{P, NP\}$ to the agent regarding which arm he should pull (“$P$” means to pull the productive arm and “$NP$” means the opposite). The agent then pulls an arm $X_t \in \mathcal{X}_t$ before he and the principal observe if any box is opened ($y_t = G$ means that a box is opened and $y_t = B$ means the opposite). If it is the first time a box is ever opened, then the relationship enters the consolidation phase where the agent receives a share $\beta_t \in [0, 1]$ of the total surplus $S$. Otherwise, the relationship remains in the experimentation phase, and the principal pays a non-negative monetary transfer $\psi_t \geq 0$ to the agent before they decide whether continue the relationship, $a_{i,t} \in \mathcal{A} = \{0, 1\}, i = p, a$. The relationship survives if they
both choose to stay \((a_{p,t} = a_{a,t} = 1)\). Otherwise, both players receive zero utility from the next period onward.

The relational contract induces a dynamic game, where a \(t\)-period public history is \(h^t = \{\{m_i\}_{i=p,a}, d^t, \beta^t, \psi^t, \{a_i\}_{i=p,a}\}\), and a \(t\)-period private history of the agent is \(h^t_a = h^t \cup \{X_s, X_s\}_{s=0}^t\). Denote by \(h^{t,c} = h^{t-1} \cup \{H, Y, G\}\) a sequence of public events where consolidation occurs at the end of period \(t\), and by \(h^{t,e} = h^{t-1} \cup \{\{m_i\}_{i=p,a}, d_t, B\}\) another sequence where experimentation continues at the end of period \(t\). Let \(h^{t,c}_a = h^{t,c} \cup \{X_s, X_s\}_{s=0}^t\) and \(h^{t,e}_a = h^{t,e} \cup \{X_s, X_s\}_{s=0}^t\) be the corresponding private histories of the agent, and use capital letters to denote the sets of all possible histories.

The principal’s strategy \(\sigma_p = \{d_t, m_{p,t}, \beta_t, \psi_t, a_{p,t}\}_{t=0}^\infty\), where

\[
(d_t, m_{p,t}) : \mathcal{H}_{t-1} \times \mathcal{M}_a \to \mathcal{D} \times \mathcal{M}_p
\]

denotes whether she contributes capital and which arm she wishes to pull,

\[
\beta_t : \mathcal{H}^{t,c} \to [0, 1]
\]

represents the share of future surplus she promises to the agent in case the relationship is consolidated, and

\[
(\psi_t, a_{p,t}) : \mathcal{H}^{t,e} \to \mathbb{R}_+ \times \mathcal{A}
\]
stands for her payment to the agent and her decision on whether to stay in case the experimentation has been failing so far. The agent’s strategy is $\sigma_a = \{m_{a,t}, X_t, d_{a,t}\}_{t=0}^\infty$, where

$$m_{a,t} : \mathcal{H}_{a,t-1} \times \mathcal{X}_t \rightarrow \mathcal{M}_a$$

denotes his message to the principal,

$$X_t : \mathcal{H}_{a,t-1} \times \mathcal{X}_t \times \mathcal{M}_a \times \{Y\} \times \mathcal{M}_p \rightarrow \mathcal{X}_t$$

represents the arm he actually pulls, and

$$a_{a,t} : \mathcal{H}_{a,t}^{t,e} \times \mathbb{R}_+ \rightarrow \mathcal{A}$$

stands for his decision on whether or not to stay. The principal’s belief system is $\eta = \{\eta_t\}_{t=0}^\infty$, where $\eta_t : \mathcal{H}_t \rightarrow \Delta(\mathcal{H}_{a,t})$ maps each $t$-period public history to a probability measure over the agent’s $t$-period private histories. Assumes that the principal does Bayesian updating whenever possible.

The relational contract is self-enforcing if it describes a perfect Bayesian equilibrium of the dynamic game it induces. Throughout the analysis, punish publicly observable deviations—e.g., failure to deliver the promised share of future surplus—by relationship severance.

### 2.2 Formulating the Problem

I now formulate the players’ problems. To begin with, I restrict the agent to public strategies whereby $\sigma_a|h_a^{t-1}$ depends solely on the public history $h_a^{t-1}$ for every $t, h_a^{t-1}$. This makes sense, since both the underlying environment and the principal’s policy depend only on the public history. Under this framework, let $W_t$ denote the agent’s
continuation value at the beginning of period \( t \) before \( X_t \) is realized, and notice that \( W_t \) is the unique state variable that the players’ continuation strategies should depend upon. Based on this observation, write the principal’s continuation value in period \( t \) as \( V(W_t) \).

The next step is to reduce the dimensionality of the policy space through the following process:

- First, I make intellectual guesses on the key properties of the optimal contract and derive their implications on how the agent’s continuation value should evolve over time. This allows me to simplify the principal’s problem considerably.

- In the next Section, I characterize the principal’s value function in the continuous time limit by the solution of an ordinary differential equation. Then by verifying that the solution satisfies the conjectures in Step one, I argue based on Cauchy-Lipschitz Theorem that it is indeed the principal’s value function under the optimal contract.

I begin with several guesses about the optimal contract:

**Conjecture 1.** Under the optimal contract, there exists \( \overline{W} \geq 0 \) such that \( W_t \in [0, \overline{W}] \) for every \( t \). At any \( W_t \in (0, \overline{W}) \),

\[
(i) \quad d(W_t, H) = Y, \quad m_p(W_t, H) = P \quad \text{and} \quad d(W_t, L) = N;
\]

\[
(ii) \quad V'(W_t) > -1.
\]

Intuitively, Part (i) means that the principal makes use of every opportunity to experiment with the productive arm before the agent becomes too expensive to incentivize (when \( W_t \) reaches \( \overline{W} \)) or the relationship is terminated permanently (when
$W_t$ hits 0), whereas Part (ii) implies that the agent is incentivized mainly through changes in continuation value, i.e.,

**Lemma 1.** Under Conjecture \(1\), $\psi(W_t, L) = \psi(W_t, H, B) = 0$ for every $W_t \in (0, \overline{W})$.

Under Conjecture \(1\) and Lemma \(1\) I write the law of motion equation of the agent’s continuation value as

$$W_t = \mathbb{E} \left[ p \max \left\{ q \Delta \beta(W_t) S + (1 - q \Delta) e^{-r \Delta} W_{t+\Delta}(H, B), \pi \Delta + e^{-r \Delta} W_{t+\Delta}(H, B) \right\} ight] + (1 - p) \max \left\{ e^{-r \Delta} W_{t+\Delta}(L), \pi \Delta + e^{-r \Delta} W_{t+\Delta}(H, B) \right\}$$

where $q \Delta \beta(W_t) S + (1 - q \Delta) e^{-r \Delta} W_{t+\Delta}(H, B)$ and $\pi \Delta + e^{-r \Delta} W_{t+\Delta}(H, B)$ represent his payoffs from pulling the productive arm and the unproductive arms, respectively, whereas $e^{-r \Delta} W_{t+\Delta}(L)$ is his continuation value from declaring that the productive arm is unavailable. For the agent to pull the productive arm whenever it is available, I need

$$q \Delta \beta(W_t) S + (1 - q \Delta) e^{-r \Delta} W_{t+\Delta}(H, B) \geq \pi \Delta + e^{-r \Delta} W_{t+\Delta}(H, B) \quad (2.1)$$

For him to tell the truth when the productive arm is unavailable, I need

$$e^{-r \Delta} W_{t+\Delta}(L) \geq \pi \Delta + e^{-r \Delta} W_{t+\Delta}(H, B) \quad (2.2)$$

Together, (2.1) and (2.2) imply that the contract is incentive compatible at $W_t$ if and only if

$$\min \{ q \beta(W_t) S \Delta + (1 - q \Delta) e^{-r \Delta} W_{t+\Delta}(H, B), e^{-r \Delta} W_{t+\Delta}(L) \} \geq \pi \Delta + e^{-r \Delta} W_{t+\Delta}(H, B)$$
Furthermore, they simplify the law of motion equation of the agent’s continuation value to

\[ W_t = p \left[ q\beta(W_t)S\Delta + (1 - q\Delta)e^{-r\Delta}W_{t+\Delta}(H, B) \right] + (1 - p)e^{-r\Delta}W_{t+\Delta}(L) \quad (2.4) \]

Based on (2.3) and (2.4), I formulate the principal’s problem as:

\[
V(W_t) = \max_{\beta(\cdot), W_{t+\Delta}(\cdot)} \mathbb{E}\left[ p \left[ -\kappa\Delta + q\Delta(1 - \beta(W_t))S + (1 - q\Delta)e^{-r\Delta}V(W_{t+\Delta}(H, B)) \right] + (1 - p)e^{-r\Delta}V(W_{t+\Delta}(L)) \right] \\
\text{s.t. (2.3), (2.4) and } V(W_t) \geq 0
\]

where the last inequality refers to her participation constraint at \( W_t \). Together with Conjecture 1 (ii), this formulation implies that under the optimal contract, the agent is exactly indifferent between all the movements he can possibly make at any point in time. This in turn pins down the motion equations of his continuation value and his surplus share in a consolidated relationship. Formally,

**Lemma 2.** Under the optimal contract,

\[ q\beta(W_t)S\Delta + (1 - q\Delta)e^{-r\Delta}W_{t+\Delta}(H, B) = e^{-r\Delta}W_{t+\Delta}(L) = \pi\Delta + e^{-r\Delta}W_{t+\Delta}(H, B) \]

for every \( W_t \in (0, \overline{W}) \). As \( \Delta \to 0 \),

(i) \( \dot{W}_{t,L} = rW_t, \dot{W}_{t,H,B} = rW_t - \pi; \)

(ii) \( \beta(W_t) = (qW_t + \pi)/qS. \)

where \( \dot{W}_{t,m_{a,t},y_t} \) denotes the motion equation of the agent’s continuation value given the period-\( t \) message and outcome \( (m_{a,t}, y_t) \).
Based on Lemma 2, I characterize the principal’s value function in the continuous time limit as the solution to an ordinary differential equation (henceforth ODE). In the next section, I discuss the key features of this value function and explore their real-world implications. In the appendix, I solve the ODE explicitly and verify that the solution indeed satisfies Conjecture 1.

3 Main Result

I now state the main result of this paper: as \( \Delta \to 0 \),

**Theorem 1.** There exists \( W \geq 0 \) such that \( W_t \in [0, W] \) for every \( t \). If \( qS \leq \pi + \kappa \), then \( W = 0 \) and the principal prefers not to initiate the relationship in the first place. If \( qS > \pi + \kappa \), then \( W > 0 \) and experimentation is profitable. In this case, the principal’s value function \( V(.) \) is the solution to the following ordinary differential equation:

\[
(r + pq)V(W) = p(qS - \pi - \kappa - qW) + V'(W)(rW - p\pi) \tag{3.1}
\]

\( s.t. \ V(0) = 0 \)

\( V(.) \) is strictly concave, single-peaked and satisfies \( V'(W) > -1 \) for every \( W \in (0, W] \).

The optimal contract terminates the relationship at \( W = 0 \). At any \( W \in (0, W] \),

(i) The principal experiments with the productive arm whenever it is claimed to be available, i.e., \( d(W,H) = Y \), \( m_p(W,H) = P \) and \( d(W,L) = N \).

(ii) The agent is incentivized mainly through changes in continuation value, since \( \psi(W,H,B) = \psi(W,L) = 0 \) for every \( W \in (0, W] \). Indeed, the only time he is paid by cash is when his continuation value reaches upperbound of the domain,
\[ \psi(W, L) = rW > 0 \text{ only if } W = \overline{W}. \]

(iii) The agent’s surplus share at the consolidation phase equals 
\[ \beta(W) = \frac{(qW + \pi)}{qS} \text{ and is increasing in } W. \]

(iv) The agent’s continuation value evolves according to the following motion equations:

\[ \dot{W}_{H,B} = rW - \pi < 0, \quad \dot{W}_L = \begin{cases} 
  rW & \text{if } W < \overline{W}; \\
  0 & \text{if } W = \overline{W}. 
\end{cases} \]

![Figure 1: The Principal’s Value Function](image)

Figure 1: The Principal’s Value Function

Theorem 1 conveys several important insights. First, by highlighting the randomness inherent in experimentation, it relates the principal’s technology choice to her unique experience with technology transfer. In this way, it generates a non-degenerate distribution of the adopted technologies even in the long-run, and thus provides a rationale for the widely documented technology difference between firms.
of similar natures.

Second, the theorem says that at the experimentation phase, the agent is incentivized mostly through changes in continuation value. One may think of this value as a score to the agent, which is upgraded if the message and output suggest that he is acting trustworthily and is downgraded otherwise. In the current setting, the agent is regarded as being trustworthy if he suspends experimentation when the underlying environment is in conducive, for this very act forgoes the opportunity to finance his private consumption. Meanwhile, he is considered as being dishonest if he insists on experimentation but fails to produce any breakthrough, as such outcome indicates that he might have diverted the resource to personal use.

Third, the result implies that technology adoption is relational, in the sense that a high trustworthiness score at the experimentation phase implies a large surplus share at the consolidation phase. In reality, this could mean a policy that favors the new technologies developed by the existing partners, which then propagages, rather than diminishes, any innate difference between initial partners (e.g., imagine that the principal and the agent are matched randomly at the outset) into future productions.

Finally, notice that the very condition for experimentation to be profitable, \( qS > \pi + \kappa \), is different than the efficiency criterion \( rS > \pi - \kappa \). In other words, there is a conflict between the efficiency of a technology and its transferability. Consequently, there are efficient technologies that can never be transferred, and everything else being equal, the principal prefers technologies that are easily transferable, i.e., those with high \( q \)'s.
4 Extension

This section extends the baseline model to address the following two questions. First, what happens when there is a constraint on the surplus share that the principal can possibly promise to the agent? Second, how does the optimal experimentation policy look like when a resource-constrained principal faces the trade-off between experimenting with new technologies and exploiting the existing technologies?

4.1 Contractual Restrictions at the Consolidation Phase

So far, one of the maintained assumptions is that the principal can promise any share of the surplus when the relationship reaches the consolidation stage. However, one may want to modify this assumption to account for the contractual frictions we face in reality. For example,

Example 1. There is a concern that the agent’s participation becomes less integral at the consolidation phase, as the principal, who no longer relies on the agent’s expertise, is now tempted to replace him with cheaper alternatives. In the most extreme case where the alternatives are plenty and the chance of future interactions is slim, the principal can simply renege and pays the agent nothing once she acquires the technology. In the other cases, she may still demand at least $1 - \beta^*$ of the future surplus for some exogenously given $\beta^*$.

Example 2. Alternatively, the parties may Nash bargain over the way that the surplus should be splitted. In this case, the agent’s share $\beta^*$ is determined by his bargaining power.

The objective of this section is to formalize these contractual restrictions and to explore their implications on the optimal experimentation policy. Specifically,
I assume that the agent’s surplus share must belong to $[\beta, \overline{\beta}]$, where $\beta$ and $\overline{\beta}$ are exogenous parameters whose determinations are beyond the scope of the current analysis. Notice that this formulation is flexible enough to subsume (1) the baseline model, where $\beta = 0$ and $\overline{\beta} = 1$, (2) Example 1 where $\beta = 0$ and $\overline{\beta} = \beta^*$, and (3) Example 2 where $\beta = \overline{\beta} = \beta^*$.

The next proposition establishes that aforementioned restrictions only modify the boundary condition of the ODE in (3.1). Intuitively, when $W$ hits $\beta S$, the principal can no longer decrease the agent’s surplus share. Instead, she resorts to a different incentive instrument: relationship severance. That is, at any $W \in [0, \beta S]$, she terminates the relationship with probability $1 - \frac{W}{\beta S}$ and implements the optimal policy at $\beta S$ with probability $\frac{W}{\beta S}$—see Figure 4.1 for a graphical illustration. For this policy to be optimal, I impose a smooth-pasting condition that equates the slope of $V(W)$ to $V(W)/W$ at $W = \beta S$. Formally,

**Proposition 1.** There exists $W \geq 0$ such that $W_t \in [0, W]$ for every $t$. If $\beta S < W$, then on $[\beta S, W]$, the principal’s value function is the solution to the following ordinary differential equation:

$$(r + pq)V(W) = p(qS - \pi - \kappa - qW) + V'(W)(rW - p\pi)$$

s.t. $V'(W)W = V(W)$ at $W = \beta S$

and the optimal policy satisfies the properties listed in Theorem 1.

On $W \in [0, \beta S]$, the principal’s value function is linear in $W$:

$$V(W) = \frac{V(\beta S)}{\beta S}W$$

The optimal policy terminates the relationship with probability $1 - \frac{W}{\beta S}$ and implements
the policy at \( W' = \beta S \) with probability \( \frac{W}{\beta S} \).

Figure 2: The Principal’s Value Function: The Case with Contractual Restrictions

4.2 Existing vs. New Technologies

This section considers the trade-off faced by a resource-constrained principal between experimenting with new technologies and exploiting the existing technologies. Specifically, now the principal can spend at most \( \kappa \Delta \) to obtain one unit of capital in every period, while there are two productive arms \( X^1 \) and \( X^2 \) and infinitely many unproductive arms \( X^3, X^4, ... \), each of which appears independently with \( p \) in every period. When being pulled, a productive arm \( X^j, j = 1, 2 \) is activated with probability \( q \Delta \) and yields a flow payoff \( y^j \Delta \) to the principal where \( y^1 > y^2 \), whereas the unproductive arms yield a private benefit \( \pi \Delta \) to the agent. For notational convenience, define

\[
S = \frac{p(qy^1 - \kappa) + (1 - p)p(qy^2 - \kappa)}{r}
\]
as the expected total surplus from pulling the most efficient arm, and

$$S^j = \frac{p(qy^j - \kappa)}{r}$$

as the expected surplus from pulling only $X^j$, $j = 1, 2$. To make the analysis interesting, let $rS^j \geq \pi - \kappa$ for $j = 1, 2$ such that the productive arms are more efficient than the unproductive arms.

At the outset, all arms except $X^2$ are covered by identical looking boxes, and only the agent knows whether $X^1$ is available and where it is located. The box containing $X^1$ is opened once and for all when $X^1$ is activated for the first time, whereas those that contain the unproductive arms can never be opened.

To maximize her ex-ante payoff, the principal proposes a relational contract that is similar to the one in the baseline model, except that she now decides not only whether to contribute capital but also which productive arm to pull. Specifically, let the agent report whether $X^1$ is available at the beginning of each period $m_{a,t} \in \{Y, N\}$ before the principal decides whether to contribute capital $d_t \in \{Y, N\}$ and which arm to pull $m_{p,t} \in \{X^1, X^2, X^{\geq 3}\}$. The agent then pulls an arm before any box is opened $y_t \in \mathcal{Y} = \{G, B\}$. If the box containing $X^1$ is opened for the first time in period $t$, then the relationship enters the consolidation phase where the agent receives $\beta_t$ of the future surplus $S$. If the principal decides not to experiment with $X^1$ anymore, then the relationship enters the exploitation phase where only $X^2$ is being pulled. Otherwise, the relationship remains at the experimentation phase.

The relational contract induces a dynamic game, where agent’s continuation value $W_t$ is the unique state variable that the players’ strategies should depend upon from period $t$ onward. To characterize the optimal contract, I begin with two basic policies: (1) the fast track policy, where the principal experiments with $X^1$ whenever it is
claimed to be available, and (2) the slow track policy, where she experiments with $X^1$ only in the absence of $X^2$. A straightforward extension of Theorem 1 establishes that under each basic policy, the principal’s value function is strictly concave and hump-shaped on the relevant domain. Then by taking the convex hull of these two value functions, I obtain the value function under the optimal policy. See Figure 4.2 for a graphical illustration.

Formally, let $V_f(\cdot)$ and $V_s(\cdot)$ denote the value functions under the fast track policy and the slow track policy, respectively. Applying the argument for Theorem 1 yields the following:

**Proposition 2.** Under the optimal policy,

(i) The principal’s value function is the convex hull of $V_f(\cdot)$ and $V_s(\cdot)$.

(ii) $V_f(0) = V_s(0) = S^2$. If $V_i'(0) > 0$, then $V_i(\cdot)$ is strictly concave and hump-shaped on the relevant domain. Otherwise, the relationship should not be initiated in
the first place.

(iii) If \( V_f \) and \( V_s \) single-cross at some \( W^* \) where \( V_i(W^*) > 0 \) for both \( i = f, s \), then 
\[ V_f(W) < V_s(W) \text{ for all } W \in (0, W^*) \text{ and } V_f(W) > V_s(W) \text{ for all } W > W^*. \]

Therefore, there exist \( W^\dagger < W^* \) and \( W^\ddagger > W^* \) such that the principal adopts the slow track policy when \( W \leq W^\dagger \), the fast tract policy when \( W \geq W^\ddagger \) and randomizes between them when \( W \in (W^\dagger, W^\ddagger) \).

In the Appendix, I solve the value functions explicitly and discuss the conditions for Parts (ii) and (iii) to hold.

According to Proposition 2 (iii), the optimal experimentation policy exhibits status-quo bias, as the principal is more prone to exploitation when the agent’s continuation value is low. Intuitively, when the new technology is sufficiently profitable, it is too costly for the principal to terminate experimentation all at once. Instead, she adjusts the speed of experimentation and implements the slow track policy more and more often as the agent’s trustworthiness score deteriorates.

### 4.3 Vertical Integration

This section examines the viability of vertical integration as a solution to the aforementioned incentive problem. Suppose that the principal can now purchase the agent at \( J \) and generate a gross benefit

\[
U = \frac{p(qS - \kappa)}{r + pq} \tag{4.1}
\]

Then purchasing is profitable if and only if

\[
U - J \geq V(W_0)
\]
where $W_0$ is the agent’s payoff from relational contracting, and $V(W_0)$ is the corresponding payoff of the principal. Based on Theorem 1, rewrite this inequality as

$$J \leq \begin{cases} 
U & \text{if } qS \leq \pi + \kappa; \\
\frac{\pi}{r} \left[ 1 - \frac{1}{1 + \frac{\pi}{qS - \kappa}} \right] & \text{if } qS > \pi + \kappa
\end{cases}$$

(4.2)

Since the RHS of (4.2) is continuous and is strictly increasing in $\pi$ when $qS > \pi + \kappa$ (see Figure 4.3 for graphical illustration), I conclude that vertical integration is more profitable when the agent benefits more from tricking the principal:

**Proposition 3.** The benefit of vertical integration is non-decreasing in $\pi$.

![Figure 4: Benefit of Vertical Integration](image-url)
5 Conclusion

This paper studies the problem of transferring tacit technologies between business partners. In the absence of a pre-existing language to describe the tacit technology, firms resort to simple reputation mechanisms to facilitate technology transfers. Under the optimal mechanism, firms of similar natures make different technology choices and such difference persists even in the long-run. The framework is used to study the trade-off between experimenting with new technologies and exploiting the existing technologies. It also sheds light on when vertical integration solves the above stated incentive problem.

A Appendix

A.1 Omitted Proofs in Section 2.2

Proof of Lemma 1

Proof. Proof by contradiction. Suppose there exists $W_t$ such that either $\psi(W_t, L) \neq 0$ or $\psi(W_t, H, B) \neq 0$. Then consider a new policy $\{\hat{\psi}(W_t, L), \hat{\psi}(W_t, H, B), \hat{W}_{t+\Delta}(L), \hat{W}_{t+\Delta}(H, B)\}$ where

\[
\hat{\psi}(W_t, L) = \hat{\psi}(W_t, H, B) = 0 \\
\hat{W}_{t+\Delta}(L) = W_{t+\Delta}(L) + \psi(W_t, L) \\
\hat{W}_{t+\Delta}(H, B) = W_{t+\Delta}(H, B) + \psi(W_t, H, B)
\]

By construction, the agent is indifferent between the old policy and the new policy, whereas the principal is strictly better-off under the new policy, since Conjecture [1]
implies that

\[
\text{Change in the Principal’s Payoff at } (H, B) = (V'(W_t) + 1)\psi(W_t, H, B)\Delta > 0 \quad - L = (V'(W_t) + 1)\psi(W_t, L)\Delta > 0
\]

However, this contradicts the optimality of the old policy. □

Proof of Lemma 2

Proof. Proof by contradiction. First, suppose there exists \( W_t \) such that

\[
q\beta(W_t)S\Delta + (1 - q\Delta)e^{-r\Delta}W_{t+\Delta}(H, B) > e^{-r\Delta}W_{t+\Delta}(L)
\]

Then consider a new policy \( \{\hat{\beta}(W_t), \hat{W}_{t+\Delta}(L)\} \) where

\[
\begin{align*}
pq\hat{\beta}(W_t)S\Delta &= pq\beta(W_t)S\Delta - \varepsilon\Delta \\
(1 - p)e^{-r\Delta}\hat{W}_{t+\Delta}(L) &= (1 - p)e^{-r\Delta}W_{t+\Delta} + \varepsilon\Delta
\end{align*}
\]

for some small positive number \( \varepsilon \). By construction, this new policy satisfies the agent’s IC constraint and the law of motion equation of his continuation value at \( W_t \). Moreover, it strictly increases the principal’s payoff at \( W_t \), since under Conjecture 1

\[
\text{Change in the Principal’s Payoff} = \varepsilon\Delta + (1 - p)e^{-r\Delta}V'(W_t)\frac{\varepsilon\Delta}{(1 - p)e^{-r\Delta}} = (V'(W_t) + 1)\varepsilon\Delta > 0
\]

However, this contradicts with the optimality of the original policy.
Second, suppose there exists $W_t$ such that

$$e^{-r\Delta}W_{t+\Delta}(L) > q\beta(W_t)S\Delta + (1 - q\Delta)e^{-r\Delta}W_{t+\Delta}(H, B)$$

Then consider a new policy $\{\hat{\beta}(W_t), \hat{W}_{t+\Delta}(L), \hat{W}_{t+\Delta}(H, B)\}$ where

$$(1 - p)e^{-r\Delta}\hat{W}_{t+\Delta}(L) = (1 - p)e^{-r\Delta}W_{t+\Delta}(L) + \varepsilon_1\Delta$$

$$p(1 - q\Delta)e^{-r\Delta}\hat{W}_{t+\Delta}(H, B) = p(1 - q\Delta)e^{-r\Delta}W_{t+\Delta}(H, B) + \varepsilon_2\Delta$$

$$pq\hat{\beta}(W_t)S\Delta = pq\beta(W_t)S\Delta - (\varepsilon_1 + \varepsilon_2)\Delta$$

for some $\varepsilon_1, \varepsilon_2$ that are small enough to satisfy the agent’s IC constraint at $W_t$. By construction, this new policy also satisfies the law of motion equation of the agent’s continuation value at $W_t$. For it to increase the principal’s payoff at $W_t$, I need

$$\text{Change in the Principal’s Payoff} = (V'(W_t) + 1)(\varepsilon_1 + \varepsilon_2)\Delta > 0$$

Under Conjecture 1, simply set $\varepsilon_1 + \varepsilon_2 > 0$ to make the RHS positive. However, the very existence of such $\varepsilon_1, \varepsilon_2$ contradicts the optimality of the old policy.

Finally, if there exists some $W_t$ such that

$$q\beta(W_t)S\Delta + (1 - q\Delta)e^{-r\Delta}W_{t+\Delta}(H, B) = e^{-r\Delta}W_{t+\Delta}(L) > \pi e^{-r\Delta}W_{t+\Delta}(H, B)$$


then consider a new policy \( \{ \hat{\beta}(W_t), \hat{W}_{t+\Delta}(L), \hat{W}_{t+\Delta}(H, B) \} \) where

\[
pq\hat{\beta}(W_t)S \Delta = pq\beta(W_t)S \Delta - \varepsilon_1 \Delta
\]

\[
(1 - p)e^{-r\Delta}\hat{W}_{t+\Delta}(L) = (1 - p)e^{-r\Delta}W_{t+\Delta}(L) - \varepsilon_2 \Delta
\]

\[
p(1 - q\Delta)e^{-r\Delta}\hat{W}_{t+\Delta}(H, B) = p(1 - q\Delta)e^{-r\Delta}W_{t+\Delta}(H, B) + (\varepsilon_1 + \varepsilon_2) \Delta
\]

for some small positive numbers \( \varepsilon_1, \varepsilon_2 \). By construction, this new policy satisfies the agent’s IC constraint and the law of motion equation of his continuation value at \( W_t \).

Furthermore, it strictly increases the principal’s payoff at \( W_t \), since

\[
\text{Change in the Principal’s Payoff} = \varepsilon_1 \Delta + V'(W_t)(\varepsilon_1 + \varepsilon_2) \Delta - V'(W_t)\varepsilon_2 \Delta
\]

\[
= (V'(W_t) + 1)\varepsilon_1 \Delta
\]

\[
> 0
\]

Again, this contradicts with the optimality of the old policy. \( \square \)

### A.2 Proof of Theorem 1

**Proof.** The proof contains three parts. The first part solves the principal’s value function explicitly. Based on Lemmas 1 and 2 I have

\[
V(W) = p \left[ -C\Delta + (1 - \beta(W))qS\Delta + (1 - q\Delta)e^{-r\Delta}V(W_{t+\Delta}(H, B)) \right]
+ (1 - p)e^{-r\Delta}V(W_{t+\Delta}(L))
\]

s.t. (1) \( \hat{W}_{H,B} = rW - \pi, \hat{W}_L = rW \)

(2) \( \beta(W) = (\pi + qW)/qS \)
for every $W \in (0, \bar{W})$ and $V(0) = 0$. Take $\Delta \to 0$ and rewrite $V(.)$ as the solution to an ODE:

$$\left(1 + \frac{pq}{r}\right)V(W) = \frac{p[qS - \pi - \kappa]}{r} - \frac{pq}{r}W + V'(W)\left(W - \frac{p\pi}{r}\right) \quad (A.1)$$

s.t. $V(0) = 0$

Guess that $V(.)$ takes the following form, with three coefficients $\alpha, \beta, \gamma$ to be determined:

$$V(W) = \alpha \left[\frac{p\pi}{r} - W\right]^{1 + \frac{pq}{r}} + \gamma W + \delta$$

Differentiating the RHS with respect to $W$ yields

$$V'(W)\left(W - \frac{p\pi}{r}\right) = \alpha \left(1 + \frac{pq}{r}\right)\left(\frac{p\pi}{r} - W\right)^{1 + \frac{pq}{r}} - \gamma \left(\frac{p\pi}{r} - W\right)$$

Plug it into the RHS of (A.1) and obtain

$$\text{RHS} = \frac{p[qS - \pi - \kappa - qW]}{r} + \alpha \left(1 + \frac{pq}{r}\right)\left(\frac{p\pi}{r} - W\right)^{1 + \frac{pq}{r}} - \gamma \left(\frac{p\pi}{r} - W\right)$$

$$\text{LHS} = \alpha \left(1 + \frac{pq}{r}\right)\left(\frac{p\pi}{r} - W\right)^{1 + \frac{pq}{r}} + \left(1 + \frac{pq}{r}\right)(\gamma W + \delta)$$

Equating the constant term and the coefficient in front of $W$ between these two equations yields

$$\gamma = -1$$

$$\delta = \frac{p(qS - \kappa)}{r + pq}$$
And pinning down $\alpha$ by the initial condition $V(0) = 0$:

$$V(0) = \alpha \left[ \frac{p\pi}{r} \right]^{1+\frac{\nu}{r}} + \frac{p(q\pi - \kappa)}{r + pq} = 0$$

$$\Rightarrow \alpha = -\frac{p(qS - \kappa)}{r + pq} \left[ \frac{p\pi}{r} \right]^{-\left(1+\frac{\nu}{r}\right)}$$

Given $\alpha, \gamma$ and $\delta$, rewrite the principal’s value function as

$$V(W) = -W + \frac{p(qS - \kappa)}{r + pq} \left[ 1 - \left( \frac{1 - rW}{p\pi} \right)^{1+\frac{\nu}{r}} \right] \quad (A.2)$$

I now proceed to the second part of the proof and characterize the key properties of the optimal contract. Specifically, differentiate (A.2) with respect to $W$ and obtain

$$V'(W) = -1 + \frac{(qS - \kappa)}{\pi} \left( 1 - \frac{rW}{p\pi} \right)^{\frac{\nu}{r}}$$

$$V''(W) = -\frac{q(qS - \kappa)}{(\pi)^2} \left( 1 - \frac{rW}{p\pi} \right)^{\frac{\nu}{r} - 1}$$

which implies that $V'(0) = -1 + \frac{qS - \kappa}{\pi}$, $V'(\frac{p\pi}{r}) = -1$ and $V''(W) < 0$ for all $W \in (0, \frac{p\pi}{r})$. These in turn imply the following on the optimal contract:

- First, the relationship should never be initiated if $V'(0) < 0$, or if $qS \leq \pi + \kappa$.

  From now on, focus on the situation where $qS > \pi + \kappa$.

- Second, $W \leq \frac{p\kappa}{r}$. To see this, notice that because $V'(\frac{p\pi}{r}) = -1$, the principal prefers to reward the agent by cash than by continuation value when $W$ reaches $\frac{p\kappa}{r}$. As a result, $W$ should never exceed $\frac{p\kappa}{r}$.

- Third, before the relationship is terminated, the principal prefers to experiment whenever possible, i.e., $d(W, H) = 1$ and $d(W, L) = 0$. For this policy to be
optimal at every $W \in (0, W)$, I need

$$\begin{align*}
-\kappa \Delta + q \Delta (1 - \beta(W)) S + (1 - q \Delta) e^{-r \Delta} V(W_{t+\Delta}(H, G)) &\geq e^{-r \Delta} V(W_{t+\Delta}(L)) \\
\end{align*}$$

In the continuous time limit, this amounts to verifying that

$$qV(W) + \pi V'(W) \leq qS - \pi - \kappa - qW, \text{ or } V(W) - WV'(W) \geq 0$$

The last condition is clearly satisfied. To see this, define $G(W) = V(W) - WV'(W)$ and notice that under (A.2), I have $G(0) = 0$ and $G'(W) = -WV''(W) > 0$ for every $W \in (0, W)$.

The final part of the proof verifies that Conjecture I is indeed true under the optimal contract. 

A.3 Omitted Proofs in Section 4

Proof of Proposition 1

Proof. The above argument implies that the principal’s value function should take the following form:

$$V(W) = \alpha \left[ \frac{p\pi}{r} - W \right]^{1+\frac{pq}{r}} - W + \frac{p(qS - \kappa)}{r + pq}$$

Pinning down $\alpha$ by the smooth-pasting condition $V(\beta S) = V'(\beta S)\beta S$ yields

$$\alpha = -\frac{r(qS - \kappa)}{(r + pq)\pi} \left( \frac{p\pi}{r} - \frac{\beta S}{\pi} \right)^{-\frac{pq}{r}}$$
For this equation to make sense, I need $\beta S < \frac{\pi r}{r}$. Yet if $\beta S \geq \frac{\pi r}{r}$, then the agent is demanding more than what the principal can possibly promise. In this case, the principal should find it unprofitable to initiate the relationship in the first place. \hfill$\square$

**Proof of Proposition 2**

*Proof.* The proof is comprised of two parts. The first part solves $V_s(.)$ and $V_f(.)$ explicitly. Based on the previous discussion, I claim that under the fast track policy, the agent’s continuation value should evolve according to

$$\dot{W}_{H,B} = rW - \pi, \quad \dot{W}_L = rW$$

and his surplus share in a consolidated relationship should equal to

$$\beta(W) = \frac{qW + \pi}{qS}$$

Now formulate the principal’s value function recursively as

$$V_f(W) = p\left[-\kappa\Delta + q\Delta(y^1 + S(1 - \beta(W))) + (1 - q\Delta)e^{-r\Delta}V_f(W_{t+\Delta}(H, B))\right]$$

$$+ (1 - p)\left[e^{-r\Delta}V_f(W_{t+\Delta}(L)) + p(qy^2 - \kappa)\Delta\right]$$

Since $rS^j = p(qy^j - \kappa)$ for $j = 1, 2$, simplify the above expression to

$$V_f(W) = rS^1\Delta + pq\Delta S(1 - \beta(W)) + p(1 - q\Delta)e^{-r\Delta}V_f(W_{t+\Delta}(H, B))$$

$$+ (1 - p)\left[e^{-r\Delta}V_f(W_{t+\Delta}(L)) + rS^2\Delta\right]$$

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Take $\Delta \to 0$ and rewrite $V_f(.)$ as the solution to the following ODE:

$$(r + pq)V_f(W) = rS^1 + p(qS - \pi) - pqW + (rW - p\pi)V'_f(W)$$

s.t. $V_f(0) = S^2$

Solving it yields

$$V_f(W) = -\frac{r(S^1 - S^2) + pq(S - S^2)}{r + pq} \left(1 - \frac{rW}{p\pi}\right)^{1+\frac{pq}{r}} - W + \frac{pqS + rS^1}{r + pq}$$

The case of slow track policy is similar. First, notice that since $V_s(.)$ is strictly concave and satisfies $V''_s(.) > -1$ in the interior of the domain, the agent’s continuation value should evolve according to

$$\dot{W}_{X^2} = \dot{W}_{X^2^{-},L} = rW, \quad \dot{W}_{X^2^{-},H,B} = rW - \pi$$

where "X^2" and "X^2^{-}" refer to the situations where $X^2$ is and isn’t available, respectively. Moreover, the agent’s surplus share at the consolidation stage should equal to

$$\beta(W) = \frac{qW + \pi}{qS}$$

Together, they imply that the principal’s value function is the solution to the following ODE:

$$(r + pq(1-p))V_s(W) = rS^2 + (1-p)[rS^1 + p(qS - \pi)] - pq(1-p)W + V'_s(W)[rW - p(1-p)\pi]$$

s.t. $V_s(0) = S^2$
Solving it yields

\[
V_s(W) = \frac{(1 - p)[pq(S - S^2) + rS^1]}{r + pq(1 - p)} \left( 1 - \frac{rW}{p(1 - p)\pi} \right)^{\frac{pq(1 - p)}{r}} - W \\
+ \frac{rS^2 + r(1 - p)S^1 + pq(1 - p)S}{r + pq(1 - p)}
\]

The second part of the proof derives the optimal contract. To begin with, notice that the principal’s value function under the optimal contract should coincide with the convex hull of \(V_s(.)\) and \(V_f(.)\), i.e.,

\[
V = \text{Conv}(V_f, V_s)
\]

since \(V_f(.)\) and \(V_s(.)\) are concave and hump-shaped on the relevant domains. Moreover, since

\[
V'_f(0) - V'_s(0) = -\frac{rS_2}{p\pi} < 0, \quad \text{and} \\
(V'_s)^{-1}(-1) = \frac{p(1 - p)\pi}{r} < \frac{p\pi}{r} = (V'_f)^{-1}(-1)
\]

then if \(V_f(.)\) and \(V_s(.)\) single-cross at some \(W^*\) such that \(V_i(W^*) > 0\) for both \(i = f, s\), it must be that \(V_f(.)\) single-crosses \(V_s(.)\) from below at \(W^*\), i.e.,

\[
V_f(W) \begin{cases} 
< V_s(W) & \text{iff } W \in (0, W^*) \\
> V_s(W) & \text{iff } W > W^*
\end{cases}
\]

Together with the concavity of \(V_f(.)\) and \(V_s(.)\), this in turn implies that there exist
\( W^\dagger < W^* \) and \( W^\ddagger > W^* \) such that

\[
V(W) = \begin{cases} 
V_s(W) & \text{if } W < W^\dagger \\
V_f(W) & \text{if } W \geq W^\ddagger 
\end{cases}
\]

At every \( W \in (W^\dagger, W^\ddagger) \), the principal implements the policy at \( W^\dagger \) with probability \( \frac{W^\ddagger - W}{W^\ddagger - W^\dagger} \), and the one at \( W^\ddagger \) with probability \( \frac{W - W^\dagger}{W^\ddagger - W^\dagger} \).

\( \square \)

\textbf{Proof of Proposition 3}

\textbf{Proof.} By construction, \( U \) satisfies

\[
U = p(qS - \kappa)\Delta + p(1 - (r + q)\Delta)U + (1 - p)(1 - r\Delta)U
\]

Simplifying this condition yields Equation 4.2.

Meanwhile, Theorem 1 implies that

\[
W_0 = \begin{cases} 
0 & \text{if } qS \leq \pi + \kappa \\
\text{Solution to } V'(W) = 0, \text{ i.e., } \frac{p\pi}{r} \left[ 1 - \left( \frac{\pi}{qS - \kappa} \right)^{\frac{r}{pq}} \right] & \text{if } qS > \pi + \kappa
\end{cases}
\]

Plugging \( W_0 \) to \( V(.) \) yields

\[
V(W_0) = \begin{cases} 
0 & \text{if } qS \leq \pi + \kappa \\
U - \frac{p\pi}{r} \left[ 1 - \frac{1}{1 + \frac{r}{pq}} \left( \frac{\pi}{qS - \kappa} \right)^{\frac{r}{pq}} \right] & \text{if } qS > \pi + \kappa
\end{cases}
\]

which implies Equation 4.2.
When \( qS > \pi + \kappa \), differentiating the RHS of Equation 4.2 with respect to \( \pi \) yields

\[
\frac{p}{r} \left[ \frac{pq - r}{pq + r} \left( \frac{\pi}{qS - \kappa} \right)^{\frac{pq}{pq + r}} + 1 \right]
\]

This expression is clearly positive if \( pq \geq r \) and is larger than the term below if \( pq < r \):

\[
\frac{p}{r} \left[ \frac{pq - r}{pq + r} + 1 \right] > 0
\]

Thus, the RHS of Equation 4.2 is increasing in \( \pi \) when \( qS > \pi + \kappa \).

References


