Statistical Evidence and the Problem of Robust Litigation

Jesse Bull and Joel Watson*

Preliminary and incomplete draft for Midwest Theory Conference
Please do not post on line.

Abstract

We develop a model of statistical evidence with a sophisticated Bayesian fact-finder. We focus on a setting of litigation, where a litigant may possess hard evidence and a jury (the fact-finder) interprets it. In addition to hard evidence, the litigant may have private unverifiable information. We study the robustness of the parties’ reasoning regarding the legal fundamentals and the litigant’s strategic behavior. The litigant’s choice of whether to disclose hard evidence entails two channels of information: the face value of the hard evidence disclosure (assuming it is disclosed whenever it exists) and a possible signal of the litigant’s private information. Our results suggest that in some situations, a desire for robust reasoning about evidence would lead the court to restrict the admissibility of some relevant evidence. We provide an explanation for the Federal Rules of Evidence Rules 403 and 404.

1 Introduction

The traditional “information theory” of evidence in litigation is that, assuming that fact-finders can process and interpret information well, evidence is always beneficial because it provides information about the underlying matter to be resolved. Thus, courts should allow a wide range of evidence, however complicated and without regard to weight. An alternative viewpoint is that fact-finders may have difficulty dealing with complicated or varied information and are better able to process evidence that is woven into a coherent and

* Bull: Florida International University; Watson: UC San Diego.
simple story—one that puts the pieces of evidence into context. This “story view” advises courts to exclude some types of relevant evidence.

We develop an alternative theory of evidence, distinct from the information and story views and having novel implications. Retaining the information theory’s basis of reasoned fact-finders, we show that constructive evidence requires alignment of the litigant’s and fact-finders’ beliefs about the meaning of evidence. This meaning is generally not fixed because it relates to the circumstances under which the litigant would choose to disclose available evidence, and the parties may have different beliefs about the litigant’s behavior. Absent coordination of beliefs and behavior, some types of evidence may be misleading in that they cause the fact-finder to update her belief in the opposite direction than she would if she knew the litigant’s actual disclosure strategy. In comparison to the story view, it is not so much a problem of fact-finders lacking the ability to process information but rather a problem of fact-finders having great latitude in how they might interpret evidence. Therefore, courts may do well to limit evidence, as in Rules 403 and 404 of the Federal Rules of Evidence and in common law, or to provide guidelines for its interpretation.

To understand the scope for interpretation, note that hard evidence is, by definition, statistical in nature: An individual piece of evidence exists with different probabilities in various states of the world. Hard evidence rarely provides definitive proof (that is, certainty) that the state is in some set, but it gives a signal that allows a fact-finder to update from a prior to a posterior probability distribution of the state. For example, on the question of whether the defendant in a trial robbed a particular store at 10:00 p.m. on a given date, the defendant may enter as evidence a time-stamped surveillance video showing him at a stadium 20 miles away at 9:20 p.m. on the same date. This piece of evidence does not prove with certainty that the defendant is innocent; it is possible that traffic conditions on the day of the crime were such that the defendant could, by leaving the stadium at 9:25 and speeding through the city, reach the store before 10:00 p.m. However, the defendant’s image on the stadium’s 9:20 p.m. surveillance video is perhaps more likely to exist in the state of the world in which the defendant did not rob the store than it would in the state of the world in which the defendant robbed the store. Therefore, disclosure of the surveillance video (the hard evidence in this illustration) may lead a sophisticated fact-finder to update her belief about the defendant’s involvement in the crime, raising the probability that the defendant

---

1Coverage of the story model is found in, for example, Pennington and Hastie (1986), Pennington and Hastie (1991), Pennington and Hastie (1992), and Hastie (1999).

2We suggest that this is fundamental to evidence law. Further discussion of this is in Section 4.

3There may also be errors in the estimate of the time of the robbery and/or the video time stamp.
is innocent but not raising it to 1.

In the robbery sketch just described, one might have imagined that the hard-evidence video would be disclosed if and only if it exists, so that the fact-finder extracts from its disclosure exactly the *face value signal* that its existence or nonexistence provides. That is, the fact-finder would then perform a Bayes’-rule update about the defendant’s guilt/innocence based on the probabilities that the video would exist conditional on guilt and conditional on innocence. However, a second channel of information operates with the surveillance video: It may signal the defendant’s private information, in particular if the fact-finder thinks that different types of defendant (such as guilty and innocent types) would act differently with regard to disclosing hard evidence.

In most settings, litigants have private unverifiable information in addition to hard evidence. Therefore, in general, a piece of hard evidence can provide information to fact-finders both through its face value signal and through its signal of the litigant’s private information. Our modeling exercise provides a simple framework to explore how these two channels of information interact. We look for conditions under which the signal of private information can outweigh the face-value signal, which leads to the possibility that evidence is misleading.

In our theory, misleading evidence is not an equilibrium phenomenon; it arises naturally in rationalizable outcomes of the litigation process. We view *rationalizability* as an appropriate solution concept for the litigation game because, realistically, one should not always expect the players’ beliefs and behavior to constitute an equilibrium. To be clear, while experienced judges and attorneys may be more likely to think and behave in coordinated ways, it is less likely that fact-finders (typically juries, whose members do not routinely hear cases) and litigants (who may not have much experience in court) are so coordinated. Even if one were to assume that players identify equilibria, they would not be expected to coordinate on the same one if there are multiple equilibria.

Thus, we promote an important consideration for evaluating the legal system that is common in the literature on social-choice/mecchanism design but has not yet been taken up in the law-and-economics literature: *robustness*, which is the requirement that the litigation process delivers the intended (socially desirable) outcomes whether or not the litigant and fact-finder are coordinated on a desired equilibrium. Our robustness criteria evaluates whether evidence is meaningful and constructive in all rationalizable strategy profiles in our litigation game. Our main point is a simple one: Robustness is difficult to achieve without imposing some restrictions on admissibility of evidence and/or on how the fact-finder may interpret evidence. Our results provide an explanation for Rules 403 and 404.
To get a feel for the analysis, consider two versions of a simple example in which a litigant faces a corruption charge and a jury must reach a finding in the case. Let us say that the state is G if the litigant committed the illegal act and it is I if the litigant did not. In the first version of the example, the litigant knows the state but there is no hard evidence. Instead the litigant has the opportunity to engage in cheap talk; he can say anything at no cost. Should the jury take the litigant’s statement as a signal of the actual state? One rationalizable outcome involves the G-type litigant making a different statement than does the I-type, and the jury correctly deducing the type from the statement, which leads to the best outcome for the court and society. However, in another rationalizable outcome, the jury gets the interpretation backward and society loses in a big way: The I-type litigant is punished, whereas the G-type gets away without punishment. If the parties were able to coordinate on an equilibrium, then no information would be transmitted via cheap talk because the G-type and the I-type litigant would both prefer not to be punished.

In the second version of the example, in place of cheap talk the litigant may have a document $d$ that would prove he did not commit the illegal act. To be precise, the document exists if and only if the state is I. In state G, the litigant does not possess $d$. In this version, there is a single rationalizable outcome and it coincides with the only equilibrium outcome: In state I, the litigant discloses $d$, and the jury finds the litigant guilty if and only if $d$ is not disclosed.\(^4\)

When reviewing this example, one’s natural response is to say that the court should not allow the jury to make inferences based on cheap talk in situations where the litigant types have similar preferences, because this is essentially a poker game that relies on the jury outsmarting the litigant types. However, there is a large continuum between the first and second versions of the example and, in general, hard evidence may have cheap-talk qualities. The problem arises in settings where litigants have private information about the state of the world, beyond what can be disclosed as hard evidence. Our modest modeling exercise provides precise conditions. This private information may be about something other than the strength of the litigant’s case. That is, it need not simply be about guilt or innocence.

Before going on to the model, let us make a few comments regarding the related literature. In the law-and-economics literature, two main approaches to modeling evidence stand out.\(^5\) The first treats evidence as statistical in nature, as just described, but it views evidence as arriving exogenously. These models are exercises in Bayes’ rule but they address neither the

\(^4\)This is a trivial case of Milgrom’s (1981) analysis.

\(^5\)Here we are summarizing Talley’s (2013) characterization of the law-and-economics literature on evidence. See also Sanchirico (2010).
parties’ incentives to disclose evidence nor the fact-finder’s evaluation of these incentives. The second modeling approach focuses on the incentives of the litigants to produce evidence, but it views the adjudicator as a mechanistic system whose judgment is an exogenous function of the quantities of evidence that the two sides produce. Evidence production is costly, and each party’s marginal cost is higher in the state of the world that favors the other party. These models treat evidence in an abstract way and they conclude that the types of litigants will be separated in equilibrium, yet the adjudicator does not utilize this signal.\textsuperscript{6}

In reality, the litigants control most pieces of evidence and disclosure is subject to their individual incentives. Evidence is discrete and statistical; a piece of evidence either exists or doesn’t exist, and the chance of existence depends on the state of the world. Producing evidence may be costly, but the cost differential between states is typically very small (for instance, if the surveillance video exists, then a culpable defendant can just as easily obtain and present it to the court as can a non-culpable defendant) or very large (such as the culpable defendant having to fabricate it).\textsuperscript{7} Juries and other fact-finders are sophisticated enough to assess the litigants’ incentives and recognize the signal inherent in the disclosure, or lack of disclosure, of evidence. That is, fact-finders routinely consider evidence to update their beliefs about the state, ideally in a way consistent with Bayes’ rule.

In the current version of this paper, we limit ourselves to very simple settings with one litigant, a jury that will impose a judgment, and one “document” (the source of statistical hard evidence) that the litigant may possess. We provide the following results. If a litigant has significant information about the state beyond what can be disclosed as hard evidence, then there is a problem of coordination of beliefs and behavior between the litigant and the jury, and hard evidence may be misleading. Further, there is an equilibrium in which the hard evidence provides no information; here we say the evidence is \textit{useless}. In contrast, if a litigant’s private information adds little to what can be disclosed as hard evidence, then there is a unique rationalizable outcome and, if and only if the document is positive evidence of the litigant’s favored state, the litigant discloses the document whenever it exists. In this

\textsuperscript{6}Some prominent entries in the literature feature both the litigants’ incentive to disclose evidence and a Bayesian decision maker, but they assume an extreme view of hard evidence as definitive proof of the state or some subset of possible states. Milgrom (1981) and Shin (1994) are classic examples. There are also mechanism-design models that seek to find the optimal judgment rule (a mechanism that maps feasible evidence sets to judgments) under the assumption that the litigants will find their way to an equilibrium in the induced evidence-production game. Bull and Watson (2004 and 2007), Green and Laffont (1986), and Kartik and Tercieux (2012) are examples of this category. Bull (2012) studies a model in which a piece of evidence can exist both when an accused is guilty and when he is innocent, but this focuses on the different issues of police interrogation and incentives for evidence fabrication.

\textsuperscript{7}In real settings, parties also may invest resources to gather evidence. We do not consider this strategic variable in the present paper.
case, we describe the evidence as *effective*.

We then discuss how undesirable outcomes can be avoided by providing ways for the court to announce how evidence sets will be interpreted or by limiting the admissibility of evidence. The latter is essentially an exercise in mechanism design, where the objective is to implement a mapping from the realized evidence to the judgment, but we restrict ourselves to the nearly trivial but realistic design element of whether to allow any particular evidence to be admissible. Otherwise, the fact-finder forms beliefs by Bayesian updating and imposes a judgment based on society’s preferences.

Our modeling exercise is based on the position that it is practically impossible for the law to commit courts to an optimal mechanism. There are too many idiosyncrasies in individual cases for overarching rules to be useful, and the law would not be able to describe exactly what the interpretation should be for every specific case. Indeed, just to determine whether a particular legal rule applies to an individual case requires analysis and interpretation. In other words, fact-finders are in the business of processing information and interpreting evidence, and this is an essential exercise in the pursuit of society’s objectives. Thus, fact-finders are ideally Bayesian and the legal system must recognize this, but the law (and courts) may optimally put some restrictions on how fact-finders interpret evidence and make judgments. A realistic instrument in this regard is the definition of *relevance*, which we take to relate to the likelihood that evidence exists in different states—and admissibility rules. It is the court’s job to evaluate the idiosyncrasies of individual cases, apply broad rules of admissibility, and conduct a Bayesian analysis.

The paper is organized as follows. Section 2 presents the model and our solution concept, which is rationalizability with plainly consistent beliefs (Watson 2017). Section 3 presents our main results. A detailed discussion of how our model and results relate to the law, including examples based on some well-known cases, is contained in Section 4. We conclude in Section 5.
2 Model

In this section, we develop our basic, abstract model.

2.1 Description of the Game

Consider a two-player disclosure game with hard evidence. The first player has information about an underlying state of the world and may be able to disclose hard evidence about it. The second player observes whatever evidence is presented and then takes an action that affects both players. Our leading application is litigation in court, where the first player is a litigant (plaintiff or defendant) and the second player is the fact-finder (typically a jury). To keep things simple, we call the players the “litigant” and “jury,” and we think of the jury as a single agent. The jury and society care about the state and also about the jury’s decision, which in practical terms is a finding in the case. Let \( \theta \) denote the state and suppose that \( \theta \in \Theta \equiv \{0, 1\} \). We will speak of \( \theta = 0 \) as the “low state” and \( \theta = 1 \) as the “high state.” Later we shall round out our description of the court by including a judge in the picture.

Whereas the jury does not observe the state, the litigant has two sources of information about it. First, the litigant privately observes an unverifiable signal \( x \in X \), where \( X \) is some arbitrary finite set. Second, the litigant may obtain hard evidence, which is verifiable and can be disclosed to the jury. Suppose hard evidence takes the following binary form: The litigant may or may not possess a single document \( d \). We represent the hard evidence that the litigant possesses by the evidentiary state \( e \in E \equiv \{d, \emptyset\} \), where \( e = d \) means the litigant possesses the document (and we say “the document exists”) and \( e = \emptyset \) means that the litigant does not possess the document (“the document does not exist”). If \( e = d \) then the litigant can choose to either disclose the document or disclose nothing. If \( e = \emptyset \), then he must disclose nothing; this is the defining characteristic of hard evidence. We sometimes describe disclosing nothing as “disclosing \( \emptyset \).” The jury observes only whether \( d \) is disclosed, not whether \( d \) exists. That is, the jury does not observe \( e \).

The underlying state \( \theta \), the evidentiary state \( e \), and the private signal \( x \) are determined exogenously by nature and in general are correlated.\(^8\) Let \( f \) denote the joint distribution, so that \( f(\theta, e, x) \) is the probability that the underlying state is \( \theta \), the evidentiary state is \( e \), and the private signal is \( x \). Defining \( f \) over sets, we also write expressions such as \( f(\theta, e, K) \) for \( K \subset X \), which is the probability that the state is \( \theta \), the evidentiary state is \( e \), and the private signal is an element of \( K \). Let \( r \equiv f(1, E, X) \) be the marginal probability that the

\(^8\)While it would be appropriate to call the entire vector \( (\theta, e, x) \) the state, it is more convenient to refer to \( \theta \) as the state because this is the part that will be payoff relevant.
state is high and assume that \( r \in (0, 1) \). It will sometimes be useful to write the probability of \( e \) and \( x \) conditional on the state \( \theta \), which is given by the standard conditional-probability formula:

\[
f(e, x \mid \theta) \equiv \frac{f(\theta, e, x)}{f(\theta, E, X)}.
\]

Assume that \( f(\Theta, d, X) \in (0, 1) \) so that there is strictly positive probability on the document existing and strictly positive probability on the document not existing.

If \( f(d, X \mid 1) > f(d, X \mid 0) \) then we say that the document is \textit{positive evidence} of the high state and the absence of the document is \textit{negative evidence} of the low state (Bull and Watson, 2004). If \( f(d, X \mid 1) < f(d, X \mid 0) \) then we say the opposite—the document is \textit{positive evidence} of the low state. Extreme cases of \textit{absolute proof} are given by \( f(d, X \mid 1) = 1 \) and \( f(d, X \mid 0) = 0 \), where disclosure of \( d \) proves that the state is high, and \( f(d, X \mid 1) = 0 \) and \( f(d, X \mid 0) = 1 \), where disclosure of \( d \) proves that the state is low.\(^9\)

The inference that the jury can draw from the litigant’s disclosure (or lack of disclosure) depends not only on the properties of the document but on the incentives of the litigant to disclose it, and the litigant’s behavior may be conditioned on his private signal \( x \). Therefore, any inference to be made from the disclosure or failure to disclose the document should incorporate both that the document cannot be disclosed if it does not exist—this is the direct information from hard evidence—but also incorporate how \( x \) influences the decision to disclose, which we refer to as the soft signaling role.

After seeing whether the litigant discloses \( d \), the jury updates its belief about the state and selects its action. To keep things simple, suppose that the jury’s action is a selection \( a \in [0, 1] \) representing, for instance, the amount of monetary damages to award the litigant. Assume that the jury’s (and society’s) payoff is decreasing in the square of the difference between the action \( a \) and the state \( \theta \). That is, the jury’s payoff is of the form \( u_J(a, \theta) = -(a - \theta)^2 \).

This implies that the jury wants to match the action to the state, and the jury’s optimal action is increasing in its posterior probability of the high state. Further, suppose that the litigant’s payoff is strictly increasing in the jury’s expected action, regardless of the state. The simplest such payoff function is \( u_L(a, \theta) = a \). It is thus in the litigant’s interest to act in whatever fashion will increase the jury’s posterior probability of \( \theta = 1 \).\(^10\) More generally,

---

\(^9\)Another extreme has \( f(d, X \mid 1) = f(d, X \mid 0) = 1 \), which is the case of a \textit{cheap document}, but the assumptions made already rule this out.

\(^10\)Another setting that leads to the same analysis and results is one in which the jury has only two actions available, such as finding the litigant guilty or not guilty, and the jury prefers or is instructed to choose not guilty in the high state and guilty in the low state. However, in addition to the information received from the litigant’s evidence choice, the jury is influenced by a separate, independent noisy information source. Therefore, the jury’s judgment is random and increasing in the posterior conditional on the litigant’s evidence.
we could allow $u_L$ to be a function of the state or even of the litigant’s private signal and realization of hard evidence, but this will not be necessary for the logical connections that we focus on. We shall comment on the more general model later.

For most of our analysis, we will not need to examine the jury’s action directly. Rather, we can formulate the analysis in terms of the jury’s posterior belief. We assume that the jury is Bayesian in that its posterior belief results from a proper application of the conditional probability formula, given the jury’s belief about the litigant’s strategy. For now, we also assume that the court cannot force the jury to commit to a decision rule in advance of the litigant’s disclosure choice.

Assume that the foregoing description is common knowledge between the players. To recap, in this incomplete-information game, an exogenous random draw determines $(\theta, e, x)$. The litigant obtains $e$ and also observes $x$. Then, if $e = d$, the litigant decides whether to disclose $d$. Finally, the jury observes whether $d$ is disclosed, forms its posterior belief about the state $\theta$, and selects its action $a$.

### 2.2 A Note on Litigant Types and Primary Activity

Our model describes a strategic situation between a litigant and jury, conditional on the case being in court. To analyze a real-world application, it can be helpful to describe how events that would lead to a court case imply the distribution $f$. Developing the context, we see that different types of litigant in our model can actually be different people in the real world. For instance, consider the following simple story about the events leading to a court case.

There is a variety of individuals in society and they may differ in their propensity to commit a crime. Their behavior and some exogenous random forces lead to an outcome of preliminary activity, which includes whether a crime is committed, evidence relevant to the crime, and the detainment by the police of an individual who is brought to trial. It is possible that this defendant (the litigant in our model) is a reasonable suspect but actually did not commit the crime, or the defendant is the one who committed the crime. These two types of defendant are different people in the society and their personal backgrounds are, to the extent not observable to law enforcement, captured in the $x$ variable. If, in this example, the defendant knows whether he or she committed the crime, then a component of $x$ would be perfectly correlated with $\theta$. Another possibility is that the litigant is unsure of whether his or her behavior was criminal, perhaps because of a lack of understanding of the law.
Consideration of the social backdrop also demonstrates how natural it is for there to be correlation between $e$ and $x$, conditional on the state $\theta$. Take the store robbery example in the Introduction and suppose $d$ denotes the litigant (defendant) being on the recording of the stadium security camera at a time that would make it challenging for him to have traveled to the store and committed the robbery. Suppose that $X$ is partitioned into four subsets representing four different groups of people in the society: $I$, $I'$, $G$, and $G'$. Types in $I$ and $I'$ would never commit a criminal act and those in $I$ happen to be on the stadium video, types in $G$ are sophisticated criminals who plan to make an appearance in front of the camera at the stadium before racing to the store to commit the crime, and types in $G'$ are naive criminals who would commit the crime on the spur of the moment and would not be on the stadium video. Assume that $x$ is randomly drawn, the crime occurs if $x \in G \cup G'$, and there is some randomness in police work so that with some probability an innocent person is the one brought to trial. Then, along with the $x$ already defined, we have $\theta = 1$ if $x \in I \cup I'$ and $\theta = 0$ otherwise. The video evidence exists, so that $e = d$, if $x \in I \cup G$. The distribution $f$ is then defined from the distribution of $x$ and the randomness induced by the police work to identify a suspect, conditional on a crime occurring. Then $x$ and $e$ are correlated overall and they are correlated conditional on $\theta$.

### 2.3 Strategies and Beliefs

A pure strategy for the litigant is a function mapping $X$ to the choice of whether to disclose, in the even that $e = d$.\textsuperscript{11} A mixed (behavior) strategy for the litigant is given by a function $\sigma : X \to [0, 1]$, where for each $x \in X$, $\sigma(x)$ is the probability that the litigant discloses the document in the event that $e = d$ and his private signal is $x$. The jury’s posterior belief regarding the state is conditioned on whether the document is disclosed. Let $b(d)$ denote the posterior probability of the high state in the event that $d$ is disclosed and let $b(\emptyset)$ be the probability of the high state in the event that $d$ is not disclosed. That is,

$$b(d) = \text{Prob}[\theta = 1 \mid d \text{ disclosed}] \quad \text{and} \quad b(\emptyset) = \text{Prob}[\theta = 1 \mid d \text{ not disclosed}].$$

These values define the jury’s interpretation of hard evidence.

It is important to recognize that $b(d)$ and $b(\emptyset)$ depend on the jury’s belief about the litigant’s strategy, as well as the jury’s understanding of the information system. Likewise, the litigant’s optimal strategy depends on the litigant’s belief about the jury’s updating rule.

\textsuperscript{11}Recall that the litigant has no choice in the event that $e = \emptyset$. 


Let $\bar{b}(d)$ denote the mean of the litigant’s belief about $b(d)$, and let $\bar{b}(\emptyset)$ denote the mean of the litigant’s belief about $b(\emptyset)$.

In the event that the document exists, it is clearly optimal for the litigant to disclose it if $\bar{b}(d) > \bar{b}(\emptyset)$ and to not disclose if $\bar{b}(d) < \bar{b}(\emptyset)$. The litigant is indifferent if $\bar{b}(d) = \bar{b}(\emptyset)$.

These incentives do not depend on the litigant’s private signal $x$ because the litigant cares only about increasing the jury’s posterior belief, which is a function of evidence only.

Let us represent the jury’s belief about the litigant’s strategy as a function $\lambda: X \rightarrow [0, 1]$, where for each $x \in X$, $\lambda(x)$ is the probability that the jury thinks the litigant discloses the document in the event that $e = d$ and the litigant’s private signal is $x$. We can determine the jury’s posterior beliefs in terms of $\lambda$ and the fundamentals of the model. If $\sum_{x \in X} f(\Theta, d, x) \lambda(x) > 0$, then the jury’s posterior belief conditional on disclosure is given by Bayes’ rule:

$$b(d) = \frac{\sum_{x \in X} f(1, d, x) \lambda(x)}{\sum_{x \in X} f(\Theta, d, x) \lambda(x)}.$$

Likewise, the Bayes’ rule expression for the jury’s posterior belief conditional on nondisclosure is:

$$b(\emptyset) = \frac{\sum_{x \in X} [f(1, \emptyset, x) + f(1, d, x)(1 - \lambda(x))]}{\sum_{x \in X} [f(\Theta, \emptyset, x) + f(\Theta, d, x)(1 - \lambda(x))]},$$

where the denominator is always strictly positive because of our assumption that the document exists with a probability strictly less than one.

Noting that $f(1, e, x) = rf(e, x|1)$ and $f(0, e, x) = (1 - r)f(e, x|0)$, we can rewrite these expressions in terms of probabilities conditional on the state:

$$b(d) = \frac{\sum_{x \in X} r f(d, x|1) \lambda(x)}{\sum_{x \in X} [r f(d, x|1) + (1 - r) f(d, x|0)] \lambda(x)} \quad (1)$$

and

$$b(\emptyset) = \frac{r \left[1 - \sum_{x \in X} f(d, x|1) \lambda(x)\right]}{r \left[1 - \sum_{x \in X} f(d, x|1) \lambda(x)\right] + (1 - r) \left[1 - \sum_{x \in X} f(d, x|0) \lambda(x)\right]} \quad (2)$$

As noted, the second equation is always valid. However, the denominator in the equation for $b(d)$ is zero if the jury’s initial belief about the litigant is $\lambda(x) = 0$ for all $x$, and in this case the expression is not valid (Bayes’ rule overall does not apply). The solution concepts we study impose constraints on the jury’s belief in such a situation, as we demonstrate in the next subsection.
2.4 Solution Concepts and Welfare

We shall analyze the game using two different solution concepts. The weaker of the two is rationalizability, which computes the beliefs and strategies that are consistent with the assumption that it is common knowledge that the players form beliefs about each other and best respond to their beliefs. In other words, the players are rational in the sense that they are sophisticated in forming their beliefs and they best respond. For instance, the jury will not put positive probability on a strategy for the litigant that itself cannot be rationalized. However, in a rationalizable outcome it is not necessarily the case that one player’s beliefs are accurate about the other player’s beliefs and behavior. For instance, depending on parameters, there may be a rationalizable outcome in which the litigant has an incorrect belief about the jury’s reasoning, so that \( \bar{b}(d) \neq b(d) \) and/or \( \bar{b}() \neq b() \). We shall adopt a version of rationalizability in which the players’ beliefs are assumed to be plainly consistent (Watson 2015); the implications and motivation are discussed below.

The rationalizability concept is appropriate for settings in which the litigant and jury lack experience in dealing with each other, and where the legal institution and social norms would not be expected to completely coordinate the litigant’s and jury’s beliefs and behavior. For example, a significant fraction of civil and criminal cases feature a litigant who has had little previous experience in court and who has not faced the same circumstances before. Most jurors also have limited experience in fact-finding. These players may be able to engage in sophisticated reasoning and understand each other’s incentives and rationality, but the two may not be coordinated in this regard.

With the rationalizability concept, it is important to further distinguish between two settings regarding the beliefs of the litigant. In the setting of aligned types, we assume that the different types of litigant have the same beliefs about the court. That is, the litigant’s beliefs \( \bar{b}(d) \) and \( \bar{b}() \) depend neither on the private signal \( x \) nor on whether document \( d \) exists. In the non-aligned types setting, in contrast, the litigant’s beliefs may depend on the litigant’s private signal and hard evidence. We shall allow for non-aligned types. The main justification for this is that, as noted already, the various litigant types (values of \( x \)) typically refer to different people in a population.\(^{12}\) The types differ not only in their culpability but in their experiences and knowledge of the court system, so they may form their beliefs in different ways. We assume that they behave rationally and understand the rationality of the jury, but they can still settle on different beliefs about how the jury would interpret evidence.

\(^{12}\)To venture into the philosophical, we one does not awake in the morning behind Harsanyi’s veil of ignorance and then receive his randomly determined type on the way to the courthouse.
and therefore be motivated to behave in different ways.

The second, stronger solution concept that we investigate is perfect Bayesian equilibrium (PBE), which for our model is equivalent to sequential equilibrium. In a PBE, the players are rational and their beliefs and behavior are aligned, so that \( \overline{b}(d) = b(d) \), \( \overline{b}(\emptyset) = b(\emptyset) \), and \( \lambda = \sigma \). Furthermore, the players’ beliefs satisfy plain consistency. The PBE concept is appropriate for settings in which some social institution serves to align beliefs and behavior. One such institution, which we discuss later, is sufficient transparency and documentation of the workings of the legal system.

In our model, plain consistency puts some structure on the jury’s beliefs. It implies that Bayes’ rule is used to calculate the posteriors \( b(d) \) and \( b(\emptyset) \) if the jury initially put positive probability on, respectively, the document being disclosed and not disclosed. That is, if \( \lambda \) gives the jury’s initial belief about the litigant’s strategy, then Equations 1 and 2 hold if the denominators are strictly positive. In fact, plain consistency implies that the posterior beliefs have this structure even in the case in which the jury initially put zero probability on the document being disclosed.

**Theorem 1** The following holds for every belief system satisfying plain consistency. The jury’s posterior belief \( b(\emptyset) \) satisfies Equation 2, where \( \lambda \) is the jury’s initial belief about the litigant’s strategy. The jury’s posterior belief \( b(d) \) satisfies Equation 1, where \( \lambda \) is the jury’s initial belief about the litigant’s strategy if it satisfies \( \sum_{x \in X} f(\Theta, d, x) \lambda(x) > 0 \) and otherwise \( \lambda \) is an arbitrary updated belief about the litigant’s strategy.

Social welfare is measured by the jury’s actual payoff. We will take the expectation with respect the distribution \( f \), calling this the *expected actual payoff of the jury*. Note that this may differ from the jury’s expected payoff—that is, the expected payoff in the mind of the jury—because what the jury expects and what actually happens may differ in expectation in a rationalizable outcome.

At times it will be useful to express this welfare in terms of how the litigant’s disclosure strategy and the jury’s interpretation of it make the jury’s posterior belief sensitive to the state itself. Note that if there were no hard evidence—indeed, no communication between the litigant and the jury—then the jury’s optimal action would be \( \hat{a} = r \) and social welfare would be \( -r(1 - r)^2 - (1 - r)(r - 0)^2 = r^2 - r \). Let us say that *hard evidence is effective* if, given the litigant’s strategy and the jury’s interpretation, the expected actual payoff of the jury is higher than in the case of no communication. We shall say that *hard evidence*

---

13 See Watson (2015) for the general description of PBE and plain consistency.

14 This is to say that the beliefs are structurally consistent (Kreps and Ramey 1987).
is useless if the expected actual payoff of the jury is exactly $r^2 - r$. Finally, we will say that hard evidence is misleading if the expected actual payoff of the jury is strictly less than $r^2 - r$. When hard evidence is misleading, the performance of the court proceeding is worse than if there were no hard evidence.

### Main Results

Whether hard evidence can turn out to be useless or misleading depends critically on the strength of the litigant’s private signal relative to the strength of the hard evidence. To explore the connection, we start with the special case in which the litigant is perfectly informed of the state. So for now assume $X$ can be partitioned into two sets $Y$ and $Z$, such that $f(d,Y | 1) + f(\emptyset,Y | 1) = 1$, $f(d,Y | 0) + f(\emptyset,Y | 0) = 0$, $f(d,Z | 1) + f(\emptyset,Z | 1) = 0$, and $f(d,Z | 0) + f(\emptyset,Z | 0) = 1$. That is, when the litigant receives private signal $x \in Y$, he knows the state is high ($\theta = 1$), and signal $x \in Z$ perfectly informs the litigant that the state is low ($\theta = 0$).

In this setting, there is a perfect Bayesian equilibrium in which the litigant never discloses the document. This behavior is rationalized by an adverse belief for the jury conditional on the document being disclosed. To be precise, the equilibrium beliefs satisfy $b(\emptyset) = r$ and $b(d) < r$. In fact, we can specify $b(d) = 0$. That is, in the event that the document is disclosed, the jury believes that the state is low for sure. This belief is consistent with the litigant’s strategy because, in the event of disclosure, the jury can believe that the litigant is much more likely to have deviated to disclose $d$ in the event that $x \in Z$ rather than when $x \in Y$. That is, Equation 1 applies but $\lambda$ is arbitrary. The intuition is similar to that for the familiar “babbling” equilibrium in cheap-talk games, but here the hard evidence setting adds a twist due to the document not always existing. In fact, as shown below, the prospects for babbling diminish as the litigant’s private signal becomes weaker and the hard evidence becomes stronger.

If $f(d,X | 0) > f(d,X | 1)$, so that $d$ is positive evidence of the low state, then the uninformative equilibrium just described gives the unique PBE outcome and hard evidence is useless.\(^{15}\) In the case of $f(d,X | 0) < f(d,X | 1)$, where $d$ is positive evidence of the high state, things get more interesting. It is straightforward to show that, in addition to the uninformative PBE, there is also a PBE in which the document is always disclosed when it

\(^{15}\)There are multiple PBE but they have the same outcome—no disclosure.
exists. In this equilibrium,

\[ b(d) = \frac{rf(d, X|1)}{rf(d, X|1) + (1-r)f(d, X|0)} > r, \]

and

\[ b(\emptyset) = \frac{r(1-f(d, X|1))}{r(1-f(d, X|1)) + (1-r)(1-f(d, X|0))} < r. \]

Turning to the analysis of rationalizable outcomes, the case of \( f(d, X|0) < f(d, X|1) \) also gives rise to the prospect of misleading hard evidence. This occurs when the players' beliefs and behavior are not coordinated. Realistically, different types of litigant (that is, for different realizations of the private signal \( x \) and/or hard evidence) may have different beliefs about the jury's reasoning and interpretation. For instance, someone who has committed a crime is likely to have different experiences and knowledge of the jury than is someone who has not committed a crime. Rationalizability allows us to study the implications of these non-aligned beliefs.

Consider, for example, the case of \( f(d, X|0) < f(d, X|1) \). Suppose that for \( x \in Y \) (knowing that \( \theta = 1 \)), the litigant believes that the jury anticipates behavior according to the uninformative PBE described above. This type of litigant believes that disclosure of the document results in a lower posterior \( b(d) \), so he does not disclose \( d \) when he has it. Further, suppose that for \( x \in Z \) (knowing \( \theta = 0 \)), the litigant believes the jury anticipates behavior according to the informative PBE, where the jury views \( d \) as statistical evidence in favor of \( \theta = 1 \). This type of litigant discloses the document when it exists. Finally, suppose that the jury's beliefs are in accordance with the informative PBE. Then \( b(d) > r \) and \( b(\emptyset) < r \), which represents updating in exactly the wrong direction for both disclosure of \( d \) and \( \emptyset \), relative to the true information content. The jury's actions lead to a lower social value than would be the case without hard evidence, and so hard evidence is misleading. Importantly, this is a rationalizable phenomenon.

We can generalize the analysis to consider any distribution of private signals, where the possibility of useless or misleading hard evidence depends on the relation between the strength of the litigant's private signal and the strength of the hard evidence. The determining factor is whether there are realizations of the private signal that provide the litigant with a strong indication that the state is low, relative how strong \( d \) indicates the high state.

The following theorems provide our results regarding equilibrium and rationalizable outcomes in the single-document model.

**Theorem 2**: If \( f(d, X|1) < f(d, X|0) \) then hard evidence is useless in every PBE. If
If \( f(d, x | 1) > f(d, x | 0) \) for all \( x \in X \), then there is a unique PBE; the litigant discloses \( d \) whenever he has it and hard evidence is effective. If \( f(d, X | 1) \geq f(d, X | 0) \) and for, some \( x \in X \), \( f(d, x | 1) \leq f(d, x | 0) \) then both kinds of PBE exist, one in which the litigant discloses \( d \) whenever he has it (hard evidence is effective) and one in which the litigant never discloses \( d \) (hard evidence is useless).

**Theorem 3**: If \( f(d, x | 1) < f(d, x | 0) \) for all \( x \in X \), then the unique rationalizable outcome has the litigant never disclosing \( d \) and hard evidence is useless. If \( f(d, x | 1) > f(d, x | 0) \) for all \( x \in X \), then the unique rationalizable outcome has the litigant disclosing \( d \) whenever he has it and hard evidence is effective. If there are values \( x, x' \in X \) such that \( f(d, x | 1) \geq f(d, x | 0) \) and \( f(d, x' | 1) \leq f(d, x' | 0) \), then there is a rationalizable outcome in which hard evidence is misleading.

### 3.1 Conditionally Independent Realization of \( d \) and \( x \)

The formulation of \( f \) allows for the realization of \( d \) and \( x \) to be correlated, which we suggest is realistic. To generate some simple intuition regarding the main results, however, it is helpful to look at the case of conditional independence. Let us denote the probability of \( e = d \) conditional on \( \theta \) as \( q_\theta \), and let us denote as \( p_\theta(x) \) the probability that \( x \in X \) conditional on \( \theta \). Thus \( f(d, K | \theta) = q_\theta p_\theta(x) \) and \( q_0 = f(d, X | \theta) \). Define:

\[
\gamma \equiv \max_{x \in X} \frac{p_0(x)}{p_1(x)}.
\]

This is the maximum of the likelihood ratio (low state to high state) over the private signal values of the litigant; it provides a bound on the highest posterior probability that the litigant can have about the low state based only on his private signal. Note that, because \( p_0(X) = p_1(X) = 1 \), we know that \( \gamma \geq 1 \).

For the conditionally independent case, we can state our previous results as follows.

**Corollary 1**: If \( q_1 / q_0 < 1 \) then there is a unique PBE; in this equilibrium, the litigant never discloses \( d \) and hard evidence is useless. If \( 1 \leq \gamma < q_1 / q_0 \) then there is a unique PBE; the litigant discloses \( d \) whenever he has it and hard evidence is effective. If \( 1 \leq q_1 / q_0 \leq \gamma \) then both kinds of PBE exist, one in which the litigant discloses \( d \) whenever he has it (hard evidence is effective) and one in which the litigant never discloses \( d \) (hard evidence is useless).

**Corollary 2**: If \( q_1 / q_0 < 1 \), then the unique rationalizable outcome has the litigant never disclosing \( d \) and hard evidence is useless. If \( 1 \leq \gamma < q_1 / q_0 \) then the unique rationalizable
outcome has the litigant disclosing \( d \) whenever he has it and hard evidence is effective. If \( 1 \leq q_1/q_0 \leq \gamma \) then there is a rationalizable outcome in which hard evidence is misleading.

This treatment provides the following intuition for the results. When the hard information conveyed by the document is strong relative to that provided by the private signal, there is no concern of evidence being misleading. However, when the information provided by the private signal is strong relative to that hard information of the document, there is scope for the evidence to be misleading. To reconcile this formulation with the previous results, consider

\[ \frac{q_1}{q_0} > \frac{p_0(x)}{p_1(x)}. \]

Rearranging yields

\[ \frac{q_1p_1(x)}{q_0p_0(x)} > 1, \]

which is equivalent to

\[ \frac{f(d,x|1)}{f(d,x|0)} > 1. \]

Thus, in the \( \gamma \) specification, \( 1 \leq \gamma < q_1/q_0 \) is equivalent to \( 1 < f(d,x|1)/f(d,x|0) \), for all \( x \in X \), and \( 1 \leq q_1/q_0 \leq \gamma \) is equivalent to \( f(d,x|1)/f(d,x|0) > 1 \) and for, some \( \tilde{x} \in X \), \( f(d,\tilde{x}|1)/f(d,\tilde{x}|0) \leq 1. \)

### 3.2 Conditional Correlation Between \( d \) and \( x \)

The case without correlation between the realization of \( d \) and the realization of \( x \) presents the opportunity for evidence to be misleading. However, when there is correlation between these two, there is much more scope for evidence to be misleading.

Consider again the robbery example discussed in Subsection 2.2, where there is correlation between \( e \) and \( x \) conditional on \( \theta \), because of the types \( I, I', G, \) and \( G' \). Note that there are two PBE: an uninformative one in which \( d \) is never disclosed and an informative one in which the litigant discloses \( d \) whenever he possesses it. Suppose that the litigant of type \( I \) anticipates play of the uninformative PBE and does not disclose \( d \), the type \( G \) litigant anticipates play of the informative PBE and discloses \( d \), and the jury anticipates play of the informative PBE and updates accordingly. This results in \( b(d) > r \), which is exactly the opposite of what is implied by the litigant’s behavior.

Note that for this negative result, we need for one just type of litigant to have a likelihood ratio less than 1. In particular, when \( \frac{f(d,x|1)}{f(d,x|0)} > 1 \) so that the document is viewed as positive.
evidence of \( \theta = 1 \), it is possible to have sufficient correlation between the realization of the document and the realization of the signal so as to allow for evidence being misleading.

**Theorem 4:** Consider any value \( \eta > 1 \) and fix \( r \). There exists a joint distribution \( f \) such that \( f(1, E, X) = r \), \( f(d, X \mid 1)/f(d, X \mid 0) = \eta \) so that \( d \) is positive evidence of \( \theta = 1 \), and yet there is a rationalizable outcome in which hard evidence is misleading.

### 4 Legal Rules and Examples

Are the prospects for misleading or useless evidence a problem for the legal system to solve? One possible response is to say that robustness is not an appreciable problem in reality, perhaps because litigants and courts find their way to the socially preferred PBE of the litigation game. In other words, the most informative PBE is the obvious way for the players to coordinate. Whereas this may be focal for game theorists evaluating the basic model of Section 2, we doubt it is so for real litigants, judges, and juries interacting in more complicated settings with many different evidentiary choices and preferences that may depend on the state and on the realization of evidence and private signals. Furthermore, based on the predictions of equilibrium refinements, the most informative PBE is generally not focal. For instance, the intuitive criterion (Cho and Kreps 1987) and the divinity criterion (Banks and Sobel 1987) have no bite in the basic model.\(^{16}\)

If one views the robustness problem as something the legal system might want to solve, then two remedies come to mind. Both describe practices of real courts. The first is for the court to be transparent regarding its interpretive rules, which means that in each case the court should make it clear how it will interpret the various possible evidentiary actions. This may help to align the beliefs of the various litigant types. However, this does not avoid the potential for the jury to fail to coordinate with the litigant, which presents a serious problem. Further, projecting complete transparency will typically be impossible because of the great many contingencies that the court would have to explain before litigation commences in a given case.\(^{17}\)

The second remedy is more realistic: The law might provide simple rules that identify the conditions under which the court should allow evidence to be evaluated in a Bayesian

\(^{16}\)More accurately, the intuitive criterion has no bite in the basic model. The divinity criterion has no bite in the slightly more general model in which the litigant’s preferences can depend slightly on the state and there is a small cost of disclosure; for instance, the litigant’s marginal value of the court’s action is slightly higher in the low state than in the high state.

\(^{17}\)Common legal representation for various litigant types—as would be the case with specialist attorneys—may serve to align the beliefs of the various types.
manner. The rules must be in a form that the court can readily apply, so they should be a function of the parameters of the evidentiary system. These are *admissibility rules*. They state whether a given document (to use our model’s terminology) shall be allowed to be considered. The determination is made on the basis of the document’s relevance, which in our model refers to the likelihood parameters.

Let us therefore assume that the policy instruments available to the framers of the law are admissibility rules and that, otherwise, the court performs Bayesian analysis and represents society’s interests in issuing judgments. There are two ways that inadmissibility might work in practice. The court could simply refuse to allow such a document to be presented. Alternatively, the court could declare and put in practice a rule that the document will be treated as though it provides no information whatsoever.

Clearly, it would be useful to make a document $d$ inadmissible only if $d$ would be misleading in some instances. We summarize with the following simple result:

**Theorem 5:** *If society wants to ensure that evidence will not be misleading, then it must adopt a rule that makes some sets of documents inadmissible.*

### 4.1 Legal Treatment of Relevance

Thayer viewed evidence law in the typical trial process as focusing on the following four issues: 1) materiality, meaning the facts to be proved; 2) relevance; 3) admissibility; and 4) the evaluation of weight or probative force of evidence.\(^18\) The first three are matters for the judge. The fourth is for the jury or other trier of fact.

Consider first the issue of relevance. In reality, most pieces of evidence presented in support of a case are available in different states with different probabilities and thus are statistical in nature. The legal terms *probative* (providing proof regarding a claim) and *relevance* (tending to strengthen the particular claim being assessed) suggest that the interpretation of evidence is often a matter of establishing a degree of confidence rather than reaching a conclusion with certainty.\(^19\) Rule 401 of the Federal Rules of Evidence states:

---

\(^{18}\)Modern reform of the law of evidence relied heavily on the ideas of James Bradley Thayer, many of which are contained in Thayer’s *A Preliminary Treatise on Evidence at the Common Law* (1898). See, for example, Anderson, Schum, and Twining (2005), Swift (2000), and Twining (1994). The discussion here draws from Anderson, Schum, and Twining (2005), which has a nice presentation of these issues.

\(^{19}\)This is not to say that near certainty is never achieved. One such actual case involved a person named Juan Catalan being accused of shooting and killing 16 year-old Martha Puebla, who was to testify in a murder case against Catalan’s brother and another Vineland Boyz gang member, Jose Ledesma. For details of the case, see Rubin and Bloomkatz (2008). The charges against Catalan were eventually dismissed when his attorney was able to acquire video showing him at a Los Angeles Dodgers game at the time of the
“Evidence is relevant if: (a) it has any tendency to make a fact more or less probable than it would be without the evidence; and (b) the fact is of consequence in determining the action.”

20 So clearly the law embraces the notion that evidence is statistical, and relevance is the determination of whether the evidence provides a signal of the claim being evaluated. We note that the law also appreciates incremental evidence—which we might call marginally relevant—at least to the extent that several such pieces can be combined to make a stronger signal.

21 On the issue of weighing the evidence (Thayer’s fourth item), note that although the judge can provide guidance to the jury, the jury weighs the evidence and this makes the issue of robustness and the potential for evidence to be misleading critical. Thayer described the analysis of evidence as being governed by “logic and experience.”

4.2 Exclusion – Rule 403

Consider next the issue of admissibility, about which Thayer focused on two principles: a) evidence that “is not logically probative of some matter requiring to be proved” should not be received, and b) all probative evidence should be allowed “unless a clear ground of policy of law excludes it.”

23 Thayer argued for judicial discretion for admissibility.

24 He suggested shooting. In this case, Puebla was shot in front of her home, and her parents and neighbors heard the gunshots, so witnesses could identify the exact time of the crime. The video evidence pinpointed Catalan in the stadium at exactly the same time.

20 Further, the Notes of Advisory Committee on Proposed Rules for Rule 401 include the following:

The rule summarizes this relationship as a “tendency to make the existence” of the fact to be proved “more probable or less probable…” The standard of probability under the rule is “more probable than it would be without the evidence.” Any more stringent requirement is unworkable and unrealistic.

21 In United States v. Pugliese (2d 1946), Pugliese was accused of possessing unstamped distilled spirits. Judge Learned Hand, when considering the relevancy of testimony by a former tenant of Pugliese that Pugliese had previously possessed unstamped spirits in the same location, said:

Its relevancy did not, and indeed could not, demand that it be conclusive; most convictions result from the cumulation of bits of proof which, taken singly, would not be enough in the mind of a fair minded person. All that is necessary, and all that is possible, is that each bit may have enough rational connection with the issue to be considered a factor contributing to an answer. (Wigmore §12)

22 Anderson, Schum, and Twining (2005) suggest that “an alternative interpretation is that the criteria for the weight of evidence are provided by probability theory, of which there are many versions.” We suggest that these views of evidence fit with our model.

23 See Thayer, pages 206 and 530.

24 Swift (2000) describes this as:

First, the exercise of judicial discretion in admitting or excluding evidence is, to use Professor
that the judge should have discretion to exclude evidence that is “slightly” or “remotely” relevant, and may “complicate the case” or “confuse, mislead, or tire the minds” of the jury.\textsuperscript{25}

In practice, admissibility may depend on the degree of relevance weighed against the potential for misleading the jury. Rule 403 of the Federal Rules of Evidence states the following:

The court may exclude relevant evidence if its probative value is substantially outweighed by a danger of one or more of the following: unfair prejudice, confusing the issues, misleading the jury, undue delay, wasting time, or needlessly presenting cumulative evidence.

Further, the notes of the Advisory Committee for Rule 403 state that “The case law recognizes that certain circumstances call for the exclusion of evidence which is of unquestioned relevance.”\textsuperscript{26}

Our modeling exercise focuses on why it may be useful to exclude evidence that has clear probative value and relevance. Exclusion is meant to avoid evidence being misleading, which can occur in situations in which the degree of relevance is low compared to the strength of the litigant’s private information. Thus, we provide a rationale for Thayer’s recommendation to exclude evidence that is slightly relevant and may confuse the jury. Further, our results may provide some guidance for judicial discretion and the balancing of relevance against the potential for misleading the jury.\textsuperscript{27}

A classic case is Robitaille v. Netoco Community Theatre Inc., 305 Mass. 265. Robitaille was injured after falling on a stairway at the Netoco Community Theatre and sued for damages. There was a thick carpet on the stairway and a critical issue in the case was whether the carpet was loose due to the tacks holding it in place having come out. A few

\textsuperscript{25}See Thayer, page 516. Further, in the case of United States v. Pugliese (2d 1946) mentioned earlier, Judge Hand writes: “It is true that evidence rationally probative ought sometimes to be rejected if it is likely unduly to complicate the issues, and prolong the trial...”\textsuperscript{26}

Swift describes Rule 403 as “the primary example of guided discretion in modern evidence law.” It is worth noting that Swift is concerned with judges being given too much discretion with little scope for review by an appellate court. These focus on judicial decisions about admissibility of expert witness testimony, scientific proof, and character evidence.

\textsuperscript{27}Interestingly, it seems, in practice, this is seen as less of an issue for bench trials. See Capra (2001). This may fit with the coordination issue we explore.
weeks before the accident two girls fell on the stairway when the carpet was loose. At
trial, evidence that the two girls fell was allowed although there was not evidence that the
condition of the stairway at the time of Robitaille’s fall were the same those at the time of
the earlier fall.

On appeal, the court noted:

In an action for personal injuries sustained in a fall on a stairway of the defen-
dant’s premises, where an issue was whether a thick carpet on the stairway was
loose at the time of the plaintiff’s fall because the tacks fastening it had pulled
out, evidence that two or three weeks previously other persons had fallen at the
same spot and the carpet afterwards had been found to be loose from pulling out
of the tacks was inadmissible, even in the discretion of the trial judge, without
evidence that the conditions existing were the same at the time of the plaintiff’s
fall and at the time of the previous falls.

It went on to note that evidence was presented that the carpet had been refastened with
tacks and that witnesses for the defendant stated that the carpet wasn’t loose at the time
of Robitaille’s accident. Further, it noted that the evidence “admitted was not merely that
on an earlier occasion the carpet had become loose under travel, which might have been
admissible to show that the tacks used were insufficient to fasten it. The evidence admitted
was of a similar fall sustained by other persons because of the loose condition of the carpet
at a different time.” The primary question of fact was whether the carpet was loose at the
time of Robitaille’s accident.

Numerical Example

We now construct a simple stylized example based on this case. This is done in the
context of the events that potentially lead to the relevant likelihood ratios. The time line
below describes the timing of events.

1. The following exogenous random draws occur at the time of a possible prior accident:

   - The rug is loose (ℓ) with probability $\frac{1}{2}$ and not loose (n) with probability $\frac{1}{2}$.
   - The “prior accident” occurs with probability $\frac{1}{2}$ if the carpet is loose and with
     probability $\frac{1}{8}$ if the carpet is not loose.

2. If the carpet was loose and there was a prior accident, the theater owner becomes aware
   of the carpet being loose. Otherwise the owner is not aware of the carpet’s condition
and has no action to take. A “good” owner, occurring with probability \( \frac{3}{4} \), repairs the loose carpet. A “bad” owner does not repair the loose carpet.\(^{28}\) Following this stage, we describe the carpet as being loose by \( L \) and not loose by \( N \), where \( L \) is the case if and only if the rug was previously loose (\( \ell \)) and either no prior accident occurred or it occurred and the owner is bad (having not repaired the carpet).

3. A random patron visits the theatre. If \( L \), an accident occurs with probability \( \frac{1}{2} \). If \( N \), an accident occurs with probability \( \frac{1}{8} \).

4. If an accident occurred at Date 3, a lawsuit is initiated and the patron (the litigant in our model) receives a private signal \( x \in \{x, \overline{x}\} = X \) about the condition of the carpet. This private signal represents the patron’s observation of the carpet just before or after the accident, and the observation occurs with noise. Assume that conditional on \( N \), \( \overline{x} \) is realized with probability \( \frac{1}{4} \) and \( x \) is realized with probability \( \frac{3}{4} \). Conditional on \( L \), \( x \) is realized with probability \( w \) and \( \overline{x} \) is realized with probability \( 1 - w \).

In addition, there is a hard-evidence document \( d \) which exists if and only if there was a prior accident that occurred with loose carpet. The patron receives this document and can disclose it to the jury.

Note that our model picks up this story at Date 4, conditional on the accident occurring. Remember that the litigant is the patron, as in the Robitaille case. The state \( \theta \) refers to whether the owner is liable, in which case the jury and society would like to reach a judgment of liability. Liability (\( \theta = 1 \)) is the event in which, conditional on the accident occurring, the carpet is loose (\( L \)), the prior accident occurred with loose carpet, and the owner did not repair the carpet. Non-liability (\( \theta = 0 \)) is the complement event, conditional on the accident occurring.

Note that we assume that in order for the case to go to trial, there must have been an accident. This does not change the qualitative results for the example. A compelling motivation for this is that if there were no accident then the case would likely be dismissed.\(^{29}\) Finally, we focus on the document \( d \) being proof of both a prior accident and the carpet being loose at the time of the prior accident.

By constructing an event tree and calculating the probabilities of the various paths, we

---

\(^{28}\)One can motivate the two types of owner on the basis of the cost of repairing the carpet, with the good owner having a small cost and the bad owner facing a large cost. Also, if the “good” owner is more likely to become aware of the loose carpet, the implications do not change.

\(^{29}\)Of course, in practice that not all frivolous lawsuits are dismissed.
obtain the following conditional probabilities:

\[ f(d, \bar{x} | 1) = w, \quad f(d, \bar{x} | 0) = \frac{1}{36}, \]
\[ f(d, \bar{x} | 1) = 1 - w, \quad \text{and} \quad f(d, \bar{x} | 0) = \frac{3}{36}. \]

Note there are two possibilities for evidence to be misleading here. Naturally, this depends on how informative the litigant’s private signal is. Here, this depends on the value of \( w \).

Consider first the case in which \( f(d, \bar{x} | 1) = w > f(d, \bar{x} | 0) = \frac{1}{36} \) and \( (d, \bar{x} | 1) = 1 - w < f(d, \bar{x} | 0) = \frac{3}{36} \). This requires \( w > \frac{33}{36} \). So for large values of \( w \), meaning that the patron has an accurate private signal of whether the carpet is loose, the conditions for evidence to be misleading are satisfied. Next consider the case in which \( f(d, \bar{x} | 1) = w < f(d, \bar{x} | 0) = \frac{1}{36} \) and \( (d, \bar{x} | 1) = 1 - w > f(d, \bar{x} | 0) = \frac{3}{36} \). This requires \( w < \frac{1}{36} \). So the conditions for evidence to be misleading are satisfied also for small values of \( w \), which again means that the patron has accurate private information about the carpet (with \( L \) now being indicated by \( x \)). We conclude that for extreme values of \( w \), where the litigant’s private signal is informative relative to the hard evidence, there is a rationalizable outcome in which evidence \( d \) is misleading.

To see the scope for evidence to be misleading, note there are two PBE: an uninformative one in which \( d \) is never disclosed and an informative one in which the litigant discloses \( d \) whenever he possesses it. Consider the case with \( w > \frac{33}{36} \) so \( \bar{x} \) suggests the carpet was loose. Suppose that the litigant of type \( \bar{x} \) anticipates play of the uninformative PBE and does not disclose \( d \), the type \( x \) litigant anticipates play of the informative PBE and discloses \( d \), and the fact-finder anticipates play of the informative PBE and updates accordingly. This results in \( b(d) > r \), which is exactly the opposite of what is implied by the litigant’s behavior.

### 4.3 Character Evidence – Rule 404

There is a general reluctance to allow character evidence or evidence of a prior conviction about a defendant in a criminal case. Rule 404 states “Evidence of a person’s character or character trait is not admissible to prove that on a particular occasion the person acted in accordance with the character or trait.”

---

30Note that since the probabilities with which \( x \) and \( \bar{x} \) are realized is fixed following \( N \), the informativeness of the private signal only depends on \( w \).

31We note that these may be more extreme than in practice since we hold the probability of \( x \) fixed and consider only \( w \), the probability of \( \bar{x} \).
Here, we do not explore the more nuanced issues concerning a criminal defendant who chooses to testify at his trial. Instead, we focus on People v. Beagle, 6 Cal. 3d 441, a well-known case in which the defendant testified, and focus on evidence of a prior conviction presented in that case.

Harvey Lynn Beagle II appealed his conviction by a jury of attempted arson and arson. The Supreme Court of California heard the case on January 5, 1972. Here is a brief summary of the relevant facts of the case. On May 25, 1969, Beagle was kicked out of Rudy’s Keg because he “became intoxicated and obnoxious while a patron in the bar.” At the time, Beagle made comments, about hiring someone to “fire bomb” the bar. Then in the afternoon of July 1st, Beagle asked the owner of the bar if he could return to the bar and was told no. Later that night, the roof of the building that housed the bar caught fire after what seemed to be an explosion. The bar owner put out the fire and discovered a soda bottle containing gasoline and a wick. Shortly after that, Beagle was arrested at his nearby apartment. He smelled of gasoline and had several books of matches in his pockets.

Beagle appealed on several grounds. One of these was that evidence of his having a prior conviction for writing a bad check was allowed. The Supreme Court of California found this inappropriate, and said:

> Although we reject all of the many contentions presented by defendant on appeal from the judgment, we nevertheless conclude, inter alia, that a trial judge must exercise his discretion to prevent impeachment of a witness by the introduction of evidence of a prior felony conviction when the probative value of such evidence is substantially outweighed by the risk of undue prejudice. (See Evid. Code, §352.)

The general idea is that a judge should have discretion to exclude some prior convictions from being admitted as evidence.

**Numerical Example – Rule 404**

There is a bit more judicial discretion when the defendant testifies as a witness since there is scope for using character evidence or evidence of prior convictions to impeach a witness. These are important details and we think the general theme fits with our results. There are potentially different rules for a civil case.

The court noted that the nature of the prior conviction and whether it reflects badly on the defendant’s honesty or integrity is a factor in determining whether it should be allowed to impeach the defendant as a witness. How recent the prior conviction was is also a factor. Additionally, it’s noted that a prior conviction for a similar crime should be “admitted sparingly.” The court cites Judge (later Chief Justice) Burger in Gordon v. United States (1967) 383 F.2d 936, 940-941 [127 App.D.C. 343] on these. The idea is that prior convictions for similar crimes may put significant pressure on a jury to convict. The court also suggested that the effects on the incentive for the defendant to testify should also be considered. We suggest that all of these issues fit with our model.
Consider a numerical example, motivated by Beagle, in which a prosecutor may choose whether to disclose $d$, which represents evidence of a prior conviction of the defendant for writing a bad check. Assume the defendant is accused of arson at a bar that he was previously kicked out of. A time line of the events follows.

1. A random draw determines whether there is a bad check prior conviction ($c$) or not ($n$). Evidence $d$ exists if and only if $c$ is realized. Assume $c$ occurs with probability $\frac{1}{2}$ and $n$ occurs with probability $\frac{1}{2}$.

2. If $c$ is realized, the person either reforms, which we denote by $g$, or has a higher propensity for criminal behavior, which we denote by $b$. Assume $g$ occurs with probability $\frac{1}{2}$ so that $b$ also occurs with probability $\frac{1}{2}$.

3. The defendant is matched with a bar/situation. The type with no prior conviction commits the crime with probability $\frac{1}{5}$, type $g$ with probability $\frac{1}{10}$, and type $b$ with probability $\frac{1}{2}$. If the defendant does not commit the crime, with probability $\frac{1}{5}$ someone else does.

4. Following commission of the crime and the defendant’s arrest, the prosecutor’s private information $x$ is realized as follows. If the defendant has no prior conviction, $x = a$. For type $g$, $\overline{x}$ is realized with probability $\frac{1}{4}$ and $x$ is realized with probability $\frac{3}{4}$. For type $b$, $\overline{x}$ is realized with probability $w$ and $x$ is realized with probability $1 - w$.

Our model picks up the story at Date 4. Regarding the probabilities with which each type commits the crime, we note the following. Someone who has been convicted of writing a bad check might learn from the experience, and wish to avoid any further legal problems. For that type of defendant, the prior conviction for writing a bad check may actually make it less likely that he would commit a crime like arson. On the other hand, it’s possible the conviction caused the defendant to feel animosity towards those in authority, including someone who owns a bar and can prevent him from drinking there. In this case, the prior conviction may make it more likely that the defendant would commit arson.

The prosecutor may be better informed than the fact-finder regarding the defendant’s type. The informativeness of $\mathbb{X}$ and $\overline{x}$ about the litigant’s type depends on $w$. Since we’re considering the prosecutor’s disclosure decision, we have that $\theta = 1$ corresponds to the defendant being guilty. We assume that the case is not brought when the crime is not committed by someone.
By constructing an event tree and calculating the probabilities of the various paths, we obtain the following conditional probabilities:

\[ f(d, \bar{x} | 1) = \frac{1}{40} + \frac{w}{2}, \quad f(d, \bar{x} | 0) = \frac{3}{40} + \frac{w}{6}, \]

\[ f(d, x | 1) = \frac{3}{40} + \frac{1}{2}[1 - w], \quad \text{and} \quad f(d, x | 0) = \frac{9}{40} + \frac{1}{6}[1 - w]. \]

As in the previous numerical example, there are two possibilities for evidence to be misleading here. Naturally, this depends on how informative the litigant’s private signal is. Here, this depends on the value of \( w \).\(^{34}\)

Consider first the case in which \( f(d, \bar{x} | 1) = \frac{1}{40} + \frac{w}{2} > f(d, \bar{x} | 0) = \frac{3}{40} + \frac{w}{6} \) and \( (d, x | 1) = \frac{3}{40} + \frac{1}{2}[1 - w] < f(d, x | 0) = \frac{9}{40} + \frac{1}{6}[1 - w] \). This requires \( w > \frac{11}{20} \). So for large values of \( w \), meaning that the prosecutor has an accurate private signal of whether the previously convicted defendant has reformed, the conditions for evidence to be misleading are satisfied. Next consider the case in which \( f(d, \bar{x} | 1) = \frac{1}{40} + \frac{w}{2} < f(d, \bar{x} | 0) = \frac{3}{40} + \frac{w}{6} \) and \( (d, x | 1) = \frac{3}{40} + \frac{1}{2}[1 - w] > f(d, x | 0) = \frac{9}{40} + \frac{1}{6}[1 - w] \). This requires \( w < \frac{3}{20} \). So the conditions for evidence to be misleading are satisfied also for small values of \( w \), which again means that the prosecutor has accurate private information about whether the previously convicted defendant has reformed (with \( b \) now being indicated by \( \bar{x} \)). We conclude that for extreme values of \( w \), where the litigant’s private signal is informative relative to the hard evidence, there is a rationalizable outcome in which evidence \( d \) is misleading. The failure to coordinate here is similar to that in the previous example.

### 4.4 Related Work on Exclusion

The idea that exclusion of evidence can improve fact-finding is unique to common law systems.\(^{35}\) Many economic models of the trial process, which typically assume that the fact-finder is able to process information without limitations, have difficulty explaining the exclusion of relevant evidence. We note that our results concerning exclusion fit very closely with those found in the Federal Rules of Evidence and the common law.

Lester, Persico, and Visschers (2009) provide a clever explanation for the exclusion of evidence that focuses on cognitive limitations of the fact-finder. In their model, it is costly for the fact-finder to evaluate evidence. This potentially results in the fact-finder’s incentives

---

\(^{34}\)Note that since the probabilities with which \( \bar{x} \) and \( \bar{x} \) are realized is fixed following \( N \), the informativeness of the private signal only depends on \( w \).

\(^{35}\)See, for example, Damaska (1997).
in evaluating evidence not being aligned with those of society in terms of accuracy of decision making. Thus, there is scope for the judge making some relevant evidence inadmissible to improve the decisions made by the fact-finder by preventing some socially unoptimal evidence evaluation by the fact-finder. We like this line of research and, although our model does not assume cognitive limitations, do not deny that fact-finders may have cognitive limitations. However, we note that Lester, Persico, and Visshers have “several results pointing to the difficulty of eliciting general principles that can inform the exclusion of specific pieces of evidence as a general rule.” In fact, they have an entire section titled “The Absence of General Principles Guiding Exclusion.”

Two other related papers that also make insightful contributions to the question of exclusion are Sanchirico (2001), and Schrag and Scotchmer (1994). Sanchirico suggests that a potential wrongdoer’s choice of action does not influence character evidence so character evidence should not be used to provide incentives.36 Schrag and Scotchmer make a related argument. While we find this argument compelling, we note that it does not explain all of the issues related to exclusion. Lester, Persico, and Visshers make the following point:

This argument relies on the predictability of exclusion on the part of the potential wrongdoer. A salient feature of Rule 403 in the US Federal Rules of Evidence, in contrast, is the latitude given the judge to exclude evidence on a case-by-case basis. That latitude seems to run counter the incentive-giving argument because it makes it difficult for the potential wrongdoer to foresee what evidence might be excluded.

5 Conclusion

We developed a model of statistical evidence with a sophisticated Bayesian fact-finder. We used rationalizability to study robustness of fact-finder decisions. Our results help to clarify the two channels of information inherent in evidence disclosure: The direct implications of the hard evidence disclosure and a signal of the litigant’s private information, which depends on the litigant’s strategy.

Further, our results suggest that it is not optimal, on the basis of robustness, to allow a fact-finder to evaluate all relevant evidence. This allows us to provide an explanation for

---

36 Sanchirico’s main example is a bar patron who is deciding whether to assault another patron who is annoying him. The argument is that if the trial decision focuses on trace evidence of whether assault occurred, even a defendant who has a history of assault in bars would have appropriate incentives.
the Federal Rules of Evidence Rules 403 and 404. Our results suggest that the evidence that should be excluded is that which is least relevant. This fits with the guidelines and language of Rule 403, and the use of Rule 404. We note that previous models have not provided this closeness of fit with Rule 403. Our future plans include studying a more general version that allows for multiple documents and multiple litigants.

A Proofs and Calculations

A.1 Proofs

The theorems are restated here, along with notes and proofs.

**Theorem 1:** The following holds for every belief system satisfying plain consistency. The jury’s posterior belief $b(\emptyset)$ satisfies Equation 2, where $\lambda$ is the jury’s initial belief about the litigant’s strategy. The jury’s posterior belief $b(d)$ satisfies Equation 1, where $\lambda$ is the jury’s initial belief about the litigant’s strategy if it satisfies $\sum_{x \in X} f(\Theta, d, x) \lambda(x) > 0$ and otherwise $\lambda$ is an arbitrary updated belief about the litigant’s strategy.

*Proof:* This is a special case of the analysis shown in Watson (2017), Section 4. □

**Theorem 2:** If $f(d, X | 1) < f(d, X | 0)$ then hard evidence is useless in every PBE. If $f(d, x | 1) > f(d, x | 0)$ for all $x \in X$, then there is a unique PBE; the litigant discloses $d$ whenever he has it and hard evidence is effective. If $f(d, X | 1) \geq f(d, X | 0)$ and for, some $x \in X$, $f(d, x | 1) \leq f(d, x | 0)$ then both kinds of PBE exist, one in which the litigant discloses $d$ whenever he has it (hard evidence is effective) and one in which the litigant never discloses $d$ (hard evidence is useless).

*Proof:* Consider the first case, where $f(d, X | 1) < f(d, X | 0)$, and consider any PBE. If the jury’s equilibrium beliefs satisfy $b(d) > b(\emptyset)$ then, because $\bar{b}(d) = b(d)$ and $\bar{b}(\emptyset) = b(\emptyset)$, the litigant must disclose $d$ regardless of $x$, but then Bayes’ rule implies that $b(d) < b(\emptyset)$ which is a contradiction. If the jury’s beliefs satisfy $b(d) < b(\emptyset)$ then the litigant never discloses and evidence is useless. If the jury’s beliefs satisfy $b(d) = b(\emptyset)$ then, by the law of iterated expectation, it must be that $b(d) = b(\emptyset) = r$ and so, even if some types of the litigant disclose with positive probability (that is, $\sigma(x) = \lambda(x) > 0$ for some $x$), it doesn’t change the jury’s belief about the state and evidence is useless.

Consider next the case of $f(d, x | 1) > f(d, x | 0)$ for all $x \in X$. By plain consistency, whether or not disclosure occurs on the equilibrium path, the conditional probability formula
in Equation 1 implies that \( b(d) > r \). If disclosure never occurs on the equilibrium path, then \( b(\emptyset) = r \), which contradicts the litigant’s rationality. Otherwise, the law of iterated expectations implies that \( b(d) > b(\emptyset) \) because a weighted average of \( b(d) \) and \( b(\emptyset) \) must equal \( r \). Thus, the only PBE has \( \sigma(x) = \lambda(x) = 1 \) for all \( x \in X \), we have \( b(d) > r > b(\emptyset) \), and evidence is useful.

In the third case, where \( f(d, X | 1) \geq f(d, X | 0) \) and for, some \( x \in X \), \( f(d, x | 1) \leq f(d, x | 0) \), it is easy to construct the two kinds of PBE. There is is a PBE in which \( \sigma(x) = 0 \) for all \( x \), and in the off-equilibrium-path event that \( d \) is disclosed, the jury believes that it is done by a type \( x \) for which \( f(d, x | 1) \leq f(d, x | 0) \). Then \( b(d) < b(\emptyset) = r \). There is clearly also a PBE in which \( \sigma(x) = \lambda(x) = 1 \) for all \( x \in X \), and \( b(d) > r > b(\emptyset) \). □

**Theorem 3:** If \( f(d, x | 1) < f(d, x | 0) \) for all \( x \in X \), then the unique rationalizable outcome has the litigant never disclosing \( d \) and hard evidence is useless. If \( f(d, x | 1) > f(d, x | 0) \) for all \( x \in X \), then the unique rationalizable outcome has the litigant disclosing \( d \) whenever he has it and hard evidence is effective. If there are values \( x, x' \in X \) such that \( f(d, x | 1) \geq f(d, x | 0) \) and \( f(d, x' | 1) \leq f(d, x' | 0) \), then there is a rationalizable outcome in which hard evidence is misleading.

**Proof:** Consider first the setting in which \( f(d, x | 1) < f(d, x | 0) \) for all \( x \in X \). By plain consistency and Equation 1, it must be that \( b(d) < r \) regardless of the jury’s belief. Furthermore, either the jury’s initial belief is that \( \lambda(x) = 0 \) for all \( x \), in which case \( b(\emptyset) = r \), or the jury’s initial belief puts positive probability on disclosure. In the first case, we have \( b(d) < b(\emptyset) \). In the second case, the jury’s initial belief puts positive probability on both disclosure and nondisclosure, which means that the law of iterated expectation applies in relating the initial belief \( r \) to \( b(d) \) and \( b(\emptyset) \), and we again conclude that \( b(d) < b(\emptyset) \). The litigant belief about the jury’s beliefs must satisfy the same condition, by common knowledge of rationality, and so the litigant strictly prefers to not disclose \( d \), whatever is the private signal \( x \). Understanding this, the jury has the belief \( b(\emptyset) = r \) and hard evidence is useless in all rationalizable outcomes.

A similar analysis applies to the setting in which \( f(d, x | 1) > f(d, x | 0) \) for all \( x \in X \). The jury must update so that \( b(d) > b(\emptyset) \), and the litigant therefore strictly prefers to disclose \( d \) regardless of \( x \).

Finally, consider the setting in which there are values \( x, x' \in X \) such that \( f(d, x | 1) \geq f(d, x | 0) \) and \( f(d, x' | 1) \leq f(d, x' | 0) \). Clearly we can find a function \( \lambda \) that makes both Equations 1 and 2 valid and also gives \( b(d) > r > b(\emptyset) \). For instance, we can specify...
\( \lambda(x) = 1 \) for all \( f(d, x \mid 1) \geq f(d, x \mid 0) \) and \( \lambda(x) = 0 \) otherwise. Likewise, we can find a function \( \lambda \) that makes both Equations 1 and 2 valid and also gives the opposite implication: \( b(d) < r < b(\emptyset) \). Thus, without restricting the litigant’s strategy space, the jury’s beliefs can go either way. Each type of litigant can therefore rationalize disclosing or not disclosing. To show that there are rationalizable outcomes in which hard evidence is misleading, note that one such outcome is the following: The jury’s beliefs satisfy \( b(d) < r < b(\emptyset) \); for litigant types satisfying \( f(d, x \mid 1) \geq f(d, x \mid 0) \), the litigant discloses \( d \) based on the belief \( b(d) > b(\emptyset) \); and for litigant types satisfying \( f(d, x \mid 1) < f(d, x \mid 0) \), the litigant does not disclose hard evidence based on the belief \( b(d) < b(\emptyset) \). The jury updates and chooses \( a \) in exactly the wrong direction compared to what is warranted by the litigant’s actual strategy, and social welfare is strictly less than \( r^2 - r \). □

**Theorem 4:** Consider any value \( f(d, X \mid 1)/f(d, X \mid 0) > 1 \) so that \( d \) is positive evidence of \( \theta = 1 \). Allow for any level of correlation. It is possible to have sufficient correlation so there is a rationalizable outcome in which hard evidence is misleading.

**Proof:** Take some \( K \) that is proper subset of \( X \). With correlation, it’s possible to specify \( f(d, K \mid 1)/f(d, K \mid 0) \leq 1 \). Note that since \( K \) is a proper subset of \( X \) it’s possible to do this in a way that does not change \( f(d, X \mid 1)/f(d, X \mid 0) \). This is done by specifying different values for \( f(d, X \setminus K \mid 1) \) and \( f(d, X \setminus K \mid 0) \). □

**Theorem 5:** If society wants to ensure that evidence will not be misleading, then it must adopt a rule that makes some sets of documents inadmissible.

**Proof:** Consider a single-litigant setting with one document \( d \) for which \( 1 \leq q_1/q_0 \leq \gamma \). To ensure that evidence is not misleading here requires making \( d \) inadmissible. □

### A.2 Rule 403 Example Calculations

Based on the timeline and probabilities with which events occur specified in the example, we can calculate joint probabilities \( f(e, x, \theta, A) \), where \( A \) denotes that the accident occurred. These are:

\[
\begin{align*}
  f(d, \overline{x}, 1, A) &= \frac{w}{32}, & f(d, \overline{x}, 0, A) &= \frac{3}{512}, \\
  f(d, x, 1, A) &= \frac{1}{32} [1 - w], & f(d, x, 0, A) &= \frac{9}{512}.
\end{align*}
\]
\[ f(\emptyset, x, 1, A) = 0, \quad f(\emptyset, x, 0, A) = \frac{w}{8} + \frac{1}{64}, \]

\[ f(\emptyset, \bar{x}, 1, A) = 0, \quad \text{and} \quad f(\emptyset, \bar{x}, 0, A) = \frac{1}{8}[1 - w] + \frac{3}{64}. \]

From these, one can calculate the conditional probabilities, which suppress the \( A \), found in the text.

### A.3 Rule 404 Example Calculations

Based on the timeline and probabilities with which events occur specified in the example, we can calculate joint probabilities \( f(e, x, \theta, A) \), where \( A \) denotes that the crime was committed. These are:

\[ f(d, a, 1, A) = 0, \quad f(d, a, 0, A) = 0, \]

\[ f(d, \bar{x}, 1, A) = \frac{1}{160} + \frac{w}{8}, \quad f(d, \bar{x}, 0, A) = \frac{9}{800} + \frac{w}{40}, \]

\[ f(d, x, 1, A) = \frac{3}{160} + \frac{1}{8}[1 - w], \quad f(d, x, 0, A) = \frac{27}{800} + \frac{1}{40}[1 - w], \]

\[ f(\emptyset, a, 1, A) = \frac{1}{10}, \quad f(\emptyset, a, 0, A) = \frac{2}{25}, \]

\[ f(\emptyset, \bar{x}, 1, A) = 0, \quad f(\emptyset, \bar{x}, 0, A) = 0, \]

\[ f(\emptyset, x, 1, A) = 0, \quad \text{and} \quad f(\emptyset, x, 0, A) = 0. \]

From these, one can calculate the conditional probabilities, which suppress the \( A \), found in the text.

### References


Federal Rules of Evidence.


Old Chief v. United States, 519 U.S. 172 (1997)


United States v. Pugliese, 153 F.2d 497 (2d Cir. 1945).