Robust Control and Persistence
in the New Keynesian Economy

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Abstract

Since Keynes no economist would deny that expectations under uncertain conditions matter for the conduct of monetary policy, but still opinions about their formation are diverse. We build a hybrid New Keynesian Framework to analyze the influence of model uncertainty on optimal interest rates under different degrees of rational forward-looking behavior, using recently developed robust control techniques. Impulse response functions illustrate that uncertainty seems to be a rationale for more aggressive interest rate reactions, but also suggest that the degree of forward-looking behavior seems to be more important than an appropriate fear about the misspecification of a given model. Furthermore, we argue that assuming to control inflation through expectations is a policy on the razor’s edge, since robust expectations overestimate shock impacts. This questions the gains from commitment under uncertainty.

Keywords: Robust Control, Knightian Uncertainty, Monetary Policy, Forward-Looking Expectations, Model Uncertainty.

JEL classification: C61; C68; D81; E5; E52; E58; E61

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1 Introduction

Since Keynes no economist would deny the importance of uncertainty for the conduct of optimal monetary policy. He explicitly stresses the role of expectations under uncertain conditions and the sluggish adjustment of prices. By combining modern macroeconomic techniques from the Real Business Cycle literature with the assumption of rigid prices, the New Keynesian framework derives forward-looking versions of the Phillips and IS curves, that allow for a short-run non-neutrality of money. However, opinions of central bankers and academic researchers, as well as results of empirical studies about inflation and output persistence are diverse, and suggest that the assumption of purely forward-looking agents seems to be too strong.\(^1\) Due to this controversy we attempt to provide some additional insights by investigating uncertainty under different assumptions on price-setting behavior and consumption habits in a hybrid New Keynesian model. A vast amount of literature on model uncertainty uses Bayesian approaches that provide a robustness analysis by drawing conclusions, given prior probability distributions over uncertain parts of a single reference model or the model as a whole.\(^2\) However, even Friedman pointed out that his knowledge about the dynamics of the economy is too small to express such personal (prior) probabilities. To reflect for these concerns, we use recently developed robust control techniques proposed by Hansen and Sargent (2007), where a robust policy is the response to a "worst case" shock process. This approach overcomes Brainard’s conservatism principle, and suggests that model uncertainty provides to be a rationale for more aggressive policy interventions.\(^3\) We investigate the influence of the private sector’s expectation formation on this result.

The remainder of this paper is organized as follows. Section 2 summarizes the Hansen-Sargent methodology. Subsequently, section 3 presents the hybrid New Keynesian framework. In section 4 we analyze the influence of uncertainty on optimal monetary policy under different assumptions on the persistence of private sector behavior, and for a wide range of policy objectives. We compute losses under robust and non-robust policies and show impulse response functions (IRFs) to illustrate the model dynamics. All simulations are done for the cases of a discretionary and a commitment policy, as well as for different simple rules of the Taylor type. The final section concludes.

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\(^2\)See for example Brainard (1967), Craine (1979) and Levin and Williams (2003).

\(^3\)Robust control applications apart from monetary policy analysis include for example a series of papers providing a solution for the "equity premium puzzle" as the increased cautiousness of investors enlarges risk premia. See for example Hansen, Sargent, and Wang (2002).
2 Hansen-Sargent Robustness

Idea The core of the idea is to treat the decision maker’s model as an approximation of the true model. Let \( x \) be a vector of state variables and let the true data follow a Markov-process with a transition density \( f(x^t|x) \). Moreover, let the approximating model be described by a transition density \( f_\alpha(x^t|x) \) (\( \alpha \in A \), where \( A \) denotes a compact set of parameter values). Then the maximum likelihood estimator \( \hat{\alpha}_0 \) would be derived by minimizing the relative entropy of \( f \) and \( f_\alpha \), which measures the "expected distance" between \( f \) and \( f_\alpha \). The Hansen and Sargent (2007) methodology inverts this approach by taking a transition density \( f_0 \) as given, and building a set of possible data generating processes around this model, so that the true model is one model in this set.

In conventional linear quadratic control problems the Certainty Equivalence Principle leads to results independent from a homoskedastic Gaussian zero mean vector shock process \( \{\varepsilon_t\} \). Thus, robust control theorists introduce an additional vector process \( \{\omega_t\} \) that depends in a possibly nonlinear way on the history of the states:

\[
x_{t+1} = Ax_t + Bu_t + C(\varepsilon_{t+1} + \omega_{t+1}),
\]

where \( u \) denotes a vector of control variables, and \( A, B \) and \( C \) are matrices, filled with appropriate structural parameters. If the loss function of the decision maker is given by \( r(x,u) = |z|^2 \), the robust problem can be reduced to the following Bellman equation:

\[
V(x) = \min_{\{u_t\}} \max_{\{\omega_{t+1}\}} \mathbb{E}_t \{ z'z - \theta \delta \omega' \omega^* + \delta V(x^*) \}
\]

subject to (1) where \( \theta > 0 \) represents the decision maker’s preference for robustness, which decreases with rising \( \theta \). The \( \{\omega_t\} \) can be interpreted as an imaginary evil agent, who embodies pessimism and tries to distort the model equations to maximize losses. The corresponding decision rule \( u = -Fx \) depends on \( C \) and Certainty Equivalence is broken. As the size of \( \omega \) is directly penalized through \( \theta \), the choice of \( \theta \) is essential for defining a plausible range of uncertainty.

Detection error probabilities Using detection error probabilities to choose a value for \( \theta \) links the choice of the worst case model with Bayesian detection theory. To derive a value for \( \theta \) log likelihood ratios are computed for a large number of simulations. Detection error probabilities are defined as \( p(\theta) = \frac{1}{2} (p_a + p_d) \), where \( p_i \) represents the frequency of simulations with a log likelihood ratio smaller or equal to zero, when the approximating \( (i = a) \) or the distorted \( (i = d) \) model is assumed to be the data generating process. Hansen and Sargent advise to set \( p(\theta) \) at a plausible value (10 – 20% in a sample of size 150) and then invert \( p(\theta) \) to find a plausible value for the robustness parameter.

\(^4\) denotes next period values.
Interpretation Unlike a Bayesian planner, who is only concerned about calculated risk (and is able to build a prior probability), a robust planner faces unorganized uncertainty and takes the least favorable prior, given a restricted set of priors. Therefore robust control theory covers a wide range of misspecified dynamics including wrong parameters ($\omega_{t+1}$ is a linear function of $x_t$), autocorrelated errors ($\omega_{t+1}$ is a linear function of $x'_t$), and nonlinearities ($\omega_{t+1}$ is a nonlinear function of $x_t$). The imaginary evil agent tries to wreak the largest possible havoc, given a budget constraint determined by $\theta$, and hits the model where the variance of the forecast error is the largest. Furthermore, for models including expectations, this methodology provides deviations from the rational expectations hypothesis.

Robust Control in monetary policy analysis Early robust control approaches study backward-looking models and suggest using more aggressive rules as shocks become larger and serially correlated under uncertainty (see for example Onatski and Stock (2002)). Giannoni (2002) concludes that parameter uncertainty in a purely forward-looking New Keynesian model leads to stronger optimal responses of the interest rate to fluctuations in inflation and the output gap and can amplify the degree of super-inertia in optimal rules (a coefficient on the lagged interest rate greater than one). We contribute to this literature the analysis of a variety of hybrid closed New Keynesian models, differing in their degree of rational forward-looking behavior.

3 Robustness in a hybrid New Keynesian model

The following framework is based on a DSGE model with optimizing behavior and Calvo (1983)-type price stickiness. To capture inflation and output persistence, a fraction of firms adjust prices using the recent history of inflation, and households preferences include consumption habits. We assume competitive good markets drawn from Dixit and Stiglitz (1977).

Households Households supply labor ($L_t$), purchase consumption goods ($C_t$), and hold money ($M_t$) and bonds ($B_t$). A representative household maximizes the expected sum of the discounted value of its utility with respect to its preferences:

$$
\max_{\{C_t,M_t,N_t,B_t\}} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{(C_{t+i} - H_{C,t})^{1-\sigma}}{1-\sigma} + \frac{\gamma (M_{t+i})^{1-b}}{1-b} - \frac{\chi N_{t+i}^{1+\eta}}{1+\eta} \right], \tag{3}
$$

where $\beta$ represents the discount factor, $b$ and $\eta$ represent the risk aversion parameters of real money holding and labor supply, respectively. $H_{C,t} = h_C C_{t-1}$ represents the habit formation in consumption: The higher $h_c$, the more the households try to differ not too much from other households.\(^5\) Thus $h_C$ is a measure of the persistence

\(^5\)See Fuhrer (2000) for details.
in consumption habits.

**Firms** For the price adjustment we assume a variant of Calvo’s price stickiness due to Galí and Gertler (1999). Conventionally a randomly selected fraction of firms \((1 - \omega)\) adjusts their price while the remaining fraction \(\omega\) does not. In addition we assume that only a fraction of \((1 - \tau)\) behave in a forward-looking way and the remaining fraction \(\tau\) uses the recent history of the aggregate price index. According to this, \(\tau\) is a measure of the degree of persistence in price setting. The backward-looking adjusters use a "rule of thumb", \(p_{t}^{bl} = \bar{p}_{t-1} + \pi_{t-1}\), where \(p_{t}^{bl}\) represents the optimal price of a backward looking firm, \(\pi_{t}\) represents inflation, and \(\bar{p}_{t}\) is the index for the prices newly set in period \(t\). So a backward-looking firm simply corrects the average price of last period’s price adjustment for inflation. The aggregate price level is an average of the price charged by the adjusters and the nonadjusters,

\[
P_{t}^{1-\theta} = \omega P_{t-1}^{1-\theta} + (1 - \omega) \left(\bar{p}_{t}^{1}\right)^{1-\theta},
\]

and the index for newly set prices is defined as the weighted average of the prices of the forward-looking and the backward-looking adjusters:

\[
\bar{p}_{t} = (1 - \tau) p_{t}^{fl} + \tau p_{t}^{bl}.
\]

**Linear approximations** Approximating the previous equations around a zero average inflation steady state gives the hybrid version of the *New Keynesian Phillips curve* (NKPC):\(^6\)

\[
\pi_{t} = \kappa (\gamma y_{t} - \sigma y_{t-1}) + \phi (\omega \beta E_{t} \pi_{t+1} + \tau \pi_{t-1}),
\]

where \(\kappa \equiv (1 - \tau) (1 - \omega) (1 - \beta \omega) \phi, \phi \equiv (\omega + \tau [1 - \omega (1 - \beta)])^{-1}, \sigma \equiv h_{C}^{1-h_{C}}, \gamma \equiv \eta + \frac{\sigma}{1-h_{C}}\), and \(y_{t}\) is the output gap. The measure of the degree of backward-looking price-setting \(\tau\) also represents a measure of the degree of persistence in inflation dynamics. The parameter \(\kappa\) represents the impact of the output gap on inflation due to its influence on real marginal costs and depends on structural parameters.

Linearizing the Euler equation, following from (3) gives the *hybrid IS curve*:

\[
y_{t} = \left(\frac{1}{1+h_{C}}\right) (E_{t} y_{t+1} + h_{C} y_{t-1}) - \sigma^{-1} \left(1 - h_{C}\right) (i_{t} - E_{t} \pi_{t+1}),
\]

This illustrates that \(h_{C}\) is also a measure of persistence in output gap dynamics.

To study impulse responses we add general demand \((e_{yt})\) and supply \((e_{\pi})\) shocks, following AR(1) processes with innovations \(\xi_{yt}\) and \(\xi_{\pi}\):

\[
e_{yt} = \rho_{t} e_{yt-1} + \xi_{yt}, \quad \xi_{yt} \sim N(0, \sigma_{j}), \quad j = y, \pi.
\]

\(^6\)See appendix for details.
Since central banker’s opinions about their optimal objective are diverse, we assume a very general quadratic loss function including the possibility for interest rate smoothing and/or stabilization:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \pi_t^2 + \lambda_y y_t^2 + \lambda_i i_t^2 + \lambda_{\Delta i} (i_t - i_{t-1})^2 \right).$$  \( (9) \)

The planner’s reaction function is derived by minimizing (9) subject to (6), (7), (8) and (??). For deriving a robust solution the problem is extended to a max-min standard RE problem.\(^7\) We derive losses and impulse response functions (IRFs) for a policymaker acting under commitment and discretion, and using a simple Taylor-type rule.

### 4 Calibration and results

<table>
<thead>
<tr>
<th>parameter</th>
<th>scenario</th>
<th>h(_c)</th>
<th>h(_c)</th>
<th>(\tau)</th>
<th>(\tau)</th>
<th>(\omega)</th>
<th>(\omega)</th>
<th>(\sigma)</th>
<th>(\sigma)</th>
<th>(\eta)</th>
<th>(\eta)</th>
<th>(\beta)</th>
<th>(\beta)</th>
<th>(\rho_y, \rho_x)</th>
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<th>(\sigma_y, \sigma_\pi)</th>
<th>(\sigma_y, \sigma_\pi)</th>
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<td>(h_c)</td>
<td>degree of persistence in habit formation</td>
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<td>0</td>
<td>0.4</td>
<td>0.8</td>
<td>0.8</td>
<td>0.1571</td>
<td>0.1571</td>
<td>0.824</td>
<td>0.824</td>
<td>0.99</td>
<td>0.99</td>
<td>0.35</td>
<td>0.35</td>
<td>1</td>
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Table 1: Model parameters

The following sections show losses and IRFs for different weights in the objective of the policymaker, inter alia the estimated objective of Rotemberg/Woodford,\(^8\) since the information about the real degree of persistence is rare, we simply experiment with the values 0.4 and 0.8. Thus, the results should be interpreted carefully, especially for \(\tau = h_c = 0.8\), which seems to be unrealistic high with respect to the corresponding \(\kappa\)-value, representing the influence of marginal costs on inflation.

\(^7\) See Söderlind (1999) and Giordani and Söderlind (2004) for details.

\(^8\) Since the information about the real degree of persistence is rare, we simply experiment with the values 0.4 and 0.8. Thus, the results should be interpreted carefully, especially for \(\tau = h_c = 0.8\), which seems to be unrealistic high with respect to the corresponding \(\kappa\)-value, representing the influence of marginal costs on inflation.
which is given by \((\lambda_y, \lambda_i, \lambda_{\Delta i}) = (0.0483, 0.2364, 0)\). Losses are computed for the approximating model using the optimal \(RE\) rule \((l_{a}^{RE})\) and the worst case model using the robust rule \((l_{a}^{r})\). Additionally, we show losses for the case of unfounded fear against mispecification, by using the robust rule although the approximating model is the true one \((l_{a}^{r})\).

4.1 Commitment

\[
\begin{align*}
\lambda_y, \lambda_i, \lambda_{\Delta i} & = (0, 0.3, 0) & (0.0483, 0.2364, 0) & (0.1, 0.5, 0) \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>Scenario</th>
<th>(p(\theta))</th>
<th>50%</th>
<th>35%</th>
<th>10%</th>
<th>50%</th>
<th>35%</th>
<th>10%</th>
<th>50%</th>
<th>35%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(h_c = \tau = 0)</td>
<td>(l_{a}^{RE})</td>
<td>153</td>
<td>166.272</td>
<td>183, (\infty)</td>
<td>831</td>
<td>ind</td>
<td>ind</td>
<td>239</td>
<td>ind</td>
<td>ind</td>
</tr>
<tr>
<td>(h_c = 0.4, \tau = 0)</td>
<td>(l_{a}^{RE})</td>
<td>45</td>
<td>48.55</td>
<td>50, (\infty)</td>
<td>219</td>
<td>ind</td>
<td>ind</td>
<td>239</td>
<td>ind</td>
<td>ind</td>
</tr>
<tr>
<td>(\tau = 0.4, h_c = 0)</td>
<td>(l_{a}^{RE})</td>
<td>47</td>
<td>45.49</td>
<td>43, (\infty)</td>
<td>220</td>
<td>ind</td>
<td>ind</td>
<td>239</td>
<td>ind</td>
<td>ind</td>
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<tr>
<td>(h_c = \tau = 0.4)</td>
<td>(l_{a}^{RE})</td>
<td>138</td>
<td>148.174</td>
<td>160, (\infty)</td>
<td>834</td>
<td>ind</td>
<td>ind</td>
<td>239</td>
<td>ind</td>
<td>ind</td>
</tr>
<tr>
<td>(h_c = \tau = 0.8)</td>
<td>(l_{a}^{RE})</td>
<td>540</td>
<td>599.4158</td>
<td>689, (\infty)</td>
<td>6151</td>
<td>ind</td>
<td>ind</td>
<td>239</td>
<td>ind</td>
<td>ind</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{objective} & = (0.5, 0, 0) & (0, 0.6, 0) & (0.5, 0.3) \\
\end{align*}
\]

<table>
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<tr>
<th>Scenario</th>
<th>(p(\theta))</th>
<th>50%</th>
<th>35%</th>
<th>10%</th>
<th>50%</th>
<th>35%</th>
<th>10%</th>
<th>50%</th>
<th>35%</th>
<th>10%</th>
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<tbody>
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<td>(l_{a}^{RE})</td>
<td>249</td>
<td>ind</td>
<td>ind</td>
<td>52</td>
<td>54.125</td>
<td>59, (\infty)</td>
<td>239</td>
<td>ind</td>
<td>ind</td>
</tr>
<tr>
<td>(h_c = 0.4, \tau = 0)</td>
<td>(l_{a}^{RE})</td>
<td>249</td>
<td>ind</td>
<td>ind</td>
<td>49</td>
<td>51.66</td>
<td>54, (\infty)</td>
<td>239</td>
<td>ind</td>
<td>ind</td>
</tr>
<tr>
<td>(\tau = 0.4, h_c = 0)</td>
<td>(l_{a}^{RE})</td>
<td>249</td>
<td>ind</td>
<td>ind</td>
<td>49</td>
<td>51.66</td>
<td>54, (\infty)</td>
<td>239</td>
<td>ind</td>
<td>ind</td>
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<tr>
<td>(h_c = \tau = 0.4)</td>
<td>(l_{a}^{RE})</td>
<td>920</td>
<td>ind</td>
<td>ind</td>
<td>176</td>
<td>187.8618</td>
<td>212, (\infty)</td>
<td>79</td>
<td>ind</td>
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<tr>
<td>(h_c = \tau = 0.8)</td>
<td>(l_{a}^{RE})</td>
<td>920</td>
<td>ind</td>
<td>ind</td>
<td>157</td>
<td>166.578</td>
<td>183, (\infty)</td>
<td>876</td>
<td>ind</td>
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</tr>
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</table>

For some scenarios the steady state of the model is indeterminate (ind).

Table 2: Losses under commitment

Losses for the commitment case are given in Table 2. The non-robust results suggest that persistence in price-setting increases losses substantially, whereas persistence in consumption dynamics does not change outcomes considerably, or even decreases losses. Whereas the impact of increasing consumption habits on the effectiveness of monetary policy is twofold, an increase in the fraction of backward-looking price-setters clearly deteriorates the transmission of monetary policy. With increasing persistence in price-setting (i) expectations are less important (this argument is also valid for an increase in consumption habits), and (ii) deviations from flexible price output play a lower role in the Phillips curve (here consumption habits imply the opposite).

Regarding the robust solutions, Table 2 clearly shows that losses under uncertainty rise. Furthermore, the results suggest that under commitment the model is

\(^9\)Robust solutions are computed for robustness parameters \(\theta\) that correspond to detection error probabilities of 10% and 35% in a sample of 150, using Monte Carlo simulations.
very sensitive with respect to uncertainty. Using the robust rule in the approximating model does not stabilize the problem for \( p(\theta) = 10\% \). For \( p(\theta) = 35\% \), corresponding to a lower degree of uncertainty, the losses in the case of an unfounded fear against misspecification already rise substantially, whereas using the robust rule in the worst case scenario implies losses near the optimal RE solution. Thus, a central banker acting under commitment is confronted with two opportunities: React on uncertainty and use a robust policy, risking an immense increase in losses, when his fear is unfounded, or ignore uncertainty, even if a robust policy provides a very good advise, when the worst case scenario actually arrives. Moreover, Table 2 suggests that already a small degree of inflation persistence increases losses more than the evil agent could, when the policymaker uses a robust strategy. This indicates that the choice of the degree of price-setting persistence is more important for the conduct of monetary policy than an appropriate degree of general model uncertainty.

Figure 1 illustrates the dynamics of the model under \((\lambda, \lambda_1, \lambda_{\Delta\lambda}) = (0, 0.3, 0)\) for the standard RE solution \((\theta = \infty)\) and the robust policy in the worst case. Furthermore, the dynamics of an unfounded fear against misspecification are illustrated by the response of the robust rule in the approximating model for \( p(\theta) = 35\% \). The IRFs show that uncertainty justifies stronger interest rate reactions for all scenarios.

In the case of a supply shock the impulse is a rise in inflation, which provokes the central banker to increase the nominal interest rate strong enough to cause a positive real interest rate in order to reduce demand. The fall in the output gap causes a period of disinflation immediately, and \( \pi_t \) undershoots its equilibrium value for some periods. Additionally, the interest rate is adjusted gradually under commitment, because of its preference for interest rate stabilization, and due to the influence on expectations. The gradual response exploits private sector expectations as it signals, that the central bank is willing to fight inflation. Furthermore, introducing persistence in consumption habits seems not to change the degree of deviations considerably, whereas persistence in price-setting clearly does. Concerning the robust solutions all graphs justify stronger interest rate reactions. The policymaker and the private sector fear stronger and more persistent shock impacts and react with a more aggressive interest rate policy. Since the effectiveness of monetary policy decreases with rising persistence, this effect becomes even stronger when we introduce consumption habits and backward-looking price-setting. Furthermore, the graphs confirm that the degree of persistence in price-setting seems to influence outcomes more than an appropriate degree of uncertainty.

\[\text{Note, that the RE and the approximating solutions share the same dynamics for the pre-determined variables (the same approximating model), but the latter uses robust expectations and a robust policy rule. Central bank and private sector share the same reference model, the same loss function, and the same degree of robustness.}\]

\[\text{As Clarida, Galí, and Gertler (1999) among others emphasize, under commitment the monetary authority chooses a path for the output gap that keeps output below its potential value for some periods to lower private sector expectations of future inflation and improve the policy trade-off.}\]
4. Calibration and results

For a demand innovation the primary impulse of an increase in consumption leads to an increase in inflation as well. The authority sets the interest rate high enough to keep output under potential for some periods to improve expectations, and $y_t$ and $\pi_t$ undershoot their equilibrium values. In opposition to the supply innovation, an increase in inflation persistence does not imply a more aggressive policy. Since the transmission of marginal costs on inflation decreases with $\tau$, the effectiveness of the central banker decreases as well, but as the primary effect on inflation becomes smaller, this calls for a more defensive policy reaction. Due to the same arguments, the opposite is true for consumption habits: Consumption habits increase the primary transmission on inflation and deteriorate the effectiveness of monetary policy, so that the optimal reaction has to be more aggressive. Once again, the degree of persistence seems to have a greater impact than an appropriate degree of uncertainty around a reference model.
However, the most exciting result is that the reactions of the robust solution in the approximating model show a highly undesirable cyclical adjustment of the interest rate and the output gap, which seems to become even stronger and more persistent when the fraction of backward-looking price-setters increases, whereas an increase in consumption habits tends to decrease the variability. This underlines the difficulty to choose an optimal interest rate reaction, since the case of unfounded fear leads to disastrous results, when the central banker and the private sector are committed to the wrong model.

4.2 Discretion

<table>
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<tr>
<th>$(\lambda_y, \lambda_i, \lambda_\Delta i)$</th>
<th>$(0, 0.3, 0)$</th>
<th>$(0.0483, 0.2364, 0)$</th>
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<td>scenario</td>
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<table>
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<tr>
<th>$(\lambda_y, \lambda_i, \lambda_\Delta i)$</th>
<th>$(0.06, 0)$</th>
<th>$(0.5, 0.3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p(\theta)$</td>
<td>50%</td>
<td>35%</td>
</tr>
<tr>
<td>scenario</td>
<td>$l^<em>_R$, $l^</em>_a$</td>
<td>$l^<em>_R$, $l^</em>_a$</td>
</tr>
<tr>
<td>$h_c = \tau = 0$</td>
<td>294</td>
<td>364,381</td>
</tr>
<tr>
<td>$h_c = 0.4, \tau = 0$</td>
<td>252</td>
<td>293,306</td>
</tr>
<tr>
<td>$\tau = 0.4, h_c = 0$</td>
<td>2215</td>
<td>5727,5876</td>
</tr>
<tr>
<td>$h_c = \tau = 0.4$</td>
<td>1360</td>
<td>1720,1759</td>
</tr>
<tr>
<td>$h_c = \tau = 0.8$</td>
<td>5529</td>
<td>6127,6018</td>
</tr>
</tbody>
</table>

Table 3: Losses under discretion

In Table 3 losses are given for a central banker, who reoptimizes every period. As the policymaker is unable to improve expectations losses rise compared to those

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12The last result is due to the increase in the transmission of output deviations on inflation. As the monetary policy becomes more effective, when the influence of consumption on real wages and marginal costs increases, robust policy and expectations do not need to be that aggressive.
of the commitment case for all scenarios and objectives. Similar to Table 2 consumption habits do not change outcomes considerably, whereas persistence in price-setting leads to an immense increase in losses. However, it seems to be easier for a discretionary central banker to handle uncertainty, since robust losses are near the optimal RE solution, and using the robust rule in the approximating model does still stabilize the model. In fact, losses under unfounded fear are near the robust solution in the worst case model. This indicates that a discretionary policymaker is well advised to use a robust policy.

Figure 2 shows the IRFs for $p(\theta) = 10\%$ and illustrates that also a discretionary policymaker should react on uncertainty with a more aggressive policy. Furthermore, the dynamics of the model clearly exhibit the well-known stabilization bias due to the inability to manage expectations: A discretionary policymaker adjust the interest rate to offset shock impacts immediately, what can only be achieved at very high
costs in terms of strong deviations from equilibrium. Furthermore, the interest rate is not adjusted gradually, but risen very sharply at the beginning, even though the authority has a preference for interest rate stabilization.

The reactions with respect to a supply innovation show similar dynamics as for the commitment solution, except for the case of $\tau = 0.4$. Similar to the observations in the case of commitment, the introduction of both, consumption habits and backward-looking price-setting, leads to a more aggressive strategy. Moreover, the responses of inflation increase with $\tau$ and decrease with $h_c$, due to the influence on the transmission of demand on inflation. The robust reactions in the worst case model illustrate again that uncertainty leads to stronger and more persistent shock impacts. In addition, for the dynamics of the robust rule in the approximating model, the graphs confirm the suggestion from Table 3: Under discretion the case of unfounded fear lies not far away from the robust solution. The IRFs illustrate that an unfounded fear would lead to stronger and more persistent reactions of the interest rate compared to the $RE$ solution, but they also show how in the absence of an evil agent all variables react with less strength than in the robust solution under the worst case. As under discretion neither the policymaker nor the evil agent are able to influence expectations, the authority earns higher losses than in the commitment case, but also less disturbances due to model misspecification, since robust expectations do not overestimate shock impacts in such a heavy manner any more.

The dynamics of the model after a demand innovation show similar primary reactions than for the commitment case, except for the size. Like in the commitment solution, rising persistence in price-setting does not imply a more aggressive policy, whereas rising consumption habits do. Furthermore, IRFs illustrate that robust reactions are more aggressive and persistent. However, using the robust policy in the approximating model does not lead to undesirably high variability, since robust expectations are unaffected and thus private sector and central banker’s overestimation of shock impacts can be handled by the approximating model. However, with respect to the comparison of uncertainty and persistence all IRFs validate again that the degree of persistence in consumption habits and price-setting seems to be more important than the general model uncertainty, described by Hansen-Sargent robustness.

### 4.3 Simple rules

At least since Taylor has shown that his hypothetical policy rule approximates the behavior of the Federal Reserve during the 1990s quite well, simple rules receive more and more popularity. Furthermore, in the context of Hansen-Sargent robust policies, studying simple rules represents an interesting deviation of the private sector from

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13See Clarida, Galí, and Gertler (1999) for a discussion

14Here the primary increase in inflation is accompanied by an increase in output, as the interest rate is not risen sharply enough to generate a positive real interest rate. As the transmission from marginal costs to inflation is very weak, this seems to be a plausible result.
the RE hypothesis: As soon as one assumes that a policymaker commits to a rule, he is no longer involved in any decision, and all differences between the optimal RE and the robust solution must be due to private sector expectations.

RE hypothesis: As soon as one assumes that a policymaker commits to a rule, he is no longer involved in any decision, and all differences between the optimal RE and the robust solution must be due to private sector expectations.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>( \theta )</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( \delta )</th>
<th>( \epsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Note that for the robust solution under the worst case scenario (\( \theta \), private expectations are robust), and for the approximating model, using robust expectations (\( \theta \)) the dynamics of the forward looking variables and the interest rate are the same. The only difference to the robust solution under the worst case scenario is that we canceled the evil agent.

Table 4: Losses using simple rules

Table 4 shows losses for six different simple rules of the form \( i_t = g_\pi \pi_t + g_y y_t + g_i i_{t-1} \) for the policy objective estimated by Rotemberg/Woodford: \((\lambda_y, \lambda_t, \lambda_{\Delta t}) = (0.0483, 0.2364, 0)\). Among other hypothetical policy rules, we also investigate the consequences of uncertainty for the famous Taylor-rule, \((g_\pi, g_y, g_i) = (1.5, 0.5, 0)\), and the optimal rules of Giannoni (2002). For the case of parameter uncertainty and a shock process on the equilibrium interest rate, he derives an approximating, (2.217, 0.5, 0), and a robust Taylor rule, (8.294, 0.5, 0).
Similar to the commitment case, the table provides that the degree of backward-looking price setters seems to cause higher problems than the degree of consumption habits, and that these measures of persistence can be more important than the policymaker’s fear about a general model misspecification. Once again, the results suggest that the authority has to decide between implementing a very aggressive policy rule that handles uncertainty quite good, but leads to very high losses in the case of no uncertainty, or ignoring uncertainty and using a more defensive rule, which leads to lower losses if the approximating model is the true one, but could imply disastrous results under uncertainty.

When we are to compare the results under simple rules with those under discretion, there is no distinct, scenario-independent regularity. For the purely forward-looking model the Taylor-rule performs worse than the optimal discretionary policy, but the introduction of a small persistence in consumption habits or price-setting behavior inverts this outcome. For $h_c = \tau = 0.8$ the simple Taylor rule implies much smaller losses than the optimal discretionary solution, and is near the optimal, but highly improbable solution under commitment. For the highly backward-looking scenario this result stays valid for all rules, except for the two most aggressive ones.

In Figure 3 the dynamics of the model for the optimal robust rule of Giannoni (2002) are given. The dynamics after a supply innovation are very similar to the impulse responses under discretion. Furthermore, the degree of persistence in price-setting behavior causes stronger deviations of inflation from equilibrium, whereas a higher degree of consumption habits causes the opposite. However, the loss in effectiveness leads to stronger and more persistent deviations of $\pi_t$ and $y_t$ and thus implies stronger and more persistent interest rate reactions. For the case of a demand innovation the IRFs illustrate again, that an increase in inflation persistence does not imply more aggressive reactions, whereas a higher degree of consumption habits does.

The robust solutions illustrate that the fear of the private sector is represented by stronger and more persistent shock impacts, which also imply stronger and more persistent interest rate responses, but it seems that unfounded robust expectations do not lead to such catastrophic outcomes as shown in the commitment case. The unfounded fear leads to an overestimation of expectations, which can be easily controlled. In addition, the graphs indicate again that the choice of the right degree of persistence can be more important than an appropriate amount of general model uncertainty.

4.4 Summary and discussion

The analysis of model uncertainty clearly provides to use more aggressive policy rules. However, the results also suggest that, when the central bank uses the robust policy and the private sector builds robust expectations in the most likely outcome, this can lead to disastrous results and infinite losses, especially in the case of commitment. As the commitment to optimal interest rate dynamics is unlikely, since a
monetary authority needs to be highly credible (see for example Clarida, Galí, and Gertler (1999) for a discussion), the results concerning uncertainty advise to use the optimal commitment solution only as a theoretical benchmark that one can not, and should not implement in reality.

Similar to the results under commitment, the analysis of discretion and simple rules also shows that using robust policies in the approximating model can lead to undesirably high losses, suggesting that the monetary authority has to decide whether he wants to reflect his concerns about misspecification and uses a more aggressive policy that works well in the worst case, but might lead to a very strong increase in losses when the more likely approximating model is the true one, or ignore his concerns and uses a standard policy, which handles the approximating model very well, but could lead to infinite losses under uncertainty. A robust policymaker should bear in mind that a good solution in the worst case can only be achieved at
costs of average performance.

**Comparison with the robust control literature** Robust control applications to purely forward-looking closed economies can be found for example in Gianonni (2002) and Kilponen (2004). Gianonni concentrates on simple Taylor rules and also concludes that these have to be more aggressive under uncertainty. The work of Kilponen combines model uncertainty and data uncertainty by combining robust control with imperfect information approaches. Robustness leads to overestimation of shocks and revisions that are more often than under standard forecasting. His investigation of robust Taylor rules leads to results similar to ours, as inflation reacts stronger on shocks under uncertainty and robust consumption implies precautionary savings.

Leitemo and Söderström (2004) investigate a forward-looking open New Keynesian economy under commitment, including a real interest rate parity condition. They follow the modelling approach of Clarida, Galí, and Gertler (2002), but assume a time-varying premium on foreign bond holding. The model converges to our closed version when the degree of openness goes to zero. Due to the simple structure of the model it can be solved analytically. To reflect that policymakers are more confident in some relationships than in others, robustness is modelled for every equation separately. Similar to our results, the solutions show that in a closed economy the interest rate reacts with more aggression as the central banker fears that inflation and output are more volatile. Furthermore, the authors also argue that the robustness against uncertainty in inflation dynamics leads to undesirably high output variability, when the robust policy is used in the more likely approximating model instead of the worst case scenario.

**Comparison with other methodologies** Studies using techniques that evaluate monetary policy in different reference models are related to ours in the sense that we also use the robust policy in the approximating model, and investigate the performance of simple rules in models, differing in the degree of persistence. Coenen (2007) focusses on inflation persistence and concludes that it pays for a policymaker to overestimate persistence, since losses increase when doing the opposite mistake. Angeloni, Coenen, and Smets (2003) extend this analysis by exploring the influence of uncertainty about nominal and real persistence in an estimated hybrid DSGE model for the Euro area, including partly backward-looking wages and prices, consumption habits, and adjustment costs in investment. They analyze both, inflation and output persistence and confirm Coenen’s result. Furthermore, optimized simple rules are quite robust against persistence in the output gap, illustrating that output

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16 Technically speaking, those policymakers are able to express their beliefs (at least partly) in probability statements. Critics would argue, that this assumption is inappropriate.

17 Only for the case of an optimal targeting rule under the Nash assumption, optimal policy is unaffected.
persistence seems to be a rather small problem. This supports our results regarding persistence in price setting and consumption habits. Moreover, the analysis shows that a commitment policy is more robust with respect to inflation persistence than a simple rule. However, optimal rules under commitment derived in a forward-looking model might work well in a backward-looking economy, but we show that commitment under uncertainty causes problems in the approximating model nevertheless, since private sector and central bank expectations of shock impacts are too strong under a preference for robustness. Thus, our analysis suggests that the costs of assuming that inflation can be controlled through expectations can be very high with respect to uncertainty.

The study of Levin and Williams (2003) uses a diagnostic approach, where non-nested models represent competing perspectives about controversial issues like expectation formation or inflation persistence. They compare optimal rules under three different models and show that rules which are derived in order to work well in the neighborhood of one model might perform very badly in one of the others. The model variants cover a standard forward-looking New Keynesian economy, the purely backward-looking model of Rudebusch and Svensson (1998), and the model of Fuhrer (2000), including persistence in output and inflation. They show that rules derived by robust control methods perform very poorly in a different model. Similar to our analysis this provides that the right reference model seems to have a stronger influence than the fear about misspecification.

5 Conclusions

This paper analyzes the influence of model uncertainty on monetary policy in a variety of closed hybrid New Keynesian models. After summarizing our results, there are a few robust conclusions to mention. First of all, the greatest fear against misspecification is represented by an overestimation of shock persistence. Therefore, robust private agents and policymakers imply more aggressive policies. However, a lot of academic work suggests that robust rules might perform very badly in models different from the reference model. We support this view by showing that the influence of persistence can be stronger than the influence of Hansen-Sargent robustness, at least with respect to price-setting behavior, and illustrate that using a robust rule in the most likely approximating model causes immense problems as well. Thus, Sims (2001) is right in his belief that robust control can only be one methodology among many others. A robust policymaker should bear in mind that to be prepared for the worst case implies higher losses when a more likely model appears to be the true one.

Our results contribute to the discussion about whether optimal policy should be build under commitment, the argument, that the costs of assuming that inflation can be controlled through expectations can be very high with respect to uncertainty, since robust expectations overestimate shocks in a manner that is difficult to handle
in a more likely model. This questions the already very difficult to achieve commitment solution to be the best advise for the monetary authority. Moreover, the results under discretion and simple rules give little support for using a moderately more aggressive Taylor rule as a benchmark. In this we support the view of Taylor (1998) who favours a mixture of a pure rule and a pure discretion scenario as it reduces uncertainty (improves predictability), and includes the necessary possibility to deviate from such a rule under unforeseeable events.

A Appendix

Approximation around the steady state

We use the same notations as in the detailed derivation in Walsh (2003) and only focus on changes due to our assumptions on persistent private sector behavior. Assuming a production function that solely depends on labor input and a stochastic zero mean productivity disturbance $Z_t$, $C_{jt} = Z_t N_{jt}$, and using the market clearing condition $b_c f_t = b_x f_t$, cost minimization of firms and the optimal leisure/labor supply decision allows us to derive an expression for percentage deviations of output from flexible price equilibrium:

$$\tilde{x}_t^f = \frac{1}{\gamma} \left[ (1 + \eta) \tilde{z}_t + \sigma \kappa \tilde{x}_{t-1}^f \right],$$

where $\gamma \equiv \eta + \frac{\sigma}{1-h_c}$ and $\kappa \equiv \frac{h_c}{1-h_c}$.

An approximation of (4) around a zero inflation steady state gives $\hat{q}_t = (\frac{\omega}{1-\omega}) \pi_t$, where $\hat{q}_t = \hat{p}_t^* - \hat{p}_t$ represents the relative price chosen by all adjusting firms, and inflation is defined as $\pi_t = \hat{p}_t - \hat{p}_{t-1}$. Using (5) and the rule of thumb to derive $\hat{q}_t = (1 - \tau) \hat{q}_t^{fl} + \tau \hat{q}_{t-1} + \tau \pi_{t-1} - \tau \pi_t$ then leads to

$$\hat{q}_t^{fl} = \left( \frac{1 - \omega}{1 - \tau (1 - \omega)} \right) \pi_t - \left( \frac{\tau}{1 - \tau (1 - \omega)} \right) \pi_{t-1}. \quad (11)$$

Using this in $\hat{p}_t^{fl} = (1 - \omega \beta) \sum_{i=0}^{\infty} \omega^i \beta^i E_t \left( \hat{\varphi}_{t+i} + \hat{p}_{t+i} \right)$ results in the hybrid New Keynesian Phillips Curve

$$\pi_t = \kappa \hat{\varphi}_t + \phi (\omega \beta E_t \pi_{t+1} + \tau \pi_{t-1}), \quad (12)$$

where $\phi \equiv (\omega + \tau [1 - \omega (1 - \beta)])^{-1}$ and $\kappa \equiv (1 - \tau) (1 - \omega) (1 - \beta \omega) \phi$.

To find a relation between marginal costs and an output gap measure we use $\hat{\varphi}_t = \hat{w}_t - \hat{p}_t - \hat{x}_t + \hat{n}_t$, which follows from the profit maximization of firms, and use this together with (10) and the assumption that output equals consumption to derive

$$\hat{\varphi}_t = \gamma y_t - \kappa \sigma y_{t-1},$$

where $y_t \equiv \hat{x}_t - \hat{x}_t^f$. Note that, if all firms use the rule of thumb for price adjustments, (5) for $\tau = 1$ implies $\hat{p}_t = \hat{p}_{t-1} + \pi_{t-1}$, and inflation is the same for all periods.
References


