Endogenous Market Structure:  
Over-the-Counter versus Exchange Trading *

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Abstract

For many assets, traders favor either over-the-counter (OTC) or centralized markets. This paper examines how traders’ choice between these trading venues depends on asset and trader characteristics. Traders’ values depend on a common and idiosyncratic component, while traders have private information with heterogeneous precision. A trader’s incentive to choose an OTC market depends on the benefit of learning the asset value and the cost due to price impact. Traders choose OTC markets over centralized exchanges when the idiosyncratic component dominates in asset values or their private information is sufficiently inaccurate, and thus, the benefit from learning is high. This paper provides comparative statics of equilibrium and efficiency for centralized and over-the-counter markets. Furthermore, when the common value component dominates in asset values, the OTC market decreases information efficiency by being conducive to trade only between informed traders.

KEYWORDS: Noncompetitive trading, Over-the-counter markets, Exchanges, Price impact, Liquidity, Efficiency

1 Introduction

Over-the-counter markets have been an important alternative trading venues for many assets, goods, commodities, financial derivatives, and securities. In over-the-counter markets, buyers and sellers are paired and privately decide their own trading terms, while public exchanges use a centralized trading mechanism such as uniform-price auctions. Many commentators have raised concerns about the implications of the various market mechanisms for their role in facilitating information aggregation and efficiency. The goal of this paper is to examine these implications when traders individually choose a trading venue and a market structure is endogenously formed by their choice. Certain types of assets appear to be traded mostly in over-the-counter markets, whereas others have been traded in exchanges. Thus, the potential challenges for the traditional role of markets will not affect all types of assets and goods equally. This paper asks what attracts traders to over-the-counter markets and whether this type of trading venue can harm the efficiency of the economy.

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A framework introduced in this paper allows characterizing Bayesian Nash equilibrium in both over-the-counter and centralized trading venues. Within the framework we can study the equilibrium incentives of market participants depending on their types, namely, institutional and retail, informed and uninformed – to choose various trading venues. An economy is modeled for two periods. In the first period \( t = 0 \), each trader chooses to enter either a centralized market or an over-the-counter market that open at \( t = 1 \); then trade takes place by traders’ bids submitted in each market. The centralized market is designed as a uniform-price double auction, in which all traders simultaneously submit their (net) demand schedules \( q_i(p) : p \mapsto \mathbb{R} \) and trades clear at price \( p^* \) such that \( \sum_i q_i(p^*) = 0 \). The over-the-counter market is designed as bilateral trading based on the uniform-price auction, with trader pairs determined by the traders’ choice of the counterparty. More precisely, upon the participation in the over-the-counter market, traders observe types of others – the correlation between his and their asset valuations (buyers and sellers) and their information precision (informed and uninformed) – and choose a counterparty to trade with.

The model accommodates the following characteristics, which determine traders’ market choice: market size (market characteristic), interdependence of traders’ asset values (asset), and precision of private information (traders). Each characteristics affects learning and liquidity of traders. Equilibrium price aggregates all market participants’ private information on asset values, and by conditioning on price, traders learn the aggregated information. The market liquidity is measured by endogenous price impact, the change of price as a trader’s demand increases by one unit.\(^1\) Larger price impacts, i.e. lower liquidity, reduce demands of traders and lower their utilities. The effects of market size on learning and liquidity have been studied in literature. With more traders participating in the market, price reveals more accurate information. Moreover, in larger markets, price impact can be smaller when other characteristics are fixed (See Rostek and Weretka (2012)). The benefit due to both learning and improved liquidity leads traders to the centralized market because of its larger size.

Unlike the market size, the effect of interdependence among asset values is ambiguous. This paper shows that certain asset characteristics can increase both benefits from learning and liquidity to trade in the over-the-counter market. Traders are uncertain about the value of a risky asset. Their valuations are interdependent and have two components: a common component, which is the same for all traders, and an idiosyncratic component, which can be correlated heterogeneously across traders. The common value captures the asset return or a future price in dynamic market. In turn, the idiosyncratic value component comes from an individual portfolio return that is correlated to the asset traded in our economy. If the portfolios of traders are correlated (e.g., they contain the same assets), then the trader’s idiosyncratic values are also correlated. When the idiosyncratic component dominates the common value, in the sense that the spread of correlations are larger than the average level of correlations,\(^2\) over-the-counter markets are more attractive to traders in terms

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\(^1\)Other frictions considered in the literature (e.g. search costs, a chance that bilateral trades fail, bid-ask spreads by dealers, etc.) can increases incentives of traders to choose centralized markets over over-the-counter markets, but the results in this paper still hold quantitatively.

\(^2\)Suppose that \( \rho_{i,j} \) denotes the correlation between individual asset values of two traders \( i,j \). A trader \( i \)'s value is correlated all other traders \( \{ \rho_{i,j} \}_{j \neq i} \). The variance of these correlation represents the idiosyncratic component in trader \( i \)'s asset value, while the average of them represents the common component. See Section 2.
of both learning and liquidity. Equilibrium price aggregates traders’ private information in a form of a weighted average of signals, which implies that it aggregates out idiosyncratic components in centralized markets. When a trader’s value relies more on the idiosyncratic value (or the common value), he learns more on this component in an over-the-counter market (in a centralized market). On the other hand, liquidity incentive is independent of dominant components. Over-the-counter market allows a trader to choose a counterparty who would more likely have an opposite trading needs (i.e., more negatively correlated asset values), so that he has a lower price impact, while the centralized price mitigates the difference between values of buyers and sellers. This effect is strengthened when traders’ asset values are interdependent through the idiosyncratic values rather than the common component.

Each trader receives a private signal about value before any market opens. Signal precision can be heterogeneous among traders; thus, traders differs in the learning needs and value of information. Traders with low information precision (i.e. uninformed traders) benefit from an over-the-counter market because it helps them learn counterparties’ information. The benefit from learning encourages any, informed or uninformed, traders to participate in the over-the-counter market. On the other hand, the over-the-counter market discourages traders with large liquidity needs (i.e., large endowments) to participate, since it decreases the likelihood of meeting a counterparty to trade with and also may increase price impact. The trade-off between information and liquidity incentives in over-the-counter markets creates a cutoff level on information precision. If a trader’s information precision is higher than the cutoff level, the liquidity incentive dominates and he chooses to trade in centralized market. With a precision lower than the cutoff, the learning incentive dominates and traders choose the over-the-counter market. This result may seem contrary to the conventional belief that informed traders tend to trade in over-the-counter trades for keeping privacy and for the benefit of information advantage at later periods, but this paper considers static markets in which the dynamic incentive of keeping privacy does not exist. The effects in this paper are indeed complementary to those in dynamic markets for informed traders. In static models, or with short-lived information in dynamic models, informed traders choose the centralized market where private information can be aggregated out in equilibrium price so that traders benefit from more precise information in a period.

This paper identifies market structures that are formed in equilibrium by traders’ choices between trading venues: the centralized market, an over-the-counter market, or both markets coexisting. I show that an over-the-counter market opens when the size of centralized market is small, the asset values are closer to idiosyncratic than common, and private information of traders is less precise. Many financial derivatives such as forwards contracts, interest rate swaps, or equity or credit linked securities are traded in over-the-counter markets, even though their trading volumes (liquidity) are large. These products are often held by traders until, or close to, the maturity, which suggests that the purpose of trading can be hedging of traders’ outside portfolios. This paper suggests that idiosyncratically valued assets tend to be traded in the over-the-counter markets. On the other hand, centralized markets attract assets traded mostly by speculators, such as stocks or bonds with short maturity, which are valued by future prices that are common to all traders.
High-yield bonds that has low credit ranking are often traded in the over-the-counter markets (e.g., Hendershott and Madhavan (2015)). Low past trading volume and volatile return prevents the traders’ access to quality information. Moreover, it is possible to increase the information asymmetry between insiders and other traders. This is consistent with this paper’s prediction that low information precision encourages traders to choose over-the-counter markets.

I examine whether traders’ individual market choice leads to an equilibrium market structure that is efficient. For the simplicity of discussion, this paper analyzes markets with two informational types, informed and uninformed, and symmetric interdependence of asset values. If no centralized market is available for traders to enter, each trader faces a trade-off between learning and liquidity in the choice of counterparty in the over-the-counter market. Namely, trades with informed counterparties improve learning, while trades with uninformed counterparties provide better liquidity. Depending on which incentive dominates, one of two structures of over-the-counter markets is formed. When a same incentive dominates for all traders, for example, when the learning incentive is stronger than the liquidity incentive, all traders prefer to trade with informed counterparties. Knowing that he cannot be matched with informed counterparty, uninformed traders choose to trade with uninformed counterparties (i.e. same-type matching). On the other hand, if the dominant incentive is different for different types, informed and uninformed traders choose to trade with each other (i.e. cross-type matching). With an available centralized market, however, a bilateral trade between these two types does not occur in equilibrium. When the learning incentive dominates the liquidity incentive for an informed trader, he is better off trading in the centralized market that provides an even lower price impact than in the over-the-counter trading with uninformed counterparties. With only uninformed traders entering in the over-the-counter market, information is not transmitted between the informed and uninformed traders and the over-the-counter market aggravates the information asymmetry in the economy.

Related Literature: There is a growing literature providing theoretical models with over-the-counter markets. How liquidity affects traders’ behavior and efficiency are studied in the literature (e.g. Duffie, Garleanu, and Pedersen (2005), Vayanos and Weill (2008), Weill (2008), Atkeson, Eisfeldt, and Weill (2014)). Other studies show how private information is aggregated in the over-the-counter markets (e.g. Duffie, Malamud, and Manso (2014), Maurin (2015), Babus and Kondor (2016), Back, Liu, and Teguia (2016)). In addition to these papers focusing on over-the-counter markets, several papers compare these two trading venues in terms of welfare or individual profit. Observing that the welfare dominance of over-the-counter markets or centralized markets is ambiguous, Acharya and Bisin (2010), Malamud and Rostek (2014), Duffie and Wang (2016), Glode and Opp (2017), and other studies consider determinants to favor either over-the-counter or centralized trading: such as default, search friction, price impacts, or information asymmetry between sellers and buyers. Praz (2015) introduces a market structure in which a centralized (liquid) and over-the-counter (illiquid) markets co-exist and characterizes trading behaviors and equilibrium prices. In his model, traders are assumed to participate in both markets, while this paper considers traders’ entry problem, based on which markets are dominant in their individual utilities.
My model is close to Babus and Parlatore (2017). The authors examine over-the-counter networks between dealers and investors, when trading is based on a uniform-price double auction, as in this paper. Similar to mine, their paper show that the interdependence of traders’ asset values is a key determinants in the comparison between over-the-counter and centralized markets. Dealers’ price impacts create a trade-off between trading in an inter-dealer market and a local market with customers and determines either dominance of over-the-counter or centralized markets in dealer’s utilities. My contribution, compared to their paper, is introducing a benefit of learning from price. An arbitrary correlation structure for traders’ values allows imperfect information aggregation by equilibrium price (see Vives (2011), Rostek and Weretka (2012), and Rostek and Yoon (2017)), while their paper focuses on independent values so that the learning benefit is absent. Furthermore, Babus and Parlatore (2017) consider traders’ behavior for a given over-the-counter market. Equilibrium market structures are endogenously formed by traders’ market choice between over-the-counter and centralized markets in this paper.

Another strand of the literature endogenizes the over-the-counter market structure. Some papers consider traders’ counterparty choice, who they want to trade with, when an over-the-counter market is the only available market. Golosov, Lorenzoni, and Tsyvinski (2014) consider the joint effects of learning and liquidity in the choice of counterparties in over-the-counter markets. Chang and Zhang (2016) show that traders’ heterogeneous preferences endogenously form core-periphery networks in equilibrium and those who have low risk exposure serve as intermediaries. The core-periphery networks are also derived in other papers (e.g. Farboodi (2015), Farboodi, Jarosch, and Shimer (2017), Hugonnier, Lester, and Weill (2016), Wang (2017)). In addition to these studies for the counterparty choices, Atkeson, Eisfeldt, and Weill (2014) and Babus and Hu (2016) consider entry and exit problems for over-the-counter markets. Similar to these papers, over-the-counter markets are endogenously formed in this paper. Traders’ heterogeneity is the key determinant for the resulting over-the-counter structures, in their information precision (as in Golosov, Lorenzoni, and Tsyvinski (2014)) and in their valuations (as in Chang and Zhang (2016)).

The objective in this paper is to understand endogenous market structures when centralized and over-the-counter markets are both available. The choice between the centralized and over-the-counter markets, when they can co-exit, has been explored by several authors. Kirilenko (2000) and Viswanathan and Wang (2002) consider a choice between trading venues for dealers by non-strategic agents (e.g. designers, authorities, consumers) maximizing profit or efficiency of the market. Bolton, Santos, and Scheinkman (2015) consider an entry problem of an informed seller to either market: a centralized (organized) market or an over-the-counter market with uninformed dealers. In this paper, all buyers and sellers, informed and uninformed strategically choose a trading venue. This lets us explore the stability of market structures: no trader has an incentive to change his market choice given chances.

costs and liquidity are key determinants on why most trades for bonds are held in over-the-counter markets, while Attanasi, Centorrino, Moscati (2016) explores the effects of lack of information in the over-the-counter market on efficiency.

2 Model

In a static economy, two trading venues open simultaneously: a centralized market where all traders' bids are executed at a single market price and an over-the-counter market where a pair of traders are matched and they trade bilaterally at a pair-specific price. Figure 1 summarizes the economy. Before the markets are open (entering period \( t = 0 \)), traders can choose which market they would trade in. If a trader choose the over-the-counter market, then he also chooses a counterparty he would like to trade with. The market choice and bilateral matching are done at the end of period \( t = 0 \). Traders can trade only once, in one market and with one counterparty if they are in the over-the-counter market. At trading period \( t = 1 \), two assets – a risky asset (asset) and a riskfree asset (numeraire) – are traded in both markets. The assets are perfectly divisible. Traders submit their demands to the market they chose at the entering period, and each market cleared independently. I describe the details below, including (1) traders and payoffs, (2) information, (3) structures of the markets, (4) strategies, and (5) equilibrium.

![Figure 1: Timing of the economy](image)

**Strategic Traders:** There are \( I < \infty \) strategic traders. Each of them has initial endowment \((w^0, q^0)\), where \( w^0 \) and \( q^0 \) are the amounts of numeraire and asset, respectively. If a trader \( i \) holds \((w_i, q_i)\) after trading in the market he participates in, his ex-post utility is define as

\[
u_i(w_i, q_i) = -\exp\left(-\mu (w_i + \theta_i q_i)\right).
\]

Here, \( \mu > 0 \) is a constant absolute risk-aversion (CARA) that is common for all traders, and \( \theta_i \) is the individual value of the risky asset for trader \( i \) that is randomly drawn from \( \theta_i \sim \mathcal{N}(E[\theta_i], \sigma^2) \).
Here, suppose that idiosyncratic value \( q_i \) is decomposed into two independent random variables: for a given common value \( v \), the idiosyncratic value component, which comes from an individual portfolio return (e.g. securities, assets, production whose returns are correlated to the asset in the market). The idiosyncratic value component, which comes from the future asset return in the market, and an idiosyncratic value component, which comes from individual portfolio return (e.g. securities, assets, goods, production whose returns are correlated to the asset in the market). The idiosyncratic value component is assumed to be independent to the common value component, but it can be correlated to other traders’ idiosyncratic value components.\(^3\) Example 1 shows how the common value and idiosyncratic value components determine \( \Sigma \) in the simplest but intuitive setting.

**Example 1 (Symmetric Interdependence in Asset Values)** There are two groups of strategic traders – buyers and sellers – with equal group sizes. Each trader has individual asset value that is decomposed into two independent random variables: for a given common value \( v_c \) and idiosyncratic value \( v_i \), a trader \( i \)'s asset value is

\[
\theta_i = v_c + v_{p,i}.
\]

Here, suppose that \( \text{Var}(v_c) = \frac{\sigma^2 + \rho \cdot \sigma^2}{2} \), \( \text{Var}(v_{p,i}) = \sigma^{2}_{\theta} - \text{Var}(v_c) = \frac{2 - (\rho_0 + \rho) \cdot \sigma^{2}_{\theta}}{2(\rho_0 - \rho)} \), and \( \text{corr}(v_{p,i}, v_{p,j}) = \frac{\rho_0 - \rho}{2(\rho_0 - \rho)} \) if both \( i \) and \( j \) are sellers or both are buyers, and \( \text{corr}(v_{p,i}, v_{p,j}) = -\frac{\rho_0 - \rho}{2(\rho_0 - \rho)} \) if one is buyer and the other is seller. Then, the correlation matrix \( \Sigma = (\text{corr}(\theta_i, \theta_j))_{ij} \) is as follows:

\[
\Sigma = \begin{bmatrix}
\rho_01_{(1/2)\times(1/2)} & \rho1_{(1/2)\times(1/2)} \\
\rho1_{(1/2)\times(1/2)} & \rho01_{(1/2)\times(1/2)}
\end{bmatrix} + (1 - \rho_0)I_d.
\]

\(^3\)Rostek and Yoon (2017) show the existence and uniqueness of equilibrium in markets with arbitrary Gaussian structure.

\(^4\)This model with arbitrary interdependence of asset values can incorporates various interpretations and settings. For instance, as another interpretation of common and idiosyncratic value components: Each trader gets a random initial endowment before he enters the market, that can be correlated with other traders endowments. This private endowment forms his idiosyncratic value component. When the market has more trading rounds \( \tau > t = 1 \) after the rounds we are considering in the model \( t = 1 \), the asset value at \( t \) is determined by the marginal value function, which is a linear combination of individual asset return and future market prices. Hence, traders’ valuation is interpreted as a combination of idiosyncratic value by holding the asset and common value by selling it at market price.
If two traders have positively correlated values, \( \text{Corr}(\bar{\theta}_i, \bar{\theta}_j) = \rho_0 > 0 \), then both of them want to either buy or sell. If their values are negatively correlated, \( \text{Corr}(\bar{\theta}_i, \bar{\theta}_j) = \rho < 0 \), then they are willing to hold opposite positions, one buyer and one seller. With this parameterization, we say that traders’ valuation are dependent more on the common value when \( \frac{\rho_0 + \rho}{2} \) is larger. Furthermore, traders’ idiosyncratic value components are more correlated when \( \frac{\rho_0 - \rho}{2 - (\rho_0 + \rho)} \) is larger. We will consider this example for further analysis in Section 4. □

**Information:** Each strategic trader gets a private information (signal) on his own valuation, \( s_i = \theta_i + \varepsilon_i \) with an independent noise \( \varepsilon_i \sim \mathcal{N}(0, \sigma_{i,\varepsilon}^2) \). The information precision \( \phi_i \equiv 1/\sigma_{i,\varepsilon}^2 \) can differ across traders, and it determines the traders’ informational type. Traders’ types and prior distribution of asset values and signals are common knowledge.

**Centralized Market (CM):** The centralized exchange is designed as a uniform-price double auction that is a canonical model for markets. A strategic trader \( i \), who enters the centralized exchange with endowment \((w^0, q^0)\), submits his demand schedule \( \Delta q_i(p) : \mathbb{R} \to \mathbb{R} \) as a continuous function of price. There exists \( L \) traders, called liquidity traders, in the centralized market who are not given the market choice. Liquidity traders strategically submit their demand schedule \( \Delta q_{lq}(p) \) as well as the traders who can choose a market,\(^5\) based on their own private information. For simplicity, precision of private information for liquidity traders are homogeneous, \( \sigma_{lq,\varepsilon}^2 \in (0, \infty] \). After all demands are collected, the centralized market is cleared at a price of which the total demand is equal to zero; \( p^* \) such that \( \sum_{i \in I} \Delta q_i(p^*) + \sum_{j \in L} \Delta q_{lq,j}(p^*) = 0 \). The equilibrium allocation is determined by the demand schedule traders submitted, \( q^*_i = q_0 + \Delta q^*_i = q_0 + \Delta q_i(p^*) \) and \( w^*_i = w_0 - p^* q^*_i \).

**Over-the-Counter Market (OTC):** In the over-the-counter market, each trader chooses a counterparty based on their types, information precision and correlation. If the choice is mutual between two traders, then they are matched as a pair. If the counterparty choice is not mutual, then traders are not given a trading opportunity and the market ends. If there are more than one trader for the chosen type of counterparty, a trader of the type is randomly matched. In a matched pair, two traders simultaneously submit their demand schedules \( \Delta q_i(p) \) as functions of price \( p \). The equilibrium price in this bilateral trading is determined as same as in the centralized market: the price \( p^* \) solves the market clearing condition \( \Delta q_i(p^*) + \Delta q_j(p^*) = 0 \). If an equilibrium price does not exist, then there is no trade and the over-the-counter market ends without any further trade. There is no liquidity trader in the over-the-counter bilateral trades.

**Strategies:** At \( t = 0 \), each strategic trader \( i \in I \) chooses a market where he enters, \( m_i \in \{\text{OTC, CM}\} \) and a type of counterparty \( \tau_i \) who he would trade with upon entering the over-the-counter exchange, \( m_i = \text{OTC} \). When the market choice of a trader is \( m_i = \text{CM} \), we will note \( \tau_i = \emptyset \) for the convenience. At \( t = 1 \), the trader chooses his demand function \( q_i(\cdot : m_i, \tau_i) \) in market \((m_i, \tau_i)\). Therefore, the strategy profile of trader \( i \) is \( \{(m_i, \tau_i), q_i(\cdot : m_i, \tau_i)\} \). A liquidity trader

\(^5\)The existence of liquidity traders make the centralized exchange more liquid than the over-the-counter market, independently of strategic traders’ market choice. In later section, I show that liquidity traders are those who do not observe other traders types so that they optimally choose to stay in the centralized exchange.
j \in L \text{ in the centralized market has a strategy } \{(CM, \emptyset), q_j(\cdot; CM, \emptyset)\} \text{ since she cannot enter the over-the-counter market.}

**Equilibrium**: Definition 1 provides two conditions for equilibrium: Bayesian Nash equilibrium in each market and no incentive to deviate from market and counterparty choices.

**Definition 1 (Equilibrium)** For each trader $i \in I$,

(i) traders’ optimal bid schedules $\{q^i(\cdot : m_i, \tau_i)\}_i$, such that,

$$\max_{q_i} E[u_i(w_0 - pq_i, q'_0 + q_i)|s, p] = \max_{q_i} E[-\exp( - \mu(w_i + \tilde{\theta}_i q_i))|s, p], \ \forall \ p \in \mathbb{R},$$

characterize a Bayesian Nash equilibrium in each market.

(ii) No trader has a strictly positive incentive to deviate from the market and counterparty. When $E[u_i(m_i, \tau_i)]$ denotes the expected utility with $(m_i, \tau_i)$ for given equilibrium distribution of traders in both markets, the optimal market and counterparty choice $(m^*_i, \tau^*_i)$ satisfies

$$E[u_i(m^*_i, \tau^*_i)] \geq E[u_i(m_i, \tau_i)], \ \forall (m_i, \tau_i) \neq (m^*_i, \tau^*_i), \ \forall i.$$

The following sections characterize equilibrium defined in Definition 1: Equilibrium bid strategies and outcomes in Bayesian Nash equilibrium for a given market - part (i) - is characterized in Section 3. The characterization allows us to develop comparative statics on traders’ expected utilities over market, asset, or traders characteristics. Section 4 shows equilibrium market structures that is endogenously formed by traders’ market and counterparty choice - part (ii) - and analyze influences of the characteristics in traders’ incentives to choose over-the-counter markets and thus in market structure. Section ?? provides efficiency properties in equilibrium.

### 3 Equilibrium in a Market: Learning and Liquidity Incentives

This section shows traders’ bidding strategies in a given market and for a given distribution on participants’ types. Equilibrium characterization in a market provides traders’ expected utility in equilibrium and how the utility depends on the characteristics of market, asset, or traders. Such comparative statics analyzes traders’ incentives to choose one market over the other.

Suppose that there are $I$ traders in a market with a correlation structure of asset values $\Sigma$ and a information precision $\{\phi_i = 1/\sigma^2_{i, \alpha}\}_i$. Each trader $i$ optimizes his expected utility (1). The first order condition is characterized as follows:

$$E[\theta_i|s, p] - \mu Var(\theta_i|s, p)q_i - p - \lambda_i q_i = 0, \ \forall \ p \in \mathbb{R},$$

where $\lambda_i \equiv \partial p/\partial q^i$ is price impact that represents the change of price when trader $i$ increases his demand by one unit. A larger price impact implies that each unit of a trader’s demand increases the equilibrium price further so that a trader places a smaller demand with higher price impact. Such
demand reduction by price impact captures *market illiquidity* that is endogenously determined by traders’ strategies. A competitive market with infinitely many traders is perfectly liquid in that the price impact is zero. In primitives, when there are fewer traders in the market or when traders are more sensitive to price changes due to inference or risk-aversion, the market becomes less liquid.

In a trader’s first-order condition (2), he takes an expectation of asset value conditioning on the equilibrium price $p$ as well as his own private information. A trader chooses his bid at each potential realization of price, and thus, his behavior incorporates the information revealed by the price as if he observes the price. Therefore, even in the static model, traders’ learning on their asset values occurs implicitly by the schedule bidding. We do not impose a value of such learning or a value of improved information. The benefit or cost of learning information from the market is valued in terms of equilibrium utility change. Endogenizing learning and illiquidity in this model allows us to consider a market choice without any assumption on exogenous frictions.

Proposition 1 states three equilibrium conditions for a given market with $I$ traders whose asset values are correlated by $\Sigma$ and whose information precisions are $\{\phi_i\}$: a trader’s strategy for a given price impact (illiquidity) and inference on asset values (learning), the consistency condition for equilibrium price impact, and inference coefficients by equilibrium price distribution. In general, there is no closed-form characterization for equilibrium, outside of models with symmetric correlation and symmetric precision. A linear Bayesian Nash equilibrium uniquely exists under conditions. Positive price impacts for all traders are imposed, which is equivalent to the submitted bidding function should be strictly decreasing in price.

**Proposition 1 (Equilibrium Representation in a Market)** In a market, a profile of demand schedules $\{q_i(\cdot)\}$ is a linear Bayesian Nash equilibrium (hereafter, equilibrium) if and only if

(i) a demand schedule $q_i(\cdot : \lambda_i)$ maximizing trader $i$’s utility is

\[
q_i = \frac{E[\theta_i|s_i,p] - p}{\mu Var(\theta_i|s_i,p) + \lambda_i} = \frac{c_{\theta,i}E[\theta_i] + c_{s,i}s_i - (1 - c_{p,i})p}{\mu Var(\theta_i|s_i,p) + \lambda_i},
\]

where $E[\theta_i|s_i,p] = c_{\theta,i}E[\theta_i] + c_{s,i}s_i + c_{p,i}p$.

(ii) price impacts satisfy the consistency condition

\[
\lambda_i = -\left(\sum_{j \neq i} \frac{\partial q_j(\cdot)}{\partial p}\right)^{-1} = \left(\sum_{j \neq i} \frac{1 - c_{p,j}}{\mu_i Var(\theta_j|s_j,p) + \lambda_j}\right)^{-1}, \quad \forall i,
\]  

(iii) inference coefficients $\{c_{\theta,i}, c_{s,i}, c_{p,i}\}$ in $E[\theta_i|s_i,p]$ and conditional variance $Var(\theta_i|s_i,p)$ are determined by the Projection Theorem, with equilibrium price distribution following

\[
p = \left(\sum_i \frac{1 - c_{p,i}}{\mu Var(\theta_i|s_i,p) + \lambda_i}\right)^{-1} \sum_i \frac{c_{\theta,i}E[\theta_i] + c_{s,i}s_i}{\mu Var(\theta_i|s_i,p) + \lambda_i}.
\]

A trader’s indirect utility in equilibrium can be written as a function of his price impact $\lambda_i$, expected asset value $E[\theta_i|s_i,p]$ conditioning on $(s_i,p)$, and conditional variable $Var(\theta_i|s_i,p)$. Ex-
ante utility of trader \(i\) in a given market is

\[
E[u_i] = E \left[ -\exp \left( -\mu \left( \frac{\mu Var(\theta|s_i, p)}{2} + 2\lambda_i \left( E[\theta|s_i, p] - p \right)^2 \right) \right) \right].
\]

Considering that the difference of individual expected asset value from equilibrium price, \((E[\theta|s_i, p] - p)\), follows a normal distribution that is generated by Gaussian structure of \(\{\theta_i, s_i\}_i\), the expectation on the right hand side of the above equation is in form of the moment generating function for \(\chi_k^2\) distribution. It provides an explicit formula for the ex-ante indirect utility:

\[
E[u_i] = -\left(1 + \frac{1 + 2\hat{\lambda}_i}{(1 + \hat{\lambda}_i)^2} \frac{Var(E[\theta|s_i, p] - p)}{Var(\theta|s_i, p)} \right)^{-1/2}, \quad \forall i.
\]

(4)

Here, \(\hat{\lambda}_i \equiv \left(\mu Var(\theta_i|s_i, p)\right)^{-1}\lambda_i\) is a normalized price impact by the quadratic coefficient \(\mu Var(\theta_i|s_i, p)\) of trader \(i\)’s mean-variance utility. Trader \(i\)’s ex-ante utility \(E[u_i]\), or a sufficient statistic \(\tau_i \equiv (E[u_i]^{-2} - 1)\), can be decomposed into two parts: the value of liquidity and learning. The benefit of liquidity is captured by the term \((1 + 2\hat{\lambda}_i)/(1 + \hat{\lambda}_i)^2 = 1 - (\hat{\lambda}_i/(1 + \hat{\lambda}_i))^2\). Recall that trader \(i\)’s demand is reduced by a fraction \(\hat{\lambda}_i/(1 + \hat{\lambda}_i)\). In that,

\[
q_i = \left(1 - \frac{\lambda_i}{\mu Var(\theta_i|s_i, p) + \lambda_i} \right) \frac{E[\theta_i|s_i, p] - p}{\mu Var(\theta_i|s_i, p)} = \left(1 - \frac{\hat{\lambda}_i}{1 + \hat{\lambda}_i} \right) q^*_i(p),
\]

where \(q^*_i(p)\) is the demand of trader \(i\) in a competitive market for a given price \(p\). The demand reduction lowers utility with the same fraction. The liquidity benefit in utility terms, called liquidity effect, is increasing in the normalized price impact \(\hat{\lambda}_i\). On the other hand, the utility (4) contains a term \(Var(E[\theta_i|s_i, p] - p)/Var(\theta_i|s_i, p)\) that captures a benefit of learning from price and private information. Equilibrium price aggregates all market participants’ private information on asset values. By conditioning on price, traders learn the aggregated information. It decreases the risk in uncertainty of the trader’s own value \(\theta_i\) and thus increases his expected utility through the term \(Var(\theta_i|s_i, p)\). In addition, the price reveals information on other traders’ asset values, that determines net surplus of buying a unit of asset, \(E[\theta_i|s_i, p] - p\). Such information on the future surplus influences the trader’s utility through \(Var(E[\theta_i|s_i, p] - p)\). The total benefit of learning trader’s own and others’ valuation, called learning effect, is incorporated in a form of ratio in the expected utility (4).

What are the key determinants of traders’ utilities? Three characteristics can be considered in this model: market size (market characteristic), interdependence of traders’ asset values (asset), and precision of private information (traders). Each characteristic affects learning and liquidity in traders’ utilities. Example 2 shows the influences of three characteristics, with symmetric interdependent asset values and symmetric information precisions across traders.

**Example 2 (Symmetric Interdependence and Precision)** Consider a market with \(I\) traders. All traders has a symmetric information precision \(\phi_i = 1/\sigma^2_{i,\epsilon} \equiv \phi\) and has a symmetric average
correlation to the residual market $\tilde{\rho}_i = \frac{1}{I-1} \sum_{j \neq i} \rho_{ij} \equiv \tilde{\rho}$ for all $i$.\(^6\) Each trader’s optimal schedule and equilibrium price are

$$q_i = \frac{E[\theta_i|s_i,p] - p}{\mu Var(\theta_i|s_i,p) + \lambda} = \frac{c_d E[\theta_i] + c_s s_i - (1 - c_p) p}{\mu Var(\theta_i|s_i,p) + \lambda}, \quad \forall i,$$

$$p = \frac{1}{1 - c_p} (c_d E[\theta] + c_s \sum_i s_i) = \frac{1}{1 - c_p} (c_d E[\theta] + c_s \bar{s}).$$

Here, the liquidity and learning effects in trader $i$’s ex-ante utility are characterized with the price impact and conditional variances:

$$\tilde{\lambda}_i = \frac{\lambda_i}{\mu Var(\theta_i|s_i,p)} = \frac{(1 + (I - 1)\tilde{\rho})(1 + \sigma^2 - \tilde{\rho})}{(I - 2)(1 + \sigma^2) + ((I - 1)^2 + 1 - 2(I - 1)(1 + \sigma^2))\tilde{\rho} - (I - 1)(I - 2)\tilde{\rho}^2},$$

$$Var(\theta_i|s_i,p) = \frac{(1 + \sigma^2) + (I - 2)\tilde{\rho} - (I - 1)\tilde{\rho}^2}{(1 + \sigma^2 + (I - 1)\tilde{\rho})(1 + \sigma^2 - \tilde{\rho})} \sigma_i^2,$$

$$Var(E[\theta_i|s_i,p] - p) = \frac{(1 - \tilde{\rho})^2}{1 + \sigma^2 - \tilde{\rho}} \frac{I - 1}{I - \sigma_i^2}.$$

Trader $i$ gets the ex-ante utility $E[u_i] = -(1 + \tau_i)^{-2}$ where

$$\tau_i = \frac{1 + 2\tilde{\lambda}_i}{(1 + \tilde{\lambda}_i)^2} \frac{Var(E[\theta_i|s_i,p] - p)}{Var(\theta_i|s_i,p)}.$$

The liquidity effect on the utility is captured by the term $\frac{1 + 2\tilde{\lambda}_i}{(1 + \tilde{\lambda}_i)^2}$. With this closed-form solution of inference parameters and price impact,\(^7\)

$$\frac{1 + 2\tilde{\lambda}_i}{(1 + \tilde{\lambda}_i)^2} = 1 - \left(\frac{\tilde{\lambda}_i}{1 + \tilde{\lambda}_i}\right)^2 = 1 - \left(\frac{(1 + \sigma^2 - \tilde{\rho})(1 + (I - 1)\tilde{\rho})}{(I - 1)(1 - \tilde{\rho})(1 + \sigma^2 + (I - 1)\tilde{\rho})}\right)^2.$$

The liquidity term is increasing as $I$ increases or $\tilde{\rho}$ decreases. Larger market size and/or more negative correlation with others on average results in more liquidity and thus higher utility for traders. When the information precision $\phi = 1/\sigma^2$ increases, the endogenous liquidity of the market increases if $\tilde{\rho} > 0$, and decreases if $\tilde{\rho} < 0$.

The effect of learning from the price on utility is measured by

$$\frac{Var(E[\theta_i|s_i,p] - p)}{Var(\theta_i|s_i,p)} = \frac{I - 1}{I} \frac{(1 - \tilde{\rho})^2(1 + \sigma^2 + (I - 1)\tilde{\rho})}{\sigma^2(1 + \sigma^2) + (I - 2)\tilde{\rho} - (I - 1)\tilde{\rho}^2},$$

which is increasing in the information precision $\phi = 1/\sigma^2$. The effect of average correlation $\tilde{\rho}$ is ambiguous. If $(1 + \sigma^2) + (2I - 3)(1 + \sigma^2)\tilde{\rho} + (I - 3)(I - 1)\tilde{\rho}^2 - (I - 1)^2\tilde{\rho}^3 > 0$, the utility component due to learning is decreasing in $\tilde{\rho}$. Otherwise, it is increasing in the average correlation $\tilde{\rho}$. \(\Box\)

The example shows how three key characteristics affect the ex-ante utility of each trader, through liquidity and learning. The same intuition can be applied to general models with asymmetric interdependent structure for asset values and asymmetric information precision. Proposition 2 states

\(^6\)Rostek and Weretka (2012) call this correlation structure an *equiconmonal model* and derive a closed form characterization of equilibrium.
effects of each of three characteristics on traders’ expected utilities, when the other characteristics are fixed.

Proposition 2 (Benefits of Learning and Liquidity) In a sufficiently symmetric market, subject to existence, the ex-ante utility of a trader $i$ increases as

(i) asset values are more negatively correlated to price, i.e. $\text{corr}(\theta_i, p)$ are more negative; or

(ii) the number of traders in market is larger so that the price impact is smaller, i.e. $\lambda_i$ is smaller.

The utility is non-monotone in the average information precision, such that

(iii) an information precision $\phi^*_{-i} \in (0, \infty]$ of other traders maximizes trader $i$’s utility.

From Proposition 2 (i), when equilibrium price is more negatively correlated to his asset valuation, traders’ utility increases in terms of both learning and liquidity. Price provides new information that is not captured in trader $i$’s private information, and with larger correlation in the absolute sense implies that the information is more relevant to his asset value. This learning effect is captured by the decrease of conditional variance $V(\theta_i|s_i, p)$. The correlation structure also affects the liquidity through the endogenous price impacts $\lambda_i$. The price impact is characterized by the slope of residual supply curve,

$$\lambda_i = -\left(\sum_{j \neq i} \frac{\partial q_j(\cdot)}{\partial p}\right)^{-1},$$

that is an inverse of aggregate reaction of other traders when price increases. With more negative correlations, trader $j \neq i$ would reply on price for his inference, in the sense that $c_{p,j}$ is more negative. It makes his demands more elastic to price change and thus trader $i$’s price impact smaller. The effects of the number of traders (part (ii)) on learning and liquidity have been studied in literature. In a sufficiently symmetric market, with more traders participating in the market, price reveals more accurate information. Moreover, in larger markets, price impact can be smaller when other characteristics are fixed (See Rostek and Weretka (2012)). From the arguments, more negative correlations between traders’ asset values and/or more number of traders in the market are beneficial to both learning and liquidity.

Information precision has an ambiguous influence on traders’ expected utilities. First, it is worth to remark that Proposition 2 (iii) states the effect of other traders’ precision rather than a trader’s own. With symmetric information precision, the effect of the traders’ own precision cannot be analyzed separately from the effect of other traders’ precision that is aggregated in equilibrium price. The trader’s own precision represents his innate needs for learning. It determines how much the trader values learning from the market. Intuitively, if a trader has a more precise information, his needs for learning new information is smaller. Others’ precision measures how much the trader can learn from the market (i.e. improved information by conditioning on price). When we fix the correlation between a trader’s asset value and the aggregate value in the market (i.e. fix $\text{cov}(s_i, \text{avg}(s_j))$), the value of learning increases as the trader’s own information precision is lower or as the (weighted) average of other traders’ information precision is higher. On the other hand, the price impact increases, and thus, the liquidity decreases, as the value of learning increases. It creates a trade-off between learning and liquidity when the precision of information from the price changes.
The trade-off between learning and liquidity over traders’ information precision is shown in Figure 2. Suppose that the average correlation and the number of traders in the market are fixed. A trader $i$ has a information precision $\phi_i = 1/\sigma_i^2 = 1/0.25 = 4$. The liquidity effect and the learning effect in his expected utility (4) are shown in each graph in Figure 2. Two effects are monotone over the other traders’ information precision $\text{avg}(\phi_j)_{j\neq i} = \text{avg}(1/\sigma_j^2)_{j\neq i}$, and show the trade-off. It results in the total effect of other traders’ information precision on trader $i$’s utility is non-monotone. The utility is maximized at a certain precision $\phi_{-i}^* \equiv \text{avg}(\phi_j)_{j\neq i}$ of other traders, and it can be at $\phi_{-i}^* < \infty$ or $\phi_{-i}^* = \infty$ depending on trader $i$’s own precision $\phi_i$. In that, if trader $i$’s own precision is high, then $\phi_{-i}^* < \infty$ because of the trade-off. If his precision is sufficiently low, the learning effect dominates liquidity effect, so that his utility is monotonically increasing in others’ precision.

These comparative effect in a give market provides some predictions on traders’ market and counterparty choice: which types of traders would enter an over-the-counter market depending on the interdependence of asset values and the precision of their information. The ambiguity in influence of information precision on traders’ utilities will be considered as a key determinants of traders’ market and counterparty choices.

4 Equilibrium Market Structure

Traders’ individual choice for markets and counterparties forms a distribution of traders’ types in centralized and over-the-counter market and matching in the over-the-counter market, which is called a market structure. There are three types of market structure: only centralized market opens, only over-the-counter market opens, and two markets co-exist. To gather some intuitions on endogenous market structures, I first consider the symmetric model in Example 2 with a competitive centralized market (i.e. perfectly liquid market with $\lambda_i = 0$ for all $i$). Assuming the competitive centralized market maximizes the difference in market sizes. The example shows that traders can be attracted to the over-the-counter market, even in such cases, depending on asset and trader characteristics.
Example 2 - Continued Equation (4) provides the explicit formula of expected utility when there are \( I \) symmetric traders. The utility at the centralized market is derived by taking \( I \) to infinity, while the utility at a bilateral trade in the over-the-counter market is by setting \( I = 2 \). With these ex-ante utilities in two exchanges, trader \( i \) chooses which exchange he wants to enter to. Under the equilibrium existence, the necessary and sufficient condition for him to enter the over-the-counter exchange is as follows:

\[
E[u_i^{CM}] < E[u_i^{OTC}] \iff \tau_i^{CM} = \frac{1 - \bar{\rho}CM}{\sigma^2} < \tau_i^{OTC} = \frac{-2\rho_{OTC}}{1 + \sigma^2 + \rho_{OTC}}
\]

where \( \bar{\rho}CM \) is the average correlation in the centralized market and \( \rho_{OTC} \) is the correlation between two traders who are matched in the over-the-counter market (Example 1).

- \( \bar{\rho}CM = 0, \rho_{OTC} = -1 \) (no common value; correlated idiosyncratic). The inequality can be written by \( \frac{1}{\sigma^2} < \frac{4}{2 + \sigma^2} \). It is satisfied if and only if \( \sigma^2 > \frac{2}{3} \). It implies that, when the interdependence of asset values are idiosyncratic, traders who has low information precision \( \phi = 1/\sigma^2 < \frac{3}{2} \) enter the over-the-counter market even when the centralized market is competitive.

- \( \bar{\rho}CM = 0, \rho_{OTC} = 0 \) (independent private value). The inequality becomes \( \frac{1}{\sigma^2} < 0 \), which never holds for any \( \sigma^2 \geq 0 \). Independent private value structure implies that the market has no valuable information to any trader (no learning occurs), so that traders choose the centralized market for the benefit of liquidity.

- \( \bar{\rho}CM = \epsilon, \rho_{OTC} = -1 + 2\epsilon \) (with both common and idiosyncratic values).

\[
\frac{1 - \epsilon}{\sigma^2} < \frac{2(1 - 2\epsilon)}{\sigma^2 + 2\epsilon} \iff \epsilon < \frac{2 + 3\sigma^2 - \sqrt{(2 + 3\sigma^2)^2 - 8}}{4}; \quad \text{or} \quad \frac{2\epsilon(1 - \epsilon)}{1 - 3\epsilon} < \sigma^2
\]

Traders prefer to trade in the over-the-counter market if the common value component in asset values is sufficiently small or information precision is sufficiently low.

When the centralized exchange is competitive, the liquidity incentive strongly derives traders to avoid higher price impacts in over-the-counter bilateral trades. Hence, a participation to over-the-counter market occurs only when benefit of learning from the market is high enough to dominate the loss from illiquidity. When the aggregated correlation \( \bar{\rho}CM = \frac{1}{I-1} \sum_{j \neq i} \rho_{ij} \) satisfies \( |\bar{\rho}CM| < |\rho| \), the price informativeness is higher in the over-the-counter exchange and thus the benefit from learning is higher.

We will show that the over-the-counter market is more attractive to certain types of traders when information accuracy is heterogeneous across traders in following sections. Section 4.1 considers a subgame where traders who are in the over-the-counter market choose an informational type of his counterparty, and then, Section 4.2 solves the whole problem and the endogenously determined market structure.
4.1 Choice of Counterparty in Over-the-Counter Market

In the over-the-counter market, each trader is given an opportunity to choose his counterparty based on informational type (informed/uninformed) and trading type (buyer/seller). The trading will occur only when the chosen counterparty also choose his type. We denote the trader who chooses the counterparty by trader $i$, while his counterparty by trader $j$.

Consider when two traders $i$ and $j$ can be successfully matched. First, no matter of information accuracy, there is no matching between two players who have positively correlated asset values. In that, if $\text{Corr}(\tilde{\theta}_i, \tilde{\theta}_j) = \rho_0 > 0$, traders optimal bid function becomes inelastic so that there is no trade. Therefore, no trader chooses the same trading type as his counterparty. On the other hand, the choice for counterparty’s informational type is not trivial. In an analogous argument with Proposition 2, Corollary 1 show the determinant of the expected utility in a bilateral trade,

$$E[u_i] = \frac{\mu + 2\lambda_i}{2(\mu + \lambda_i)^2} \cdot \text{Var}(E[\theta_i|s_i, p] - p) :$$

the correlation between two trader $\text{corr}(\tilde{\theta}_i, \tilde{\theta}_j) = \rho < 0$ and information accuracy $(\sigma_i, \sigma_j)$.

**Corollary 1 (Learning and Liquidity in the OTC Market)** Upon traders’ participation in the over-the-counter market, as the counterparty’s information precision increases,

(i) the liquidity incentives decreases, i.e. $\frac{\mu + \lambda_i}{2(\mu + \lambda_i)^2}$ decreases in $(1/\sigma_j^2)$; and

(ii) the learning incentive increases, i.e. $\text{Var}(E[\theta_i|s_i, p^*] - p^*)$ increases in $(1/\sigma_j^2)$.

Liquidity incentive dominates learning incentive, if and only if the trader’s own accuracy $(1/\sigma_i^2)$ is sufficiently high.

As we can see in Corollary 1, the uninformed trader is better off than the informed trader in a bilateral over-the-counter market. It is because the price in bilateral trade is fully revealing about the counterparty’s private information, so that it removes informational advantage of informed trader. Furthermore, learning from the price is more valuable for uninformed trader, which leads him to be more sensitive to price change. The trader who chooses an uninformed counterparty faces a higher price impact. We conclude that a trader who has a sufficiently accurate private information prefers an uninformed counterparty in the benefit of smaller liquidity friction, while a trader who has less accurate private information prefers an informed counterparty for learning benefit. The cutoff value of his own accuracy with which a trader is indifferent between informed and uninformed counterparties are dependent on the set of accuracy $\{\sigma_k\}_{k=I,U}$.

With two informational types, the over-the-counter matching can be formed in two ways: same-type matching where traders are matched with a trader with the same type, and cross-type matching where informed and uninformed traders are matched to each other. Figure 3 presents regions of $\{\sigma_I, \sigma_U^2\}$ for the same-type and cross-type matching in equilibrium. First, let us consider the case where informed trader has sufficiently high accuracy (small $\sigma_I$) and uninformed trader has
sufficiently low accuracy (large $\sigma_U$). Both types of traders choose to trade with the opposite type of counterparty, with a different incentives: informed traders get benefit from liquidity incentive, and uninformed traders get from learning incentive. Hence, the cross-type matching is equilibrium. Outside of this region, equilibrium shows the same-type matching. When the accuracy levels for both traders are high (small $\sigma_I$ and $\sigma_U$), liquidity incentives dominates and all traders want to be matched with an uninformed counterparty. It results in that some informed traders may not be matched with their preferred counterparty. Since there is no more chance of another match and trade, the informed traders optimally shift their counterparty choice to less preferred counterparty, informed traders. Hence, the same-type matching occurs in equilibrium. Lastly, when the accuracy is low for both types (large $\sigma_I$ and $\sigma_U$), all traders prefer to trade with informed counterparty for learning incentive. With the similar argument with the high accuracy case, uninformed traders should shift their counterparty choice into the second best, and equilibrium shows the same-type matching.

Proposition 4 in Appendix shows a sufficient and necessary condition for the cross-type matching equilibrium, in terms of the set of information accuracy ($\sigma_I, \sigma_U$) and correlation between buyers and sellers $\rho$. We emphasize the cross-type matching, because it provides information transmission from informed to uninformed traders. The transmission reduce the information asymmetry across traders at the end of market. However, with the same-type matching, informed traders share their information between themselves, not with uninformed traders. It worsen the information asymmetry. We will discuss the efficiency in later sections with market choice.

4.2 Equilibrium with Market Choice

Based on the comparative statics on equilibrium utility for a given market in section 3 and the counterparty choice in section 4.1, we now characterize equilibrium for the whole problem considering traders’ choice between over-the-counter market and centralized market.

We recall that a trader faces a counterparty in the over-the-counter market who has a more
strongly and negatively correlated asset value than the aggregate residual market in the centralized market. It is because (i) he can choose his own counterparty in the over-the-counter market based on their correlation and information accuracy, and because (ii) the aggregated asset value in the centralized market includes both buyers and sellers, and both informed and uninformed, so that its correlation to the trader’s value is mitigated. A trader who chooses to enter the over-the-counter market gets sufficient benefit from this information incentives, since he faces higher price impact in a bilateral trade in over-the-counter market than in centralized market (where sufficiently many liquidity traders present). From Proposition 2, we know that these traders exists when the asset valuations are more relying on the correlated private value rather than the common value, and their private information has sufficiently inaccurate.

**Proposition 3 (Information Sharing in OTC)** With two informational types \( \{\sigma_I < \sigma_U\} \) and with positive common value (i.e. \( \rho_0 + \rho > 0 \)), in equilibrium, the over-the-counter matching between informed and uninformed traders (i.e. cross-type matching) does not occur.

Figure 4 is an example of equilibrium market structure through traders’ market and counterparty choice. The figure on left shows three types of equilibrium: all traders choose the centralized market (when both \( \sigma_I \) and \( \sigma_U \) are small, and learning is not sufficiently valuable to neither of them); only uninformed traders choose to trade in the over-the-counter market (large \( \sigma_U \) but small \( \sigma_I \)); and all traders choose the over the counter market (both types of traders has inaccurate information). Since learning incentive is more likely dominating for uninformed traders, there is no equilibrium where only informed traders enter the over-the-counter market.

![Figure 4: Equilibrium Market Structure: \( \sigma_\theta^2 = 1, \rho = -0.5 \)](image)

Traders in the over-the-counter market always trade with a same-type counterparty. The sub-game equilibrium for over-the-counter matching are shown in the figure on right. When an informed trader has learning incentive dominated by liquidity incentive, he would choose an uninformed counterparty if he is already in the over-the-counter market. However, with his own trade-off between two incentives, it is more better off for him to trade in the centralized market, rather than the cross-
type bilateral trade in over-the-counter market. Since informed traders do not enter, uninformed traders in the over-the-counter market are matched with another uninformed.

Non-existence of matching between informed and uninformed is interpreted as information asymmetry worsening by over-the-counter market. With a random match mechanism, information can be transmitted from informed to uninformed when they are met and thus information asymmetry disappears or diminishes over time. However, when traders choose their own counterparty based on how accurate information they have, informed traders do not want to be matched with uninformed traders. The information is shared only with each type, and the asymmetry between types increases after trades in our setting. It results in the informational inefficiency by allowing an over-the-counter market into the economy.

5 Discussion

Connection to Markets. I show that an over-the-counter market opens when the size of centralized market is small, the asset values are closer to idiosyncratic than common, and private information of traders is less precise. Many financial derivatives such as forwards contracts, interest rate swaps, or equity or credit linked securities are traded in over-the-counter markets, even though their trading volumes (liquidity) are large. These products are often held by traders until, or close to, the maturity, which suggests that the purpose of trading can be hedging of traders’ outside portfolios. This paper suggests that idiosyncratically valued assets tend to be traded in the over-the-counter markets. On the other hand, centralized markets attract assets traded mostly by speculators, such as stocks or bonds with short maturity, which are valued by future prices that are common to all traders. High-yield bonds that has low credit ranking are often traded in the over-the-counter markets (e.g., Hendershott and Madhavan (2015)). Low past trading volume and volatile return prevents the traders’ access to quality information. Moreover, it is possible to increase the information asymmetry between insiders and other traders. This is consistent with this paper’s prediction that low information precision encourages traders to choose over-the-counter markets.

Alternative Over-the-Counter Designs. This paper design over-the-counter markets by uniform-price double auction, as well as centralized markets. The benefit of designing centralized markets and over-the-counter markets consistently with the same mechanism is that the results on market choice is based on the characteristics of market, asset, or traders, rather than the difference between mechanisms. When conventional mechanisms and/or frictions in literature on over-the-counter markets are imposed, the results in this paper do not change. For instance, suppose that the over-the-counter market is operated by random matching instead of traders’ counterparty choice. Traders’ expected utilities in the over-the-counter market would strengthen the effect of asymmetric interdependence of traders’ asset values and heterogeneous information precisions. Uninformed traders has a chance to meet an informed traders and to learn more precise information, while informed traders’s liquidity can be improved with a higher chance of meeting uninformed counterparty. It implies that imposing random matching mechanism in the over-the-
counter market does not affect the endogenous market structure qualitatively, but it can increase traders’ incentive to enter the over-the-counter market when the heterogeneity across traders is present. Exogenous frictions in the over-the-counter markets - a probability that a trader does not trade, cost of waiting, etc. - can decrease traders’ incentive to trade in over-the-counter markets.

Another alternative mechanism can arise based on the timeline of the model. In this paper, traders’ individual market choice occurs ex-ante, in that traders choose where to trade before their private information is realized. This model is appropriate when traders’ individual asset values \( \theta_i \) contains future returns of the asset in the markets and/or future returns of traders’ individual portfolios, as interpreted in Section 2. The future returns of certain assets are often unobservable or costly to observe, to those who are not in the market in order to keep market participants privacy. If the traders asset values or their private information are interdependent by other sources - for example, traders’ endowments before markets, pre-trades, macroeconomic information, cheap talk, etc. - the model may incorporate interim market choice, in that, traders choose a market to participate in after they observe the private information. Interim market choice increases the dimension of heterogeneity, in addition to the correlation structure and information precision. A trader whose realized signal is high can be more attracted to the over-the-counter market since the difference between asset values and price, \( |E[\theta_i|s_i,p] - p| \), is larger in a bilateral trades. \(^7\)

The additional heterogeneity of realized private information with interim market choice influence equilibrium distribution of traders’ types in each market and the distribution of equilibrium prices, but the trade-off between learning and liquidity for each trader still exists.

References


\(^7\)Boyarchenko, Lucca, and Veldkamp (2015) study the effect of realized private signal in market structure in a different market mechanism from this paper. They consider an inter-dealer market and dealer-customer market. Traders are given a choice to be either a dealer or a customer. They show that traders who has a high private signal choose to trade directly in the inter-dealer market to keep his information private. Although the conjecture on interim market choice in my model is related to the intuitions in Boyarchenko, Lucca, and Veldkamp (2015), their results is not applied. This paper considers traders’ learning on the values due to demand schedule conditioning on equilibrium prices, while Boyarchenko, Lucca, and Veldkamp (2015) assumes that traders reveal their private information truthfully to the dealers. Hence, the learning incentive of traders in over-the-counter markets is absent.


A Proofs

Proof of Proposition 1 (Equilibrium Representation in a Market). For a given price impact \( \lambda_i > 0 \) and inference \( E[\theta_i|s_i,p] = c_{\theta,i}E[\theta_i] + c_{s,i}s_i + c_{p,i}p \), trader \( i \)'s first order condition gives his best response, i.e. demand schedule.

\[
q_i = \frac{E[\theta_i|s_i,p] - p}{\mu Var(\theta_i|s_i,p) + \lambda_i} = \frac{c_{\theta,i}E[\theta_i] + c_{s,i}s_i - (1 - c_{p,i})p}{\mu Var(\theta_i|s_i,p) + \lambda_i}.
\]

With positive price impacts, the second order condition holds for all \( i \):

\[
-\mu Var(\theta_i|s_i,p) - 2\lambda_i < 0.
\]

The market clearing condition \( \sum_i q_i(\cdot) = 0 \) determines equilibrium price from the demand function.

\[
p = \left( \sum_i \frac{1 - c_{p,i}}{\mu Var(\theta_i|s_i,p) + \lambda_i} \right)^{-1} \sum_i \frac{c_{\theta,i}E[\theta_i] + c_{s,i}s_i}{\mu Var(\theta_i|s_i,p) + \lambda_i}.
\]

Since the price is a linear function of traders’ private information \( \{s_i\}_i \), it follows a normal distribution as well as the signals. This Gaussian-linear structure allows us to use the Projection Theorem in order to derive traders’ conditional expectation on asset value. First, the unconditional expectation of price is equal to \( E[\theta_i] \) which is same across traders. It results in \( c_{\theta,i} + c_{s,i} + c_{p,i} = 1 \) for any \( i \). The inference coefficient \( \{c_{s,i}, c_{p,i}\} \) is

\[
\begin{bmatrix}
c_{s,i} \\
c_{p,i}
\end{bmatrix}
= \begin{bmatrix}
Var(s_i) & Cov(s_i,p) \\
Cov(s_i,p) & Var(p)
\end{bmatrix}^{-1}
\begin{bmatrix}
Cov(\theta_i, s_i) \\
Cov(\theta_i,p)
\end{bmatrix},
\]
and the conditional variance of $\theta_i$ over $(s_i, p)$ is

$$
\text{Var}(\theta_i|s_i, p) = \text{Var}(\theta_i) - \frac{1}{\text{Var}(\theta_i|s_i, p)} \left[ \frac{\text{Cov}(\theta_i, s_i)}{\text{Var}(\theta_i, p)} \right] \left[ \frac{\text{Var}(s_i)}{\text{Cov}(s_i, p)} \frac{\text{Cov}(s_i, p)}{\text{Var}(p)} \right]^{-1} \left[ \frac{\text{Cov}(\theta_i, s_i)}{\text{Var}(\theta_i|s_i, p)} \frac{\text{Cov}(\theta_i, p)}{\text{Var}(p)} \right].
$$

We denote $\sigma_i^2 = \sigma_{s,i}^2/\sigma_{p,i}^2$, the relative variance of noise in private information compared to variance of asset values. By plugging the following variance and covariance of $(s_i, p)$ into equation (5),

$$
\text{Var}(p) = (\sum \frac{1 - c_{p,i}}{\mu \text{Var}(\theta_i|s_i, p) + \lambda_i})^{-2} \left( \frac{c_{s,i}}{\mu \text{Var}(\theta_i|s_i, p) + \lambda_i} \right) \left( \frac{\sigma_i^2 \Sigma + \text{diag}(\sigma_i^2)}{\mu \text{Var}(\theta_i|s_i, p) + \lambda_i} \right),
$$

$$
\text{Cov}(s_i, p) = (\sum \frac{1 - c_{p,i}}{\mu \text{Var}(\theta_i|s_i, p) + \lambda_i})^{-1} \left( \frac{c_{s,i}}{\mu \text{Var}(\theta_i|s_i, p) + \lambda_i} \right) \left( \frac{\sigma_i^2 \rho_{ij} + \sigma_i^2 1_{j = i}}{\mu \text{Var}(\theta_i|s_i, p) + \lambda_i} \right),
$$

we get a fixed point problem for the inference coefficients $\{c_{s,i}, c_{p,i}\}_i$, 

$$
c_{s,i} = \frac{\sum_j \frac{c_{s,j}c_{s,k}(\rho_{jk} + \sigma_i^2 1_{j = k}) - c_{s,j}c_{s,k}(\rho_{jk} + \sigma_i^2 1_{j = k})}{\mu \text{Var}(\theta_j|s_j, p) + \lambda_j}}{\sum_j \frac{(1 + \sigma_i^2)c_{s,j}c_{s,k}(\rho_{jk} + \sigma_i^2 1_{j = k}) - c_{s,j}c_{s,k}(\rho_{jk} + \sigma_i^2 1_{j = k})}{\mu \text{Var}(\theta_j|s_j, p) + \lambda_j}}, \quad \forall i, \quad (6)
$$

$$
c_{p,i} = \frac{\sum_j \frac{1 - c_{p,j}}{\mu \text{Var}(\theta_j|s_j, p) + \lambda_j} \sum_j \frac{(1 + \sigma_i^2)c_{s,j}c_{s,k}(\rho_{jk} + \sigma_i^2 1_{j = k}) - c_{s,j}c_{s,k}(\rho_{jk} + \sigma_i^2 1_{j = k})}{\mu \text{Var}(\theta_j|s_j, p) + \lambda_j}}{\sum_j \frac{(1 + \sigma_i^2)c_{s,j}c_{s,k}(\rho_{jk} + \sigma_i^2 1_{j = k}) - c_{s,j}c_{s,k}(\rho_{jk} + \sigma_i^2 1_{j = k})}{\mu \text{Var}(\theta_j|s_j, p) + \lambda_j}}, \quad \forall i, \quad (7)
$$

In addition, the price impacts are characterized by

$$
\lambda_i = \left( \sum_{j \neq i} \frac{1 - c_{p,j}}{\mu \text{Var}(\theta_j|s_j, p) + \lambda_j} \right)^{-1}, \quad \forall i, \quad (8)
$$

for a given inference coefficients $\{c_{p,j}\}_j$ and the conditional variance of asset values,

$$
\text{Var}(\theta_i|s_i, p) = \sigma_i^2 \left( 1 - c_{s,i} - c_{p,i} \sum_j \frac{1 - c_{p,j}}{\mu \text{Var}(\theta_j|s_j, p) + \lambda_j} \right)^{-1} \sum_j \frac{c_{s,j}\rho_{ij}}{\mu \text{Var}(\theta_j|s_j, p) + \lambda_j}, \quad \forall i. \quad (9)
$$

Equations (6) - (9) solves $\{c_{s,i}, c_{p,i}, \lambda_i, \text{Var}(\theta_i|s_i, p)\}_i$, and thus, characterizes equilibrium. [Existence and Uniqueness proof from Rostek and Yoon (2017). If it is not done in time, delete this part.]

With the equilibrium characterization, a trader’s indirect interim utility is

$$
E[u_i|s_i, p] = -\exp \left( -\mu(-pq_i + E[\theta_i|s_i, p]q_i - \frac{\mu}{2} \text{Var}(\theta|s_i, p)q_i^2) \right) = -\exp \left( -\mu \left( \frac{\mu \text{Var}(\theta|s_i, p) + 2\lambda_i}{(2\mu \text{Var}(\theta|s_i, p) + \lambda_i)^2} (E[\theta_i|s_i, p] - p)^2 \right) \right),
$$

\[23\]
while his ex-ante utility is

$$E[u_i] = E\left[-\exp\left(-\mu\frac{\mu Var(\theta|s_i,p) + 2\lambda_i}{2(\mu Var(\theta|s_i,p) + \lambda_i)^2}(E[\theta_i|s_i,p] - p)^2\right)\right].$$

Considering that the difference of individual expected asset value from equilibrium price, $(E[\theta_i|s_i,p] - p)$, follows a normal distribution that is generated by Gaussian structure of $\{\theta_i, s_i\}$, the expectation on the right hand side of the above equation is in form of the moment generating function for $\chi^2$ distribution. It provides an explicit formula for the ex-ante indirect utility:

$$E[E[u_i|s_i,p]] = E\left[-\exp\left(-\mu\frac{\mu Var(\theta|s_i,p) + 2\lambda_i}{2(\mu Var(\theta|s_i,p) + \lambda_i)^2}(E[\theta_i|s_i,p] - p)^2\right)\right].$$

$$= E\left[-\exp\left(-\mu\frac{\mu Var(\theta|s_i,p) + 2\lambda_i}{2(\mu Var(\theta|s_i,p) + \lambda_i)^2}Var(E[\theta_i|s_i,p] - p)\chi^2_{k=1}\right)\right].$$

$$= -\left(1 - 2\left(-\mu\frac{\mu Var(\theta|s_i,p) + 2\lambda_i}{2(\mu Var(\theta|s_i,p) + \lambda_i)^2}Var(E[\theta_i|s_i,p] - p)\right)\right)^{-1/2}$$

$$= -\left(1 + 2\mu\frac{\mu Var(\theta|s_i,p) + 2\lambda_i}{2(\mu Var(\theta|s_i,p) + \lambda_i)^2}Var(E[\theta_i|s_i,p] - p)\right)^{-1/2}$$

We introduce a measure for the ex-ante utility. For each $i$,

$$\tau_i \equiv \left(\frac{1}{E[u_i]} - 1\right) = \frac{1}{\mu Var(\theta|s_i,p) + \lambda_i)^2}Var(E[\theta_i|s_i,p] - p).$$

The ex-ante utility is strictly increasing and strictly concave in $\tau_i$. ■

**Proof of Proposition 2 (Benefits of Learning and Liquidity).** In equilibrium for a given market, subject to existence, the ex-ante indirect utility of a trader $i$ is strictly increasing and strictly concave in the following measure:

$$\tau_i = \frac{\mu\frac{\mu Var(\theta|s_i,p) + 2\lambda_i}{2(\mu Var(\theta|s_i,p) + \lambda_i)^2}Var(E[\theta_i|s_i,p] - p)}{1 + \frac{2\lambda_i}{(1 + \lambda_i)^2}Var(\theta_i|s_i,p).}$$

Intuitively, more negative asset correlation increases the variance of difference between individual asset value and price, $Var(E[\theta_i|s_i,p] - p)$; the higher information precision of other traders, on average, improves trader $i$’s learning from price so that decreases $Var(\theta_i|s_i,p)$; and the larger market size decreases price impact $\lambda_i$. Suppose that the correlation of traders’ asset values is symmetric across traders: when the average correlation is defined by $\bar{\rho}_i \equiv (\frac{\sum_{j \neq i} c_{s,i} c_{s,j}}{\mu Var(\theta_i|s_i,p) + \lambda_i})^{-1} \frac{1}{I-1} \sum_{j \neq i} c_{s,i} c_{s,j} \rho_{ij}$ for each $i$, the symmetric market assumes

$$\frac{c_{s,i}}{\mu Var(\theta_i|s_i,p) + \lambda_i}^2 \bar{\rho}_i (1 + \sigma_i^2 + (I-1)^2 \bar{\rho}_i) = \frac{c_{s,j}}{\mu Var(\theta_j|s_j,p) + \lambda_j}^2 \bar{\rho}_j (1 + \sigma_j^2 + (I-1)^2 \bar{\rho}_j), \ \forall i \neq j.
In general, the inference coefficients are

\[
c_{s,i} = \frac{1}{1 - \frac{1}{\mu Var(\theta_i|s_i,p) + \lambda_i}} \left(1 + \sigma_i^2 + (I - 1)\hat{\rho}_i\right)(1 + \sigma_i^2 + (I - 1)\hat{\rho}_i)
\]

\[
c_{p,i} = \frac{1}{\mu Var(\theta_i|s_i,p) + \lambda_i} \sigma_i^2 + (I - 1)\hat{\rho}_i\rho_i\sigma_i^2
\]

In symmetric markets, for each \(i\),

\[
c_{s,i} = \frac{1 - \hat{\rho}_i}{1 + \sigma_i^2 - \hat{\rho}_i}, \quad c_{p,i} = \frac{\hat{\rho}_i\sigma_i^2}{(1 - \hat{\rho}_i)(1 + \sigma_i^2 + (I - 1)\hat{\rho}_i) + \rho_i\sigma_i^2}
\]

\[
\lambda_i = \left(\sum_{j \neq i} \frac{1 - c_{p,j}}{\mu Var(\theta_j|s_j,p) + \lambda_j}\right)^{-1}.
\]

Furthermore, the inference coefficients characterize the learning effect in expected utility.

\[
Var(E[\theta_i|s_i,p] - p) = \left(\sigma_i^2 + (1 - c_{p,i})^2 Var(p) - 2c_{s,i}(1 - c_{p,i}) Cov(s_i, p)\right)
\]

\[
= \sigma_i^2(1 + \sigma_i^2 + (1 - c_{p,i})^2\left(\sum_i \frac{1 - c_{p,i}}{\mu Var(\theta_i|s_i,p) + \lambda_i}\right)^{-2} \sum_{j,k} \frac{c_{s,j}c_{s,k}r_{jk} + \sigma_j^2\sigma_k^2}{\mu Var(\theta_j|s_j,p) + \lambda_j})
\]

\[
-2c_{s,i}(1 - c_{p,i})\left(\sum_i \frac{1 - c_{p,i}}{\mu Var(\theta_i|s_i,p) + \lambda_i}\right)^{-1} \sum_j \frac{c_{s,j}(\rho_{ij} + \sigma_i^2\delta_{j=i})}{\mu Var(\theta_j|s_j,p) + \lambda_j}.
\]

\[
Var(E[\theta_i|s_i,p] - p)\left(Var(\theta_i|s_i,p)\right)^{-1} = \left(\frac{\sigma_i^2(1 + \sigma_i^2 + (1 - c_{p,i})^2\left(\sum_i \frac{1 - c_{p,i}}{\mu Var(\theta_i|s_i,p) + \lambda_i}\right)^{-2} \sum_{j,k} \frac{c_{s,j}c_{s,k}r_{jk} + \sigma_j^2\sigma_k^2}{\mu Var(\theta_j|s_j,p) + \lambda_j}}}{1 - c_{s,i} + c_{p,i}(\sum_j \frac{1 - c_{p,j}}{\mu Var(\theta_j|s_j,p) + \lambda_j})^{-1} \sum_j \frac{c_{s,j}(\rho_{ij} + \sigma_i^2\delta_{j=i})}{\mu Var(\theta_j|s_j,p) + \lambda_j}}\right).
\]

From the above equilibrium characterization, we can explore influences of key characteristics on trader \(i\)'s expected utility \(E[u_i]\), that are the market size \(I\), average correlation \(\hat{\rho}\), and information precision of other traders. We impose another simplicity on information precision: \(\sigma_j^2 = \sigma_i^2\) for any \(j \neq i\). It implies that all other traders \(j \neq i\) except for trader \(i\) have symmetric strategy. This assumption is to make the derivation simpler, but it does not affect conclusions.

\[
\tilde{\lambda}_i = \frac{1}{(I - 1)(1 - c_{p,-i})} \frac{\mu Var(\theta_{-i}|s_{-i},p) + \lambda_{-i}}{\mu Var(\theta_i|s_i,p)}
\]

\[
c_{s,i} = \frac{1 - \hat{\rho}_i}{1 + \sigma_i^2 - \hat{\rho}_i}, \quad c_{p,i} = \frac{\hat{\rho}_i\sigma_i^2}{(1 - \hat{\rho}_i)(1 + \sigma_i^2 + (I - 1)\hat{\rho}_i) + \rho_i\sigma_i^2}
\]

\[
\frac{Var(E[\theta_i|s_i,p] - p)}{Var(\theta_i|s_i,p)} = \frac{\sigma_i^2Var(s_i) + (1 - c_{p,i})^2Var(p) - 2c_{s,i}(1 - c_{p,i}) Cov(s_i, p)}{1 - c_{s,i} - c_{p,i}(\sum_j \frac{1 - c_{p,j}}{\mu Var(\theta_j|s_j,p) + \lambda_j})^{-1} \sum_j \frac{c_{s,j}(\rho_{ij} + \sigma_i^2\delta_{j=i})}{\mu Var(\theta_j|s_j,p) + \lambda_j}}.
\]

(i) Market size, equivalently, the number of traders \(I\): We can see that \(I\) affects utility only
through the normalized price impact $\hat{\lambda}_i$ and the inference coefficient on price $c_{p,i}$. As $I$ increases, $\hat{\lambda}_i$ decreases and $|c_{p,i}|$ increases for sufficiently large $I$. The liquidity effect in utility, $(1+2\hat{\lambda}_i)/(1+\hat{\lambda}_i)^2$, increases by the decrease of $\hat{\lambda}_i$. The learning effect in equation (10) increases when $\bar{\rho}_i > 0$ and decreases when $\hat{\rho}_i < 0$. With sufficiently symmetric market, the effect of liquidity dominates the learning effect, so that the expected utility $E[u_i]$ increases in the market size $I$.

(ii) Average correlation $\bar{\rho}_i = \bar{\rho}$: As more negative correlation $\bar{\rho}_i$, i.e., as $\bar{\rho}_i$ decreases, the inference coefficient on private information $c_{s,i}$ decreases and the absolute value of the coefficient on price $|c_{p,i}|$ increases:

$$\frac{\partial c_{p,i}}{\partial \bar{\rho}_i} = \frac{\sigma_i^2(1 + \sigma_i^2 + (I - 1)\bar{\rho}_i^2)}{(1 - \bar{\rho}_i)(1 + \sigma_i^2 + (I - 1)\bar{\rho}_i) + \bar{\rho}_i\sigma_i^2)^2} \frac{1 + 2\hat{\lambda}_i}{\hat{\lambda}_i} > 0.$$  

It implies that the price impact $\hat{\lambda}_i$ and the conditional variance $Var(\theta_i|s_i,p)$ both decrease. In addition, it increases $Var(E[\theta_i|s_i,p] - p)$ by decreasing $Cov(s_i,p)$. Hence, the more negative correlation $\bar{\rho}$ increases both liquidity and learning effects and thus increases traders’ expected utility.

(iii) Information precision $\sigma_{-i}^2$ of other traders: as the information precision $1/\sigma_{-i}^2$ increases (i.e., $\sigma_{-i}^2$ decreases), trader $j \neq i$’s inference coefficient on private information $c_{s,-i}$ increases and the absolute value of the coefficient on price $|c_{p,-i}|$ decreases.

$$\frac{\partial c_{p,-i}}{\partial \sigma_{-i}^2} = \frac{\bar{\rho}(1 - \bar{\rho})(1 + (I - 1)\bar{\rho})}{((1 - \bar{\rho})(1 + \sigma_{-i}^2 + (I - 1)\bar{\rho}) + \bar{\rho}\sigma_{-i}^2)^2} \frac{1 + 2\hat{\lambda}_{-i}}{\hat{\lambda}_{-i}}.$$  

It makes $\hat{\lambda}_i$ decreasing if $\bar{\rho}_{-i} > 0$ and increasing $\hat{\lambda}_i$ otherwise. Hence, the liquidity effect of trader $i$’s utility changes depending on the correlation of other traders: as $\sigma_{-i}^2$ decreases, the liquidity effect $(1 + 2\hat{\lambda}_i)/(1+\hat{\lambda}_i)^2$ increases when $\bar{\rho}_{-i} > 0$ and decreases when $\bar{\rho}_{-i} > 0$. The learning effect in trader $i$’s utility is increasing in other traders’ information precision by the decrease of $Var(\theta_i|s_i,p)$. The liquidity and learning can create a trade-off with respect to others’ information precision. ■

**Lemma 1** Upon traders’ participation in the over-the-counter exchange, there is no matching between two players who have positively correlated asset values. In that, if $Corr(\tilde{\theta}_i, \tilde{\theta}_j) = \rho_0 > 0$, then neither of trader $i$ nor $j$ choose the other as his counterparty.

**Proof.** What we want to prove is that there is no equilibrium in the bilateral trade. From the assumption that no equilibrium leads to no trade, the incentive to choose the other as a counterparty is zero. The equilibrium price is

$$\hat{p}^* = \left(\frac{1 - c_{p1}}{\mu + \lambda_1} + \frac{1 - c_{p2}}{\mu + \lambda_2}\right)^{-1} \left(\frac{c_{s1}s_1}{\mu + \lambda_1} + \frac{c_{s2}s_2}{\mu + \lambda_2} + \left(\frac{c_{q1}}{\mu + \lambda_1} + \frac{c_{q2}}{\mu + \lambda_2}\right)\theta\right).$$  

Therefore, the equilibrium price impact is characterized by

$$\lambda_i = \frac{\mu + \lambda_j}{1 - c_{pj}} = \frac{\mu(2 - c_{pj})}{(1 - c_{pi})(1 - c_{pj}) - 1} > 0, \quad i \neq j \in \{1, 2\}.$$  

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The characterization implies that the positivity condition for price impact $\lambda_i > 0$, $i = 1, 2$ implies that $c_{pi} < 1$ for both $i = 1, 2$. By the projection theorem, the endogenous coefficients $(c_{si}, c_{pi}, c_{b}, \lambda_i)_{i=1,2}$ are the fixed-point solution of the following system of equations: with $c_{si} + c_{pi} + c_{b} = 1$,

$$c_{si} = \frac{((c_{si} \mu + \lambda_i)(1 - \rho^2 + \sigma_{s,i}^2) - c_{si} \mu + \lambda_i \rho^2}{(c_{si} \mu + \lambda_i)((1 + \sigma_{s,i}^2)(1 + \sigma_{s,i}^2) - \rho^2)}, \quad c_{pi} = \frac{1 - c_{pi} \mu + \lambda_i}{(c_{si} \mu + \lambda_i)((1 + \sigma_{s,i}^2)(1 + \sigma_{s,i}^2) - \rho^2)}, \quad \forall i \neq j \in \{1, 2\}.$$

From the further calculation, we get the following explicit solution$^8$

$$c_{si} = \frac{(1 - \rho^2 + \sigma_{s,i}^2)(1 - \rho^2 + \sigma_{s,j}^2) - \rho^2 \sigma_{s,i}^2 \sigma_{s,j}^2}{(1 - \rho^2 + \sigma_{s,i}^2)(1 + \rho + \sigma_{s,i}^2)} = \frac{1 - \rho^2}{4 \rho \sigma_{s,i}^2(1 + \rho + \sigma_{s,i}^2)}$$

$$c_{pi} = \frac{(1 - \rho^2 + \sigma_{s,j}^2)(1 + \rho + \sigma_{s,j}^2) + (1 - \rho^2 + \sigma_{s,j}^2)(1 + \rho + \sigma_{s,j}^2) + 2 \rho \sigma_{s,j}^2(1 + \rho + \sigma_{s,j}^2)}{2 \rho \{ \sigma_{s,i}^2(1 + \rho + \sigma_{s,i}^2) + \sigma_{s,j}^2(1 + \rho + \sigma_{s,j}^2) \}} - \frac{\sigma_{s,j}^2(1 + \rho + \sigma_{s,j}^2)}{\sigma_{s,i}^2(1 + \rho + \sigma_{s,i}^2) + \sigma_{s,j}^2(1 + \rho + \sigma_{s,j}^2)}$$

Here, the positivity condition for the price impacts, $\lambda_i > 0, \lambda_j > 0$, is equivalent to $\rho < 0$. Hence, when two traders’ valuations have a non-negative correlation, there is no equilibrium.

**Proposition 4 (OTC matching equilibrium)** The over-the-counter market is in form of cross-type matching, if and only if

$$\frac{\sigma_{s,i}^2(1 + \rho + \sigma_{s,j}^2) + \sigma_{s,j}^2(1 + \rho + \sigma_{s,i}^2)}{4(1 + \rho)^2(1 + \sigma_{s,i}^2)(1 + \sigma_{s,j}^2) - \rho^2(1 + \rho + \sigma_{s,j}^2) - \rho^2(1 + \rho + \sigma_{s,i}^2)} + \frac{4 \rho \sigma_{s,i}^2(1 + \rho + \sigma_{s,i}^2)}{2 \rho \{ \sigma_{s,i}^2(1 + \rho + \sigma_{s,i}^2) + \sigma_{s,j}^2(1 + \rho + \sigma_{s,j}^2) \}} - \frac{\sigma_{s,j}^2(1 + \rho + \sigma_{s,j}^2)}{\sigma_{s,i}^2(1 + \rho + \sigma_{s,i}^2) + \sigma_{s,j}^2(1 + \rho + \sigma_{s,j}^2)}$$

for all $i \neq j$, when $L = (1 - \rho^2 + \sigma_{s,i}^2)(1 + \rho + \sigma_{s,j}^2) + (1 - \rho^2 + \sigma_{s,j}^2)(1 + \rho + \sigma_{s,i}^2)$.

**Proof.** The same-group matching gives a trader with $\sigma_{s,i}^2$ his expected utility as follows:

$$E[u_{si}^{same}] = \frac{\mu + 2 \lambda_{si} c_{si}^2}{(\mu + \lambda_{si})^2} 4(1 + \sigma_{s,i}^2) - \rho - \frac{\rho \sigma_{s,i}^2(1 - \rho^2 + \sigma_{s,i}^2)}{\mu(1 + \rho + \sigma_{s,i}^2)(1 + \rho + \sigma_{s,i}^2)}$$

$^8$As special cases, the above fixed-point problem provides that (a) in a symmetric case, $c_{si} = \frac{1 - \rho}{\mu + \lambda_{si}}, c_{pi} = 0$ and $c_{b} = \frac{2 \rho}{(1 + \rho)^2(1 + \rho + \sigma_{s,i}^2)}$, and that (b) if $\sigma_{s,i}^2 = \infty$, $c_{si} = \frac{1}{\mu + \lambda_{si}}, c_{pi} = 0$ and $c_{b} = \frac{2 \rho}{(1 + \rho)^2(1 + \rho + \lambda_{si})}$.
where \( \lambda = -\mu \frac{(1+\rho)(1-\rho+\sigma_i^2)}{2\rho \sigma_i^2}, c_s = \frac{1-\rho}{1-\rho+\sigma_i^2} \). Hence, the condition for that the cross-group matching is the equilibrium in OTC,

\[
E[u_i^{\text{cross}}] \geq -\frac{\rho \sigma_i^2(1-\rho^2+\sigma_i^2)}{\mu(1+\rho+\sigma_i^2)^2(1-\rho+\sigma_i^2)} , \quad \forall \ i \in \{i,n,un\}.
\]

\[
E[u_i^{\text{cross}}] = \frac{\mu + 2\lambda_i}{2(\mu + \lambda_i)^2} \left( \frac{1}{\lambda_i} + \frac{1}{\lambda_j} \right)^{-2} \left( \frac{c_{si}}{\lambda_i} \right)^2(1+\sigma_i^2) + \left( \frac{\mu + \lambda_i}{\mu + \lambda_j} \right)^2(1+\sigma_j^2) - 2\frac{c_{si} \mu + \lambda_i}{\lambda_i(\mu + \lambda_j)} \rho \sigma_i^2(1-\rho^2+\sigma_i^2)
\]

\[
L = (1-\rho^2+\sigma_i^2)(1+\rho+\sigma_j^2) + (1-\rho^2+\sigma_j^2)(1+\rho+\sigma_i^2)
\]

\[
\mu + 2\lambda_i = \mu + \frac{\rho(1+\rho)(\sigma_i^2 - \sigma_j^2) - L}{\rho \left( \sigma_i^2(1+\rho+\sigma_j^2) + \sigma_j^2(1+\rho+\sigma_i^2) \right)}
\]

\[
\mu + \lambda_i = \frac{2\rho \sigma_i^2(1+\rho+\sigma_j^2) - L}{2\rho \left( \sigma_i^2(1+\rho+\sigma_j^2) + \sigma_j^2(1+\rho+\sigma_i^2) \right)}
\]

\[
\frac{1}{\lambda_i} + \frac{1}{\lambda_j} = -\frac{4\rho \left( \sigma_i^2(1+\rho+\sigma_j^2) + \sigma_j^2(1+\rho+\sigma_i^2) \right)}{\mu(2\rho \sigma_i^2(1+\rho+\sigma_j^2) - L)^2} \left( \sigma_i^2(1+\rho+\sigma_j^2) + \sigma_j^2(1+\rho+\sigma_i^2) \right)^2
\]

\[
\frac{\mu + 2\lambda_i}{2(\mu + \lambda_i)^2} \left( \frac{1}{\lambda_i} + \frac{1}{\lambda_j} \right)^{-2} = \frac{(\rho(1+\rho)(\sigma_i^2 - \sigma_j^2) - L)}{(L - 2\rho \sigma_i^2(1+\rho+\sigma_j^2))^2} \left( \sigma_i^2(1+\rho+\sigma_j^2) + \sigma_j^2(1+\rho+\sigma_i^2) \right)^2
\]

\[
\mu + \lambda_i = \frac{L - 2\rho \sigma_i^2(1+\rho+\sigma_j^2)}{L - 2\rho \sigma_i^2(1+\rho+\sigma_j^2)}
\]

\[
c_{si} = \frac{2\rho(1-\rho^2) \left( \sigma_i^2(1+\rho+\sigma_j^2) + \sigma_j^2(1+\rho+\sigma_i^2) \right)}{\mu(2\rho \sigma_i^2(1+\rho+\sigma_j^2) - L)^2} \left( \sigma_i^2(1+\rho+\sigma_j^2) + \sigma_j^2(1+\rho+\sigma_i^2) \right)^2
\]

\[
\frac{\mu + \lambda_i}{\mu + \lambda_j} c_{sj} = \frac{2\rho(1-\rho^2) \left( \sigma_i^2(1+\rho+\sigma_j^2) + \sigma_j^2(1+\rho+\sigma_i^2) \right)}{\mu(2\rho \sigma_i^2(1+\rho+\sigma_j^2) - L)^2} \left( \sigma_i^2(1+\rho+\sigma_j^2) + \sigma_j^2(1+\rho+\sigma_i^2) \right)^2
\]

\[
E[u_i^{\text{cross}}] = \frac{-\rho \left( \sigma_i^2(1+\rho+\sigma_j^2) + \sigma_j^2(1+\rho+\sigma_i^2) \right)(1+\rho+\sigma_j^2)(1-\rho^2+\sigma_j^2)(L - 2\rho \sigma_i^2(1+\rho+\sigma_j^2))^2}{4\rho(1+\rho)^2((1+\sigma_i^2)(1+\sigma_j^2) - \rho^2)^2(L - 2\rho \sigma_i^2(1+\rho+\sigma_j^2))^2}
\]

\[
\times \left\{ \frac{(1+\sigma_i^2)(L + 2\rho \sigma_i^2(1+\rho+\sigma_j^2))^2}{(1+\rho+\sigma_i^2)(2(1+\rho+\sigma_j^2)(1-\rho+\sigma_i^2) + \rho(\sigma_i^2 - \sigma_j^2))^2 + 2(1+\rho+\sigma_i^2)(L + 2\rho \sigma_i^2(1+\rho+\sigma_j^2))^2} \right\}
\]

\[
- \left\{ \frac{(1+\rho+\sigma_j^2)(2(1+\rho+\sigma_j^2)(1-\rho+\sigma_i^2) + \rho(\sigma_i^2 - \sigma_j^2))(1+\rho+\sigma_i^2)(L + 2\rho \sigma_i^2(1+\rho+\sigma_j^2))^2}{(1+\rho+\sigma_i^2)(2(1+\rho+\sigma_j^2)(1-\rho+\sigma_i^2) + \rho(\sigma_i^2 - \sigma_j^2))^2}
\right\}
\]
\[
\frac{\left\{ \sigma_i^2(1 + \rho + \sigma_j^2) + \sigma_j^2(1 + \rho + \sigma_i^2) \right\} (1 + \rho + \sigma_i^2)(1 - \rho^2 + \sigma_j^2)(L - 2\rho\sigma_j^2(1 + \rho + \sigma_i^2))^2}{4(1 + \rho)^2 \{(1 + \sigma_i^2)(1 + \sigma_j^2) - \rho^2\}^2(L - 2\rho\sigma_j^2(1 + \rho + \sigma_i^2))^2} \times \left\{ \frac{(1 + \sigma_i^2)(L + 2\sigma_j^2(1 + \rho + \sigma_i^2))^2}{[(1 + \rho + \sigma_i^2)(2(1 + \rho + \sigma_j^2)(1 - \rho + \sigma_j^2) + \rho(\sigma_i^2 - \sigma_j^2))^2 + \frac{1}{2}\rho(L + 2\sigma_j^2(1 + \rho + \sigma_i^2))(L + 2\sigma_j^2(1 + \rho + \sigma_i^2))} - \frac{1}{(1 + \rho + \sigma_i^2)(2(1 + \rho + \sigma_j^2)(1 - \rho + \sigma_j^2) + \rho(\sigma_i^2 - \sigma_j^2))^2[(1 + \rho + \sigma_j^2)(2(1 + \rho + \sigma_i^2)(1 - \rho + \sigma_i^2) + \rho(\sigma_j^2 - \sigma_i^2)]\} \right\} \geq \frac{\sigma_i^2(1 - \rho^2 + \sigma_j^2)}{(1 + \rho + \sigma_i^2)^2(1 - \rho + \sigma_i^2)}
\]