Macroprudential Policy in the New Keynesian World*

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Abstract

We integrate banks and the coexistence of bank and bond financing into an otherwise standard New Keynesian Framework. Macroprudential policies in the form of varying aggregate capital requirements and interest rate policies of the central bank are used to stabilize shocks, to moderate bank credit cycles, and to induce more efficient allocation of resources across sectors. We study the interplay of these instruments. We examine how the economy reacts to policy shock, productivity shock and financial shock. Finally, we investigate the optimal policy rules for monetary and macroprudential policy-makers. The optimal policy rules indicate that the central bank should focus exclusively on price stability and the macroprudential policy-maker should react to both output variation and financial instability. For the latter, the state of the credit cycle is a better indicator than direct measures of financial risk.

Keywords: central banks, banking regulation, capital requirements, optimal monetary policy.

JEL: E520, E580, G280

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1 Introduction

1.1 Motivation

Macroprudential and monetary policies are two policy areas with three objectives: banking stability, price stability, and output stability. How these policies should be conducted and organized is a major issue, and since the global financial meltdown, policy-makers and academics have been working on solutions.

This task is complicated by two problems. First, pursuing one objective may impede fulfilling the other objectives. This is well-known for potential inflation-output trade-offs. It has also been a long-standing issue whether and how changes in the monetary stance can strengthen or undermine financial stability. Pursuing financial stability, in turn, may affect output. For instance, if aggregate capital requirements are relaxed in a downturn to counteract an output decline, the banks’ balance sheets may weaken, thereby exacerbating vulnerabilities in the financial system.

Second, policy-makers do not have enough instruments to attain the three key objectives: price, output, and financial stability. Typically, the central bank’s instrument is the short-term interest rate. While this instrument is sufficient to stabilize demand shocks, the stabilization of supply shocks such as cost-push shocks involves trade-offs between output and price stability. Macroprudential policies have one additional macro-tool—varying bank capital requirements. Hence there are only two independent macro-instruments for attaining three objectives.

Several proposals on how monetary and macroprudential policies could be organized and conducted are reviewed in Section 1.3. The purpose of this paper is to integrate the banking sector into an otherwise standard New Keynesian Framework in order to develop a microfounded model that enables us to study the interplay of monetary policy and macroprudential policy. Specifically, we focus on an economy in which one portion of the intermediate firms has to rely on bank financing to produce. Shocks to the production of these firms may hamper their ability to repay loans and ultimately affect the solvency of banks. Macroprudential policies, in the form of aggregate capital requirements for the banking system, can increase banking stability. Policy-makers
have started to use a variant of this tool. Interest-rate policies can aim at stabilizing demand or supply shocks in the entire economy.

We are interested in these classic questions:

- How should monetary policy and macroprudential policy be used to stabilize shocks in the economy and to safeguard financial stability, price stability, and output stability?
- Should macroprudential policy-makers focus on price and output stability as well?
- Which policy-maker should focus on financial instability, and which variable is a better indicator of financial instability?

To address these questions, we need an appropriate framework. In this paper we explore how the New Keynesian approach to monetary policy can be extended to encompass banks and the coexistence of bank and bond financing and thus pave the way for the analysis of the interplay between monetary and macroprudential policies. We will first integrate bank and bond financing in the New Keynesian Framework. Firms in one sector rely on bank financing, firms in the other on bond financing. We then provide a first round of applications.

1.2 Approach and results

To integrate banks and financing modes into the New Keynesian Model, we start from four key observations.

- One subset of firms is financed by banks. The remaining firms obtain funds directly from households through the capital market. The ratio of bank-financed firms is generally important, but varies widely across countries. It is particularly high in many continental European countries. Moreover, the relative size of financing modes depends on the state of the economy (see e.g. De Fiore and Uhlig (2011) and Laeven and Valencia (2013) on these differences).

\footnote{Today it comes in the form of countercyclical capital requirements and has been included in national law in many countries. It is already used in some countries (see https://www.bis.org/bcbs/ccyb/, retrieved on 15th March 2017).}
• Firms that turn to banks and obtain bank loans are more risk-prone than firms that finance themselves through bond markets. These risks translate into default risk of banks and risk premia on loan interest rates.2

• Defaults of banks impose additional costs on households, e.g. in the form of bailout costs.

• Households face costs when they acquire and hold risky assets like bank equity. For example, households need time (or pay fees) to assess the return prospects on risky assets. As a consequence, safe assets are preferred to risky assets, unless the returns on the latter compensate these costs beyond standard risk premium.

We embed these features into a standard New Keynesian Model. There is a banking sector and there are two types of intermediate firms: risky firms that can only obtain funds from banks and safe firms that can issue bonds to finance themselves. In addition, the government implicitly or explicitly backs deposits and thereby makes these assets safe.

There are two major policy instruments. Macroprudential policy, in the form of time-varying aggregate capital requirements, balances the costs of bank equity, the costs of bank defaults, and the potential misallocation of resources across sectors. Typically, higher aggregate capital requirements increase the costs of bank equity, reduce bank defaults, and reduce investments channeled through the banking sector. Interest-rate policies serve the typical purpose of influencing aggregate output and inflation.

We proceed as follows: In the first part, we derive the relationships that must hold in equilibrium. In this equilibrium—among other things—banks choose capital structures in each period, such that capital requirements are binding and loan rates increase with the stringency of capital requirements, while loan sizes decline. We next log-linearize the model around the steady state.

Moreover, we illustrate the properties of the steady states for different levels of capital requirements. Typically, for low levels of aggregate capital requirements, the allocation of resources across sectors is inefficient: Too many resources are channeled through the

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banking sector, and too few through the capital market. Appropriate macroprudential policies that prescribe particular levels of aggregate capital requirements (and as a consequence, individual capital requirements) are welfare-improving, as they can correct the misallocation of resources across sectors and improve resource allocation over time. If aggregate capital requirements are too high, however, this will excessively shift resources to safe firms and entail large equity management costs.

In the second part we provide a first round of applications of the log-linearized framework. We begin with some examples of how the economy reacts to various shocks, where we consider exogenous disturbances to aggregate productivity, monetary policy, macroprudential policy as well as shocks to financial intermediation. We next derive a welfare formula for arbitrary policies by deriving a second-order approximation to welfare around the steady state that is implied by unconditionally optimal policies. This provides a starting point for a wide variety of further possible policy-making investigations. In particular, we determine optimal rules for monetary policy as well as macroprudential policy rules from a fairly general class of rules. Our calibration exercise reveals that the central bank should focus exclusively on price stability. The macroprudential policy-maker, however, should react to both output variation and financial instability. The latter is best proxied by changes in the credit cycle. Direct measures of default risk, such as the loan rate premium and the fraction of defaulted banks, are less suitable.

1.3 Literature

We integrate banks and financial stability concerns into the New Keynesian Framework, leaving all other essential parts of this framework intact. However, integrating the four features referred to above into the New Keynesian Framework is a major undertaking. Several papers already combine monetary and macroprudential policy-making. Angeloni and Faia (2013) show how bank capital requirements can mitigate the consequences of bank runs when monetary policy follows interest-rate rules. Christensen et al. (2011) link risky bank projects to the aggregate-credit-to-GDP ratio.3 Another Loisel (2014) assesses the conclusions that can be drawn from the early literature on what monetary and macroprudential policy rules can achieve.
branch of this literature has focused on optimal policies. Among others, De Paoli and Paustian (2013) and Collard et al. (2017) characterize Ramsey-optimal policies. The latter, in particular, shows that prudential policies prevent excessive risk-taking by banks while monetary policy aims at smoothing business cycles. This is an important benchmark for how such policies could or should be conducted jointly.

Our contribution to this literature is as follows: First, we embed the four features referred to above into an otherwise standard New Keynesian Framework. This allows for a comparison with standard results in monetary policy-making. Second, we investigate how such an economy responds to shocks affecting bank-financed or capital-market-financed firms and aggregate shocks. Third, we derive the unconditional optimal welfare formula and focus on institutional questions of how policy-making can be operationalized by Taylor-type rules for monetary policy and macroprudential policy.

Several frameworks for monetary and macroprudential policies have been proposed.\footnote{A more extended framework would also include microeconomic regulation and supervision of banks (see Gersbach and Hahn (2011) for such a framework).} Detailed outlines, rationales, and assessment can be found in Gersbach and Hahn (2011), Schoenmaker and Wierts (2016), Borio (2014), Claessens et al. (2013), and Jonsson and Moran (2014). Like these frameworks, our model shares the view that constraints on leverage and credit expansion are a key angle of macroprudential policies. Authors, however, differ as to how effective countercyclical policies can be and whether macroprudential policies should aim at smoothing credit cycles. We use macroprudential policies in the form of varying aggregate capital requirements for the banking system and investigate how such policies need to be conducted and organized.

1.4 Structure of the paper

The remainder of this paper proceeds as follows: In the next section we present the model. After establishing the equilibrium conditions in Section 3, we characterize the steady state of the economy and show the impulse responses to various types of shock in Section 4. In Section 5, we explore optimal policy rules. Section 6 concludes.
2 Model

2.1 Overview

There are six sectors in the model: households, banks, safe and risky intermediate firms, final firms, and the public sector made up of a fiscal agency, a central bank, and a macroprudential policy-maker. We start with the timeline of events in each period $t = 0, 1, 2, \ldots$. Then we describe the agents’ optimization problems, the firms’ technologies, and the market structure in more detail.

The sequence of events is shown in Figure 1. Each period $t$ is divided into two stages. At the beginning of the first stage, households own bonds $B_t$, which they acquired in the previous period. In addition, aggregate shocks materialize. In particular, we will consider aggregate shocks to the productivity of intermediate firms, financial shocks, shocks to macroprudential policy and monetary policy. After observing the aggregate shocks, the monetary policy-maker (mon) chooses $I_t$, the nominal interest rate on bonds that mature in $t + 1$, where one unit of the bond represents a claim on one nominal unit at maturity. Households rent out part of their labor to safe firms in return for real claims (safe firms’ bonds) against these firms in the second stage of period $t$. Risky intermediate firms cannot issue claims to households to hire labor because they have to be monitored closely. They rely on bank loans instead, where we assume that each bank serves exactly one risky firm. Hence risky firms take loans from banks and receive bank deposits at the same time. Risky firms use these deposits to hire labor from households. Deposits are riskless because they are insured by the government. The macroprudential policy-maker (mac) sets a capital requirement $\Gamma_t$ which banks have to fulfill in order to be allowed to operate. We also assume that households incur costs when acquiring and holding equity. This makes equity financing more costly for banks than debt financing.

At the beginning of the second stage, idiosyncratic shocks occur, affecting risky firms’ productivities. Subsequently, safe and risky intermediate firms choose their prices, taking the amount of labor hired in the first stage as given. While safe firms face Rotemberg price adjustment costs, risky firms live only for one period and can choose the prices of their outputs freely. Safe firms can always repay their bonds, whereas some
risky intermediate firms with adverse shock realizations cannot repay their loans in full. As a consequence, the corresponding bank may fail if its equity buffer is insufficient. These banks are bailed out by the fiscal agency, which uses a lump-sum tax on all households to guarantee that deposits are always repaid. All banks are dissolved, and the remaining funds are distributed to equity holders. Profits of intermediate firms also accrue to households, and bonds $B_t$ mature. Perfectly competitive final-good firms purchase the intermediate goods and use them to produce final goods. Households acquire new bonds $B_{t+1}$ at a price $1/I_t$ as well as final goods. In the following, we describe the different agents in more detail.

### 2.2 Households

Each household has the instantaneous utility function:

$$u(c_t, n_t) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma} - \psi \frac{n_t^{1+\varphi}}{1 + \varphi},$$  \hspace{1cm} (1)

where the relative risk aversion of consumption $\sigma > 0$, the inverse of the Frisch labor supply elasticity $\varphi > 0$, the relative utility weight of labor $\psi > 0$, and $c_t$ and $n_t$ denote consumption and total labor, respectively. It is sufficient to focus on the behavior of a representative household.

In the first stage of each period, the household provides labor to both safe and risky intermediate firms. Safe intermediate firms provide safe intra-period bonds $\tilde{s}_t$ with gross return $R_t^s$ to the household in exchange for the labor $n_t^s$,

$$\tilde{s}_t = \tilde{w}_t n_t^s,$$  \hspace{1cm} (2)
where $\tilde{w}_t$ represents the real market wage.

The household saves in bank deposits $\tilde{d}_t$ with gross return $R^d_t$ and bank equity $\tilde{e}_t$. Thus, the amount of loans the bank can issue is

$$\tilde{l}_t = \tilde{d}_t + \tilde{e}_t.$$  \hspace{1cm} (3)

Deposits are backed by the government and therefore riskless. Hence the following no-arbitrage condition holds

$$R^d_t = R^s_t.$$  \hspace{1cm} (4)

As a consequence, we will not distinguish between $R^d_t$ and $R^s_t$ for the remainder of this paper.

The representative household acquires equal amounts of equity from each bank.\footnote{We are effectively looking for symmetric bank equity allocations. Since banks are identical at this stage, this allocation is rationalized in the equilibrium.} Because the aggregate shocks are realized at the beginning of each period and already known when asset allocations are made, the equity portfolio held by the household is riskless. The gross return on the equity portfolio is denoted by $R^e_t$.\footnote{The representative household holds all assets in the economy and is fully diversified. We could allow for heterogeneity of bank equity holding across households to further rationalize the costs of acquiring and holding risky bank equity.}

Using loans $\tilde{l}_t$ granted by banks, risky intermediate firms hire labor $n^r_t$ from the household

$$\tilde{l}_t = \tilde{w}_t n^r_t.$$  \hspace{1cm} (5)

It will be useful to introduce $s_t := R^s_t \tilde{s}_t$, $d_t := R^s_t \tilde{d}_t$, $l_t := R^s_t \tilde{l}_t$, and $e_t := R^s_t \tilde{e}_t$. Intuitively, $s_t$ and $d_t$ are the funds the household receives in the second stage from investing in safe bonds $\tilde{s}_t$ and deposits $\tilde{d}_t$, respectively. $l_t$ and $e_t$ correspond to the hypothetical funds that one would receive in stage 2 if one invested $\tilde{l}_t$ and $\tilde{e}_t$ at rate $R^s_t$ in stage 1.

We model the costs of equity financing by assuming that the household needs to spend resources to monitor and manage equities. For simplicity, we assume that the resources necessary for equity management are proportional to the dividend payments $R^e_t \tilde{e}_t$:

$$m_t = \chi_t R^e_t \tilde{e}_t.$$  \hspace{1cm} (6)
where \( \chi_t \) is an exogenous positive random variable.

We introduce the premium on equity financing as \( \Delta^e_t := R^e_t/R^s_t \). In the second stage, the household receives the gross returns on deposits, equities, and safe bonds, i.e. \( d_t, \Delta^e_t e_t, \) and \( s_t \), respectively. In addition, safe and risky intermediate firms’ profits \( z^s_t \) and \( z^r_t \) also go into the household’s pocket. On the expense side, the household consumes goods \( c_t \) and pays lump-sum taxes \( \tau^l_t \).

Using \( w_t = R^s_t \bar{w}_t \) and \([2]-[6]\), we can write the total funds the household receives from safe bonds, deposits, and equity net of equity management costs as

\[
s_t + d_t + \Delta^e_t e_t - \chi_t \Delta^e_t e_t = w_t n_t + (\Delta^e_t (1 - \chi_t) - 1) e_t, \tag{7}
\]

where \( n_t = n^s_t + n^r_t \) denotes total labor.

We are now in a position to state the household’s budget constraint in the second stage of period \( t \) as

\[
c_t + B_{t+1} \frac{I tp_{t+1}}{I tp_t} \leq \frac{B_t}{p_t} + z^s_t + z^r_t - \tau^l_t + w_t n_t + (\Delta^e_t (1 - \chi_t) - 1) e_t. \tag{8}
\]

The representative household maximizes the overall utility

\[
\max_{\{c_t,B_{t+1},n_t,e_t\}} \left\{ \sum_{t=0}^{\infty} \beta^t \left( \frac{c_1 - \sigma - 1}{1 - \sigma} - \psi \frac{n^1 + \varphi}{n_t + \varphi} \right) \right\} \text{ s.t. } (8), \tag{9}
\]

where \( \beta \) is the discount factor with \( 0 < \beta < 1 \). Let \( \lambda_t \) be the current-value Lagrange multiplier associated with \( [8] \). Then we obtain the following first-order conditions of the household problem:

\[
c_t : \quad \lambda_t = c_t^{-\sigma}, \tag{10}
\]
\[
B_{t+1} : \quad \frac{\lambda_t}{I tp_{t+1}} = \mathbb{E}_t \left[ \beta \frac{\lambda_{t+1}}{p_{t+1}} \right], \tag{11}
\]
\[
n_t : \quad \psi n^r_t = \lambda_t w_t, \tag{12}
\]
\[
e_t : \quad \Delta^e_t = \frac{1}{1 - \chi_t}, \tag{13}
\]

where the last equation is a no-arbitrage condition, which involves that investing an additional unit of funds into equity delivers the same additional payoff—net of equity management costs—as investing the same amount in a deposit.
2.3 Final-good firms

There are infinitely many, perfectly competitive firms that purchase intermediate goods \( y_t(i) \) at prices \( p_t(i) \) and assemble them to a final good \( y_t \), which can be used for consumption:

\[
y_t = \left( \int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta-1}},
\]

where \( \theta > 1 \) represents the elasticity of substitution between differentiated intermediate goods.

Hence each firm’s profit maximization problem can be formulated as

\[
\max_{\{y_t(i)\}_{i=0}^1} \left\{ p_t y_t - \int_0^1 p_t(i)y_t(i) \, di \right\} \quad \text{s.t. (14).}
\]

This problem leads to the following demand for intermediate good \( i \):

\[
y_t(i) = \left( \frac{p_t(i)}{p_t} \right)^{-\theta} y_t \quad \text{(15)}
\]

and the price level is

\[
p_t := \left( \int_0^1 p_t(i)^{1-\theta} \, di \right)^{\frac{1}{1-\theta}}.
\]

2.4 Intermediate firms

In the first stage, monopolistically competitive intermediate firms attract loans from banks as well as households and hire labor. In the second stage, output materializes, prices are chosen, loans are repaid, and profits are transferred to households. The total number of intermediate firms is normalized to 1. A fixed proportion \( \nu \) constitutes the sector of safe firms. These firms are infinitely-lived and face quadratic price-adjustment costs. The remaining \( 1 - \nu \) intermediate firms, the sector of risky firms, live for one period and can choose their prices freely.
2.4.1 Safe firms

The safe firms’ production function is

\[ y_s(i) = a_t n_s(i), \]  

where \( a_t \) is aggregate productivity, which is driven by an exogenous stochastic process.

In the first stage of each period \( t \), safe firms take loans from (or issue bonds to) households at a gross real rate \( R_s \) and use them to hire labor

\[ \tilde{s}_t(i) = \tilde{w}_t n_s(i). \]  

Taking into account that the value of loans in stage 2 is \( s_t(i) = R_s \tilde{s}_t(i) \), safe firm \( i \)'s real profit in the second stage of period \( t \) can be written as

\[ z_s(i) = \frac{p_s(i)}{p_t} y_s(i) - s_t(i) - \gamma p \left( \frac{p_s(i)}{p_{t-1}(i)} - 1 \right)^2 y_t, \]  

where \( p_s(i) \) is the price for firm \( i \)'s output, and \( \gamma p > 0 \) is the coefficient for price adjustment cost \( \gamma p \left( \frac{p_s(i)}{p_{t-1}(i)} - 1 \right)^2 y_t \).

Taking the wage as given, firms maximize the expected sum of discounted real profits

\[ \max_{\{n_s(i), p_s(i)\}_{t=0}^{\infty}} \left( \mathbb{E}_0 \sum_{t=0}^{\infty} Q_t z_s(i) \right), \]  

subject to

\[ a_t n_s(i) \geq \left( \frac{p_s(i)}{p_t} \right)^{-\theta} y_t, \]  

where future profits are discounted by

\[ Q_t = \frac{\beta^t \lambda_t}{\lambda_0} = \beta_t \frac{c^0}{c_t}. \]

The optimal behavior of safe firms is quite standard and is examined and described in Appendix A.
2.4.2 Risky firms

The risky firms’ production function is

\[ y^*_t(i) = (\phi(i))^\alpha Aa_t n^*_t(i), \]  

(22)

where \( \phi(i) \in [0, 1] \) with uniform distribution represents an idiosyncratic shock to firm \( i \)’s productivity. Parameter \( \alpha (\alpha > 0) \) affects the riskiness of production, where lower values involve less risk. Parameter \( A \) affects the relative productivity of risky firms compared to the safe firms.

Note that while the aggregate shock, \( a_t \), becomes commonly known at the beginning of each period \( t \), the idiosyncratic shock \( \phi(i) \) is realized at the beginning of the second stage of the corresponding period. Hence, in the first stage of each period it is unknown both to the risky firms and to the banks funding these firms.

Risky firms get loans from banks

\[ \tilde{L}_t(i) = \tilde{w}_t n^*_t(i) \]  

(23)

to finance the wage bill. We assume that like banks, risky intermediate firms live for one period. Consequently, they do not face price-adjustment cost. Thus, the risky firms’ real profit in period \( t \) can be written as

\[ z^*_t(i) = \frac{p^*_t(i)}{p_t} y^*_t(i) - R^*_t(i) \tilde{L}_t(i) \]  

(24)

subject to

\[ \phi(i)^\alpha Aa_t n^*_t(i) \geq \left( \frac{p^*_t(i)}{p_t} \right)^{-\theta} y_t. \]  

(25)

\( R^*_t(i) \) represents the gross return on loans paid by risky firms. If firms default, it is smaller than the market loan rate \( R^l_t \), i.e. \( R^*_t(i) < R^l_t \). Otherwise, \( R^*_t(i) = R^l_t \).

Aggregate profits, \( z^*_t = \int\int z^*_t(i) d\phi(i) di \), are paid out to households as dividends.

We analyze the optimal behavior of risky firms in Appendix B and obtain

**Proposition 1**

(i) A risky firm’s demand for bank loans is

\[ l_t(i) = \frac{(Aa_t)^{\theta-1}}{w_t^{\theta-1}(\Delta L^*_t)^\theta} L^* y_t, \]  

(26)
where \( \Delta_l^i := \frac{R_l^i}{R_s^i} \) represents the loan rate premium\(^7\) and \( L^* \) is the root of

\[
g(L) := (\theta - 1)L^{-\frac{1}{\theta}} + (1 + \alpha(\theta - 1))L^{\frac{1}{\theta(\theta - 1)}} - (\theta + \alpha(\theta - 1))
\]

that satisfies \( 0 < L^* < 1 \).

(ii) Defaulted firms are those with realized \( \phi(i) \) below

\[
\phi^c = (L^*)^{\frac{1}{\theta(\theta - 1)}}.
\]

We observe that the risky firms’ demand for loans is a decreasing function of the loan rate \( \Delta_l^i \) and the real wage \( w_t \). It increases with aggregate productivity \( a_t \) and aggregate output \( y_t \). Moreover, we note that the fraction of defaulted firms, \( \phi^c \), is constant over the business cycle. However, we will see that the same is not true for the fraction of banks that default.

### 2.5 Banks

The banking system is characterized as follows: There is a continuum of banks.\(^8\) An individual bank lives for one period and is specialized in granting loans to a particular risky firm. Accordingly, banks do not hold a well-diversified loan portfolio.\(^9\) The banking system is competitive and operates as follows:

- Banks attract equity from households. A bank is founded if it receives a positive amount of equity.
- Banks attract deposits from households and decide on their capital structure.
- The market for loans opens. The pool of ex-ante identical risky firms demands loans \( l_t(i) \) for a firm \( i \). If a bank satisfies capital requirements, it is allowed to operate and decides whether to offer its intermediation services, offering loans coupled with monitoring. Market clearing yields the loan rate \( R_l^i_t \).

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\(^7\)The premium stems from two sources: default risk and higher costs of equity financing.

\(^8\)Since, ultimately, one risky firm will obtain a loan from one bank, we assume for measurement consistency that the measure of banks is at least 1 − \( \nu \).

\(^9\)We focus on banks without well-diversified loan portfolios and make a simple assumption about the specialization of banks. If banks held fully diversified portfolios, either all banks would collapse or they would all be solvent.
The productivity of risky firms is affected by idiosyncratic shocks. If a risky firm cannot pay the loan back, banks will secure the liquidation value.

To examine the equilibrium in the loan market, we assume that perfect monitoring prevails, i.e. banks can enforce the terms of the contract to ensure that they either get the repayment of the loan or the liquidation value if the firm cannot pay back. Monitoring is costless for banks.

Once banks have received equity, the objective of a bank is to maximize returns on equity, taking into account limited liability, i.e. the fact that equity holders do not bear losses. In doing this, they decide on the capital structure, i.e. how many deposits they want to attract, whether they want to attract more equity, and whether they want to offer loans to risky firms. We assume that an individual bank can attract equity and deposits as long as it offers expected returns with which equity holders and depositors, respectively, are at least as well off as with other investment opportunities. Of course, given such individual choices, aggregate supply and demand for equity and deposits have to match in equilibrium.

Note that in our model, the maximization of the expected return on equity is equivalent to the maximization of the utility of shareholders. The reasons are as follows: First, as banks are perfectly competitive, an individual bank’s choice will not alter prices in the economy. Second, the bank’s decision to lend does not open up new insurance opportunities for households. As a consequence, all shareholders will agree that the bank should maximize its expected return on equity in order to contribute the maximal expected amount to the budget of shareholders.

We now consider a representative bank’s problem in more detail. Since loan and deposit markets are perfectly competitive, the bank demands an amount of deposits \( d_t(i) \) at the prevailing deposit rate without worrying about whether this is consistent with market clearing for deposits and loans. Once the bank has chosen its capital structure \( \frac{e_t(i)}{d_t(i)} \), it decides whether to offer \( e_t(i) + d_t(i) \) as loans to risky firms or to invest in safe firms’ bonds. As a tie-breaking rule, we assume that the bank will grant loans to risky firms if they generate at least the same expected return on equity as for other investment opportunities.
With these remarks, the problem of a representative bank can be formulated as in Appendix C. Three results that are crucial for our model are summarized in the next three propositions. First, we characterize the equilibrium capital structures.

**Proposition 2**

* Banks will always choose their capital structure to be equal to the aggregate capital requirement\(^{10}\)

\[
e_t = \Gamma_t d_t.
\]  

(29)

Similarly to the fraction of defaulted firms, we obtain the following proposition for the fraction of defaulted banks:

**Proposition 3**

* The fraction of defaulted banks is

\[
\phi_t^\Gamma = \frac{\phi^c}{(\Delta^l_t (1 + \Gamma_t))^{\theta/(\theta - 1)}} \in [0, \phi^c).
\]  

(30)

This proposition implies that the fraction of defaulted banks decreases with \(\Gamma_t\) and reaches 0 when banks are fully financed by equity, i.e. for \(\Gamma_t \to \infty\). The number of defaulted banks is also a decreasing function of \(\Delta^l_t\), i.e. the difference between the interest rates on loans and on deposits.

Accounting identities lead to the following relationship between loan rate \(\Delta^l_t\) and return on equity \(\Delta^e_t\):

**Proposition 4**

* The market loan rate satisfies

\[
\Delta^l_t = \frac{h^{-1}(\Gamma_t \Delta^e_t)}{1 + \Gamma_t},
\]  

(31)

where

\[
h(x) := \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} \left( \frac{1}{x^{\alpha(\theta - 1)}} - x \right) \phi^c + x - 1.
\]  

(32)

\(^{10}\)See Gersbach et al. (2015a) on the uniqueness of bank capital structure in more general setups.
As shown in Appendix C, function \( h(\cdot) \) is a monotonically increasing function that satisfies \( h(1) = 0 \) and goes to infinity for large values of its argument. Hence, (31) establishes that the return on equity that the banks can generate, \( \Gamma_{t}^{-1} h \left( (1 + \Gamma_{t}) \Delta_{t}^{l} \right) \), is an increasing function of the loan rate that banks charge risky intermediate firms. Moreover, we can conclude that for \( \Gamma_{t} \rightarrow 0 \), \( \Delta_{t}^{l} \rightarrow 1 \). Therefore, if banks are fully financed by deposits, the rate on bank loans equals the rate on deposits. Positive values of \( \Gamma_{t} \) result in values of \( \Delta_{t}^{l} \) that are strictly larger than one.

### 2.6 The government

The sole function of the government is to use lump-sum taxes \( \tau_{t}^{l} \) from households to bail out banks that have failed in the second stage. We assume a fraction \( \mu \) of the bailout fees is dissipated when the government bails out the defaulted banks. Denoting the aggregate costs of these bailouts as \( bo_{t} \), the government’s budget constraint is

\[
\tau_{t}^{l} + \frac{B_{t+1}}{I_{t}p_{t}} = (1 + \mu)bo_{t} + \frac{B_{t}}{p_{t}}.
\]

(33)

An expression for the bailout fees \( bo_{t} \) is presented in Appendix D.

We assume that monetary policy can be described by the following augmented Taylor rule

\[
\frac{I_{t}}{I^{*}} = \left( \frac{\Pi_{t}}{\Pi^{*}} \right) \left( \frac{y_{t}}{y^{*}} \right)^{1.5} \left( \frac{l_{t}}{l^{*}} \right)^{0.5} \left( \frac{\phi_{t}}{\phi^{*}} \right)^{0.5} e^{\xi_{t}},
\]

(34)

where \((v_{\text{mon}}^{\pi}, v_{\text{mon}}^{y}, v_{\text{mon}}^{l}, v_{\text{mon}}^{\phi})\) represent the Taylor-rule coefficients and \(\xi_{t}\) stands for a monetary policy shock. A priori, we allow the Taylor rule to depend on inflation, the output gap, the aggregate volume of loans, and the share of bank defaults, where the gross rate of inflation is \( \Pi_{t} := \frac{p_{t}}{p_{t-1}} \). While the first two variables are standard, the other two variables serve as indicators of financial stability, which the central bank may also take into account.\(^{11}\) We note that, for \( v_{\text{mon}}^{\pi} = 1.5, v_{\text{mon}}^{y} = 0.5, v_{\text{mon}}^{l} = 0, \) and \( v_{\text{mon}}^{\phi} = 0 \), equation (34) simplifies to the standard Taylor rule:

\[
\frac{I_{t}}{I^{*}} = \left( \frac{\Pi_{t}}{\Pi^{*}} \right)^{1.5} \left( \frac{y_{t}}{y^{*}} \right)^{0.5} e^{\xi_{t}}.
\]

(35)

\(^{11}\)In a model with two sectors, one with flexible prices and one with sticky prices, Aoki (2001) shows that the central bank should target inflation in the sticky-price sector rather than aggregate inflation. In the present paper, we assume that the central bank focuses on a broad price index rather than sector-specific prices, which is in line with common practice among central banks. Exploring Taylor rules based on sector-specific inflation rates would be an interesting extension to our model.
We now turn to macroprudential policy-making. We assume the macroprudential policy-maker’s instrument is the capital requirement $\Gamma_t$. Analogously to the central bank’s augmented Taylor rule (34), we write down a fairly general policy rule

$$\frac{\Gamma_t}{\Gamma^*} = \left(\frac{\Pi_t}{\Pi^*}\right)^{\nu_{mac}} \left(\frac{y_t}{y^*}\right)^{\nu_{y}} \left(\frac{l_t}{l^*}\right)^{\nu_{l}} \left(\frac{\phi_t^G}{\phi^G}\right)^{\nu_{\phi}} e^\eta,$$

(36)

where $(\nu_{mac}, \nu_{y}, \nu_{l}, \nu_{\phi})$ describe how vigorously the macroprudential policy-maker responds to the respective economic variables. Variable $\eta_t$ represents a shock to macroprudential policy.

### 2.7 Market clearing

Finally, we state the market clearing conditions. Goods-market clearing implies that output equals the sum of consumption, the adjustment costs for prices, the equity management costs, and the dissipation when defaulted banks are bailed out,

$$y_t = c_t + \text{adj}_t^p + m_t + \mu b_0 t,$$

(37)

where $\text{adj}_t^p = \frac{p^p}{2} \nu \left(\frac{p_t^2(i)}{p_{t-1}^2(i)} - 1\right)^2 y_t$.

Equilibrium on the labor market implies that the total supply of labor has to equal the demand from both safe and risky intermediate firms:

$$n_t = n_t^s + n_t^r,$$

(38)

where $n_t^s = \int_0^\nu n_t^s(i)di$ represents total labor demand by safe firms, $n_t^r = \int_\nu^1 n_t^r(i)di$ total labor demand by risky firms.

The market for intra-period debt issued by safe firms is balanced if

$$\tilde{s}_t = \int_0^\nu \tilde{s}_t(i)di.$$

(39)

Finally, the following accounting identity must hold for banks:

$$\tilde{l}_t = \tilde{d}_t + \tilde{e}_t,$$

(40)

where $\tilde{l}_t = \int_\nu^1 \tilde{l}_t(i)di$ represents total loans, $\tilde{d}_t = \int_\nu^1 \tilde{d}_t(i)di$ total deposits, and $\tilde{e}_t = \int_\nu^1 \tilde{e}_t(i)di$ total equity.
3 Equilibrium

In this section we summarize the equations describing the equilibrium. For this purpose, we observe that all safe firms are identical and thus set the same price $p_s^t := p_s^t(i)$. This enables us to introduce $q_t := p_s^t/p_t$, the ratio between the price of the intermediate goods produced by safe firms with respect to the aggregate price, and $\Pi_s^t := p_t^{i-1}/p_{t-1}^{i-1}$, the gross inflation for the intermediate goods produced by safe firms. With the help of this notation we now state the equations describing the evolution of endogenous variables for the paths of exogenous shocks $\{a_t, x_t, \xi_t, \eta_t\}_{t=0}^{\infty}$.

As shown in Appendix B, a risky firm’s demand for bank loans is

$$l_t(i) = \left( \frac{Aa_t}{w_t} \right)^{\theta-1} \frac{L^*}{(\Delta_t^{\theta})^n} y_t,$$

where the loan is used to finance the wage bill, which implies

$$l_t(i) = w_t n_r^t(i).$$

The optimal price-setting of safe firms results in the following standard condition for price-setting in the presence of quadratic adjustment costs (see Appendix A):

$$0 = \frac{y_t}{(q_t)^{\theta-1}} \left[ (1 - \theta) + \theta \frac{w_t}{q_t a_t} \right] - \gamma^p y_t \Pi_s^t (\Pi_s^t - 1)$$

$$+ \mathbb{E}_t \left[ \beta \frac{c_t^2}{c_{t+1}^2} \gamma^p y_{t+1} \Pi_{s+1}^t (\Pi_{s+1}^t - 1) \right].$$

Note that, in the absence of price adjustment costs, i.e. for $\gamma^p = 0$, each safe intermediate firm would charge a constant markup over real marginal costs, i.e.

$$q_t = \frac{p_t^s}{p_t} = \frac{\theta}{(\theta-1)} mc_t^s,$$

where real marginal costs are $mc_t^s = \frac{w_t}{a_t}$.

Inflation, $\Pi_t := \frac{p_t^s}{p_{t-1}^s}$ can be formulated as

$$\Pi_t = \frac{q_t^{-1}}{q_t} \Pi_t^s.$$

and, according to (30), the fraction of defaulted banks is

$$\phi_t^\Gamma = \frac{\phi^c}{(\Delta_t^{\theta}(1 + \Gamma_t))^{\frac{\theta}{\theta-1}}}.$$
The demand function for the intermediate goods produced by safe firms results in the following equation (see (15) and (17)):

\[ a_t n_t^s(i) = q_t^{-\theta} y_t. \]  

(46)

Aggregate output can be computed from the final-good firms' production function and the production functions for intermediate firms (see (14), (17), and (22)):

\[ y_t = a_t \left( \nu(n_t^s(i))^\theta + (1 - \nu) \frac{\theta}{\theta + \alpha(\theta - 1)} (A n_t^r(i))^{\theta-1} \right)^{\frac{\theta}{\theta-1}}. \]  

(47)

Combining (13) and (31) yields a relationship between the loan rate \( \Delta_l^t \) and the equity management cost \( \chi_t \)

\[ \frac{1}{1 - \chi_t} = (\Gamma_t)^{-1} h \left( (1 + \Gamma_t) \Delta_l^t \right), \]  

(48)

where \( h(\cdot) \) is defined in (32).

Optimal bond holdings entail a standard consumption Euler Equation (see (10) and (11)):

\[ \frac{1}{I_t} c_{t-\sigma}^{-\sigma} = \mathbb{E}_t \left[ \beta c_{t+1}^{-\sigma} \frac{1}{\Pi_{t+1}} \right]. \]  

(49)

Moreover, the marginal disutility from work has to equal the wage rate times the marginal utility of consumption (see (10) and (12)):

\[ \psi(n_t^s(i) + (1 - \nu)n_t^r(i))^\phi = w_t. \]  

(50)

Equilibrium on the goods market involves

\[ y_t = c_t + \frac{1}{2} \gamma^p \nu (\Pi_t^s - 1)^2 y_t + \frac{\chi_t}{1 - \chi_t (1 + \Gamma_t)} (1 - \nu) l_t(i) \]

\[ + \frac{\mu}{\theta + \alpha(\theta - 1)} (\Delta_l^t)^{\frac{\phi}{\phi-1}} (1 + \Gamma_t)^{\theta+\alpha(\theta-1)} \]  

(51)

which is to be interpreted as output equals consumption, the costs for price adjustment and equity management, and the dissipations when the government bails out the defaulted banks. The expression for the bailout costs is derived in Appendix D.

To sum up, the equilibrium dynamics are described by the private-sector equilibrium conditions (41)–(51) and the policy rules for monetary and macroprudential policy, i.e. (34) and (36). It is then straightforward to determine the other variables not contained in this system of equations. We summarize the findings in the following proposition:
Proposition 5

For given shocks \( \{a_t, \chi_t, \xi_t, \eta_t\}_{t=0}^{\infty} \), the equilibrium \( \{n_s(i), n_r(i), w_t, l_t(i), \Delta^i_t, \phi^\Gamma_t, \Pi_s, \Pi_t, q_t, y_t, c_t, \Gamma_t, I_t\}_{t=0}^{\infty} \) is described by the system of Equations (34), (36), and (41)-(51).

3.1 Log-linearized equations

In this section we state log-linearized versions of the conditions (41)-(51). Observe that we log-linearize around a steady state that does not necessarily involve zero inflation. We use the symbol \( \hat{\cdot} \) to denote log deviations from steady-state values and \( \ast \) for steady-state values. For the details, we refer to Appendix E.

Equation (41) can be approximated as

\[
\hat{l}_t(i) = (\theta - 1)(\hat{a}_t - \hat{w}_t) - \theta \hat{\Delta}^i_t + \hat{y}_t,
\]

where

\[
\hat{l}_t(i) = \hat{w}_t + \hat{n}_r^s(i).
\]

The Phillips curve, Equation (43), has the following log-linearized approximation:

\[
0 = (1 - \beta) \gamma^p \Pi^s (\Pi^s - 1) (\hat{y}_t - \theta \hat{q}_t + \hat{w}_t - \hat{a}_t) \\
+ (q^s)^{1-\theta} (\theta - 1) (\hat{w}_t - \hat{a}_t - \hat{q}_t) \\
+ \gamma^p (\Pi^s)^2 \hat{\epsilon}_t \left[ \beta (\sigma \hat{c}_t - \sigma \hat{c}_{t+1} + \hat{y}_{t+1} + 2\pi^s_{t+1}) - 2\pi^s_t - \hat{y}_t \right] \\
+ \gamma^p \Pi^s \hat{\epsilon}_t \left[ \pi^s_t + \hat{y}_t - \beta (\sigma \hat{c}_t - \sigma \hat{c}_{t+1} + \hat{y}_{t+1} + \pi^s_{t+1}) \right],
\]

where we write \( \pi^s_t = \hat{\Pi}^s_t \). Using \( \pi_t := \hat{\Pi}_t \) for the inflation rate, we obtain

\[
\pi_t = \hat{q}_{t-1} - \hat{q}_t + \pi^s_t.
\]

Moreover, \( \hat{\phi}^\Gamma_t \), the relative deviation of the fraction of defaulted banks from the steady-state value satisfies

\[
\hat{\phi}^\Gamma_t = \frac{\theta}{\alpha(\theta - 1)} \hat{\Delta}^i_t - \frac{\theta}{\alpha(\theta - 1)} \frac{\Gamma^s_0}{1 + \Gamma^s_0} \hat{\Gamma}_t.
\]

Hence higher interest rates on loans and higher capital requirements are associated with fewer bank failures. While the relationship between capital requirements and
bank failures is clear, the relationship between bank failures and loan rates is more subtle. It relies on the observations that higher interest rates on loans reduce the demand for loans and that the return on loans, given that a firm defaults, decreases with the size of the loan.

Equation (46) has the following log-linear approximation:

\[ \hat{a}_t + \hat{n}_t^s(i) = -\theta \hat{q}_t + \hat{y}_t. \]  

(57)

We obtain the following approximation to (47):

\[ \hat{y}_t = \hat{a}_t + \kappa_1 \hat{n}_t^s(i) + (1 - \kappa_1) \hat{n}_t^r(i), \]  

(58)

where \( \kappa_1 \in (0, 1) \) is given in Appendix E. According to (58), aggregate output increases with the quantities of labor employed in both sectors.

Equation (48) can be approximated as

\[ \hat{\Delta}_t = \tilde{\kappa}_2 \hat{\Gamma}_t + \chi^\ast \kappa_2 (1 - \chi^\ast) \hat{\chi}_t, \]  

(59)

where \( \kappa_2 > 0 \) and \( \tilde{\kappa}_2 > 0 \) are given in Appendix E. This equation implies that the interest rate that banks charge on loans increases with capital requirements and equity management costs.

A log-linear approximation to the consumption Euler Equation, Equation (49), is

\[ \hat{c}_t = -\sigma^{-1} \left( \hat{I}_t - \mathbb{E}_t[\hat{\pi}_{t+1}] \right) + \mathbb{E}_t[\hat{c}_{t+1}], \]  

(60)

Equation (50) can be approximated as

\[ \hat{w}_t - \sigma \hat{c}_t = \varphi \kappa_3 \hat{n}_t^s(i) + \varphi (1 - \kappa_3) \hat{n}_t^r(i), \]  

(61)

where \( \kappa_3 \in (0, 1) \) is given in Appendix E.

A log-linearized approximation to the resource constraint, Equation (51), is given by

\[ \hat{y}_t = \frac{1}{1 - \nu \frac{\gamma^p}{2} (\Pi^* - 1)^2} \left( \frac{c^s}{y^s} \hat{c}_t + \nu \gamma^p \Pi^* (\Pi^* - 1) \pi_t^s + \left( 1 - \frac{c^s}{y^s} - \frac{1}{2} \gamma^p \nu (\Pi^* - 1)^2 \right) \hat{I}_t(i) \right. \]

\[ + \left. \frac{\kappa_4}{1 - \chi^s} \hat{\chi}_t - \kappa_5 \theta \hat{\Delta}_t + \frac{\kappa_4 - \kappa_5 (\theta + \alpha (\theta - 1))}{1 + \Gamma^*} \hat{\Gamma}_t \right), \]  

(62)
where $\kappa_4$ and $\kappa_5$ are given in Appendix E.

The policy rules of the central bank and the macroprudential policy-maker can be approximated as:

\[
\hat{I}_t = v^\sigma \sigma_t + v^y y_t + v^\rho \rho_t + \hat{\xi}_t, \quad (63)
\]
\[
\hat{\Gamma}_t = v^\sigma \sigma_t + v^y y_t + v^\rho \rho_t + \hat{\xi}_t, \quad (64)
\]

We complement the equations describing the equilibrium of the log-linearized economy by the following specification of shocks:

\[
\hat{a}_t = \rho^a a_{t-1} + \epsilon^a_t, \quad (65)
\]
\[
\hat{\chi}_t = \rho^\chi \chi_{t-1} + \epsilon^\chi_t, \quad (66)
\]
\[
\hat{\xi}_t = \rho^\xi \xi_{t-1} + \epsilon^\xi_t, \quad (67)
\]
\[
\hat{\eta}_t = \rho^\eta \eta_{t-1} + \epsilon^\eta_t, \quad (68)
\]

where the $\rho$’s are coefficients of persistence that are strictly smaller than one and the $\epsilon$’s are serially uncorrelated normally distributed error terms with zero mean.

After these steps, we are in a position to describe the equilibrium of the log-linearized economy as follows:

**Proposition 6**

For given shocks $\{\hat{a}_t, \hat{\chi}_t, \hat{\xi}_t, \hat{\eta}_t\}_{t=0}^\infty$, whose evolution is given by (65)-(68), the equilibrium $\{\hat{n}_s(i), \hat{n}_r(i), \hat{w}_t, \hat{l}_t(i), \hat{\Delta}_t, \hat{\phi}_t, \sigma_t, \pi_t, \hat{\phi}_t, \hat{\sigma}_t, \hat{\eta}_t, \hat{\xi}_t, \hat{\Gamma}_t\}_{t=0}^\infty$ of the log-linearized economy is described by the system of Equations (52)-(64).

### 4 Numerical Findings

#### 4.1 Calibration

For the parameter values we follow Collard et al. (2017) in setting the discount rate $\beta = 0.993$, the inverse of labor supply elasticity $\varphi = 0.276$, the relative utility weight of labor $\psi = 3.409$, the relative risk aversion of consumption $\sigma = 1$, and the elasticity of substitution of intermediate goods $\theta = 7$ (which corresponds to a 17% markup). The persistence and the standard deviation of the productivity shocks are set at 0.966 and
0.0068, respectively. We use the simulated method of moments to find plausible values for the persistence and the standard deviation of the financial shocks. The persistence of the financial shocks is pinned down by the empirical value of the persistence of the return on bank equity (see Goddard et al. (2011) for the U.S. data). The standard deviation of the financial shocks is pinned down by the standard deviation of the equity premium in the U.S. over 1900 – 2015 (see Damodaran (2016)). We set $\gamma^p = 74.55$, which is the average of the values found in Table 1 in Ireland (2001).

We choose the fraction of safe firms $\nu = 0.616$ such that the total revenue of the safe (bond-financed) firms is 1.5 times the total revenue of the risky (bank-financed) firms in steady state (see De Fiore and Uhlig (2011) and Gersbach et al. (2015b)). We select $\alpha = 0.118$ such that the charge-off rate on loans is equal to the empirical value 0.97%. In addition, we set the steady-state value for the coefficient of equity management $\chi_t$ at 0.0521, which results in an equity premium of 5.5% (U.S. data over 1900-2015). The coefficient of bailout dissipation is set at $\mu = 0.93$ capturing output losses and tax distortions, which amounts to 0.34% quarterly output losses. We normalize the steady-state productivity of safe firms to $a^* = 1$ and set $A$ such that it satisfies $\frac{\theta}{\theta + \alpha (\theta - 1)} A^{\frac{1}{\alpha (\theta - 1)}} = 1$, which means that, loosely speaking, safe firms and risky firms are on average equally productive (see (47)). We assume $B_t = 0$, so that the government relies on the lump-sum taxes to bail out defaulted banks.

We log-linearize the model around the unconditionally optimal (UO) steady state (see Damjanovic et al. (2008) and Damjanovic et al. (2015)), i.e. we determine the steady state associated with the UO policy, which maximizes the unconditional expectation of the utility (1) subject to constraints (41)-(51). Given the other steady-state parameters, the corresponding numerical optimization problem results in the optimal capital requirement $\Gamma^* = 5.8\%$ and inflation rate $\Pi^* - 1 = 0.02\%$. Notably, the UO steady

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12 See https://fred.stlouisfed.org/series/CORALACBN for the charge-off rate on loans for all commercial banks in the U.S. 1985—2016. Another approach is to choose $\alpha$ such that the fraction of non-performing loans to total loans, i.e. $\phi^c$, matches the ratio of non-performing loans to total loans for small enterprises in developed countries (see Beck et al. (2011)). Both approaches yield a very similar value for $\alpha$.

13 See Damodaran (2016) for a review of the equity premiums across countries and over different periods.

14 We take the U.S. data from Laeven and Valencia (2012).
state features a mildly positive net rate of inflation. This is plausible as a mildly positive inflation rate alleviates the distortions arising from monopolistic competition to some extent. With this optimal steady-state capital requirement and inflation, the corresponding fraction of defaulted banks is $\phi \Gamma^* \approx 5.7\%$.

### 4.2 Comparative statics

In this section we characterize and illustrate the properties of the steady state. Figure 2 shows the steady-state variables as functions of the aggregate capital requirement. In the following we interpret Figure 2 from left to right and from top to bottom.
• The first two graphs show the significant impact on the risky sector when aggregate capital requirements rise. Since banks face higher financing costs, the loan rate increases. As a consequence, loan demand by risky firms decreases and less labor is hired. The next graph indicates that labor is shifted to the safe sector.

• The sharp contraction in the risky sector dominates the reallocation to safe firms and aggregate labor used in production decline, albeit only mildly, as displayed in the first panel in the second row.

• As resources are shifted to safe firms, the safe firms’ profits grow while the risky firms’ profits shrink with the aggregate capital requirement.

• As indicated in the first panel in the third row (the horizontal dashed line represents $\phi^c$, the fraction of defaulted risky firms), the contraction in the risky sector when aggregate capital requirements are raised entails less bank failure.

• There are two reasons why the bailout fees decrease with aggregate capital requirement. First, a higher $\Gamma^*$ leads to fewer loans to risky firms, and thus fewer rescue funds are needed when banks default. Second, higher aggregate capital requirement results in a smaller fraction of defaulted banks.

• As aggregate labor decreases with $\Gamma^*$, final output and consumption do the same.

• High aggregate capital requirements have only a moderately negative impact on wages.

• The last plot in Figure 2 shows that utility increases with the aggregate capital requirements for low aggregate capital requirements and attains a maximum at $\Gamma^* = 5.8\%$. The reason is that limited liability of share-holders and the government guarantee for deposits leads to significant overinvestment in risky firms with low aggregate capital requirements. Higher aggregate capital requirements reduce this inefficiency, thereby increasing the aggregate output per unit of labor input. However, excessive aggregate capital requirements entail large equity management costs, and shift too many resources to safe firms.
4.3 Impulse responses

In this section we examine how the economy reacts to shocks. More specifically, we consider shocks to monetary policy as well as macroprudential policy, productivity shock, and financial shock. The persistence of these shocks is set at 0.9. Monetary policy follows the standard Taylor rule (35), and macroprudential policy follows Equation (36) with (0, 0, 0, 0) policy rule coefficients.

Figure 3 shows the dynamics of the economy after an interest-rate shock $\epsilon_0^\xi = 0.01$ (solid lines). High interest rates lead to lower output, consumption, declining wages,
and lower total labor supply. Declining demand results in falling prices. However, due to price stickiness, safe firms are relatively slow to lower prices compared to risky firms. Thus the ratio between the price of the intermediate goods produced by safe firms with respect to the aggregate price, i.e., $\hat{q}_t$, hikes. With relatively high prices of their goods, safe firms face low demand, so less labor is hired by safe firms, and labor is shifted to risky firms. As a consequence, risky firms request and receive larger loans, which leads to slightly more resources being devoted to bailing out banks.

It is instructive to study the dynamic effects of a shock to the macroprudential policy instrument next (dashed lines in Figure 3). Due to an increase in capital requirements $\epsilon^\eta_0 = 1$, banks charge high loan rate premium to compensate the increased equity-financing cost. The high loan rate reduces total loan demanded by risky firms. Thus, risky firms hire less labor, which leads to a decline in wages. As a consequence, safe firms hire more labor and therefore produce more goods, which leads to a decline in the relative prices of their goods, $\hat{q}_t$. Aggregate output drops moderately, as safe firms are overly invested and too few resources are channeled through the banking sector. It is also clear that the total funds that the government uses to bailout banks ($bo_t$) decline, because large equity buffer reduces the fraction of defaulted banks. An increase in the macroprudential policy instrument is inflationary, as it makes bank-financing more expensive and thereby increases the costs of production for risky firms.

It is noteworthy that the two policy instruments have very different effects on the economy. As is common in models of monetary policy, changes in the interest rate always move both inflation and output in the same direction. By contrast, changes in the macroprudential policy instrument affect inflation and output differently. For example, more strict capital requirements increase inflation, while at the same time leading to lower output. These observations show that the macroprudential instrument is complementary to the monetary-policy instrument and that adding the macroprudential policy instrument to the policy-makers’ tool box might be welfare enhancing, as the joint use of both tools allows, for example, to influence output and inflation independently.

We next examine the impact of a productivity shock $\epsilon^\alpha_0 = 0.01$ (solid lines in Figure 4). The increase in productivity yields higher output and consumption, which gradually
Figure 4: Impulse responses to productivity shock (solid lines) and financial shock (dashed lines).

decline to the steady level. Higher marginal productivity raises wages. Higher wages decrease aggregate labor supply, since the income effect dominates the substitution effect. Lower marginal costs lead to deflation. However, due to price rigidity, safe firms cannot lower the price as fast as risky firms. Thus the ratio between the price of the intermediate goods produced by safe firms with respect to the aggregate price, i.e., $\hat{q}_t$, displays a hump-shape hike. We observe a reallocation of resources between sectors: labor is shifted from safe firms to risky firms. Due to price rigidity, the process of labor reallocation displays a hump shape. A high wage rate and more labor employed in risky firms imply more bank loans granted to risky firms. Hence more rescue funds are needed, as shown by the increase of bailout fees. In other words, a positive productivity shock on firms increases the severity of bank failures.

Dashed lines in Figure 4 show the evolution of the economy in response to a shock on
equity management costs $\epsilon^X_0 = 1$. As the costs of holding equity rise, investors will request a higher return on equity. This, in turn, implies that loan rate premium will increase. Thus total loan demand declines, and labor is shifted from the risky sector to the safe sector. Furthermore, high loan rate premium leads to low fraction of defaulted banks and bailout fees. In addition, an increase in equity management costs leads to inflation, lower relative prices for intermediate goods produced by safe firms (safe firms adjust prices more slowly than risky firms) and declines in wage rates, aggregate labor, output (countercyclical equity premium), and consumption.

5 Optimal Policy Rules

The global financial crisis 2007-2009 and its aftermath have cast doubt on the effectiveness of monetary policy and have raised questions about how to include financial stability measures—proxied by e.g. credit aggregates or non-performing loans—into macroeconomic policy-making. The crisis has also rekindled the debate on how to govern and coordinate monetary and macroprudential policies during financial crises and economically more tranquil times. The last crisis has also cast considerable doubt on the consensus formed in the “Great Moderation” that central banks should pursue inflation targeting, where monetary policy can be described by a Taylor rule which aims at stabilizing inflation and output. The crisis has shown that, despite stable inflation and output for a considerable period, unsustainable sectoral booms and gradual buildups of financial risks—e.g. excessive leverage of banks—may lead to a financial meltdown with adverse macroeconomic consequences.

As a response, one strand of the literature studies modified Taylor rules. Blanchard et al. (2013) and Woodford (2014) point out that monetary policy should incorporate multiple targets and multiple instruments. Woodford (2012) demonstrates that a temporary departure of monetary policy from the inflation and output target path due to financial stability concerns can be socially optimal.

As documented in Goodhart et al. (1988), the original purpose of establishing central banks in certain countries was to prevent financial instability. Several economists, such as King (1997), Bernanke and Mishkin (1997), and Svensson (1999), propose more flexible inflation targeting. Käfer (2014) reviews the literature on Taylor rules augmented with a financial stability term.
Another strand of literature studies the optimal proxy or indicator for financial instability. Bernanke and Gertler (1999) and Cecchetti et al. (2002) use asset prices; Agnor et al. (2011) and Christiano et al. (2010) use credit aggregates; and Carlstrom et al. (2010), Angelini et al. (2014), Curdia and Woodford (2010), Quint and Rabanal (2014), and Ueda and Valencia (2014) use credit spreads and leverage. We contribute to both strands of literature by studying the interplay between monetary policy and macroprudential policy and investigating optimal policy rules for central banks and macroprudential policy-makers.

We derive the unconditionally optimal (UO) policies in order to be able to obtain a welfare measure for different policy stances. As shown in Damjanovic et al. (2015), it is possible to derive a purely quadratic approximation to welfare around the UO steady state by using approximations to the social planner’s constraints up to the second order to eliminate all linear terms in the approximation of the household’s utility function up to the second order. This purely quadratic measure can be evaluated for constraints and policies that are correct up to the first order. For this purpose, one has to compute the variances and covariances of the log-deviations of the endogenous variables and exogenous shocks from the UO steady state. As the computation of the welfare measure requires the computation of second-order derivatives of the constraints and the utility function, which is quite cumbersome, we perform the respective calculations with the help of a computer algebra system.\textsuperscript{16}

We next determine the optimal Taylor rule for monetary policy-making under the assumption that the aggregate capital requirement is constant. This investigation breaks down into three questions:

- What are the optimal weights on inflation and the output gap?
- Should the central bank take financial instability into account?
- If so, what is the optimal indicator for financial instability? Is it the financial risk represented by the fraction of defaulted banks $\phi_t$, or the credit cycle represented by the change in the aggregate amount of bank loans $l_t$?
Figure 5: Social losses under different policy rules.
We first shut down the financial instability terms in (34), i.e. \( \upsilon_{\text{mon}} = \upsilon_{\phi \text{mon}} = 0 \), and run the program for \( \upsilon_{\text{mon}} \in [1,1,2] \) and \( \upsilon_{\phi \text{mon}} \in [0,1] \). Figure 5a shows the unconditional expectation of social losses under different coefficient constellations \((\upsilon_{\text{mon}}, \upsilon_{\phi \text{mon}}, 0, 0)\). We observe that social losses are minimal for \( \upsilon_{\phi \text{mon}} = 0 \). Thus, given that the capital requirement is fixed at its steady-state level and the monetary policy-maker does not react to financial instability, the central bank should focus solely on price stability.

We next examine whether the central bank should take financial instability into account and if so, which variable serves better as the indicator of financial instability, the fraction of defaulted banks or the total credit supply? We run the program for \( \upsilon_{\text{mon}} \in [0,1] \) and \( \upsilon_{\phi \text{mon}} \in [0,1] \). Figure 5b shows the social losses under different coefficient constellations \((2, 0, \upsilon_{\text{mon}}, \upsilon_{\phi \text{mon}})\). The optimal Taylor rule does not involve any concern about financial instability. This finding backs the prevailing view that monetary policy cannot serve as an effective tool to safeguard financial stability, as it primarily affects the aggregate amount of lending through the banking system and capital market.

We then study the optimal macroprudential rule for a given standard Taylor rule (35). We first consider the scenario where the macroprudential policy-maker focuses on financial instability only. We investigate which financial indicator should be given more weight, i.e. we study the optimal constellation of coefficients \((0, 0, \upsilon_{\text{mac}}, \upsilon_{\phi \text{mac}})\). Figure 5c shows that the macroprudential policy maker should react to both the credit cycle and the financial risk. However, for a larger coefficient range (see Figure 7c in Appendix C), the simulation shows that the use of time-varying capital requirements should be based on the credit cycle, as suggested in Basel III. The event analysis of credit booms by Mendoza and Terrones (2012) shows that credit booms occur with a frequency of only 2.8 percent in a sample of 61 industrial and emerging economies for the 1960-2010 period, but conditional on a credit boom, the probability of banking or currency crises is one-third. For the connection between abnormal credit expansion and financial instability, see Borio and Drehmann (2009) and Reinhart and Rogoff (2008).
suitable indicators for financial instability. Figures 5e and 5f show that the credit-to-
GDP ratio could significantly improve social welfare, while the loan rate premium has
a negligible impact.

Finally, by running $\nu_{mon}$, $\nu_{mac}$ and $\nu_{mac}$ from 0 to 1 and $\nu_{mon}$ from 1.1 to 2, we
find that social loss is minimized when $\nu_{mon} = 0$, $\nu_{mon} = 2$, $\nu_{mac} = 1$ and $\nu_{mac} = 1$.\(^\text{19}\) That is, social welfare attains its maximum when the monetary authority focuses
exclusively on price stability and when the macroprudential authority reacts to both
the output variation and the credit cycles. This observation provides some support
for the separation of monetary policy and macroprudential policy in different policy-
making bodies, since optimal policy requires both policies to focus on different economic
variables.

6 Conclusion

We have integrated banks and the coexistence of banks and bond financing into an
otherwise standard New Keynesian Model. While interest rate policies stabilize shocks
that affect aggregate variables, they are less suitable for stabilizing macroeconomic
events driven by sectoral shocks. If they affect firms primarily financed by banks, such
shocks are best dealt with by time-varying aggregate capital requirements.

While we have pursued a small number of applications, numerous extensions of the basic
framework and further applications can and should be pursued. Regarding applications,
a variety of alternative shocks could be investigated. For instance, markup shocks and
demand shocks originating from preference shocks or government spending shocks are
obvious candidates for an in-depth analysis of how monetary policy and aggregate
bank-equity policies can jointly stabilize such shocks.

Regarding extensions, one could introduce aggregate shocks occurring after households
have acquired equity, which would introduce risk premium to equity returns. Also,
the banking sector could be modeled in more sophisticated ways, e.g. by introducing
monitoring costs or a more complex funding and asset structure.

\(^\text{19}\)Our results are robust for a larger range of coefficients ($\nu_{mon}$, $\nu_{mac}$ and $\nu_{mac}$ from 0 to 10 and
$\nu_{mon}$ from 1.1 to 10.1), see Figure 7 in Appendix F.
A Optimal Behavior of Safe Intermediate Firms

Stage 2: Price setting In line with (15), (19), and \( s_t(i) = w_t n^s_t(i) = \frac{w t^s(i)}{a_t} \), the real profits of a safe intermediate firm \( i \) in period \( t \) are given by

\[
\begin{align*}
  z^s_t(i) &= \left( \frac{p^s_t(i)}{p_t} - \frac{w_t}{a_t} \right) \left( \frac{p^s_t(i)}{p_t} \right)^{-\theta} y_t - \gamma p_t \left( \frac{p^s_t(i)}{p^s_{t-1}(i)} - 1 \right)^2 y_t. \\
  (69)
\end{align*}
\]

Consequently, the present value of discounted profits can be written as

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} Q_t \left\{ \left( \frac{p^s_t(i)}{p_t} - \frac{w_t}{a_t} \right) \left( \frac{p^s_t(i)}{p_t} \right)^{-\theta} y_t - \gamma p_t \left( \frac{p^s_t(i)}{p^s_{t-1}(i)} - 1 \right)^2 y_t \right\}. \\
(70)
\]

The first-order condition with respect to \( p^s_t(i) \) yields safe firms’ optimal price dynamics

\[
0 = \left( \frac{p^s_t(i)}{p_t} \right)^{-\theta} y_t \left[ (1 - \theta) + \theta \frac{w_t}{a_t} \frac{p_t}{p^s_t(i)} \right] - \gamma p_t \left( \frac{p^s_t(i)}{p^s_{t-1}(i)} - 1 \right) + \mathbb{E}_t \left[ \beta \frac{c^s_t}{c^s_{t+1}} \left( \frac{\gamma p^s_{t+1}(i) y_{t+1}}{(p^s_t(i))^2} \left( \frac{p^s_{t+1}(i)}{p^s_t(i)} - 1 \right) \right) \right].
\]

(71)

\]

B Optimal Behavior of Risky Intermediate Firms

Stage 2: Price setting In stage 2, after the realization of the idiosyncratic shock \( \phi(i) \), production is determined in line with (22). Each risky firm’s revenues are maximized by selecting the maximum price for which it can sell its output. Rewriting (15), we obtain the optimal price set by risky firms

\[
p^r_t(i) = \left( \frac{y_t}{y^r_t(i)} \right)^{\frac{1}{\theta}} p_t.
\]

(72)

It will be convenient to introduce the premium on bank financing:

\[
\Delta^l_t := \frac{R^l_t}{R^s_t}.
\]

(73)

Combining (24) and (72) yields the real profit, conditional on the firm’s being able to repay the loan:

\[
z^r_t(i) = y^\frac{1}{\theta} t^r(i) \frac{\theta + 1}{\theta} - \Delta^l_t l_t(i) \geq 0.
\]

(74)
Let $\phi^c$ be the level of $\phi(i)$ below which risky intermediate firms default and hence cannot fully repay the loan. We can solve the critical value $\phi^c$ from $l_t(i) = R_t^i l_t(i)$, $w_t = R_t^i \tilde{w}_t$, (22), (23) and (74):

$$
\phi^c = \left( \frac{1}{A_{a_t} n_t^i(i)} \left( \frac{\Delta^i_l(i)}{y_t^{\theta}} \right)^{\frac{\theta-1}{\theta}} \right)^{\frac{1}{\alpha}} = \left( \frac{w_t}{A_{a_t}} \left( \frac{l_t(i)}{y_t} \right)^{\frac{1}{\theta-1}} (\Delta^i_l)^{\frac{\theta}{\theta-1}} \right)^{\frac{1}{\alpha}} \cdot (75)
$$

For risky intermediate firms with $\phi(i) \in [0, \phi^c)$, i.e. firms that cannot repay the loan in full, profit is zero

$$
z^*_t(i) = 0, \quad (76)
$$

and all the revenue goes to the bank. Thus, provided that the firm defaults, the gross return on the bank loan is

$$
R^*_t(i) = \frac{y_t^{\frac{1}{\theta}} y_t^i(i)^{\frac{\theta-1}{\theta}}}{l_t(i)^{\theta}} R^*_t \in [0, R^*_t]. \quad (77)
$$

**Stage 1: Attraction of loans** Given the price (72) set in stage 2, firms determine the optimal amount of loan $\tilde{l}_t(i)$.

The risky firms’ expected real profit is

$$
\int_0^1 z^*_t(i) d\phi(i) = \int_{\phi^c}^1 \left[ y_t^{\frac{1}{\theta}} y_t^i(i)^{\frac{\theta-1}{\theta}} - \Delta^i_l(i) \right] d\phi(i), \quad (78)
$$

where we have taken into account the fact that profits are zero if $\phi(i) < \phi^c$.

Equation (75) reveals that we have to restrict the choice of $l_t(i)$ to values that involve $\phi^c \leq 1$, i.e.

$$
l_t(i) \leq \left( \frac{A_{a_t}}{w_t} \right)^{\theta-1} y_t \left( \Delta^i_l \right)^{\theta} =: \bar{l}. \quad (79)
$$

Using (22), (23), (75), and (78), we can state the firm’s profit maximization problem in the following way:

$$
\max_{l_t(i)} \left( \frac{\theta}{\theta + \alpha(\theta - 1)} l_t^i(i)^{\frac{\theta-1}{\theta}} + \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} \frac{l_t^i(i)^{1+\alpha(\theta-1)}}{\bar{l}^{1+\alpha(\theta-1)}} - l_t(i) \right) \Delta^i_l
$$

s.t. $l_t(i) \leq \bar{l}. \quad (80)$
Obviously, the condition $\phi^c \leq 1$ will be slack at the optimal choice of $l_t(i)$ because profits are zero when $\phi^c = 1$, which means that the firm defaults with probability one.

An optimal choice of $l_t(i)$ implies

$$
(\theta - 1) \left( \frac{l_t(i)}{\bar{L}} \right)^\frac{1}{\bar{b}} + (1 + \alpha(\theta - 1)) \left( \frac{l_t(i)}{\bar{L}} \right)^\frac{1}{\alpha(\theta - 1)} - (\theta + \alpha(\theta - 1)) = 0. \tag{81}
$$

Consequently, the optimal value of $l_t(i)$ can be written as

$$
l_t(i) = \bar{L}^* = \left( \frac{Aa_t}{w_t} \right)^{\theta - 1} \frac{L^*}{(\Delta_i)^{\theta}} y_t, \tag{82}
$$

where $L^*$ is the root of

$$
g(L) := (\theta - 1)L^{-\frac{1}{\bar{b}}} + (1 + \alpha(\theta - 1))L^{\frac{1}{\alpha(\theta - 1)}} - (\theta + \alpha(\theta - 1)) \tag{83}
$$

that satisfies $0 < L^* < 1$.\(^{20}\)

For arbitrary $\theta > 1$, the existence of such a solution can be readily established. Function $g(L)$ has at least one root on $(0, 1)$ because (i) $\lim_{L \to 0} g(L) = \infty$, (ii) $g(1) = 0$, and (iii) $g'(1) > 0$. The uniqueness of the solution follows from the additional observation that $g(L)$ has a single minimum on $(0, 1)$, which is straightforward to verify. To sum up, the risky firms’ expected profit is maximized when the real loan is given by $l_t(i) = L^*\bar{L}$, where $L^*$ is the solution to (83).

We observe that (82) also allows us to use a particularly simple expression for $\phi^c$. Inserting (82) into (75) results in

$$
\phi^c = (L^*)^{\frac{1}{\alpha(\theta - 1)}}. \tag{84}
$$

Hence $\phi^c$ will be constant in equilibrium and will not depend on the central bank’s policy rate $I_t$ or the capital requirement $\Gamma_t$.

Using (22), (23), and (82), we can write the output of risky firms as

$$
y^*_r(i) = (\phi(i))^\alpha \left( \frac{Aa_t}{w_t\Delta_i^t} \right)^\theta L^* y_t. \tag{85}
$$

The aggregate profits of all risky firms can be computed with the help of (78), (82), (83), (84), and (85) as

$$
z^*_r = (1 - \nu) \left( \frac{Aa_t}{w_t\Delta_i^t} \right)^{\theta - 1} \frac{L^*}{\theta - 1} \left( 1 - (L^*)^{\frac{1}{\alpha(\theta - 1)}} \right) y_t. \tag{86}
$$

\(^{20}\)For $\theta = 2$ and $\alpha = 1$, for example, the unique solution is $L = \frac{1}{4}(\sqrt{3} - 1)^2$. \hspace{1cm} \Box
C Optimal Behavior of Banks

We examine the problem of a representative bank in three steps.

Step 1: Loan provision for given capital structure

In the first step, we examine loan provision by a representative bank if it has a capital structure equal to the aggregate capital requirement and can provide a loan to one risky firm. For convenience, we denote the risky firm by $i$ and use the same index $i$ for the representative bank that lends to firm $i$. On a bank’s balance sheet, the asset (loan to a risky firm) is equal to the sum of liabilities (deposits and equity). Thus we have

$$\tilde{l}_t(i) = \tilde{d}_t(i) + \tilde{e}_t(i) = (1 + \Gamma_t)\tilde{d}_t(i).$$

(87)

We define $R_{t}^\Gamma$ as the smallest gross return on bank loans $R_{t}^\Gamma(i)$, such that the corresponding bank does not default. At this rate, the following condition must hold:

$$\tilde{d}_t(i)R_{t}^\Gamma = \tilde{l}_t(i)R_{t}^\Gamma,$$

(88)

which means that the total repayment to depositors just equals the funds received from the risky firm. Combining (87) and (88) yields

$$\Delta_t^\Gamma := \frac{R_{t}^\Gamma}{R_{t}^s} = \frac{1}{1 + \Gamma_t}.$$  

(89)

As a next step, we compute $\phi^\Gamma$, the value of $\phi(i)$ below which the bank defaults. According to (22) and (77), $\Delta_t^\Gamma$ has to satisfy

$$\Delta_t^\Gamma = \frac{y_t^\frac{1}{\theta} \left[ (\phi^\Gamma)^\alpha Aa_t n_t^\Gamma(i) \right]^{\frac{\theta - 1}{\theta}}}{l_t(i)}.$$  

(90)

Equating (89) and (90) and solving for $\phi^\Gamma$ results in

$$\phi^\Gamma = \left( \frac{1}{Aa_t n_t^\Gamma(i)} \left( \frac{l_t(i)}{(1 + \Gamma_t)^{\frac{1}{\theta}}} \right) \right)^{\frac{1}{\theta}}.$$  

(91)

Using (75) and (91) entails
\[ \phi^\Gamma = \frac{1}{(\Delta_t^i(1 + \Gamma_t))^{\frac{\theta}{(\theta - 1)}}} \phi^c = \left( \frac{L^*}{(\Delta_t)^\theta(1 + \Gamma_t)^\theta} \right)^{\frac{1}{\alpha(\theta - 1)}}. \]  

(92)

Note that \( \phi^\Gamma \) decreases with \( \Gamma_t \), which indicates that a high equity-to-debt ratio reduces the fraction of banks that fail. When \( \Gamma_t \to \infty \), i.e. banks are fully financed by equity, we obtain \( \phi^\Gamma \to 0 \), i.e. banks never default.

We note that the above equation implies

\[ \phi^\Gamma \leq \phi^c. \]  

(93)

Hence we have to distinguish between three ranges of \( \phi(i) \). For \( \phi(i) \geq \phi^c \), the firm can fully repay the loan and so the bank does not default. For an intermediate range of \( \phi(i) \), \( \phi(i) \in [\phi^\Gamma, \phi^c) \), the firm cannot fully repay the loan. However, the bank will not default because it can simply reduce dividends. For \( \phi(i) < \phi^\Gamma \), the repayment on the loan is not sufficient to repay depositors. In this case, the government has to bail out the bank and equity holders receive nothing.

The expected return on equity

\[ R_t^e = \int_0^{\phi^\Gamma} 0 \, d\phi(i) + \int_{\phi^\Gamma}^{\phi^c} R_t^e(i) \, d\phi(i) + \int_{\phi^c}^{1} \tilde{R}_t^e \, d\phi(i), \]

where

\[ R_t^e(i) = \frac{R_t^r(i) - R_t^d(i)}{\tilde{e}_t(i)} = \frac{(1 + \Gamma_t)R_t^r(i) - R_t^d}{\Gamma_t}, \]

\[ \tilde{R}_t^e = \frac{R_t^d(i)}{\tilde{e}_t(i)} = \frac{(1 + \Gamma_t)R_t^d - R_t^d}{\Gamma_t}. \]

We can rewrite the expected return on equity as

\[ R_t^e = \int_0^{\phi^c} \frac{(1 + \Gamma_t)R_t^r(i) - R_t^d}{\Gamma_t} \, d\phi(i) + \int_{\phi^c}^{1} \frac{(1 + \Gamma_t)R_t^d - R_t^d}{\Gamma_t} \, d\phi(i). \]  

(94)

We observe that (77) can be combined with (22), (23), (82), and (84) to yield

\[ R_t^e(i) = \frac{\alpha(\theta - 1)}{\phi^c} R_t^d(i). \]  

(95)

Inserting (95) into (94) yields the following relationship between \( \Delta_t^e \) and \( \delta_t^l \), where \( \delta_t^l := (1 + \Gamma_t)\Delta_t^l \):

\[ \Gamma_t\Delta_t^e = h(\delta_t^l) := \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} \left( \frac{1}{(\delta_t^l)^{\frac{\theta}{(\theta - 1)}}} - \delta_t^l \right) \phi^c + \delta_t^l - 1. \]  

(96)
Step 2: Uniqueness of loan rate

For $\Gamma_t \to 0$, which implies that banks would be entirely financed by deposits, (96) becomes

$$0 = \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} \left( \frac{1}{(\Delta^e_t) - \Delta^l_t} - \Delta^l_t \right) \phi^c + \Delta^l_t - 1. \tag{97}$$

In this case the solution is $\Delta^l_t = 1$. In the case of $\Gamma_t \to \infty$, i.e. with very strict capital requirements that lead to banks being financed entirely through equity, we obtain

$$\Delta^l_t = \frac{1}{1 - \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} \phi^c} \Delta^e_t > \Delta^e_t. \tag{98}$$

For general values of $\Gamma_t$, (96) is more difficult to analyze. Recall that $(1 + \Gamma_t)\Delta^l_t \geq 1$ and therefore $\delta^l_t \geq 1$. It can be easily verified that $h(1) = 0$ and that $\lim_{\delta^l_t \to \infty} h(\delta^l_t) = \infty$. Moreover, $h'(\delta^l_t) > 0$, $\forall \delta^l_t \in [1, \infty)$. As a consequence, for all combinations of $\Gamma_t$ with $\Gamma_t \geq 0$ and $\Delta^e_t$ with $\Delta^e_t \geq 1$, there is a unique solution for $\Delta^l_t$ given by $\Gamma_t \Delta^e_t = h((1 + \Gamma_t)\Delta^l_t)$ or $\Delta^l_t = \frac{1}{1 + \Gamma_t} h^{-1} (\Gamma_t \Delta^e_t)$. For fixed $\Gamma_t$, $\Delta^l_t$ is an increasing function of $\Delta^e_t$.

Step 3: Optimal capital structure

Finally, we show that it is optimal for banks to choose a capital structure that is equal to the aggregate capital requirement $\Gamma_t$ in each period. Suppose that except for one deviating bank, all banks choose $\Gamma_t$ in period $t$. Then the market loan rate $\Delta^l_t$ is given by (96) and illustrated in Graph 1 in Figure 2, since the deviating bank has no impact on equilibrium interest rates. Suppose that the deviating bank chooses a capital structure $\tilde{\delta}^l_t(i) > \Gamma_t$ and finances a loan to the risky firm $\tilde{l}_t(i) = \tilde{d}_t(i) + \tilde{e}_t(i)$. It is profitable for this bank to do so if the deviation strictly increases the return on equity. Hence, we have to verify whether for a given $\Delta^l_t$, $\Delta^e_t$ is increasing in the bank-specific capital structure that we denote by $\Gamma_t(i)$. Such a deviation cannot be profitable. For a given loan size and market loan rate $\Delta^l_t$, choosing $\Gamma_t(i) > \Gamma_t$ implies that in the case of default and bailout, expected transfers from the government are lower than for $\Gamma_t(i) = \Gamma_t$. The reason is that both the likelihood of default and the bailout transfer in the case of default are lower. Since bank revenues are unaffected by different choices of capital structure and return on equity is higher than deposit rates,
the preceding observation implies necessarily that the expected return on equity is lower with choice $\Gamma_t(i) > \Gamma_t$ than with $\Gamma_t(i) = \Gamma_t$. This is illustrated in Figure 6, which displays expected returns on equity for aggregate capital requirement ratios of 4%, 8%, and 12%, respectively. For instance, the solid black line represents the expected return on equity when the aggregate capital requirement ratio $\Gamma_t$ is 4% (represented by the vertical dashed black line). The expected return on equity decreases with the equity-to-deposit ratio $\Gamma_t(i)$. Thus the individual bank would select the lowest possible $\Gamma_t(i)$, i.e. $\Gamma_t(i) = \Gamma_t$. The realized return on equity is represented by the horizontal gray line at value $\Delta^* = \frac{1}{1 - \chi^*} = 1.055$.

\[ \square \]

## D The Government

The real bail-out fees amount to

$$b_{0t} = (1 - \nu) \int_{0}^{\phi^c} (d_t(i) - R_t^c(i) \tilde{l}_t(i)) \, d\phi(i).$$

With the help of $\phi^c / \phi^r = (\Delta_t^{\frac{1}{\theta}})^{\frac{\theta}{\theta - 1}} (1 + \Gamma_t) \frac{\theta^{\frac{1}{\theta - 1}}}{\alpha^{\frac{\theta}{\theta - 1}}} \cdot \phi^c = (L_t^{\frac{1}{\theta}})^{\frac{1}{\theta - 1}} \cdot l_t(i) = w_t n_t \Gamma(i)$, and (95), this expression can be stated as

$$b_{0t} = \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} \frac{(1 - \nu) w_t n_t(i)(L_t^{\frac{1}{\theta}})^{\frac{1}{\theta - 1}}}{\alpha(\theta - 1)} \cdot (\Delta_t^{\frac{1}{\theta}})^{\frac{\theta}{\theta - 1}} (1 + \Gamma_t) \frac{\theta + \alpha(\theta - 1)}{\alpha^{\frac{\theta}{\theta - 1}}}.$$

(99)
E Linearized Dynamics

All variables with an asterisk denote steady-state values. Variables with a “hat” stand for relative deviation from steady-state values. We explicitly allow non-zero inflation in the steady state.

Equation (41)

\[ w_t n_r^i(i) = \left( \frac{Aa_t}{w_t} \right)^{\theta - 1} \frac{1}{(\Delta^i)^{\theta}} L^* y_t. \]  

(100)

Steady state:

\[ w^* n_r^{*i}(i) = \left( \frac{Aa^*}{w^*} \right)^{\theta - 1} \frac{1}{(\Delta^{i*})^{\theta}} L^* y^*. \]  

(101)

Log-linearization:

\[ w^* (1 + \hat{w}_t) n_r^{*i}(i)(1 + \hat{n}_r^i(i)) = \left( \frac{Aa^*(1 + \hat{a}_t)}{w^*(1 + \hat{w}_t)} \right)^{\theta - 1} \frac{1}{(\Delta^{i*}(1 + \hat{\Delta}_t))^{\theta}} L^* y^* (1 + \hat{y}_t). \]  

(102)

Using (101) to simplify the equation above yields

\[ \hat{w}_t + \hat{n}_r^i(i) = (\theta - 1)(\hat{a}_t - \hat{w}_t) - \theta \hat{\Delta}_t + \hat{y}_t. \]  

(103)

Equation (43)

We can write (43) as

\[ 0 = q_t^{1 - \theta} y_t \left[ (1 - \theta) + \frac{\theta w_t}{a_t q_t} \right] - \gamma^p y_t \frac{q_t}{q_{t-1}} \Pi_t \left( \frac{q_t}{q_{t-1}} \Pi_t - 1 \right) \]

\[ + \beta E_t \left[ \frac{c_i^{*\sigma}}{c_{i+1}^{*\sigma}} \gamma^p y_{t+1} \frac{q_{t+1}}{q_t} \Pi_{t+1} \left( \frac{q_{t+1}}{q_t} \Pi_{t+1} - 1 \right) \right]. \]  

(104)

Steady state:

\[ (1 - \beta) \gamma^p \Pi^* (\Pi^* - 1) = (q^*)^{1 - \theta} \left[ (1 - \theta) + \frac{\theta w^*}{a^* q^*} \right]. \]  

(105)
Log-linear approximation around steady state:

\[
0 = (q^*)^{-\theta} y^* \left[ (1 - \theta) + \frac{\theta w^*}{a^* q^*} \right] (\hat{\pi}_t - (\theta - 1) \hat{\pi}_t)
\]

\[+ (q^*)^{-\theta} y^* \frac{\theta w^*}{a^* q^*} (\hat{\pi}_t - \hat{\pi}_t - \hat{\pi}_t)\]

\[- \gamma^p y^* \Pi^* \left( \Pi^* - 1 \right) (\hat{\pi}_t - \hat{\pi}_t + \pi_t + \hat{\pi}_t) - \gamma^p y^* (\Pi^*)^2 (\hat{\pi}_t - \hat{\pi}_t - \pi_t)\]

\[+ \gamma^p \beta y^* (\Pi^* - 1) \mathbb{E}_t \left[ \sigma \hat{\pi}_t - \sigma \hat{\pi}_t + \hat{\pi}_t + \hat{\pi}_t + \hat{\pi}_t - \hat{\pi}_t + \pi_t + \pi_t \right]\]

\[+ \gamma^p \beta y^* (\Pi^*)^2 \mathbb{E}_t \left[ \hat{\pi}_t + \hat{\pi}_t - \hat{\pi}_t + \pi_t + \pi_t \right],\]

(106)

where \(\pi_t = \hat{\Pi}_t\) is the relative deviation of inflation from its steady-state value. Note that \(\pi_t + \hat{\pi}_t - \hat{\pi}_t = \hat{\Pi}_t - \hat{\Pi}_t - \pi_t^*\) represents the relative deviation of the growth rate of the price of goods produced by safe firms from the corresponding steady-state inflation rate.

Combining (105) and (106) yields

\[
0 = (1 - \beta) \gamma^p \Pi^* (\Pi^* - 1) (\hat{\pi}_t - (\theta - 1) \hat{\pi}_t)
\]

\[+ \left[ (1 - \beta) \gamma^p \Pi^* (\Pi^* - 1) + (q^*)^{-\theta} (\theta - 1) \right] (\hat{\pi}_t - \hat{\pi}_t - \hat{\pi}_t)\]

\[- \gamma^p \Pi^* \left( \Pi^* - 1 \right) (\hat{\pi}_t - \hat{\pi}_t - \hat{\pi}_t - \hat{\pi}_t + \pi_t + \hat{\pi}_t) - \gamma^p (\Pi^*)^2 (\hat{\pi}_t - \hat{\pi}_t + \pi_t)\]

\[+ \gamma^p \beta \Pi^* (\Pi^* - 1) \mathbb{E}_t \left[ \sigma \hat{\pi}_t - \sigma \hat{\pi}_t + \hat{\pi}_t + \hat{\pi}_t + \hat{\pi}_t - \hat{\pi}_t + \pi_t + \pi_t \right]\]

\[+ \gamma^p \beta (\Pi^*)^2 \mathbb{E}_t \left[ \hat{\pi}_t + \hat{\pi}_t - \hat{\pi}_t + \pi_t + \pi_t \right],\]

(107)

which can be re-arranged as

\[
0 = (1 - \beta) \gamma^p \Pi^* (\Pi^* - 1) (\hat{\pi}_t - \theta \hat{\pi}_t + \hat{\pi}_t - \hat{\pi}_t)
\]

\[+ (q^*)^{-\theta} (\theta - 1) (\hat{\pi}_t - \hat{\pi}_t - \hat{\pi}_t)\]

\[+ \gamma^p (\Pi^*)^2 \mathbb{E}_t \left[ \beta (\sigma \hat{\pi}_t - \sigma \hat{\pi}_t + \hat{\pi}_t + \hat{\pi}_t - 2 \pi_t^* - 2 \pi_t^* - \hat{\pi}_t) \right] \]

\[+ \gamma^p \Pi^* \mathbb{E}_t \left[ \pi_t^* + \hat{\pi}_t - \beta (\sigma \hat{\pi}_t - \sigma \hat{\pi}_t + \hat{\pi}_t + \hat{\pi}_t + \pi_t^*) \right].\]

(108)
For a steady-state gross inflation rate of $\Pi^* = 1$, (108) simplifies to

$$\pi^s_t = \frac{(\theta - 1)}{\gamma^p(q^*)^{\theta - 1}}(\hat{w}_t - \hat{a}_t - \hat{q}_t^+) + \beta E_t[\pi^s_{t+1}].$$  (109)

Equation (46)

$$a_t n^s_t(i) = q_t^{-\theta} y_t.$$  (110)

Steady state:

$$a^* n^{s*} (i) = (q^*)^{-\theta} y^*.$$  (111)

Log-linearization yields

$$\hat{a}_t + \hat{n}_t^s(i) = -\theta \hat{q}_t + \hat{y}_t.$$  (112)

Equation (47)

$$y_t = \left(\nu(n^s_t(i))^{\frac{\theta - 1}{\theta}} + (1 - \nu)\frac{\theta}{\theta + \alpha(\theta - 1)}(An^r_t(i))^{\frac{\theta - 1}{\theta}}\right)^{\frac{\theta}{\theta - 1}} a_t.$$  (113)

Steady state:

$$y^* = \left(\nu(n^{s*}(i))^{\frac{\theta - 1}{\theta}} + (1 - \nu)\frac{\theta}{\theta + \alpha(\theta - 1)}(An^{r*}(i))^{\frac{\theta - 1}{\theta}}\right)^{\frac{\theta}{\theta - 1}} a^*.$$  (114)

Log-linearization:

$$(1 + \hat{y}_t)^{\frac{\theta - 1}{\theta}} (y^*)^{\frac{\theta - 1}{\theta}}$$

$$= \left(\nu(1 + \hat{n}_t^s(i))^{\frac{\theta - 1}{\theta}} (n^{s*}(i))^{\frac{\theta - 1}{\theta}} + (1 - \nu)\frac{\theta}{\theta + \alpha(\theta - 1)}(1 + \hat{n}_t^r(i))^{\frac{\theta - 1}{\theta}} (An^{r*}(i))^{\frac{\theta - 1}{\theta}}(1 + \hat{a}_t)^{\frac{\theta - 1}{\theta}} (a^*)^{\frac{\theta - 1}{\theta}}\right)$$

An approximation that disregards all terms of order two and higher is

$$\hat{y}_t (y^*)^{\frac{\theta - 1}{\theta}} = \left(\nu(n^{s*}(i))^{\frac{\theta - 1}{\theta}} \hat{n}_t^s(i) + (1 - \nu)\frac{\theta}{\theta + \alpha(\theta - 1)}(An^{r*}(i))^{\frac{\theta - 1}{\theta}} \hat{n}_t^r(i)\right)(a^*)^{\frac{\theta - 1}{\theta}}$$

$$+ \left(\nu(n^{s*}(i))^{\frac{\theta - 1}{\theta}} + (1 - \nu)\frac{\theta}{\theta + \alpha(\theta - 1)}(An^{r*}(i))^{\frac{\theta - 1}{\theta}}\right)\hat{a}_t (a^*)^{\frac{\theta - 1}{\theta}}.$$  (115)

Simplifying yields

$$\hat{y}_t = \hat{a}_t + \kappa_1 \hat{n}_t^s(i) + (1 - \kappa_1) \hat{n}_t^r(i),$$  (117)
where $\kappa_1 = \frac{\nu(\nu^*(i))^{\frac{\theta - 1}{\theta}}}{\nu(\nu^*(i))^{\frac{\theta - 1}{\theta}} + (1 - \nu)\frac{\theta}{\gamma(\theta -1)}(A \nu^*(i))^{\frac{\theta - 1}{\theta}}} \in (0, 1)$.

**Equation (48)**

\[
\frac{\Gamma_t}{1 - \chi_t} = h((1 + \Gamma_t)\Delta_t^l),
\]

where $(1 + \Gamma_t)\Delta_t^l = \delta_t^l$. With steady-state identity $\delta^l = (1 + \Gamma^*)\Delta^l$, the log-linearized version of $(1 + \Gamma_t)\Delta_t^l = \delta_t^l$ can be written as

\[
\hat{\delta}_t^l = \frac{\Gamma_t \hat{\Gamma}_t}{1 + \Gamma^*} + \hat{\Delta}_t^l.
\]

**Equation (32)**:

\[
\frac{\Gamma_t}{1 - \chi_t} = h(\delta_t^l) = \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} \left( \frac{1}{(\delta_t^l)^{\frac{\theta}{\gamma(\theta - 1)}}} - \delta_t^l \right) \phi^c + \delta_t^l - 1.
\]

with steady state:

\[
\frac{\Gamma^*}{1 - \chi^*} = \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} \left( \frac{1}{(\delta^l)^{\frac{\theta}{\gamma(\theta - 1)}}} - \delta^l \right) \phi^c + \delta^l - 1.
\]

Log-linearization:

\[
\frac{\Gamma^*(1 + \hat{\Gamma}_t)}{1 - \chi^*(1 + \hat{\chi}_t)} = \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} \left[ \frac{1}{(1 + \hat{\delta}_t^l)^{\frac{\theta}{\gamma(\theta - 1)}}(\delta^l)^{\frac{\theta}{\gamma(\theta - 1)}}} - (1 + \hat{\delta}_t^l)\delta^l \right] (L^*(\theta))^{\frac{1}{\gamma(\theta - 1)}} + (1 + \hat{\delta}_t^l)\delta^l - 1.
\]

Using the following equation:

\[
\frac{1}{(1 + \hat{\delta}_t^l)^{\frac{\theta}{\gamma(\theta - 1)}}(\delta^l)^{\frac{\theta}{\gamma(\theta - 1)}}} - (1 + \hat{\delta}_t^l)\delta^l = \frac{1}{(\delta^l)^{\frac{\theta}{\gamma(\theta - 1)}}} - \delta^l - \frac{\theta}{\alpha(\theta - 1)} \frac{1}{(\delta^l)^{\frac{\theta}{\gamma(\theta - 1)}}} \hat{\delta}_t^l - \delta^l \hat{\delta}_t^l - 1
\]

yields

\[
\frac{\Gamma^*(1 + \hat{\Gamma}_t)}{(1 - \chi^*)(1 - \hat{\chi}_t)} = \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} \left[ \frac{1}{(\delta^l)^{\frac{\theta}{\gamma(\theta - 1)}}} - \delta^l \right] (L^*(\theta))^{\frac{1}{\gamma(\theta - 1)}} + \delta^l - 1
\]

\[
- \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} \left( \frac{\theta}{\alpha(\theta - 1)} \frac{1}{(\delta^l)^{\frac{\theta}{\gamma(\theta - 1)}}} + \delta^l \right) (L^*(\theta))^{\frac{1}{\gamma(\theta - 1)}} \hat{\delta}_t^l + \delta^l \hat{\delta}_t^l.
\]
Dividing by the steady-state equation yields

\[ \hat{\Gamma}_t + \frac{\chi^*}{1 - \chi^*} \hat{\chi}_t = \kappa_2 \hat{\delta}_t, \quad (125) \]

where \( \kappa_2 = \frac{1 - \chi^*}{\Gamma^*} \left( \frac{\delta^* - \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)}}{\theta - 1} \right) \left( \frac{1}{\alpha(\theta - 1)} + \frac{\delta^*}{\alpha(\theta - 1)} \right) \left( L^*(\theta) \frac{1}{\alpha(\theta - 1)} \right). \)

With the help of (119), (125) can be restated as

\[ \frac{\chi^*}{1 - \chi^*} \hat{\chi}_t = \frac{(\kappa_2 - 1)\Gamma^* - 1}{1 + \Gamma^*} \hat{\Gamma}_t + \kappa_2 \hat{\delta}_t. \quad (126) \]

Rewriting (126) yields

\[ \hat{\Delta}_t = \hat{\kappa}_2 \hat{\Gamma}_t + \frac{\chi^*}{\kappa_2(1 - \chi^*)} \hat{\chi}_t, \quad (127) \]

where \( \kappa_2 > 0 \) and \( \hat{\kappa}_2 = \frac{1 - (\kappa_2 - 1)\Gamma^*}{\kappa_2(1 + \Gamma^*)} > 0 \) in our calibration.

Equation (49)

\[ \frac{1}{I_t p_t} c_t^{-\sigma} = \mathbb{E}_t \left[ \beta \frac{c_{t+1}^{-\sigma}}{p_{t+1}} \right] \quad (128) \]

is equivalent to

\[ 1 = I_t \beta \mathbb{E}_t \left[ \frac{c_t^{\sigma}}{c_{t+1}^{\sigma} \Pi_{t+1}} \right]. \quad (129) \]

Steady state:

\[ I^* = \frac{\Pi^*}{\beta}. \quad (130) \]

Log-linearization:

\[ \hat{c}_t = -\sigma^{-1} \left( \hat{I}_t - \mathbb{E}_t[\pi_{t+1}] \right) + \mathbb{E}_t[\hat{c}_{t+1}], \quad (131) \]

Equation (50)

\[ \psi(\nu n_t^s(i) + (1 - \nu)n_t^r(i)) = c_t^{-\sigma} w_t. \quad (132) \]

Steady state:

\[ \psi(\nu n^s(i) + (1 - \nu)n^r(i)) = (c^*)^{-\sigma} w^*. \quad (133) \]

\[ \hat{w}_t = \sigma \hat{c}_t = \varphi \kappa_3 \hat{n}_t^s(i) + \varphi (1 - \kappa_3) \hat{n}_t^r(i), \quad (134) \]

where \( \kappa_3 = \frac{\nu n^s(i)}{\nu n^s(i) + (1 - \nu)n^r(i)} \in (0, 1). \)

Equation (51)
\begin{equation}
y_t = c_t + \frac{1}{2} \gamma^p \nu \left( \Pi_t \frac{q_t}{q_{t-1}} - 1 \right)^2 y_t + \frac{\chi_t}{1 - \chi_t} \frac{\Gamma_t}{1 + \Gamma_t} (1 - \nu) w_t n_t^r (i) \\
\quad + \mu \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} (1 - \nu) w_t n_t^r (i) (L^*) \frac{1}{\alpha(\theta - 1)}
\end{equation}

Steady state:

\begin{equation}
y^* = c^* + \frac{1}{2} \gamma^p \nu (\Pi^* - 1)^2 y^* + \frac{\chi^*}{1 - \chi^*} \frac{\Gamma^*}{1 + \Gamma^*} (1 - \nu) w^* n^r (i) \\
\quad + \mu \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} (1 - \nu) w^* n^r (i) (L^*) \frac{1}{\alpha(\theta - 1)}
\end{equation}

Log-linearization:

\begin{equation}
y^* \frac{\hat{y}_t}{y_t} = c^* \hat{c}_t + \nu y^* \frac{\gamma^p}{2} (\Pi^* - 1)^2 \hat{y}_t + \nu y^* \gamma^p \Pi^* (\Pi^* - 1) (\pi_t + \hat{q}_t - \hat{q}_{t-1}) \\
\quad + \frac{\chi^*}{1 - \chi^*} \frac{\Gamma^*}{1 + \Gamma^*} (1 - \nu) w^* n^r (i) (\hat{w}_t + \hat{n}_t^r (i) + \frac{\chi_t}{1 - \chi_t} + \frac{\hat{\Gamma}_t}{1 + \hat{\Gamma}_t}) \\
\quad + \mu \frac{\alpha(\theta - 1)}{\theta + \alpha(\theta - 1)} (1 - \nu) w^* n^r (i) (L^*) \frac{1}{\alpha(\theta - 1)} \left( \hat{w}_t + \hat{n}_t^r (i) - \frac{\theta}{\alpha(\theta - 1)} \hat{\Delta}_t - \frac{\theta + \alpha(\theta - 1)}{\alpha(\theta - 1)} \frac{\Gamma^*}{1 + \Gamma^*} \hat{\Gamma}_t \right).
\end{equation}

Further simplification yields

\begin{equation}
\hat{y}_t = \frac{1}{1 - \nu \frac{\gamma^p}{2} (\Pi^* - 1)^2} \left( c^* \hat{c}_t + \nu y^* \gamma^p \Pi^* (\Pi^* - 1) \pi_t + (1 - c^* \frac{1}{y^*} - \frac{1}{2} \gamma^p \nu (\Pi^* - 1)^2) (\hat{w}_t + \hat{n}_t^r (i)) \\
\quad + \frac{\kappa_4}{1 - \chi^*} \Lambda_t - \kappa_5 (1 + \alpha(\theta - 1)) \frac{\Gamma^*}{1 + \Gamma^*} \hat{\Gamma}_t \right),
\end{equation}

where $\kappa_4 = \frac{(1 - \nu) \chi^*}{1 - \chi^*} \frac{\Gamma^*}{1 + \Gamma^*} \frac{w^* n^r (i)}{y^*}$ and $\kappa_5 = \frac{\mu}{\theta + \alpha(\theta - 1)} \frac{(1 - \nu) (L^*)}{y^* (L^*)} \frac{\gamma^p \Pi^* (\Pi^* - 1) \pi_t}{\alpha(\theta - 1)} \frac{1}{\alpha(\theta - 1)}$.

\section{F The Unconditional Expectation of Social Losses under Different Policy Rules}

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Figure 7: Social losses under different policy rules for large ranges of coefficients.
References


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