Advertising and Business Cycle Fluctuations*

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Abstract

This paper provides new empirical evidence regarding quarterly U.S. aggregate advertising expenditures, showing that advertising follows a well-defined pattern over the course of the business cycle. To understand this pattern, we develop a general equilibrium model where targeted advertising increases the marginal utility of the advertised good. Advertising intensity is endogenously determined by profit maximising firms. This assumption is embedded into an otherwise standard model of the business cycle with monopolistic competition. We find that advertising significantly affects the aggregate dynamics, and that it exacerbates the welfare costs of fluctuations for the consumer. Finally, we estimate the model to test our results.

JEL Classification: E32, D11, J22, M37

Key words: Advertising, DSGE model, Business Cycle Fluctuations, Bayesian estimation, Habits persistence.

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"... as a matter of fact, the scale of expenditures on advertising varies positively with the general level of economic activity, so that, insofar as the effect of marginal expenditures is positive, advertising itself tends to accentuate the amplitude of economic fluctuations..."

Nicholas Kaldor (1950)

1 Introduction

In 2005 firms spent 230 billion dollars to advertise their products in the U.S. media, around 1000 dollars per U.S. citizen. The U.S. advertising industry accounts for 2.2% of GDP, absorbs around 20% of firms’ budgets for new investments, and uses 13% of their corporate profits.\(^1\) Despite the sizeable amount of resources absorbed, advertising has traditionally been analysed in microeconomic contexts, receiving scarce attention in the macroeconomics literature. Advertising is typically viewed as a selling cost\(^2\) that potentially redistributes consumers’ demand across firms without affecting the total market size, and that therefore does not play any significant role in macroeconomic theory.\(^3\)

This paper challenges such opinions by arguing that advertising can have a significant impact on the aggregate dynamics after accounting for its effect on the demand for goods. The rationale for firms’ advertising decisions has been identified in the literature as the positive effect of advertisements on sales. Firms realise that the demand they face is not exogenously a product of consumers’ preferences, but instead that it can be tilted toward their own products through advertisements. The effectiveness of advertising in enhancing demand is not only revealed by firms’ willingness to spend money on it, but is also supported by a large number of empirical studies.\(^4\) Overall, a positive relationship between firms’ advertising and sales is widely accepted based on robust empirical evidence.

Building on this fact, we ask whether such relationships would hold in the aggregate. Since the reason for advertising is to increase consumers’ demand, as targeted advertising increases the sales of single goods, will aggregate advertising enhance aggregate consumption? If so, will it also increase aggregate demand and production? Moreover, does advertising affect other aspects of the aggregate economy? In analysing aggregate advertising, we first focus on the relationship between advertising and consumption because of the pivotal role that consumption plays in assessing the impact of advertising on the aggregate dynamics. As we will show, aggregate consumption is the main avenue by which a variation in aggregate advertising can have an economy-wide effect. If this causative channel is shut down, the macroeconomic effect of aggregate advertising becomes negligible.

\(^1\)Statistics refer to the year 2005. Investments are fixed non-residential investments (source: Bureau of Economic Analysis of the U.S.). Profits are taken from The Economist (Economic and Financial Indicators).

\(^2\)A selling cost is defined as a cost that firms bear in order to enhance demand, but that neither enters as a factor in the production function like investment in equipment and machinery does, nor affects production technology like R&D does.

\(^3\)From this perspective, advertising is intended as a combative and dissipative cost. However, it is interesting to note that the Industrial Organisation literature widely accepts the idea that advertising is market-enhancing at the industry level. For instance, see Friedman (1983) or Martin (1993, Ch. 6).

\(^4\)A survey of these studies can be found in Bagwell (2005) and Schmalensee (1972).
The question of whether aggregate advertising is a determinant of aggregate consumption has already been posed in the literature, and the widespread opinion is that it is not. Building on Solow (1968) and Simon (1970), macro-economists argued that it would be incorrect to assume aggregate advertising and aggregate consumption to have a causal relationship identical to that between targeted advertising and sales, since advertising raises a firm’s level of demand by stealing customers from competitors, not by increasing the overall size of markets. Because of this “competition” effect, advertising affects the composition but not the size of aggregate consumption. This view is usually referred to as spread-it-around advertising. In the literature, however, there is also an opposite view that supports the enhancing effect of advertising on aggregate consumption, the market-enhancing hypothesis (Galbraith, 1958). Several papers have attempted to empirically test the relationship between advertising and the amount of consumption, among them Ashley, Granger, and Schmalensee (1980), Jacobson and Nicosia (1981), or more recently, Jung and Seldom (1995). Despite the large amount of empirical evidence considered, none of these studies were conclusive. Additionally, the literature lacks a theoretical model that could be used to analyse aggregate advertising such as the one developed in this paper, which reveals evidence that a positive relationship between advertising and consumption alone is not enough to predict the overall effect of advertising on aggregate demand. Moreover, once we assume such a relationship to hold, we find that advertising has several other significant effects on equilibrium.

This paper analyses aggregate advertising using a general equilibrium model that incorporates the two hypotheses mentioned above, and uses the model with a twofold objective. First, we aim to analyse the effect of advertising on the aggregate dynamics under each of the two hypotheses. While the impact of spread-it-around advertising has been shown to be negligible in the aggregate, market-enhancing advertising can have a significant impact by generating a work and spend cycle, where a consumer who wants to consume more because of the advertising incentive but faces an intertemporal budget constraint ends up working more hours. Second, we aim to estimate the model parameters in order to test which hypothesis fits better with U.S. postwar macroeconomic data. The results show that aggregate advertising does affect aggregate consumption, as originally suggested by Galbraith.

This paper considers advertising as a way of manipulating consumer’s preferences. As in Dixit and Norman (1978) and Benhabib and Bisin (2002), advertising is modeled as increasing the marginal utility of the advertised good through a modification of parameters in the utility function. Note, however, that this assumption by itself is not a sufficient condition to conclude that aggregate advertising enhances aggregate demand. If the consumer used savings to pay for the extra consumption generated by advertisements, then advertis-

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5 There are some exceptions in this regard. Benhabib and Bisin (2002, manuscript) analyse under which conditions advertising can affect the aggregate labour supply in a neoclassical general equilibrium model, and Grossmann (2007) studies the link between advertising and in-house R&D expenditures in a quality-ladder model of endogenous growth.

6 There is controversy over how to integrate advertising into consumer’s choice theory. In general, there are three different views in the literature about what advertising does: the Persuasive, the Informative, and the Complementary views. See Bagwell (2005) for an excellent survey. Taste-shifter advertising as it is modelled here fits with the Persuasive view of advertising as originally proposed by Marshall (1890,1919), Chamberlain (1933), Robinson (1933), and Kaldor (1950) and as used later on by Dixit and Norman (1978) and Benhabib and Bisin (2002).
ing would at the same time increase consumption and crowd out investments, and the net effect on the demand would be unclear. Also, if advertising shifted purchases towards more expensive goods, then an increase in advertising could imply a reduction in real consumption, and therefore in the aggregate demand. Moreover, advertising is not just a matter of demand; it can affect economic activity in various ways—for instance, increasing substitutability among goods and therefore affecting the market power of firms (price effects), or in a dynamic framework, reducing consumer’s savings and hence reducing future demand.

In order to cope with all the effects mentioned above, we embed the candidate utility function with advertising into a dynamic stochastic growth model with monopolistic competition, which is then simulated to analyse the general equilibrium effects of advertising. In general, advertising absorbs resources, can increase firms’ monopolistic power, and can eventually shift upward the responses of consumption, labour, and output to exogenous shocks. In particular, we find that market-enhancing advertising operates through three channels. The first one is the work and spend cycle: in the presence of advertising, people work more in order to be able to afford greater consumption, where the perceived need for higher consumption is due to the advertising signals they are exposed to. The second mechanism operates through prices. Advertising increases firms’ markup, therefore reducing consumer’s wages, and with all else being equal, the quantity of labour supplied. The third operates through the resource constraint. By absorbing resources, advertising puts a wedge between gross production and net GDP, which is defined as consumption plus investment.

We show that for a reasonable set of calibrations, the first mechanism prevails over the other two. At equilibrium, both labour and output increase, where part of the extra production is used to produce advertising and the rest is sold as consumption. As a consequence, after an exogenous shock, the responses of consumption and labour from the model are larger than those from a benchmark model economy where advertising is banned. Thus, advertising could tend to accentuate the amplitude of business cycle fluctuations, as argued by Kaldor. We quantify the impact of advertising on fluctuations by comparing the welfare costs of fluctuations when firms can advertise their products to those when advertising is banned. The welfare analysis points out that a 2% of GDP level of spending on advertising increases the cost of fluctuations to the consumer by 124%.

The method we use to include advertising in the utility function is akin to that used in the macroeconomic literature on consumption habits. Like external deep habits, advertising creates dissatisfaction in the consumer about his actual level of consumption, pushing him to buy more. This modelling strategy has the side result that the optimal demand for goods for consumption turns out to be a function of past sales, as in the model with deep habits or in customers’ market models. The result appears particularly appealing because it rationalises the persistence observed in actual data of consumption within a context of profit-maximising firms with rational representative consumer. From this perspective, advertising improves on models with habits because consumption persistence arises endogenously at equilibrium based on the interaction between firms’ optimal advertising policies and consumer’s optimal demand for goods, while in the case of habits it is exogenously assumed in the utility function.

The paper is structured as follows. Section 2 characterises the cyclical behaviour of

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quarterly aggregate advertising expenditures in the U.S. postwar economy. Section 3 introduces the DSGE model for advertising and shows that firms optimally use advertising as a complementary tool for price-setting. This section also presents the side contribution of a dynamic version of the Dorfam-Steiner (1954) theorem of optimal advertising spending with monopolistic competitive markets. Simulation results are reported in Section 4. Section 5 analyses the implications of advertising on the welfare cost of fluctuations, and Section 6 estimates a log-linearised version of the model to test for the effect of advertising on aggregate consumption. Finally, a counterfactual exercise and the variance decomposition of the estimated model are used to show that advertising improves the internal propagation mechanism of the standard real business cycle model. Section 7 presents our conclusions.

2 Stylized Facts

In what follows we define aggregate advertising as the total spending of domestic and foreign firms that advertise their products in U.S. media. Quarterly data for aggregate advertising are not included among standard business cycle indicators. Appendix A lists the sources used to collect the data. The resulting database is novel in the literature, and is to our knowledge the only up-to-date free-of-charge quarterly series for U.S. aggregate advertising.\(^8\) Our data report firms’ expenditures for advertisements in 7 media types, namely cable and network television, radio, newspapers, magazines and Sunday magazines, billboards, direct mail, and outdoor advertising. The sample starts in the first quarter of 1976 and ends in the second quarter of 2006 (122 quarters).

In order to check whether the series provided is actually representative of all U.S. aggregate advertising expenditures, we compute the cumulative yearly expenditures from our data set and compare them with annual data for total advertising expenditures constructed by Robert Coen of Universal McCann; advertising experts consider this to be the most reliable source of data on aggregate advertising. In the considered sample, our series accounts on average for 30% of Coen’s aggregate advertising, with a minimum of 25%, and an in-sample standard deviation of 2.95%.

Coen’s annual data are also useful in assessing the magnitude of aggregate advertising. Figure (1) plots the ratio of advertising over GDP (panel 1), which measures the relative importance of advertising as a component of GDP, and per-capita real advertising expenditures (panel 2), which are commonly used in the literature as a measure of the number of advertising messages that reach the consumer — i.e., a proxy for the intensity of advertising in the economy. The first statistics fluctuate around 2.1% throughout the sample, with a maximum peak in year 2000, while the second show a steady and strong upward trend, implying that the number of advertising messages per individual has constantly grown during the second half of the last century.

The novel series of quarterly data is used in figure (2) to represent the cyclical component of real advertising expenditures along with that of real GDP, real total consumption, and

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\(^8\)The U.S. Federal administration used to collect quarterly data for aggregate advertising, but it stopped after 1968 when advertising was dismissed from the list of relevant variables used by the Fed to analyse the cycle.
Figure 1: Advertising in Postwar U.S. economy. Panel 1. Advertising as share of GDP. Panel 2. Per-capita real advertising. Coen’s annual data, sample from 1948 to 2005.

real fixed private investment.\(^9\) Basically, the figure shows that (i) advertising is pro-cyclical; and that (ii) it is more volatile than GDP and consumption and less so than investment. Table 1 reports some related statistics, which confirm these findings: advertising displays a high and positive correlation with GDP (0.59), and it is 2.62 times more volatile than GDP. In addition, it appears to be very persistent over the cycle, with a point estimate of first-order autocorrelation of 0.89. Besides, the positive correlation (0.26) between the advertising-GDP ratio and GDP itself suggests that advertising cannot be simply assumed as a constant proportion of output.

Regarding the other aggregates, advertising displays the strongest correlation with total consumption expenditures (0.68), and it has a very high standard deviation, with a point estimate 3.64 times higher than the one for consumption. Specifically, advertising is 4 times more volatile than non-durable consumption, slightly more volatile than expenditures in durable goods (the relative standard deviation is equal to 1.12), and 23% less volatile than investment.

Since we have only a partial series of aggregate advertising expenditures, we check the robustness of the previous findings by computing the same statistics with Coen’s annual data. Results are provided in the second panel of Table 1. Annual data confirm the quarterly

\(^9\)All the quarterly figures used in this section are in logs and per capita units. In figure (2), the cyclical components have been extracted using a Band Pass (BP) filter with 6-32 as bands. For advertising, we previously eliminated the seasonal component from raw data using the X11 filter. Also, to control for spurious facts, we calculated all the statistics in this section with both BP and Hodrick-Prescott filters. The main empirical evidence presented hereafter does not change when one or the other filter is used.
Figure 2: Advertising and the main Business Cycle Indicators. Quarterly figures. Data sample from 1976q1 to 2006q2.

evidence: aggregate advertising is pro-cyclical – $\text{corr}(\text{Adv}_t, \text{GDP}_t) = 0.72$ –, and more volatile than GDP – $\sigma(\text{Adv}_t)/\sigma(\text{GDP}_t) = 1.62$.

Finally, we analyse the dynamic cross-correlations between advertising, GDP, consumption, and investment. Dynamic correlations are useful in providing empirical evidence in order to support or dismiss the idea that advertising can be a leading indicator of the cycle. As we see from Table 2, advertising only slightly leads GDP: the cross-correlation coefficient is almost the same at $k=0$ (0.59) and $k=1$ (0.60). Also, advertising appears to move contemporaneously with consumption (i.e., the strongest correlation occurs at $k=0$), while it strongly leads investment (higher correlations occur at $k=-2$ and $k=-1$). Overall, the dynamic cross-correlations seem to dismiss the idea that advertising can be used as a leading indicator of the cycle. The fact that advertising slightly leads GDP could be due to the fact that it moves with consumption, which itself has been shown to slightly lead GDP in actual data.\footnote{This evidence is not clear in our data, where the correlation between consumption and output is almost the same at $k=0$ and $k=1$, but it has been analysed and supported in several papers, e.g. Wen and Benhabib (2004).}

Overall, the main findings of this section can be summarised as follows:

- The amount of resources invested in advertising in the U.S. accounts for roughly 2% of GDP.
- Advertising is strongly procyclical and positively correlated with both consumption and investment.
Table 1: **Second order moments**

<table>
<thead>
<tr>
<th>$X_t$</th>
<th>$\frac{\sigma(X_t)}{\sigma(GDP_t)}$</th>
<th>$corr(X_t, Adv_t)$</th>
<th>$corr(X_t, GDP_t)$</th>
<th>$corr(X_t, X_{t-1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarterly Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Advertising</td>
<td>2.62</td>
<td>1</td>
<td>0.59</td>
<td>0.90</td>
</tr>
<tr>
<td>GPD</td>
<td>1</td>
<td>0.59</td>
<td>1</td>
<td>0.93</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.72</td>
<td>0.68</td>
<td>0.91</td>
<td>0.94</td>
</tr>
<tr>
<td>Non-Dur.</td>
<td>0.60</td>
<td>0.67</td>
<td>0.79</td>
<td>0.93</td>
</tr>
<tr>
<td>Durables</td>
<td>2.33</td>
<td>0.60</td>
<td>0.90</td>
<td>0.92</td>
</tr>
<tr>
<td>Investment</td>
<td>3.41</td>
<td>0.64</td>
<td>0.93</td>
<td>0.94</td>
</tr>
<tr>
<td>$\frac{Adv}{GDP}$</td>
<td>2.18</td>
<td>0.93</td>
<td>0.26</td>
<td>0.88</td>
</tr>
<tr>
<td>Annual Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>1</td>
<td>0.72</td>
<td>1</td>
<td>0.08</td>
</tr>
<tr>
<td>Advertising</td>
<td>1.62</td>
<td>1</td>
<td>0.72</td>
<td>0.12</td>
</tr>
<tr>
<td>$\frac{Adv}{GDP}$</td>
<td>1.14</td>
<td>0.79</td>
<td>0.15</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Note: $\sigma(\cdot)$ is in-sample standard deviation. Annual data have been detrended using the BP(2,8)

Table 2: **Dynamic cross correlations.**

<table>
<thead>
<tr>
<th>$corr\left(X_t, GDP_{t+k}\right)$</th>
<th>$k$</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advertising</td>
<td></td>
<td>0.01</td>
<td>0.20</td>
<td>0.38</td>
<td>0.52</td>
<td>0.59</td>
<td>0.60</td>
<td>0.55</td>
<td>0.47</td>
<td>0.38</td>
</tr>
<tr>
<td>Consumption</td>
<td></td>
<td>0.16</td>
<td>0.39</td>
<td>0.62</td>
<td>0.81</td>
<td>0.91</td>
<td>0.90</td>
<td>0.78</td>
<td>0.58</td>
<td>0.35</td>
</tr>
<tr>
<td>Investment</td>
<td></td>
<td>0.27</td>
<td>0.54</td>
<td>0.76</td>
<td>0.91</td>
<td>0.93</td>
<td>0.84</td>
<td>0.66</td>
<td>0.42</td>
<td>0.18</td>
</tr>
</tbody>
</table>

| $corr\left(X_t, Adv_{t+k}\right)$ | | | | | | | | | | |
|-----------------------------------| | | | | | | | | | |
| Consumption | | 0.35 | 0.46 | 0.57 | 0.65 | 0.68 | 0.63 | 0.51 | 0.34 | 0.13 |
| Non-Dur. | | 0.34 | 0.47 | 0.59 | 0.67 | 0.67 | 0.60 | 0.46 | 0.28 | 0.08 |
| Durables | | 0.16 | 0.26 | 0.38 | 0.50 | 0.60 | 0.64 | 0.58 | 0.44 | 0.25 |
| Investment | | 0.51 | 0.63 | 0.70 | 0.71 | 0.64 | 0.51 | 0.32 | 0.12 | -0.09 |
• Advertising is highly volatile, more volatile than GDP and consumption, but less volatile than investment. Also, it is persistent over the cycle.

3 A DSGE model with Advertising

This section describes the model economy and displays the problems of households and firms. The market consists of a continuum of differentiated goods produced by monopolistically competitive producers that possess the technology to advertise their products. Advertising is assumed to generate an urge to consume the advertised good. We obtain this effect by introducing advertising as an argument of the utility function that is complementary to consumption (we support this modelling strategy in section 3.5). We then embed the modified utility function with advertising into an otherwise standard dynamic stochastic growth model with no nominal or real friction, and we study the dynamics of this model in reaction to: (i) a shock to production technology; (ii) a shock to preferences; (iii) a shock to exogenous government spending; (iv) an idiosyncratic shock to the production of advertising.

3.1 The household and the role of advertising

We assume that a representative consumer exists with preferences defined for consumption and hours worked, which are described by the utility function:

\[ U(\tilde{C}_t, H_t) = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \tilde{C}_t^{1-\sigma} - \frac{1}{1-\sigma} \xi_t - \frac{\phi_t H_t^{1+\phi}}{1+\phi} \right] \]  

(1)

where \( \tilde{C}_t \) is the consumption aggregate, \( H_t \) is the time devoted to work, and \( \xi_t \) is a preference shock. The composite consumption aggregate \( \tilde{C}_t \) is defined as follows:

\[ \tilde{C}_t = \left( \int_0^1 \left( c_{i,t} + B(g_{i,t}) \frac{\xi_i}{\epsilon} \right) di \right)^{1-\epsilon} \]  

(2)

where \( \epsilon > 1 \) is the pseudo-elasticity of substitution across varieties; \( g_{i,t} \) is the goodwill associated with good \( i \), where goodwill is meant to represent the stock of the firm’s advertising accumulated over time; and \( B(\cdot) \) is a decreasing and convex function controlling for the impact of goodwill on consumer’s preferences, satisfying \( B(0) = a \geq 0. \) We introduce the concept of goodwill because several empirical studies have shown that advertising campaigns affect product sales for several periods, evidence that seems robust across different goods, countries, and time periods.\(^{11}\)

\(^{11}\)The consumption aggregate (2) is a Stone-Geary-type non-homothetic utility function. Depending on whether the term \( B(g_{i,t}) \) is assumed to be positive or negative, the utility displays a saturation point or a subsistence level with respect to each variety consumed.

\(^{12}\)In particular, see Clarke (1976) for an empirical study of the dynamic effects of advertising in the U.S. and Bagwell (2005) for a survey.
Building on Arrow and Nerlove (1962), we model the dynamic effect of advertising by assuming that current and past advertising combine to create a reputation for a good, the producer’s goodwill, which is defined as the intangible stock of advertising that affects the consumer’s utility at time \( t \), as shown in (2). The stock of goodwill evolves according to the law of motion:

\[
g_{i,t} = z_{i,t} + (1 - \delta_g) g_{i,t-1}
\]

where \( z_{i,t} \) is a firm’s investment in new advertising at time \( t \) and \( \delta_g \in (0, 1) \) is the depreciation rate of the goodwill. The law of motion (3) implies that current sales could be affected not only by current advertising expenditures, but also by past advertising, with a decreasing intensity over time.

In this setup, the positive link between the producer’s goodwill and sales operates through the marginal utility of consumption. Notice that from (2) follows:

\[
\frac{\partial^2 \tilde{C}_t}{\partial c_{i,t} \partial g_{i,t}} \propto -\frac{1}{\varepsilon} (c_{i,t} + g_{i,t})^{-1/(\varepsilon+1)} B'(g_{i,t}) \geq 0
\]

where the last inequality comes from the assumption that \( B(\cdot) \) is decreasing in \( g_{i,t} \). This setup reflects what is known in the literature as the persuasive role of advertising: advertisements create some added value for the good that would otherwise not exist. Consequently, the promoted good is worth more to consumers, as if it were a new or different good. The intuition behind this effect is that advertising creates dissatisfaction in the consumer about his current level of consumption. The consumption aggregate (2) is modelled in the spirit of the "catching up with the Joneses" hypothesis of Abel (1990), or better, is based on the single-good habits version proposed by Ravn Schmitt-Grohe and Uribe (2006).\(^{13}\)

The rest of the model is standard. We assume that the representative consumer holds one asset, the capital stock \( K_t \), which he rents to firms, and that he supplies labour services per unit of time. Labour and capital markets are perfectly competitive, with a wage \( W_t \) paid per unit of labour services and a rental rate \( R_t \) paid per unit of capital. In addition, the consumer receives net profits \( \Pi_t \) from firms and pays lump sum taxes \( T_t \) to finance the exogenous government spending. Under these assumptions, the representative agent’s nominal budget constraint is defined as:

\[
\int_0^1 p_{i,t} (c_{i,t} + i_{i,t}) \, di \leq W_t H_t + R_t K_{t-1} + \Pi_t - T_t
\]

The utility maximisation problem for the representative consumer can be stated as a matter of choosing the processes \( \tilde{C}_t, H_t \) in order to maximise the utility function (1) subject to the standard law of the motion of capital, i.e., \( K_{t+1} = (1 - \delta_k) K_t + I_t \), and the budget constraint (5).\(^{14}\) Note that in our setup the consumer does not choose the desired goodwill,

\(^{13}\)As in the case of external habits, goodwill works in the utility as a negative externality for the consumer. With respect to other theories of advertising, one advantage of this modelling strategy is that it allows advertising to affect consumer behaviour maintaining a certain analytical tractability when solving for the general equilibrium.

\(^{14}\)To solve the maximisation problem, it is useful to write the budget constraint in the Lagrangian as a function of \( \tilde{C}_t, I_t \). Note that at the optimum, \( \int_0^1 p_{i,t} i_{i,t} \, di = P_t I_t \) and \( \int_0^1 p_{i,t} c_{i,t} \, di = P_t \tilde{C}_t - \int_0^1 p_{i,t} g_{i,t} \, di \).
but instead passively receives the whole amount of advertising determined by the firms.\footnote{This feature distinguishes our model from Becker’s (1993) complementary theory of advertising. Following the Persuasive view of advertising, we assume that the agent passively receives the advertising signals without being aware of the effect they have on his preferences. On the contrary, in Becker (1993), the agent actively demands the informative content of advertising, since it raises the utility he gets from consumption.}

The first-order conditions for an interior maximum are:

$$\frac{\tilde{C}_t}{P_t} = \lambda_t$$

(6)

$$\lambda_t = \beta E \{ \lambda_{t+1} [ R_t + (1 - \delta_k)] \}$$

(7)

$$\xi_t H_t^\phi = W_t \lambda_t$$

(8)

where $\lambda_t$ is the Lagrange multiplier associated with the budget constraint, and $P_t$ is the aggregate price index. Equation (7) is the familiar Euler equation that gives the intertemporal optimality condition, while equation (8) describes the labour supply schedule.

The optimality conditions (6), (7), and (8) mimic those of the standard neoclassical growth model, but with the remarkable difference that the definition of the shadow price $\lambda_t$ depends not only on aggregate consumption but also on aggregate goodwill. Consequently, consumer’s decisions about labour and investment are affected by the level of aggregate advertising.\footnote{In particular, insofar as $\tilde{C}_t$ has a negative first derivative with respect to the aggregate goodwill, then advertising will increase both the marginal utility of aggregate consumption and the opportunity cost of leisure.}

This mechanism plays a pivotal role in determining the general equilibrium results that we will explore in the next section. A partial equilibrium analysis is useful for understanding how advertising affects demand. Suppose, for instance, that advertising expenditures increase exogenously for a sufficiently large fraction of firms. Given our assumptions, $\int B (g_{i,t}) \, di$ decreases, and as a consequence, the consumer’s shadow price $\lambda_t$ increases. Consider now the labour supply schedule (8). An increase in $\lambda_t$ implies that the agent values consumption more than leisure, since for any given wage the marginal rate of substitution increases. Hence, the labour supply schedule shifts to the right, or the agent is willing to work more in order to consume more.

An increase in $\lambda_t$ also affects the consumer’s saving decisions by changing the intertemporal elasticity of substitution in the Euler equation (7). However, since (7) is a function of the ratio of current $\lambda_t$ over future $\lambda_{t+1}$ marginal utility, the sign of the effect of higher advertising depends on the relative response of current over future goodwill. In this simple example, the eventual effect is easily predictable. The goodwill is an $AR(1)$ process, and we assumed a one-time increase in advertising: current consumption will increase. In general, an increase in advertising due to an exogenous shock, while unambiguously shifting the labour supply to the right, has an effect on the saving function that is determined by the dynamic response of expected future goodwill to a shock, which itself depends on several different general equilibrium effects that combine together. In particular, however,
whenever the growth rate of the goodwill is positive, the consumer finds it more convenient to postpone his consumption, since he foresees that his marginal utility will be higher in the future. Conversely, when the growth rate of the goodwill is negative, the consumer experiences an urge to consume and increases his demand for current consumption.

Overall, this analysis suggests that from the standpoint of a consumer, aggregate advertising can be interpreted as an exogenous state variable that modifies its own supply of labour and savings modifying, respectively, the elasticity of the wage and the intertemporal elasticity of substitution.

3.2 Firms

There is a continuum of firms indexed \( i \in [0, 1] \), each producing a differentiated product that is sold as an item for consumption, an investment, or a government good.

The optimal demand function for consumption goods is the solution to the consumer’s problem of minimising consumption expenditures subject to the aggregate constraint (2), i.e.,

\[
    c_{i,t} = \max \left\{ \left( \frac{p_{i,t}}{P_t} \right)^{-\varepsilon} \hat{C}_t - B(g_{i,t}) ; 0 \right\}
\]

where

\[
P_t = \left[ \int_0^1 p_{i,t}^{1-\varepsilon} di \right]^{1/(1-\varepsilon)}
\]

is the nominal price index. Equation (9) has two key implications for this paper, which we shall explore in turn.

Firstly, the demand for consumption goods increases with the level of advertising. A positive investment in \( z_{i,t} \) raises the stock of goodwill \( g_{i,t} \), which in turns decreases \( B(g_{i,t}) \), thus shifting the demand function (9) to the right. The prediction of a positive relationship between sales and advertising is in line with a large number of empirical studies about advertising at the firm level,\(^{17}\) and in our model derives from the assumption that advertising enters into the utility function. However, it is worth noting that this assumption is not arbitrary once we restrict our attention to models with Walrasian demand functions and perfect information. In this case, the only way that advertising can enhance demand is through a modification of the preference relation.\(^{18}\)

\(^{17}\) Actually, a positive relationship between advertising and sales is one of the few non-controversial pieces of evidence regarding advertising. See Bagwell (2005), section 3.2, for more references.

\(^{18}\) The argument proceeds by contradiction. First, recall that advertising according to our assumptions is not a productive factor, nor does it affect the production technology. As a result, it does not alter the quality of goods, thus implying that pre- and post-advertising, the consumer chooses among the same bundle of goods. Moreover, the condition that advertising does not affect the marginal cost, again because it does not enter into the production function, together with the assumption of perfect information, rules out any direct effect of advertising on prices. In this case, any bundle the agent chooses post-advertising must also have been affordable pre-advertising. Now, suppose that advertising shifts the demand, meaning that the consumer chooses two different bundles of goods pre- and post-advertising, but that the preferences relation remains unchanged pre and post advertising. Since the bundle chosen post-advertising was affordable pre-
Secondly, the price elasticity of the demand diminishes with the level of advertising. Specifically, the demand function (9) is composed of two terms: the first one, $\left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \tilde{C}_t$ with elasticity $\varepsilon$, and the second one $B(g_{i,t})$, which is inelastic. Overall price elasticity is then a weighted average between the elasticity of these two terms, and we can show that its value will depend on the relative importance of the goodwill over the total demand, i.e.,

$$\eta(c_{i,t}, g_{i,t}) = \left| \frac{\partial c_{i,t}}{\partial p_{i,t}} \frac{p_{i,t}}{c_{i,t}} \right| = \varepsilon \left( 1 + \frac{B(g_{i,t})}{c_{i,t}} \right) \quad (11)$$

In particular, notice that the elasticity of demand (11) is always smaller than the elasticity of the demand without advertising, i.e., with $g_{i,t} = 0$, since $B(g_{i,t})$ is decreasing over $g_{i,t}$. This feature of the model replicates a well-known effect of advertising in the literature: firms advertise their products to develop consumers’ loyalty. The intuition is that advertising, although it does not modify the quality of the advertised good, increases the differentiation among goods perceived by consumers. Thus, firms can use advertisements to manipulate the elasticity of the demand, thus increasing market power, and eventually profits.

The goods produced by firms are sold as consumption, investment, and government purchases. Unlike consumption, investments and government purchases are assumed not to be affected by advertising. The assumption about the demand for investment goods fits naturally into our setup because by assumption we modelled a positive effect of advertising on consumer’s willingness to consume, whereas investment represents the alternative option for the consumer who does not want to consume. The second assumption about government spending is conservative with respect to the results we will find, since it can be shown that modelling a positive effect of advertising on government expenditures would strengthen the effect of advertising on the aggregate dynamics.

Altogether, the demand for consumption $c_{i,t}$, investment $i_{i,t}$, and government expenditures $f_{i,t}$ forms the total demand of firm $i$ at time $t$, i.e.:

$$y_{i,t} \equiv c_{i,t} + i_{i,t} + f_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \left( \tilde{C}_t + I_t + F_t \right) - B(g_{i,t}) \quad (12)$$

Accordingly, firm $i$ chooses a price for sales and a level of advertising in order to maximise the discounted flow of future profits subject to the constraint given by (12). The optimal policy rules, which are derived formally in Appendix B, are:

$$p_{i,t} = \frac{\varepsilon \left( 1 + \frac{B(g_{i,t})}{y_{i,t}} \right)}{\varepsilon \left( 1 + \frac{B(g_{i,t})}{y_{i,t}} \right) - 1} \varphi_t \equiv \mu_{i,t} \phi_t$$

$$- (p_{i,t} - \varphi_t) B'(g_{i,t}) + E_t \left[ \left( 1 - \delta_g \right) \left( \nu_{i,t+1} r_{t+1,t+1} \right) \right] = \nu_{i,t} \quad (13)$$

advertising, it must yield lower utility than the one chosen pre-advertising, since the preference relation is unchanged. As a result, post-advertising the agent is choosing a bundle that is not preferred to the pre-advertising one, violating the Weak Axiom of Revealed Preferences. Hence, if the agent chooses two different bundles pre and post advertising, then the preferences relation must change pre- and post-advertising, which justifies the assumption of advertising as an argument of the utility function. Note that, in general, for this argument to hold true, the model utility function must be derived from a strictly convex preference relation.

These demands are derived in Appendix B.
where \( \varphi_t \) is the marginal cost of production and \( \nu_{i,t} \) is the marginal cost of producing new advertising \( z_{i,t} \).

Equation (13) describes the familiar pricing policy in monopolistic competition models: the firm exploits its monopolistic power by charging a positive markup \( \mu_{i,t} \) over the marginal cost. Unlike the standard case, \( \mu_{i,t} \) is not constant but increases with the level of goodwill due to the negative relation between price elasticity (11) and goodwill.

Equation (14) describes the optimal advertising policy. It states that the firm invests in advertising until the marginal benefit from an extra dollar of advertising equals the marginal costs of producing it. Given the dynamic nature of goodwill, the marginal benefit on the LHS of (14) has two components: the increase in current revenues associated with a marginal increase in advertising, and the discounted opportunity cost of not producing tomorrow the surviving goodwill produced today.

According to (14), advertising is sensitive to variations in the conditions of both supply and demand. On the one hand, reductions in marginal costs lead to higher investments in advertising. On the other hand, the marginal benefit of advertising depends on markup that is positively affected by aggregate demand (see equation 13). Hence, any exogenous increase in the demand simultaneously raises markup and advertising.

Besides, note that (13) and (14) together imply that advertising and price-setting are complementary policies, in accordance with the theory of optimal advertising as the outcome of firms playing a supermodular game, as shown in Tremblay (2005).

Interestingly, we can establish an equivalence result between the optimal advertising policy (14) and the seminal Dorfman-Steiner (1954) theorem about firms’ optimal spending on advertising, which states that the optimal budget for advertising expenditures is equal to the ratio between the elasticity of the demand with respect to advertising and the elasticity of demand with respect to price. The equivalence result is contained in the following proposition.

**Proposition 1.** Let the demand function for firm \( i \) be defined as in (12), and denote \( \eta_{g,t}(i) \) and \( \eta^*_{p,t}(i) \) as the elasticity of the demand with respect to goodwill and the elasticity of the demand with respect to price, respectively. Then, the optimal level of goodwill for firm \( i \) will be a proportion of the ratio of \( \eta_{g,t}(i) \) over \( \eta^*_{p,t}(i) \).

**Proof.** First notice that from (12) follows:

\[
-B'(g_{i,t}) = \eta_{g,t}(i) \frac{y_{i,t}}{g_{i,t}}
\]

Using this result into (14) to substitute out \( B'(g_{i,t}) \) and rearranging, it yields:

\[
g_{i,t} \frac{y_{i,t}}{y_{i,t}} = \eta_{g,t}(i) \left\{ \frac{p_{i,t} - \varphi_t}{\nu_{i,t} - E_t[1 - \delta_g(1 + r_{t+t+1})]} \right\}
\]

\[20\] Also, note that in the extreme case where advertising has no effect on demand, (i.e., \( B'(\cdot) = 0 \)), equation (14) implies that optimal advertising is equal to zero. Therefore, in this framework the only incentive for firms to advertise is the potential that they will have to manipulate demand. In particular, no strategic reason is considered, like for instance entry deterrence.
or, substituting out $p_{i,t}$ using the optimal pricing rule (13),

$$g_{i,t} = \frac{\eta_{g,t}(i)}{\eta_{p,t}^*(i)} \Omega_{i,t}$$

(15)

where $\Omega_{i,t} = \frac{\nu_{i,t} - E_{t-1}(1 - \delta_g)\nu_{i,t+1\mid t+1}}{\nu_{i,t} - \nu_{i,t}}$ and $\eta_{p,t}^*(i) = (\eta(y_{i,t}, p_{i,t}) - 1)$. Thus, the optimal goodwill intensity is proportional to the ratio $\eta_{g,t}(i)/\eta_{p,t}^*(i)$. \hfill $\Box$

It is straightforward to see that equation (15) is a general result that nests the Dorfman-Steiner theorem as a particular case when $\delta_g = 1$ and $\nu_{i,t} = \nu_{i,t}$, i.e., the goodwill fully depreciates in each period, and firms use the same technology to produce goods and advertising.

3.3 Advertising and consumption persistence

Another result embedded in equation (15) is that advertising generates persistence in the dynamics of consumption. To prove this claim, first note that at the symmetric equilibrium,$^{21}$ $\eta_{p,t}^*(i) = \eta_{p,t}^*(j)$, $\eta_{g,t}(i) = \eta_{g,t}(j)$, and $\nu_{i,t} = \nu_{j,t}$ $\forall$ $i, j$. Then, rewrite equation (15) as:

$$g_{i,t} = \Phi_t y_{i,t}$$

(16)

Using (16) lagged one period to work out $g_{i,t-1}$ from the law of motion of goodwill (3), and plugging the result into the demand function (9) to work out $g_{i,t}$, we obtain:

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{-\varepsilon} \left(\tilde{C}_t + I_t + F_t\right) + \psi(\Phi_t, y_{i,t-1}, z_{i,t})$$

(17)

where $\psi(\cdot)$ is a non-linear function with a non-negative partial derivative with respect to the last two arguments.$^{22}$

Equation (17) reveals that the demand faced by each producer depends on past sales, as in customers market models or models that include habits of consumption.$^{23}$ This result is determined by two properties of our setup: (i) the assumption that the consumer’s preferences are affected by advertising; and (ii) the dynamic nature of the goodwill. As noted in the introduction, advertising naturally rationalises the presence of consumption persistence within the theory of rational consumer and profit-maximising firms. Compared with other models that have explained consumption persistence, the advantage of using advertising is that persistence is endogenously determined at equilibrium based on the optimising behaviour of firms, once we explicitly incorporate into the model the observable and well-known phenomenon of advertising, instead of being derived from an ad hoc assumption like "the costumers" in the market or "the habits" in the utility.

$^{21}$See section 3.4.

$^{22}$This follows immediately from the fact that $B(\cdot)$ is assumed to be strictly decreasing together with the fact that non-negative optimal advertising requires $\Phi_t > 0$.

$^{23}$For more about this issue, see Ravn, Schmitt-Grohé and Uribe (2006), and the ”habit persistence” entry of the Palgrave Economic Dictionary, written by Schmitt-Grohé and Uribe (2006).
3.4 The Symmetric Equilibrium

In this model the market for production factors is perfectly competitive, and all firms share the same production technology. Thus, all firms face the same marginal cost.\footnote{The reader can check this by inspecting the RHS of equation (25).} Moreover, all goods have the same pre-advertising elasticity of substitution, i.e., $\varepsilon$. These two conditions imply together that a symmetric equilibrium exists where all firms set the same price, produce the same quantities, and invest the same amount of resources in advertising.\footnote{This equilibrium requires the extra assumption that the initial stock of goodwill is the same across firms.} In addition, the equilibrium (common) price of goods is normalised to unity in each period, i.e., $p_t = 1 \ \forall t$. Thus, all other prices in the model (e.g., wage, rental rate) are expressed in terms of contemporaneous consumption.

Let $X_t$ be the vector of all endogenous variables,\footnote{Specifically, $X_t = (\lambda_t, G_t, \mu_t, Z_t, H_t, H_{a,t}, H_{p,t}, C_t, K_t, I_t, Y_t, R_t, W_t, Q_{t,t+1})$.} then the symmetric equilibrium is a process $\{X_t\}^{\infty}_{t=0}$ that satisfies: (6)-(8), (13)-(14), plus the production function of consumption goods and advertising, the optimal factors demand for productions,\footnote{See Appendix B for details.} the laws of motion of capital and goodwill, the market clearing condition on the goods market, $Y_t = C_t + I_t + F_t$, and the market clearing condition on the labour market, $H_t = H_{p,t} + H_{a,t}$.

3.5 Advertising in Utility Function: Functional Forms Assumptions

In order to fully specify the utility function, we need to parameterise the function $B(\cdot)$ in a way that satisfies all assumptions made so far (see equation (2) and the following discussion). In addition, we are interested in some specificati\on of $B(\cdot)$ that nests market-enhancing and spread-it-around advertising.

In the following, we assume that the function $B(g_{i,t})$ is defined as:

$$B(g_{i,t}) \equiv S(g_{i,t}) + \gamma \int_0^1 (1 - S(g_{i,t})) \, di \quad \text{with } \gamma \in [0, 1] \quad (18)$$

where

$$S(g_{i,t}) \equiv \frac{1}{1 + \theta g_{i,t}} \quad (19)$$

It is easy to verify that the function (19) is strictly decreasing and convex in goodwill. More importantly, the goodwill in (18) enters in quasi-difference from its market average, meaning that the effectiveness of firm’s advertising on its own demand will depend on the level of advertising of competitors.\footnote{This formulation implies that advertising is combative.} Now, we consider in detail the role of $\gamma$. At symmetric equilibrium, (18) and (19) imply:

$$B(G_t) \equiv \frac{1 + \gamma \theta G_t}{1 + \theta G_t}$$
If $\gamma = 1$, then $B(g_{i,t}) = 1$. Thus, the aggregate goodwill disappears from the marginal utility of consumption (see equations (6) and (2)), and does not directly affect the representative consumer’s decisions about labour supply (7) and savings (8), and therefore the spread-it-around hypothesis holds. In this case, the effect of advertising on the whole economy is easily predictable. It absorbs resources without enhancing demand, and it has no direct effect on prices. Thus, it is a deadweight loss both for firms and for the consumer. Note that firms are still employing resources to advertise their products because in the non-cooperative solution, they do not internalise the effect of their decisions on the mean level of advertising. As a result, they keep wasting money in an unproductive way while the effect of their advertisements on their own demand is offset by other firms’ advertising.

If $\gamma = 0$ the goodwill enters in level in the utility function. Accordingly, each firm’s advertising affects the marginal utility of its product no matter what the other firms do. In this case advertising directly affects consumption, labor, investment, and firms’ markup, and its overall effect in the general equilibrium will be the object of the analysis in next section 4. Finally, any value of $\gamma \in (0,1)$ implies a convex combination between the two extreme cases (complete spread-it-around vs. market enhancing).

A consideration apart deserves the choice of $S(\cdot)$. Equation (19) implies that the marginal utility of consumption is bounded (hence we will refer to it as “bounded marginal utility”).$^{29}$ Due to this bound, in the demand function (9) there exists a maximum price above which the demand is zero: when the price is too high the marginal benefit of consuming that good is smaller than its cost, and the consumer drops it from his basket of purchases. In this fashion, firms have incentive to advertise their products for reducing the bound. In the absence of advertising, the bound is constant, while with advertising it depends on the level of goodwill, whose effect is larger with larger $\theta$. Hence, this parameter is interpreted as a measure of the effectiveness of advertising in manipulating consumer’s tastes.

4 Impulse-Response Analysis

This section considers a log-linear approximation of the model’s policy functions in the neighbourhood of the non-stochastic steady state. Rational expectations are solved to obtain the dynamic responses of endogenous variables as functions of state variables. We characterise the response of the model’s variables to several endogenous shocks, namely: a technology shock (figure 3), a preferences shock (figure 4), a shock on exogenous government spending (figure 5), and an idiosyncratic shock to the production function of advertising (figure 6).

To compute the impulse-response functions (hereafter IRFs), we need to assign values to the parameters $\{\beta, \sigma, \phi, \Xi, \varepsilon, \theta, \alpha, \rho_z, \rho_a, \rho_f, \rho_h, \sigma_k, \sigma_a, \sigma_f, \delta_g, \delta_h, \gamma\}$. The parameters that are standard in real business cycle (hereafter RBC) models are calibrated using the values commonly used in the literature, while the others are chosen such that steady states of model variables match selected long-run moments of U.S. postwar data. In particular, the discount parameter $\beta$ is set to $(1.04)^{-25}$, implying a yearly nominal interest rate of about $29$ As a matter of fact, preference featuring bounded marginal utility have been already used in the literature. See for instance Melitz and Ottaviano (2008)
4%. The depreciation rate of capital $\delta_k$ is equal to 2.5% per quarter, and the gross elasticity of substitution across varieties is equal to 6. Following Prescott (1986), the preference parameter $\Xi$ is chosen to ensure that in the steady state, the consumer devotes 1/4 of his time to labour activities. Following Ravn, Schmitt-Groh and Uribe (2006), we set the intertemporal elasticity of substitution to 0.5, the labour elasticity of output $\alpha$ to 0.75, the Frisch elasticity of labour supply to 1.3, and the government expenditures-GDP ratio $s_f$ to 0.12. These restrictions imply that the preference parameters $\sigma$ and $\phi$ are 2 and 0.77, respectively, and the steady state labour share is 0.71.\(^\text{30}\)

The values of advertising related parameters have been assigned using the following strategy. The goodwill depreciation rate has been fixed to 0.3, implying that the half life of goodwill stock is about two quarters. This value is consistent with the empirical evidence provided in Clarke (1976): the effect of advertising on the firm’s demand basically vanishes after one year. As a benchmark case, we set the parameter $\gamma$ to zero, while the intensity of advertising in the utility function $\theta$ is chosen such that conditional to all other parameters, the steady-state value of the advertising over GDP ratio is equal to 2.27%, consistent with the U.S. average over the period 1948-2005.\(^\text{31}\)

The autoregressive parameters for all the endogenous process have been set to 0.95. This number is intermediate among the values normally used in the RBC literature. For the simulations, following Rebelo and King (1998) and Collard (2006), we set the standard deviations of technology shock $\sigma_a$ and government expenditures shock $\sigma_f$ to 0.0079 and 0.0089, respectively. Finally, the standard deviation of the preference shock $\sigma_h$ is chosen such that the volatility of hours worked in the model matches its empirical counterpart of 0.91%.\(^\text{32}\) The time period in the model is one quarter. Table 3 summarises the set of calibrated parameters.

We plot the IRFs for different values of the spread-it-around parameter $\gamma$, and we use the associated model economy where advertising is banned as a benchmark to evaluate the impact of advertising. The IRFs appear in Figures (3) − (6), and we will emphasise a number of these results.

First, advertising responds positively to any shock considered. This result follows directly from equation (16), which establishes a positive relationship between aggregate goodwill and aggregate demand. Whenever a shock increases demand, the marginal benefit of goodwill also increases, pushing firms to invest more in advertising. In particular, of the shocks considered, advertising reacts mostly to the technology shock, as is apparent by comparing figures (3) and (4). The response of advertising to a 1% technology shock is twice as large

\(^\text{30}\)In our framework, the steady state labour share denoted by $s_h$ takes the following form:

\[
s_h = \frac{W (H_p + H_a)}{Y} = \alpha \mu^{-1} \left[ 1 + \frac{H_a}{H_p} \right]
\]

so that the usual relationship between the intensity of labour in the production function and the labour share no longer necessarily holds. Note that in the last equation, $\mu$ denotes the average long run markup.

\(^\text{31}\)This number refers to the ratio of advertising expenditures to net GDP, where exports are subtracted from GDP because exported goods are not sold based on domestic advertising.

\(^\text{32}\)This number refers to the standard deviation of the bandpass filtered hours worked in our sample.
Table 3: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>.9902</td>
<td>Subjective discount factor</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>6</td>
<td>Elasticity of substitution across varieties</td>
</tr>
<tr>
<td>$\delta_k$</td>
<td>0.025</td>
<td>Capital depreciation rate</td>
</tr>
<tr>
<td>$\Xi$</td>
<td>2.49</td>
<td>Steady State of the preference shock</td>
</tr>
<tr>
<td>$\delta_g$</td>
<td>0.3</td>
<td>Goodwill depreciation rate</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.77</td>
<td>Preference parameter</td>
</tr>
<tr>
<td>$\theta$</td>
<td>2.54</td>
<td>Intensity of advertising in the utility function</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.75</td>
<td>Labor elasticity of output</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
<td>Preference parameter</td>
</tr>
<tr>
<td>$s_f$</td>
<td>0.12</td>
<td>Government expenditures-Gdp ratio</td>
</tr>
<tr>
<td>$\rho_a, \rho_h, \rho_g, \rho_z$</td>
<td>0.95</td>
<td>Persistence of exogenous shocks</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>$7.9e-3$</td>
<td>Standard error of the technology shock</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>$9.8e-3$</td>
<td>Standard error of the government spending shock</td>
</tr>
<tr>
<td>$\sigma_h$</td>
<td>$6.2e-3$</td>
<td>Standard error of the preference shock</td>
</tr>
</tbody>
</table>

as the response to a 1% preference shock. This is due to the double effect of an unexpected increase in productivity; the firm revises its advertising spending on the one end because the demand increases, and on the other end because the marginal cost of advertising diminishes. Note that this second effect is further amplified by the dynamic nature of advertising, which modifies the optimal plan of producing advertising to stock future goodwill.\(^{33}\)

Second, advertising in the model is pro-cyclical, as it becomes apparent comparing the IRFs of advertising and output in figures (3) – (6). After each shock, the pairs of IRFs display the same sign both at impact and afterward during the transition back to the steady state, thus replicating the positive correlation between advertising and GDP observed in real data. Note that this feature of the model is independent of the value assigned to $\gamma$.\(^{34}\)

Third, spread-it-around and market-enhancing advertising play two very different roles in the aggregate dynamics. When $\gamma = 1$ (spread-it-around), the IRFs of the main economic aggregates essentially coincide with the benchmark ones (compare dashed versus circle lines): i.e., the effect of advertising on the aggregate becomes negligible. As intuition suggests, in this case advertising does not influence consumer’s decisions, and its effect on the aggregate

\(^{33}\)Clearly, in the event of a transitory positive technology shock, producing advertising today becomes cheaper than doing it tomorrow, thus pushing firms to produce today the advertising that they will need to maintain future goodwill at the optimal level.

\(^{34}\)It has often been argued in the literature that the correlation between GDP and advertising is not a relevant statistic to the disentangling of market-enhancing advertising from spread-it-around advertising. This also remains true in our model.
Figure 3: Impulse Response Functions to technology shock. Each plot displays percent deviation from steady state of the corresponding variable in response to a 1% increase in the rate of productivity.

dynamics is determined only by the excess of labour demand that results from producing advertising. The numerical analysis shows that such an effect is negligible since the absorption of resources due to advertising is too small to affect the other aggregates in a relevant way. Hence, the conjecture of Simon and Solow is confirmed: under the spread-it-around hypothesis, advertising is irrelevant in the aggregate.

Oppositely, when $\gamma \neq 1$ (market-enhancing), advertising operates on the aggregate dynamics as a mechanism that amplifies and propagates the effects of exogenous shocks (compare the continuous versus dashed lines), and the effect turns out to be stronger when $\gamma$ is lower (continuous versus dotted lines). The effect of advertising on the labour supply is most important.\textsuperscript{35} Despite the fact that wages are lower at equilibrium than in the benchmark model, the upward pressure that advertising puts on the supply of labour is so strong that worked hours increase.\textsuperscript{36} This mechanism is called the work and spend cycle (Schor, 1992), and it has been empirically supported by Brack and Cowling (1983) for the U.S., and by Fraser and Paton (2003) for the UK.

As a result of the higher level of hours worked, production increases, and the response

\textsuperscript{35}See Molinari and Turino 2007 for a detailed analysis.

\textsuperscript{36}To save space, the impulse-response functions of wages are not reported, but they are available upon request.
of output to any shock considered is stronger than the benchmark. The effect of advertising appears quantitatively relevant when $\gamma = 0$. Figure (6) shows that an unexpected 1% increase in the productivity of advertising affects fluctuations in a non-negligible way; consumption increases by 0.25%, output by 0.12%, investment by 1.07%, and labour by 0.18%.\textsuperscript{37}

Overall, the analysis of the aggregate dynamics reveals that demand-shifting advertising works as a built-in mechanism of the transmission of exogenous shocks. The case of a positive technology shock is particularly interesting in light of the RBC literature. Intuitively, an unexpected increase in productivity leads to a higher level of desired goodwill.\textsuperscript{38} In turn, a higher level of goodwill implies an upward shift in the demand of consumption goods, according to (17). Hence, after a technology shock, the level of the aggregate demand increases not only because of the traditional transmission channels (higher wage and higher present value of wealth), but also because of the higher spending on advertising, which makes the consumer willing to consume more for any given price. In the RBC literature, it has often been argued that technology shocks cannot generate business cycles such as

\begin{footnotesize}
\textsuperscript{37}Recall that advertising in the model is not a component of the output, which is defined as consumption plus investment and government spending.
\textsuperscript{38}This result follows immediately from the optimal advertising policy (14).
\end{footnotesize}
Figure 5: Impulse Response Functions to government spending shock. Each plot displays percent deviation from steady state of corresponding variable in response to a 1% shock on government spending.

Lastly, we want to emphasise the behaviour of consumption, whose dynamics in the model with respect to the benchmark display a lower response to the impact of the shock, but a larger, hump-shaped impulse response function during the transition. Given that by assumption advertising raises the marginal utility of consumption in the model, the relative reduction of consumption at impact may not be intuitive and deserves further explanation. The basic intuition is that with advertising the consumer experiences temporal variations in the intertemporal elasticity of substitution, which determines the relative reduction of consumption with respect to the benchmark economy. To see this point, it is useful to rewrite the log-linearised Euler Equation in terms of expected consumption growth, that is:

$$E_t\{\Delta \hat{c}_{t+1}\} = (1 - \gamma)\eta_{c,g} E_t\{\Delta \hat{g}_{t+1}\} + \frac{\eta_{c,p}}{\varepsilon} E_t\{\hat{r}_{t+1}\}$$

(20)

where $\eta_{c,p}$ and $\eta_{c,g}$ are, respectively, the steady state demand elasticity with respect to price and goodwill. According to (20), aggregate advertising modifies the savings decision along two different dimensions. On the one hand, since the elasticity $\eta_{c,g}$ is positive (the first term on the RHS of (20), expected variations in the stock of goodwill modify the expected
consumption growth in the same direction. Intuitively, the consumer correctly anticipates the effect of future goodwill on the utility of future consumption and modifies the degree of consumption smoothness over time accordingly. Clearly, the effect is stronger for lower $\gamma$, since for values of $\gamma$ that approach 1, the evaluation of future utility becomes independent from the level of goodwill. On the other hand, the elasticity of expected consumption growth to the interest rate is lower than that used in the benchmark model (second term in the RHS of 20), since the long run price elasticity $\eta_{c,p}$ is lower than the benchmark one $\varepsilon$. Hence, if interest rate and goodwill growth respond to a shock in the same direction, the two effects tend to offset each other, and the overall impact will depend on which one prevails.

In our calibrations, the first effect dominates the second, making the response of consumption to a shock larger than in the benchmark model. This feature of the model becomes apparent upon inspecting the behaviour of investment. In all the cases considered, the IRF of investment is stronger than the benchmark one up to the quarter in which the goodwill peaks. This effect is particularly interesting because it shows that contrary to the conventional wisdom, a positive link between consumption and advertising does not necessarily need to be associated with a crowding-out effect on investment. In fact, once we account for the dynamic effect of advertising and let the supply of labour be endogenously determined by the consumer, an equilibrium in which consumption, hours and investment all increase is indeed possible.

Overall, since the impulse response function of the goodwill stock is hump-shaped, during the transition to the steady state, the consumer experiences a time-varying intertemporal elasticity of substitution, as previously claimed. Accordingly, the sensitivity of the savings function to the interest rate is not constant as in the benchmark model.
5 Model Estimation

This section estimates the DSGE model with advertising using a quarterly time series of U.S. macroeconomic data and a Bayesian estimation method. The data sample goes from the first quarter of 1976 to the second quarter of 2006, the interval over which we have data on aggregate advertising.

Bayesian estimation is preferred over other techniques for several reasons. First, it naturally accommodates the unobservable goodwill in the estimation algorithm, which is a crucial variable for estimating advertising related parameters. Second, it is preferred to Maximum Likelihood estimation because we want to pin down model parameters exploiting combined information from business cycle frequency data and long-run moments of the data, as we showed that advertising also has a relevant effect in the long run.\textsuperscript{40} Bayesian priors are a convenient tool for including extra information in the estimation. Third, the effect of advertising spreads in the economy through various transmission channels that can only be assessed completely by computing the general equilibrium solution of the model, as we showed in section 4. Therefore, any estimation that exploits only partial equilibrium relationships, such as a GMM estimation of the Euler Equation, would neglect to consider some potentially important information.

To estimate the model, we proceed as follows. First, we derive the VARMA representation of the log-linearised model used in section 4:\textsuperscript{41}

$$\hat{x}_t = \Psi_x(\omega) \hat{x}_{t-1} + \Psi_\epsilon(\omega) \epsilon_t$$

(21)

where $\hat{x}_t$ is the vector of partially latent endogenous variables in log-deviations from their steady state values, and $\Psi_x(\omega)$ and $\Psi_\epsilon(\omega)$ are matrices containing the structural parameters of the model, which are listed in vector $\omega$.\textsuperscript{42} Then, we add four measurement equations that link model variables to four key observable macroeconomic variables. In particular, we use log differences of real consumption, real output net of exports, total hours worked, and aggregate real advertising expenditures.\textsuperscript{43} The corresponding vector of measurement equations is:

$$x^*_t = \begin{bmatrix} dlCONS_t \\ dlGDP_t \\ dlHOURS_t \\ dlADV_t \end{bmatrix} = \begin{bmatrix} \tau \\ \tau \\ 0 \\ \tau \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{c}_t - \hat{c}_{t-1} \\ \hat{h}_t - \hat{h}_{t-1} \\ \hat{z}_t - \hat{z}_{t-1} \end{bmatrix}$$

(22)

where $\tau = log(\tau)$ is the common quarterly growth rate trend, and $\tau$ is the theoretical deterministic trend of technology. Finally, using the state-space representation (21)-(22), model parameters are estimated in order to maximise the likelihood of observed data, or in other words, we choose the vector $\omega$ that maximises the log posterior kernel of $\omega$ conditional to $x^*_t$.\textsuperscript{44}

\textsuperscript{40}This point is made in a companion paper. See Molinari and Turino (2007).

\textsuperscript{41}See the appendix C for a detailed explanation of the model used in the estimation.

\textsuperscript{42}Specifically, $\omega = \{ \beta, \sigma, \phi, \Xi, \varepsilon, \theta, \alpha, \alpha_z, \rho_h, \rho_o, \rho_f, \rho_p, \sigma_h, \sigma_o, \sigma_f, \sigma_p, \delta_g, \delta_k, \gamma \}$.

\textsuperscript{43}Details of the data set are available in Appendix A.

\textsuperscript{44}The log posterior kernel $lnK(\omega \mid x^*_t)$ is a linear combination of our prior knowledge about the distribution of $\omega$, $lnp(\omega)$, and the log likelihood of $\omega$ conditional to the observed data, $lnL(\omega \mid x^*_t)$. 

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Table 4: Prior Distributions of Structural Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Density</th>
<th>Domain</th>
<th>Mean</th>
<th>90% interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Gamma</td>
<td>$\mathbb{R}_+$</td>
<td>2.00</td>
<td>[1.39, 2.70]</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Gamma</td>
<td>$\mathbb{R}_+$</td>
<td>0.77</td>
<td>[0.19, 1.46]</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Gamma</td>
<td>$\mathbb{R}_+$</td>
<td>2.50</td>
<td>[1.88, 3.19]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Uniform</td>
<td>[0, 1]</td>
<td>0.5</td>
<td>[0.10, 0.90]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Beta</td>
<td>[0, 1]</td>
<td>0.75</td>
<td>[0.68, 0.81]</td>
</tr>
<tr>
<td>$\alpha_z$</td>
<td>Beta</td>
<td>[0, 1]</td>
<td>0.75</td>
<td>[0.68, 0.81]</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Normal</td>
<td>$\mathbb{R}$</td>
<td>.005</td>
<td>[.0017, .0083]</td>
</tr>
</tbody>
</table>

The shocks used to estimate the model are different from those used for simulations in section 4, since we eliminated the shock from the production function of advertising and introduced a shock in the elasticity of demand, which is usually interpreted in the literature as a cost push shock. The new shock is introduced so that the number of structural shocks equals the number of observable variables to which the model is fitted, while the shock on advertising is dismissed because it appeared to work in the estimation algorithm as a measurement error that absorbs all the variations in actual advertising series that could not be explained by the model. We wanted instead to use the information contained in the series of advertising data to influence all of the estimates, in order to use joint information from all the observables to identify all the parameters.

In the estimation, we keep some parameters fixed, namely the discount rate $\beta$, the gross elasticity of demand $\epsilon$, the depreciation rate of capital $\delta_k$, the depreciation rate of goodwill $\delta_g$, and the steady state value of hours worked $H$. The first three parameters are typically difficult to identify in RBC models, while $\delta_g$ turns out to be not identifiable separately from $\theta$. $H$ is fixed because we are using data on worked hours in first difference, which leaves the mean level of hours worked undetermined. Fixed parameters are calibrated according to the values in table 3. The other parameters are estimated by combining the information contained in actual data $x_t^*$ with that contained in the priors. Prior distributions are chosen according to what is used in the literature, while prior means are chosen according to table 3. Details about the priors are given in tables 4 and 5.

In general, our priors for structural parameters are quite flat. The prior on $\theta$ is a gamma distribution — i.e., $\theta$ is bounded away from zero — with a mean of 2.50 and a variance of 0.4. Given $\delta_g = 0.3$, this value of $\theta$ implies a ratio of advertising over GDP equal to 0.02, in line with the empirical evidence presented in Section 2. The prior for $\gamma$ is a uniform (0,1) distribution, which reflects our neutral stand between spread-it-around and market-enhancing advertising. For the processes of the shocks, we use standard priors, following Smets and Wouters (2007), Chang Doh and Schorfheide (2006), and An and Schorfheide (2007).

The algorithm for the Bayesian estimation works as follows. First, the posterior kernel

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45The estimation works as follows: at each iteration where the algorithm tries a new vector $\omega$, the steady state value of the preference shock $\Xi$ adjusts for $H$ to remain fixed.
is maximised in order to find the mode of the posterior distribution. Second, starting from a random perturbation around the mode, a random-walk Metropolis-Hastings algorithm is used to sample from the posterior distribution.\textsuperscript{46} We run this algorithm 4 times from different perturbation points, eventually building 4 chains of 70,000 draws each. This strategy seems to ensure a relatively fast convergence of the Markov chains generated from the algorithm, at least compared to what is usually reported in related literature. Convergence diagnostics indicate that around 40,000 drawings are sufficient to attain convergence for all the parameters.\textsuperscript{47} Finally, we report selected statistics for the posterior distributions by computing the average of correspondent moments from all the chains, wherein we discard the initial 40\% of observations from each chain.

### 5.1 Results

Tables 6 and 7 report the mean and 90\% interval from posterior distributions, and Appendix C.3 provides a set of figures of prior and posterior distributions represented together for each parameter.

Our first concern is for the estimates of $\gamma$. As we showed in section 4, $\gamma$ is a crucial parameter for assessing whether and how advertising matters in the aggregate. Any estimate of $\gamma$ significantly close to 1 would imply that the model fits the data better with spread-it-around advertising, while any estimate significantly lower than 1 would imply that market-enhancing advertising is preferred. The estimate of $\gamma$ seems quite informative, and the posterior has less variance than the corresponding prior. The posterior mean is 0.39, and

\[\sigma_y \text{ InvGamma } \mathbb{R}_+ \quad .008 \quad [.001, .022]\]
\[\sigma_h \text{ InvGamma } \mathbb{R}_+ \quad .034 \quad [.007, .096]\]
\[\sigma_f \text{ InvGamma } \mathbb{R}_+ \quad .099 \quad [.020, .279]\]
\[\sigma_{mk} \text{ InvGamma } \mathbb{R}_+ \quad .039 \quad [.008, .109]\]
\[\rho_y \text{ Beta } [0, 1) \quad 0.6 \quad [0.25, 0.90]\]
\[\rho_h \text{ Beta } [0, 1) \quad 0.6 \quad [0.25, 0.90]\]
\[\rho_f \text{ Beta } [0, 1) \quad 0.6 \quad [0.25, 0.90]\]
\[\rho_{mk} \text{ Beta } [0, 1) \quad 0.6 \quad [0.25, 0.90]\]

\textsuperscript{46}The variance of the jumping distribution is the inverse of the Hessian from the maximisation of the mode, multiplied by 0.35. The acceptance rate is around 35\%. The initial perturbation is a single draw from a normal distribution with zero mean and a variance equal to 5 times the variance of the jumping distribution. This larger variance helps to ensure that together, the chains cover the entire parameter space.

\textsuperscript{47}See Appendix C.2 for details about the convergence diagnostic. To double-check our results, we also estimate the model running the algorithm to build 2 chains of 500,000 draws each. The estimates remained close to the values found with the shorter chains, but the estimation took much longer, since the number of draws increased from 280,000 to 1 million.
the upper bound of 1 is rejected with certainty based on the data.\footnote{We estimated several specifications of the model; in all cases, $\hat{\gamma}$ was significantly different from 1, ranging over $\hat{\gamma} \in (0.00, 0.39)$.} This value suggests that aggregate advertising is a significant explanatory variable of aggregate consumption. Hence, market-enhancing advertising is preferred to spread-it-around advertising, thereby yielding all the consequences of the importance of aggregate advertising highlighted in section 4.

Our interpretation of this result hinges on the effect of advertising on the marginal utility of consumption. Typically, RBC models tend to predict an excess of consumption smoothing with respect to what is observed in the data.\footnote{See Attanasio (1989).} Here, advertising has an effect on the marginal utility opposite to the effect of consumption, while actual advertising data co-move with those of consumption. Thus, the effect of cyclical variations in advertising offsets that of consumption leaving the overall argument of the marginal utility more stable.
Table 8: Posterior Distributions for \{\sigma, \phi\} in different models.

<table>
<thead>
<tr>
<th>Model</th>
<th>\sigma</th>
<th>\phi</th>
<th>LogMarginal DataDensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dixit-Stiglitz</td>
<td>1.55</td>
<td>2.48</td>
<td>1,312</td>
</tr>
<tr>
<td></td>
<td>[1.12, 1.98]</td>
<td>[1.54, 3.33]</td>
<td></td>
</tr>
<tr>
<td>Bounded Marginal Utility</td>
<td>1.88</td>
<td>2.65</td>
<td>1,314</td>
</tr>
<tr>
<td></td>
<td>[1.24, 2.47]</td>
<td>[1.85, 3.14]</td>
<td></td>
</tr>
<tr>
<td>B.M.U. with Advertising</td>
<td>2.48</td>
<td>2.52</td>
<td>1,530</td>
</tr>
<tr>
<td></td>
<td>[1.86, 3.07]</td>
<td>[1.76, 3.37]</td>
<td></td>
</tr>
<tr>
<td>B.M.U. with Advertising restricted (\gamma = 1)</td>
<td>2.43</td>
<td>2.63</td>
<td>1,528</td>
</tr>
<tr>
<td></td>
<td>[1.83, 3.01]</td>
<td>[1.76, 3.43]</td>
<td></td>
</tr>
</tbody>
</table>

Note: 90% confidence interval in parentheses.

over the cycle, and in particular, less volatile than the single series of consumption. This feature reconciles the evidence of stable marginal utility and volatile consumption data.

Our second concern is about \(\theta\), which is the parameter that controls for the effect of advertising on preferences. Again, the estimate seems informative since the posterior has less variance than the corresponding prior. Its posterior mean suggests that a value of 2.5 is reasonable for this parameter, supporting the previous calibration. This estimate implies a ratio of advertising to GDP of 1.6% in the estimated model, which is a notable result because it is obtained using first difference data and remains stable when we increase the variance of the prior of \(\theta\).

Among the other parameters estimates, the mean values from posterior distributions of \(\sigma\) and \(\phi\) raise our attention because they are relatively high compared with other estimates appeared in the literature, e.g., Smets and Wouters (2007), or with the calibrations typically used in the RBC literature. In order to better understand whether this result depends on our model or on the data set employed, we estimate two more RBC models without advertising, one based on a utility function with the standard Dixit-Stiglitz consumption aggregate, and one with a Bounded Marginal utility. Mean estimates of \(\sigma\) and \(\phi\) are reported in the table (8).

While the evidence on \(\phi\) is mixed, the estimates of \(\sigma\) clearly indicates that \(\sigma\) is always higher with advertising. This result can be explained again by observing the behavior of the marginal utility of consumption. Since in the model with advertising there is no one-to-one correspondence between the volatility of marginal utility and consumption, then the estimation algorithm does not pin down the value of \(\sigma\) to match the volatility of the marginal utility with that of consumption, as usually happens in standard RBC models.

As final remark, we point out that the estimates of shocks processes appear in line with the ones found in the empirical literature for similar DSGE models, e.g. Smets and Wouters (2007), suggesting that our model is able to treat uncertainty in the data with the same degree of accuracy of other estimations reported in the literature.
Table 9: Counterfactual Analysis

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Tech. Shock</th>
<th>Gov. Shock</th>
<th>Pref. Shock</th>
<th>Mark Up Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>10.6</td>
<td>20.5</td>
<td>-1.91</td>
<td>19.9</td>
<td>798</td>
</tr>
<tr>
<td>GDP</td>
<td>0.40</td>
<td>0.86</td>
<td>5.38</td>
<td>0.38</td>
<td>24.1</td>
</tr>
<tr>
<td>Investment</td>
<td>-1.48</td>
<td>-4.18</td>
<td>-0.34</td>
<td>-4.65</td>
<td>251</td>
</tr>
<tr>
<td>Hours</td>
<td>1.16</td>
<td>9.72</td>
<td>6.22</td>
<td>1.13</td>
<td>-50.5</td>
</tr>
<tr>
<td>Mark-Up</td>
<td>8.91</td>
<td>9.71</td>
<td>11.8</td>
<td>10.2</td>
<td>8.50</td>
</tr>
<tr>
<td>Welfare Loses(a)</td>
<td>1.31</td>
<td>-0.37</td>
<td>0.10</td>
<td>0.40</td>
<td>1.18</td>
</tr>
</tbody>
</table>

(125%) (17.1%) (359%) (11.2%) (98.8%)

The figures indicate the percent increase in the volatility of corresponding variable when going from the counterfactual economy to the model economy. They are computed as \(\frac{\text{var}(x|\text{adv} > 0)}{\text{var}(x|\text{adv} = 0)} - 1\) * 100, and \(\text{var}(x|\cdot)\) are simulated standard deviations obtained as averages out of 3000 simulated samples of 151 periods length. Simulated data are detrended with BP filter BP(6,32). All refers to the full model with parameters at estimated value. Other columns refer to the same model where all the shocks are shut down but the one indicated in the heading of the column.

\(a\) Welfare losses in units of steady state consumption. In parenthesis, the increase in welfare losses when switching from the counterfactual economy without advertising to the model economy.

5.2 Applications

In section 4, we argued that advertising works in the model economy as a built-in mechanism of propagations of exogenous shocks and, in particular, as a mechanism of transmission of technology shocks to the demand. In the following, we make use of the estimated model to test our assertion. We examine how standard deviations of model variables would change if advertising were suddenly banned from the economy. The counterfactual exercise computes the percent difference of standard deviations from the model economy to the counterfactual economy without advertising. Table 9 reports the figures. Last line of table 9 gives an overall assessment about the impact of advertising on the aggregate dynamics, by comparing the welfare costs of fluctuations from the model economy with those from the counterfactual economy. As usual in the literature on welfare,\(^{50}\) we define the costs of fluctuations as the units of steady state consumption that the consumer would be willing to pay to eliminate all the variability from his consumption stream. Such costs are computed as the difference between the 2\(^{nd}\) order Taylor approximated utility of the agent endowed with the equilibrium bundle \(\{C_t, H_t\}\) and the 2\(^{nd}\) order Taylor approximated utility of an agent that consumes at the steady state level and works the steady state amount of hours in every period.

In general, advertising is confirmed to amplify the volatility of endogenous variables, where the result is stronger for consumption and markup, +10.6% and +8.91%, respectively, relatively mild for worked hours, +1.16%, and does not apply to investment, −1.48%. The difference between the mild effect on labour encountered here and the relatively large effect observed in the IRF of labour in section 4 is explained by the difference between the

\(^{50}\)See Erceg, Henderson and Levin (2000).
estimated mean value of $\phi$ and the calibration used in section 4. The estimated Frish labour elasticity is much lower than in the calibration, implying a series of worked hours that is fairly stable over the cycle and reacts little to any shocks.

The welfare analysis supports the results of the counterfactual exercise. The increase in the costs of fluctuations clearly indicates that when firms advertise their products, the consumer is willing to pay an higher percentage of his consumption to get rid of fluctuations. The extra cost due to advertising ranges between 11.2% and 359%, and worths 124% in the baseline case where all the shocks are active. Overall, our results confirm that even a small advertising industry can have a relevant impact on the aggregate dynamics and, in particular, on fluctuations of the endogenous variables.

To isolate and shed light on the effect of technology shocks alone, we also provide a counterfactual analysis where the model is simulated by shutting down all but one shock at a time. As expected, technology shocks generate more volatile variables in the model economy than in the benchmark. In particular, the effect on the variance of consumption and hours worked is stronger than in the case where all shocks are active. Only the effect on the variance of output remains mild, which is clearly due to the lower volatility of investment, which compensates the higher volatility of consumption. Beyond the impact on technology, advertising also stimulates consumption by partially offsetting the usual crowding out effect of government spending shocks on consumption. A positive government spending shock that increases demand leads firms to raise investments in advertising. This in turn creates urge to consume and pushes the supply of labour upward, which together reduce the crowding out effect on consumption.

The greater importance of technology shocks in generating the cyclical fluctuations in
model variables can be further investigated using the variance decomposition of estimated model variables. Figure (7) depicts the variance decomposition of the estimated model compared to that of the estimated benchmark, i.e., models 3 and 2 of table 8, and confirm previous results. Technology shocks account for a bigger proportion of the volatility of consumption, output, and labor. In particular, with respect to the benchmark, technology shocks account for roughly 10% more of consumption volatility and 17% more of output volatility, whereas exogenous preference shocks account for 15.3% less of consumption volatility and around 15% less of output volatility.

6 Conclusions

This paper provides a model to understand the observed behavior of firms’ expenditures on advertising over the business cycle and to quantify the effect of advertising on the aggregate dynamics. To this end, we first characterised the pattern of actual U.S. data of aggregate advertising expenditures over the business cycle. Second, we built a model that can rationalise this pattern within the neoclassical growth model theory. Third, we showed that under some conditions advertising can have a relevant impact on aggregate dynamics, and we isolated this impact with a simulation exercise which makes use of a calibrated version of the model. Fourth, we showed that a log-linearised version of the model fits well with actual data of aggregate advertising. Finally, we estimated the model to test whether the conditions mentioned above do actually hold in the U.S. economy. In our framework, this was equivalent to testing the spread-it-around (Solow, 1968) versus market-enhancing (Galbraith, 1958) hypotheses of aggregate advertising. We find that the data fit the second hypothesis better. From an econometric point of view, this result hinges on the significance of aggregate advertising as an explanatory variable of the volatility of aggregate consumption.

The general equilibrium results from the model show that the effectiveness of advertising in enhancing aggregate consumption yields an important corollary: advertising affects agents choices about desired savings and labor supplies. Through this channel, advertising is shown to have a non-negligible impact on the aggregate dynamics of the whole economy. Despite the modest size of the advertising industry in comparison with total production, i.e., roughly 2% of GDP in the U.S., its short run impact on business cycle fluctuations turns out to be quantitatively important, exacerbating the welfare costs of fluctuations by 124%. Overall, the model goes in the direction suggested by Kaldor (1950): advertising works to amplify and propagate fluctuations of economic activity.

One final remark is worth underlining. The model proposed in this paper uses the worst-case scenario to obtain effects of advertising on the aggregate. In a model with nominal rigidities, the consumer would further increase the supply of labor in response to new advertising because of his low wage variability, which would strengthen the work and spend cycle mentioned above. Also, in a model with fully flexible prices like the one used here, any increase of the markup due to advertising makes investment goods more expensive, thus reducing the real return on capital and therefore household savings. As a result, the general equilibrium effect of advertising on investment is negative, and it partially offsets the positive effect of the work and spend cycle, thus reducing the overall effect of advertising.
Lastly, we show that advertising in this model behaves as an endogenous taste shock whose intensity is controlled by firms and varies whenever a shock to productivity occurs. This feature leads to the observation that technology shocks shift the aggregate demand through advertising. In fact, we find evidence that disregarding the advertising channel in an RBC model may lead to an underestimation of the effect of technology shocks in generating business cycle fluctuations.

\footnote{In an early draft of this paper, we showed that in a two-sector model where consumption and investment consist of different goods, advertising increases not only consumption but also investment, since it lowers the relative price of investment goods. Thus, the overall effect on the aggregate demand was stronger than the effects presented in this paper.}
References


[34] Solow, Robert (1968): "The truth further refined: a comment on Marris", The Public Interest vol. 11, pp. 47-52


A Sources of Data

A.1 Data on Advertising

Advertising expenditures in TV, Cable, Radio, Magazines, and Outdoor:
Ad$Summary, quarterly issues from 1976:1 to 2006:2, issued by Media Market New York City

Advertising expenditures in newspaper:
Newspaper Association of America. Data available on the official website of the Association: http://www.naa.org/

Annual advertising expenditures and its components:

A.2 Macroeconomic Data

Source: Database "FRED II" of the Federal Reserve Bank of St. Luise available at the website: http://research.stlouisfed.org/fred2

Real Gross Domestic Product (GDPC96)
Real Exports of Goods & Services (EXPGSC96)
Real Personal Consumption Expenditures (PCECC96)
Real Personal Consumption Expenditures: Durable Goods (PCDGCC96)
Real Personal Consumption Expenditures: Nondurable Goods: (PCNDGC96)
Real Private Fixed Investment (FPIC96)
GDP Implicit Price Deflator (GDPDEF)
Civilian Employment-Population Ratio (EMRATIO)
Civilian Non-Institutional Population (CNP160V)

Source: Bureau of Labor Statistics
Available at the website: http://www.bls.gov/data/home.htm

Total Private Average Weekly Hours of Production Workers (CES050007)
Total Non-farm Employment (CES050001)

Note: The series of worked hours used in the estimation is

\[ H = \frac{CES050007 \times EMRATIO}{168} \]

where 168 normalizes weekly hours to agents’ total endowment of hours in a week. Alternatively we use the series:

\[ H = \frac{CES050001}{CNP160V \times 168} \]
B Technical Appendix

B.1 Firm’s costs minimization problem

To produce its good each firm employs two inputs, labor and capital, combined according to the following production function:

\[ y_{i,t} = A_{t} k_{i,t}^{1-\alpha} (\Gamma_{t} H_{p,t}(i))^{\alpha} \]  

(23)

where \( y_{i,t} \), \( k_{i,t} \), \( H_{p,t}(i) \), denote respectively firm’s output, capital stock, and production-related labor. \( A_{t} \) measures the stochastic technological progress of the Total Factor Productivity, and \( \Gamma_{t} \) is the labor augmenting technological progress, which follows a deterministic trend, i.e. \( \Gamma_{t} = \tau \Gamma_{t-1} \).

Firm’s demand of production-related inputs is the solution to the dual problem of total cost minimization, given by \( W_{t} H_{p,t} + R_{t} k_{i,t} \), and subject to the production function constraint (23).

As result, firm’s total cost function, and marginal cost are given respectively by:

\[ CT(y_{i,t}) = \frac{D}{A_{t}} W_{i}^{\alpha} R_{t}^{1-\alpha} (y_{i,t}) \]  

(24)

and

\[ \phi_{i,t} = \frac{D}{A_{t}} W_{i}^{\alpha} R_{t}^{1-\alpha} \]  

(25)

where \( D = \frac{(1-\alpha)^{\alpha}}{(1-\alpha)(1-\alpha)} \) is a positive constant.

Also, each firm promotes its sales by incurring in advertising expenditures. As in Grossmann (2007), we assume that advertising is produced by the marketing department of the firm using the following technology:

\[ z_{i,t} = A_{t} \Gamma_{t} (H_{a,t}(i))^{\alpha} U_{t}^{z} \]  

(26)

where \( z_{i,t} \), \( H_{a,t}(i) \) denote respectively the new investment in advertising and the marketing-related labor. \( U_{t}^{z} \) is a purely transitory idiosyncratic shock on advertising productivity.

B.2 Profits maximization problem

Each producer faces three demands for its product. One for consumption, i.e. (9), one for investment, and one for government purchases.

The demand function for investment goods derives from the solution to consumer’s dual problem of expenditures minimization, subject to the technology used to combine the goods into a unit of capital,

\[ I_{t} \geq \left( \int i_{i,t}^{\frac{1}{1-\epsilon}} \right)^{\frac{\epsilon}{1-\epsilon}} \]  

(27)

and it will be

\[ i_{i,t} = \left( \frac{P_{t}}{P_{t}} \right)^{-\epsilon} I_{t} \]  

(28)
The demand function for government purchases derives from the solution of the consumer’s dual problem of expenditures minimization, subject to the constraint:

\[ F_t \geq \left( \int f_{i,t}^{\varepsilon-1} \right)^{\varepsilon-1} \]  

where for simplicity we set the bound in utility to zero. It will be:

\[ f_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} F_t \]  

Thus, the total demand for good \( i \) can be written as:

\[ y_{i,t} = c_{i,t} + i_{i,t} + f_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \left( \bar{C}_t + I_t + F_t \right) - B(g_{i,t}) \]  

where (31) uses (9) (30) and (28).

Firm’s problem of profits maximization can be stated as choosing a sequences of prices \( p_{i,t} \) and advertising-related labor \( H_{a,t}(i) \) in order to maximize:

\[
\max_{\{H_{a,t}(i), p_{i,t}\}} E \sum_{t=0}^{\infty} Q_{0,t} \left( p_{i,t} y_{i,t} - CT(y_{i,t}) - W_t H_{a,t}(i) \right) 
\]

subject to

\[ g_{i,t} = z_{i,t} + (1 - \delta_g) g_{i,t-1} \]

\[ z_{i,t} = A_t \Gamma_t (H_{a,t}(i))^\alpha U_t^z \]

\[ y_{i,t} = \left( \frac{p_{i,t}}{P_t} \right)^{-\varepsilon} \left( \bar{C}_t + I_t + F_t \right) - B(g_{i,t}) \]

where \( Q_{0,t} \) is the discount factor. \( CT(y_{i,t}) \) is defined as in equation (24).

The first order conditions for an interior maximum of (32) are:

\[
P_{i,t} = \frac{\varepsilon \left( 1 + \frac{B(g_{i,t})}{y_{i,t}} \right)}{\varepsilon \left( 1 + \frac{B(g_{i,t})}{y_{i,t}} \right) - 1} \varphi_t \equiv \mu_{i,t} \varphi_t \quad (33)
\]

\[ \nu_t = \frac{W_t}{\alpha \Gamma_t U_t^z A_t} H_{a,t}(i)^{1-\alpha} \]

\[ - (p_{i,t} - \varphi_t) B'(g_{i,t}) + E \left[ (1 - \delta_g) (\nu_{t+1} Q_{t+1}^t) \right] = \nu_t \quad (35) \]
C Estimation

C.1 The estimated model

To ensure that the economy evolves along a balanced growth path we have to modify consumer’s preference. First, we assume that representative household derives utility from the object $\tilde{C}_t$ relative to the level of technology, $\Gamma_t$. That is, equation (1) is replaced with the following utility function:

$$
U(\tilde{C}_t, H_t) = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \left( \frac{\tilde{C}_t}{\Gamma_t} \right)^{(1-\sigma)} - 1 - \xi_t \frac{H_t^{1+\phi}}{1+\phi} \right]
$$

(36)

As in An and Schorfheide (2007), the term $\Gamma_t$ in the previous equation can be interpreted as an exogenous habit stock component. Second, we modify equation (18) as follows:

$$
B(g_{i,t}) \equiv \Gamma_t \left( S(g_{i,t}) + \gamma \int_0^1 (1 - S(g_{i,t})) \, di \right)
$$

(37)

where the function $S(g_{i,t})$ is now defined as

$$
S(g_{i,t}) \equiv \frac{1}{1 + \theta g_{i,t}/\Gamma_t}
$$

(38)

One can easily show that these assumptions, together with the assumed production functions, guarantee that existence of a balanced growth equilibrium in which all the endogenous variable growth a the same rate, $\tau$, with the exception of mark-up, interest rate and labor which are instead constant.

It is therefore convenient to express the model in terms of detrended variables, for which there exists a deterministic steady state.\footnote{An equilibrium in which all the stochastic innovation are zero all the time.} Let $\hat{X}_t = X_t/\tau$ denote the ratio of a variable $X_t$ with respect to its deterministic trend, $\tau$. The model can be expressed as:

$$
\hat{\tilde{C}}_t = \hat{C}_t + \frac{1 + \gamma \tilde{G}_t}{1 + \theta \tilde{G}_t} \quad (39)
$$

$$
\hat{\tilde{C}}_{t}^{-\sigma} = \frac{\beta}{\tau} E_t \left[ \hat{\tilde{C}}_{t+1}^{-\sigma} \left( R_{t+1} + 1 - \delta_k \right) \right] \quad (40)
$$

$$
\xi_t H_t^{\phi} = \hat{W}_t \hat{\tilde{C}}_{t}^{-\sigma} \quad (41)
$$

$$
\hat{W}_t = \alpha \mu_t^{-1} \left( \frac{\hat{Y}_t}{H_{p,t}} \right) \quad (42)
$$
\[ R_t = (1 - \alpha) \mu_t^{-1} \left( \frac{\tilde{Y}_t}{\tilde{K}_{t-1}} \right) \tau \]  

\[ \mu_t = \frac{\varepsilon_t \left( 1 + \frac{1+\theta\tilde{G}_t}{(1+\theta\tilde{G}_t)Y_t} \right) (\frac{\tilde{Y}_t}{\tilde{K}_{t-1}})\tau}{\varepsilon_t \left( 1 + \frac{1+\theta\tilde{G}_t}{(1+\theta\tilde{G}_t)Y_t} \right) - 1} \]  

\[ (1 - \mu_t^{-1}) \frac{\theta}{(1 + \theta\tilde{G}_t)}^2 + E_t [(1 - \delta_g) Q_{t,t+1} \nu_{t+1}] = \nu_t \]  

\[ Q_{t,t+1} = \frac{\beta}{\tau} \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\sigma} \]  

\[ H_t = H_{a,t} + H_{p,t} \]  

\[ \nu_t = \frac{\tilde{W}_t H_{a,t}}{\alpha_z \tilde{Z}_t} \]  

\[ \tilde{Z}_t = A_t H_{a,t}^\alpha \]  

\[ \tilde{G}_t = \frac{1 - \delta_g}{\tau} \tilde{G}_{t-1} + \tilde{Z}_t \]  

\[ \tilde{K}_t = \frac{1 - \delta_k}{\tau} \tilde{K}_{t-1} + \tilde{I}_t \]  

\[ \tilde{Y}_t = \tilde{C}_t + \tilde{I}_t + \tilde{F}_t \]  

\[ \tilde{Y}_t = A_t \tilde{K}_{t-1}^{1-\alpha} H_{p,t}^\alpha \tau^{\alpha-1} \]  

\[ \log(A_t) = \rho_A \log(A_{t-1}) + \epsilon_t^A \]  

\[ \log(\xi_t) = (1 - \rho_h) \log(\xi) + \rho_h \log(\xi_{t-1}) + \epsilon_t^h \]  

\[ \log(F_t) = (1 - \rho_f) \log(F) + \rho_f \log(F_{t-1}) + \epsilon_t^f \]  

\[ \log(\varepsilon_t) = (1 - \rho_p) \log(\varepsilon) + \rho_p \log(\varepsilon_{t-1}) + \epsilon_t^p \]  

where the exogenous shocks processes are assumed to satisfy: \( \rho_a, \rho_h, \rho_f, \rho_m \in [0, 1) \) and
\[
\begin{bmatrix}
\epsilon_t^a \\
\epsilon_t^h \\
\epsilon_t^f \\
\epsilon_t^p \\
\end{bmatrix}
\sim N
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix};
\begin{bmatrix}
\sigma_a^2 & 0 & 0 & 0 \\
0 & \sigma_h^2 & 0 & 0 \\
0 & 0 & \sigma_f^2 & 0 \\
0 & 0 & 0 & \sigma_p^2 \\
\end{bmatrix}
\]

The system of equations (39)-(57) fully describes the model economy. Let \( \hat{x}_t = \log(\hat{X}_t) - \log(X) \) denote the percentage deviation of the variable \( \hat{X}_t \) with respect to its deterministic steady-state. The VARMA representation of the model is determined by solving the linear system of first order stochastic difference equations obtained by log-linearized equations (39)-(57) around the deterministic steady-state:

\[
\hat{x}_t = \Psi_x(\omega) \hat{x}_{t-1} + \Psi_\epsilon(\omega) \epsilon_t
\] (58)

where \( \hat{x}_t \) is a vector containing all the endogenous variables in percentage deviation with respect to its steady state, \( \epsilon_t \) is a vector containing all the exogenous innovations, and \( \Psi_x \) and \( \Psi_\epsilon \) are matrixes whose entries are functions of the model structural parameters.

To estimate the model, the VARMA representation is augmented by adding four measurement equations that link model variables to four key macroeconomic observable variables. In particular, we use log difference data of: real consumption, real output net of exports, total hours worked, and aggregate real advertising expenditures. The corresponding vector of measurement equations is:

\[
x_t^* =
\begin{bmatrix}
\text{dlCONS}_t \\
\text{dlGDP}_t \\
\text{dlHOURS}_t \\
\text{dlADV}_t \\
\end{bmatrix} =
\begin{bmatrix}
\tau \\
0 \\
0 \\
\tau \\
\end{bmatrix} +
\begin{bmatrix}
\hat{y}_t - \hat{y}_{t-1} \\
\hat{c}_t - \hat{c}_{t-1} \\
\hat{h}_t - \hat{h}_{t-1} \\
\hat{z}_t - \hat{z}_{t-1} \\
\end{bmatrix}
\] (59)

where \( \tau = \log(\tau) \) is the common quarterly growth rate trend. Equations (58)-(59) form the state-space representation of the model economy through which the structural parameters are estimated in order to maximize the likelihood of observed data conditional to our model.
C.2 Convergence diagnostics for selected parameters

Figures 8 and 9 plot the tests of convergence for our Markov chains. Following section 3 in Gelman and Brooks (1998), we employ three criteria of converge for each parameter (i.e. univariate diagnostic):

**Interval** First column of figures 8 and 9. For each chain, take the 80% central interval of the draws from the simulation, compute the length of the interval over the parameter value, and form the mean of the interval lengths. This is the red line, i.e. the mean length of the within-chain intervals, calculated and plotted for increasing number of draws $n$. After, from the whole set of draws gained from all the chains (280,000 draws), calculate the 80% central interval length. This is the blue line, i.e. the length of total-sequence interval, plotted for increasing number of draws $n$.

**$m_2$** Second column of figures 8 and 9. Repeat the same procedure, calculating the central second moment of the chain instead of the interval. Hence, the red line is the mean of central second moments, i.e. the mean of the variance of chains, while the blue line is the total-sequence (all chains) variance.

**$m_3$** Third column of figures 8 and 9. Repeat the same procedure as in $m_2$, calculating the central third moment instead of the second moment.

Gelman and Brooks (1998) showed that in the three criteria the ratio between the two statistics goes to 1 as convergence is approached. Thus, we expect the blue and the red lines to stabilize and shrink to the same value for an increasing number of $n$ on the x-axis.

![Figure 8: Gelman-Brooke Test](image-url)
Figure 9: Gelman-Brooke Test
Figure 10: **Priors and Posteriors distributions.** Priors are plotted in gray, Posteriors in black, and the mode values computed to initiate the Chains are in dashed green.
Figure 11: **Priors and Posteriors distributions.** Priors are plotted in gray, Posteriors in black, and the mode values computed to initiate the Chains are in dashed green.