The Extensive Margin of Trade in the UK

Preliminary and incomplete

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Abstract

This paper investigates whether the extensive margin of trade plays a role in determining the behaviour of UK exports over the business cycle. We set up a model in which firms are heterogeneous in terms of their productivity and face fixed export costs so that exports vary on both the intensive and extensive margins. We estimate the model to UK data and find that the extensive margin plays a role in explaining UK exports behaviour. We also investigate whether the model can help us understand the behaviour of UK exports during the great trade collapse.

Keywords: Export dynamics, heterogeneous firms, extensive margin of trade, international business cycles.

JEL codes: F41; F44; E32.

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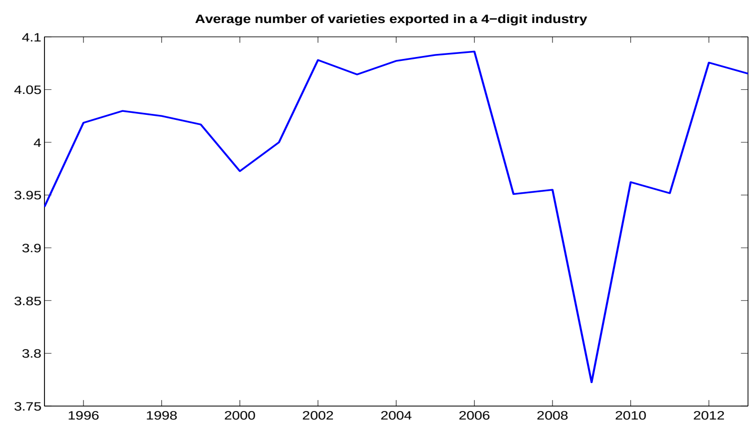
1 Introduction

Standard international business cycle models fail to replicate the observed volatility of exports.[1]. This failure became particularly visible during the recent financial crisis as trade collapsed. One possible reason for the failure of standard international business cycle models to explain the observed trade dynamics is that those models do not explicitly take into account that trade dynamics are not only driven by changes in the intensive margin but also in the extensive margin.

Recent evidence suggests that the extensive margin of trade plays a crucial role in export dynamics at business cycle frequency. In particular, Bricongne et al. (2012) find that on average between 2000 and 2007, the net extensive margin contributed to more than 50% of the growth rate of French exports. Suggestive evidence for the UK also point to the extensive margin playing an important role over the business cycle. Indeed, Figure 1 shows that the net extensive margin of UK exports, as proxied by the average number of varieties exported in each 4-digit industry, varies over the business cycle.[2] Interestingly, the fall in the average number of varieties exported during the great trade collapse was around 10%, the same amount as the fall in UK exports over that period.



Source: UN Comtrade and own calculations

In this paper, we investigate whether the extensive margin of trade plays an important role in determining UK export dynamics. We set up an international business cycle model in which firms are heterogenous in terms of their productivity and face fixed and variable exports costs as in Ghironi and Melitz (2005). Firms decide on their intensive as well as extensive margin of exports, and only the most productive firms export. Our model extends Ghironi and Melitz (2005) on several dimensions. First, we add nominal rigidities in the form of wage stickiness. This allows monetary policy and shocks to the exchange rate to affect the extensive margin of trade over the business cycle, alongside productivity shocks. Second, we generalize the model by allowing for unequal country sizes. We can thus estimate the model to UK data and evaluate its performance. We show that taking into account the extensive margin improves the model predictions of UK export behaviour. We also contribute to the literature sparked by a new interest in models which give insight to the causes of the 'great trade collapse'. In particular, we investigate whether a shock to financial conditions within our model with an extensive margin of trade can help explain the behaviour of UK exports during the great trade collapse.

[1]See e.g. Corsetti et al. (2008)

[2]The number of varieties has been computed using UN Comtrade data for UK exports.

The international macroeconomic literature on the extensive margin of trade mainly builds on Ghironi and Melitz (2005), as do we. Ghironi and Melitz (2005) set up an international business cycle model in which firms are heterogenous with respect to their individual levels of productivity. Firms face fixed export costs, and therefore only the most productive firms will find it profitable to export. The productivity threshold at which firms can export depends on productivity and demand conditions, such that the number of firms exporting varies with economic conditions. Within this framework, the authors analyse the implications of their model for international prices, focusing in particular on the Harrod-Balassa-Samuelson effect.
The bulk of the literature building on the framework developed by Ghironi and Melitz (2005) analyses the effects of trade policies such as changes in tarifs and other fixed costs, but has not focused on business cycle analysis. Ruhl (2008) reconciles the high trade elasticity found at the micro level with the low trade elasticty found at the macro level by showing how permanent changes in trade costs implies large changes in trade while temporary changes in productivity imply small changes in trade within a model with an extensive margin of trade; Cacciatore and Ghironi (2013) study the effects of trade integration for monetary policy.
Our paper relates to two other papers investigating the business cycle implications of the extensive margin of trade, namely Ghironi and Melitz (2005) and Devereux and Hnatkovska (2012). The former study the implications of their model for trade dynamics and business cycle correlations. The latter focuses on the fluctuations in traded and non-traded output shares. Though we have in common with those papers to focus on the business cycle implications of the extensive margin of trade, our work differs in one important way. We do not hinge our results on the occurrence of productivity shocks exclusively but consider the possibility of monetary policy shocks and exchange rate shocks alongside productivity shocks. We can thus test the predictions of our model more broadly.
In a similar spirit to Chaney (2007), we consider the implications of liquidity constraints for export dynamics. His analysis builds on the assumption that fixed exporting costs are denominated in foreign currency such that the extensive margin can play a dampening effect on export dynamics. We find no evidence for this in the data we consider, and therefore do not model the liquidity channel in that manner. Instead, we build on the evidence showed in Figure 1, which is in line with e.g. Manova (2013). Exploring a cross-country dataset, she finds that exporting companies face credit constraints and that these have an effect on the extensive margin of trade (as well as the intensive). She investigates a model consistent with those implications, but focus on the long-run implications for trade patterns rather than on the business cycle implications.

The next section presents the two-country model we use for our analysis of exports behaviour. Section 3 analyses exports behaviour over the business cycle within that model and points out what the extensive margin of trade implies for business cycle fluctuations. Section 4 investigates whether the extensive margin of trade might have played an important role in explaining UK exports behaviour. Section 5 investigates whether a financial shock can explain the fall in UK exports during the great trade collapse.

2 The Model

The model we present here closely follows Ghironi and Melitz (2005), but expands that model in several ways. It is a two country general equilibrium model in which firms are heterogeneous with respect to their relative productivity levels. Firms face fixed and variable costs of exporting so only more productive firms are able to export. Financial markets are incomplete at the international level. Different from Ghironi and Melitz (2005), we introduce nominal rigidities in labour markets such that there is a role for monetary policy in our framework. In particular, wages are staggered a la Calvo (1983) in both countries. And monetary policy is conducted according to a Taylor type rule. We also allow the Home and Foreign country sizes to differ; both countries are populated by a mass of infinitely lived households with a fraction of n and $1-n$ of the world total, respectively. We will denote the foreign country variables with an asterisk (*).

2.1 Households

The representative home household's lifetime utility function can be expressed as a function of consumption (C_t) and labour (L_t):

$$U_t = \mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \kappa \frac{L_t^{1+\eta}}{1+\eta} \right] \tag{1}$$

where E_t denotes the expectations at time t, β is the discount factor. $1/\gamma$ is the intertemporal elasticity of substitution and the parameter η is the inverse of the Frisch elasticity of labour supply.

Consumers can consume differentiated goods (ω) defined over a continuum set of goods, Ω:

$$C_t = \left[\int_{\omega \epsilon \Omega} c_t(\omega)^{\frac{\theta-1}{\theta}} d\omega \right]^{\frac{\theta}{\theta-1}}$$

such that θ denotes the elasticity of substitution between the differentiated goods. Each period only a subset of varieties are actually available for consumption and that subset is allowed to be different across countries and vary over time.

The corresponding consumer price index for the Home economy is:

$$P_t = \left[\int_{\omega \in \Omega} p_t(\omega)^{1-\theta} d\omega \right]^{\frac{1}{1-\theta}}$$

The optimal allocation of nominal expenditure of the representative household in the Home country for each differentiated good ω yields the following demand function:

$$c_t(\omega) = \left(\frac{p_t(\omega)}{P_t} \right)^{-\theta} C_t$$

2.2 Firms

A continuum of monopolistically competitive firms in each country produce differentiated goods $\omega \in \Omega$ with labour being the only production input. To start producing i.e. to enter the market, firms need to pay a sunk cost ($f_{E,t}$) in the form of a labour requirement which is equal to $w_t f_{E,t}/Z_t$ in real terms. w_t is the real wage and Z_t is the aggregate productivity in the Home country. Once a firm has entered the market, it draws its productivity from a common distribution $G(z)$ where $z \in [z_{min}, \infty)$, so firms are heterogeneous with respect to their productivities. This productivity does not change over the life of the firm. A firm produces in every period until it is hit by an exogenous "death" shock. In every period, firms face this death shock with probability $\delta \in (0, 1)$, independently of their relative productivity.

Once firms enter, they produce and sell in the domestic market. Firms can also export, and every period firms decide whether they'll do so. To export in a given period, firms need to pay a fixed export cost ($f_{X,t}$) in effective labour units as well as an iceberg cost, τ. Firms decide to export if they extract positive profits from exporting, and this depends on their relative productivity and demand conditions. Only the most productive firms - who can afford this fixed cost as well as the variable iceberg cost - will export.

Each firm produces one variety ω with associated productivity level z and maximises profits subject to a downward sloping demand curve. Profits obtained by selling to the domestic (Π_t^D) and the export market (Π_t^X) repsectively are:

$$\Pi_t^D(\omega) = \frac{p_{D,t}(\omega)}{P_t} y_t(\omega) - w_t l_t(\omega) = \frac{p_{D,t}(\omega)}{P_t} y_t(\omega) - \frac{w_t}{Z_t\, z(\omega)} y_t(\omega)$$

$$\Pi_t^X(\omega) = Q_t \frac{p_{X,t}(\omega)}{P_t^*} y_t(\omega) - \tau_t w_t l_t(\omega) - \frac{w_t f_{X,t}}{Z_t} = Q_t \frac{p_{X,t}(\omega)}{P_t^*} y_t(\omega) - \frac{\tau_t w_t}{Z_t\, z(\omega)} y_t(\omega) - \frac{w_t f_{X,t}}{Z_t}$$

where $y_t(\omega)$ denotes production of variety ω, $l_t(\omega)$ denotes the amount of labour required to produce it, and w_t the wage. Z_t is the country-specific productivity level, and $z(\omega)$ denotes the firm-specific productivity level. $p_{D,t}(\omega)$ denotes the price charged in the domestic market for variety ω, and $p_{X,t}(\omega)$ denotes the corresponding price charged in the export market.

Because each firm produces a single variety associated with a level of productivity z, such that z is a good summary statistics for a firm, we index variables by z in the rest of the paper rather than by ω. Prices are flexible, and therefore set as a mark-up over marginal costs. The firm producing variety ω with associated productivity level z sets the following prices:

$$\rho_{D,t}(z) \equiv \frac{p_{D,t}(z)}{P_t} = \frac{\theta}{(\theta-1)} \frac{w_t}{Z_t\, z}, \qquad \rho_{X,t}(z) \equiv \frac{p_{X,t}(z)}{P_t^*} = \frac{\theta}{(\theta-1)} \frac{\tau_t\, w_t}{Q_t\, Z_t\, z} \tag{2}$$

where $Q_t = \frac{S_t\, P_t^*}{P_t}$ is the real exchange rate (RER) and S_t is the nominal exchange rate defined as the home currency price of buying one unit of foreign currency.

By plugging in the prices and the demand functions, and defining $\rho_{D,t}(z) \equiv \frac{p_{D,t}(z)}{P_t}$ and $\rho_{X,t}(z) \equiv \frac{p_{X,t}(z)}{P_t^*}$, we can express the profits as:

$$\Pi_t^D(z) = \frac{(\rho_{D,t}(z))^{(1-\theta)}\, C_t}{\theta}, \qquad \Pi_t^X(z) = \left(\frac{1-n}{n}\right) \frac{Q_t(\rho_{X,t}(z))^{(1-\theta)}\, C_t^*}{\theta} - \frac{w_t f_{X,t}}{Z_t} \tag{3}$$

All firms have the choice to export, but only those with a relative productivity z above a cutoff level $z_{X,t} = inf\{z : \Pi_t^X(z) > 0\}$ ensuring non-negative profits from exporting, will do so. The exporting firms sell their goods both in local and foreign markets. So, while all firms can export, a firm with productivity between z_{min} and the export cutoff level, $z_{X,t}$, will decide to serve only the local market.[3] The export productivity cut-off level $z_{X,t}$ varies with economic conditions, and therefore so does the number of exporting firms. The size of the non-traded sector is thus determined endogenously.

In every period there is a mass $N_{D,t}$ of firms in the home country and $N_{X,t} = [1 - G(z_{X,t})]N_{D,t}$ of

[3]The lower bound for idiosyncratic productivity, z_{min}, is below $z_{X,t}$.

them can also export.

2.3 Firm Averages

Average productivity for all domestic firms and for only exporting firms respectively are:

$$\tilde{z}_D \equiv \left[\int_{z_{min}}^{\infty} z^{\theta-1} dG(z)\right]^{\frac{1}{\theta-1}}, \qquad \tilde{z}_X \equiv \left[\frac{1}{1-G(z_{X,t})}\int_{z_{X,t}}^{\infty} z^{\theta-1} dG(z)\right]^{\frac{1}{\theta-1}} \tag{4}$$

$N_{D,t}$ local firms will have an average nominal price of $\tilde{p}_{D,t} = p_{D,t}(\tilde{z}_D)$ and $N_{X,t}$ exporters will charge $\tilde{p}_{X,t} = p_{X,t}(\tilde{z}_X)$. Thus the consumer price index of the home country will be:

$$P_t = \left[\frac{N_{D,t}}{N_{D,t} + \frac{1-n}{n}N^*_{X,t}}(\tilde{p}_{D,t})^{1-\theta} + \frac{\frac{1-n}{n}N^*_{X,t}}{N_{D,t} + \frac{1-n}{n}N^*_{X,t}}(\tilde{p}^*_{X,t})^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{5}$$

Equivalently:

$$1 = \frac{N_{D,t}}{N_{D,t} + \frac{1-n}{n}N^*_{X,t}}(\tilde{\rho}_{D,t})^{1-\theta} + \frac{\frac{1-n}{n}N^*_{X,t}}{N_{D,t} + \frac{1-n}{n}N^*_{X,t}}(\tilde{\rho}^*_{X,t})^{1-\theta} \tag{6}$$

The average total profits in the Home country are:

$$\widetilde{\Pi}_t = \widetilde{\Pi}^D_t + \left[\frac{N_{X,t}}{N_{D,t}}\right]\widetilde{\Pi}^X_t = \widetilde{\Pi}^D_t + [1 - G(z_{X,t})]\widetilde{\Pi}^X_t \tag{7}$$

Following Ghironi and Melitz (2005), we assume that relative productivity is drawn from a Pareto distribution with lower bound z_{min} and shape parameter k which is higher than $\theta - 1$[4]: $G(z) = 1 - (z_{min}/z)^k$. By defining $\phi \equiv \{k/(k-(\theta-1))\}^{1/(\theta-1)}$ and integrating (4), one can obtain: $\tilde{z}_D = \phi z_{min}$ and $\tilde{z}_{X,t} = \phi z_{X,t}$. So the share of home exporting firms, the extensive margin of trade, can be written as:

$$\frac{N_{X,t}}{N_{D,t}} = [1 - G(z_{X,t})] = \left(\frac{z_{min}\phi}{z_{X,t}}\right)^k \tag{8}$$

The marginal exporting firm, with productivity $z_{X,t}$, will make zero profits: $\Pi^X_t(z_{X,t}) = 0$. Using equation (3) the zero profit condition implies:

$$\left(\frac{1-n}{n}\right)\frac{Q_t(\rho_{X,t}(z_{X,t}))^{(1-\theta)}\,C^*_t}{\theta} = \frac{w_t f_{X,t}}{Z_t}$$

Plugging in the optimal prices from equation (2) yields:

$$\frac{\theta}{\theta-1}\frac{\tau_t w_t}{Q_t Z_t z_{X,t}} = \left(\frac{n}{1-n}\right)^{1/1-\theta}\left(\frac{w_t f_{X,t}\theta}{Q_t Z_t\,C^*_t}\right)^{1/1-\theta}$$

[4]Ghironi and Melitz (2005) assumes Pareto distribution for firm productivity as this distribution fits firm level data quite well. Pareto distribution is a skewed and heavy-tailed distribution. As it is heavy-tailed for a finite mean and variance the shape parameter needs to be sufficiently high: $k > 1$ ensures a finite mean and $k > 2$ ensures a finite variance. In our case the mean firm size/sale will be finite when $k/\theta - 1 > 1$.

Solving for $z_{X,t}$ yields:

$$z_{X,t} = \left(\frac{n}{1-n}\right)^{\frac{1}{\theta-1}} \left(\frac{\tau_t}{\theta-1}\right) \left(\frac{w_t\theta}{Z_t Q_t}\right)^{\theta/\theta-1} \left(\frac{f_{X,t}}{C_t^*}\right)^{1/\theta-1}$$

The average profits of exporters will satisfy the following:

$$\widetilde{\Pi}_t^X(\tilde{z}_{X,t}) = \left(\frac{1-n}{n}\right) \frac{Q_t(\tilde{\rho}_{X,t}(\tilde{z}_{X,t}))^{(1-\theta)}\, C_t^*}{\theta} - \frac{w_t f_{X,t}}{Z_t}$$

Now we use $\tilde{z}_{X,t} = \phi z_{X,t}$ and the zero-profit condition to obtain:

$$\widetilde{\Pi}_t^X = \left(\frac{w_t f_{X,t}}{Z_t}\right)(\phi^{\theta-1} - 1)$$

Since $(\phi^{\theta-1} - 1) \equiv [k/(k-(\theta-1))]^{(\theta-1)/(\theta-1)} - 1$, we can express the above equation as:

$$\widetilde{\Pi}_t^X = (\theta-1)\left(\frac{w_t f_{X,t}}{Z_t}\right)\left(\frac{\phi^{\theta-1}}{k}\right) \tag{9}$$

This is the average profits from exports.

2.4 Free Entry

Firms enter to the market at time t and start their production at $t+1$, so some of these new entrants will die (with probability δ) before starting the production at the end of period t. The total number of firms at period t in home country will be equal to the new entrants and established firms who survived from the previous period:

$$N_{D,t} \;=\; (1-\delta)(N_{D,t-1} + N_{E,t-1}) \tag{10}$$

Households will decide to start a business by calculating the expected present discounted value of future profits which gives simply the post entry value:

$$\tilde{v}_t = \mathrm{E}_t \sum_{s=t+1}^{\infty} [\beta(1-\delta)]^{s-t} \left(\frac{C_{t+s}}{C_t}\right)^{-\gamma} \tilde{d}_s \tag{11}$$

The free entry condition implies that, firms will enter until the average firm value is equal to the entry cost; $\tilde{v}_t = w_t f_{E,t}/Z_t$.

2.5 Budget Constraint

Households finance their expenditure through labour income, holdings of risk free bonds and from the shares, (x_t), they own in the mutual fund. We assume that labour income is subsidised at a constant rate, σ. It is assumed that the international asset markets are incomplete in the sense that households are able to trade only nominal bonds. We follow Benigno (2009) in modelling the incomplete asset market structure. Households in the Home country can hold two kinds of nominal bonds; one is denominated in units of the home currency and the other is denominated in foreign currency. However, the bonds issued by the Home country are not traded internationally for simplicity. Households in the home country face an additional cost when they take a position in the foreign asset market. As discussed in Schmitt-Grohe and Uribe (2003), we thus avoid non-stationarity in the model, as the cost function $\Theta(.)$ ensures a stationary distribution of wealth across countries [5]. The budget constraint in real terms is:

$$\begin{aligned} &C_t + B_{H,t+1} + \tilde{v}_t(N_{D,t} + N_{E,t})x_{t+1} + \frac{Q_t B_{F,t+1}}{\Theta(Q_t B_{F,t+1})} = \\ &(1+r_t)B_{H,t} + (1+\sigma)w_t L_t + Q_t(1+r_t^*)B_{Ft} + (\widetilde{\Pi}_t + \tilde{v}_t)N_{D,t}x_t \end{aligned} \tag{12}$$

Households make the intertemporal decision by maximising (1) subject to (12). This yields the following Euler equations for bonds and share holdings respectively and, combined with the analogous foreign conditions, the UIP condition:

$$1 = \mathrm{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} (1 + r_{t+1}) \right] \tag{13}$$

$$\tilde{v}_t = \mathrm{E}_t \left[\beta(1-\delta) \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} (\tilde{v}_{t+1} + \widetilde{\Pi}_{t+1}) \right] \tag{14}$$

$$1 = \beta(1 + r_{t+1}^*)\Theta(Q_t B_{F,t+1})\mathrm{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{Q_{t+1}}{Q_t} \right] \tag{15}$$

The situation of foreign households is analogous.

2.6 Labour Supply and Wage Setting

Expanding on GM, we introduce nominal rigidities through labour market frictions. There is monopolistic competition among households in the labour market, in the sense that households offer differentiated labour services. As described in Erceg et al. (2000), an "employment agency" combines individual household's supply in the following Dixit-Stiglitz form:

$$L_t = \left[\int_0^1 L_t(i)^{\frac{\theta_w - 1}{\theta_w}} di \right]^{\frac{\theta_w}{\theta_w - 1}}$$

where $\theta_w > 1$ is the elasticity of substitution between labour inputs.
The aggregate nominal wage index in home country can be defined as:

$$W_t = \left[\int_0^1 W_t(i)^{1-\theta_w} di \right]^{\frac{1}{1-\theta_w}}$$

[5] In order to have a well-behaved steady state in the model, we impose the following restrictions on the cost function: $\Theta(.)$ is a differentiable decreasing function in the neighbourhood of steady state level of net foreign assets and when the net foreign assets are in the steady state level ($B_{F,t} = 0$), the cost function is equal to 1 ($\Theta(0) = 1$). See Benigno (2009) for a more detailed explanation.

The cost minimisation problem of producers gives the downward sloping labour demand curve. The total demand for household i's labour services by all firms is:

$$L_t(i) = \left[\frac{W_t(i)}{W_t}\right]^{-\theta_w} L_t \tag{16}$$

Wages are staggered á la Calvo (1983); in a given period $(1-\xi)$ of households are able to adjust their wages. To choose the optimum wage $\widetilde{W}_t(i)$, households maximise the expected lifetime utility (1) subject to the budget constraint and the labour demand curve (16). The first order condition for this nominal wage setting problem is:

$$\sum_{k=0}^{\infty}(\beta\xi)^k E_t\left[L_{t+k}(i)U_C(C_{t,t+k})\left((1+\sigma)\frac{\widetilde{W}_t(i)}{P_{t+k}} - \frac{\theta_w}{\theta_w - 1}MRS_{t,t+k}\right)\right] = 0 \tag{17}$$

where $MRS_{t,t+k}$ is the marginal rate of substitution between consumption and labour in period $t+k$ for the household resetting the wage in period t, i.e. $MRS_{t,t+k} \equiv -\dfrac{U_L(L_{t,t+k})}{U_C(C_{t,t+k})}$.

When wages are flexible ($\xi \to 0$), the real wage multiplied by the subsidy will be equal to the mark-up over the marginal rate of substitution:

$$(1+\sigma)\frac{W_t}{P_t} = \frac{\theta_w}{\theta_w - 1}MRS_{t,t+k} \tag{18}$$

In order to ensure a flexible wage equilibrium in steady state, we assume that the subsidy cancels out the monopolistic distortion in steady state, implying that $(1+\sigma) = \dfrac{\theta_w}{\theta_w - 1}$.

2.7 Monetary Policy

The monetary policy instrument is the nominal interest rate paid on bonds. We assume that monetary policy is conducted using a Taylor type rule, and that monetary policy is subject to monetary policy shocks denoted ε^m.

$$\frac{i_t}{i} = \left(\frac{i_{t-1}}{i}\right)^{\Gamma_{i-1}}\left(\pi_t^{\Gamma_{\pi_t}}\right)^{1-\Gamma_{i-1}} exp(\varepsilon_t^m) \tag{19}$$

2.8 Market Clearing and the Current Account

Aggregating across household budget constraints, shows that revenue from production (labour income and profits) and bond holdings is invested in new firms as well as used for consumption of domestic and imported goods and bond purchases:

$$w_t L_t + N_{D,t}\widetilde{\Pi}_t = C_t + N_{E,t}\tilde{v}_t + \frac{Q_t B_{F,t+1}}{\Theta(Q_t B_{F,t+1})} - Q_t(1+r_t^*)B_{Ft}$$

Labour markets will clear when labour supply is equal to labour demand - to be specified (see GM Technical appendix)!:

$$L_t = \frac{\theta-1}{w_t}N_t^d\,\widetilde{\Pi}_t^d + \frac{\theta-1}{w_t}N_t^X\,\widetilde{\Pi}_t^X + \frac{\theta}{Z_t}N_t^X f_{X,t} + \frac{1}{Z_t}N_t^E f_{E,t} \tag{20}$$

We define the current account as the change in claims on foreign agents:

$$CA_t = \frac{Q_t B_{F,t}}{(1+r_t^*)\Theta(Q_t B_{F,t})} - Q_t B_{F,t-1} \tag{21}$$

2.9 Definition of Some International Variables

The RER we have been using so far is a welfare based RER, $Q_t = \frac{S_t P_t^*}{P_t}$ which includes the variety effect. We construct the CPI based RER to be consistent with the data as the statisticians report only the weighted averages. The CPI based transformation of Q_t discounts the impact arising from the changes in the variety. This can be done by simply using the average prices $\tilde{P}_t$, $\tilde{P}_t^*$, as they are simple weighted averages and correspond to the data much closer: $P_t = N_t^{1/1-\theta}\tilde{P}_t$ and $P_t^* = (N_t^*)^{1/1-\theta}\tilde{P}_t^*$. Hence the CPI based RER will be: $\tilde{Q}_t = \frac{S_t \tilde{P}_t^*}{\tilde{P}_t}$.

Following Ghironi and Melitz (2005), we will now re-write the RER to be able understand the predictions of the model about deviations from PPP. We will use the price index equation (equation (5)) since we know that $P_t = N_t^{-1/1-\theta}\tilde{P}_t$:[6]

$$\tilde{Q}_t^{1-\theta} = \frac{(N_t^*)^{-1} S_t^{1-\theta}[N_{D,t}^*(\tilde{p}_{D,t}^*)^{1-\theta} + N_{X,t}(\tilde{p}_{X,t})^{1-\theta}]}{(N_t)^{-1}[N_{D,t}(\tilde{p}_{D,t})^{1-\theta} + N_{X,t}^*(\tilde{p}_{X,t}^*)^{1-\theta}]}$$

Plug in the optimal prices from equation (2) and their foreign counterparts:

$$\tilde{Q}_t^{1-\theta} = \frac{(N_t^*)^{-1} S_t^{1-\theta}\left[N_{D,t}^*\left(\frac{\theta}{\theta-1}\frac{W_t^*}{Z_t^*\tilde{z}_{D,t}^*}\right)^{1-\theta} + N_{X,t}\left(\frac{\theta}{\theta-1}\frac{\tau_t W_t}{S_t Z_t \tilde{z}_{X,t}}\right)^{1-\theta}\right]}{(N_t)^{-1}\left[N_{D,t}\left(\frac{\theta}{\theta-1}\frac{W_t}{Z_t\tilde{z}_{D,t}}\right)^{1-\theta} + N_{X,t}^*\left(\frac{S_t\tau_t^* W_t^*}{Z_t^*\tilde{z}_{X,t}^*}\right)^{1-\theta}\right]}$$

Note that, for this expression we are not using the consumption goods as units of measure any more, hence, we are using nominal wages now, W_t, not the real wages, w_t.

Dividing the above expression by $\tilde{p}_{D,t} = \frac{\theta}{(\theta-1)}\frac{W_t}{Z_t\tilde{z}_{D,t}}$ and simplifying a bit yields:

$$\tilde{Q}_t^{1-\theta} = \frac{(N_t^*)^{-1} S_t^{1-\theta}\left[N_{D,t}^*\left(\frac{W_t^*}{Z_t^*\tilde{z}_{D,t}^*}\right)^{1-\theta}\left(\frac{Z_t\tilde{z}_{D,t}}{W_t}\right)^{1-\theta} + N_{X,t}\left(\frac{\tau_t W_t}{S_t Z_t\tilde{z}_{X,t}}\right)^{1-\theta}\left(\frac{Z_t\tilde{z}_{D,t}}{W_t}\right)^{1-\theta}\right]}{(N_t)^{-1}\left[N_{D,t}\left(\frac{W_t}{Z_t\tilde{z}_{D,t}}\right)^{1-\theta}\left(\frac{Z_t\tilde{z}_{D,t}}{W_t}\right)^{1-\theta} + N_{X,t}^*\left(\frac{S_t\tau_t^* W_t^*}{Z_t^*\tilde{z}_{X,t}^*}\right)^{1-\theta}\left(\frac{Z_t\tilde{z}_{D,t}}{W_t}\right)^{1-\theta}\right]}$$

Note that an increase in real exchange rate, both for $\tilde{Q}$ and Q, means a depreciation.

As in Ghironi and Melitz (2005), we define *terms of labour* which measures the cost of effective labour across countries: $ToL_t \equiv S_t(W_t^*/Z_t^*)/(W_t/Z_t)$. We now re-write what we obtained previously:

$$\tilde{Q}_t^{1-\theta} = \frac{\frac{N_{D,t}^*}{N_t^*}(\text{ToL}_t)^{1-\theta}\left(\frac{\tilde{z}_{D,t}}{\tilde{z}_{D,t}^*}\right)^{1-\theta} + \frac{N_{X,t}}{N_t^*}\left(\frac{\tau_t\tilde{z}_{D,t}}{\tilde{z}_{X,t}}\right)^{1-\theta}}{\frac{N_{D,t}}{N_t} + \frac{N_{X,t}^*}{N_t}(\text{ToL}_t)^{1-\theta}\left(\frac{\tau_t^*\tilde{z}_{D,t}}{\tilde{z}_{X,t}^*}\right)^{1-\theta}} \tag{22}$$

[6]The total variety availabale at home is reflected by: $N_t = N_{D,t} + N_{X,t}^*$

We can also define the terms of trade, the average price of exports relative to average price of imports: $ToT_t = S_t \tilde{p}_{X,t}/\tilde{p}^*_{X,t}$. By plugging in the optimal prices we can express ToT as:

$$ToT_t = \left(\frac{\tau_t}{\tau_t^*}\right)\left(\frac{\tilde{z}^*_{X,t}}{\tilde{z}_{X,t}}\right) ToL^{-1} \tag{23}$$

An decrease in ToT implies a deterioration of terms of trade, as imports become more expensive.

3 The role of the extensive margin of trade over the business cycle

In this section, we look at the dynamics of our model following temporary shocks. We consider three type of shocks: productivity shocks, monetary policy shocks, and shocks to the exchange rate (UIP shocks). We compare our baseline model presented in the previous section to a similar model where the extensive margin of trade is constant. This allows us to understand how the extensive margin of trade changes the model dynamics in response to shocks.

For this exercise, we use the baseline parameter values reported in Table 1. We set the discount factor to 0.99 so that the steady state interest rate is 4% per year. The two countries assumed to have the equal size ($n = 1 - n = 0.5$). The value of both the coefficient of risk aversion and Frisch elasticity is chosen consistently with the calibration in DSGE models. We take the value of elasticity of substitution between differentiated labour supply from Erceg et al. (2000) meaning 13% mark-up. We assume 4 quarters for the duration of wage contracts. We set the autoregressive component of productivity shocks and the UIP shock to 0.8. The Taylor rule coefficients are calibrated in a standard way.

We follow Ghironi and Melitz (2005) for the rest of the calibration. The value chosen of elasticity of substitution parameter ($\theta = 3.8$) implies $k = 3.4$. The value of fixed export costs and iceberg costs chosen such that the proportion of exporting firms is 21%. z_{min} and f_E are normalized to 1 without loss of generality. This calibration implies that exporters on average 58% more productive than non-exporters. For the model in which extensive margin is constant; i.e. there is a constant number of exporting firms, we fix the z_x to 0.71 so that it is equal to steady state value of z_x of our benchmark model.

3.1 Productivity shocks

The impulse responses of selected variables to a temporary productivity shock are shown in Figure 1.
A temporary aggregate productivity improvement in the Home country causes the real exchange rate to appreciate through three channels. First, following a positive productivity shock in home country, the home market becomes more attractive for firms. The number of firms in the home country therefore increases ($N_D \uparrow$) - this is the home market effect. As labour demand increases, wages in the domestic market increase so that the price of home produced non-traded goods rises. The Balassa-Samuelson effect is a consequence of the endogenous entry mechanism. The increase in the price of goods imply a real exchange rate (RER) appreciation, ($RER \downarrow$) . Second, as the increase in entry pushes the wages up in the home country relative to foreign country ($ToL \downarrow$), the profitability of exporting firms decrease. Only more productive firms can export now, so that the cut-off productivity level increases at home and decrease in the foreign country, ($\tilde{z}_X \uparrow$, $\tilde{z}^*_X \downarrow$). Since more productive firms charge lower prices while

Table 1: Parameter Values

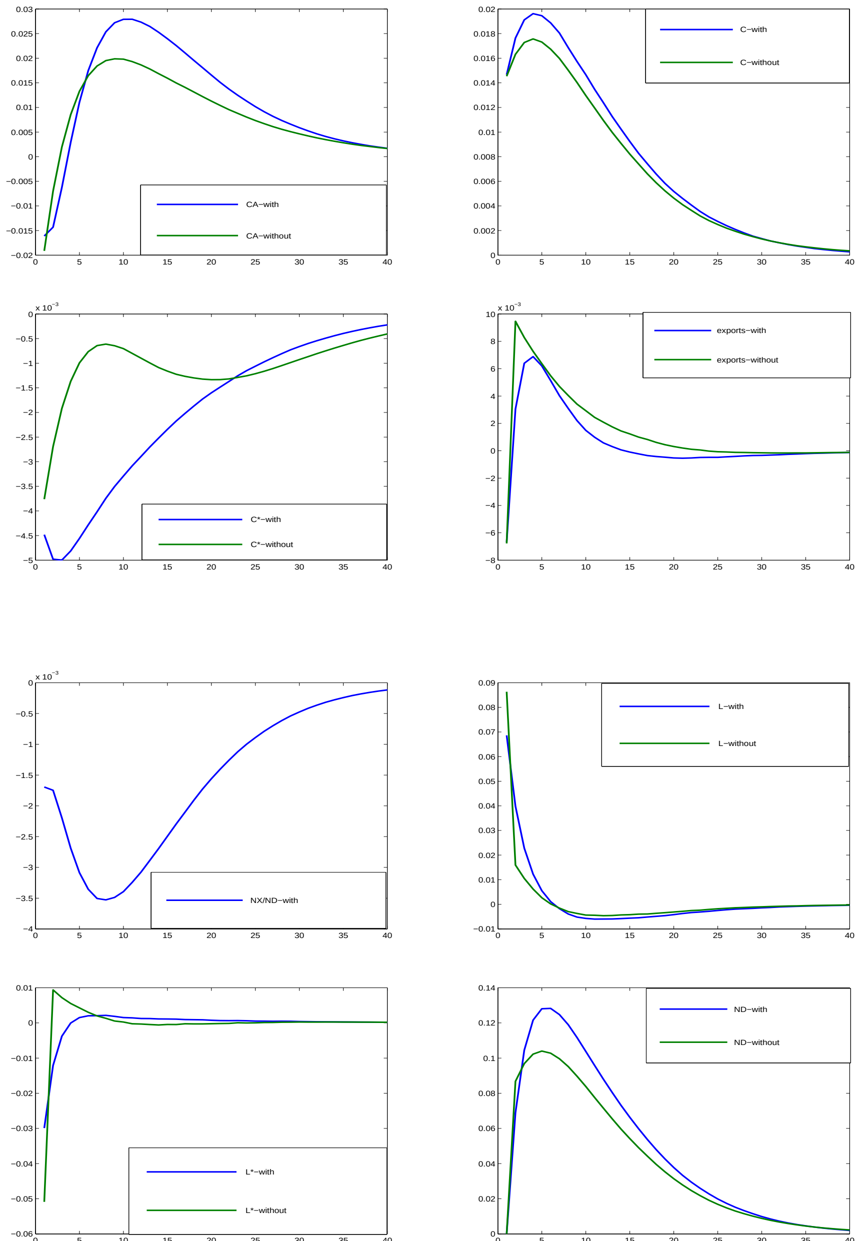
β	0.99	discount factor
n	0.5	country size
γ	2	degree of risk aversion
η	1.5	inverse Frisch elast. of labour supply
θ	3.8	elast. of subs. btw. goods, mark-up
ω	0.0025	cost of intermediation
σ_w	4	elast. of subs. across types of labour
$\xi_w = \xi_w^*$	0.75	Calvo wages
$\Gamma_i = \Gamma_i^*$	0.75	interest rate smoothing
$\Gamma_\pi = \Gamma_\pi^*$	1.5	response to inflation
δ	0.025	prob. of death
k	3.4	dispersion of distr.
$fe = fe^*$	1	entry cost
$fx = fx^*$	0.0085	fixed export cost
$\tau = \tau^*$	1.3	iceberg cost
$z_{min} = z_{min}^*$	1	lower productivity bound
$z_x = z_x^*$	0.71	share of exporters

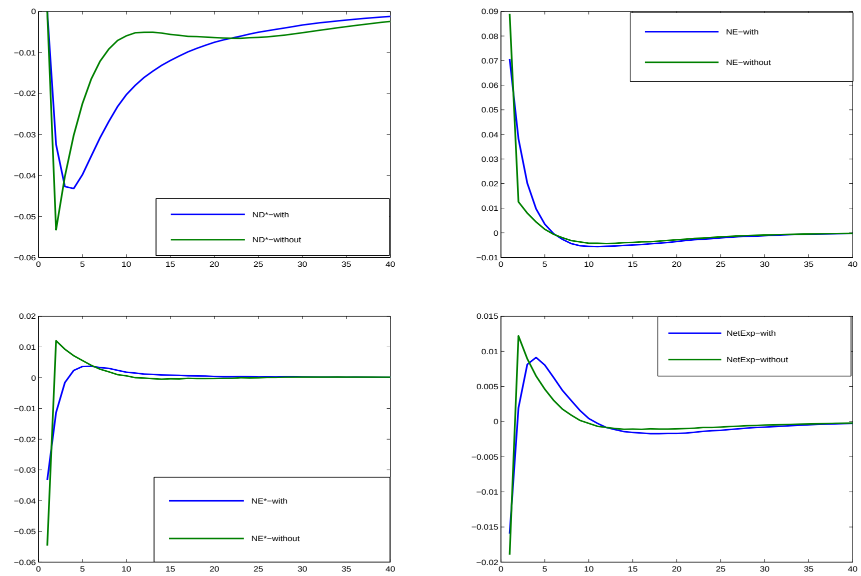
making higher profits (See equations (2) and (3)), export prices fall. Similarly, foreign exporting firms now have a lower average productivity level and will on average charge higher prices. Therefore, home import prices will increase relative to home export prices causing ther RER to decrease. Third, the increase in domestic firms corresponds to an increase in the number of varieties available to domestic consumers. As consumers love variety the demand for home produced goods will increase causing further RER appreciation. However, note that while the RER appreciates, the welfare based definition of the RER - including the variety effect - depreciates ($Q \uparrow$). This is because consumers experience an increase in utility from consuming a wider range of home produced goods which dominates the decrease in utility resulting from higher prices.

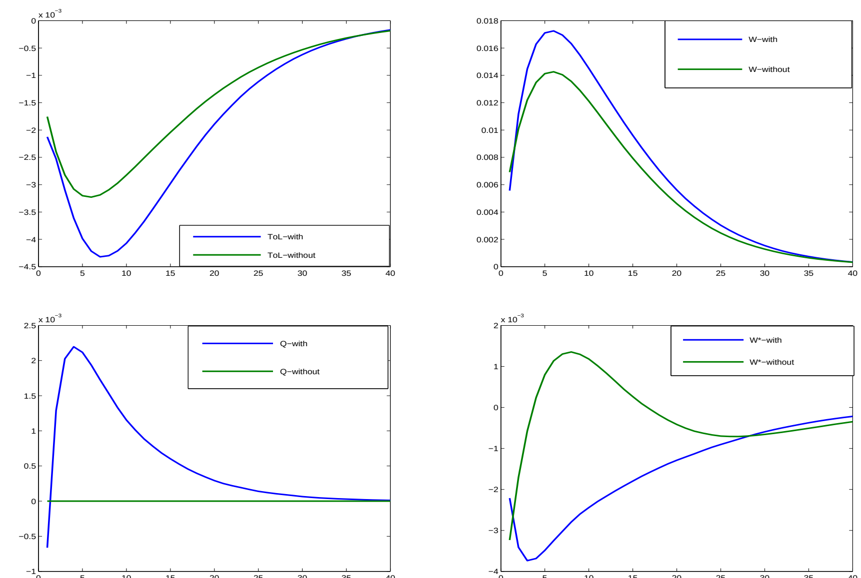
In response to a productivity shock, the real exchange rate varies more when the extensive margin of trade is allowed to respond, alongside the intensive margin (blue lines). In general, most variables fluctuate more, except for exports.

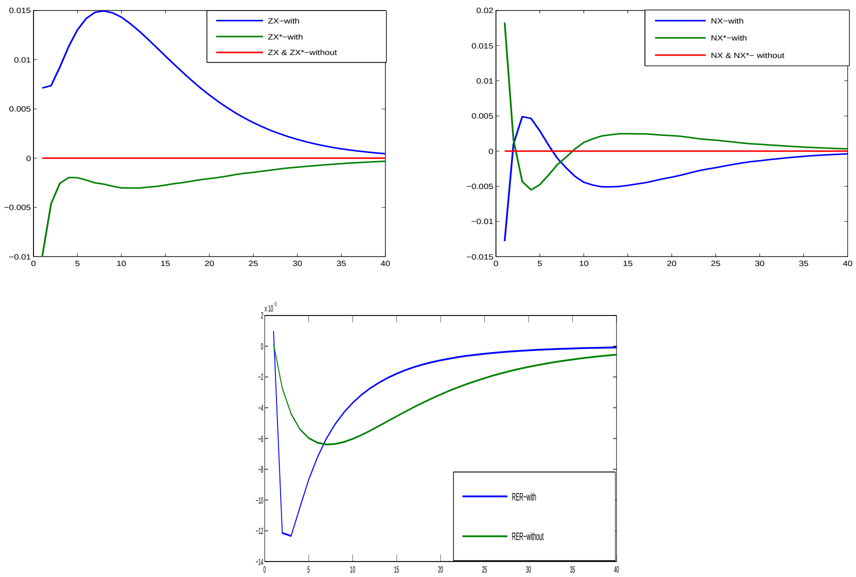
With/out Endo. Export Decision under bond economy with sticky wages

Figure 1: Aggregate Temporary Productivity Shock: Sticky wage Bond Economy-With/out Endo. Export Decision




ND*−with
ND*−without
NE−with
NE−without
NE*−with
NE*−without
NetExp−with
NetExp−without

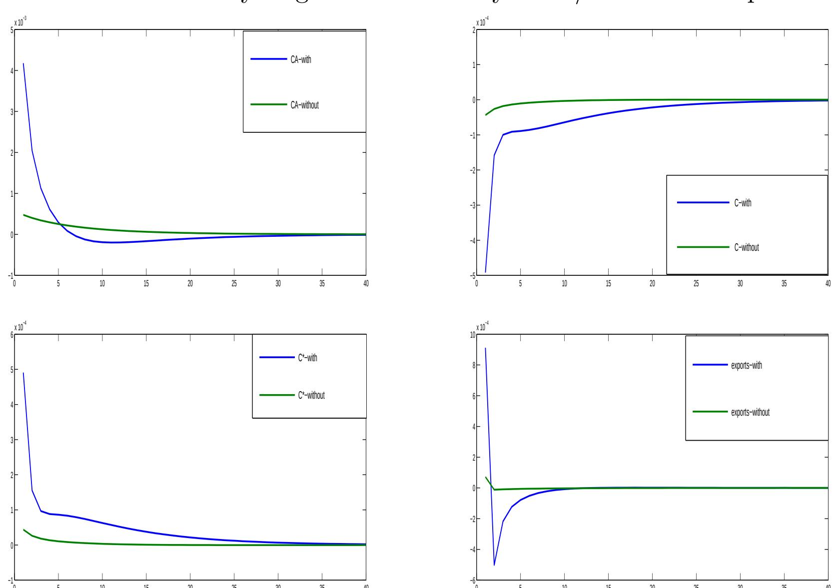

ToL−with
ToL−without
W−with
W−without
Q−with
Q−without
W*−with
W*−without

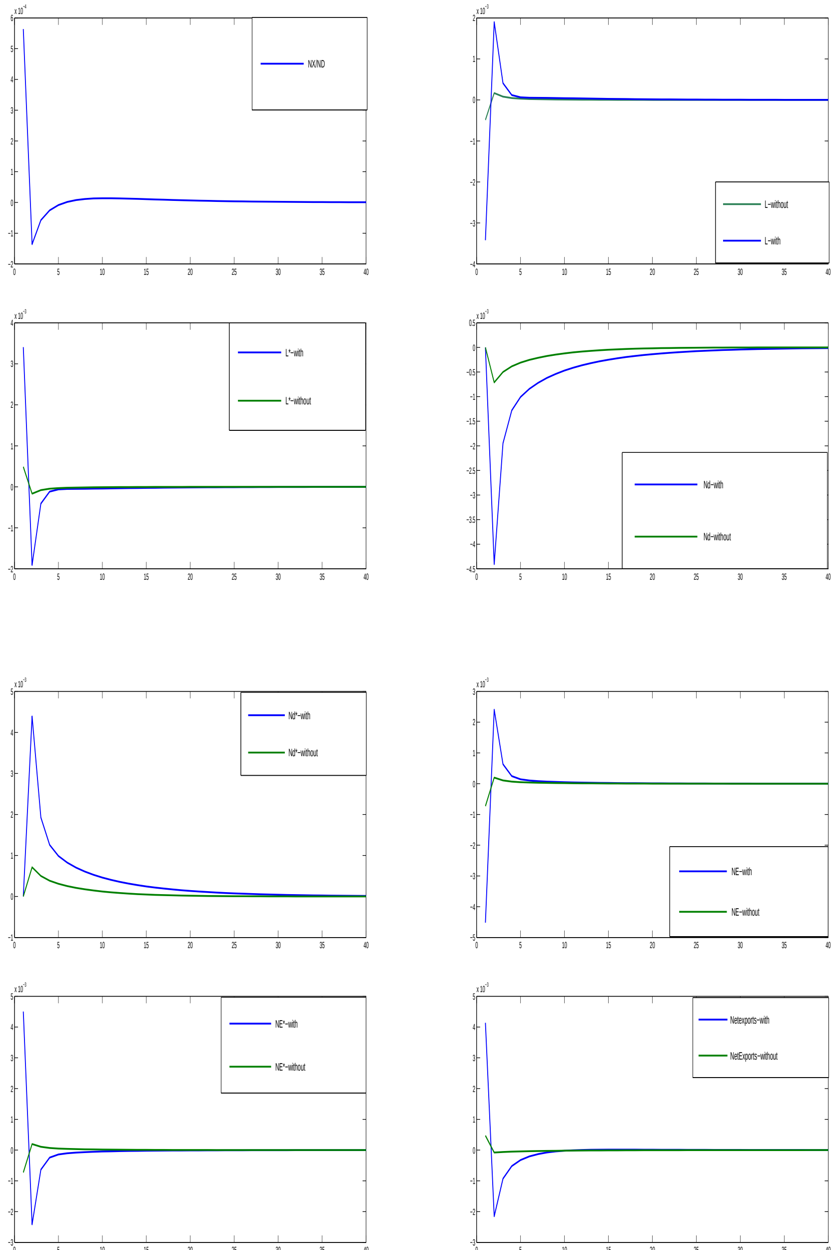


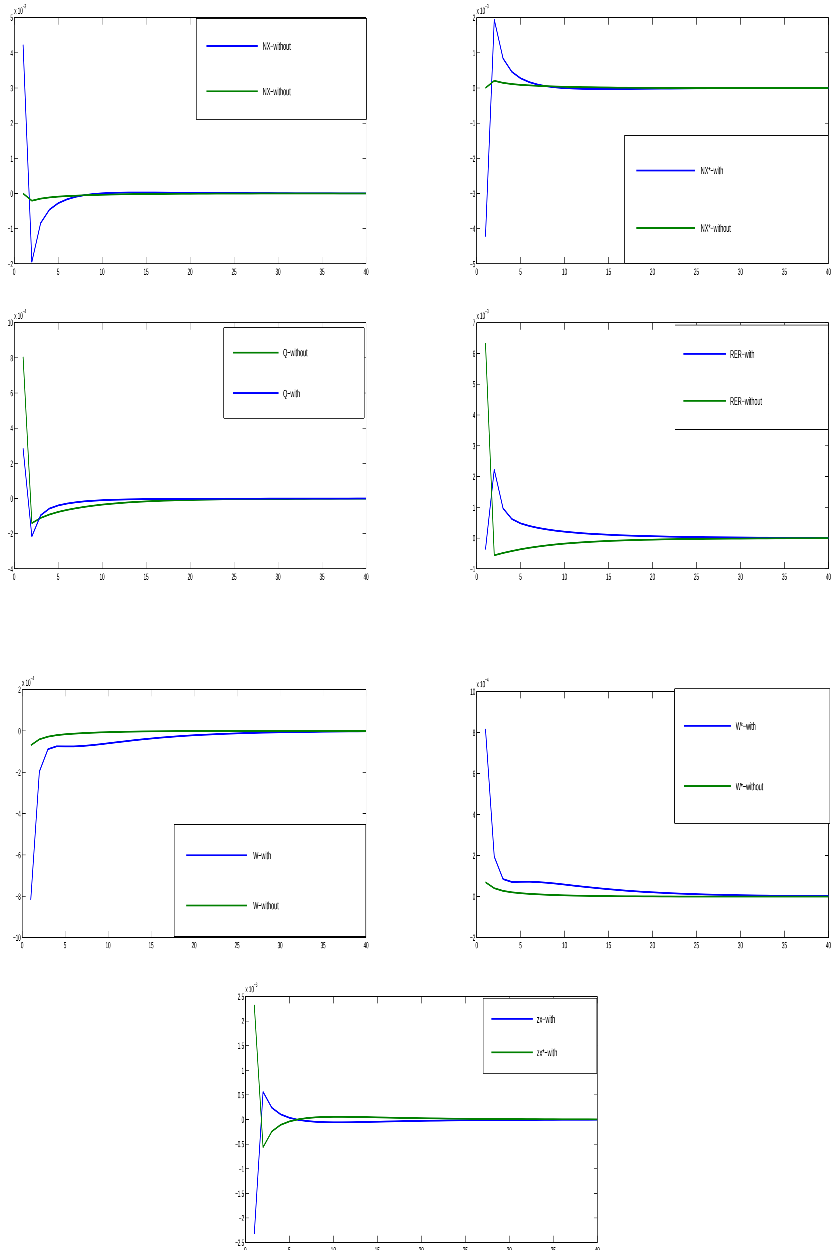
3.2 UIP shock

In response to an exogenous shock to the exchange rate all variables are much more responsive when the extensive margin of trade is allowed to respond. This can be seen by comparing the blue and green lines in the figures below.

Figure 2: UIP Shock: Sticky wage Bond Economy-With/out Endo. Export Decision



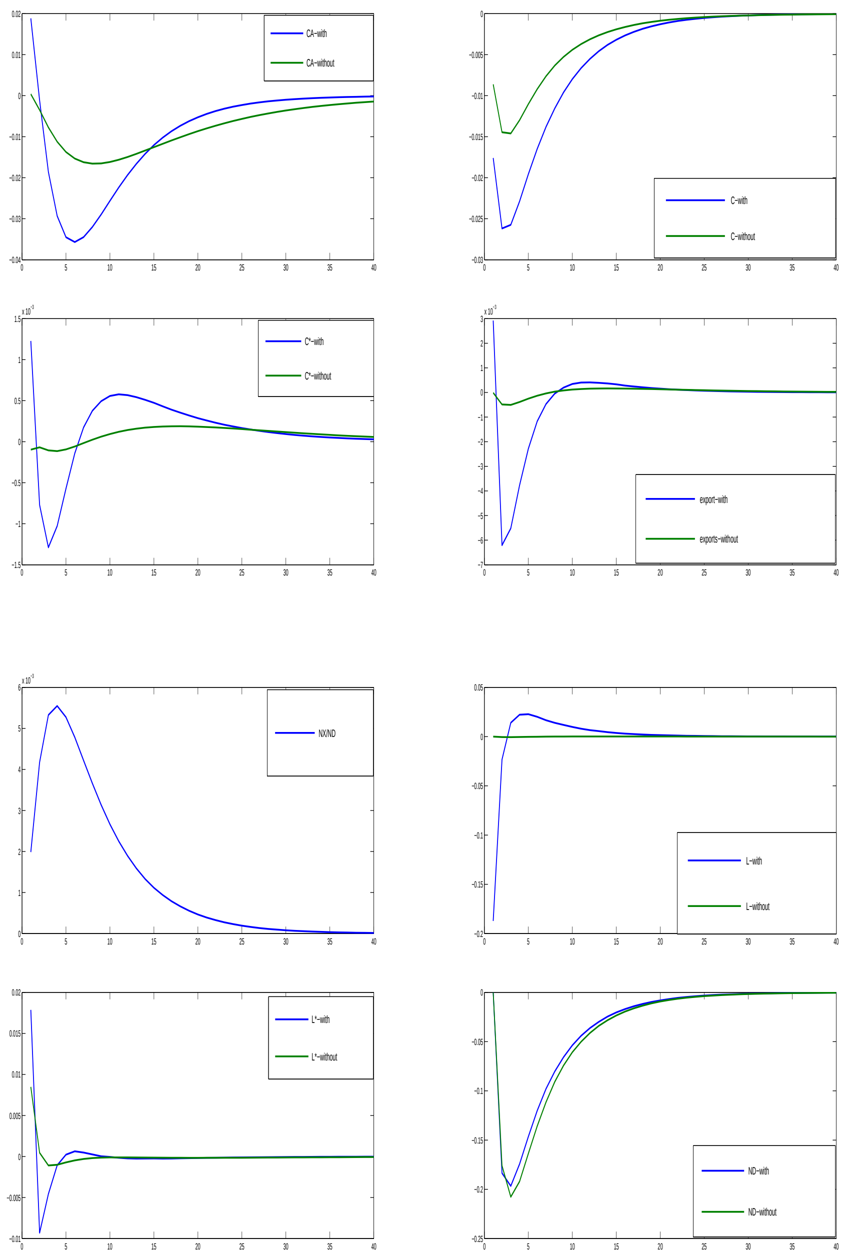

NX/ND
L−without
L−with
L*−with
L*−without
Nd−with
Nd−without
Nd*−with
Nd*−without
NE−with
NE−without
NE*−with
NE*−without
Netexports−with
NetExports−without

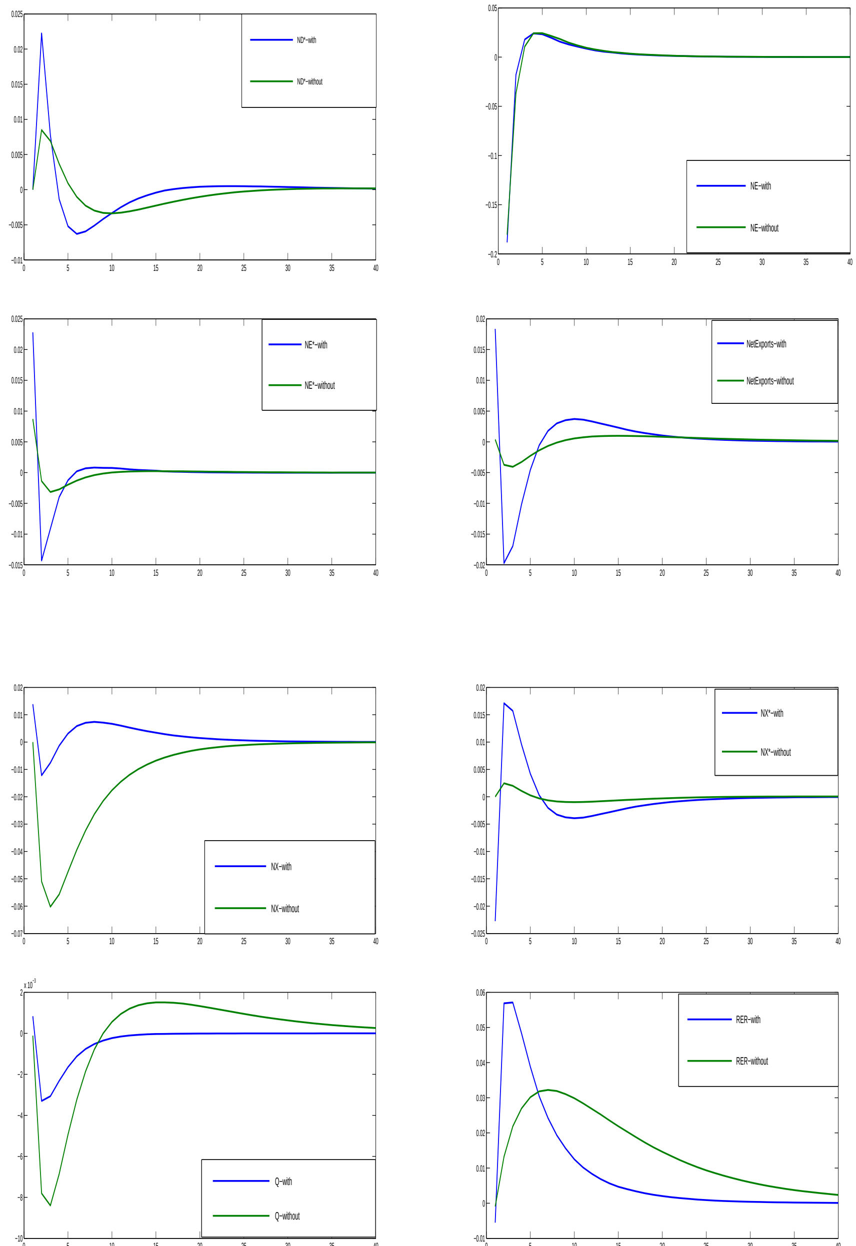

NX−without
NX−without
NX*−with
NX*−without
Q−without
Q−with
RER−with
RER−without
W−with
W−without
W*−with
W*−without
zx−with
zx*−with

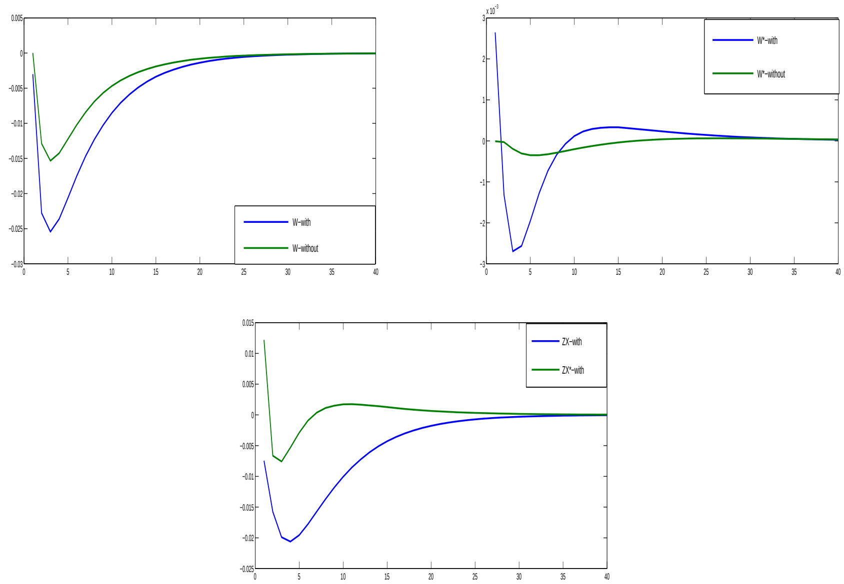
3.3 Monetary policy shock

The dynamics following an unanticipated monetary policy change also differs depending on whether the extensive margin of trade is allowed to respond or not. When the extensive margin of trade is allowed to vary this implies a higher volatility in all variables, as the figures below show.

Figure 3: Monetary Shock: Sticky wage Bond Economy-With/out Endo. Export Decision




ND*−with
ND*−without
NE−with
NE−without
NE*−with
NE*−without
NetExports−with
NetExports−without
NX−with
NX−without
NX*−with
NX*−without
Q−with
Q−without
RER−with
RER−without



4 Application to the UK

In this section we investigate whether introducing a role for the extensive margin of trade improves our understanding of the business cycle fluctuations in the UK economy, specifically the dynamics of the UK exports.

As we do not know the true underlying shock processes and shock variances, we will estimate these parameters by using Bayesian estimation techniques. We estimate the model for the UK and RoW data. Our data covers the period between 1988:Q1 and 2013:Q1. We use output, inflation, interest rates and the real exchange rate as observables. To be consistent with the model specification, we take the logs of the data and detrend the output by using the HP-filter[7]. As the model is log-linearised in deviations from steady state, we demean the rest of the variables.

We estimate the autoregressive component and the standard deviation of productivity shocks, preference shocks and the UIP shock as well as the standard deviation of monetary policy shocks . Estimating these parameters allows us to understand which shocks matter the most for the UK economy over the period and which shocks we should focus on in our IRF analysis. Table 2 shows the prior values we chose for our estimations. These are quite standard with a wide standard deviation which allows degrees of freedom to be large in our estimations.

[7] As we are working with quarterly data, we choose 1600 for the smoothing parameter of the HP filter.

Table 2: Priors

Description	Distribution	Mean	Standard Deviation
Ar(1) coefficients of shocks	Beta	0.75	0.1
Standard Deviation of shocks	Gamma	0.5	0.1

We estimate two models: the model with variable number of exporters (*Model 1*) and fixed number of exporters (*Model 2*) . The results of the estimations are presented in table 3.

Table 3: Posterior Distributions

Posterior Mean					
	Model-1	Model-2		Model-1	Model-2
AR(1) Coeff.s			***Std. Dev.***		
ρ_{pref}	0.94	0.93	ε_{pref}	1.4168	1.4154
ρ_{pref^*}	0.93	0.66	ε_{pref^*}	1.4184	1.4030
ρ_z	0.10	0.95	ε_z	0.7603	0.5566
ρ_{z^*}	0.10	0.96	ε_{z^*}	0.6784	0.4950
ρ_{uip}	0.10	0.74	ε_{uip}	1.3316	1.1750
			ε_m	0.3554	0.1950
			ε_{m^*}	0.7465	0.1168

As we have the estimated parameters for both models, we can now investigate which one fits the business cycle features of the UK economy and the export dynamics best. To do so, we make a Bayesian model comparison.

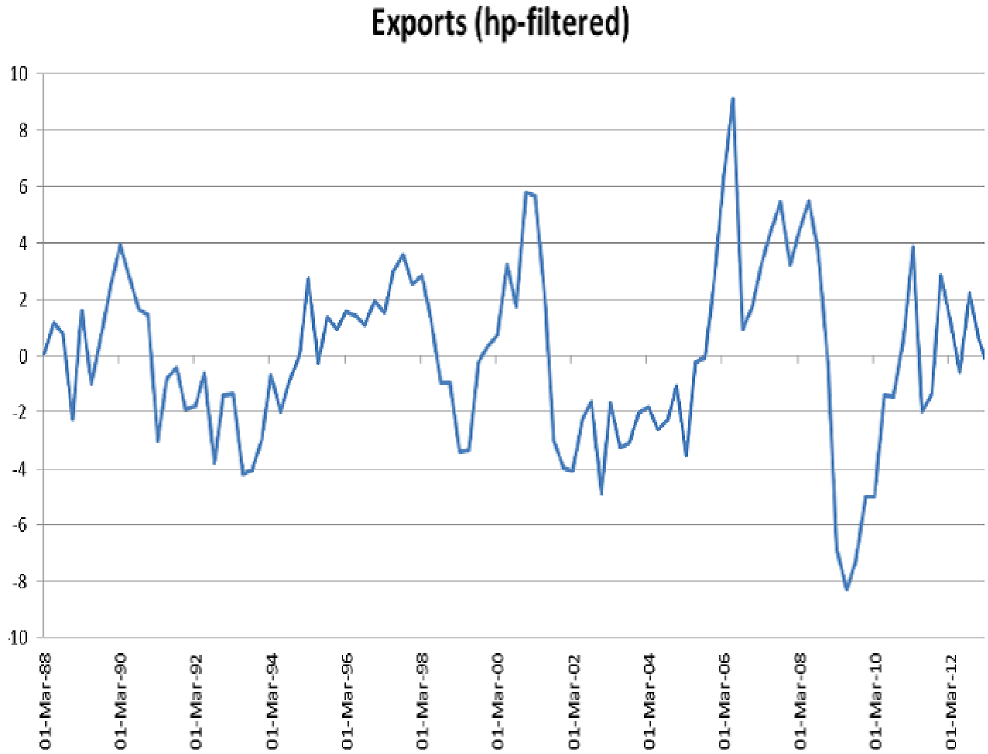
Moreover, we can compare our model predictions on exports with those observed in the data. Indeed, even though, we do not use exports as an observable we are interested in understanding which model does best in predicting the dynamics of exports.

SECTION TO BE COMPLETED

5 Understanding the great trade collapse

In order to understand whether the extensive margin of trade played a role during the great trade collapse, we make a shock decomposition for exports in both models and compare that with the UK exports dynamics during the great trade collapse. As shown in Figure 4, UK exports fell by around 8% with respect to its trend during the great trade collapse.

Figure 4: UK Exports-Volume



SECTION TO BE COMPLETED

6 Conclusion

This paper shows that the extensive margin of trade plays a role in explaining the volatility of trade over the business cycle. Using a model in which firms are heterogenous with respect to their productivity and face fixed exports cost, we show how the extensive margin of trade contributes to explaining trade dynamics over the business cycle. Focusing on the role of the extensive margin of trade in explaining UK export dynamics during the great trade collapse, we find that it plays a role which cannot be ignored.

Appendices

A Steady State

In this section we list the steady state of the system. The steady state of the level variables will be indicated by overbars[8]. There are no technological improvements in steady state: $\bar{Z} = \bar{Z}^* = 1$. We further assume: $\bar{\tau} = \bar{\tau}^*$, $\bar{f_E} = \bar{f_E}^*, \bar{C} = \bar{C}^*$ and $\bar{L} = \bar{L}^*$.
We first pin down the export cut-off, $\tilde{z}_x$, and import prices, $\tilde{\rho}_X$, then solve for the other endogenous variables.

In steady state, the free entry equation implies that $\bar{\tilde{v}} = \bar{w}\bar{f}_E$. We can re-write the Euler equation for share holdings (11) as:

$$\bar{\tilde{v}} = \bar{w}\bar{f}_E = \frac{\beta(1-\delta)}{1-\beta(1-\delta)}\bar{\tilde{\Pi}} \tag{A-1}$$

We use equation (9) to get the steady state average profit:

$$\bar{\tilde{\Pi}}_X = (\theta-1)\left(\bar{w}\bar{f_X}\right)\left(\frac{\phi^{\theta-1}}{k}\right) \tag{A-2}$$

Using equations (3), the average profits of home firms in steady state can be written as[9]:

$$\bar{\tilde{\Pi}}_D = \frac{(\bar{\rho}_D)^{(1-\theta)}\,\bar{C}}{\theta}, \qquad \bar{\tilde{\Pi}}_X = \frac{1-n}{n}\frac{(\bar{\rho}_X)^{(1-\theta)}\,\bar{C}}{\theta} - \bar{w}\bar{f_X}$$

This implies:

$$\bar{\tilde{\Pi}}_D = \left(\frac{\bar{\rho}_X}{\bar{\rho}_D}\right)^{(\theta-1)}\left(\frac{n}{1-n}\right)\left(\bar{\tilde{\Pi}}_X + \bar{w}\bar{f_X}\right)$$

We can replace $\bar{\tilde{\Pi}}_X$ by equation (A-2) and plug in the steady state prices, $\bar{\rho}_D = \frac{\theta}{(\theta-1)}\frac{\bar{w}}{\bar{\tilde{z}}_D} = \frac{\theta}{(\theta-1)}\frac{\bar{w}}{\phi z_{min}}$, $\bar{\rho}_X = \frac{\theta}{(\theta-1)}\frac{\bar{\tau}\,\bar{w}}{\bar{\tilde{z}}_X}$:

$$\bar{\tilde{\Pi}}_D = \left(\frac{\bar{\tau}\phi z_{min}}{\bar{\tilde{z}}_X}\right)^{(\theta-1)}\left(\frac{n}{1-n}\right)\left((\theta-1)\bar{w}\bar{f_X}\left(\frac{\phi^{\theta-1}}{k}\right) + \bar{w}\bar{f_X}\right) \tag{A-3}$$

The steady state share of exporting firms among all domestic firms is:

$$\frac{\bar{N}_X}{\bar{N}_D} = \left(\frac{z_{min}\phi}{\bar{\tilde{z}}_X}\right)^k \tag{A-4}$$

We know that, the average total dividends in home country are:

$$\bar{\tilde{\Pi}} = \frac{1-\beta(1-\delta)}{\beta(1-\delta)}\bar{w}\bar{f}_E = \bar{\tilde{\Pi}}_D + \left[\frac{\bar{N}_X}{\bar{N}_D}\right]\bar{\tilde{\Pi}}_X \tag{A-5}$$

[8] This section follows the online appendix provided by the authors for the Ghironi and Melitz (2005) paper.
[9] Note, we are using the symmetric steady state assumption: $\bar{C} = \bar{C}^*$

We can now plug in equations (A-2), (A-3) and (A-4) into equation (A-5) and solve for $\bar{\tilde{z}}_X$:

$$\frac{1-\beta(1-\delta)}{\beta(1-\delta)}\frac{\bar{f}_E}{\bar{f}_X} = \left(\frac{n}{1-n}\right)(\bar{\tilde{z}}_X)^{1-\theta}(\bar{\tau} z_{min})^{\theta-1}\phi^{2(\theta-1)} + (\bar{\tilde{z}}_X)^{-k}(z_{min})^k\phi^k\left(\frac{\theta-1}{k-(\theta-1)}\right) \tag{A-6}$$

where $\phi \equiv \{k/(k-(\theta-1))\}^{1/(\theta-1)}$. We can solve the steady state value of $\bar{\tilde{z}}_X$ numerically by plugging in the value of parameters.

We will now solve for the steady state value of $\tilde{\rho}_X$.

The steady state market clearing condition is:

$$\bar{C} = \bar{w}\bar{L} + \bar{N}_D\bar{\tilde{\Pi}} - \bar{N}_E\bar{\tilde{v}}$$

The total number of firms in steady state becomes:

$$\bar{N}_E = \frac{\delta}{1-\delta}\bar{N}_D$$

By using this, we can re-write the market clearing as:

$$\frac{\bar{C}}{\bar{w}} = \bar{L} + \bar{N}_D\bar{f}_E\frac{1-\beta}{\beta(1-\delta)} \tag{A-7}$$

Also from $\bar{\tilde{\Pi}}_X = \dfrac{1-n}{n}\dfrac{(\bar{\rho}_X)^{(1-\theta)}\bar{C}}{\theta} - \bar{w}\bar{f}_X = (\theta-1)\left(\bar{w}\bar{f}_X\right)\left(\frac{\phi^{\theta-1}}{k}\right)$ as:

$$\frac{\bar{C}}{\bar{w}} = \left(\frac{n}{1-n}\right)(\bar{\rho}_X)^{(\theta-1)}\,\theta\,\phi^{(\theta-1)}\,\bar{f}_X \tag{A-8}$$

Combining equations (A-7) and (A-8) yields:

$$\left(\frac{n}{1-n}\right)(\bar{\rho}_X)^{(\theta-1)}\,\theta\,\phi^{(\theta-1)}\,\bar{f}_X = \bar{L} + \bar{N}_D\bar{f}_E\frac{1-\beta}{\beta(1-\delta)} \tag{A-9}$$

We will use the price index in symmetric steady state: $1 = \bar{N}_D(\bar{\tilde{\rho}}_D)^{1-\theta} + \bar{N}_X(\bar{\tilde{\rho}}_X)^{1-\theta}$. When we multiply this expression with $(\bar{\tilde{\rho}}_X)^{\theta-1}/\bar{N}_D$, we obtain:

$$\frac{(\bar{\tilde{\rho}}_X)^{\theta-1}}{\bar{N}_D} = \left(\frac{\bar{\rho}_X}{\bar{\rho}_D}\right)^{(\theta-1)} + \frac{\bar{N}_X}{\bar{N}_D} \tag{A-10}$$

Plugging in equation (A-4) and using $\bar{\tilde{\rho}}_X/\bar{\tilde{\rho}}_D = \bar{\tau}\, z_{min}\,\phi/\bar{\tilde{z}}_x$ gives:

$$\frac{(\bar{\tilde{\rho}}_X)^{\theta-1}}{\bar{N}_D} = \left(\frac{\bar{\tau}\, z_{min}\,\phi}{\bar{\tilde{z}}_x}\right)^{\theta-1} + \left(\frac{z_{min}}{\bar{\tilde{z}}_x}\right)^k\phi^k \tag{A-11}$$

We can now solve for $\bar{\tilde{\rho}}_X$ by plugging in equation (A-11) into the equation (A-9):

$$(\bar{\tilde{\rho}}_X)^{1-\theta} = \frac{\left\{\left(\frac{n}{1-n}\right)\theta\phi^{\theta-1}\bar{f}_X - A^{-1}\bar{f}_E\frac{1-\beta}{\beta(1-\delta)}\right\}}{L} \tag{A-12}$$

where A is the right hand side of the equation (A-11).

The list of all the steady state relationships is as follows:

- $1+r = 1/\beta \rightarrow$ Consumption Euler equation

- $\dfrac{1-\beta(1-\delta)}{\beta(1-\delta)} \frac{\bar{f}_E}{\bar{f}_X} = \left(\dfrac{n}{1-n}\right)(\bar{\tilde{z}}_X)^{1-\theta}(\bar{\tau} z_{min})^{\theta-1}\phi^{2(\theta-1)} + (\bar{\tilde{z}}_X)^{-k}(z_{min})^k \phi^k \left(\frac{\phi^{\theta-1}}{k}\right)$

- $(\bar{\tilde{\rho}}_X)^{1-\theta} = \dfrac{\left\{\left(\dfrac{n}{1-n}\right)\theta\phi^{\theta-1}\bar{f}_X - A^{-1}\bar{f}_E \dfrac{1-\beta}{\beta(1-\delta)}\right\}}{L}$

- $\bar{N}_D = A^{-1}(\bar{\tilde{\rho}}_X)^{\theta-1} \rightarrow$ equation (A-11)

- $\bar{N}_E = \frac{\delta}{1-\delta}\bar{N}_D$

- $\bar{\tilde{\rho}}_D = \bar{\tilde{\rho}}_X \frac{\bar{\tilde{z}}_x}{\bar{\tau}\, z_{min}\, \phi} \rightarrow \bar{\tilde{\rho}}_X / \bar{\tilde{\rho}}_D = \bar{\tau}\, z_{min}\, \phi / \bar{\tilde{z}}_x$

- $\bar{w} = \frac{\theta-1}{\bar{\tau}\theta}\bar{\tilde{\rho}}_X \bar{\tilde{z}}_X \rightarrow \bar{\rho}_X = \frac{\theta}{(\theta-1)}\frac{\bar{\tau}\,\bar{w}}{\bar{\tilde{z}}_X}$

- $\bar{C} = \bar{w}\left(\bar{L} + \bar{N}_D \bar{f}_E \dfrac{1-\beta}{\beta(1-\delta)}\right) \rightarrow$ equation (A-8)

- $\bar{\tilde{v}} = \bar{w}\bar{f}_E \rightarrow$ free entry condition

- $\bar{\tilde{\Pi}} = \dfrac{1-\beta(1-\delta)}{\beta(1-\delta)}\bar{w}\bar{f}_E \rightarrow$ Share holdings Euler equation

- $\bar{\tilde{\Pi}}_D = \dfrac{(\bar{\rho}_D)^{(1-\theta)}\,\bar{C}}{\theta}$

- $\bar{\tilde{\Pi}}_X = \left(\dfrac{1-n}{n}\right)\dfrac{(\bar{\rho}_X)^{(1-\theta)}\,\bar{C}}{\theta} - \bar{w}\bar{f}_X$

- $\bar{N}_X = \left(\frac{z_{min}\phi}{\bar{\tilde{z}}_X}\right)^k \bar{N}_D \rightarrow$ equation (A-4)

B Endogenous Labour

Here, we are considering the labour that is hired by exporting firms and domestic firms separately. For the domestic markets firms will hire l_t^D of labour and for export markets they will use l_t^X of labour. In addition to the labour used in the production, each firm will need to hire $f_{E,t}/Z_t$ units of labour as an entry cost, to start production. In addition each exporter will need to hire $f_{X,t}/Z_t$ of labour to pay the fixed exporting cost. The profits from domestic sales becomes:

$$\Pi_t^D(\omega) = \frac{p_t^D(\omega)}{P_t} y_t^D(\omega) - w_t l_t^D(\omega) = \frac{p_{D,t}(\omega)}{P_t} Z_t z l_t^D - w_t l_t^D(\omega)$$

Plugging the optimal prices, $\rho_{D,t}(z) = \frac{p_{D,t}(z)}{P_t} = \frac{\theta}{(\theta-1)}\frac{w_t}{Z_t\, z}$ yields:

$$\Pi_t^D(\omega) = \frac{1}{\theta-1} w_t l_t^D(\omega) \tag{B-1}$$

The average amount of labour hired to cover the sales at home for the firm with average productivity of $\tilde{z}^D$ can be shown as: $\tilde{l}_t^D = (\theta - 1)\, \widetilde{\Pi}_t^D / w_t$

The profts of an exporting firm is:

$$\Pi_t^X(\omega) = Q_t \frac{p_t^X(\omega)}{P_t^*} y_t^X(\omega) - w_t l_t^X(\omega) - \frac{w_t f_{X,t}}{Z_t} = Q_t \frac{p_{X,t}(\omega)}{P_t^*} \frac{Z_t z l_t^X}{\tau_t} - w_t l_t^X(\omega) - \frac{w_t f_{X,t}}{Z_t}$$

Again, when we plug in optimal prices, $\rho_{X,t}(z) = \frac{p_{X,t}(z)}{P_t^*} = \frac{\theta}{(\theta-1)} \frac{\tau_t\, w_t}{Q_t\, Z_t\, z}$, we obtain:

$$\Pi_t^X(\omega) = \frac{1}{\theta - 1} w_t l_t^X(\omega)) - \frac{w_t f_{X,t}}{Z_t} \tag{B-2}$$

The average amount of labour hired to cover the export sales for the firm with average productivity of $\tilde{z}_t^X$ can be shown as: $\tilde{l}_t^X = (\theta - 1)\, \widetilde{\Pi}_t^X / w_t + (\theta - 1)\, f_{X,t}/Z_t$

The total labour demand in home country then:

$$L_t = \frac{\theta - 1}{w_t} N_t^d\, \widetilde{\Pi}_t^d + \frac{\theta - 1}{w_t} N_t^X\, \widetilde{\Pi}_t^X + \frac{\theta}{Z_t} N_t^X f_{X,t} + \frac{1}{Z_t} N_t^E f_{E,t} \tag{B-3}$$

where $\frac{1}{Z_t} N_t^E f_{E,t}$ reflects the total amount of labour hired by the new entrants.

Actually, the balanced trade condition combined with the market clearing ensures the market clearing for the labour market. To show this we will re-write the balanced trade condition, $Q_t N_{X,t}(\tilde{\rho}_{X,t})^{1-\theta} C_t^* = N_{X,t}^*(\tilde{\rho}_{X,t}^*)^{1-\theta} C_t$, by using the price index, $1 = N_{D,t}(\tilde{\rho}_{D,t})^{1-\theta} + N_{X,t}^*(\tilde{\rho}_{X,t}^*)^{1-\theta}$.

$$Q_t N_{X,t}(\tilde{\rho}_{X,t})^{1-\theta} C_t^* = \left[1 - N_{D,t}(\tilde{\rho}_{D,t})^{1-\theta}\right] C_t$$

We can express this by using the profit functions: $\widetilde{\Pi}_t^D = \dfrac{(\tilde{\rho}_t^D)^{(1-\theta)}\, C_t}{\theta}$, $\widetilde{\Pi}_t^X = \dfrac{Q_t(\tilde{\rho}_t^X)^{(1-\theta)}\, C_t^*}{\theta} - \frac{w_t f_{X,t}}{Z_t}$.

$$C_t = \theta N_t^X \left(\widetilde{\Pi}_t^X + \frac{w_t f_{X,t}}{Z_t}\right) + \theta N_t^D \widetilde{\Pi}_t^D$$

Plugging in this into the market clearing, $C_t = w_t L_t + N_{D,t}\widetilde{\Pi}_t - N_{E,t}\tilde{v}_t$ and using $\tilde{v}_t = w_t f_{E,t}/Z_t$ and $\widetilde{\Pi}_t = N_t^d\, \widetilde{\Pi}_t^d + N_t^X\, \widetilde{\Pi}_t^X$ gives the equation (B-3):

$$L_t = \frac{\theta - 1}{w_t} N_t^d\, \widetilde{\Pi}_t^d + \frac{\theta - 1}{w_t} N_t^X\, \widetilde{\Pi}_t^X + \frac{\theta}{Z_t} N_t^X f_{X,t} + \frac{1}{Z_t} N_t^E f_{E,t}$$

C List of Equilibrium Conditions

- Price Index: $1 = \frac{N_{D,t}}{N_{D,t}+\frac{1-n}{n}N^*_{X,t}}(\tilde{\rho}_{D,t})^{1-\theta} + \frac{\frac{1-n}{n}N^*_{X,t}}{N_{D,t}+\frac{1-n}{n}N^*_{X,t}}(\tilde{\rho}^*_{X,t})^{1-\theta}$

- Prices: $\tilde{\rho}_{D,t}, = \frac{\theta}{(\theta-1)}\frac{w_t}{Z_t\, z_{min}\phi}$ and $\tilde{\rho}_{X,t} = \frac{\theta}{(\theta-1)}\frac{\tau_t\, w_t}{Q_t\, Z_t \tilde{z}_{X,t}}$

  where $\phi \equiv \{k/(k-(\theta-1))\}^{1/(\theta-1)}$

- Free Entry: $\tilde{v}_t = w_t f_{E,t}/Z_t$

- Number of Firms: $N_{D,t} = (1-\delta)(N_{D,t-1}+N_{E,t-1})$

- Average Profits: $\widetilde{\Pi}_t = \widetilde{\Pi}^D_t + \left[\frac{N_{X,t}}{N_{D,t}}\right]\widetilde{\Pi}^X_t$

- Profits from domestic markets: $\widetilde{\Pi}^D_t = \dfrac{(\tilde{\rho}_{D,t})^{(1-\theta)}\, C_t}{\theta}$

- Profits from export markets: $\widetilde{\Pi}^X_t = \left(\frac{1-n}{n}\right)\dfrac{Q_t(\tilde{\rho}_{X,t})^{(1-\theta)}\, C^*_t}{\theta} - \frac{w_t f_{X,t}}{Z_t}$

- Bond Euler Equation:$1 = \mathrm{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(1+i_t)\left(\dfrac{P_t}{P_{t+1}}\right)\right]$

- Share holdings Euler Equation: $\tilde{v}_t = \mathrm{E}_t\left[\beta(1-\delta)\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(\tilde{v}_{t+1}+\widetilde{\Pi}_{t+1})\right]$

- Share of exporting firms: $\frac{N_{X,t}}{N_{D,t}} = \left(\frac{z_{min}\phi}{\tilde{z}_{X,t}}\right)^k$

- Zero profit export cut-off: $\widetilde{\Pi}^X_t = (\theta-1)\left(\dfrac{w_t f_{X,t}}{Z_t}\right)\left(\frac{\phi^{\theta-1}}{k}\right)$

- Market Clearing: $w_t L_t + N_{D,t}\widetilde{\Pi}_t = C_t + N_{E,t}\tilde{v}_t + \dfrac{Q_t B_{F,t+1}}{\Theta(Q_t B_{F,t+1})} - Q_t(1+r^*_t)B_{Ft}$

- Wage Phillis curve: $\sum_{k=0}^{\infty}(\beta\xi)^k E_t\left[L_{t+k}(i)U_C(C_{t,t+k})\left((1+\sigma)\dfrac{\widetilde{W}_t(i)}{P_{t+k}} - \dfrac{\theta_w}{\theta_w-1}MRS_{t,t+k}\right)\right] = 0$

- Labour Market Clearing:

$$L_t = \frac{\theta-1}{w_t}N^d_t\,\widetilde{\Pi}^d_t + \frac{\theta-1}{w_t}N^X_t\,\widetilde{\Pi}^X_t + \frac{\theta}{Z_t}N^X_t f_{X,t} + \frac{1}{Z_t}N^E_t f_{E,t}$$

- UIP: $1 = \beta(1+r^*_{t+1})\Theta(Q_t B_{F,t+1})\mathrm{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{Q_{t+1}}{Q_t}\right]$

- Current Account: $CA_t = \dfrac{Q_t B_{F,t}}{(1+r^*_t)\Theta(Q_t B_{F,t})} - Q_t B_{F,t-1} = \left(\frac{1-n}{n}\right)Q_t N_{X,t}(\tilde{\rho}_{X,t})^{1-\theta}C^*_t - N^*_{X,t}(\tilde{\rho}^*_{X,t})^{1-\theta}C_t$

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