Systemic Risk and Network Spillovers in the European Sovereign CDS Market:

A Spatial Autoregressive Approach

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Abstract:
This paper proposes an alternative explanation for the non-linear relationship between the theoretical probability of default and observed sovereign CDS spreads: "the credit spread puzzle". Government CDS spreads in the euro area feature a time-varying pattern of comovement that constitutes a serious challenge for econometric modeling and forecasting. Standard specifications, which model spreads as a persistent mean-reverting process determined by fiscal fundamentals and market's appetite for risk, are unable to capture this pattern. This paper argues that a systemic factor based on network interlinkages between countries, has become increasingly important. The paper rationalizes the use of this new factor by developing a simple structural network model with financial cross-holdings and multiple equilibria. The theoretical framework considers a "balance sheet" type of contagion, where spillovers from a default or a severe financial shock occur via direct losses to assets held by creditors. Next, the paper shows that the theoretical network model naturally translates into a Spatial Autoregressive Model (SAR), which models the interdependence between spreads by making each sovereign's CDS spread a function of the CDS spreads of its "network neighbors". The main findings of the paper are: (1) there is evidence of significant network spillovers in CDS markets; (2) the SAR model consistently outperforms the standard model in out-of-sample prediction tests and improves forecasting accuracy by 15\% to 20\%; (3) exogenous financial shocks propagate in the network of sovereigns and 40\% to 50\% of the total effect is due to indirect (network) effects

Keywords: networks, financial contagion, CDS spreads, spatial autoregressive model

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I. Introduction

Sovereign credit default spreads (CDS) in the Euro-area feature a time-varying pattern of comovement, which constitutes a major challenge for econometric modelling and forecasting. During the recent European Sovereign Debt Crisis of 2010-2012 spreads have reached levels that cannot be explained by standard models, which typically model spreads as a persistent mean-reverting process driven by two factors: a local and a global factor. The local factor is determined by fundamentals, whereas the global factor captures risk aversion i.e. proxies for global market conditions. Predicted spreads from these models cannot match the pattern in the data and are, on average, 100 basis points lower than realized values. This empirical evidence suggests a non-linear relationship between a sovereign’s theoretical probability of default and observed credit spreads, a phenomenon dubbed as the “credit spread puzzle” (Amato & Remolona (2003); Chen, Collin-Dufresne, & Goldstein (2008); Longstaff, Pan, Pedersen, & Singleton (2011)).

Until recently, the probability of a developed country defaulting on its sovereign debt was considered to be close to zero. However, with the onset of the Global Liquidity and the subsequent European Sovereign Debt Crisis many governments had to step in and save their financial sectors, as a result of which fiscal deficits reached levels unseen since World War II. This led to a revision of credit markets and raised a discussion about the true probability of sovereign default. Credit rating agencies responded with a series of downgrades, notwithstanding developed countries. For example, Germany was the only country in Europe, which retained its AAA rating.

Furthermore, in the aftermaths of the European Sovereign Debt crisis economists, policymakers and the media have raised concerns over the different forms of contagion in the financial system. One common source of anxiety is that given the interconnectedness of the European financial systems, the default of one country would have spillover effects that would result in higher borrowing costs for other sovereigns, and potentially would trigger a series of other defaults. The recent theoretical literature on financial networks and contagion has proposed default spillovers as an important contagion channel for European sovereigns (Elliot, Golub, & Jackson, 2014). However, the magnitude of network spillovers resulting from financial
interconnectedness and their impact on CDS markets remains an open empirical question.

This paper addresses the three points outlined above by considering an extension to the standard models of credit spreads and proposing a new factor, which captures time-varying financial linkages among European sovereigns. The new systemic factor reflects important nonlinearities in CDS markets, because it allows that the credit risk of one sovereign depends not only on its own fundamentals, but also on the credit risk of the countries to which it is financially exposed. Empirically, this factor is operationalized as a country-specific weighted average of CDS spreads, where the weighting scheme is determined by financial network connections.

This work fulfills three research purposes. First, it motivates the use of the new factor by developing a network model of credit risk with asset value interdependencies in the spirit of recent theoretical models (Acemoglu, Ozdaglar, & Tahbaz-Salehi (2015); Elliot, Golub, & Jackson, 2014; Glaserman & Young (2015); Rogers & Veraart (2013)). Second, it shows that the theoretical network model empirically translates into a system of simultaneous equations and proposes to estimate the model using spatial autoregressive model (SAR). Third, the paper tests for the presence of network spillover effects in CDS markets and quantifies their importance and shows that financial shocks do propagate through sovereign networks.

The model is estimated using data on CDS spreads and financial linkages among 10 European sovereigns from 2006 to 2016. To construct the network I use data from Bank of International Settlements (BIS), which provides detailed information on bilateral lending and borrowing relationships between BIS-reporting countries. A directed link in this network exists if country $i$ holds a claim vis-à-vis country $j$ and the strength of the connection, $x_{ij}$, is given by dollar value of the outstanding debt to country $j$, divided by the total amount that country $i$ borrows from all the countries in the sample. It is important to note that this data exists on an aggregate level i.e. how much do all banks of country $i$ borrow/lend to country $j$. BIS provides a split by sector, which allows me to uncover the total amount borrowed from general governments, but not the identity of the countries. I call this amount $D_{i}^{govt}$. Finally, to obtain the link between two sovereigns I weigh $D_{i}^{govt}$ by the strength of the connection $x_{ij}$.
The theoretical framework considers “balance sheet” mechanism of contagion, where spillovers from a default or a severe financial shock occur via direct losses to assets held by creditors. Using a fixed-point argument, it is possible to show that in the presence of asset interdependencies and discontinuities in value multiple equilibrium solutions for organization’s values are possible. In this context of multiple equilibria, contagion emerges because of linkages and the joint determination of asset prices: organizations fail because people expect that other connected organizations will fail as well and this then becomes self-fulfilling.

From an empirical standpoint, the paper offers strong statistically significant evidence for the presence of credit risk spillovers in CDS markets. Introducing the systemic risk factor considerably improves in-sample model fit and explanatory power. The results indicate that network linkages account for 15 % to 20 % of the CDS variance. In out-of-sample predictive tests, the SAR model consistently outperforms standard models. The SAR model is better able to match monthly changes in CDS spreads and leads to 25 % to 35% improvement in predictive accuracy, measured in the root mean squared error (RMSE) sense. Finally, the paper shows that the constructed network of financial linkages between sovereigns is an important mechanism for the propagation of exogenous financial shocks. Using the SAR model, it is possible to decompose the total effect of financial shocks into direct and indirect effects. The results of an event study around announcement dates indicate that as much as 45% of the overall effect of shocks is due to indirect (network) effects. The findings of the paper are robust to the chosen time period, data frequency and alternative specifications of the network links.

II. Related Literature

This paper contributes to three main strands of literature. First, it is related to the literature studying contagion in financial networks. Allen and Gale (2000) and Freixas, Parigi and Rochet (2000) consider a simple interbank lending network, where liquidity shocks arise due to consumers. They study contagion of insolvencies as a

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result of one bank failing and reach the conclusion that the more connected the network is, the more robust to contagion it is. More recently, Allen, Babus and Carletti (2010) develop a model, where institutions form connections by swapping projects in order to diversify their risks. The authors find that these type of connections lead to two different network topologies: a clustered one, in which institutions hold identical portfolios and fail together, and an unclustered dispersed network. Eisenberg and Noe, (2001) look at default of firms as a part of a clearing mechanism. They develop an computationally efficient algorithm that clears the financial system and, at the same time, provides information about the systemic risk faced by the individual firms. Rogers and Veraart (2013) extend the seminal work of Eisenberg and Noe (2001) by introducing costs of default and offer a rigorous analysis of those situations in which banks will have an incentive to cooperate and bail-out other distressed banks. Elliot, Golub and Jackson (2014) study how failures cascade in a network of interconnected financial organizations and how discontinuous changes in asset value trigger further failures. Whereas Elliot, Golub and Jackson assume that the non-inflated market value of an organization is well-captured by the value of equity held by outside investors, this paper takes a different approach. Building on recent work by Barucca, et al. (2016), the market value of organizations is obtained via a valuation function that takes into consideration at the same time interdependencies and uncertainty. Furthermore, Acemoglu, Ozdaglar and Tahbaz-Salehi (2015) consider an exogenous network and derive results based on the topology of the network. On a related note, Glasserman and Young (2015) derive bounds on the magnitude of network effects of contagion.

Despite this plethora of theoretical works on financial networks and contagion, there is relatively little empirical research drawing on these models. Allen and Gale (2000), Elliot, Jackson and Golub (2014) and Glasserman (2015) suggest that their models can be applied in an empirical setting. Although both Elliot, Jackson and Golub (2000) and Glasserman (2015) use empirical data to illustrate their models, these results are intended as complementing the theoretical results and are not intended as an econometric exercise. This paper contributes to the literature by

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3 Examples are Cohen-Cole, Patachini and Zenou (2011);
applying the theoretical frameworks to a network of countries and proposing methods from *spatial econometrics* to estimate the models in an empirically rigorous manner.

Spatial autoregressive models (SAR) have been traditionally developed in regional and social sciences. Important theoretical contributions in the field include: Prucha and Kelejian (1998; 1999; 2006; 2010); Anselin and Bera (1998); Anselin, Bera, Florax, and Yoon (1996); LeSage and Pace (2006); (Arbia, 2006); Lee (2004) etc. On the empirical side, Kim, Phipps and Anselin (2003) apply a spatial approach to study the benefits of air quality improvement. In a political economy setting, (Bordignon, Cerniglia, & Revelli, 2003) how spatial dependencies affect the way property taxes are set. Furthermore, Bloningen et al. (2007) study how foreign direct investment (FDI) into a host country depends on the FDI in proximate countries.

SAR models in finance are not widespread, but have began to gather momentum in the recent years. Examples of recent applications are Ozdagli and Weber (2016), who study how monetary policy shocks propagate through the input-output production network. Fernandez (2011) derives a spatial version of the CAPM model and uses the results to perform value-at-risk simulations. This paper is closest to Blasques et al. (2016) and Eder and Keiler (2015) The former investigates a network of eight European sovereigns and their financial linkages and finds time-varying spatial dependence. However, in contrast to this paper, Blasques et al. (2015) are more concerned with the statistical properties of the model and do not provide a microfoundation for their results. The article by Eder and Keiler looks at network connections determined by asset correlations on the stock market between systemically important institutions. The paper finds strong empirical evidence of spillovers in CDS markets and concludes that around 10 % to 15 % of the CDS variance can be explained by network connections. The results of this paper are consistent with Eder and Keiler (2015). Differently from Eder and Keiler (2015), the focus here is on a network of sovereigns, the connections between which are given by cross-border borrowing and lending exposures.

Finally, this paper is related to research that studies modeling and forecasting credit spreads in the euro area. The standard specification adopted for sovereign spreads in the Euro Area models them as a persistent processs reverting to a time-varying mean explained by two factors: a local country-specific factor, related to fiscal
fundamentals, and a global factor, which measures market appetite for risk (Favero, Pagano and von Thadden (2010); Beber, Brandt and Kavajecz (2009); Laubach (2009, 2011)). Another common finding in this literature, starting with Codogno, Favero and Missale (2003) and Geier, Kossmeier and Pichler (2004) among others, is that the sovereign spread yields in the Euro area are strongly comoving. This comovement given the heterogenous liquidity of bonds issued by different countries suggests either that credit risk is dominating credit risk and that the two strongly move together (Favero, Pagano and von Thadden (2010); Beber, Brandt and Kavajecz (2009)). Credit risk should theoretically depend on fiscal fundamentals, but the empirical literature suggests that a linear relationship between the credit risk and fundamentals has been largely illusive (Attinasi, Checherita and Nickel (2010); Laubach (2009)). Finally, this paper contributes to the literature on market spillovers. Diebold and Yilmaz (2009, 2011) measure spillovers by employing variance decomposition of Vector Autoregressive models (VARs). In particular, they look at the proportion of the (conditional) variance of the returns to an asset, which is explained by the (conditional) variance of other assets. Another approach is the Global Vector Autoregressive model (GVAR) advanced by Peseran, Schuermann and Weiner (2004). The model provides a flexible reduced-form framework, which allows for accommodating time-varying co-movement between local and global country-variables. An important contribution in this field is Favero (2013), who augments the standard GVAR framework by introducing two variables. These variables define for each country a global spread, which is a weighted average of the spreads of other countries, where the weights are given by distances in the fiscal fundamentals (debt and deficit) between countries.

III. Theoretical Model

This section develops a simple input-output model with interbank cross-holdings. The network model builds on earlier work of Eisenberg and Noe (2001), Suzuki (2002) and more recently, Rogers & Veraart (2013), Elliot, Golub and Jackson (2014) and Barucca et al. (2016). The model investigates cascades of failures in a network composed of interdependent institutions and shows that discontinuous changes in the asset value of organizations trigger further failures and that this depends on the properties of the network structure. Contagion is discussed in the context of multiple
equilibria, the source of which is interdependencies of asset values: organizations fail because people *expect* that other organizations will fail as well and this then becomes *self-fulfilling*.

The equilibrium concept is investigated in the context of a process, which repeats itself over $t = 1,\ldots,T$ number of periods. Each period is treated independently and is assumed to unfold in the following steps:

**Step 1:** The financial sectors in each country are endowed with bilateral claims to foreign sovereigns, which are established in the previous period

**Step 2:** The financial sector collects deposits and equity and invests these in primitive assets (e.g. loans, equities, bonds, commodities etc.)

**Step 3:** Exogenous financial shocks are realized

**Step 4:** Valuation of claims and investments is performed: *market value* obtains

**Step 5:** If the market value hits a threshold value (exogenously given), failure occurs and default costs are incurred

**Step 6:** Claims are established for the next period

**Step 7:** CDS contracts are traded for credit events in the next period

The following sections describe in detail *Steps 1:7*.

**A. Primitive Assets, Organizations and Cross-Holdings**

Consider a financial system composed of $n$ countries making up a set $N = \{1,\ldots,n\}$. In each country, the financial sector collects deposits ($d$) and equity ($e$) and invests these in primitive assets $M = \{1,\ldots,m\}$. To fix ideas, a primitive asset may be thought of as a project that generates net cash flow over time. Let the amount of the primitive asset $k$ of bank $i$ at time $t$ be $\pi_{ikt}$ and $p_{kt}$ be its price, then $\pi_{ikt}p_{kt}$ is the book value of the primitive assets. One can think of these $k$ assets as different asset classes: e.g. loans to firms and households, equities, bonds, commodities etc., or simply as factors of production. Banks/sovereigns also lend to each other in the interbank market. Let $a_{ijt}$ is the amount that country $i$ lends to the general government of country $j$. The quantity $a_{ijt}$ represents sovereign debt claims and is of main interest in the model. Both sovereign debt and interbank lending values are determined in the previous

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*The terms “financial sector” is understood here as the collection of all BIS-reporting banks in a given country*
period. All magnitudes are expressed in monetary terms, euros, and represent \textit{book value}. The balance sheet of bank $i$ is given by:

$$d_i + e_i + a_{1i} + \cdots + a_{ni} = a_{i1} + \cdots + a_{in} + \sum_k \pi_{ik} p_k$$

(1)

It is possible to aggregate the balance sheets of all the $n$ banks and express it in matrix form:

$$d + e + A_M t_n = A'_M t_n + \Pi p$$

(2)

where $d$, $e$ and $p$ are column vectors, $t_n$ is column vector of ones, $\Pi$ is $n \times m$ matrix, $A_M$ is bilateral interbank exposure and everything is expressed in monetary terms. Let $v$ be a vector representing total book value of bank assets:

$$v = d + e + A_M t_n = A'_M t_n + \Pi p$$

(3)

Finally, sovereigns are exposed to exogenous financial shocks, which is the only source of stochasticity in the model.

\textbf{B. Introducing Market Value}

A proper valuation of country $i$, denoted by $V$ in the model, depends on how much the country values its assets. Such valuation can substantially differ from the book value of assets and depends on assets of other nodes in the model, and more precisely, on how much they value their assets. Let $T_{ij}$ denote the maturity of the debt contract between $i$ and $j$ and let $t$ denote the time at which the evaluation of the financial claim takes place. I assume that $t = T_{ij}$ for all $i$ and $j$ i.e. the evaluation takes place at maturity. Capitalizing on recent work by Barucca and co-authors (2016), I introduce the following valuation function:

\textbf{Definition 1:} Given an integer $q \leq n$, a function $\Upsilon : \mathbb{R}^q \rightarrow [0,1]$ is called feasible valuation function if and only if:

1. It is non-decreasing: $V \leq V' \Rightarrow \Upsilon(V) \leq \Upsilon(V'), \forall V, V' \in \mathbb{R}^q$
2. $I$ is continuous from above

The intuition behind the definition is that the market value of any asset can be written as the product of its book value multiplied by the valuation function. Thus, the
market value of an asset ranges from its face value to zero. I assume that the valuation depends only on banks’ asset values. If \( \mathcal{U}^{IB}(V) \) and \( \mathcal{U}(V)^{PA} \) are the valuation functions for the interbank assets and the proprietary assets respectively, then the market value of assets is given by:

\[
V(t) = A'Mt_n \mathcal{U}^{IB}(V(t)) + \Pi p \mathcal{U}^{PA}(V(t))
\]  

(4)

In the interest of readability, in the following sections the explicit dependence on the time at which the valuation is carried out will be dropped.

Let \( \hat{V} \) be the corresponding diagonal matrix such that \( \hat{V}t_n = V \). Then, the right hand side of equation (11) can be represented as:

\[
V = \hat{V}^{-1}A'M \hat{V}t_n \mathcal{U}^{IB}(V) + \Pi p \mathcal{U}^{IB}(V) = ZV \mathcal{U}^{IB}(V) + \Pi p \mathcal{U}^{PA}(V)
\]  

(5)

where \( Z = \hat{V}^{-1}A'M \) is a matrix such that each row is divided by the total assets of the lending bank. The entries of the vector \( z = Zt_n \) are fractions of unity, which give the proportion of country \( i \)'s interbank lending to its total assets. Then, the parameter \( \psi = \frac{1}{n} \sum z_{ij} \) gives the average contribution of interbank lending to the total value of the organization. This parameter is endogenously determined from the model as it depends on the face value of interbank claims and assets and their market valuations. Furthermore, since \( z \in [0,1) \), then by definition \( \psi \in [0,1) \). Rewrite equation (12) as:

\[
V = \psi WV \mathcal{U}^{IB}(V) + \Pi p \mathcal{U}^{PA}(V)
\]  

(6)

where \( W = \frac{1}{\psi}Z \).

Since all valuation functions take values in the interval \([0,1]\), then organization’s values \( V \) are bounded both from above and from below:

\[
V_{\min} \equiv 0 \leq V \leq V_{\max}
\]  

(7)

C. Introducing Discontinuities in Value and Failure Costs

An important aspect of the model is that organizations can lose value in a discontinuous way if their values hit certain critical thresholds. These discontinuities
can trigger cascading failures and multiple equilibria. There could be many sources of such discontinuities: for example, if an airline cannot pay for its fuel, then its fleet has to sit idle, which leads to a discontinuous drop in revenues because of taxes paid to the ground operators, lost bookings, failure to meet delivery obligations to courier companies and so forth. When a country’s sovereign debt is downgraded, then it experiences a discontinuous jump in the cost of capital. An increase in the borrowing cost could lead to an interruption in the ability to pay or acquire other factors of production, which could then lead to a decrease in revenue and loss of value. The particular source of discontinuity that this paper considers is an exogenous financial shock.

The model assumes that if the value of an organization \( V \) falls below some threshold \( V_0 \), then it is said to fail and incurs a failing cost of \( \gamma \) proportionate to the price of the proprietary asset and expressed as fraction of cents on the euro. The organization incurs this cost, because it needs to liquidate its asset in order to cover its liabilities. Since debt is given priority over equity in this setting, it can be assumed that organizations are always able to recover the market value of their interbank claims. Such discontinuities could easily be accommodated in the valuation function \( U(V) \) using the following rule:

1. \( U^B = 1 \)

2. \( U^A = I_{V > V_0} + (I_{V \leq V_0} - \gamma I_{V \leq V_0}) \)

The discontinuous drop imposes a loss on the organization and so its value becomes:

\[
V = \psi WV + \Pi p \left( I_{V > V_0} + (I_{V \leq V_0} - \gamma I_{V \leq V_0}) \right)
\]

(8)

Alternatively, \( V \) can be expressed as:

\[
V = (I_n - \psi W)^{-1} \Pi p \left( I_{V > V_0} + (I_{V \leq V_0} - \gamma I_{V \leq V_0}) \right) = \mathcal{A} \Pi p \left( I_{V > V_0} + (I_{V \leq V_0} - \gamma I_{V \leq V_0}) \right)
\]

(9)

The matrix \( \mathcal{A} \) determines how the costs of a failing organization \( j \) are distributed among other organizations in the system and how this affects their value. Note that the dependency matrix \( \mathcal{A} \) is not row-stochastic, which is not surprising. In this model, there are two types of assets: interbank claims, expressed as fractions of total value,
and proprietary assets. The entries of $\mathcal{A}$ describe total value as a fraction of the proprietary asset and, so its rows sum up to more than unity. A final caveat of the model regards the matrix $\mathbb{I}_n - \psi W$. In order to show that it is not singular and that the spatial multiplier $(\mathbb{I}_n - \psi W)^{-1}$ exists, I use Lemma 1.

**Lemma 1:**

If $Z$ is a matrix with $\|Z\| < 1$, where $\|Z\|_1 = \sup_x = \frac{\|Zx\|_1}{\|x\|_1}$, then $\mathbb{I}_n - Z$ is invertible and

$$(\mathbb{I}_n - Z)^{-1} = \sum_{k=1}^{\infty} Z^k$$

Given that $\|\psi W\| = \|Z\|$ and that the entries of the matrix $Z$ are all fractions of unity, the conditions in Lemma 1 are fulfilled and the matrix is invertible. Appendix A offers a simple proof of the lemma.

**D. Relationship to Leontief’s Input-Output Model**

Equation (13) is reminiscent of the famous input-output analysis of Leontief (1951). In the traditional application, each firm or industry sector $i$ uses $z_{ij}$ units of output from sector $j$ ($j = 1, ..., n$), labor and other units of primary inputs to produce one unit of final output. Here, each bank borrows, i.e. “uses” funds from other banks in the model, deposits and equities and invests these in assets (e.g. production factors

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5 For example, if no organization fails, $V = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and the value of the interbank asset is $\psi WV = \\
\begin{pmatrix} 0 & 0.5 \\ 0.5 & 0 \end{pmatrix} V$, then the value of the proprietary asset as a fraction of total value is $\Pi p = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix} V$.

Finally, $V = \mathcal{A} \Pi p = \begin{pmatrix} 1.33 & 0.67 \\ 0.67 & 1.33 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} V = \begin{pmatrix} 0.67 & 0.33 \\ 0.33 & 0.67 \end{pmatrix} (1) = (1)$. The obtained matrix $\begin{pmatrix} 0.67 & 0.33 \\ 0.33 & 0.67 \end{pmatrix}$ is row-stochastic, which means that the values of the underlying assets sum up to the values of the organizations. Hence, no value is lost and no value is created.

6 In practice, in order to ensure that $\mathbb{I}_n - \psi W$ is not singular and that the spatial multiplier $(\mathbb{I}_n - \psi W)^{-1}$ exists, the model imposes that the dependency matrix $W$ is normalized to a row-stochastic matrix $W$. Kelejian and Prucha (2004; 2010) discuss regularity conditions for spatial models and show that for a row-stochastic spatial weights matrix and for a spatial autoregressive parameter belonging to the interval $(-1, 1)$, the spatial multiplier exists and is well defined.
and/or loans). If the $z_{ij}$ are relatively constant, then the relation between $V$ and the fundamental asset is given by the well-known Leontief Inverse $\mathcal{A} = (I_n - \psi W)^{-1}$.

It is interesting to notice that the literature on social networking interprets the vector $b = (I_n - \psi W)^{-1}e_n$ as the centrality of a node (Katz, (1953); Bonacich, (1987)). The vector $b$ is also known as Katz-Bonacich centrality, and in the context here, measures the number of direct and indirect connections that a country in the network of bilateral exposures has and the parameter $\psi$ reflects a discount factor, which assigns less influence to distant nodes.

### E. Equilibrium Existence and Multiplicity

A solution for the values of organizations in equation (16) is an *equilibrium set of values* that takes into consideration dependencies between countries. Invoking Tarski’s fixed point theorem (Tarski, 1955), it is possible to show that there always exists a solution to the problem in (16), and moreover, that there is a least and a greatest solution. In fact, the set of solutions forms a complete lattice, which follows from the fact that failures are strategic complements. *Appendix A* offers a simple proof of the result.

The presence of discontinuities and equilibrium multiplicity can come from two distinct sources. The first type of discontinuity arises when the failure of organization $i$ is caused by a drop in the value of its underlying assets. The second type of discontinuity is triggered when another organization $j$, in which $i$ holds claims, hits the failure threshold: the value of $i$'s assets drops discontinuously, and so does its total value. Consequently, these two sources of discontinuity result in two different types of multiplicities of equilibria. The first is when the values of other organizations and their assets are taken as given: there could be multiple values of organization $i$ consistent with equation (16). There may exist a value $V_i$ that solves equation (16) such that $i$ does not fail and another value $V_i$ satisfying equation (16) such that $i$ fails. This mechanism generates the type of multiple equilibria corresponding to the classical models of self-fulfilling bank runs (e.g. Dybvig and Dybvig, (1983)). The second type of multiple equilibria arises due to interdependencies between organizations: the value of $i$ depends on the value of $j$ and vice versa. There might be two consistent values for $i$ and $j$: one in which both fail, and another one in which none
fail. This source of equilibrium multiplicity is distinct from the bank run story, as in this case organizations fail because people expect that other organizations fail, which then becomes self-fulfilling.

F. A Simple Microfoundation

Consider a network composed of two countries only: country 1 and country 2. The dependence relation between the two countries is given by the matrix $W$, which is assumed to be for simplicity row-stochastic:

$$W = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

The two countries in the model lend each other 1 million. Let $\psi = 0.5$. Then according to Lemma 1 $(I_n - \psi W)^{-1}$ exists and is equal to:

$$(I_n - \psi W)^{-1} = \begin{pmatrix} 1.33 & 0.67 \\ 0.67 & 1.33 \end{pmatrix}$$

If the assumption is made that the fundamental asset that each country owns is its fiscal stream, then by exchanging cross-holdings, countries obtain holdings whose value depends not only on the value of their own fiscal stream, but also the fiscal stream of other countries. Thus, $m = n$ and $\Pi = I$. Finally, assume that countries fail if their value falls below 50 and that in this case they incur a cost of 50$^7$.

Given the dependence relation, the conditions for a country failing are given by:

**Country 1:**

$$1.33p_1 + 0.67p_2 < 50$$

**Country 2:**

$$0.67p_1 + 1.33p_2 < 50$$

These inequalities define two failure frontiers $FF_1$ and $FF_2$ and four regions (only Country 1 Fails, only Country 2 Fails, Both fail, None Fail), which are graphically illustrated in Figure 1 Panel A. In this situation, the prices for which one of the four possible scenarios occur are uniquely determined.

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$^7$ This means that $py = 50$
It is interesting to discuss what happens to the failure frontier of Country 1, conditional on Country 2 failing and vice versa. In this case the conditions for failing are given by:

**Country 1:**
\[1.33p_1 + 0.67(p_2 - 50) < 50\]

**Country 2:**
\[0.67(p_1 - 50) + 1.33p_2 < 50\]

The two new *conditional* failure frontiers \(FF_1'\) and \(FF_2'\) identify a region of multiple equilibria given by the shaded area in Panel B. Multiplicities arise here because the value of Country 1 decreases discontinuously when Country 2 hits the failure threshold and defaults and the value of Country 2 decreases discontinuously when Country 1 fails. It then becomes consistent for both Country 1 and Country 2 *not* to fail, in which case the relevant frontiers are the unconditional ones. However, it is also consistent for both Country 1 and Country 2 *to fail*, in which case the relevant frontiers are the conditional frontiers. This mechanism is of particular importance from a policy-making perspective because for the same price of the fundamental asset, a “good” and a “bad” equilibrium are possible.

**Figure 1: Conditional and Unconditional Failure Frontiers**

As discussed above, the source of multiplicities is interdependencies of asset values. Organizations fail because people expect that other connected organizations fail, and this then becomes self-fulfilling.
To summarize, this paper considers a network model with inter-related asset values. Default occurs when the value of a sovereign’s assets hits a critical threshold. In this framework contagion is understood as defaults or other significant losses transmitted via the network of financial linkages. This is a direct, “balance sheet” type of contagion. To be clear, in order to estimate the structural model of spillovers in financial networks outlined in this section, this paper applies a static equilibrium concept on repeated observations within a fixed set of countries. In order to treat each period independently, the paper ignores any dynamic aspect of decision making. Additionally, unobserved shocks are assumed to be independent over the time period. As in Denbee, et al. (2014), this paper treats financial linkages as exogenous if the network is determined by actions in previous periods.

This paper proposes a novel econometric technique to quantify spillover effects in financial networks. The particular choice of an empirical strategy is directly motivated by the solution to the structural model outlined in Section III. Recall that the first-order condition writes as:

\[ V = \psi WV + \Pi p \left( I_{V > V} + (I_{V \leq V} - \gamma I_{V \leq V}) \right) \]

It is immediate to observe that the equation above has exactly the form of a *spatial autoregression*. The next chapter introduces in detail the empirical framework.

### IV. Empirical Framework

#### A. Spatial Autoregressions

I use methods from spatial econometrics to decompose the overall reaction of CDS spreads into a direct effect and higher-order effects, due to spillovers transmitted via connections in the financial system.

The spatial autoregressive model (SAR) is given by:

\[ y = \rho W y + \beta X + \varepsilon \quad (10) \]

---

8 By applying a static equilibrium concept, the paper follows the existing literature. See Denbee et al. (2014); Cohen-Cole, Patachini and Zenou (2011); Bonaldi, Hortacsu and Kastl (2014)
with data-generating process:

\[ y = (\mathbb{I}_n - \rho W')^{-1}(\beta X + \varepsilon) \]

\[ \varepsilon \sim N(0, \sigma^2 \mathbb{I}_n) \]

where \( y \) is a vector of CDS spreads, \( X \) is a vector of controls and \( W \) is a row-normalized spatial weights matrix. In my application \( W \) corresponds to a cross-holdings matrix, which gives the consolidated foreign claims of banks from one country on the debt obligations of the general government of another country. All entries on the main diagonal of \( W \) are zero, because I rule out dependence of an observation on its own value. The spatial parameter \( \rho \) indicates the relevance of a country’s connectedness for its probability of default and in this sense can be regarded as a measure for network spillover effects. Testing for the presence of spillovers is tantamount to the following hypotheses:

\[ H_0: \rho = 0 \]

\[ H_1: \rho \neq 0 \]

Here, I assume that \( \text{abs}(\rho) < 1 \). I use the term \( N(0, \sigma^2 \mathbb{I}_n) \) to denote a zero mean disturbance process with constant variance \( \sigma^2 \) and zero covariance between the observations. This results in a diagonal variance-covariance matrix \( \sigma^2 \mathbb{I}_n \) with \( \mathbb{I}_n \) representing an \( n \times n \)-dimensional identity matrix. The term \( W'y \) is called a spatial lag and is constructed as a linear combination of neighboring values to each observation.

**Example 1:** Consider the following dependence matrix \( C \):

\[ C = \begin{pmatrix}
0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 \\
1 & 0 & 0 & 0
\end{pmatrix} \]

In this case, there are four countries, the column gives the country whose debt is being held and the row is the country, which holds the debt obligation. Where there exists a dependence the value of the debt is 1 million. The dependence relationship reads like this: country 1 lends to country 2, 3 and 4; country 2 lends only to country 4; country 3 lends to country 1 and 2; country 4 lends to country 1. In order to form a spatial lag
from neighboring observations, we can normalize $C$, such that its rows sum up to unity. This results in a row-stochastic matrix, which I label $W$:

$$W = \begin{pmatrix} 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1 \\ 1/2 & 1/2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Having defined the spatial weights matrix $W$, it is easy to see that the spatial lag $Wy$ corresponds to:

$$Wy = \begin{pmatrix} 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1 \\ 1/2 & 1/2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} (y_2 + y_3 + y_4)/3 \\ y_4 \\ (y_1 + y_2)/2 \\ y_1 \end{pmatrix}$$

The presence of spatial lags of the dependent variable renders the OLS parameter estimates and standard errors inconsistent. On the other hand, maximum likelihood is consistent for the spatial autoregressive model in (10) (Lee, 2004).

**B. Spatial Autoregressions: Parameter Interpretation**

Parameter estimates in a linear regression have a straightforward interpretation as partial derivatives of the dependent variable with respect to the independent variable. This arises because of assumed linearity and independence of the observations in the model. When the model contains spatial lags of the dependent variable, interpretation of the parameters becomes more complicated, because the model incorporates a richer information set. In a spatial context, a change in any given explanatory variable (e.g. GDP) will have an impact on the CDS spread of the country itself (direct effect) and potentially an impact of the CDS spreads of other related countries (indirect effect). In fact, a large number of researchers have argued that spatial autoregressive models require special attention to the interpretation of the parameters (Anselin & LeGallo (2006); Kelejian, Tavlas and Hondronyiannis (2006); Kim, Phipps and Anselin (2003); Le Gallo, Ertur and Baumont (2003)).

To see more clearly the complication of parameter interpretation, rewrite the model in (10) as:
\[(\mathbb{I}_n - \rho W) y = \beta X + \varepsilon\]

\[y = S(W)X + V(W)\varepsilon\]

where

\[S(W) = V(W) \mathbb{I}_n \beta\] (11)

\[V(W) = (\mathbb{I}_n - \rho W)^{-1} = \mathbb{I}_n + \rho W + \rho^2 W^2 + \cdots\] (12)

To illustrate the point, consider again the previous example with four countries, but for simplicity assume that there is only one covariate: fiscal deficit. The data-generating process can be expanded to the following:

\[
\begin{pmatrix}
  y_1 \\
  y_2 \\
  y_3 \\
  y_4
\end{pmatrix} =
\begin{pmatrix}
  S(W)_{11} & S(W)_{12} & S(W)_{13} & S(W)_{14} \\
  S(W)_{21} & S(W)_{22} & S(W)_{23} & S(W)_{24} \\
  S(W)_{31} & S(W)_{32} & S(W)_{33} & S(W)_{34} \\
  S(W)_{41} & S(W)_{42} & S(W)_{43} & S(W)_{44}
\end{pmatrix}
\begin{pmatrix}
  X_1 \\
  X_2 \\
  X_3 \\
  X_4
\end{pmatrix} + V(W)\varepsilon
\]

with \(S(W)_{ij}\) indicating the \(ij^{th}\) element of the matrix \(S(W)\). Focusing on country 1, the following obtains:

\[y_1 = S(W)_{11}X_1 + S(W)_{12}X_2 + S(W)_{13}X_3 + S(W)_{14}X_4 + V(W)_1\varepsilon\] (13)

with \(V(W)_1\) referring to the first row of the matrix \(V(W)\). An immediate implication from equation (13) is that the CDS spread of country 1 depends not only on changes in its own fiscal deficit, but also on the fiscal deficit of other countries it is connected to. In this sense, \(S(W)_{11}\) gives the reaction of country 1’s CDS spread to a change in its own fiscal deficit. Similarly, \(S(W)_{12}\) denotes the reaction of country 1’s CDS spread to a change in the fiscal deficit of country 2. Therefore, \(S(W)_{11}\) gives the direct effect of the shock and \(S(W)_{12}, S(W)_{13}\) and \(S(W)_{14}\) give the indirect effect due to country 1’s exposure to countries 2, 3 and 4 through the network of debt cross-holdings.

The response of a country’s CDS spread to changes in \(X\) is determined by the cross-holdings matrix \(W\) through its effect on liquidity provision, the spatial autoregressive parameter \(\rho\), which denotes the strength of the network spillover effects, and the parameter \(\beta\). The own derivative of \(y\) with respect to \(X\) results in \(S(W)_{ii}\) and measures the direct effect. These elements are located on the diagonal of the matrix \(S(W)\). It is
important to note that this impact takes into consideration feedback effects, where observation \( i \) affects observation \( j \) and \( j \) affects \( i \), as well as longer paths i.e. from \( i \) to \( j \) to \( k \) and back to \( i \). The off-diagonal elements of \( S(W) \) represent indirect effects.

Following LeSage and Pace (2006), it is possible to define three scalars, which summarize the total, direct and indirect effects:

1. **Average Direct Impact**: the average of the diagonal elements of \( S(W) \), which equals \( \frac{1}{n} \text{tr}(S(W)) \) with \( \text{tr} \) being the trace of a matrix.

2. **Average Total Impact from an Observation**: the sum down the \( j \)th column of \( S(W) \) gives the impact on all \( y \) as a result of changing the credit rating variable by an amount in the \( j \)th observation (e.g. Greece’s rating going from A to A-).

   There are \( N \) of these sums given by the row vector \( r = \iota_n'S(W) \), where \( \iota_n' \) is a vector of ones. The average of these effects is equal to \( \frac{1}{n}r\iota_n \).

3. **Average Indirect Effect**: the difference between average total impact and average direct impact.

LeSage and Pace (2009) show that for the SAR model (1) with a row-stochastic matrix \( W \) and \( \text{abs}(\rho) < 1 \), the summary measure of total impacts, \( \frac{1}{n}t_n'S(W)t_n \), takes the simple form of:

\[
\frac{1}{n}t_n'S(W)t_n = \frac{1}{n}t_n'S(W)(I_n - \rho W)^{-1} \beta t_n = (1 - \rho)^{-1} \beta \tag{14}
\]

It is computationally inefficient to calculate the summary measures defined in (i)-(iii), because this would require inverting the \( n \times n \) matrix \( (I_n - \rho W) \) in \( S(W) \). It is possible to approximate the infinite expansion in (12) using traces of the powers of \( W \). In this case, the highest power considered has to be large enough in order to ensure that there is approximate convergence.

In order to make inferences about the statistical significance of the direct and indirect effects of changing the explanatory variables, the distribution of the scalars (i)-(iii) is required. I produce the empirical distribution of the parameters \( \beta, \rho, \sigma \) using a Bayesian Markov Chain Monte Carlo (MCMC) estimation method proposed by LeSage (1997). The idea is that since MCMC yields draws from the posterior distribution of the model parameters, these then can be used in (i) and (ii) to generate
the posterior distribution of the summary measures. Importantly, Gelfand & Smith (1990) show that MCMC yields valid inference in the case of a non-linear function of the parameters, such as (i) and (ii). The only requirement is the evaluation and storage of non-linear combinations of parameter values.

The SAR model (10) allows for a differential impact of a change in the explanatory variable depending on the order of the neighbors. Approximating the infinite series expansion of \((I_n - \rho W)^{-1}\) using the first \(q\) powers of \(W\), it is possible to represent \(S(W)\):

\[
S(W) \approx (I_n + \rho W + \rho^2 W^2 + \ldots + \rho^q W^q) \beta
\]  

(15)

Such a representation allows to the study the effect associated with each power of \(W\). The powers in equation (15) correspond to observations themselves (zero-order), immediate neighbors (first-order), neighbors of neighbors (second-order) etc. Consider again Example 1: the first-order neighbor of country 4 is country 1. The second-order neighbors are neighbors to the first-order neighbors. In this sense second-order neighbors to country 4 are the first-order neighbors to country 1: i.e. country 2, 3 and 4.

\[
W^1 = \begin{pmatrix} 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1 \\ 1/2 & 1/2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, W^2 = \begin{pmatrix} 0.50 & 0.167 & 0 & 0.333 \\ 1 & 0 & 0 & 0 \\ 0 & 0.167 & 0.167 & 0.667 \\ 0 & 0.333 & 0.333 & 0.333 \end{pmatrix}
\]

In the case of \(W^2\), positive elements appear on the diagonal. This is so, because, for example, country 4 is a second-order neighbor to itself. This is not surprising because by definition of a second-order neighbor, country 1 is a neighbor to its neighbor. Importantly, given that \(\text{abs}(\rho) < 1\), the data-generating process assigns decreasing influence to higher-order neighbors, where the decay declines geometrically as the order increases. If \(\rho = 0.5\), then \(\rho^2 = 0.25\), \(\rho^3 = 0.125\) etc. Stronger dependence reflects bigger values for \(\rho\), which on its term means that more importance will be assigned to distant neighbors (higher-order).

C. Example

To illustrate how the SAR model operates in the context of the theoretical network model, consider again Example 1 and the matrix \(W\). Recall that:
\[ V = \psi W V + \Pi p \left( I_{V > \underline{V}} + (I_{V \leq \underline{V}} - \gamma I_{V \leq \underline{V}}) \right) \leftrightarrow (I_n - \psi W)^{-1} \Pi p \left( I_{V > \underline{V}} + (I_{V \leq \underline{V}} - \gamma I_{V \leq \underline{V}}) \right) \]

Consider the extreme case, in which country 1’s value falls below the payment obligation: country 1 fails and all creditor countries are rationed in proportion to \( V_1 \) with countries 3 and 4 claiming \( w_{31} V_1 \) and \( w_{41} V_1 \) respectively of its value. For simplicity, I assume that all creditors are of equal seniority. If country 1 cannot meet its debt obligations, it is forced to liquidate its fundamental asset, i.e. its fiscal stream, at a cost \( \gamma_1 = 0.10 \). Let \( p_1 = p_2 = p_3 = p_4 = 10 \) and let us assume for the moment that countries 2, 3 and 4 are solvent and so \( \gamma_2 = \gamma_3 = \gamma_4 = 0 \). Countries have only one fundamental asset, hence \( \Pi = I \) and \( p\Pi = \vec{p} \). Further, assume \( \psi = 0.5 \). Equation (14) rewrites:

\[
\begin{pmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4
\end{pmatrix} =
\begin{pmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34} \\
a_{41} & a_{42} & a_{43} & a_{44}
\end{pmatrix}
\begin{pmatrix}
p_1 \\
p_2 \\
p_3 \\
p_4
\end{pmatrix}
\begin{pmatrix}
0 \\
1 \\
1 \\
1
\end{pmatrix} =
\begin{pmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34} \\
a_{41} & a_{42} & a_{43} & a_{44}
\end{pmatrix}
\begin{pmatrix}
0.1 \\
0 \\
0 \\
0
\end{pmatrix}
\begin{pmatrix}
9 \\
10 \\
10 \\
10
\end{pmatrix}
\]

The immediate loss in asset value equals \( p_1 \gamma_1 = 1 \). Using the infinite series approximation from equation (12), it is possible to study how the costs are shared and transmitted across the network. The direct effect here is given by \( I_n p\gamma \) and equals -1: this is the failure cost that country 1 pays. However, this is not the end of the story: column 1 of \( \psi W \) gives the effect of a failure of country 1 on the countries that hold its debt; here, these are country 3 and 4. The incurred cost for country 3 is \( \psi w_{31}^1 \times (-1) = 0.5 \times 0.5 \times (-1) = -0.25 \text{ million} \) and the cost for country 4 is \( \psi w_{41}^1 \times (-1) = 0.5 \times 1 \times (-1) = -0.5 \text{ million} \). The second-order neighbors of country 1 are country 1 itself and country 2. Hence, the second-order effects of the shock are \( \psi^2 w_{21}^1 \times (-1) = -0.125 \text{ million} \) for country 1 and \( \psi^2 w_{21}^2 \times (-1) = -0.25 \text{ for country 2} \). Please notice that if a linear model without dependence relationships was assumed the only cost incurred would be given by \( I_n p\gamma \). This would be largely imprecise and misleading: e.g. due to feedback effects, the costs for country 1 are amplified and it incurs second-order costs.

---

9 Country 1 borrows from Country 3 and 4; Country 3 borrows from country 1 and Country 4 borrows from Country 1 and 2. The second-order neighbors of Country 1 are the first-order neighbors of its immediate neighbors i.e. Country 1 and Country 2.
of substantial magnitude. The spatial autoregressive model relaxes the independence assumption and allows to study network spillover effects in an easy and intuitive way.

V. Data

A. CDS Spreads:

In this paper financial health of the sovereign is proxied by CDS spreads. CDS spreads offer a hedge against credit risk, in which the protection sellers agrees to compensate the buyer if the underlying defaults before the contract matures. The fee, which the seller charges, is paid up to end of the contract or until the buyer defaults. This fee is denoted as a CDS spread and is usually quoted in basis points. The way CDS contracts are designed makes them a suitable proxy to assess the probability of default of the borrower. Another advantage of CDS spreads is that they are market-based instruments. As such, they are forward-looking and any price changes today reflect anticipated future performance.

Data for 10 euro-zone sovereigns is collected from Credit Market Analytics (CMA) for the period 2006-2016. The sovereigns included in the sample are: Austria, Belgium, France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal and Spain. All spreads are on 5-year contracts and are used in the analysis either on a monthly or on a daily basis. Since Greek CDS spreads are flat for a large part of the sample (2010-2012), for the main part of the analysis the paper works with 9 countries and uses a 9x9 weight matrix. Greek spreads play a crucial role in the section on predictive regressions.

B. Financial Linkages

BIS reports consolidated asset holdings of the financial sector vis-à-vis entities in other countries at quarterly frequency. This measure includes all financial assets held by the financial sector and offers a breakdown according to the country that issues the claim. This information is contained in Table 9B of the Quarterly BIS Bulletin. A directed link in this network exists if country $i$ holds a claim vis-à-vis country $j$ and the strength of the connection, $x_{ij}$, is given by dollar value of the outstanding debt to country $j$, divided by the total amount that country $i$ borrows from all the countries in the sample. The problem is that Table 9B reports all financial claims, not only
sovereign debt. On the other hand, Table 4B provides a breakdown on a country level by sector of the counterparty: banks, public sector\(^{10}\), non-bank and private sector. According to the definition by BIS, international public sector claims refer to “claims to the general government”, which matches the empirical purpose of this paper. Table 4B gives the amount of sovereign debt held abroad, but it does not provide the nationalities of the foreign creditors. I call this amount \(D_i^\text{Gvmt}\). Finally, to obtain the link between two sovereigns I weigh \(D_i^\text{Gvmt}\) by the strength of the connection \(x_{ij}\). These weighted directed links are collected in the matrix \(W\), which is the main input into the SAR model. Appendix B1 gives an example of how the matrix is constructed.

**C. Identification of Financial Shocks**

In order to identify exogenous financial shocks, this paper focuses on the US financial market and uses two approaches. The first approach is to conduct an event study around announcement dates of major US financial events. In particular, changes in daily CDS spreads in the interval \([t - \Delta t^- , t + \Delta t^+]\) around even time \(t\) are examined. The use of daily data and a tight event window allows to minimize contamination problems, which might bias the results. Going through newspaper articles and press releases, 18 events are identified (e.g. collapse of Lehman Brothers, etc.). Appendix B2 provides a detailed list of these events. However, one problem with this approach is that most of these events occur in 2007-2008, which leaves a considerable part of the data on CDS spreads unused. An alternative approach is to use financial shocks, based on a financial conditions index.

In this paper I use data on the National Financial Conditions Index (NFCI), which is published by the Federal Reserve Bank of Chicago. This index provides a comprehensive weekly update on US financial conditions in money markets, debt markets, equity markets and the traditional and “shadow” banking systems. The NFCI is published every week at 08:30 a.m. ET on Wednesday, and reflect information for the time period through Friday. To construct financial shocks, innovations of the NFCI are taken and values between announcement days are linearly interpolated.

\(^{10}\) In more recent reports the term “public sector” is substituted with the term “official sector”
D. Other data

This section lists all other variables used as controls in regression models:

**Fundamentals:**

- **GDP, Debt-to-GDP, Fiscal Deficit:** measured at a quarterly frequency; seasonally adjusted. Values are linearly interpolated to match the frequency of CDS spreads data. Data is obtained from Datastream

**Liquidity:**

- **MRO:** the interest rate on Main Refinancing Operations in the euro-zone, which is set by the Governing Council of the European Central Bank (ECB). MROs provide the bulk of liquidity to the banking system. Updates are made on a monthly basis following regular meetings of the Governing Council. Data is obtained from the ECB Data Warehouse

**Market conditions:**

- **OIS:** Overnight Indexed Swap, where the floating payment is chained to the Eonia rate. Available at daily frequency. Data is obtained from the ECB Data Warehouse
- **LIBOR:** London Interbank Offered Rate for short-term loans. Available at daily frequency. Data is obtained from Datastream. LIBOR carries more risk than the “risk-free” OIS rate.
- **LIBOR-OIS:** the difference between OIS and LIBOR rates
- **Eonia:** Euro OverNight Index Average. This is the 1-day interbank interest rate for the euro-zone. Available at daily frequency. Data is obtained from Datastream.

**Risk aversion:**

- **Baa-Aaa Spread:** the yield spread of Moody’s Baa corporate bonds over Moody’s Aaa bonds. A wide spread indicates worsening economic conditions. This occurs because more investors switch to safer Aaa bonds, which pushes down the yield. The money flowing into Aaa bonds typically comes from lower-
rated Baa bonds, which at the same time increases the rate of Baa bonds. As economic conditions improve, more investors invest into low-rated bonds, which narrows the spread. For example, the highest value of the Baa-Aaa spread in the sample is 3.50 and it obtains on December 3rd 2008. Its lowest value is 0.53 and it obtains on the June 13th 2014. In this paper it is used as a measure of global risk aversion. Data is daily and comes from Federal Reserve Bank St. Louis.

- **VSTOXX**: Measures the implied 30-day volatility of the EURO STOXX 50. It reflects investor sentiment and overall economic uncertainty in Europe. It is used as a measure of local risk-aversion. Data is daily and comes from Datastream.

**VI. Empirical Results**

**A. Summary Statistics**

In line with prior empirical evidence, CDS spreads of euro-zone sovereigns exhibit significant degree of comovement throughout the sample (Figure 2). Prior to the financial crisis, spreads of all countries move closely together. During the peak of the Sovereign Debt Crisis, two groups of countries are noticeable: central countries (France, Germany, Belgium, Austria and the Netherlands) with low CDS spreads and peripheral countries (Portugal, Ireland, Italy and Greece) with high CDS spreads.

Table 1 shows summary statistics of CDS spreads over the sample period from 2006-2016. The results are in line with Figure 2. Countries such as Austria, France, Germany and the Netherlands have mean spreads between 30 and 50 basis points, whereas the spreads of Ireland, Italy, Portugal and Spain exhibit spreads that are on average 5 to 10 times bigger. Germany has the lowest mean and lowest standard deviation. In fact, Germany was the only country, which retained its AAA rating during the Sovereign Debt Crisis of 2010-2012. For this reason, it is reasonable to use German CDS spreads as a proxy for the risk-free asset. In all subsequent sessions, any reference to spreads should be understood to be on an *adjusted* basis i.e. the spread over Germany.
Figure 2: CDS spreads of Euro-zone Countries

Table 1: Summary Statistics of CDS Spreads

<table>
<thead>
<tr>
<th>Country</th>
<th>Mean</th>
<th>Min</th>
<th>Max</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>55.81</td>
<td>1.4</td>
<td>255.81</td>
<td>51.90</td>
</tr>
<tr>
<td>Belgium</td>
<td>76.51</td>
<td>1.7</td>
<td>307.41</td>
<td>73.02</td>
</tr>
<tr>
<td>France</td>
<td>57.08</td>
<td>1.5</td>
<td>214.86</td>
<td>49.16</td>
</tr>
<tr>
<td>Germany</td>
<td>30.58</td>
<td>1.5</td>
<td>114.35</td>
<td>25.11</td>
</tr>
<tr>
<td>Greece</td>
<td>10504.24</td>
<td>4.5</td>
<td>37030.49</td>
<td>15899.13</td>
</tr>
<tr>
<td>Ireland</td>
<td>202.50</td>
<td>2</td>
<td>866.19</td>
<td>232.82</td>
</tr>
<tr>
<td>Italy</td>
<td>158.85</td>
<td>5.6</td>
<td>563.40</td>
<td>125.92</td>
</tr>
<tr>
<td>The Netherlands</td>
<td>38.53</td>
<td>1.3</td>
<td>128.27</td>
<td>30.55</td>
</tr>
<tr>
<td>Portugal</td>
<td>304.35</td>
<td>3.4</td>
<td>1471.74</td>
<td>323.74</td>
</tr>
<tr>
<td>Spain</td>
<td>152.34</td>
<td>2.5</td>
<td>595.93</td>
<td>133.00</td>
</tr>
</tbody>
</table>
B. Unit Root Tests

If the variables in the spatial regressions are not stationary, then standard assumptions on asymptotic behavior and inference are not valid. Table 2 reports the results of panel unit root tests based on Augmented Dickey-Fuller (ADF) regressions. The value of the test-statistic for $CDS$ suggests the presence of a unit root. However, the adjusted measure, $CDSadj$ is stationary with p-value close to zero. For the rest of the variables, with the exception of $Debt$, $MRO$ and $Eonia$, the null hypothesis of a non-stationarity is rejected. Therefore, regression results, where these three variables are included should be interpreted with caution.

Table 2: Unit Root Tests

The Table reports results of Panel Unit Root Tests (Levin-Lin-Chu). Values of the Augmented Dickey-Fuller (ADF) statistic and p-value are reported. ADF regressions are performed with one lag.

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF Statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CDS$</td>
<td>17.34</td>
<td>0.63</td>
</tr>
<tr>
<td>$CDSadj$</td>
<td>60.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$GDP$</td>
<td>-1.66</td>
<td>0.04</td>
</tr>
<tr>
<td>$Debt$</td>
<td>-1.21</td>
<td>0.11</td>
</tr>
<tr>
<td>$Deficit$</td>
<td>-5.75</td>
<td>0.00</td>
</tr>
<tr>
<td>spread</td>
<td>-3.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$VSTOXX$</td>
<td>-3.17</td>
<td>0.00</td>
</tr>
<tr>
<td>$MRO$</td>
<td>-0.68</td>
<td>0.77</td>
</tr>
<tr>
<td>$Libor-OIS$</td>
<td>-2.82</td>
<td>0.05</td>
</tr>
<tr>
<td>$Eonia$</td>
<td>-0.77</td>
<td>0.82</td>
</tr>
<tr>
<td>$STOXX$</td>
<td>-1.35</td>
<td>0.08</td>
</tr>
</tbody>
</table>
C. Spatial Dependence

A useful visual tool for exploratory spatial analysis is the Moran Scatter Plot (Anselin, 1996). It allows to assess how similar an observed value is to its neighboring observations. The horizontal axis gives the values of the observations and is called the response axis. The vertical axis is based on the weighted average or spatial lag of the corresponding observations on the response axis. On Figure 3, the horizontal axis gives monthly changes in CDS spreads, the vertical axis gives the spatial lag, where the weighting scheme is defined based on financial linkages. In this particular example, data from 2005 Q3 is used to construct the spatial lag. Two observations are immediate. First, if the spatial weights matrix did not contain any relevant information, observations would be randomly scattered in the plot. This is not the case and data appears to be centered. Second, the majority of the data points are concentrated in Quadrant I and in Quadrant III. High values surrounded by high neighboring values are located Quadrant I and low values surrounded by low values are located in Quadrant III. Together, these two quadrants account for roughly 75% of the data points. These results are intuitive: countries that are connected to countries with high credit risk are riskier themselves and this is reflected in a higher CDS spread. Thus, Moran Scatter Plot offers preliminary evidence for the relevance of spatial dependence for credit risk.
D. In-sample Results

Table 3 compares the in-sample performance of a standard model of CDS spreads (columns (1)-(2)) and the SAR model (columns (3)-(5)). The specification of the standard model is motivated by the existing literature (See Section II. Literature Review): spreads are modeled as a persistent mean-reverting process determined by local factors, driven by fundamentals, and a global factor, driven by risk-aversion. To capture the impact of fundamentals, I use debt-to-GDP ratio (Debt) and deficit-to-GDP ration (Deficit). Risk aversion is proxied by Moody’s Baa-Aaa spread (spread). To fulfill stationarity conditions, lags of adjusted CDS spreads and risk aversion are included. Column (1) reports the results of OLS regressions, consistent with prior results in the literature: the higher the indebtedness of a sovereign, the higher the CDS spread; the smaller the fiscal deficit, the smaller the CDS spread; the higher the risk aversion and market uncertainty, the higher is the CDS spread. The standard model is able to explain around 19 % of the total variation, which is markedly low.

In column (2), other variables, which have been found relevant in the empirical literature are included. GDP is found to be negatively correlated with CDS spreads.
Table 3: In-sample Regression Analysis

The table reports the results of regressing changes in CDS spreads on innovations of the National Financial Conditions Index (NFCI) published by the Federal Reserve Bank of St. Louis (FRED), a set of controls and a spatial lag (column 3). The sample ranges from 2006 to 2017. T-stats are reported in parentheses. Significance at the 1%, 5% and 10% is given by ***, ** and * respectively. \( |\rho| < 1 \)

\[
\Delta CDS_t = \beta_0 + \rho W \Delta CDS_t + \beta_1 Debt_t + \beta_2 Deficit_t + \beta_3 RiskAv_t + \beta_4 CDS_{t-1} + \beta_5 RiskAv_{t-1} + \epsilon_t
\]

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>OLS Standard Model</th>
<th>OLS Standard Model</th>
<th>SAR: 2005Q3</th>
<th>SAR: 2005Q3</th>
<th>SAR: Pseudo W</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.64***</td>
<td>(28.93)</td>
<td>0.61***</td>
<td>(25.88)</td>
<td>0.13***</td>
<td>(18.09)</td>
</tr>
<tr>
<td>Debt</td>
<td>0.08</td>
<td>0.34***</td>
<td>0.035</td>
<td>0.111**</td>
<td>0.124**</td>
</tr>
<tr>
<td>(1.39)</td>
<td>(5.29)</td>
<td>(0.80)</td>
<td>(2.14)</td>
<td>(2.26)</td>
<td></td>
</tr>
<tr>
<td>Deficit</td>
<td>-1.18***</td>
<td>-1.25***</td>
<td>-1.12***</td>
<td>-1.32***</td>
<td>-1.42***</td>
</tr>
<tr>
<td>(-3.49)</td>
<td>(-5.11)</td>
<td>(-4.16)</td>
<td>(-4.58)</td>
<td>(-4.67)</td>
<td></td>
</tr>
<tr>
<td>RiskAv</td>
<td>45.00***</td>
<td>6.09</td>
<td>19.46***</td>
<td>0.40</td>
<td>2.78</td>
</tr>
<tr>
<td>(5.80)</td>
<td>(3.10)</td>
<td>(0.4)</td>
<td>(0.289)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CDS_{t-1}</td>
<td>-0.11***</td>
<td>-0.14***</td>
<td>-0.06***</td>
<td>-0.06***</td>
<td>-0.06***</td>
</tr>
<tr>
<td>(-12.80)</td>
<td>(-12.99)</td>
<td>(-8.47)</td>
<td>(-8.75)</td>
<td>(-8.67)</td>
<td></td>
</tr>
<tr>
<td>RiskAv_{t-1}</td>
<td>-56.16***</td>
<td>-32.98***</td>
<td>-23.75***</td>
<td>-13.15*</td>
<td>-15.46***</td>
</tr>
<tr>
<td>(-7.84)</td>
<td>(-3.66)</td>
<td>(-3.76)</td>
<td>(-1.75)</td>
<td>(-1.96)</td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>-0.82***</td>
<td>-0.40*</td>
<td>-0.206</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-2.74)</td>
<td>(-1.64)</td>
<td>(-0.78)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VSTOXX</td>
<td>1.07***</td>
<td>0.60***</td>
<td>0.51***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4.07)</td>
<td>(2.72)</td>
<td>(2.19)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>StockInd</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.33)</td>
<td>(0.03)</td>
<td>(0.20)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MRO</td>
<td>2.47***</td>
<td>0.84</td>
<td>1.27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4.19)</td>
<td>(1.58)</td>
<td>(0.80)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eonia</td>
<td>-3.95</td>
<td>-1.55</td>
<td>-1.94***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-1.44)</td>
<td>(-0.67)</td>
<td>(-2.27)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Libor – OIS</td>
<td>-2.06***</td>
<td>-0.95</td>
<td>-0.97</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-2.95)</td>
<td>(-1.55)</td>
<td>(-1.51)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Size FinSector</td>
<td>0.23***</td>
<td>0.20***</td>
<td>0.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4.75)</td>
<td>(3.75)</td>
<td>(3.00)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adj R², %</td>
<td>18.64</td>
<td>26.42</td>
<td>48.70</td>
<td>49.50</td>
<td>44.52</td>
</tr>
<tr>
<td>Observations</td>
<td>1080</td>
<td>1080</td>
<td>1080</td>
<td>1080</td>
<td>1080</td>
</tr>
</tbody>
</table>
The volatility of the stock market (VSTOXX) carries a positive premium, although the magnitude of the coefficient is lower than the one for the global risk aversion. A potential explanation could be that the two are positively correlated. The coefficient on the European stock market index is close to zero and insignificant. The ECB’s interest rate (MRO) is positively related to CDS spreads and the coefficient is statistically significant. The overnight interbank offered rate (Eonia) carries a negative sign, which is not surprising because the rate was considerably lowered during the peak of the crisis, when CDS spreads were at their highest. The spread LIBOR-OIS captures marker uncertainty and in this model is negatively related to CDS spreads. Finally, to control for the size of the financial system, the variable SizeFinSector is included, which is defined as total banking assets over GDP. Countries, where the financial sector accounts for a bigger fraction of total GDP are more exposed to systemic risk and, hence are characterized by higher levels of CDS spreads. It is interesting to note that even though the model is saturated with many variables, the improvement in explanatory power is low.

Column (3) reports the results of the SAR model estimated by maximum likelihood. To address endogeneity issues, the spatial weights matrix $W$ is calculated using data from BIS 2005 Q3, which is entirely pre-determined with respect to the sample. The spatial autoregressive parameter $\rho$ is positive and strongly statistically significant. This is interpreted as evidence for strong network spillover effects. Since, unlike the OLS estimates, the SAR coefficients have the interpretation of average partial derivatives, it is not possible to discuss directly their magnitude, only their sign. Comparing columns (1) and (2) to column (3), note that the direction of the coefficients is the same. Importantly, the explanatory power of the model increases more than two times with an R-squared of 49%.

One concern could be that the spatial lag does not capture new information, but it is correlated to other variables, previously found in the literature. To discard this doubt, controls are included in the SAR model (column (4)). The parameter $\rho$ continues to be positive and statistically significant and its magnitude is largely unchanged. This leads to the conclusion that the spatial lag is an important factor, which is omitted from standard specifications.
Given the high values for $\rho$, another concern naturally emerges. It could be that this value is mechanically attributed because changes in CDS spreads are present both on the right and the left hand side of the equation. A random pseudo $W$ is generated, whose entries are drawn from an uniform distribution, is used in column(5). The value of $\rho$ is 0.13, which is 5 times lower than the values in the previous two specifications. Therefore, it is safe to assume that the results on network spillovers are not driven by randomness.

In Table 4, the effect of the SAR coefficients is decomposed into direct and indirect effects using the formula from Section IV B. Except for Debt, all decompositions are statistically significant. Indirect effects constitute approximately 60% of the overall effect. Comparing Total and OLS, it is noticed that OLS coefficients are systematically lower than SAR coefficients and the difference is roughly 20%.

**Table 4: Decomposition: Direct, Indirect and Total Effects**

This table reports the decomposition to and indirect effects of the coefficients of the SAR model in column (3) of Table 2

<table>
<thead>
<tr>
<th></th>
<th>Debt</th>
<th>Deficit</th>
<th>Spread</th>
<th>$\Delta$$\text{CDS}_{t-1}$</th>
<th>Spread$_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct</td>
<td>0.04</td>
<td>-1.28***</td>
<td>22.42***</td>
<td>-0.06***</td>
<td>-27.26***</td>
</tr>
<tr>
<td></td>
<td>(0.85)</td>
<td>(-4.14)</td>
<td>(3.24)</td>
<td>(-8.34)</td>
<td>(-3.85)</td>
</tr>
<tr>
<td>Indirect</td>
<td>0.05</td>
<td>-1.82***</td>
<td>31.605***</td>
<td>-0.09***</td>
<td>-38.43***</td>
</tr>
<tr>
<td></td>
<td>(0.84)</td>
<td>(-3.89)</td>
<td>(3.24)</td>
<td>(-6.91)</td>
<td>(-3.85)</td>
</tr>
<tr>
<td>Total</td>
<td>0.10</td>
<td>-3.10***</td>
<td>54.02***</td>
<td>-0.15***</td>
<td>-65.70***</td>
</tr>
<tr>
<td></td>
<td>(0.85)</td>
<td>(-4.04)</td>
<td>(3.26)</td>
<td>(-7.71)</td>
<td>(-3.90)</td>
</tr>
<tr>
<td>OLS</td>
<td>0.08</td>
<td>-1.18***</td>
<td>45.00***</td>
<td>-0.11***</td>
<td>-56.16***</td>
</tr>
<tr>
<td></td>
<td>(1.39)</td>
<td>(-3.49)</td>
<td>(5.80)</td>
<td>(-12.80)</td>
<td>(-7.84)</td>
</tr>
</tbody>
</table>

To summarize, the in-sample results provide evidence for the systemic nature of credit risk and for strong network spillovers in CDS markets. The SAR model introduces a new factor, which improves significantly the explanatory power of the model. The results are intuitive and are in line with the predictions of the theoretical model outlined in Section III.

Having addressed the first research question, the paper moves on to study how exogenous financial shocks are transmitted through the network of financial linkages.
E. Financial Shocks

To answer this second research question, the paper uses exogenous financial shocks identified using the methodology described in Section V C: (1) using announcement dates of negative financial events and (2) using innovations of a financial conditions index (NFCI). All results in this section use daily CDS spreads or changes in the CDS spreads.

To study the effects of exogenous financial events, I conduct an event study around announcement dates. Changes in CDS spreads in a two-day event window \([-1, 1]\) bracketing US financial shocks are regressed on an event dummy (\(\text{FinShock}\)) and a spatial lag. Table 5 tabulates the results of the event study.

Panel A gives point estimates of three models: OLS without a spatial lag, a SAR model with a weight matrix constructed from BIS data 2005 Q3 and a SAR model with a \(pseudo W\) for robustness. The OLS estimate of the event dummy is positive and significant: a negative financial shock in the US financial market affects positively the European sovereign CDS market. Introducing the spatial lag in column (2), the event dummy remains positive and significant. The spatial autoregressive parameter equals 0.50 and is strongly statistically significant. In the last column a test with a \(pseudo spatial weights matrix\) is performed, which yields results consistent with the statement that the results of the SAR model are not random.

In order to be able to compare the OLS and SAR results, Panel B offers a decomposition of \(\text{FinShock}\). For the OLS model, the \(Total Effect\) equals the point estimate from Panel A. In the case of the SAR model, the \(Total Effect\) is a sum of the \(Indirect Effect\) and the \(Direct Effect\). It is interesting to note that indirect network effects account for nearly 46% of the overall effect. Comparing the overall effect, SAR model yields effects, which are 25% higher than the OLS estimate. The difference between the two is attributed to propagation effects through the network of sovereign financial linkages.
Table 5: Event study: Response of CDS Spread to Exogenous Negative Financial Events

The table reports the results of regressing changes in CDS spreads in a two-day event window [-1, 1] bracketing US financial shocks on an event dummy and a spatial lag. The sample ranges from 2006 to 2017. T-stats are reported in parentheses. Significance at the 1%, 5% and 10% is given by ***, ** and * respectively.

\[ \Delta CDS_{[t-1,t+1]} = \beta_0 + \rho W \Delta CDS_{[t-1,t+1]} + \beta_1 \text{FinShock}_t + \epsilon_t \]

<table>
<thead>
<tr>
<th></th>
<th>OLS No Spatial Lag</th>
<th>SAR: 2005Q3</th>
<th>SAR: Pseudo W</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(\rho)</strong></td>
<td>0.50***</td>
<td>0.13***</td>
<td></td>
</tr>
<tr>
<td>(88.96)</td>
<td>(67.95)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(\text{FinShock})</strong></td>
<td>1.49***</td>
<td>0.89**</td>
<td>0.75**</td>
</tr>
<tr>
<td>(2.36)</td>
<td>(1.89)</td>
<td>(1.96)</td>
<td></td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td>0.11**</td>
<td>0.005</td>
<td>0.006</td>
</tr>
<tr>
<td>(5.32)</td>
<td>(0.02)</td>
<td>(0.06)</td>
<td></td>
</tr>
<tr>
<td><strong>Adj R², %</strong></td>
<td>0.40</td>
<td>17.00</td>
<td>17.3</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>27408</td>
<td>27408</td>
<td>27408</td>
</tr>
</tbody>
</table>

Panel B: Decomposition of \(\text{FinShock}\)

<table>
<thead>
<tr>
<th></th>
<th>OLS No Spatial Lag</th>
<th>SAR: 2005Q3</th>
<th>SAR: Pseudo W</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Direct Effect</strong></td>
<td>0.98**</td>
<td>0.76**</td>
<td></td>
</tr>
<tr>
<td>(1.88)</td>
<td>(1.95)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Indirect Effect</strong></td>
<td>0.86**</td>
<td>0.11**</td>
<td></td>
</tr>
<tr>
<td>(1.84)</td>
<td>(1.88)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total Effect</strong></td>
<td>1.49***</td>
<td>1.84**</td>
<td>0.87**</td>
</tr>
<tr>
<td>(3.23)</td>
<td>(1.96)</td>
<td>(1.96)</td>
<td></td>
</tr>
</tbody>
</table>

Although significant, the overall effect of event study is very small in magnitude. One reason could be the small number of relevant events in the sample, which reduces the statistical power of the test. Another approach is to use innovations in the NFCI index computed by the Federal Reserve Bank of St. Louis. This allows to identify a continuum of shocks throughout the entire sample 2006:2016.

The main variable of interest in \(\text{ShockNFCI}\), defined as weekly innovations in the NFCI index linearly interpolated between announcement days (Thursdays). High values of the NFCI signify economic instability, whereas low values indicate downturn and recession. The mean value of the index is -0.33, the minimum is -0.94 and the
maximum is 2.86. The average value of the innovations is 0.001 with occasional positive spikes (positive shocks) and negative spikes (negative shocks). Table 6 reports the average effect of exogenous financial shocks.

**Figure 4: The NFCI Index**

The figure plots the NFCI Index: Panel A in levels, Panel B innovations.
Table 6: Response of CDS Spreads to Exogenous Financial Shocks

The table reports the results of regressing changes in CDS spreads on innovations of the National Financial Conditions Index (NFCI) published by the Federal Reserve Bank of St. Louis (FRED), a set of controls and a spatial lag (column 3). The sample ranges from 2006 to 2017. T-stats are reported in parentheses. Significance at the 1%, 5% and 10% is given by ***, ** and * respectively.

\[
\Delta CDS_t = \beta_0 + \rho W \Delta CDS_t + \beta_1 FinShock_t + \beta_2 Debt_t + \beta_3 Deficit_t + \beta_4 Spread_t + \beta_5 \Delta CDS_{t-1} + \beta_6 Spread_{t-1} + \epsilon_t
\]

<table>
<thead>
<tr>
<th>OLS No Spatial Lag</th>
<th>SAR: 2005Q3</th>
<th>SAR: Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
</tbody>
</table>

Panel A: Point Estimates

<table>
<thead>
<tr>
<th></th>
<th>OLS No Spatial Lag</th>
<th>SAR: 2005Q3</th>
<th>SAR: Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>\rho</td>
<td>0.49***</td>
<td>0.40***</td>
<td>(77.58)</td>
</tr>
<tr>
<td>ShockNFCI</td>
<td>\textbf{15.37***}</td>
<td>\textbf{8.40***}</td>
<td>\textbf{9.15***}</td>
</tr>
<tr>
<td>Debt</td>
<td>0.32</td>
<td>0.75</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>(0.115)</td>
<td>(0.37)</td>
<td>(0.34)</td>
</tr>
<tr>
<td>Deficit</td>
<td>-0.067***</td>
<td>-0.067***</td>
<td>-0.06***</td>
</tr>
<tr>
<td></td>
<td>(-4.72)</td>
<td>(-4.75)</td>
<td>(-4.39)</td>
</tr>
<tr>
<td>Spread</td>
<td>-0.89</td>
<td>-0.16</td>
<td>-0.48</td>
</tr>
<tr>
<td></td>
<td>(-0.44)</td>
<td>(-0.08)</td>
<td>(-0.25)</td>
</tr>
<tr>
<td>\Delta CDS_{t-1}</td>
<td>-0.002***</td>
<td>-0.002***</td>
<td>-0.002***</td>
</tr>
<tr>
<td></td>
<td>(-5.87)</td>
<td>(-4.85)</td>
<td>(-5.57)</td>
</tr>
<tr>
<td>Spread_{t-1}</td>
<td>0.88</td>
<td>0.14</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>(0.43)</td>
<td>(0.07)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.026</td>
<td>-0.032</td>
<td>-0.038</td>
</tr>
<tr>
<td></td>
<td>(-0.17)</td>
<td>(-0.25)</td>
<td>(-0.26)</td>
</tr>
<tr>
<td>Adj R², %</td>
<td>00.25</td>
<td>17.16</td>
<td>14.02</td>
</tr>
<tr>
<td>Observations</td>
<td>23490</td>
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<td>23490</td>
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</table>

Panel B: Decomposition of Fin Shock

<table>
<thead>
<tr>
<th></th>
<th>Direct Effect</th>
<th>Indirect Effect</th>
<th>Total Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8.95***</td>
<td>7.73***</td>
<td>15.37***</td>
</tr>
<tr>
<td></td>
<td>(2.25)</td>
<td>(2.24)</td>
<td>(3.23)</td>
</tr>
<tr>
<td></td>
<td>9.49***</td>
<td>5.80***</td>
<td>15.30***</td>
</tr>
<tr>
<td></td>
<td>(2.45)</td>
<td>(2.45)</td>
<td>(2.45)</td>
</tr>
</tbody>
</table>

Changes in CDS spreads react significantly to the unexpected component of the NFCI index. The effect is between 15 to 16 basis points with nearly 45% of it due to indirect
network effects. In column (3), the SAR model uses an averaged $W$. This matrix is computed by averaging the bilateral sovereign exposures from 2006 Q1 to 2016 Q4 and using these values to compute the relative weights. The results are qualitatively unchanged, which I interpret as evidence of the robustness of the results.

F. Out-of-sample Prediction

Finally, this paper investigates whether the forecasting performance of models of CDS spreads is improved when a spatial lag is introduced. To test this, I use data from 2006 to 2012. I use monthly data, because data at a higher frequency normally contains a lot of noise. I split the sample into two parts: an estimation sample 2006:2010 and an evaluation sample 2011-2012. The evaluation period is chosen deliberately to cover the Sovereign Debt Crisis. I estimate the standard and the SAR model on data from 2006 to 2010. The estimation includes the Greek CDS spread in the sample and conditions on the spatial lag, fundamentals and risk aversion. Conditioning on the same set of information allows to attribute differences in forecasting performance to the way the two models process the same information. The evaluation sample consists of Austria, Belgium, France, Ireland, Italy, the Netherlands, Portugal and Spain i.e. Germany and Greece are excluded for obvious reasons. The German CDS spread is considered to be the safe asset, whereas in the case of Greece the sharp increase in CDS was due to misrepresentation of fundamentals and, hence, the Greek spread is unlikely to be predicted from a weighted average of neighboring countries\(^{11}\).

Table 7 reports the results for predicted CDS spreads in changes (Panel A) and in levels (Panel B). Let’s focus on first on Panel A. The mean observed change in CDS spread is -70.88. The standard model is unable to match the pattern of comovement in the data with a predicted mean of -6.38, which is nearly 63 basis points away from the observed one. The SAR model predicts -31.83, which is an improvement of 25.45 basis points. Furthermore, the SAR model improves the forecasting accuracy by 20 % in the root-mean-squared-error (RMSE) sense. Turning to predictions in levels (Panel B), the results are even more impressive. The observed mean CDS spread is 237.60 basis points. The prediction of the standard model, 77.52 basis points, misses the true mean

\(^{11}\) Neighbors in the financial network sense
by 160 basis points. The SAR model on the other hand predicts spreads, which at mean value of 159.99 are considerably closer to the true data. In terms of accuracy, the SAR model reduces RMSE by around 15%.

The SAR model consistently outperforms the standard model in this out-of-sample prediction exercise both in terms of matched values and accuracy. This suggests that the SAR model with a spatial weights matrix based on sovereign financial linkages offers a powerful, yet intuitive tool for econometric modeling and forecasting of CDS spreads.

Table 7: Predictive Regressions
The table reports the results of out-of-sample predictive performance of the standard model and the SAR model during the period 2011 to 2012. Parameters have been estimated using data from 2006:2010. The standard model and SAR model simulations are conditioned to the same information set consisting of the behavior of the Greek CDS spread, fiscal fundamentals in all euro area countries and the Baa-Aaa spread. The weight matrix used in the SAR model is constructed using BIS data from 2005 Q3. p-values are reported in parentheses. Significance at the 1%, 5% and 10% is given by ***, ** and * respectively.

<table>
<thead>
<tr>
<th></th>
<th>Observed Values</th>
<th>Standard Model</th>
<th>SAR: 2005Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Spreads Changes $\Delta CDS_t$</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-70.88***</td>
<td>-6.38***</td>
<td>-31.83***</td>
</tr>
<tr>
<td></td>
<td>(&lt;0.00)</td>
<td>(&lt;0.00)</td>
<td>(&lt;0.00)</td>
</tr>
<tr>
<td>SD</td>
<td>72.17</td>
<td>9.15</td>
<td>84.02</td>
</tr>
<tr>
<td>Min</td>
<td>-541.72</td>
<td>-28.51</td>
<td>-140.00</td>
</tr>
<tr>
<td>Max</td>
<td>215.99</td>
<td>5.026</td>
<td>43.00</td>
</tr>
<tr>
<td>RMSE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel B: Spreads Levels</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>237.60</td>
<td>77.52***</td>
<td>159.99***</td>
</tr>
<tr>
<td></td>
<td>(&lt;0.00)</td>
<td>(&lt;0.00)</td>
<td>(&lt;0.00)</td>
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<tr>
<td>SD</td>
<td>281.45</td>
<td>44.40</td>
<td>70.36</td>
</tr>
<tr>
<td>Min</td>
<td>-9.36</td>
<td>-18.47</td>
<td>18.035</td>
</tr>
<tr>
<td>Max</td>
<td>1386.55</td>
<td>203.14</td>
<td>358.45</td>
</tr>
<tr>
<td>RMSE</td>
<td>295.86</td>
<td>253.35</td>
<td></td>
</tr>
</tbody>
</table>
VII. Robustness

This section discusses additional results and demonstrates that the findings reported in the previous section are robust to alternative specifications of time periods and the spatial weights matrix.

A. Structural Breaks

I test for structural breaks in the coefficients by employing a simple Wald Test approach. I split the sample into five periods: 2006:2007, 2008:2010, 2011:2012, 2013:2014, 2015:2016. These correspond roughly to the Pre-crisis Period, the Global Liquidity Crisis, the European Sovereign Debt Crisis and 2 Post-crisis periods. I create five dummy variables Period1, Period2, Period3, Period 4 and Period 5 that take values equal to one if one of the five periods applies. Then variables are interacted with the dummies. I run the following regression:

$$\Delta CDS_t = \beta_0 + \sum_{j=1}^{5} \rho W \Delta CDS_t \times Period_j + \sum_{j=1}^{5} \beta_j Debt_t \times Period_j + \sum_{j=1}^{5} \beta_{2j} Deficit_t \times Period_j$$

$$+ \sum_{j=1}^{5} \beta_{3j} RiskAv_t \times Period_j + \sum_{j=1}^{5} \beta_{4j} CDS_{t-1} \times Period_j$$

$$+ \sum_{j=1}^{5} \beta_{5j} RiskAv_{t-1} \times Period_j + \epsilon_t$$

Then, I conduct Wald tests for each set of variables. For example, $\beta_{11} = \beta_{12} = \beta_{13} = \beta_{14} = \beta_{15}$ etc. Judging from the values of the F-statistic and the corresponding p-values, there is significant evidence for structural breaks in the coefficients induced by the Global Financial Crisis and the European Sovereign Debt Crisis (Table 8).
Table 8: Structural Breaks

The Table reports the results of tests of structural breaks in the regression coefficients. The data sample is split into four parts, separate SAR regressions are performed on each and then the coefficients are compared using a series of Wald tests.

<table>
<thead>
<tr>
<th>Variable</th>
<th>F-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spatial Lag</td>
<td>0.01</td>
<td>0.98</td>
</tr>
<tr>
<td>Debt</td>
<td>0.14</td>
<td>0.96</td>
</tr>
<tr>
<td>Deficit</td>
<td>2.23</td>
<td>0.10</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>0.16</td>
<td>0.84</td>
</tr>
</tbody>
</table>

B. Alternative Time Periods

The results of the tests in Section A suggest changes in the time-series of the variables from one period to the other. One concern could be that the presence of network spillovers in CDS markets is an artefact of the long sample or that it is driven by very strong spillovers during a sub-sample of the data. It will be useful to see whether the spatial autoregressive parameter remains significant when the data is split into subsamples and how its values change.

Table 9 documents significant differences in the spatial autoregressive parameter throughout the samples. It is lowest during 2006:2007, when CDS spreads are low and the levels of risk are small. It skyrockets during 2008:2010 to 0.67 at the peak of the Liquidity Crisis and the onset of the Sovereign Debt Crisis. During 2011:2012, the value of the parameter is reduced nearly in half, which is a result of government interventions and macroeconomic policies aiming at decoupling the financial system. During the next two periods after the crisis, values of the parameter increase, which reflects the strong comovement and narrowing down of the differences in CDS spreads between countries in the sample. These results are consistent with the graphical evidence in Figure 2.
Table 9: SAR Model: Sub-samples

The table reports the results of regressing changes in CDS spreads on a spatial lag and a set of controls over four sub-samples of the data. The full sample ranges from 2006 to 2017. T-stats are reported in parentheses. Significance at the 1%, 5% and 10% is given by ***, ** and * respectively.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td></td>
</tr>
<tr>
<td>(\rho)</td>
<td>0.25**</td>
<td>0.67***</td>
<td>0.38***</td>
<td>0.52***</td>
<td>0.46***</td>
</tr>
<tr>
<td></td>
<td>(2.08)</td>
<td>(18.11)</td>
<td>(4.07)</td>
<td>(13.46)</td>
<td>(6.39)</td>
</tr>
<tr>
<td>(\text{Debt})</td>
<td>-0.3</td>
<td>0.026</td>
<td>0.17</td>
<td>0.06</td>
<td>0.09*</td>
</tr>
<tr>
<td></td>
<td>(-0.46)</td>
<td>(0.42)</td>
<td>(0.69)</td>
<td>(0.94)</td>
<td>(1.71)</td>
</tr>
<tr>
<td>(\text{Deficit})</td>
<td>-0.002</td>
<td>-1.08***</td>
<td>-1.83</td>
<td>-0.22</td>
<td>-0.006</td>
</tr>
<tr>
<td></td>
<td>(-0.02)</td>
<td>(-4.27)</td>
<td>(-1.09)</td>
<td>(-0.54)</td>
<td>(-0.01)</td>
</tr>
<tr>
<td>(\text{RiskAv})</td>
<td>3.42</td>
<td>16.93***</td>
<td>43.25</td>
<td>14.68</td>
<td>4.43</td>
</tr>
<tr>
<td></td>
<td>(1.40)</td>
<td>(3.34)</td>
<td>(1.05)</td>
<td>(0.72)</td>
<td>(0.49)</td>
</tr>
<tr>
<td>(\text{CDS}_{t-1})</td>
<td>-0.18***</td>
<td>-0.003</td>
<td>-0.04**</td>
<td>-0.06***</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>(-2.96)</td>
<td>(-0.14)</td>
<td>(-1.99)</td>
<td>(-5.69)</td>
<td>(-1.02)</td>
</tr>
<tr>
<td>(\text{RiskAv}_{t-1})</td>
<td>6.33</td>
<td>-21.11***</td>
<td>-132.32***</td>
<td>-9.97</td>
<td>1.79</td>
</tr>
<tr>
<td></td>
<td>(1.39)</td>
<td>(-4.02)</td>
<td>(-3.01)</td>
<td>(-0.42)</td>
<td>(0.19)</td>
</tr>
<tr>
<td>(\text{Adj R}^2, %)</td>
<td>17.49</td>
<td>54.04</td>
<td>29.98</td>
<td>41.85</td>
<td>18.38</td>
</tr>
<tr>
<td>Observations</td>
<td>117</td>
<td>327</td>
<td>216</td>
<td>216</td>
<td>207</td>
</tr>
</tbody>
</table>

C. Alternative Specifications of the Spatial Weights Matrix

This section establishes the robustness of the results using different specifications of the spatial weights matrix.

In column (1) of Table 10, I use a categorical spatial matrix, which is computed in the following way. Data for financial linkages is averaged over all periods from 2006 Q1 to 2016 Q4 and collected in a matrix \(\text{Average}\). Then the following rule is applied:

\[
\begin{align*}
    w_{ij} &= \begin{cases} 
    1 & \text{if } \text{average}_{ij} \leq p_{25} \\
    2 & \text{if } p_{25} < \text{average}_{ij} \leq p_{50} \\
    3 & \text{if } \text{average}_{ij} \geq p_{75}
    \end{cases}
\end{align*}
\]

where \(p_{25}, p_{50}\) and \(p_{75}\) stand for the 25th, 50th and 75th percentile of the matrix \(\text{Average}\). In order to ensure that \(\text{abs}(\rho) < 1\), row-normalization is applied. The results indicate significant spillover effects, but the magnitude of the spatial autoregressive parameter is lower, because some information is lost when the categorical scheme above is carried out.
In column (2) of Table 10, I use data from BIS 2010 Q4 to capture changes in the dynamics of cross-border borrowing and lending. The network parameter remains significant and positive.

Another potential source of bias could arise due to the assumptions of the construction method. Recall that the entry \( w_{ij} = x_{ij} D_{i}^{Gvmnt} \), \( x_{ij} = \frac{a_{ij}}{\sum a_{ij}} \), \( a_{ij} \) is claims of \( i \) vis-à-vis \( j \) in 2005 Q3 and \( j = 1,..,9 \). This assumes a closed system i.e. no lending and borrowing outside of the network of countries in the sample. Since, it is reasonable to believe that in reality countries in the sample hold claims vis-à-vis countries outside of the euro-zone, the weighting scheme \( x_{ij} \) might overstate the effect. To check whether this is true, the following is introduced:

\[
    x_{ij}^{BIS} = \frac{a_{ij}}{\sum_{j=1}^{BIS} \sum a_{ij}}
\]

where \( BIS \) stands for the total number of BIS-reporting banks. Then, let \( W^{BIS} = x_{ij}^{BIS} D_{i}^{Gvmnt} \). The results of column (3) alleviate this concern: the sign and magnitude of all coefficients in the model remain almost the same.
Table 10: Robustness Results for the Spatial Weights Matrix

The Table reports robustness checks using different spatial weight matrices. See the text for details on how the matrices (1)-(4) are constructed.

<table>
<thead>
<tr>
<th></th>
<th>Categorical W</th>
<th>2010 Q4</th>
<th>$W^{BIS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.44***</td>
<td>0.59***</td>
<td>0.62***</td>
</tr>
<tr>
<td></td>
<td>(16.20)</td>
<td>(27.03)</td>
<td>(29.28)</td>
</tr>
<tr>
<td>Debt</td>
<td>1.10**</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(2.28)</td>
<td>(0.81)</td>
<td>(0.88)</td>
</tr>
<tr>
<td>Deficit</td>
<td>-1.08***</td>
<td>-1.07***</td>
<td>-1.12***</td>
</tr>
<tr>
<td></td>
<td>(-3.61)</td>
<td>(-3.95)</td>
<td>(-4.14)</td>
</tr>
<tr>
<td>RiskAv</td>
<td>25.09***</td>
<td>20.02***</td>
<td>16.83***</td>
</tr>
<tr>
<td></td>
<td>(3.69)</td>
<td>(3.15)</td>
<td>(2.67)</td>
</tr>
<tr>
<td>CDS$_{t-1}$</td>
<td>-0.09***</td>
<td>-0.05***</td>
<td>-0.06***</td>
</tr>
<tr>
<td></td>
<td>(-12.23)</td>
<td>(-8.29)</td>
<td>(-8.73)</td>
</tr>
<tr>
<td>RiskAv$_{t-1}$</td>
<td>-31.02***</td>
<td>-24.52***</td>
<td>-21.24***</td>
</tr>
<tr>
<td></td>
<td>(-4.53)</td>
<td>(-3.84)</td>
<td>(-3.36)</td>
</tr>
<tr>
<td>Adj R$^2$, %</td>
<td>39.93</td>
<td>47.62</td>
<td>47.83</td>
</tr>
<tr>
<td>Observations</td>
<td>1080</td>
<td>1080</td>
<td>1080</td>
</tr>
</tbody>
</table>

VIII. Conclusion

To summarize, this paper addresses the question of systemic risk in sovereign CDS spreads markets both from a theoretical, as well as from an empirical standpoint.

In the spirit of recent theoretical work on networks, this paper develops a network model with asset interdependencies. A “balance sheet” mechanism of contagion is considered, where spillovers from a default or a severe financial shock occur via direct losses to assets held by creditors. Using a fixed-point argument, it is possible to show that in the presence of asset interdependencies and discontinuities in value multiple equilibrium solutions for organization’s values are possible. In this context of multiple equilibria, contagion emerges because of linkages and the joint determination of asset prices: organizations fail because people expect that other connected organizations will fail as well and this then becomes self-fulfilling.

Next, the paper shows that the theoretical model can be empirically operationalized via a spatial autoregressive (SAR) model. Using methods from spatial
econometrics, the paper makes three empirical findings. First, the paper offers strong statistically significant evidence for the presence of credit risk spillovers in CDS markets. Furthermore, Introducing the systemic risk factor considerably improves in-sample model fit and explanatory power. The results indicate that network linkages account for 15 % to 20 % of the CDS variance. Second, the paper shows that the constructed network of financial linkages between sovereigns is an important mechanism for the propagation of exogenous financial shocks. Using the SAR model, it is possible to decompose the total effect of financial shocks to direct and indirect effects. The paper finds that as much as 45% of the overall effect of shocks is due to indirect (network) effects. Third, in out-of-sample predictive tests, the SAR model consistently outperforms standard models. The SAR model is better able to match monthly changes in CDS spreads and leads to 25 % to 35% improvement in predictive accuracy, measured in the root mean squared error (RMSE) sense.
References


Appendix A: Theoretical Model

A1. Tarski’s Fixed Point Theorem

**Tarski’s Fixed Point Theorem (1955):** Let $X$ be a non-empty complete lattice. If $\Phi: X \to X$ is non-decreasing, then the set of fixed points of $\Phi$ is a non-empty complete lattice. Moreover, there exists a least fixed point $\underline{x}$ and a greatest fixed point $\overline{x}$ such that for any solution $x^*$, $\underline{x} \leq x^* \leq \overline{x}$.

**Proof:**

Recall that by the definition of a valuation function, organizations’ values are bounded from above and from below. Introduce the following map:

$$\Phi: [V_{\min}, V_{\max}]^n \to [V_{\min}, V_{\max}]^n$$

$$\Phi(V) = \psi W V^I B(V) + \Pi p^P A(V)$$

In order to prove that Tarski’s Fixed Point Theorem applies, we must show that the function $\Phi$ maps a complete lattice onto itself (i) and that the function $\Phi$ is a non-decreasing function (ii). To prove (i), notice that if the valuation functions are feasible, then:

$$V_{\min} \leq \Phi(V) \leq \psi W V + \Pi p \leq V_{\max}$$

and so $X = [V_{\min}, V_{\max}]^n$ is a complete lattice such that $\Phi: X \to X$. Since $\Phi$ is a linear combination of monotonic non-decreasing functions in $V$, then $\forall V, V’$ if $V < V’$, then $\Phi(V) \leq \Phi(V’)$. Since both (i) and (ii) hold, Tarski’s Fixed Point Theorem applies.

A2. Invertibility of the dependency matrix $\mathcal{A}$

**Lemma 1:**

If $Z$ is a matrix with $\|Z\| < 1$, where $\|Z\|_1 = \sup_x \frac{\|Zx\|_1}{\|x\|_1}$, then $I_n - Z$ is invertible and

$$(I_n - Z)^{-1} = \sum_{k=1}^{\infty} Z^k$$

**Proof:**
First, note that if \( \vec{v} \neq \vec{0} \), then \( \|Z\vec{v}\| < \|\vec{v}\| \) and so

\[
\|(I_n - Z)\vec{v}\| \geq \|I_n\vec{v}\| - \|Z\vec{v}\| > 0
\]

This means that \( I_n - Z \) has a trivial kernel, and therefore it is invertible. Letting \( S = \sum_{k=0}^{N} Z^k \), then \( S(I_n - Z) = I_n - Z^{N+1} \). Therefore:

\[
\sum_{k=0}^{N} (I_n - Z)^{-1}(I_n - Z^{N+1})
\]

Letting \( N \to \infty \) and observing that \( Z^{N+1} \to 0 \), the desired result obtains.

**Appendix B: Data**


Financial Linkages are constructed in three simple steps. This example uses BIS data from 2005 Q4

**Step 1**: Obtain raw bilateral links from BIS Quarterly Bulletin. Entries \( a_{ij} \) give the dollar amount of claims of banks headquartered in country \( i \) borrow from banks headquartered in country \( j \). Values are in millions of US dollars. Entries on the diagonal are zero (Table 1).
Table 11: Bilateral claims in millions of US dollars, BIS Table 9B 2005 Q4

<table>
<thead>
<tr>
<th></th>
<th>AUS</th>
<th>BEL</th>
<th>FRA</th>
<th>GER</th>
<th>IRE</th>
<th>ITA</th>
<th>NET</th>
<th>PRT</th>
<th>SPA</th>
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<tbody>
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<td>14996</td>
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<tr>
<td>PRT</td>
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<td>81661</td>
<td>0</td>
<td>48114</td>
<td></td>
</tr>
<tr>
<td>SPA</td>
<td>3379</td>
<td>20967</td>
<td>84677</td>
<td>133490</td>
<td>21892</td>
<td>12587</td>
<td>81661</td>
<td>13448</td>
<td>0</td>
</tr>
</tbody>
</table>

**Step 2:** Row-normalize to obtain fractions: \( x_{ij} = \frac{a_{ij}}{\sum_j a_{ij}} \)

**Step 3:** Weigh sovereign debt by the strength of the connection \( w_{ij} = x_{ij}D_i^{Govt} \). The first column of Table 2 gives the dollar amount that the banks in country \( i \) borrow from the general government of foreign countries (Table 2).

**Step 4:** Transpose the matrix: focus on lending relationships \( W = W' \)

**Step 5:** Construct spatial weights matrix: row-normalize \( W \). This ensures that the spatial multiplier exists.

---

\( ^{12} \) Data for borrowing from Portugal to Ireland are not available in the BIS Bulletin 2005 Quarter 4 and the reported value is missing. I approximate the entry by using the first data available, which is from 2008 Quarter 1.
Table 12: Sovereign Debt Matrix, BIS 2005 Q4

<table>
<thead>
<tr>
<th>$a_{ij}$</th>
<th>$D_{i}^{Gvmnt}$</th>
<th>AUS</th>
<th>BEL</th>
<th>FRA</th>
<th>GER</th>
<th>IRE</th>
<th>ITA</th>
<th>NET</th>
<th>PRT</th>
<th>SPA</th>
</tr>
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B1. Financial Shocks

The Table below lists financial events used in the empirical analysis to identify financial shocks.

Table 13: List of Event Dates

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<th>Date</th>
<th>Event Description</th>
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<td>2/27/2007</td>
<td>Mortgage giant Freddie Mac says it will no longer buy the most risky subprime loans</td>
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<td>Subprime mortgage lender New Century Financial files for bankruptcy-court</td>
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<td>4/2/2007</td>
<td>Investment bank Bear Stearns liquidates two hedge funds that invested in risky</td>
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<td>securities backed by subprime mortgage loans</td>
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<tr>
<td>7/31/2007</td>
<td>American Home Mortgage Investment, which specializes in adjustable-rate</td>
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<td>8/6/2007</td>
<td>Fitch Ratings cuts the credit rating of giant mortgage lender Countrywide Financial</td>
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<td>to its third-lowest investment-grade rating</td>
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<tr>
<td>8/16/2007</td>
<td>Bank of America, the biggest U.S. bank by market value, agrees to buy Countrywide</td>
</tr>
<tr>
<td>1/11/2008</td>
<td>Financial for about $4 billion</td>
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</tbody>
</table>
The Federal Reserve agrees to guarantee $30 billion of Bear Stearns’ assets in connection with the government-sponsored sale of the investment bank to JPMorgan Chase.

Federal regulators seize IndyMac Federal Bank after it becomes the largest regulated thrift to fail.

Mortgage giants Fannie Mae and Freddie Mac are taken over by the government.

Lehman Brothers files for bankruptcy-court protection.

American International Group, the world’s largest insurer, accepts an $85 billion federal bailout that gives the government a 79.9% stake in the company.

Goldman Sachs and Morgan Stanley, the last two independent investment banks, will become bank holding companies subject to greater regulation by the Federal Reserve.

Federal regulators close Washington Mutual Bank and its branches and assets are sold to JPMorgan Chase in the biggest U.S. bank failure in history.

Congress rejects a $700 billion Wall Street financial rescue package, known as the Troubled Asset Relief Program or TARP, sending the Dow Jones industrial average down 778 points, its single-worst point drop ever.

Congress passes a revised version of TARP and President Bush signs it. Wells Fargo & Co., the biggest U.S. bank on the West Coast, agrees to buy Wachovia for about $14.8 billion.

Ford, General Motors and Chrysler executives testify before Congress, requesting federal loans from TARP.

The Treasury Department, Federal Reserve and Federal Deposit Insurance Corp. agree to rescue Citigroup with a package of guarantees, funding access and capital. Citigroup will issue preferred shares to the Treasury and FDIC in exchange for protection against losses on a $306 billion pool of commercial and residential securities it holds.

The U.S. Treasury authorizes loans of up to $13.4 billion for General Motors and $4.0 billion for Chrysler from TARP.