Informative fundraising: The signaling value of seed money and matching gifts

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Abstract

While existing theory predicts that a matching leadership gift raises more donations than seed money, recent experiments find otherwise. We aim to reconcile the two by studying a model of sequential fundraising under incomplete information about the charity’s quality, in which both the lead donor’s gift and the charity’s fundraising scheme may inform donors about the charity’s quality. The charity decides on the fundraising scheme and a potentially informed lead donor makes the first contribution. The uninformed subsequent donors can then infer quality and make simultaneous donations.

We find that with exogenously informed lead donor each charity type relies solely on the leader to reveal its quality through the donation size. Thus, the charity chooses the matching gift scheme since it raises more funds. With costly information acquisition, however, the lead donor is less reliable in conveying information. Thus, a high quality charity may engage in costly quality signaling by opting for seed money. Consistent with experimental data, seed money becomes a signal of high quality and raises more donations relative to a matching gift.

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1 Introduction

The charitable giving market is notorious for a significant presence of charity scams. According to Charity Navigator, the largest charity rating agency in the USA, one third of charities fail to meet industry standards\(^1\). This is largely due to donors’ lack of information. According to Money for Good 2015 report, “49% [of donors] don’t know how nonprofits use their money”\(^2\). The same report states that “Only 9% [of donors] compare nonprofits before giving.”

At first glance, this lack of information might be attributed to the donors’ lack of interest. However, survey evidence suggests otherwise. Money for Good 2015 reveals that donors “want clearer communication with nonprofits” regarding the charitable services that their money provides. The lack of information is attributed to the fact that “[donors] are often uncertain where to start, don’t have the information they want, feel pressed for time, ...”. This points to significant information costs facing donors. While the presence of rating agencies such as Charity Navigator, CharityWatch, and GiveWell can help donors, the mere number of charitable organizations\(^3\) makes the available information imperfect and costly to obtain. Consequently, a big challenge for good charities is finding ways to credibly inform donors of their quality and distinguish themselves from scams and low quality organizations.

In this paper, we investigate the role that leadership giving plays in conveying information to donors. Leadership giving refers to a fundraising strategy by charities of soliciting a large donation by a wealthy donor, whose donation announcement aims to incentivize giving by other donors. Leadership gifts can be in the form of an unconditional lump sum called “seed money” or a promise of matching small donations by a fixed ratio called “matching gift”.

The impact of the size and the form of the leadership gift has attracted significant interest in the theoretical and empirical literature. Most of the theoretical literature, however, has focused on analyzing leadership giving under complete information. In this environment, seed money is equivalent to sequential fundraising, which results in significant free-riding by downstream donors on the lead donor’s gift Varian (1994). Thus, seed money is predicted to reduce donations relative to no leadership giving. In contrast, a matching gift is associated with weaker free-riding incentives as the lead donor’s giving is contingent on the subsequent donors’ giving. Thus, the prediction generated by this literature is that matching leadership gifts are likely to raise more donations than seed money.

\(^1\)For more information, visit [www.charitynavigator.org](http://www.charitynavigator.org).
\(^2\)For the full survey conducted by Camber Collective, visit [http://www.cambercollective.com/moneyforgood/](http://www.cambercollective.com/moneyforgood/).
\(^3\)According to National Center for Charitable Statistics, there are more than 1.5 million tax-exempt organizations in USA. For more information, visit [http://nccs.urban.org/data-statistics/quick-facts-about-nonprofits](http://nccs.urban.org/data-statistics/quick-facts-about-nonprofits).
This finding of the theoretical literature has recently been challenged by experimental studies (see, for example, Karlan et al. (2011); List and Lucking-Reiley (2002); Rondeau and List (2008)). These studies find that donors’ giving is not very responsive to a matching gift. In contrast, seed money is found to increase giving. One plausible explanation for this finding is that the structure of the leadership gift itself conveys information about the charity’s quality. In particular, if seed money is associated with high quality, while matching gift with lower quality, donors may respond favorably to an announcement of seed money, but would respond little or even reduce donations in response to a matching gift.

To investigate the signaling impact of leadership giving, we propose a model of charitable fundraising, in which a charity is privately informed about its quality. It chooses its fundraising mechanism to maximize donations by two donors that are approached sequentially. In particular, the charity has the option of employing a leadership giving mechanism by soliciting the lead donor for seed money or a matching gift; alternatively, the charity can opt out of leadership giving by choosing not to announce the lead donor’s donation. Subsequently, given the fundraising strategy of the charity, the lead donor decides whether to acquire costly information about the charity’s quality before making a donation decision. Under leadership giving, the information acquired would not only benefit the lead donor directly, as it would result in more informed giving, but it may enable the lead donor to signal the charity’s quality to the downstream donor through the size of her contribution.

We find that the charity’s choice of a fundraising mechanism depends on the lead donor’s information. In general, both the charity’s fundraising strategy and the lead donor’s donation size has the potential of conveying information about the charity’s quality. If the large donor is exogenously informed about the quality, the charity can always rely on the lead donor to reveal the quality to the subsequent donor through the size of her donation. As a result, the charity finds it optimal to solicit the lead donor for a matching gift independent of the charity’s quality. This is because under either leadership scheme, the charity’s quality will be revealed, but the matching gift has the advantage of reducing the free-rider’s incentives by the downstream donor. Thus, to understand the use of seed money, one needs to consider a model of costly information acquisition by the lead donor.

Extending the model to allow for endogenous information acquisition by the lead donor results in equilibrium behavior that is consistent with the experimental findings. As the lead donor becomes less reliable in signaling the charity’s quality, the high quality charity needs to engage in costly signaling through the fundraising scheme. In particular, we show that there exists a (partially) separating equilibrium, in which the high quality charity fundraises for seed money with high probability to differentiate itself from the low quality charity. The low quality charity finds such strategy more costly and chooses seed money with lower probability. Consequently, it relies more on a matching gift as a fundraising scheme. Since the scheme
itself reveals information about the charity, the lead donor acquires information with lower probability than she would have chosen if the fundraising scheme was not informative.

This theoretical finding provides a novel explanation for the desirability of seed money. It predicts that donors associate seed money with better charities and thus announcing a large lump sum donation tends to increase giving by small donors. In contrast, a matching gift is associated with a lower quality, causing a weak or no response by donors. Moreover, it suggests that the low incentives for information acquisition may be partially attributable to the informational value of the fundraising mechanism employed by the charities. These predictions provide powerful testable hypotheses to be explored in the lab or the field.

**Related Literature** Our theoretical model builds upon large theoretical literature. Early theoretical work on private provision of public goods, such as Warr (1983) and Bergstrom et al. (1986), are focused on simultaneous contributions. They show the equivalence of Nash equilibrium of the simultaneous donations to Lindhal equilibrium. Admati and Perry (1991) expand the analysis to a mechanism of alternating sequential contributions towards a threshold public good. They find that this can lead to an inefficient outcome. Similarly, Varian (1994) considers sequential fundraising and finds that it results in a lower provision of the public good compared to simultaneous contributions due to stronger free-riding incentives of donors further down in the queue. However, the possibility of a donor subsidizing others’ contributions can alleviate this problem. The implication of these findings is that seed money is bad for fundraising, but matching gifts increase donations.

In the context of complete or symmetric information, the rationale for the use of seed money has been found in the case of threshold public goods, other regarding preferences, and significant risk aversion by donors. Andreoni (1998) shows that charities can use seed money to avoid a bad no-donations equilibrium for threshold public goods. Romano and Yildirim (2001) consider alternative preferences and show that when donors’ utility function goes beyond the standard altruistic forms and includes more general preferences such as a warm-glow, sequential donations can result in more funds raised compared to simultaneous donations. Gong and Grundy (2014) explain the use of seed money by a significant risk aversion of donors in the public good. They study the two schemes in a full information model, and show that when donors are very risk averse in the public service, their giving is decreasing in the size of the leadership gift in both seed money and matching gift schemes. Furthermore they find that since a matching gift counters free-riding incentives, the donations are less decreasing compared to when seed money has been granted. As a result, lead donor could be incentivized to contribute more in seed money scheme to make up for donor’s less willingness to pay and result in more funds being raised, compared to the matching gift scheme.

There is sparse theoretical literature that has considered incomplete information about
the public good. Krasteva and Yildirim (2013) consider an independent value threshold public good, in which each donor can choose whether to contribute informed or uninformed. They find that announcing seed money discourages informed giving while a matching gift encourages it. However, the independence of donors’ valuations precludes the possibility of signaling through scheme or contributions of lead donors. In this respect, the closest work to ours is Vesterlund (2003) and Andreoni (2006).

Similar to our model, Vesterlund (2003) and Andreoni (2006) consider the use of seed money as a signaling device to convey the charity’s quality. They demonstrate that announcing seed money incentivizes lead donors to signal high quality by a large donation. The intuition behind this finding is that the information provided to potential donors through the signal has a positive effect on their incentives for giving and can outweigh their incentives to free-ride on the large initial donation. However, an important distinction between these papers and ours is that they only allow for signaling with seed money and ignore the possible signaling value of a matching gift. By enabling charities to choose between seed money and a matching gift, we allow them to use the fundraising mechanism itself to convey quality to donors. Such quality signaling through scheme becomes an important tool of information transmission when acquiring information about the charity’s quality is costly for donors.

In the area of experimental economics, Silverman et al. (1984) and Frey and Meier (2004) find that donors respond positively to information about other donors’ gift. Furthermore, field experiments by List and Lucking-Reiley (2002) and Landry et al. (2006) demonstrate that both the probability and size of donations significantly increase when the seed money increases. All of these findings support the theory of seed money having a signaling value.

Matching gift schemes have also been studied in experiments. Eckel and Grossman (2003, 2006a,b); Eckel et al. (2007); Eckel and Grossman (2008) find evidence in support of matching gifts being effective in boosting donations. Meier (2007) finds that a matching gift increases donations in the short run, but has a negative impact on the long run giving. Karlan and List (2007) find evidence that presence of a matching gift causes the probability and size of donations to increase but increasing the match ratio does not have any additional effect.

The result of recent experiments is more surprising. Rondeau and List (2008) compare seed money to matching gift in a field experiment and find that seed money can have positive impact on giving but not matching gift. In another field experiment, Karlan et al. (2011) find that a matching gift has very little overall effect on giving. Huck et al. (2015) also run a field experiment and find evidence in support of donors better responding to seed money than to matching gifts in terms of donation levels. These findings leave a gap between the current theoretical and experimental literature regarding the effect of a matching gift, which we address by considering information signaling through the fundraising scheme.

In the following sections, we present our model and findings. Section 2 describes the
theoretical model. Section 3.1 considers the benchmark case of complete information and describes how the fundraising schemes rank in terms of money raised. Section 3.2 considers the signaling benchmark, in which the lead donor is exogenously informed and shows that in this case the charity will always choose a matching gift fundraising. Finally, section 3.3 presents the case of endogenously informed donor and discusses the possibility of signaling through the fundraising scheme. Section 4 concludes.

2 Model description

A single charity, $C$, aims to maximize the amount of money raised, $G$, to a continuous public good of quality $q \in \{q_l, q_h\}$ with $0 < q_l < q_h$ and a prior distribution $\pi_C = \{1 - \pi_h, \pi_h\}$ where $\pi_h \in (0, 1)$ denotes the likelihood of high quality.

There are $n \geq 2$ potential donors. Each donor $i$ is endowed with wealth $w_i$ drawn from an iid distribution with continuous density $f(w_i)$ and has the following preferences over private and public consumption:

$$u(g_i, G, q) = h(w_i - g_i) + qv(G), \quad i = L, F$$  \hspace{1cm} (1)$$

where $h'(\cdot) \geq 0, h''(\cdot) \leq 0, v'(\cdot) > 0, v''(\cdot) < 0$. Moreover, we assume that $\left| \frac{Gv''(G)}{v'(G)} \right| \leq 1$ so that the donors’ marginal utility from the public good is not diminishing too rapidly as provision increases.$^4$

The charity solicits donors by employing leadership giving, in which it first solicits a lead donor, denoted by $L$. The lead donor’s gift is announced to the remaining donors, denoted by $F$. Moreover, we let $w_L \geq \max_{i \in F} w_i$ so that the lead donor is the richest individual in the economy. This is consistent with Andreoni (2006) who finds that the wealthy individuals have stronger incentives to become leaders in charitable campaigns.$^5$

While the charity is privately informed about $q$, the donors are initially uninformed and only know the probability distribution $\pi_C$ over charity types. The charity moves first and publicly commits to a fundraising mechanism ($Z$), which takes either the form of seed money ($S$), or a matching gift ($M$). Under $S$, $L$’s unconditional gift $g^S_L$ is publicly announced and thus the follower donors can condition their donations on $L$’s gift. Under $M$, $L$ commits to a match ratio $m$, which is publicly announced, and results in a contribution $g^M_L = m \sum_{i \in F} g^M_i = mG^M_F$ by $L$.

$^4$This condition is a sufficient condition for the matching scheme to eliminate the free-riding incentives by the follower donors. It is satisfied by a large class of utility functions commonly used in economics, such as the CES utility function. For a discussion of the consequence of violating this condition, see Gong and Grundy (2014).

$^5$The lead donor being the wealthiest individual also guarantees that in a limit economy with $n \to \infty$, the matching grant scheme converges to a strictly higher giving compared to seed money.
Conditional on the fundraising mechanism, the lead donor forms a belief about the charity’s type. We denote by \( \mu_L(Z) \) the lead donor’s posterior belief of high quality \((q = q_h)\) conditional on observing scheme \(Z\). Moreover, consistent with Vesterlund (2003), we assume that \(L\) is able to verify the charity’s quality at cost \(k\) and may choose to signal her information through her donation size. Thus, follower donors’ posterior belief that the charity is of high quality, denoted by \( \mu_F(Z, g^Z_L) \), is both a function of the charity’s scheme choice \(Z\) and the lead donors’ gift size \(g^Z_L\) since both choices may carry information about the charity’s quality.

The timing of the game is as follows. First, \(C\) privately observes \(q\) and commits to a fundraising scheme \(Z\). Then, it solicits \(L\) for a donation. \(L\) decides whether to learn \(q\) at cost \(k\) and how much to donate, \(g^Z_L\). All follower donors then observe \(Z\) and \(g^Z_L\), and simultaneously choose their donations \(g^Z_F\).

In the following section, we provide the equilibrium analysis of the game. We focus on characterizing the sequential equilibria of this dynamic signalling game. Moreover, as commonly adopted in the literature, we refine equilibria using the intuitive criterion.

3 Equilibrium characterization

To study the impact of asymmetric and costly information, we provide two benchmarks that are instructive in understanding how the fundraising scheme affects donations by the two donor. Section 3.1 discusses the benchmark case of complete information about the charity’s quality, \(q\), and establishes that under complete information, the matching gift is optimal from the fundraiser’s point of view. Section 3.2 expands the analysis to an uninformed follower, but an exogenously informed leader. This introduces the possibility of the lead donor signaling the charity’s quality to the follower donors through the size of her donation. This benchmark illustrates that with a large contributing donor base, the matching gift is still the only mechanism that emerges in equilibrium. These two benchmarks lay the foundation for introducing endogenous information as the environment, in which seed money emerges in equilibrium. In particular, we show that with costly information, seed money necessarily emerges as a costly signal of high quality.

3.1 Benchmark: observable quality

Given an observable quality and a fundraising scheme \(Z\) by the charity, each follower donor chooses her donation to maximize her payoff given by eq. (1). Consider the best response of a follower donor \(i \in F\). For seed money, equating the marginal cost and benefit of donating results in

\[
h'(w_i - g^S_i) = qv'(G^S) \tag{2}
\]
Inverting $h'(\cdot)$ in eq. (2) and rearranging terms results in

$$g_i^{S} = \max\{w_i - h'^{-1}(qv'(G^S)), 0\} \quad (3)$$

where $h'^{-1}(\cdot)$ is a decreasing function in its argument. Clearly, $i$’s contribution is increasing in her individual wealth $w_i$ and thus the set of contributors $F^S$ comprises of the wealthiest individuals. If $n^S$ denotes the number of contributing donors in equilibrium, then the aggregate follower donors best response is derived by summing eq. (3) across all contributors:

$$G^S_F(q, G^S) = \sum_{i \in F^S} w_i - n^S h'^{-1}(qv'(G^S)) \quad (4)$$

For the matching scheme, the optimal donation by a contributing follower donor solves

$$h'(w_i - g_i^M) = qv'(G^M)(1 + m), \quad (5)$$

where $m$ corresponds to the matching ratio announced by the lead donor. Note that $\frac{GM}{GF} = 1 + m$ and thus rearranging terms and summing across all contributing donors $F^S$ results in the following equation for the aggregate best response function:

$$G^M_F(q, G^M) = \sum_{i \in F^S} w_i - n^M h'^{-1}(qv'(G^M)) \frac{GM}{GF_F(q, GM)} \quad (6)$$

where $n^M$ denotes the number of contributing donors under $M$. Comparing eq. (4) and eq. (6), it is easy to verify that for the same amount of total giving, i.e. $G^M = G^S$, the follower donors must be contributing more under a matching scheme relative to a seed money scheme\(^6\). Intuitively, while the marginal cost of giving for a follower donor is the same across the two types of leadership gift, the marginal benefit of donating an additional dollar is higher under the matching grant due to the lead donor’s commitment to multiply each donation. As a result, the follower donor has stronger incentives to give under a matching scheme compared to a seed money scheme.

Turning to the lead donor’s contribution choice, her utility function given by eq. (1) for a fundraising scheme $Z$ can be re-written as

$$u_L(q, G^Z) = h(w_L - G^Z + G^Z_F(q, G^Z)) + qv(G^Z). \quad (7)$$

\(^6\)For a formal proof, see Lemma A1 in the appendix.
Thus, the lead donor’s optimization problem can be postulated as choosing the total contributions $G^Z$ given the follower donor’s equilibrium best response. The equilibrium total donation amount then solves

$$h'(w_L - G^Z + G^Z_F(q, G^Z)) \left(-1 + \frac{dG^Z_F(q, G^Z)}{dG^Z}\right) + q v'(G^Z) = 0 \quad (8)$$

Eq. (8) reveals that the marginal cost of increasing total donations depends not only on the follower donors’ total contributions $G^Z_F$ to the public good, but also on how the follower donors’ contributions change with the rise in the total contributions, i.e. $\frac{dG^Z_F(q, G^Z)}{dG^Z}$. The following lemma highlights the follower’s response to increasing $G^Z$ under the two schemes.

**Lemma 1** The follower donors’ contributions decrease as the total donations increase under seed money (i.e. $\frac{dG^S_F(q, G^S)}{dG^S} < 0$), while they increase under a matching gift (i.e. $\frac{dG^M_F(q, G^M)}{dG^M} \in [0, 1)$).

Lemma 1 highlights the standard free-rider’s problem present in public good provision. Under seed money, the incentives for the follower donors to give as total donations increase are diminishing due to the decreasing marginal utility of the public good. Under seed money, the free-rider’s incentives are mitigated since the lead donor’s giving hinges on the other donors’ contributions. This causes the follower donors’ contributions to increase with the lead donors’ match ratio and thus with the rise in total donations.

The weaker free-riders’ incentives under matching makes the lead donor more willing to contribute to the public good herself, leading to the following observation.

**Proposition 1** Under complete information about $q$, the equilibrium total donations satisfy $G^M(q) > G^S(q)$ for all $q$ and all $n$ with $\lim_{n \to \infty} G^M(q) > \lim_{n \to \infty} G^S(q)$. As a result, the equilibrium fundraising scheme $\overline{Z}(q) = M$ for all $q$ and $n$. Moreover, $G^M(q_h) > \overline{G}^M(q_l)$.

Proposition 1 states that matching dominates seed money from the charity’s point of view. Therefore, in absence of asymmetric information about the quality of the charity, both charity types would prefer to fundraise for a matching gift. Moreover, for any fundraising scheme, the high quality charity will raise more money, which suggests that with imperfect information about the quality type, the low quality charity would have incentives to mimic the high quality charity in order to increase its donations.

To understand the high quality charity’s incentives to credibly signal its type through the fundraising mechanism, we next extend the model to include incomplete information about

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7The follower donors’ giving is increasing in the match ratio as long as $\left| \frac{G^Z v'(G)}{v'(G)} \right| \leq 1$. As shown by Gong and Grundy (2014), this is a sufficient condition for the matching scheme to raise more total donations.
the charity’s quality. The next section presents the case, in which only the lead donor is informed about the charity’s quality, turning the contribution decisions of the lead donor into a signaling game.

### 3.2 Benchmark: exogenously informed leader and uninformed follower

Given that the follower donors are uniformed about the charity’s quality, their belief regarding the likelihood of high quality, \( \mu_F(Z, g^L_Z) \), may be affected both by the charity’s equilibrium choice of a fundraising scheme as well as the leader’s donation choice since both choices may convey information in equilibrium.

As a first step in our analysis, consider the incentives for the informed leader to signal the charity’s quality through the size of her leadership gift. If in equilibrium the two types of charities pool on a leadership fundraising scheme, i.e. \( Z = \{S, M\} \), the corresponding donation subgame is equivalent to a signaling game, in which the leader donor can credibly disclose the charity’s type through the size of his donation. Due to the single crossing property, \( \frac{\partial^2 u(g_i, G, q)}{\partial G \partial q} > 0 \), there exists a unique separating equilibrium that satisfies the intuitive criterion. Therefore, even if the two types of charities choose the same fundraising scheme in equilibrium, the lead donor’s equilibrium contribution choice would inform the follower donors of the charity’s quality. This argument can be easily extended to any equilibrium leadership charity scheme both on and off the equilibrium path.

**Lemma 2** Suppose that the lead donor is exogenously informed about \( q \). Then, for any fundraising scheme \( Z \), the lead donor’s corresponding equilibrium donation \( g^L_Z(q) \) in the contribution subgame will be such that \( \bar{\mu}_F(Z, g^L_Z(q)) = \begin{cases} 1 & \text{if } q = q_h \\ 0 & \text{if } q = q_l \end{cases} \).

Lemma 2 reveals that with an exogenously informed lead donor, the charity’s quality will be perfectly revealed to the follower donors independent of the charity’s fundraising scheme. However, the equilibrium contributions would differ from the complete information benchmark as the low type of lead donor, i.e. the one that observes a low quality charity, would have incentives to mimic the high type, causing the high type to increase his equilibrium donation.

In particular, consider a scheme \( Z \) that is associated with a posterior belief \( \mu_L(Z) \in (0, 1) \). Since both types are present in this scheme with positive probability, the contribution subgame constitutes a signaling game, in which the lead donor’s contribution causes the follower donors to update their belief about the charity’s quality. Given a fixed belief of the follower donors \( \hat{q} \), the lead donor’s utility function for a realized quality \( q \) is given by

\[
\bar{u}_L(q, \hat{q}, G^Z) = h(w_L - G^Z + G^F(\hat{q}, G^Z)) + qv(G^Z).
\]
Given eq. (9), the least costly separating equilibrium contributions \((G^Z(q_l), G^Z(q_h))\) solve

\[
G^Z(q_l) = \arg \max_{G^Z} \bar{u}_L(q_l, q_l, G^Z) \tag{10}
\]

\[
\bar{u}_L(q_l, q_h, G^Z(q_h)) = \bar{u}_L(q_l, q_l, G^Z(q_l)) \tag{11}
\]

Clearly, since the low type’s fundraising amount \(G^Z(q_l)\) coincides with the amount raised under complete information \(G^Z(q_l)\), the matching scheme should always generate more total donations for the low type relative to the seed money scheme. The high type, on the other hand, engages in costly signaling by increasing her donations above \(G^Z(q_h)\) in order to dissuade imitation by the low type.

Since under either scheme the charity’s quality will be revealed with certainty by the lead donor, similar to the complete information benchmark, the charity will choose the scheme that raises the highest donation amount. In general, eliminating the free-rider’s incentives under matching does not guarantee anymore that matching maximizes donations for every quality type. Nevertheless, if the number of contributing donors is large, then the equilibrium giving \(\overline{G}^{Z}(q)\) approaches the giving under full information \(\overline{G}^{Z}(q)\) since \(G^Z(q)\) has a finite asymptote in the number of contributors (see Andreoni 1988). Since a large donor population seems to be the more empirically relevant scenario, it is the focus on our further analysis\(^9\). Then, as the following Proposition indicates, matching should be the preferred fundraising scheme for both charity types.

**Proposition 2** There exists \(\pi \in [2, \infty)\) such that if \(n \geq \pi\), the matching scheme raises more money, i.e. \(\overline{G}^{M}(q) > \overline{G}^{S}(q)\) for all \(q\). Consequently, both types of charities pool on a matching scheme in equilibrium, \(Z(q) = M\) for \(q = \{q_l, q_h\}\).

Proposition 2 reveals that in a large economy with exogenously informed leader, both charity types will solicit the lead donor for a matching gift. Intuitively, since the lead donor

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\(^8\)The equilibrium condition given by eq. (11) assumes that signaling high quality is costly for the high type, meaning that \(G^Z(q_h) > \overline{G}(q_h)\). This occurs whenever \(q_h\) is not too high relative to \(q_l\), making imitation of the high type attractive for the low type. Since costless separation \((G^Z(q_h) \leq \overline{G}(q_h))\) makes the problem trivially equivalent to the complete information benchmark, here we only consider the case of costly signaling.

\(^9\)As alluded to in the Introduction, the charitable market in the USA is rather large with a very significant portion of contributions coming from private donations. This makes our focus on a large donor base more plausible. Nevertheless, it is important to point out that the alternative is still consistent with the general prediction that seed money generates more giving. This is because it is the high quality lead donor that engages in costly signaling and numerical analysis reveals that with a relatively few contributing donors, costly signaling by the lead donor may result in seed money raising more donations than matching.
has incentives to credibly reveal the charity’s type to the downstream donor, there is no need for the high quality charity to opt for seed money since it is a more costly fundraising scheme as a result of the free-riding incentives.

The finding in this section suggests that in a large economy, seed money should never be observed if the lead donor is already informed about the quality. This raises the question of whether reducing the lead donor’s information by making it costly for him to learn the charity’s quality may induce the charity to choose seed money. Intuitively, by weakening the lead donor’s reliability in conveying quality, it may be possible to strengthen the incentives of the high quality charity to use the fundraising mechanism itself to convey quality. The following section endogenizes the lead donor’s information acquisition.

3.3 Endogenous information acquisition by the lead donor

Suppose that instead of costlessly observing the charity’s quality, the lead donor has to pay $k > 0$ in order to learn $q$. Thus, the lead donor can either choose to stay uninformed, denoted by $D = U$, or pay for information, denoted by $D = I$. Clearly, the lead donor’s choice to acquire information depends on the value of such knowledge. That, in turn, rests upon the leader’s belief about the charity being of high quality $\mu_L(Z)$ and the corresponding expected quality

$$q_U(Z) = \mu_L(Z)q_h + (1 - \mu_L(Z))q_l,$$

(12)

Thus, unlike with exogenously informed lead donor, in which the charity’s and the lead donor’s type space $q \in \{q_l, q_h\}$ coincide, with endogenous information the lead donor can be one of three types, $q_L(Z) \in \{q_l, q_U(Z), q_h\}$, where the second element corresponds to the uninformed donor. Moreover, since, in equilibrium, the two charity types may choose different fundraising strategy, the expected quality may vary with the fundraising scheme so that the lead donor’s type space itself is endogenous and a function of the charity’s equilibrium fundraising strategy. Clearly, for any $\mu_L(Z) \in [0,1]$, $q_U(Z) \in [q_l, q_h]$ and $q_U(Z)$ is increasing in the posterior belief $\mu_L(Z)$. The corresponding distribution over the lead donor’s types $\pi_L(Z) = \{\pi_{L,l}(Z), \pi_{L,u}(Z), \pi_{L,h}(Z)\}$ is also impacted by the charity’s equilibrium fundraising strategy as well as the lead donors’ information acquisition strategy.

The donation subgame following the charity’s fundraising decision and the lead donor’s information acquisition decision corresponds to a signaling game with 3 types. While the type space is endogenously determined, the equilibrium strategies by the two charity types and the subsequent information acquisition strategy by the lead donor uniquely pin down the lead donor’s type space $q_L(Z)$ and the corresponding distribution $\pi_L(Z)$. Thus, as in any standard signaling game, the donation subgame has a unique separating equilibrium that
satisfies the intuitive criterion, in which the lead donor perfectly reveals her type.

**Lemma 3** For a given fundraising strategy $Z$, the corresponding posterior belief $\mu_L(Z)$, and the equilibrium information acquisition decision $\bar{D}(Z)$, the lead donors’ equilibrium donation $\bar{g}_{SL}(q_L)$ in the contribution subgame will be such that:

$$\bar{\mu}_F(Z, \bar{g}_{SL}(q_L)) = \begin{cases} 1 & \text{if } q_L = q_h \\ \mu_L(Z) & \text{if } q_L = q_U(Z) \\ 0 & \text{if } q_L = q_l \end{cases}$$

Thus, analogous to section 3.2, the three types of leaders separate in equilibrium with types $q_h$ and $q_U(Z)$ engaging in (potentially) costly signaling through the size of their donation. Let $\bar{G}^Z(q_L)$ denote the corresponding equilibrium total donations for type $q_L$. Then, analogous to the exogenous information case, given eq. (7), the least costly separating equilibrium contributions $(\bar{G}^Z(q_l), \bar{G}^Z(q_U(Z)), \bar{G}^Z(q_h))$ solve the following system of equations:

$$\bar{G}^Z(q_l) = \arg \max_{G^Z} \bar{\pi}_L(q_l, q_l, G^Z)$$

(13)

$$\bar{\pi}_L(q_l, q_U(Z), \bar{G}^Z(q_U(Z))) = \bar{\pi}_L(q_l, q_l, \bar{G}^Z(q_l))$$

(14)

$$\bar{\pi}_L(q_U(Z), q_h, \bar{G}^Z(q_h)) = \bar{\pi}_L(q_U(Z), q_U(Z), \bar{G}^Z(q_U(Z)))$$

(15)

It is worth noting that while the low type’s total contributions coincide with the ones from exogenously informed leader, the presence of an uninformed donor with $q_U(Z)$ changes the equilibrium condition for the total donations of the high quality charity. This is because the lead donor of high type is choosing her donation to separate both from the low and the uninformed type.

Analogous to the exogenous information benchmark, our focus is on a market with a large number of contributors. Thus, the equilibrium giving satisfies the following relationship.

**Condition 1** $\bar{n}^Z$ is sufficiently large such that $\bar{G}^M(q_L) > \bar{G}^S(q_L)$ for $q_L = \{q_l, q_h\}$ and $\bar{G}^M(q_U(M)) > \bar{G}^S(q_U(S))$ if $q_U(M) > q_U(S)$.

As discussed in Section 3.2, the large donor base guarantees that the equilibrium total donations are not too far from the optimal amounts under symmetric information. This, in turn, implies that the matching scheme should raise more donations compared to seed money for both the high and the low quality type whenever the lead donor is informed. Moreover, since the equilibrium donations are increasing in quality, matching donations would exceed
seed money donations under uninformed lead donor as long as the expected quality under matching is higher.

To obtain an expression for the value of information, let $\nabla_L(q_L) = \pi_L(q_L, q_L, G^Z(q_L))$ denote the corresponding equilibrium utility for the lead donor. Then, the value of information for the lead donor is simply the difference between the expected informed and uninformed utility:

$$\Delta^Z(\mu_L(Z)) = \mu_L(Z) \nabla_L(q_h) + (1 - \mu_L(Z)) \nabla_L(q_l) - \nabla_L(q_U(Z)).$$  \hspace{1cm} (16)

The value of information depends crucially on the charity’s equilibrium fundraising strategy. In particular, the following lemma points out that $\Delta^Z(\mu_L(Z))$ is positive if and only if the two charity types (partially) pool in equilibrium, thus leaving the lead donor uncertain of the charity’s quality.

**Lemma 4** $\Delta^Z(\mu_L(Z))$ is continuous in $\mu_L(Z) \in [0, 1]$. Moreover, $\Delta^Z(\mu_L(Z)) = 0$ for $\mu_L(Z) \in \{0, 1\}$ and $\Delta^Z(\mu_L(Z)) > 0$ for all $\mu_L(Z) \in (0, 1)$.

Clearly, if the two charity types follow a fully separating fundraising strategy, the fundraising scheme would be perfectly informative, i.e. $\mu_L(Z) \in \{0, 1\}$. Then, by eq. (12), $q_U(Z) = \{q_h, q_h\}$, and thus the value of information would be zero. Since the scheme would be perfectly revealing of the charity’s type, it must be the case that the low quality charity chooses a matching scheme, while the high quality charity chooses seed money. Then, by eq. (12), the uninformed lead donor under seed money coincides with the high quality type, i.e. $\tilde{q}_U(S) = q_h$, implying that the only consistent belief in the donation subgame corresponds to the two type case under exogenous information, i.e. $\pi_F(S, \tilde{G}^S(q_h))$. \hspace{1cm} (16) Therefore, $G^S(q_h) = G^M(q_h)$. Given a fully uninformed lead donor, to prevent deviation by either type of charity, the two schemes must raise the same amount of money ($G^S(q_h) = G^M(q_h)$). Therefore, while in a fully separating equilibrium seed money emerge as a signal of high quality, such equilibrium is purely incidental and requires that each scheme raises the same amount of money.

Apart from the fully separating equilibrium, there are other possible equilibria, in which the lead donor chooses to stay uninformed either due to a high information cost or low value of information. A common feature of such equilibria is that both charity types raise the same amount of money, as the following Proposition highlights.

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10 More technically, recall that the equilibrium beliefs in a sequential equilibrium are derived as a limit of fully mixed strategies at every information set. Consequently, the type space under seed money is the converging limit of a sequence $q^k_L(S) \in \{q_l, q_U(S), q_h\}$ with corresponding probability distribution $\tilde{\pi}^k_L(S) = \left\{ \pi^k_L(S), \pi^k_{LU}(S), \pi^k_{LH}(S) \right\}$ where $q^k_L(S) \rightarrow q_h$, and $\tilde{\pi}^k_L(S) \rightarrow \{0, 1, 0\}$. Thus, the corresponding equilibrium belief about the likelihood of high quality given the lead donor’s contribution, $\tilde{\pi}_F(S, \tilde{G}^S_L(q_h))$, converges to the least costly separating equilibrium with two types of lead donor, $\tilde{\pi}_F(S, \tilde{G}^S_L(q_h))$.  

---
Proposition 3  In every equilibrium with no information acquisition, i.e., $\tilde{D}(\tilde{Z}) = U$, each scheme on the equilibrium path results in the same total donations and each charity raises the same amount of money.

In absence of information acquisition, the high quality charity is not able to effectively separate from the low quality charity since imitation by the low type is always possible. Consequently, the two charities will either pool on the scheme or the two schemes would be equally attractive to prevent profitable deviation. Since this is not consistent with the experimental evidence alluded to in the Introduction, we instead focus on equilibria, in which information acquisition occurs with positive probability.

In order for information acquisition to take place, the value of information should be sufficiently high relative to the cost. In particular, if the value of information at the prior distribution $\Delta^M(\pi_h)$ exceeds the cost $k$, fully informed equilibrium always exists. In such equilibrium, the two charity types necessarily pool on the matching fundraising scheme.

Proposition 4  (Fully informed equilibrium) Fully informed equilibrium exists if and only if $\Delta^M(\pi) \geq k$. Moreover, the fully informed equilibrium is unique with $\tilde{Z}(q) = M$ for all $q \in \{q_l, q_h\}$, $\tilde{D}(M) = I$, and $\tilde{G}^M(q_h) \geq G^M(q)$.

The intuition behind Proposition 4 coincides with the one under exogenously informed donor- as long as the lead donor obtains information with certainty, the high quality charity can rely on the lead donor to signal the charity’s quality through her donation size. As a result, matching will be preferred by the charity since it incentivizes more giving. Interestingly, the amount of money raised by the high quality charity in equilibrium exceeds the amount raised under an exogenously informed donor $(\tilde{G}^M(q_h) \geq G^M(q))$. This is because the donation is tailored to not only signal away from the low quality type, but also the uninformed type who is more willing to mimic the high type.

Proposition 4 implies that the lead donor must have reduced incentives to acquire information in order for the high quality charity to find seed money attractive. However, Proposition 3 indicates that the other extreme of no information acquisition also does not provide strict incentives for seed money fundraising. Thus, we next turn to partial information acquisition. In particular, we focus on equilibria with partial information acquisition, in which seed money is on the equilibrium path$^{11}$. We refer to such equilibria as SPI (seed-partial info) equilibria. More formally,

$^{11}$As typical for most signaling games, there is multiplicity of equilibria, including an equilibrium, in which seed money is off the equilibrium path due to very pessimistic beliefs about the charity’s quality. For our purposes, however, the more relevant equilibria involve seed money being utilized by charities in equilibrium since it allows us to address the question of which type of charity is more likely to employ seed money fundraising.
**Definition 1** SPI equilibrium satisfies $\alpha(Z) = \text{Prob}(\tilde{D}(Z) = 1) \in (0,1)$ for some $Z$ and $\beta^S(q) = \text{Prob}(\tilde{Z} = S|q) > 0$ for some $q$.

The following Lemma provides sufficient conditions for the existence of an SPI equilibrium.

**Lemma 5** (Existence of an SPI equilibrium) If $\Delta^S(\pi_h) \geq k$ and $\bar{G}^S(q_u(\pi_h)) > \bar{G}^M(q_l)$, there exists an SPI equilibrium. Moreover, every SPI equilibrium satisfies:

1) $\beta^S(q) > 0$ for all $q$;
2) $\alpha(S) < 1$;
3) $\alpha(M) < 1$.

Lemma 5 states that an SPI equilibrium exists as long as the cost of information is low enough ($\Delta^S(\pi_h) \geq k$) and seed money has some attraction for the low type ($\bar{G}^S(q_u(\pi_h)) > \bar{G}^M(q_l)$). This is because in equilibrium both type of charities need to be present in seed money. To see this, note first that the low type would never unilaterally choose seed money since it would perfectly reveal its quality. The high type, on the other hand, may find seed money attractive if it is perfectly revealing of its quality, but the resulting zero value of information and no quality verification by the lead donor, would make seed money also attractive for the low type. Thus, in equilibrium, both types need to utilize seed money, resulting in strictly positive value of information (Lemma 4). Since too much information acquisition would make seed money unattractive for the low type, the lead donor must be not fully informed under seed, i.e. $\alpha(S) < 1$. This can provide an incentive for both charity types to choose seed money. Of course the lead donor might be discouraged from choosing seed money when the lead donor is very unlikely to obtain information, but that can be offset with seed money being a much stronger signal of quality. Moreover, information acquisition under matching should also be less than perfect, i.e. $\alpha(M) < 1$ to reduce the appeal of the matching scheme for the high type.

Given that both types of charity are present in seed money, we are interested in how seed money compares to matching in terms of conveying quality information to donors. The following Proposition delivers a sharp prediction by establishing that in any SPI equilibrium, seed money is a stronger signal of high quality compared to matching.

**Proposition 5** In every SPI equilibrium, the seed money scheme is associated with a higher posterior belief about the likelihood of high quality compared to the matching scheme, i.e. $\mu_L(S) > \mu_L(M)$, and higher expected donations, i.e. $E[\tilde{G}^S] > E[\tilde{G}^M]$.

16
Proposition 5 is consistent with the experimental evidence alluded to in the Introduction. It reveals that in every SPI equilibrium, the high quality charity chooses seed money with higher probability relative to the low quality charity. This leads to more optimistic beliefs regarding the quality type under seed money and higher expected quality. Consequently, the expected donations under seed money are also higher. Intuitively, the attraction of seed money for the high quality type is in its ability to signal the charity’s type more reliability. Thus, seed money must be either associated with higher expected quality for the uninformed lead donor or induce more information acquisition by the lead donor relative to matching. However, if the benefit is coming purely from more information acquisition, such that \( \alpha(S) > \alpha(M) \) and \( q_U(M) > q_U(S) \), then the low quality charity would strictly prefer to fundraise for matching. This is because unlike the high type, the low type dislikes information acquisition and would find matching more attractive if it is less informative and associated with more optimistic belief regarding its type. Thus, a necessary condition for both types to find seed money attractive is for seed money to signal higher quality to the donors.

Overall, the analysis in this section illustrates that with costly information acquisition, seed money is likely to be used by the high quality charity to credibly signal its quality. More importantly, we illustrate that with both schemes being utilized in equilibrium, the seed money scheme is always indicative of a higher expected quality and results into more funds being raised compared to the matching gift scheme. This is a rather strong result that corroborates the finding in the experimental literature.

4 Conclusion
To be added.

Appendix

Proof of Lemma 1

Part 1: Seed Money

Eq. (4) holds for followers’ donation in scheme \( S \). Taking derivative with respect to \( G^S \) gives:

\[
\frac{\partial G_F^S}{\partial G^S} = -n^S h^{-1} (q v' (G^S)) q v'' (G^S) = \frac{-n^S q v'' (G^S)}{h'' (h^{-1} (q v' (G^S)))}
\] (A-1)
Remembering that \( h''() < 0 \) and \( v''() < 0 \), this gives:

\[
\frac{\partial G_S}{\partial G_S} < 0
\]

Part 2: Matching gift

Eq. (6) holds for followers’ donation in scheme \( M \). Taking derivative with respect to \( G_M \) gives:

\[
\frac{\partial G_M}{\partial G_M} = -n_M h'^{-1}(q v'(G_M) G_M) \left[ \frac{q v''(G_M) G_M}{G_F} + \frac{q v'(G_M) G_M (\frac{\partial G_M}{\partial G_M})}{G_F^2} \right]
\]

\[
= \frac{-n_M \left[ \frac{q v''(G_M) G_M}{G_F} + \frac{q v'(G_M) G_M (\frac{\partial G_M}{\partial G_M})}{G_F^2} \right]}{h''(h'^{-1}(q v'(G_M) G_M))}
\]

Rearranging and solving for \( \frac{\partial G_M}{\partial G_M} \) gives:

\[
\frac{\partial G_M}{\partial G_M} = \frac{-q v''(G_M) G_M - q v'(G_M)}{G_M h''(h'^{-1}(q v'(G_M) G_M)) - q v'(G_M) G_M (\frac{\partial G_M}{\partial G_M}) G_F^2}
\]  \( \text{(A-2)} \)

Remembering that \( h''() \leq 0 \) and \( v''() < 0 \) and \( v'(()) > 0 \) and \( \left| \frac{G_M}{v'(G_M)} \right| \leq 1 \), this gives:

\[
0 \leq \frac{\partial G_M}{\partial G_M} < 1
\]

This completes the proof. ■

**Lemma A-2** (Followers’ donations in matching an seed money) The follower donors’ contributions under seed money is less than under matching (i.e. \( G_S^S(q) \leq G_M^M(q) \)) as long as the donors’ preference satisfies \( \left| \frac{G_M}{\sigma'(G)} \right| \leq 1 \).

**Proof of Lemma A-2**
For a given quality \( q \):

Since \( G_F^S \) is decreasing in \( G^S \) and since \( G_F^S \leq G^S \), the highest possible value for \( G_F^S \) occurs at \( G_F^{max} = G^S \) when \( g_F^S = 0 \).
Similarly, as $G^M_F$ is increasing in $G^M$ and since $G^M_F \leq G^M$ the lowest possible value for $G^M_F$ occurs at $G^M_F = G^M$ when $s^M_L = 0$.

But if the leadership gift is zero the two schemes become the same game of simultaneous donations played by the followers. So:

$$G^S_F^{\text{max}} = G^M_F^{\text{min}} \Rightarrow G^S_F \leq G^M_F$$

Since by Lemma 1, $\frac{\partial G^S_F}{\partial G^S} < 0$, whenever the lead donor is contributing a positive amount and as a result $G^S_F < G^S_F^{\text{max}}$ the inequality will be strict:

$$G^S_F < G^M_F$$

This completes the proof. ■

**Proof of Proposition 1**

For a given quality $q$:

From Lemma 1

$$\frac{\partial G^S_F}{\partial G^S} < 0 \leq \frac{\partial G^M_F}{\partial G^M} < 1 \Rightarrow \left( -1 + \frac{dG^S_F}{dG^S} \right) < \left( -1 + \frac{dG^M_F}{dG^M} \right) < 0$$

From Lemma A-2:

$$G^S_F(\overline{G}) \leq G^M_F(\overline{G}) \Rightarrow w_L - \overline{G}^S + G^S_F(\overline{G}) \leq w_L - \overline{G}^S + G^S_F(\overline{G})$$

Combining the last two results and remembering that $h''() < 0$:

$$h'(w_L - \overline{G}^S + G^S_F(\overline{G})) \left( -1 + \frac{dG^S_F}{dG^S} \right) < h'(w_L - \overline{G}^S + G^M_F(\overline{G})) \left( -1 + \frac{dG^M_F}{dG^M} \right)$$

Plugging this into Eq. (8) for $Z = S$ gives:

$$h'(w_L - \overline{G}^S + G^M_F(\overline{G})) \left( -1 + \frac{dG^M_F}{dG^M} \right) + qv'(\overline{G}) > 0$$

Comparing this result to Eq. (8) for $Z = M$ and remembering that both $h()$ and $v()$ are concave functions gives:

$$\overline{G}^S < \overline{G}^M$$

This completes the proof. ■
Proof of Lemma 2

For a given scheme choice by the charity:

In a separating equilibrium, when the leader has observed $q_l$, she knows that follower infers $q_l$ after observing the leader’s donation i.e. $\mu_F(G^Z(q_l)) = 0$. Anticipating this, the leader’s optimal choice has to satisfy her unconstrained maximization problem. That will be Eq. (10). So:

$$G^Z(q_l) = G^Z(q_l)$$

When the leader has observed $q_h$, she wants the follower to infer $q_h$ after observing the leader’s donation i.e. $\mu_F(G^Z(q_h)) = 1$. Taking it for granted that this happens, the leader’s optimal choice has to satisfy her unconstrained maximization problem. That will be Eq. (8).

On the other hand, the leader’s signal of $q_h$ has to be credible i.e. it has to be high enough so that had the leader observed $q_l$ is would not have had the incentive to donate this amount. The least costly credible signal of $q_h$, $G^Z(q_h)$, has to satisfy Eq. (11).

Let’s call the left hand side of Eq. (11) $L(G^Z(q_h))$ and the right hand side $R(G^Z(q_l))$. Due to concavity of $u()$ and $v()$, $L()$ is concave. So, there always exist two roots to the equation:

$$L(x) = R(G^Z(q_l))$$

Let’s call them $G_1$ and $G_2$ and assume $G_1 < G_2$. Noting that $h’(>) > 0$, it is easy to see:

$$G^Z_F(q_l,G^Z(q_l)) < G^Z_F(q_h,G^Z(q_h)) \Rightarrow R(G^Z(q_l)) < L(G^Z(q_l)) \Rightarrow G_1 < G^Z(q_l) < G_2 \Rightarrow G^Z(q_h) = G_2$$

So noting that $v’(>') > 0$:

$$G^Z(q_h) > G^Z(q_l) \Rightarrow (q_h - q_l)v(G^Z(q_h)) > (q_h - q_l)v(G^Z(q_l))$$

Adding this to the equation $R(G^Z(q_h)) = L(G^Z(q_l))$ gives:

$$h(w_l - G^Z(q_h) + G^Z_F(q_h,G^Z(q_h))) + q_hv(G^Z(q_h)) > h(w_l - G^Z(q_l) + G^Z_F(q_l,G^Z(q_l))) + q_hv(G^Z(q_l))$$

So, while $G^Z(q_h)$ is not too high a signal when the leader has observed $q_h$, it is high enough to discourage the leader when she has observe $q_l$. In other words any signal greater than or equal to $G^Z(q_h)$ is equilibrium dominated for a leader of type $q_l$ and not for a leader of type $q_h$. Imposing the intuitive criterion requires:

$$\forall G^Z \geq G^Z(q_h) \quad \mu_F(G^Z) = 1$$

So, in a separating equilibrium, the optimal choice of a leader of type $q_h$ has to satisfy her constrained maximization problem. As explained before when signaling is not costly the
problem becomes trivial, so we concentrate on cases where signaling is costly. As a result, the leader’s problem and solution will be:

\[
\mathcal{G}^Z(q_h) = \arg \max_{G^Z} \pi_L(q_h, q_h, G^Z) \text{ s.t. } \pi_L(q_l, q_h, G^Z(q_h)) \leq \pi_L(q_l, q_l, G^Z(q_l))
\]

Thus far it has been established that there exists one and only one separating sequential equilibrium. Furthermore, in any pooling equilibrium, both types of leader will choose the same amount of total funds \(G^S_p\). Consistent belief on the path of such equilibrium for the follower is that the expected quality of the leader is \(q_p < q_h\). But going through the same process as above and replacing \(q_l\) by \(q_p\), it can be shown that there does exist a gift \(G^Z\) that is not too high a signal when the leader has observed \(q_h\), but it is high enough to discourage the leader when she has observe \(q_l\). In other words any signal greater than or equal to \(G^Z\) would be equilibrium dominated for a leader of type \(q_l\) and not for a leader of type \(q_h\). Imposing the intuitive criterion requires:

\[
\forall G^Z \geq G^Z \quad \mu_F(G^Z) = 1
\]

This gives a leader of type \(q_h\) an incentive to deviate to \(G^Z\). So no pooling equilibrium exists. This completes the proof.

**Proof of Proposition 2**

Consistent with Andreoni (1988) let’s consider a case of seed money scheme with a given quality \(q\) and a lead donor with a finite total contribution goal of \(G^S\). With \(n\) followers, it will be the case that \(0 < n^S \leq n\) followers each contribute a positive amount that sums up to \(G^S_F(q, G^S, n)\). If another follower \(j\) joins the pool of followers, by Eq. (3), she should be contributing either \(g_j^S = w_j - h'\left(qv'(G^S)\right)\) or zero. Also by Eq. (3) all other donors must be donating as before \(j\) joined the pool. So:

\[
G^S_F(q, G^S, n + 1) = G^S_F(q, G^S, n) + \max\{g_j^S, 0\}
\]

Thus, as more followers join the pool, for a given \(G^S\) and \(q\), \(G^S_F\) increases. This gives:

\[
\lim_{n \to \infty} G^S_F(q, G^S) \to G^S
\]  

(A-3)

This means that the size of the leadership gift \(g_L^S\) required to raise \(G^S\) becomes very small compared to the followers’ gift \(G_F\). So when quality is \(q_h\), even though the lead donor engages in costly signaling by increasing his gift from optimal \(\overline{G}^S_L(q_h)\) to \(\overline{G}^S_L(q_h)\), his role becomes solely to provide information to the followers and his cost of signaling becomes negligible.
relative to the total funds raised $G$. So $G^M(q_h)$ and $G^M(q_h)$ must converge. To see this consider the trivial equality:

$$\lim_{n \to \infty} G^S(q_h) - \bar{G}^S(q_h) = \lim_{n \to \infty} \bar{G}^S(q_h) + G^S(q_h, \bar{G}^S(q_h)) - \bar{G}^S(q_h) - G^S(q_h, \bar{G}^S(q_h))$$

By Eq. (A-3):

$$\lim_{n \to \infty} \bar{G}^S(q_h) - \bar{G}^S(q_h) = 0$$

Remembering from Eq. 1 and 7 that there is a one to one relationship between $G^S$ and $g^S_L$:

$$\lim_{n \to \infty} G^S(q_h) - \bar{G}^S(q_h) = 0 \quad (A-4)$$

Also costly signaling in matching gift scheme requires that:

$$G^M(q_h) \geq \bar{G}^M(q_h)$$

From this and Eq. (A-4) and by Proposition 1 it can be seen that:

$$\lim_{n \to \infty} G^S(q_h) < \lim_{n \to \infty} G^S(q_h)$$

So, there exists some $\pi$ such that when $n \geq \pi$:

$$\bar{G}^M(q_h) > \bar{G}^S(q_h)$$

This completes the proof. ■

Proof of Lemma 3

The proof follows the same pattern as the proof of Lemma 2. Given a scheme $Z$ chosen by the charity, in a separating equilibrium, a leader that has observed $q_l$ knowing that follower infers $q_l$ after observing the leader’s donation will solve her unconstrained maximization problem as in Eq. (13) so:

$$\tilde{G}^Z(q_l) = \bar{G}^Z(q_l), \quad \mu_F(\tilde{G}^Z(q_l)) = 0 \quad (A-5)$$

The same method as in the proof of Lemma 2 can be used to show that there exists a least costly credible signal of $q_U$ and $q_h$, $\tilde{G}^Z(q_U)$ and $\tilde{G}^Z(q_h)$ respectively, that each satisfies Eq. (14) and Eq. (15) and also the following:

$$\tilde{u}_L(q_U, q_l, \tilde{G}^Z(q_l)) < \tilde{u}_L(q_U, q_U, \tilde{G}^Z(q_U)) \quad \tilde{u}_L(q_h, q_U, \tilde{G}^Z(q_U)) < \tilde{u}_L(q_h, q_h, \tilde{G}^Z(q_h)) \quad (A-6)$$
Imposing the intuitive criterion requires:
\[ \forall G^Z \geq \tilde{G}^Z(q_h) \quad \mu_F(G^Z) = 1 \quad \forall G^Z \in [\tilde{G}^Z(q_U), \tilde{G}^Z(q_h)) \quad \mu_F(G^Z) = \mu_L(Z) \]

Given a leader’s (of type \( q_U \) or \( q_h \)) constrained maximization problem, and concentrating on cases where signaling is costly, the leader’s choice will be unique:
\[ \tilde{G}^Z(q_h) > \bar{G}^Z(q_h) \quad \tilde{G}^Z(q_U) > \bar{G}^Z(q_U) \]

Furthermore, going through the same process as in proof of Lemma 2 it can be shown that no pooling equilibrium exists. This completes the proof. ■

**Proof of Lemma 4**

Manipulating Eq. (16) gives:
\[ \Delta^Z(\mu_L(Z)) = \mu_L(Z)(\nabla_L(q_h) - \nabla_L(q_U(Z))) + (1 - \mu_L(Z))(\nabla_L(q_U - \nabla_L(q_U(Z))) \]

Plugging in for \( \nabla_L(q_U(Z)) \) and \( \nabla_L(q_i) \) from Eq. (14) and Eq. (15) and simplifying gives:
\[ \Delta^Z(\mu_L(Z)) = \mu_L(Z)[(q_h - q_U(Z))\nabla(\tilde{G}^Z(q_h))] - (1 - \mu_L(Z)){(q_U(Z) - q_i)\nabla(\tilde{G}^Z(q_U(Z)))} \]
\[ = \mu_L(Z)(1 - \mu_L(Z))(q_h - q_i)[\nabla(\tilde{G}^Z(q_h)) - \nabla(\tilde{G}^Z(q_U(Z))] \]

It is evident that \( \Delta^Z(\mu_L(Z)) \) is continuous in \( \mu_L(Z) \). It is also obvious that \( \Delta^Z(\mu_L(Z)) = 0 \) for \( \mu_L(Z) \in \{0, 1\} \) and \( \Delta^Z(\mu_L(Z)) > 0 \) for all \( \mu_L(Z) \in (0, 1) \). The proof is complete. ■

**Lemma A-6 (Extension of Proposition 2)** There exists \( \bar{n} \geq 1 \) such that if \( n \geq \bar{n} \), then \( \tilde{G}^M(q_h) > \bar{G}^S(q_h) \) and also if additionally \( q_U(M) \geq q_U(S) \), then \( \tilde{G}^M(q_U(M)) > \bar{G}^S(q_U(S)) \).

**Proof**

By comparing Eq. (10) and Eq. (11) to Eq. (13) and Eq. (14), it can be seen that if \( q_U(M) = q_U(S) \) the problem for an uninformed leader will become identical to the problem of a high quality leader in Proposition 2. So by the same logic there exists some \( \bar{n} \) s.t. when \( n \geq \bar{n} \):
\[ q_U(M) = q_U(S) \Rightarrow \tilde{G}^M(q_U(M)) > \bar{G}^S(q_U(S)) \quad \text{(A-7)} \]

Let’s now consider the followers’ response to change in quality. Eq. (4) holds for followers’ donation in scheme 5. Taking derivative with respect to \( q \) gives:
\[ \frac{\partial G^S}{\partial q} = -n^5h''^{-1}(qv'(G^S))v'(G^S) = -\frac{n^5v'(G^S)}{h''(h'^{-1}(qv'(G^S)))} \quad \text{(A-8)} \]
Eq. (6) holds for followers’ donation in scheme $M$. Taking derivative with respect to $q$ gives:

$$\frac{\partial G^M_F}{\partial q} = -n^M h'^{-1}(q\nu'(G^M)\frac{G^M}{G^M_F})[\nu'(G^M)\frac{G^M}{G^M_F} - \frac{q\nu'(G^M)G^M M(\frac{\partial G^M}{\partial q})}{G^M^2}]$$

$$= \frac{-n^M[\nu'(G^M)\frac{G^M}{G^M_F} - \frac{q\nu'(G^M)G^M M(\frac{\partial G^M}{\partial q})}{G^M^2}]}{h''(h'^{-1}(q\nu'(G^M)\frac{G^M}{G^M_F})))}$$

Rearranging and solving for $\frac{\partial G^M_F}{\partial q}$ gives:

$$\frac{\partial G^M_F}{\partial q} = \frac{-\nu'(G^M)\frac{G^M}{G^M_F}}{n^M h'^{-1}(q\nu'(G^M)\frac{G^M}{G^M_F})} - \frac{q\nu'(G^M)G^M M(\frac{\partial G^M}{\partial q})}{G^M^2} \tag{A-9}$$

Now let’s see how the total funds raised change when quality changes. The least costly signal of $q_U(Z), \tilde{G}^Z(q_U(Z))$, has to satisfy Eq. (11). For a given level of $q_l$ deriving both sides with respect to $q_U(Z)$ gives:

$$h'(w_L - \tilde{G}^Z(q_U) + G^Z(q_U, \tilde{G}^Z(q_U)))\frac{\partial \tilde{G}^Z(q_U)}{\partial q_U} - \frac{\partial G^Z(q_U, \tilde{G}^Z(q_U))}{\partial q_U} - \frac{\partial G^Z(q_U, \tilde{G}^Z(q_U))}{\partial \tilde{G}^Z(q_U)} \frac{\partial \tilde{G}^Z(q_U)}{\partial q_U}$$

Solving for $\frac{\partial \tilde{G}^Z(q_U(Z))}{\partial q_U(Z)}$ gives:

$$\frac{\partial \tilde{G}^Z(q_U(Z))}{\partial q_U(Z)} = \frac{-h'(w_L - \tilde{G}^Z(q_U) + G^Z(q_U, \tilde{G}^Z(q_U)))\left(\frac{\partial \tilde{G}^Z(q_U, \tilde{G}^Z(q_U))}{\partial q_U(Z)}\right)}{q\nu'(\tilde{G}^Z(q_U)) + h'(w_L - \tilde{G}^Z(q_U) + G^Z(q_U, \tilde{G}^Z(q_U)))\left(\frac{\partial \tilde{G}^Z(q_U, \tilde{G}^Z(q_U))}{\partial \tilde{G}^Z(q_U)} - 1\right)} \tag{A-10}$$

By Lemma 1 $\frac{\partial \tilde{G}^Z}{\partial \tilde{G}^Z} - 1 < 0$ and from Eq. (A-8) and Eq. (A-9) it can be seen that $\frac{\partial \tilde{G}^Z}{\partial q} > 0$. So from Eq. (A-10):

$$\frac{\partial \tilde{G}^Z(q_U)}{\partial q_U} > 0$$

Based on this result, Inequality (A-7) can be extended to the following: there exists some $\tilde{n}$ s.t. when $n \geq \tilde{n}$:

$$q_U(M) \geq q_U(S) \Rightarrow \tilde{G}^M(q_U(M)) > \tilde{G}^S(q_U(S))$$
Let’s plug in for \( \frac{\partial \widetilde{G}^S}{\partial q_U} \) from Eq. (A-1) and for \( \frac{\partial \widetilde{G}^S}{\partial q_U(S)} \) from Eq. (A-8), then simplify and take the limit:

\[
\lim_{n \to \infty} \frac{\partial \widetilde{G}^S(q_U)}{\partial q_U} = \lim_{n \to \infty} \frac{-h'(w_L - \widetilde{G}^S(q_U) + G_F^S(q_U, \widetilde{G}^S(q_U))) \left( \frac{-v'(\widetilde{G}^S(q_U))}{h'(h^{-1}(q_U v'(\widetilde{G}^S(q_U)))} \right)}{q_U(S) v''(\widetilde{G}^S(q_U(S)))}
\]

By comparing Eq. (14) to Eq. (15) it can be seen that similar process yields the following:

\[
\lim_{n^S \to \infty} \frac{\partial \widetilde{G}^S(q_h)}{\partial q_h} = \frac{-v'(\widetilde{G}^S(q_h))}{q_h v''(\widetilde{G}^S(q_h))}
\]

The last two asymptotic derivatives do not depend on \( q_L \) and \( q_U(Z) \) respectively. So \( \widetilde{G}^S() \) is a smooth function and as a result:

\[
\lim_{n^S \to \infty} \widetilde{G}^S(q_h) \to \widetilde{G}^S(q_h)
\]

(A-11)

Also by using similar methods as in Lemma 3 and noting to the single crossing property of \( u() \), from Eq. (15) it can be shown that:

\[
\overline{u}_L(q_l, q_h, \widetilde{G}^M(q_h)) < \overline{u}_L(q_l, q_h, \widetilde{G}^M(q_l))
\]

By comparing this to Eq. (11) it can be seen that:

\[
\widetilde{G}^M(q_h) \geq \overline{G}^M(q_h)
\]

From this result and Eq. (A-11) and by Proposition 2 it can be seen that:

\[
\lim_{n \to \infty} \widetilde{G}^S(q_h) < \lim_{n \to \infty} \widetilde{G}^M(q_h)
\]

So, there exists some \( \tilde{n} \) such that when \( n \geq \tilde{n} \):

\[
\widetilde{G}^M(q_h) > \widetilde{G}^S(q_h)
\]

This completes the proof. ■

**Proof of Proposition 3**
Since the leader is always uninformed in equilibrium, for all \( Z \) on the equilibrium path, \( q_L = q_U(Z) \) with probability 1. So:

\[
\forall Z, E(\tilde{G}^Z) = \tilde{G}^Z(q_U(Z)) \tag{A-12}
\]

If only one scheme \( S \) or \( M \) is on the equilibrium path, then the proof is trivial. If both schemes are on the equilibrium path then for some type \( q_1 \in \{q_l, q_h\} \), \( \tilde{Z}(q_1) = S \) with probability \( \beta^S(q_1) > 0 \). This requires:

\[
E(\tilde{G}^S|q = q_1) \geq E(\tilde{G}^M|q = q_1)
\]

By Eq. (A-12):

\[
\tilde{G}^S(q_U(S)) \geq \tilde{G}^M(q_U(M)) \tag{A-13}
\]

There are possibilities: \( \beta^S(q_1) < 1 \) or \( \beta^S(q_1) = 1 \).

**Case 1:** If \( \beta^S(q_1) = 1 \) then for scheme \( M \) to be on the equilibrium path it must be that for the other type \( q_2 \neq q_1 \), \( \tilde{Z}(q_2) = M \) with probability \( \beta^M(q_2) > 0 \). This requires:

\[
E(\tilde{G}^M|q = q_2) \geq E(\tilde{G}^S|q = q_2)
\]

By Eq. (A-12):

\[
\tilde{G}^M(q_U(M)) \geq \tilde{G}^S(q_U(S))
\]

And by Eq. (A-12) and Eq.(A-13):

\[
E(\tilde{G}^M) = E(\tilde{G}^S)
\]

**Case 2:** If \( \beta^S(q_1) < 1 \) then type \( q_1 \) is mixing both schemes or \( \tilde{Z}(q_1) = M \) with probability \( 1 - \beta^S(q_1) > 0 \). This requires:

\[
E(\tilde{G}^M|q = q_1) = E(\tilde{G}^S|q = q_1)
\]

And by Eq. (A-12):

\[
E(\tilde{G}^M) = E(\tilde{G}^S)
\]

This completes the proof. □

**Proof of Proposition 4**

Since the leader is always informed in equilibrium, for all \( Z \) on the equilibrium path, \( q_L = q \) with probability 1. So:

\[
\forall Z, E(\tilde{G}^Z|q) = \tilde{G}^Z(q) \tag{A-14}
\]
By Condition 1 and from Lemma A-6 for all \( q \):

\[
\bar{G}^M(q_U(M)) \geq \bar{G}^M(q_l) > \bar{G}^S(q_l)
\]

If \( S \) is on the equilibrium path then if leader’s choice in \( M \) is \( D(M) = I \) with any probability \( \alpha(M) \) \(^{12}\) the previous equation Combined by Eq. (A-14) gives:

\[
E(\bar{G}^S|q_l) = \bar{G}^S(q_l) < \alpha(M)\bar{G}^M(q_l) + (1-\alpha(M))\bar{G}^M(q_U(M)) = E(\bar{G}^M|q_L)
\]

So a low quality charity strictly prefers matching gift to seed money and \( \bar{Z}(q_L) = M \). As a result if \( S \) is on the equilibrium path, it must have been chosen only by type \( q_h \). Then the consistent belief is: \( \mu_L(S) = 1 \). But by Lemma 4:

\[
\Delta^Z(\mu_L(S)) = \Delta^Z(1) = 0 < k
\]

This gives the leader an incentive to deviate to \( D = U \) in \( S \). So \( S \) cannot be on the equilibrium path or at equilibrium it must be that \( \bar{Z}(q) = M \) with probability 1 for all \( q \). This means that such equilibrium can only exit in a unique form: pooling on matching gift.

Furthermore since both types are choosing \( M \) for sure, consistency of beliefs on the equilibrium path requires:

\[
\mu_L(M) = \pi_h
\]

For the leader to choose \( D = I \) in equilibrium, the value of information must be weakly higher than its cost as a necessary condition:

\[
k \leq \Delta^M(\mu_L(M)) = \Delta^M(\pi_h)
\]

Now, let’s assume \( k \leq \Delta^M(\pi_h) \) and consider the equilibrium where \( \bar{Z}(q) = M \) for all \( q \) and \( \bar{D}(M) = I \) all with probability 1 and also the off equilibrium path belief is \( \mu_L(S) = 0 \). It is then easy to see that since both types are pooling on \( M \), constant belief is \( \mu_L(M) = \pi_h \) and so \( \Delta^M(\mu_L(M)) \geq k \). So the leader will not have an incentive to deviate.

By Lemma 4:

\[
\Delta^Z(\mu_L(S)) = \Delta^Z(0) = 0 < k
\]

So the lead donor never verifies \( q \) in scheme \( S \):

\[
E(\bar{G}^S|q) = \bar{G}^Z(q_L)
\]

\(^{12}\)Since \( M \) may be off equilibrium path it is not necessarily the case that \( \alpha(M) = 1 \).
Also since the leader always verifies \( q \) in scheme \( M \) by Eq. (A-14) and by Lemma A-6, for any charity type \( q \):

\[
E(\tilde{G}^M|q) = \tilde{G}^M(q) > \tilde{G}^S(q) \geq \tilde{G}^S(q_l) = E(\tilde{G}^S|q)
\]

So no charity type has an incentive to deviate. So \( k \leq \Delta^M(\pi_h) \) is a sufficient condition for the existence of an informed equilibrium.

Finally from Lemma A-6, for all \( q \), \( \tilde{G}^M(q) \geq G^M(q) \). This completes the proof. ■

**Proof of Lemma 5**

Let’s consider the following SPI equilibrium:

\[
\tilde{Z}(q_h) = S, \quad \tilde{Z}(q_l) = \begin{cases} S & \text{with prob. } \beta^S(q_l) \\ M & \text{with prob. } 1 - \beta^S(q_l) \end{cases} \quad \text{with } 0 < \beta^S(q_l) \leq 1
\]

and also:

\[
D(S) = I \text{ with probability } 0 < a(S) < 1
\]

By Lemma 4:

\[
\Delta^S(1) = 0 < k \leq \Delta^S(\pi_h)
\]

By Lemma 4 \( \Delta^S() \) is a continuous function, so there exists a root in the range \((0,1]\) to the equation:

\[
k = \Delta^S\left(\frac{\pi_h}{\pi_h + (1 - \pi_h)x}\right)
\]

Let’s set \( \beta^S(q_l) = x \). Consistency of beliefs requires:

\[
\Delta^S(\mu_L(S)) = \Delta^S\left(\frac{\pi_h}{\pi_h + (1 - \pi_h)\beta^S(q_l)}\right) = k \quad (A-15)
\]

Also the consistent belief is \( \mu_L(M) = 0 \) and by Lemma 4:

\[
\Delta^M(\mu_L(M)) = \Delta^Z(0) = 0 < k \quad (A-16)
\]

So the leader is always uninformed in \( M \) and so \( q_L = q_U(M) = q_l \) with probability 1. So:

\[
E(\tilde{G}^M) = \tilde{G}^M(q_l) \quad (A-17)
\]

Furthermore:

\[
\beta^S(q_l) \leq 1 \Rightarrow \mu_L(S) = \frac{\pi_h}{\pi_h + (1 - \pi_h)\beta^S(q_l)} > \pi_h \Rightarrow q_U(S) \geq q_U(\pi)
\]
So by Condition 1 and Lemma A-6 and the Proposition assumption:

\[
\frac{\partial \tilde{G}^Z(q_U)}{\partial q_U} > 0 \Rightarrow \tilde{G}^S(q_U(S)) \geq \tilde{G}^S(q_U(\pi)) > \tilde{G}^M(q_l) > \tilde{G}^S(q_l)
\]

So there exists \(a(S) \in (0, 1)\) such that:

\[
a(S)\tilde{G}^S(q_l) + (1 - a(S))\tilde{G}^S(q_U(S)) = \tilde{G}^M(q_l)
\]

(A-18)

By Eq. (A-15) and Eq. (A-16). The leader has no incentive to deviate. Furthermore a low quality charity has no incentive to deviate since by Eq. (A-17) and Eq. (A-18):

\[
E(\tilde{G}^S|q = q_l) = E(\tilde{G}^M|q = q_l)
\]

Also a high quality charity has no incentive to deviate since by Eq. (A-17) and Eq. (A-18):

\[
E(\tilde{G}^S|q = q_h) = a(S)\tilde{G}^S(q_h) + (1 - a(S))\tilde{G}^S(q_U(S)) > a(S)\tilde{G}^S(q_l) + (1 - a(S))\tilde{G}^S(q_U(S)) = E(\tilde{G}^M|q = q_h)
\]

So the Proposition conditions are sufficient for an SPI equilibrium to exist. Moreover, to see how the 3 properties hold in any SPI equilibrium, one should note that in every SPI equilibrium for some scheme \(Z, a(Z) \in (0, 1)\). Let’s check two possible cases.

**Case 1:** If \(Z = S\), property 2 is trivial. Also since \(S\) is on the equilibrium path if by contradiction for some type \(\beta^S(q_j) = 0\), then consistency of beliefs requires \(\mu_L(S) \in \{0, 1\}\) so by Lemma 4:

\[
\Delta^S(\mu_L(S)) = 0 < k
\]

So the leader will have an incentive to deviate to \(a(S) = 0\). So property 1 holds. Finally if by contradiction \(a(M) = 1\) then the leader is always informed in \(M\) and so \(q_L = q\) with probability 1. So:

\[
E(\tilde{G}^M|q = q_h) = \tilde{G}^M(q_h)
\]

So by Condition 1 and Lemma A-6

\[
E(\tilde{G}^M|q = q_h) > a(S)\tilde{G}^S(q_h) + (1 - a(S))\tilde{G}^S(q_U(S)) = E(\tilde{G}^S|q = q_h)
\]

So a charity of type \(q_h\) will have an incentive to deviate. So Property 3 holds.

**Case 2:** If \(Z = M\), property 3 is trivial. If by contradiction \(\beta^S(q_h) = 0\), since \(S\) is on the equilibrium path \(\beta^S(q_l) > 0\) and consistency of beliefs requires \(\mu_L(S) = 0\) so by Lemma 4:

\[
\Delta^S(\mu_L(S)) = 0 < k
\]
So the leader will choose \( \alpha(S) = 0 \) and then the leader is always uninformed in \( S \) and so \( q_L = q_l \) with probability 1. So:

\[
E(\tilde{G}^S) = \tilde{G}^S(q_l)
\]

So by Condition 1 and Lemma A-6

\[
E(\tilde{G}^S|q = q_l) < \alpha(M)\tilde{G}^M(q_l) + (1 - \alpha(M))\tilde{G}^S(q_U(M)) = E(\tilde{G}^M|q = q_l)
\]

So a charity of type \( q_l \) will have an incentive to deviate. So it has to be that \( \beta^S(q_h) > 0 \). If by contradiction \( \beta^S(q_l) = 0 \), since \( S \) is on the equilibrium path \( \beta^S(q_h) > 0 \) and consistency of beliefs requires \( \mu_L(S) = 1 \) so by Lemma 4:

\[
\Delta^S(\mu_L(S)) = 1 < k
\]

So the leader will choose \( \alpha(S) = 0 \) and then the leader is always uninformed in \( S \) and so \( q_L = q_h \) with probability 1. So:

\[
E(\tilde{G}^S) = \tilde{G}^S(q_h)
\]

For a high quality charity to choose \( S \) with positive probability it must be that:

\[
E(\tilde{G}^S) \geq E(\tilde{G}^M|q = q_h) = \alpha(M)\tilde{G}^M(q_h) + (1 - \alpha(M))\tilde{G}^S(q_U(M))
\]

Then for a low quality charity:

\[
E(\tilde{G}^M|q = q_l) = \alpha(M)\tilde{G}^M(q_l) + (1 - \alpha(M))\tilde{G}^S(q_U(M)) < E(\tilde{G}^M|q = q_h) \leq E(\tilde{G}^S)
\]

But this means that a charity of type \( q_l \) prefers \( S \) over \( M \) and it contradicts \( \beta^S(q_l) = 0 \). So it has to be that \( \beta^S(q_l) > 0 \). So property 1 holds. Finally, if by contradiction \( \alpha(S) = 1 \) then the leader is always informed in \( S \) and so \( q_L = q \) with probability 1. So:

\[
E(\tilde{G}^S|q = q_l) = \tilde{G}^S(q_l)
\]

So by Condition 1 and Lemma A-6

\[
E(\tilde{G}^S|q = q_l) < \alpha(M)\tilde{G}^M(q_l) + (1 - \alpha(M))\tilde{G}^M(q_U(M)) = E(\tilde{G}^M|q = q_l)
\]

But this means that a charity of type \( q_l \) prefers \( M \) over \( S \) and it contradicts Property 1. So Property 2 holds. This completes the proof. ■

**Proof of Proposition 5**

In every SPI equilibrium, seed money is a on the equilibrium path but matching may not be on the equilibrium path.
**Case 1:** $M$ is on the equilibrium path. If by contradiction $\beta^S(q_l) = 1$, since $M$ is on the equilibrium path $\beta^S(q_h) > 0$ and consistency of beliefs requires $\mu_L(M) = 1$ so by Lemma 4:

$$\Delta^S(\mu_L(M)) = 0 < k$$

So the leader will choose $\alpha(M) = 0$ and then the leader is always uninformed in $M$ and so $q_L = q_h$ with probability 1. So:

$$E(\tilde{G}^M) = \tilde{G}^M(q_h)$$

So by Condition 1 and Lemma A-6 for all $q$

$$E(\tilde{G}^M|q) > \alpha(S)\tilde{G}^S(q) + (1-\alpha(S))\tilde{G}^S(q_{U}(S)) = E(\tilde{G}^S|q)$$

But this means that any charity prefers $M$ over $S$. This contradicts $S$ being on the equilibrium path. So by Property 1 of Lemma 5, $0 < \beta^S(q_l) < 1$. So type $q_l$ must be indifferent between $S$ and $M$:

$$E(\tilde{G}^M|q = q_l) = E(\tilde{G}^S|q = q_l)$$

$$\Rightarrow \alpha(M)\tilde{G}^M(q_l) + (1-\alpha(M))\tilde{G}^M(q_{U}(M)) = \alpha(S)\tilde{G}^S(q_l) + (1-\alpha(S))\tilde{G}^S(q_{U}(S)) \quad (A-19)$$

**Case 2:** $M$ is off the equilibrium path. Type $q_l$ must weakly prefer $S$ over $M$:

$$E(\tilde{G}^M|q = q_l) \leq E(\tilde{G}^S|q = q_l)$$

$$\Rightarrow \alpha(M)\tilde{G}^M(q_l) + (1-\alpha(M))\tilde{G}^M(q_{U}(M)) \leq \alpha(S)\tilde{G}^S(q_l) + (1-\alpha(S))\tilde{G}^S(q_{U}(S)) \quad (A-20)$$

Let’s assume that by contradiction that $\mu_L(S) \leq \mu_L(M)$. Then by Condition 1 and Lemma A-6 for any $q$:

$$\tilde{G}^M(q) > \tilde{G}^S(q) \quad (A-21)$$

Comparing this with Eq. (A-19) or Eq. (A-20) reveals that the probability of leader’s information acquisition must be strictly higher in matching gift compared to seed money:

$$\alpha(S) < \alpha(M)$$

But then this combined with Eq. (A-21) reveals

$$\alpha(M)\tilde{G}^M(q_h) + (1-\alpha(M))\tilde{G}^M(q_{U}(M)) > \alpha(S)\tilde{G}^S(q_h) + (1-\alpha(S))\tilde{G}^S(q_{U}(S))$$

$$\Rightarrow E(\tilde{G}^M|q = q_h) > E(\tilde{G}^S|q = q_h)$$
But this means that a charity of type $q_h$ prefers $M$ over $S$. This contradicts Property 1 of Lemma 5 for type $q_h$. So it must be that:

$$\mu_L(S) > \mu_L(M)$$

Since charity of type $q_h$ chooses scheme $S$ in equilibrium it must be that:

$$E(\tilde{G}^S|q = q_h) \geq E(\tilde{G}^M|q = q_h)$$

It is straightforward to see that for all $Z$

$$E(\tilde{G}^Z|q = q_h) \geq E(\tilde{G}^Z|q = q_l)$$

And since for some scheme $Z$ in SPI equilibrium, $\alpha(Z) > 0$ then for that $Z$

$$E(\tilde{G}^Z|q = q_h) > E(\tilde{G}^Z|q = q_l)$$

From the last four results and Eq. (A-19) and Eq. (A-20):

$$\mu_L(S)E(\tilde{G}^S|q = q_h) + (1 - \mu_L(S))E(\tilde{G}^S|q = q_l) > \mu_L(M)E(\tilde{G}^M|q = q_h) + (1 - \mu_L(M))E(\tilde{G}^M|q = q_l)$$

$$\Rightarrow E(\tilde{G}^S) > E(\tilde{G}^M)$$

The proof is complete.
References


