Dynamic-Stochastic Contracts, Signal Dampening and the Ratchet Effect
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1 Introduction

In a static setting, incentive contracts between a regulator and a regulated firm have several very well known characteristics. The most efficient firms operate at “first best” cost levels, and less efficient firms have their costs inflated in an attempt to recoup some informational rent from the best firms. Importantly, these static contracts are impervious to noisy cost observations.

One of the main conclusions of (Laffont & Tirole, 1993) is that when the interaction between the regulator and firm extends over multiple periods, it is difficult to induce efficient types to give up their future informational rents when using short term contracts. Thus, separating equilibria are difficult to obtain. If a separating contract is feasible, one common theme between dynamic and static interactions is the “no distortion on top” result; efficient firms still operate at first best levels, and less efficient firms have their costs distorted in an attempt to extract some rent from the efficient types.

A crucial difference between the static and dynamic cases, however, is that in a dynamic setting, the presence of noise in cost observations may dramatically alter the optimal contract relative to the deterministic setting. This is due to the fact that in a deterministic setting, cost observations are perfectly type-revealing in a separating equilibrium. In a sufficiently noisy environment, however, any given cost realization could have been generated by any type of firm, even in a separating equilibrium. Thus, the regulator’s learning process
is slowed in a stochastic setting. This preserves efficient firms’ informational rents, in turn relaxing the dynamic incentive problem.

This paper examines a dynamic interaction between a regulator and regulated firm when cost observations are stochastic. In contrast to the deterministic setting studied by Laffont and Tirole (1987, 1993), separation is feasible as long as cost observations are sufficiently noisy. Additionally, the ratchet effect behaves in a different manner when costs are stochastic. The regulator manipulates both the good and bad firm’s effort levels away from the first best; in a deterministic setting, the best type of firm always exerts the first best effort.

Both Jeitschko and Mirman and Jeitschko, Mirman and Salguiero have examined dynamic principal-agent interactions when the observable outcome is noisy. The authors show that the principal has competing incentives to either increase or decrease the informativeness of the observable outcome. The regulator dampens the informativeness of the first period signal (decreases informativeness) in order to decrease the good agent’s first period transfer. Conversely, the principal experiments (increases informativeness) because her second period payoff is increasing in the accuracy of her information.

In Jeitschko, Mirman and Salguiero, the authors study a setting in which the principal and agent contract over output. They assume the noise in output follows a uniform distribution, and are able to determine that the signal dampening effect dominates the experimentation effect. Jeitschko and Mirman also study a setting in which the principal and agent contract over output, but they do not specify a distribution of noise. In this setting, it is not possible to conclude which effect dominates.

Interestingly, we are able to find that in a regulatory setting, the signal dampening effect wins even with a general distribution of noise. This result holds because the regulator includes the firm’s utility in her objective function, and the good type of firm enjoys a second period rent in a separating equilibrium.
2 Model

In this paper, we consider the two period interaction between a welfare-maximizing regulator and a regulated firm.\(^1\) In each period, the regulator offers the firm a contract to complete a project that has gross-benefit \(S\). Contracts are short term; thus, when designing the second period contract, the regulator cannot commit to ignore any information she learns about the firm’s type from observing the realized first period project cost.

The project cost in each period depends on the firm’s intrinsic cost parameter, it’s cost-reducing effort, and a homoskedastic, zero mean noise term, \(\varepsilon\):

\[
c_t = \beta - e_t + \varepsilon_t, \quad t = 1, 2.
\]  

We assume that \(\varepsilon\) is distributed over the entire real line according to the distribution function \(G(\varepsilon)\) with associated density \(g(\varepsilon)\). We further assume that \(g\) satisfies the monotone likelihood ratio property. The firm’s type can be either \(\beta\) or \(\bar{\beta}\), where \(0 < \beta < \bar{\beta}\). We will refer to type \(\bar{\beta}\) as the “low cost type” or “good type”, and type \(\beta\) as the “high cost type” or “bad type.” The regulator’s prior belief that the firm is the low cost type is given by \(\rho\). The firm’s type is unknown to the regulator at the beginning of the game, and remains the same in each period.

The regulator compensates the firm for the project’s costs, and pays an additional transfer in order to incentivize unobservable, cost-reducing effort. We assume that the cost reimbursement and the incentive transfer are raised via distortionary taxation, and \(\lambda > 0\) denotes the shadow cost of public funds.

It is important to note that the random shock \(\varepsilon\) occurs after the firm has exerted its cost reducing effort. Thus, \(\varepsilon\) does not affect the firm’s private cost of effort, but only the overall project cost. Therefore, at the beginning of each period, the regulator offers the firm a contract that specifies an expected cost for each type of firm, and a transfer function that rewards the firm for realized costs. Thus, the incentive compatibility and participation constraints throughout the paper will be over expected costs and expected transfers.

Because \(\varepsilon\) is realized after the firm has chosen its effort, the firm’s private disutility from effort depends only on the cost target and the firm’s type. If, for example, the firm is the low cost type, and in the first period the firm chooses the cost target designed for her by the

\(^1\)See (Laffont & Tirole, 1993)
regulator, her effort is
\[ e_1 = \beta - \zeta_1. \]  
(2)

In this way, the firm’s private cost of effort is deterministic, while project costs are stochastic. The firm’s cost of effort is given by
\[ \psi(e) = \begin{cases} \gamma e^2, & e > 0, \\ 0, & e \leq 0. \end{cases} \]
(3)

Continuing the example, the first period cost realization given that the firm is the low cost type and chooses its effort according to (2) is
\[ c_1 = \zeta_1 + \epsilon. \]
(4)

From (4), we see that the distribution of first period costs is \( g(c_1 - \zeta_1) \) if the firm targets \( \zeta_1 \), and \( g(c_1 - \bar{\zeta}_1) \) if the firm targets \( \bar{\zeta}_1 \).

### 3 Second period

We begin the second period analysis by assuming that the first period equilibrium cost targets were separating in actions. Once the regulator observes the first period cost realization, she is able to update her beliefs using Baye’s rule:
\[ \rho_2 = \frac{\rho g(c_1 - \zeta_1)}{\rho g(c_1 - \zeta_1) + (1 - \rho) g(c_1 - \bar{\zeta}_1)}. \]
(5)

In (5), \( c_1 \) is the realized first period cost, \( \zeta_1 \) is the first period cost target for the low cost type, and \( \bar{\zeta}_1 \) is the first period cost target for the high cost type. In the remainder of the paper, we use the following notation: \( g = g(c_1 - \zeta_1) \), and \( \bar{g} = g(c_1 - \bar{\zeta}_1) \).

When \( \rho_2 \) is not too large, the second period game is the standard static game when the regulator’s beliefs are \( P(\beta = \beta) = \rho_2 \). Thus, the equilibrium efforts for the low and high cost firm, respectively, are given by
\[ e_2 = \beta - \zeta_2 = \frac{1}{\gamma} \]
(6)

and
\[ \bar{e}_2 = \bar{\beta} - \bar{\zeta}_2 = \frac{1}{\gamma} - \frac{\rho_2}{1 - \rho_2} \frac{\Delta \beta}{1 + \lambda}. \]
(7)

\[ \text{For the regulator’s second period problem, see the appendix.} \]
Thus, the low cost type exerts the first best effort in the second period, while the high cost type’s effort is distorted below the first best. The low cost type’s equilibrium rent is given by
\[
U_2(\rho_2) = \Delta \beta \left[ 1 - \frac{\gamma}{2} \frac{1 + \lambda - \rho_2(1 - \lambda)}{(1 - \rho_2)(1 + \lambda)} \Delta \beta \right].
\]
(8)

Notice that the good type’s rent depends on the first period cost realization and the first period cost targets through the second period belief function, \( \rho_2 \).

When \( \rho_2 \) is large, the low cost firm’s cost of effort from mimicking the high cost firm is negative. Since posterior beliefs are monotone decreasing in \( c_1 \), there is a critical value of \( c_1 \) to consider, denoted \( c_0^1 \). For \( c_1 \leq c_0^1 \), the posterior beliefs are such that the low cost firm’s effort from mimicking the high cost type is negative.\(^3\) The corresponding beliefs are
\[
\rho_2^0 \equiv \rho_2(c_1^0) = \frac{(1 + \lambda)(1 - \gamma \Delta \beta)}{1 + \lambda - \gamma \Delta \beta} < 1.
\]
(9)

In this case, the low cost type’s incentive compatibility constraint is written
\[
t_2 - \frac{\gamma}{2} (\beta - \bar{c}_2)^2 = \bar{t}_2.
\]
(10)
The high cost firm’s participation constraint remains unchanged. Together, this implies that
\[
U_2 = \frac{\gamma}{2} (\bar{\beta} - \bar{c}_2)^2,
\]
(11)
and the equilibrium efforts and rent are given by
\[
\bar{\beta} - \bar{c}_2 = \frac{1}{\gamma},
\]
(12)
\[
\bar{e}_2 = \bar{\beta} - \bar{c}_2 = \frac{1}{\gamma} \frac{(1 - \rho_2)(1 + \lambda)}{1 + \lambda - \rho_2},
\]
(13)
and
\[
U_2 = \frac{1}{2 \gamma} \frac{(1 - \rho_2)^2(1 + \lambda)^2}{(1 + \lambda - \rho_2)^2}.
\]
(14)

This change in the functional form of the low cost type’s rent at \( c_1 = c_0^1 \) makes it difficult to obtain tractable first period results. Consider the following assumption, which simplifies the first period analysis:

**Assumption 1.**
\[
0 \geq S - (1 + \lambda) \left( \frac{\bar{\beta} + \beta}{2} \right),
\]
(15)
\(^3\)The case when \( c_1 > c_0^1 \) is considered above.
Assumption 1 guarantees that the regulator finds it optimal to shutdown the high cost type when \( c_1 \leq c_0 \). To see this, note that it is optimal to shut down the high cost type in general when

\[
\rho_2 \left[ S - (1 + \lambda) \left( \frac{\beta - \frac{1}{2\gamma}}{2\gamma} \right) \right] \geq \rho_2 \left[ S - (1 + \lambda) \left( \frac{\beta - \frac{1}{2\gamma}}{2\gamma} \right) - \lambda U_2(\rho_2) \right] + (1 - \rho_2) \left[ S - (1 + \lambda) \left( \tilde{\beta} - \tilde{e}_2 + \frac{\gamma}{2\tilde{e}_2^2} \right) \right].
\]

(16)

Thus, shutdown is optimal when \( c_1 = c_0 \) if

\[
\frac{\rho_2^0}{1 - \rho_2^0} \lambda U_2(\rho_2^0) + (1 + \lambda) \left( \tilde{\beta} - \tilde{e}_2(\rho_2^0) + \frac{\gamma}{2}(\tilde{e}_2(\rho_2^0))^2 \right) \geq S.
\]

(17)

After substituting for the necessary terms from (9), (13) and (14), this simplifies to

\[
0 \geq S - (1 + \lambda) \left( \frac{\beta + \beta_2}{2} \right).
\]

(18)

Additionally, one can show that

\[
\frac{d}{d\rho_2} \left[ \frac{\rho_2}{1 - \rho_2} \lambda U_2(\rho_2) + (1 + \lambda) \left( \tilde{\beta} - \tilde{e}_2(\rho_2) + \frac{\gamma}{2}(\tilde{e}_2(\rho_2))^2 \right) \right] > 0.
\]

(19)

Thus, Assumption 1 is sufficient for the regulator to shut down the high cost type when \( c_1 \leq c_0 \). One interpretation of Assumption 1 is that it is only worth it to have the high cost firm complete the project when he exerts enough cost reducing effort.\(^4\) The higher is \( \rho_2 \), the more the regulator distorts the high cost type’s effort in an attempt to reduce the low cost firm’s informational rent. As we can see from (13), as posterior beliefs approach one, the high cost type’s effort approaches zero. By making Assumption 1, we are assuming that \( \rho_2^0 \) is sufficiently large that the regulator prefers the high cost type not undertake the project.

Related to the intuition underlying Assumption 1, a standard result that is important for the first period analysis is that the low cost type’s rent is decreasing in the regulator’s belief that he is the high cost type,

\[
\frac{dU_2}{d\rho_2} = -\gamma \frac{\lambda}{1 + \lambda} \frac{1}{(1 - \rho_2)^2} \Delta \beta^2 < 0
\]

(20)

This is the classic rent extraction efficiency trade-off present in static adverse selection models. The difference in a dynamic setting when costs are stochastic is that the regulator influences her second period belief function, \( \rho_2 \), by her choice of first period cost targets.

\(^4\)If \( 0 \geq S - (1 + \lambda) \left( \frac{\beta + \beta_2}{2} \right) \), then \( 0 \geq S - (1 + \lambda)\tilde{\beta} \).
The regulator can increase (decrease) the informativeness of the first period signal by increasing (decreasing) the distance between $c_1$ and $\bar{c}_1$. As we show in the next section, the regulator takes into consideration the effects that first period cost targets have on the low cost firm’s second period rent when choosing the first period contract.

The regulator also takes into account the impact that first period cost targets have on expected second period welfare. By making Assumption 1, the functional form of second period welfare depends on the first period cost realization. For $c_1 > c_0^1$,

$$W_2(\rho_2) = \rho_2 \left[ S - (1 + \lambda)(\beta - \frac{1}{2}\gamma) - \lambda W_2(\rho_2) \right] + (1 - \rho_2) \left[ S - (1 + \lambda)(\bar{\beta} - \bar{e}_2 + \frac{\gamma}{2}(\bar{e}_2)^2) \right], \quad (21)$$

and for $c_1 \leq c_0^1$,

$$W_2(\rho_2) = \rho_2 W^{FB}. \quad (22)$$

Thus, for low cost observations, the regulator offers only one contract, the first best for the low cost type. The high cost type does not undertake the project, resulting in welfare of zero.

4 First period

In Jeitschko and Mirman, the authors show that the principal has two competing incentives when it comes to setting the first period output targets. First, decreasing the good type’s output target (increasing the bad type’s output target) decreases the good type’s first period transfer. However, doing so also decreases the principal’s expected second period payoff. The extension we make is to show that these incentives still exist when the principal (in this case, regulator) includes the firm’s utility in her objective function.

The regulator still balances her desires to decrease the first period transfer and increase expected second period welfare, but in addition to these two concerns, she considers the impact that the first period contract has on the low cost firm’s expected second period rent. Because learning is not immediate, the good type of firm gets a second period rent even in equilibrium. Further, the firm is not receiving this rent via the first period transfer, so it is not socially costly to the regulator in the first period.

To see this point, consider the low cost firm’s first period incentive compatibility con-
straint:
\[
\bar{t}_1 - \frac{\gamma}{2} (\beta - \bar{c}_1)^2 + \delta \int_{c_1}^{\infty} U_2(\rho_2)\,g(c_1)\,dc_1 = \bar{t}_1 - \frac{\gamma}{2} (\beta - \bar{c}_1)^2 + \delta \int_{c_1}^{\infty} U_2(\rho_2)\,\bar{g}(c_1)\,dc_1. \tag{23}
\]

The left hand side of (23) is the low cost firm’s equilibrium expected utility. He receives expected transfer \(t_1\), exerts deterministic effort \(\beta - c_1\), and expects a second period rent, where expectations are taken over the distribution \(g(c_1 - \bar{c}_1)\). Similarly, the right hand side is what he would get if he targeted \(\bar{c}_1\) instead of \(c_1\) in the first period. The distribution induced by targeting \(\bar{c}_1\) first order stochastically dominates the distribution induced by targeting \(c_1\), so that his expected second period rent from deviating is higher than in equilibrium. Thus, the first period transfer must compensate him for this opportunity cost in order to get him to choose the appropriate cost target:
\[
t_1 = \frac{\gamma}{2} (\beta - \bar{c}_1)^2 + \frac{\gamma}{2} (\beta - \bar{c}_1)^2 - \frac{\gamma}{2} (\beta - \bar{c}_1)^2 + \delta \int_{c_1}^{\infty} U_2(\rho_2)\,(\bar{g} - g)\,dc_1. \tag{24}
\]

As discussed in Theorem 1 of Jeistchko Mirman, this expected first period payment is larger than in a static game, but less than in a dynamic setting in which the regulator fully learns the firm’s type after the first period.

For the high cost firm, the first period game is static since the second period game is designed to extract all the rent from the bad type. Therefore, the low cost firm’s participation constraint is
\[
\bar{t}_1 - \frac{\gamma}{2} (\bar{\beta} - \bar{c}_1)^2 = 0. \tag{25}
\]

To ensure that the different firm types target unique cost levels in the first period, we assume that the single crossing property is satisfied:
\[
\psi'(\bar{\beta} - c) \geq \psi'(\bar{\beta} - c) + \delta \int \frac{dU_2}{d\rho_2} \frac{d\rho_2}{dc_1} g(c_1 - c)\,dc_1. \tag{26}
\]

Thus, the high cost type’s marginal cost of decreasing the cost target \(c\) is higher than the high cost type’s marginal cost of decreasing the cost target for every \(c\). This condition is satisfied in noisy enough environments.

Because the low cost firm gets an expected second period rent in equilibrium, the regulator’s first period problem looks different than in Laffont Tirole 1993. Therefore, we carefully derive the regulator’s full first period problem before discussing her incentive to signal dampen or experiment. We write the low cost firm’s portion of the regulator’s first
After consolidating, this is equivalent to

\[ S - (1 + \lambda)(c_1 + t_1) + L, \]

as follows:

\[
S - (1 + \lambda)\left(c_1 + \frac{\gamma}{2}(\beta - c_1)^2 + t_1 - \frac{\gamma}{2}(\beta - c_1)^2\right) + t_1 - \frac{\gamma}{2}(\beta - c_1)^2 + \delta \int_{c_1}^{\infty} U_2(\rho_2)gdc_1
\]

\[ = S - (1 + \lambda)\left(c_1 + \frac{\gamma}{2}(\beta - c_1)^2\right) - \lambda\left(t_1 - \frac{\gamma}{2}(\beta - c_1)^2\right) + \delta \int_{c_1}^{\infty} U_2(\rho_2)gdc_1. \tag{27} \]

Once we substitute for the low cost firm’s expected transfer, the low cost firm’s portion of the regulator’s first period objective function becomes

\[
S - (1 + \lambda)\left(c_1 + \frac{\gamma}{2}(\beta - c_1)^2\right) - \lambda\left(\frac{\gamma}{2}(\beta - c_1)^2 - \frac{\gamma}{2}(\beta - c_1)^2 + \delta \int_{c_1}^{\infty} U_2(\rho_2)(g - g)dc_1\right) + \delta \int_{c_1}^{\infty} U_2(\rho_2)gdc_1. \tag{28} \]

Since the game is static for the high cost firm, the high cost firm’s portion of the regulator’s first period objective is simply

\[
S - (1 + \lambda)\left(c_1 + \frac{\gamma}{2}(\beta - c_1)^2\right). \tag{29} \]

To finish writing the first period problem, we need an expression for expected second period welfare:

\[
E[W_2(\rho_2)] = \rho \left(1 - \mathcal{G}(c_1^0)\right) \int_{c_1^0}^{\infty} \left[ S - (1 + \lambda)\left(\beta - \frac{1}{2}\right) - \lambda U_2 - \frac{\rho}{1 - \mathcal{G}(c_1^0)}W_{FB}c_1 + \mathcal{G}(c_1^0)W_{FB}c_1 \right] dc_1 \]

\[ + (1 - \rho) \left(1 - \mathcal{G}(c_1^0)\right) \int_{c_1^0}^{\infty} \left[ S - (1 + \lambda)\left(\beta - \bar{c}_2 + \frac{\gamma}{2}(\bar{c}_2)^2\right) - \lambda U_2 - \frac{\rho}{1 - \mathcal{G}(c_1^0)}W_{FB}c_1 \right] dc_1. \tag{30} \]

After consolidating, this is equivalent to

\[
E[W_2(\rho_2)] = \rho \left(W_{FB} - \lambda \int_{c_1^0}^{\infty} U_2gdc_1\right) + (1 - \rho) \left(\int_{c_1^0}^{\infty} \left[ S - (1 + \lambda)\left(\beta - \bar{c}_2 + \frac{\gamma}{2}(\bar{c}_2)^2\right) - \lambda U_2 - \frac{\rho}{1 - \mathcal{G}(c_1^0)}W_{FB}c_1 \right] dc_1\right). \tag{31} \]

Thus, the regulator’s full first period problem is given by

\[
\max_{c_1, \bar{c}_1} \rho \left[S - (1 + \lambda)\left(c_1 + \frac{\gamma}{2}(\beta - c_1)^2\right) - \lambda\left(\frac{\gamma}{2}(\beta - c_1)^2 - \frac{\gamma}{2}(\beta - c_1)^2 + \delta \int_{c_1}^{\infty} U_2(\rho_2)(g - g)dc_1\right) + \delta \int_{c_1}^{\infty} U_2(\rho_2)gdc_1\right] \]

\[ + (1 - \rho) \left[ S - (1 + \lambda)\left(\bar{c}_1 + \frac{\gamma}{2}(\beta - \bar{c}_1)^2\right) + \delta \int_{c_1}^{\infty} U_2(\rho_2)gdc_1\right] + \delta E[W_2(\rho_2)]. \tag{32} \]

In the following analysis, we at first separately discuss the regulator’s first and second period incentives when choosing \(c_1\) and \(\bar{c}_1\). We do so because the first period contract affects the first and second period portions of the regulator’s problem in opposite directions. In the signal dampening subsection, we show that the regulator prefers to move \(c_1\) and \(\bar{c}_1\) closer.
together because doing so allows her to decrease the first period transfer to the low cost type and increase the low cost type’s expected second period welfare.

In the experimentation subsection, we show that the regulator would like to move $c_1$ and $\bar{c}_1$ further apart. This is because information is valuable to the regulator (see Mirman and Matthews). Finally, we consider the two effects together, and show that the signal dampening effect dominates the experimentation effect for any $\lambda$ less than one.

4.1 Signal dampening

When we ignore the first period contract’s effects on expected second period welfare, the regulator has an incentive to decrease the distance between $c_1$ and $\bar{c}_1$ relative to the deterministic (or separating in messages) setting. As we show, this signal dampening effect is due to the fact that the regulator can both reduce the socially costly first period transfer and increase the low cost firm’s expected second period rent by decreasing the informativeness of the first period signal.

To show this, we ignore the impact of $c_1$ and $\bar{c}_1$ on $E[W_2(\rho_2)]$. When we do so, the first order condition of (32) with respect to $c_1$ is

$$\frac{\beta - c_1}{\gamma} = \frac{\delta}{\gamma(1 + \lambda)} \left[ \lambda \frac{d}{dc_1} \int_{c_1}^{\infty} U_2(\rho_2)(\bar{g} - g)dc_1 - \frac{d}{dc_1} \int_{c_1}^{\infty} U_2(\rho_2)gdc_1 \right],$$

and the first order condition for the high cost type is

$$\frac{\beta - \bar{c}_1}{\gamma} = \frac{\rho \lambda}{(1 - \rho)(1 + \lambda)} \Delta \beta + \frac{\rho \delta}{\gamma(1 - \rho)(1 + \lambda)} \left[ \lambda \frac{d}{dc_1} \int_{\bar{c}_1}^{\infty} U_2(\rho_2)(\bar{g} - g)dc_1 - \frac{d}{dc_1} \int_{\bar{c}_1}^{\infty} U_2(\rho_2)gdc_1 \right].$$

If the regulator dampens the informativeness of the first period signal, she will decrease the effort of the low cost type and increase the effort of the high cost type. Therefore, we hope to show that

$$\lambda \frac{d}{dc_1} \int_{c_1}^{\infty} U_2(\rho_2)(\bar{g} - g)dc_1 < \frac{d}{dc_1} \int_{c_1}^{\infty} U_2(\rho_2)gdc_1$$

and

$$\lambda \frac{d}{dc_1} \int_{\bar{c}_1}^{\infty} U_2(\rho_2)(\bar{g} - g)dc_1 > \frac{d}{dc_1} \int_{\bar{c}_1}^{\infty} U_2(\rho_2)gdc_1.$$
and
\[ \frac{d}{dc_1} \int_{c_1}^\infty U_2(\rho_2)(\bar{g} - g)dc_1 > 0, \quad (38) \]
so to show that the regulator signal dampens, we show that the low cost type's equilibrium expected second period rent is increasing in \( c_1 \) and decreasing in \( \bar{c}_1 \).

**Proposition 1.** The low cost firm's equilibrium expected second period rent is increasing in \( c_1 \) and decreasing in \( \bar{c}_1 \). Thus, the regulator dampens the informativeness of the first period signal.

**Proof.** First, we consider the effects of \( c_1 \) on the low cost firm's expected second period rent:
\[ \frac{d}{dc_1} \int_{c_1}^\infty U_2(\rho_2)\bar{g}dc_1 = \int \frac{dU_2}{d\rho_2} \frac{d\rho_2}{dc_1} \bar{g} - U_2g'dc_1. \quad (39) \]
Once we integrate the second term under the right hand side integral by parts, we are left with
\[ \int_{c_1}^\infty \frac{dU_2}{d\rho_2} \left[ \frac{d\rho_2}{dc_1} + \frac{d\rho_2}{dc_1} \right] gdc_1. \quad (40) \]
Now,
\[ \frac{d\rho_2}{dc_1} = -\frac{\rho(1-\rho)\bar{g}g'}{D^2}, \quad (41) \]
and
\[ \frac{d\rho_2}{dc_1} = \frac{\rho(1-\rho)(\bar{g}g' - g\bar{g})}{D^2}, \quad (42) \]
where \( D = \rho \bar{g} + (1-\rho)\bar{g} \). Thus,
\[ \frac{d\rho_2}{dc_1} + \frac{d\rho_2}{dc_1} = -\frac{\rho(1-\rho)\bar{g}g'}{D^2}. \quad (43) \]
Substituting into (40) yields
\[ \int_{c_1}^\infty \frac{dU_2(\rho_2)}{d\rho_2} - \frac{\rho(1-\rho)\bar{g}^2}{D^2} \bar{g}' = -\frac{1-\rho}{\rho} \int_{c_1}^\infty \frac{dU_2(\rho_2)}{d\rho_2} \rho \bar{g} dc_1. \quad (44) \]
Once again, we integrate the right hand side integral by parts. Doing so yields
\[ \frac{1-\rho}{\rho} \int_{c_1}^\infty \left[ \frac{d^2U_2}{d\rho_2^2} \rho_2 + 2 \frac{dU_2}{d\rho_2} \rho_2 \frac{d\rho_2}{dc_1} \bar{g} dc_1, \right. \quad (45) \]
which we would like to show is positive. From (20), we know that \( \frac{dU_2}{d\rho_2} < 0 \), and \( \frac{d\rho_2}{dc_1} < 0 \) by the monotone likelihood ratio property. Thus, \( \frac{d^2U_2}{d\rho_2^2} < 0 \) is sufficient for the regulator to signal dampen. From (8),
\[ \frac{d^2U_2}{d\rho_2^2} = -\gamma \frac{\lambda}{1 + \lambda (1-\rho_2)^3} \Delta \beta^2 < 0. \quad (46) \]
Therefore,
\[
\frac{d}{d\hat{c}_1} \int_{c_1^0}^{\infty} \frac{d}{d\rho} \left( U_2(\rho) \right) g dc_1 = \frac{1 - \rho}{\rho} \int_{c_1^0}^{\infty} \left( \frac{d^2 U_2}{d\rho^2} \rho_2 + 2 \frac{dU_2}{d\rho} \right) \rho_2 \frac{d\rho_2}{d\rho} \bar{g} dc_1 > 0, \tag{47}
\]
as desired. One can show via a similar proof that
\[
\frac{d}{d\hat{c}_1} \int \frac{d}{d\rho} \left( U_2(\rho) \right) g dc_1 = -\frac{1 - \rho}{\rho} \int \left[ \frac{d^2 U_2}{d\rho^2} \rho_2 + 2 \frac{dU_2}{d\rho} \right] \rho_2 \frac{d\rho_2}{d\rho} \bar{g} dc_1 < 0. \tag{48}
\]
Thus, the regulator signal dampens.

The regulator’s incentives to signal dampen in the first period are clear. By bringing the equilibrium cost targets closer together than in a deterministic setting, the regulator is able to decrease the first period transfer to the low cost type and increase the low cost type’s equilibrium expected second period rent. Since this equilibrium rent is not a component of the socially costly first period transfer, it increases first period welfare.

### 4.2 Experimentation

Now, we will consider the impacts of the first period cost targets on expected second period welfare, \((31)\). Mirman and Matthews and Jeischko and Mirman rely on the fact that second period welfare is convex in second period beliefs (“information is valuable”) in order to show that the principal experiments.

While that approach is valid here, our model assumptions allow us to explicitly illustrate the impacts of \(c_1\) and \(\hat{c}_1\) on expected second period welfare. In particular, we are able to show that while increasing (decreasing) \(c_1\) (\(\hat{c}_1\)) increases the high cost type’s expected second period effort, thus increasing expected second period welfare, this benefit is outweighed by the increase in the low cost type’s expected second period rent. This straightforward approach is useful in the next section when we analyze the regulator’s full first period problem.

When ignoring the first period portion of the regulator’s first period problem, equilibrium first period effort is higher for the low cost firm and lower for the high cost firm when compared to the deterministic setting if
\[
\frac{dE[W_2(\rho_2)]}{d\rho} < 0, \tag{49}
\]
and
\[
\frac{dE[W_2(\rho_2)]}{d\rho} > 0, \tag{50}
\]
respectively.
Proposition 2. Second period welfare is decreasing in $c_1$ and increasing in $\bar{c}_1$. Thus, regulator increases the informativeness of the first period signal (experiments).

Proof. First, notice that

$$
\frac{dE[W_2(\rho_2)]}{dc_1} = -\rho \lambda \frac{d}{dc_1} \int_{c_1}^{\infty} U_2(\rho_2) g dc_1 + (1 - \rho)(1 + \lambda) \frac{d}{dc_1} \int_{c_1}^{\infty} \left( \bar{e}_2 - \frac{\gamma}{2}(\bar{e}_2)^2 \right) \bar{g} dc_1. \tag{51}
$$

From the proof of Proposition 1, we know that the low cost firm’s expected second period rent is decreasing in $c_1$. Thus, if

$$
\frac{d}{dc_1} \int_{c_1}^{\infty} \left( \bar{e}_2 - \frac{\gamma}{2}(\bar{e}_2)^2 \right) \bar{g} dc_1 < 0, \tag{52}
$$

the proof is done. However, this is not the case. The high cost firm’s expected second period effort is increasing in $c_1$. Therefore, we show that the increase in the low cost firm’s expected second period rent outweighs the benefit of higher expected effort from the high cost type.

To see this, note that

$$
(1 - \rho)(1 + \lambda) \frac{d}{dc_1} \int_{c_1}^{\infty} \left( \bar{e}_2 - \frac{\gamma}{2}(\bar{e}_2)^2 \right) \bar{g} dc_1 = (1 - \rho)(1 + \lambda) \int_{c_1}^{\infty} (1 - \gamma \bar{e}_2) \frac{d\bar{e}_2}{d\rho_2} \frac{d\rho_2}{dc_1} \bar{g} dc_1
$$

$$
= -\rho(1 + \lambda) \int_{c_1}^{\infty} (1 - \gamma \bar{e}_2) \frac{d\bar{e}_2}{d\rho_2} (1 - \rho_2)^2 \bar{g} dc_1. \tag{53}
$$

Integrating the last term in (53) by parts yields

$$
\rho(1 + \lambda) \int_{c_1}^{\infty} \left[ -\gamma \frac{(d\bar{e}_2)^2}{d\rho_2} (1 - \rho_2)^2 + (1 - \gamma \bar{e}_2) \frac{d^2\bar{e}_2}{d\rho_2^2} (1 - \rho_2)^2 - (1 - \gamma \bar{e}_2) \frac{d\bar{e}_2}{d\rho_2} (1 - \rho_2) \right] \frac{d\rho_2}{dc_1} g dc_1,
$$

and simplifying the term in brackets leaves us with

$$
-\rho(1 + \lambda) \int_{c_1}^{\infty} \gamma \frac{1}{(1 - \rho_2)^2} \left( \frac{\lambda}{1 + \lambda} \Delta \beta \right)^2 \frac{d\rho_2}{dc_1} g dc_1 > 0. \tag{55}
$$

Now,

$$
-\rho \lambda \frac{d}{dc_1} \int_{c_1}^{\infty} U_2(\rho_2) g dc_1 = (1 - \rho) \lambda \int_{c_1}^{\infty} 2\gamma \frac{\rho_2}{(1 - \rho_2)^3} \frac{\lambda}{1 + \lambda} \Delta \beta \frac{d\rho_2}{dc_1} \bar{g} dc_1. \tag{56}
$$

Thus

$$
\frac{dE[W_2(\rho_2)]}{dc_1} = \int_{c_1}^{\infty} 2\gamma \frac{\rho_2}{(1 - \rho_2)^3} \frac{\lambda^2}{1 + \lambda} \Delta \beta \frac{d\rho_2}{dc_1} (1 - \rho) \bar{g} dc_1 - \int_{c_1}^{\infty} \gamma \frac{1}{(1 - \rho_2)^2} \frac{\lambda^2}{1 + \lambda} \Delta \beta \frac{d\rho_2}{dc_1} \rho g dc_1. \tag{57}
$$
Using the fact that $\rho g = \rho_2 D$ and $(1 - \rho)\bar{g} = (1 - \rho_2)D$,

$$\frac{dE[W_2(\rho_2)]}{d\bar{c}_1} = \int_{\bar{c}_1}^{\infty} 2\gamma \frac{\rho_2}{(1 - \rho_2)^2} \frac{\lambda^2}{1 + \lambda} \Delta \beta^2 \frac{d\rho_2}{d\bar{c}_1} Dd\bar{c}_1 - \int_{\bar{c}_1}^{\infty} \gamma \frac{\rho_2}{(1 - \rho_2)^2} \frac{\lambda^2}{1 + \lambda} \Delta \beta^2 \frac{d\rho_2}{d\bar{c}_1} Dd\bar{c}_1 < 0.$$  

(58)

One can show via a similar proof that

$$\frac{dE[W_2(\rho_2)]}{d\bar{c}_1} = - \int_{\bar{c}_1}^{\infty} \gamma \frac{\rho_2}{(1 - \rho_2)^2} \frac{\lambda^2}{1 + \lambda} \Delta \beta^2 \frac{d\rho_2}{d\bar{c}_1} Dd\bar{c}_1 > 0.$$  

(59)

Thus, the regulator experiments. □

The intuition is clear. Spreading the cost targets further apart than in a deterministic setting reduces the welfare loss from the low cost firm’s expected second period rent, but increases the welfare loss from the low cost firm’s distorted second period effort. The net effect is a gain in expected second period welfare, as the rent effect dominates the effort effect.

### 4.3 Which effect wins?

So far, we have shown that the regulator has an incentive to decrease the distance between $\bar{c}_1$ and $\bar{c}_1$ relative to the deterministic setting in order to decrease the low cost firm’s first period transfer and increase his first period utility. Conversely, the regulator has an incentive to increase the difference between the first period cost targets relative to the deterministic benchmark because doing so increases her expected second period welfare.

The key thread connecting these two conflicting incentives is the low cost firm’s expected second period rent. The fact that the good firm gets a second period rent in equilibrium increases first period welfare, because the second period rent is not a component of the socially costly first period transfer. Clearly, however, the second period rent is socially costly in the second period, which is what drives the regulator to experiment. It would seem logical that for some $\lambda$ large enough, the benefit of increasing the low cost firm’s first period utility would come at too high a price in the second period, and the experimentation effect would win out over the signal dampening effect.
However, we show that the effect of the first period cost targets on the low cost type’s first period transfer perfectly offsets the increase in the low cost firm’s second period rent, so that the only tow effects that determine whether the regulator signal dampens are the low cost firm’s first period utility, and the high cost firm’s second period effort distortion. Thus, we are able to show that the regulator will choose to signal dampen no matter the size of \( \lambda \).

To see this, we will show that the effect of the first period cost targets on the dynamic portion of the first period transfer is due entirely to their effect on the low cost firm’s second period rent.

**Lemma 1.**

\[
\frac{d}{dc_1} \int_{c_1^0}^\infty U_2(\rho_2)(\bar{g} - g)dc_1 = -\frac{d}{dc_1} \int_{c_1^0}^\infty U_2(\rho_2)gdc_1. \tag{60}
\]

**Proof.** To see this, decompose the first period transfer effects

\[
\frac{d}{dc_1} \int_{c_1^0}^\infty U_2(\rho_2)(\bar{g} - g)dc_1 = \frac{d}{dc_1} \int_{c_1^0}^\infty U_2(\rho_2)\bar{g}dc_1 - \frac{d}{dc_1} \int_{c_1^0}^\infty U_2(\rho_2)gdc_1 \tag{61}
\]

and focus on

\[
\frac{d}{dc_1} \int_{c_1^0}^\infty U_2(\rho_2)\bar{g}dc_1 = \int_{c_1^0}^\infty \frac{dU_2}{d\rho_2} \frac{d\rho_2}{dc_1} \bar{g}dc_1
\]

\[
= -\frac{\rho}{1 - \rho} \int_{c_1^0}^\infty \frac{dU_2}{d\rho_2} (1 - \rho_2)^2 \bar{g}'dc_1 \tag{62}
\]

\[
= \frac{\rho}{1 - \rho} \int_{c_1^0}^\infty \left[ \frac{d^2U_2}{d\rho_2^2} (1 - \rho_2) - 2 \frac{dU_2}{d\rho_2} \right] (1 - \rho_2) \frac{d\rho_2}{dc_1} \bar{g}dc_1.
\]

Now,

\[
\frac{d^2U_2}{d\rho_2^2} (1 - \rho_2) - 2 \frac{dU_2}{d\rho_2} = -2\gamma \frac{\lambda}{1 + \lambda (1 - \rho_2)^2} \Delta \beta^2 - \left( -2\gamma \frac{\lambda}{1 + \lambda (1 - \rho_2)^2} \Delta \beta^2 \right) = 0. \tag{63}
\]

Thus,

\[
\frac{d}{dc_1} \int_{c_1^0}^\infty U_2(\rho_2)\bar{g}dc_1 = 0, \tag{64}
\]

so we can ignore this term in the regulator’s full first period problem, which we consider below.

Earlier we argued that the dynamic portion of the low cost firm’s first period transfer is decreasing in \( c_1 \) by a proof that follows directly from Theorem 2 in Jeitschko Mirman.
For the purposes of showing whether signal dampening or experimentation dominates in the regulator’s full first period problem, however, it is useful to use Lemma 1. From (32),

\[
\beta - c_1 = \frac{1}{\gamma} + \frac{\delta}{\gamma(1 + \lambda)} \left[ \lambda \frac{d}{dc_1} \int_{c_1}^{\infty} U_2(\rho_2)(\bar{g} - g)dc_1 - \frac{d}{dc_1} \int_{c_1}^{\infty} U_2(\rho_2)gdc_1 - \frac{1}{\rho} \frac{dE[W_2]}{dc_1} \right].
\] (65)

Thus, the the incentive to signal dampen dominates the incentive to experiment when

\[
\frac{\lambda \frac{d}{dc_1} \int_{c_1}^{\infty} U_2(\rho_2)(\bar{g} - g)dc_1 - \frac{d}{dc_1} \int_{c_1}^{\infty} U_2(\rho_2)gdc_1 - \frac{1}{\rho} \frac{dE[W_2]}{dc_1} < 0.
\] (66)

From the proof of Proposition 1, we know that

\[
\frac{d}{dc_1} \int_{c_1}^{\infty} U_2(\rho_2)gdc_1 > 0,
\] (69)

and from the proof of Proposition 2,

\[
\frac{d}{dc_1} \int_{c_1}^{\infty} (\bar{e}_2 - \frac{\gamma}{2}(\bar{e}_2)^2)gdc_1 > 0.
\] (70)

Thus, we know that (68) is true. We have shown that no matter how costly it is to leave the low cost firm a rent in the second period, that cost is perfectly offset by the reduction in the first period transfer. Thus, the signal dampening effect dominates the experimentation effect.

5 Appendix

5.1 Second period game

The low cost firm’s second period incentive compatibility constraint is

\[
t_2 - \frac{\gamma}{2}(\beta - c_2)^2 = \bar{t}_2 - \frac{\gamma}{2}(\beta - \bar{c}_2)^2;
\] (71)

and the high cost firm’s participation constraint is

\[
\bar{t}_2 - \frac{\gamma}{2}(\bar{\beta} - \bar{c}_2)^2 = 0.
\] (72)
Thus, the regulator’s second period problem is to maximize
\[
\rho_2 \left[ S - (1+\lambda) \left( \zeta_2 + \frac{\gamma}{2} (\bar{\beta} - \bar{c}_2)^2 \right) - \lambda \left( \frac{\gamma}{2} (\bar{\beta} - \bar{c}_2)^2 - \frac{\gamma}{2} (\bar{\beta} - \bar{c}_2)^2 \right) \right] + (1 - \rho_2) \left[ S - (1+\lambda) \left( \bar{c}_2 + \frac{\gamma}{2} (\bar{\beta} - \bar{c}_2)^2 \right) \right]
\]  
with respect to \( \zeta_2 \) and \( \bar{c}_2 \). The first order conditions are given by
\[
\bar{\beta} - \bar{c}_2 = \frac{1}{\gamma}
\]  
and
\[
\bar{\beta} - \bar{c}_2 = \frac{1}{\gamma} - \frac{\rho_2}{1 - \rho_2} \frac{\lambda}{1 + \lambda} \Delta \beta.
\]  
The low cost type’s effort from mimicking the high cost type is
\[
\bar{\beta} - \bar{c}_2 = \frac{1}{\gamma} - \frac{1 + \lambda - \rho_2}{(1 - \rho_2)(1 + \lambda)} \Delta \beta.
\]  
Thus, the high cost type’s rent is given by
\[
U_2(\rho_2) = \frac{\gamma}{2} \left( \frac{1}{\gamma} - \frac{\rho_2}{1 - \rho_2} \frac{\lambda}{1 + \lambda} \Delta \beta \right)^2 - \frac{\gamma}{2} \left( \frac{1}{\gamma} - \frac{1 + \lambda - \rho_2}{(1 - \rho_2)(1 + \lambda)} \Delta \beta \right)^2
= \Delta \beta \left[ 1 - \frac{\gamma}{2} \frac{1 + \lambda - \rho_2(1 - \lambda)}{(1 - \rho_2)(1 + \lambda)} \Delta \beta \right].
\]  

5.2 First period problem

In order to make the signal dampening proof easier, we will write the first period problem the following way. First, notice that the game is static for the high cost type, so his portion of the regulator’s first period problem is the same as in the standard static problem. It is convenient to write the low cost portion of the first period problem in the following way:
\[
S - (1+\lambda) \left( \zeta_1 + t_1 \right) + U_1 = S - (1+\lambda) \left( \zeta_1 + \psi(\bar{\beta} - \zeta_1) + t_1 - \psi(\bar{\beta} - \zeta_1) \right) + \zeta_1 - \psi(\bar{\beta} - \zeta_1) + \delta \int U_2(\rho_2) g dc_1
= S - (1+\lambda) \left( \zeta_1 + \psi(\bar{\beta} - \zeta_1) \right) - \lambda \left( t_1 - \psi(\bar{\beta} - \zeta_1) \right) + \delta \int U_2(\rho_2) g dc_1.
\]  
After substituting for \( t_1 \) from the low cost type’s first period transfer, we are left with
\[
S - (1+\lambda) \left( \zeta_1 + \frac{\gamma}{2} (\bar{\beta} - \zeta_1)^2 \right) - \lambda \left( \frac{\gamma}{2} (\bar{\beta} - \zeta_1)^2 - \frac{\gamma}{2} (\bar{\beta} - \zeta_1)^2 + \delta \int U_2(\rho_2) (\bar{g} - g) dc_1 \right) + \delta \int U_2(\rho_2) g dc_1
\]  
Thus, the first period portion of the regulator’s problem is
\[
\rho \left( S - (1+\lambda) \left( \zeta_1 + \frac{\gamma}{2} (\bar{\beta} - \zeta_1)^2 \right) - \lambda \left( \frac{\gamma}{2} (\bar{\beta} - \zeta_1)^2 - \frac{\gamma}{2} (\bar{\beta} - \zeta_1)^2 + \delta \int U_2(\rho_2) (\bar{g} - g) dc_1 \right) + \delta \int U_2(\rho_2) g dc_1 \right)
+ (1 - \rho) \left( S - (1+\lambda) \left( \zeta_1 + \frac{\gamma}{2} (\bar{\beta} - \zeta_1)^2 \right) \right).
\]
From the perspective of the first period, expected second period welfare is

\[ E[W_2(\rho_2)] \equiv \rho \int \left[ S - (1+\lambda)(\frac{\beta - \frac{1}{2\gamma}}{2}) - \lambda U_2(\rho_2) \right] gdc_1 + (1-\rho) \int \left[ S - (1+\lambda)\left(\beta - \bar{e}_2 + \frac{\gamma}{2}(\bar{e}_2)^2\right) \right] \bar{g}dc_1. \tag{81} \]

Thus, the regulator’s full first period problem is

References


