Unbalanced growth, secular stagnation and the relative price of investment goods

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Abstract

Over the past four decades, real interest rates have risen then fallen across the industrialised world. Over the same period, nominal investment rates are down, while house prices and household debt are up. I explain these four trends with a fifth - the widespread fall in the relative price of investment goods. I present a simple closed-economy OLG model in which households finance retirement in part by selling claims on the corporate sector (capital goods) accumulated over their working lives. As capital goods prices fall, a given quantity of capital goods buys less retirement consumption, so households accumulate more of them, putting downward pressure on interest rates. This reduces the user cost of housing, raising house prices and, given that housing is bought early in life, increasing household debt. The debt of the young provides an alternative outlet for the retirement savings of the old. The analysis in this paper shows recent debates on macroeconomic imbalances and household and government indebtedness in a new light. In particular, low real interest rates, accompanied by pressure for governments and households to become indebted, may be the new normal.

1 Introduction

The financial crisis that began in 2007 pushed central banks in much of the industrialised world to the zero lower bound on nominal policy rates. Much ink has been spilled about how this happened, what central banks should have done when they got there, and how to avoid it happening again. But real interest rates had been trending down across the industrialised world for at least twenty years before this, and had already reached historic lows on the eve of the crisis (Summers (2013), King and Low (2014)). Alongside this fall in interest rates, much of the industrialised world saw house prices and household debt rise to historic highs before the crisis. While these series have subsequently fallen back somewhat, they appear at the time of writing to have stabilised at elevated levels in relation to GDP and real incomes in many countries.

There have been many explanations for this fall in industrialised-world interest rates, among which are three leading candidates. The first is demographics - in particular a rise in the weight of high-saving age-groups as baby-boomers enter late middle age. The second is inequality, whereby a rise in the share of income or wealth accruing to the high-saving rich has raised aggregate saving. And the third is emerging markets, whereby an excess of saving in the developing world has pushed down on rich-world interest rates.

Each of these explanations has merit. But what they all have in common is a rise in domestic or foreign saving as a cause of the fall in interest rates. They all predict, therefore, a rise in investment in the industrialised world.[1] But in contrast, nominal investment rates have fallen sharply across the industrialised world over the past thirty years, a fall which again long predates the recent financial crisis.

This paper proposes a new explanation for the falls in real interest rates and rises in household debt across the industrialised world, complementary to those which rely on higher saving, but which also explains the fall in investment rates. The story is

[1] With the caveat that some demographic models that featuring slowing population growth may predict falling investment rates

based on the widespread fall in the price of investment goods relative to consumption over the past thirty or so years documented in Karabarbounis and Neiman (2014). I extend their data back in time for some countries, to show that this fall has not been a feature of the very long run, but rather began a few years either side of 1980.

In the model, as in the real world, the corporate sector invests the savings of the household sector in capital goods. As the prices of these goods have fallen, a given quantity of capital goods buys less retirement consumption. Households targeting a certain amount of retirement consumption will therefore build up claims on a bigger physical capital stock. This places downward pressure on the marginal product of capital, interest rates and thus the user cost of housing, boosting housing demand. Housing supply is fixed, so house prices (or at least land prices) must rise. Houses are bought early in life and largely on credit, so household debt also increases. Acquiring these debt claims is an alternative form of retirement saving, so the capital investment rate falls in the steady state, as we see in the data.

We parameterise the model with a less-than-unit elasticity of substitution between labour and capital, in line with most estimates but at variance with those in Karabarbounis and Neiman (2014). Consistent with the predictions of the model at these parameter values, we present cross-country evidence showing that nominal investment rates have fallen further in countries where the relative price investment rates and prices. Depending on parameterisation and the exact nature of the shocks hitting the model, the dynamics of the transition to this new steady state can involve a temporary rise in interest rates, as households attempt to bring forward the extra consumption afforded by the fall in the relative price of capital. This provides a new interpretation of the period of historically high world real interest rates experienced in the 1980s. But the new steady state is one of lower interest and investment rates and higher household debt ratios, even after investment goods prices have stopped falling.

These findings cast recent debates on macroeconomic imbalances and household and government indebtedness in a new light, and has important policy implications. Some prominent policymakers (see, for example, Ingves (2014)) are seeking to prevent what

they see as 'excessive' levels of household debt. But if low rates of interest and investment, accompanied by pressure for governments and households to become indebted, represent the transition to a new steady state in which the corporate sector's demand for household savings is weak, then attempts by macroprudential or monetary authorities to prevent this may be futile or counterproductive.

The mechanism in this paper builds on a long history of related ideas in the literature. Summers (2013) recently raised the issue of the pre-crisis falls in real interest rates and the possibility that they would stay low for an extended period in the future. But the idea that capitalist economies could be plagued by chronically low returns on capital, and that this could result from an overaccumulation, in some sense, of physical capital goes back at least to Marx (1867) and Hansen (1938). The fall in capital goods prices in the face of a need for retirement savings creates a form of asset shortage reminiscent of Caballero et al. (2008), which is satisfied by the endogenous creation of debt claims on the young. The focus on the fall in the relative price of investment goods builds on the important contribution of Karabarbounis and Neiman (2014), whose data I draw on for this study. Finally, two papers are particularly close, methodologically speaking, to the present study. Giglio and Severo (2012) examine the effect of a change in production technology in an OLG model and find that the conditions in which asset bubbles may exist are modified. Like the present study, Eggertsson and Mehrotra (2014) address the issue of secular stagnation in an OLG model. They show that a tightening of the debt limits facing young households, reduced population growth and increased income inequality can reduce the equilibrium real interest rate in such a model.

The remainder of this paper is structured as follows. Section 2 sets out the key facts the model aims to explain. Section 3 describes the model. Section 4 shows the results of model simulations in which we vary the relative price of investment and generate movements in interest rates, investment rates and household debt which are qualitatively similar to those presented in section 2. Section 5 examines the sensitivity of these findings to parameter values, and extends the model to allow for bequests. Section 6 concludes.

2 Motivating facts

This section sets out the key stylised facts that the model aims to connect. We focus on the widest possible set of industrialised countries for each data series, but also, where possible, show data for a subset consisting of the 11 advanced countries [2] for which the EU-KLEMS database has sufficient data to calculate long time-series of nominal and real capital-GDP ratios.

2.1 Falling real interest rates

Figure 1 shows two measures of the world real interest rate taken from King and Low (2014) - simple and GDP-weighted average 10-year yields on index-linked government liabilities for the G7 excluding Italy. The figure shows that interest rates have been trending generally downwards for the past 30 years, and had already halved from 4% to 2% before the onset of the financial crisis.

Ex-post real interest rates are known to have been low in the 1970s. But it is hard to measure ex ante real interest rates before the 1980s, and especially in the 1970s, given the absence of index-linked securities and the volatility of realised inflation. The model-based series in IMF (2014) suggest that US ex ante real rates were close to current levels in the early 1970s, fell below zero in the middle part of that decade, before rising sharply in the late 1970s-early 1980s.

2.2 Rising household debt ratios

Figure 2 shows the change in the ratio of household debt to GDP since 1971 for a broad sample of industrialised countries and our restricted sample of 11 countries. The figure shows a rise in the average ratio of around 50pp since 1971.[3]

[2] Australia, Austria, Denmark, Finland, Germany, Italy, Japan, Netherlands, Sweden, the UK and the US

[3] The chart is constructed from an unbalanced panel of data by running a fixed-effects panel regression of the household debt ratio on year dummies, then adding the dummy for each year to

2.3 Price and quantity of capital investment and stock

Figure 3 shows the simple average across OECD countries and across our restricted sample of the ratio of nominal investment to nominal GDP. The nominal investment rate has been trending downwards since at least the mid-1970s. Figure 4 shows that the corresponding stock ratio (the current replacement cost of the capital stock as a proportion of GDP) had also fallen from nearly 4 times annual GDP around 1980 to nearly 3 times by 2007 for the 11 countries in the EU-KLEMS database for which data are available.

Figure 5 shows the real investment - GDP ratio across the same two sets of countries since 1970. The series show no strong trend over the whole sample, although there is weak evidence of an upward trend since the early 1980s. Figure 4 shows that the ratio of the real capital stock to real GDP (both at 1995 prices) has been trending upwards since the 1970s.

These divergent patterns in the nominal and real ratios are of course a manifestation of a trend fall in the price of investment goods relative to consumption or GDP, documented in Karabarbounis and Neiman (2014). Figure 6 shows four series of the ratio of the investment deflator to the consumption deflator. The red and blue lines are taken from the respective countries' national accounts data. The green line is the average change across all the countries in the dataset, and the purple line is the average among our restricted sample in this dataset. All three lines show that the relative price of investment goods has been falling in recent decades, with a fall of perhaps 30% since the mid-1970s. The longer series show that, prior to this fall, there has not been a secular trend in this relative price.

The changes in relative investment goods prices and nominal investment rates are positively correlated across countries at low frequency. To quantify this link, I estimate country-specific time trends of nominal investment-GDP rates and the relative price of investment across the 37 countries with more than 15 years of data in the

the intercept of the equation. This allows other countries to affect the change in the ratio in years after they have been added to the sample

Karabarbounis and Neiman (2014) dataset. I then perform a cross-country regression of time trend in investment rate on time trend in relative price of capital goods, analogous to their paper, using both the full and 11-country samples.

$$\overline{dln\left(\frac{I}{Y}\right)_i} = \alpha + \beta \, \overline{dln\left(\frac{P_I}{P_C}\right)_i}$$

The results are given in table 1. I find a positive and highly significant coefficient on the relative price of investment goods: a 1 % fall in the relative price of capital is associated with an 0.2-0.3pp fall in the nominal investment rate. On the face of it, these results suggest a less-than-unit elasticity of substitution between capital and labour, in line with most estimates in the literature Chirinko (2008) but at variance with those presented in Karabarbounis and Neiman (2014).

3 The model

3.1 Households

The economy is closed and comprises three overlapping generations of constant and equal size. Each generation has a standard separable CES utility function over consumption and housing

$$U(c_1, c_2^{'}, c_3^{''}, h) = \frac{1}{1-\theta}\left(c_1^{1-\theta} + \beta_2 c_2^{'1-\theta} + \beta_3 c_3^{''1-\theta}\right) + \phi\frac{h^{1-\gamma}}{1-\gamma} \tag{1}$$

where we denote leads one and two periods hence with primes and double-primes respectively. In period 1 (young adulthood), the household supplies η units of labour inelastically (remunerated at wage W), consumes goods and buys a house. She can borrow or save a net amount S_1 at rate r. In period 2 (middle age), the household remains in said house, supplies $(1-\eta)$ units of labour, and can again borrow or save $S_2^{'}$. In period 3 (retirement), she sells her house and consumes the proceeds plus

her accumulated savings. So each of the three periods is associated with a budget constraint as follows

$$c_1 + hp_h + S_1 = \eta W \tag{2}$$

$$c_2^{'} + S_2^{'} = (1-\eta)W + (1+r)S_1 \tag{3}$$

$$c_3^{''} = (1+r^{''})S_2^{'} + hp_h \tag{4}$$

Forming and solving the Lagrangean we get consumption Euler equations thus

$$\frac{c_1^{-\theta}}{(1+r^{'})(1+r^{''})} = \frac{\beta_2 c_2^{'-\theta}}{(1+r^{''})} = \beta_3 c_3^{''-\theta} \tag{5}$$

We also get a housing demand equation that depends on future house prices and consumption as you would expect

$$\phi h^{-\gamma} + \beta_3 c_3^{''-\theta} p_h^{''} = c_1^{-\theta} p_h \tag{6}$$

This is intuitive. The LHS is the marginal utility of housing plus the discounted marginal utility of the consumption from the sale of the house . The RHS is the consumption utility cost of buying a unit of housing. Note that this means that housing demand depends on consumption and house prices two periods into the future.

3.2 Firms

A measure of perfectly competitive firms produce intermediate goods, combining capital and labour with a CES production technology

$$Y = A[(1-\alpha)L^{\frac{\sigma-1}{\sigma}} + \alpha K^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \tag{7}$$

These intermediates can then either be consumed directly, or transformed into capital goods at rate p units of intermediate for every one unit of capital. The relative price

of investment goods - the key exogenous parameter in our model - is therefore p. This means of introducing investment-specific technological change is isomorphic to that in Greenwood et al. (1997).

Wages are set equal to the marginal product of labour

$$W = \frac{\partial Y}{\partial L} \tag{8}$$

Firms equate the user cost of capital to its marginal product, both denominated in consumption goods

$$1 + r^{'} = \frac{1}{p_K}\frac{\partial Y^{'}}{\partial K^{'}} + \frac{p_K^{'}}{p_K}(1-\delta) \tag{9}$$

3.3 Market clearing

At the end of each period, the net savings of households of young and middle age are transformed into next period's capital stock, such that the following capital-market clearing condition holds

$$S_1 + S_2 = K^{'} p_K \tag{10}$$

There is a fixed measure $\bar{H}$ of housing for each of the first two generations to live in, so that in equilibrium

$$h = \bar{H} \tag{11}$$

4 Results

4.1 Parameterisation

Each of the three periods of adult life lasts twenty years. The standard parameters $\{\beta_1, \beta_2, \alpha, \delta\}$ are set at standard values ($\left\{.99^{20}, .99^{40}, \frac{1}{3}, .05\right\}$). The elasticities $\{\theta, \gamma\}$ in the utility function are set to unity (log utility), while the production elasticity σ is set to three-quarters, consistent with most estimates and with the econometric

evidence presented above. I set$\{\phi, p_K\}$ to hit sensible values for the ratios of housing wealth and nominal capital to GDP respectively. The steady-state growth rate of intermediate production technology A is set to 1% per year.

We simulate the effects of a 30% fall in p_K on the steady state of the model over two generations, shown in the black line in the top-left panel of figure 7. This amounts to an improvement in the overall level of technology, in the sense that the lower is p_K, the larger is the total volume of consumption and investment goods a given factor endowment can produce. So we reduce the level of technology A to keep potential GDP unchanged given existing factor endowments.

4.2 Comparative statics

The blue lines in figure 7 show the results of the simulation on the steady state of the model (the red lines, showing the dynamic effects, will be discussed below). The top left panel shows the assumed exogenous value for the relative price of capital. The fall in the relative price of capital gives rise to an increase in the real capital-output ratio (top right panel). The left panel in the second row shows that this increase is large enough to reduce the real interest rate by about 0.3pp, outweighing the direct positive effect of the fall in capital goods prices on the return denominated in consumption goods.

This effect would not be present in a model of infinite-horizon consumers, where the interest rate would be pinned down by the household discount factor. In our model, households are concerned with providing for consumption in retirement. Abstracting for now from debt and housing, the resources available to the retired generation are the profits Π generated from the capital stock, plus the resale value of the stock itself

$$c_3 = \Pi + (1 - \delta) K p_K \tag{12}$$

With an unchanged quantity of investment goods, the fall in the relative price of capital reduces the amount of consumption that can be financed by selling the stock.

Depending on the elasticity of substitution between capital and labour σ, the profit share may fall too. To protect retirement consumption, households must therefore accumulate a larger volume of capital goods, pushing down the return on capital. Depending on the assumed value of σ, this can offset the positive direct effect of lower capital goods prices on the return denominated in consumption units.

The right panel in the second row shows the impact on the steady-state nominal investment rate - a fall of around 0.5pp - which translates into a fall in the corresponding stock ratio, shown in the left panel of the third row, of around 7%. The right panel in the third row shows that the household debt-GDP ratio increases substantially, by around 20pp. The bottom-left panel shows that the share of housing wealth in total wealth increases: lower real interest rates reduce the user cost of housing, boosting house prices and therefore the borrowing needed to finance house purchases.

4.3 Dynamic results

What are the dynamic consequences of the experiment considered above? The results are shown in the red lines of figure 7.

The difference between the static and dynamic results is most apparent in the behaviour of the real interest rate.[4] In the dynamic simulation, the capital stock is inertial, so the fall in the relative price of capital has a direct impact on the interest rate denominated in consumption units, which is only partly offset by a negative effect from expected capital losses as the price of the installed stock falls. Over time, the capital stock grows and the real interest rate falls. The capital deepening in real terms takes three or four generations to complete, during which time the nominal investment- and capital-output ratios undershoot their long-run averages. The household debt ratio remains close to its steady state along the transition path. The bottom-right panel shows that, in the dynamic simulation, the capital share is

[4]In these graphs, the interest rate at date t is the return paid in period t on an investment in period $t-1$

broadly flat over the period in which capital prices are falling, in contrast to the rise observed in most countries documented in Karabarbounis and Neiman (2014).

Overall, the simulation results generate a qualitatively similar pattern in the real interest rate, the real and nominal investment- and capital-output ratios, and the household debt-GDP ratio to those which we have observed over the past two decades. Furthermore, the simulations provide forecasts of what may happen in years to come. In particular, even if the relative price of capital has stopped falling, the interest rate may continue to fall, even as the nominal investment rate begins to recover, as the capital deepening process brought on by the fall in the relative price of capital runs its course.

5 Sensitivity analysis

5.1 Parameterisation

A key parameter in this model is the elasticity of substitution in the production function σ between capital and labour. Figure 8 presents the results of the same experiment as above but sets $\sigma = 1.3$, in line with Karabarbounis and Neiman (2014). Broadly speaking, the model now hits the behaviour of the profit share, but gets wrong the behaviour of the interest rate and the stocks of capital and debt. The behaviour of the interest rate and the household debt ratio are qualitatively opposite to those in the low-elasticity baseline and in the data - the interest rate falls temporarily, then rises in the steady state, while the household debt ratio falls (further below zero on this parameterisation).

This can be viewed as evidence against the generality of the model. However, the model only gets the direction of the change in investment, interest rates and debt right when the elasticity of substitution is low, and is supported by the cross-sectional evidence in table 1. The results can therefore equally well be viewed as evidence in favour of a below-unit substitution elasticity, in line with the central tendency of the

several estimates reported in Chirinko (2008).

5.2 Bequests

In the baseline model, households spend all their wealth by the end of their lives, including their housing wealth. In practice, bequests form a large part of households' total resources and a large fraction of GDP is bequeathed in any one year (Piketty (2011)). Retirees often live in owner-occupied housing until the end of life (Yang (2009). These features can be introduced into our framework by adding bequests $\{b^{'}, b\}$ respectively given and received to the utility function and budget constraints as follows[5]

$$U(c_1, c_2^{'}, c_3^{''}, h, b) = \frac{1}{1-\theta}\left(c_1^{1-\theta} + \beta_2 c_2^{'1-\theta} + \beta_3 c_3^{''1-\theta}\right) + \phi\frac{h^{1-\gamma}}{1-\gamma} + \xi\frac{b^{'1-\zeta}}{1-\zeta} \tag{13}$$

$$c_1 + hp_h + S_1 = \eta W \tag{14}$$

$$c_2^{'} + S_2^{'} = (1-\eta)W + (1+r)S_1 + b \tag{15}$$

$$c_3^{''} + b^{'} = (1+r^{''})S_2^{'} + hp_h \tag{16}$$

Again forming and solving the Lagrangean we get Euler equations thus

$$\frac{c_1^{-\theta}}{(1+r^{'})(1+r^{''})} = \frac{\beta_2 c_2^{'-\theta}}{(1+r^{''})} = \beta_3 c_3^{''-\theta} = \xi b^{'-\zeta} \tag{17}$$

Piketty (2011) finds that bequests in France are around 15% of GDP. With log utility bequests will be a constant fraction of old-age consumption. Consistent with this, we set $\xi = 0.75\beta_3$. A preference for bequests reduces consumption and increases the demand for saving, turning the interest rate negative. We accordingly reduce the

[5]These are 'warm glow' preferences over bequests. Households still care about the consumption value of their assets in retirement, because they evaluate their bequests in consumption rather than utility terms. Adding later generations' utility directly to the utility function would collapse the model into an infinite horizon setup.

discount factor β to an annualised 0.98, but the simulation still starts from a point of higher investment and household debt, and lower interest rates, reflecting the fact that middle-aged households receive their bequests after they have taken out loans, and need to store them (in the form of mortgages and capital goods) before they can be bequeathed to the next generation. The parameterisation is otherwise identical to the baseline.

Figure 9 shows the results of the simulation allowing for bequests. The impact of the fall in p_K is similar to that in the baseline.

6 Conclusion

This paper presents a model of 'secular stagnation' - persistently low real interest rates - driven by the interaction of life-cycle savings motives and an improvement in the technology for producing investment goods. The model is complementary to other explanations for low real interest rates that rely on demographics, emerging markets and inequality. Using standard parameter values and the observed path for capital goods prices over the past few decades, it is able to reproduce part of the rising-falling pattern in real interest rates, the falling ratios of nominal investment and capital to GDP, and the rise in household debt observed across the industrialised world. The dynamic simulations predict that the investment rate will recover somewhat over coming decades, but that the real interest rate will stay low.

Later versions of this model will feature a more granular account of the life-cycle, incorporate demographic shocks, deal with open-economy considerations, and consider optimal fiscal and macroprudential policy responses.

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7 Tables and charts

Table 1: Coefficient of regression of investment rate on relative investment price

Sample	Full		EU-KLEMS subset	
Estimator	OLS	Robust	OLS	Robust
$\hat{\beta}$	0.34***	0.29***	0.20***	0.20***
se$[\hat{\beta}]$	0.0624	0.0046	0.0222	0.024
num obs	37	37	9	9

Figure 1: World real interest rate

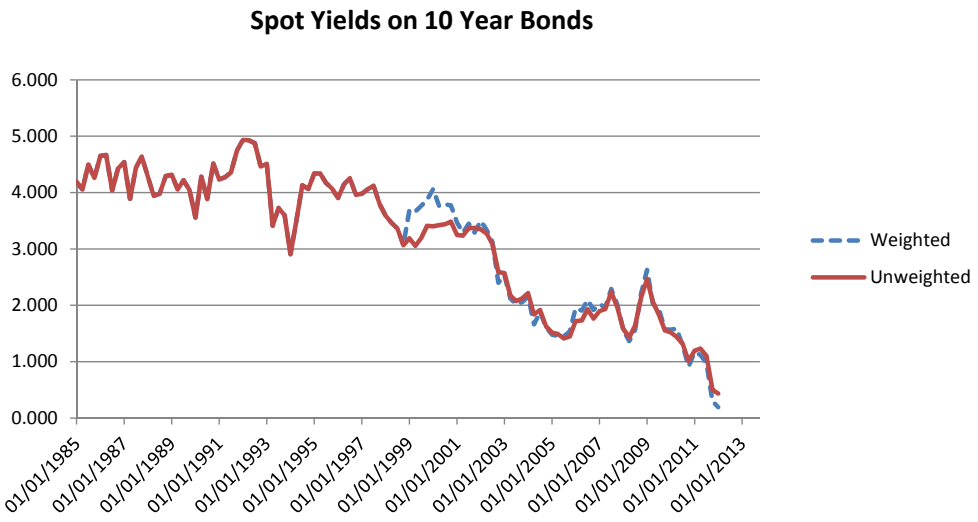

Figure 2: HH debt-GDP ratio, % of GDP

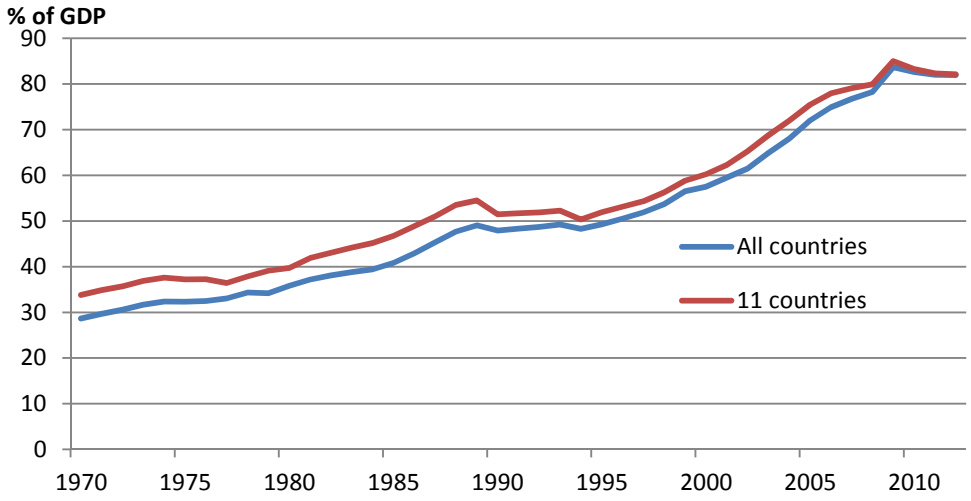

Figure 3: Nominal investment-GDP ratios

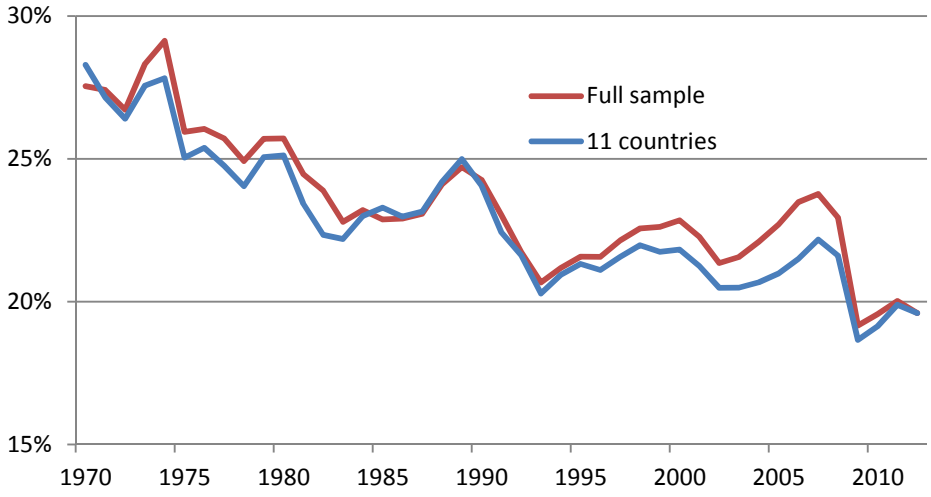

Figure 4: Nominal and real capital stock-GDP ratio

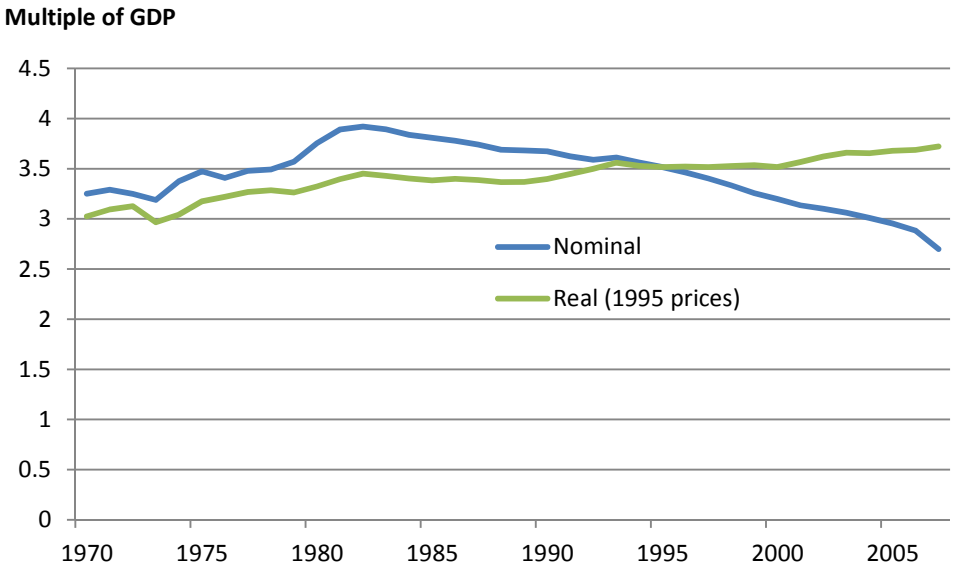

Figure 5: Real capital-GVA ratio, 11 industrialised countries, 2007=1

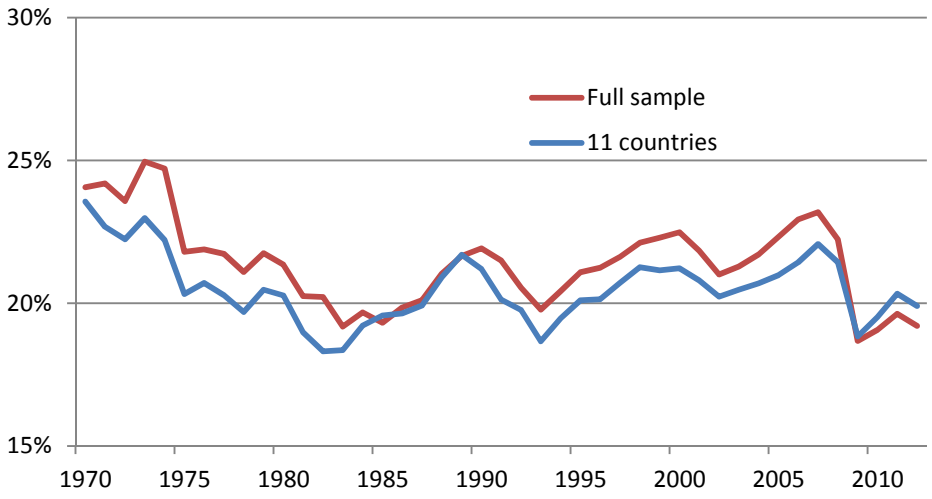

Figure 6: Price of investment relative to consumption

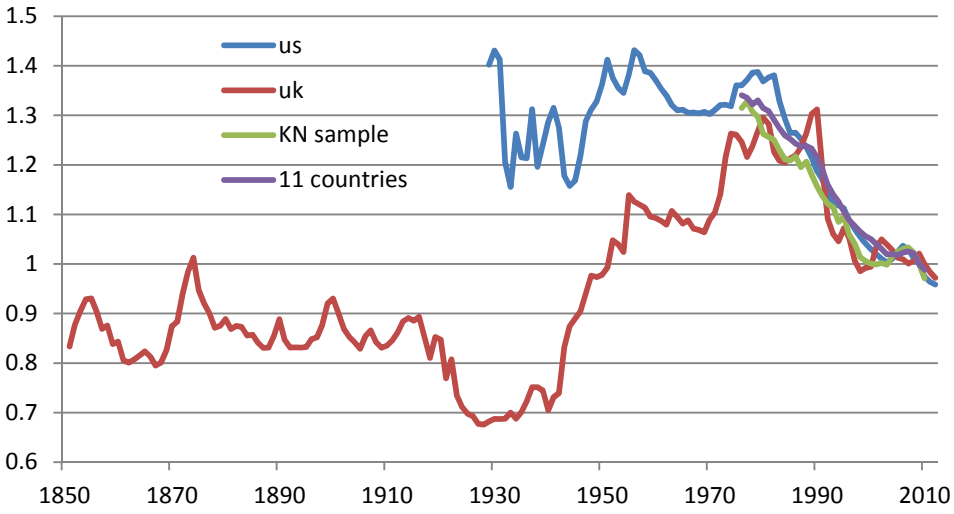

Figure 7: Baseline results

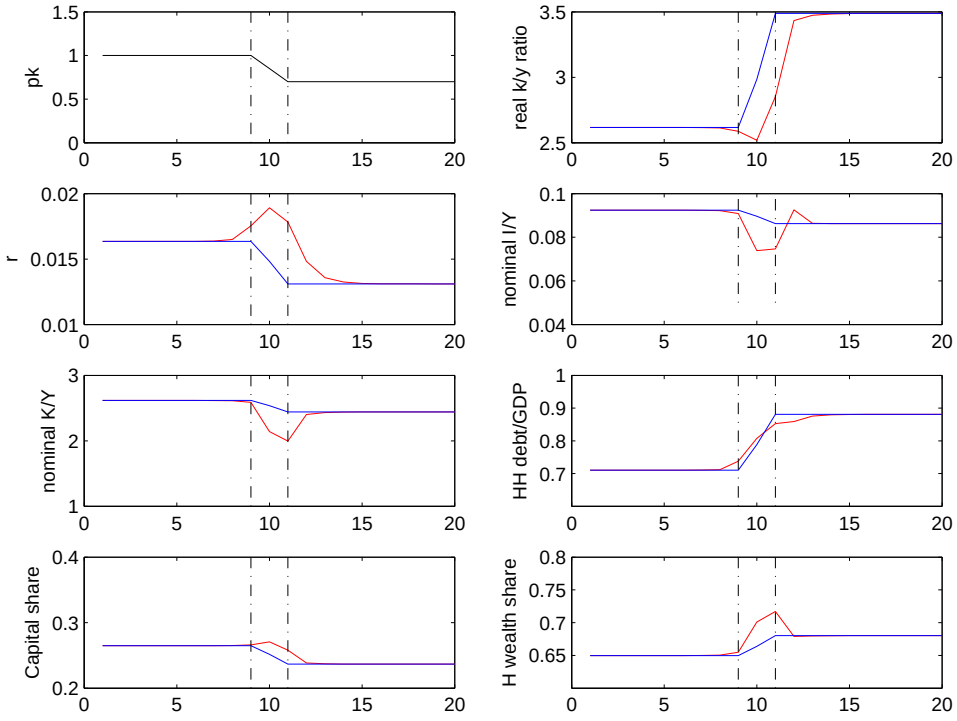

Figure 8: Highly elastic capital-labour substitution ($\sigma = 1.3$)

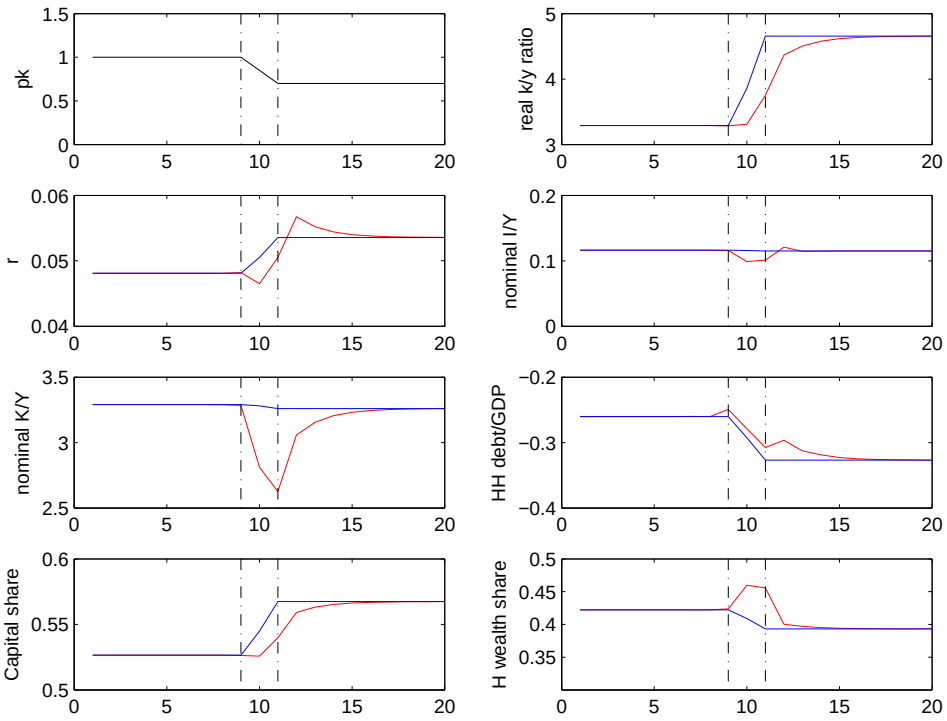

Figure 9: Bequests ($\xi = 0.75$)