I study optimal unconventional monetary policy under commitment in a two-country model where domestic policy entails larger spillovers to foreign countries. Equity injections into financial intermediaries turn out to be more efficient than discount window lending and the large-scale asset purchases that has been employed in many countries. Due to precautionary effects of future crises, a central bank should exit from its policy more slowly than the speed of deleveraging in the financial sector. The optimal policy can be changed considerably if cross-country policy cooperation is not imposed. In this case, interventions tend to be too strong in one country but too weak in the other. Gains from cooperation become positive if using unconventional monetary policy is costly enough, then correlates positively with the cost. Finally, I evaluate several simple rules and find that the rule responding to gaps in asset prices mimics the optimal policy.

JEL: E59, F41, F42

I. Introduction

The recent financial crisis involved a significant disruption to financial intermediation, as evidenced by limited access to credit (e.g. Ivashina and Scharfstein, 2010) and high credit spreads. Such disruption can propagate internationally via integrated financial markets. To stabilize the financial system, fiscal and monetary authorities in major economies acted jointly and introduced new policy tools. These new tools included the provision of large-scale liquidity, and resulted in the balance sheets of some central banks expanding 20 percent to 30 percent of GDP. The new tools break away from conventional monetary policy, which may have reached its effective lower bound, and are commonly known as unconventional monetary policies (UMPs). In the last ten years, different sets of UMPs have been employed at dif-

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1These policies may not be strictly monetary policy. For example, Kollmann et al. (2013) consider government support for banks as fiscal policy. I use the terms unconventional policy and unconventional monetary policy interchangeably.
ferent stages. For example, the Federal Reserve mainly used an expanded discount window in the early stage of the 2007-2008 crisis. After the Lehmann failure, the Fed started its asset purchase programs (quantitative easing, or QE) and injected equity into the financial system. The Fed started to taper the QE at the end of 2013, and ceased it in October 2014, after which the Federal Reserve has kept the size of its balance sheet constant by buying just enough to replace maturing securities. The European Central Bank initialized its asset purchase programs at a relatively small scale, slightly later than the Federal Reserve. The ECB formally introduced QE in 2015 and further increased the amount purchased in late 2016. The ECB also used equity injections during this period.

By employing UMP, policymakers hope to reduce long-term interest rates, boost lending, and stimulate real activity. In the meantime, conventional monetary policy can play an active role if its transmission channel via credit and financial markets is restored (Altavilla, Canova and Ciccarelli, 2016). As a probably unintended consequence, domestic interventions into markets of intentionally traded assets also affect financial conditions in foreign countries. Although much work has been devoted to evaluating the effectiveness of UMP, a normative analysis is still missing, namely, what is the optimal way of conducting unconventional policy, especially when foreign countries may take a free-ride on domestic policy. A normative analysis does not only provide a natural benchmark against which we can evaluate certain policy implementations but also sheds some light on critical policy decisions.

In this paper, I study optimal unconventional policy under commitment (Ramsey policy) and aim to address three questions. First, which of the three unconventional policies, namely public asset purchases, discount window lending, and equity injections into banks, is more effective. More generally, what are the factors that affect policy effectiveness. Second, how should the optimal policy respond to a foreign or domestic shock that may trigger a global financial crisis. Then, how should a central bank exit from its policy. Since the policy may not be permanent, the exit from the policy is particularly interesting after a decade of the crisis because central banks start to discuss shrinking their large balance sheet. For instance, in a blog article, Bernanke (2017) argues from a policy communications perspective that the shrinkage should be done in a passive and predictable way. Third, what is the difference between the cooperative policy coordinated across countries and the non-cooperative policy conducted strategically by independent central banks.

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2 Brunnermeier and Koby (2017) build a model where a effective lower bound of the conventional monetary policy is determined partially by financial constraints that banks face. The financial constraints can potentially be addressed by UMP.

3 It is relatively well established that UMP reduces long-term interest rates. See, among many others, Gagnon et al. (2011) and Krishnamurthy and Vissing-Jorgensen (2011) for the Federal Reserve’s QE, and Joyce et al. (2011) and Christensen and Rudebusch (2012) for the Bank of England’s QE. However, UMP can have insignificant or unintended real effects through the bank lending channel, as shown by Chakraborty, Goldstein and MacKinlay (2017) and Acharya et al. (2017). Negro et al. (2017) evaluate the Fed’s QE using a DSGE model with a borrowing constraint and a resaleability constraint. Mouabbi and Sahuc (2016) estimate a DSGE model in which the macroeconomic effects of the ECB’s UMP are equivalent to that of a negative shadow short-term rate. Borio and Zabai (2016) provide a review of a border range of issues related to UMP, such as a taxonomy of unconventional measures, diminishing returns and long-term effectiveness of UMP.
The literature has identified several channels through which UMP works. One is a signalling channel (Christensen and Rudebusch, 2012; Bauer and Rudebusch, 2014), which means an announcement of interventions lowers market expectations about future short-term rates, and therefore current long-term rates. Christensen and Gillan (2017) argue that introducing the central bank as a large committed buyer to financial markets lowers liquidity premiums of targeted assets. However, Kandrac (2014) finds evidence that this effect can be negative when trading among private participants decreases too much. With segmentation of the market for reserves, Christensen and Krogstrup (2016) demonstrate that reserve expansions associated with UMP can affect long-term rates through a portfolio channel even in the absence of interventions in the long-term asset market. (Unconventional) monetary policy may affect banks willingness take on risk exposures, and hence affect financial conditions via the risk-taking channel (Borio and Zhu, 2012; Bruno and Shin, 2015; Angeloni, Faia and Duca, 2015; Coimbra and Rey, 2017). Chakraborty, Goldstein and MacKinlay (2017) discuss an origination channel that is specific to the Fed’s purchases of mortgage-backed securities.

As a first step toward a comprehensive analysis of optimal unconventional policy, however, this paper focuses on a capital gain channel, which has been well tested in data yet is not fully understood in terms of optimality. This channel is particular relevant in a multi-country context because, with financial market integration, asset returns are synchronised internationally. To this aim, I consider a simple two-country model where each country has a stylized multinational banking sector similar to that in Gertler and Kiyotaki (2010). Banks face a balance sheet constraint (financial constraint) derived from an agency problem between the banks and their depositors. The constraint is slack in normal times but binds endogenously in a financial crisis, which constitutes the systemic risk in this model. Given their high leverage, these banks are vulnerable to shocks having negative impact on the value of their assets and to financial shocks that tighten their balance sheet. When the balance sheet constraint binds, banks have difficulty rolling over their short-term debts, which leads to a collapse in asset prices and investment. With multinational banks, the deteriorated balance sheet condition has a global impact, which also means that there will be large spillovers of unconventional policy from one country to another. The basic mechanism of the capital gain channel is as follows. Unconventional policy provides liquidity to support asset prices. Banks holding these assets have an improved balance sheet condition. Consequently, the policy leads to more lending to the non-financial sector.

My main findings are as follows. First, unconventional policy effectively crowds

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4Policies that do not work via this channel, such as forward guidance, are not considered.
5Evidence for increasingly global banking and integrated financial markets can be found in Devereux and Yetman (2010), Perri and Quadrini (2011), Fillat, Garetto and Götz (2015), and Bank for International Settlements' international banking statistics.
6While the banks in this model will never actually default, Coimbra and Rey (2017) precisely define systemic risk as a state in which generalized solvency issues take place.
7In models with imperfectly substitute assets, especially assets with different maturities, this channel is also referred to as the portfolio balance channel. And the effect of interventions on asset prices is reflected by the yield curve. See, for example, Negro et al. (2017) and Quint and Rabanal (2017).
out deposits received by banks. The crowding-out effect is mitigated if the policy relaxes the financial constraint that banks face. I find public asset purchases, while constitute the main policy that has been employed in many countries, have relatively larger crowding-out effect. The most efficient policy, i.e. equity injections, has the smallest crowding-out effect and allows the fastest exit from the policy. Second, domestic and foreign policy respond asymmetrically to shocks. The degree of asymmetry depends on the nature of the shock, the cost of interventions, and the bank’s portfolio. After a relatively strong response in same period when a shock hits (i.e., in a non-prudential manner), the central bank exits from its interventions in accordance with the deleveraging of the banks, the speed of which depends on the crowding-out effect. Overall, due to the precautionary effects of the occasionally binding constraints, the exit is slow and lasts even after the economy has escaped from the financial constraints. Third, the difference between cooperative and non-cooperative policy is a weakly increasing function of the intervention cost. There is no cooperation gain if the intervention cost is small. Increasing the intervention cost to a certain point generates positive gains to cooperation. In the noncooperative equilibrium, the interventions are too strong in one country, but too weak in the other.

As is well known, the Ramsey solution is silent regarding implementation. I proceed to compare several simple feedback rules. I find that the Ramsey policy can be characterised by a rule responding to gaps in asset prices. However, this rule requires knowledge of the asset prices that would be realized in a frictionless world. Among other rules that respond to observable variables, the autoregressive rule targeting the expected credit spread gives the best results.

I contribute to the literature that examines the capital gain channel of unconventional policy. The literature has focused on simple rules or a particular policy scheme. The work of Dedola, Karadi and Lombardo (2013), which is most closely related to this paper, studies the international dimension of public asset purchases in an economy where the financial constraint in each country is always binding. In their consideration of a credit spread rule, the lack of policy cooperation reduces the policy responses in both countries, which is in sharp contrast to my result. My discussion about the exit problem is linked to Foerster (2015) who also suggests slowly unwinding the central bank’s balance sheet. However, Foerster (2015) only examines one policy, public asset purchases, in a closed economy. I show that other policies with a smaller crowding-out effect can exit relatively faster. He and Krishnamurthy (2013) compare multiple policies: borrowing subsidies, equity injections, and public asset purchases. They also find that equity injections lead to the fastest recovery. Their comparison is based on a particular policy scheme that corresponds to the actual policy employed during the recent crisis. Ellison and Tischbirek (2014) and Quint and Rabanal (2017) investigate if asset purchase programs could be valuable in normal times. The former paper jointly optimises the parameters in the interest rate and the asset purchase rules while the latter optimises the asset purchase rules conditional on a estimated Taylor rule.

This paper also relates to the literature that works on occasionally binding con-
straints (OBCs) as a source of nonlinearity. The benefits of this setting is to introduce asymmetry such that we can capture the sudden and discrete nature of a financial crisis, and eliminate the financial accelerator mechanism during normal times (Del Negro, Hasegawa and Schorfheide, 2016; Swarbrick, Holden and Levine, 2017). In this paper, the OBC setting also means that unconventional policy may be conducted only when the constraint is binding. More importantly, the risk of the constraint being binding in the future has strong precautionary effects on the Ramsey policy.

The rest of the paper is organized as follows. The next section presents a two-country model with occasionally binding financial constraints. After describing my numerical method in section III, sections IV and V report the main results for cooperative and noncooperative policy, respectively. I evaluate the performance of simple rules in section VI. The last section concludes.

II. The Model

The model mostly follows Dedola, Karadi and Lombardo (2013), which extends the two-country real business cycle model (Backus, Kehoe and Kydland, 1992) by including a Gertler and Karadi (2011) style financial friction. The world economy consists of two countries, Home and Foreign, that are symmetric before being hit by a shock. In each country, domestic labour and capital are used to produce homogeneous goods, which can be used for consumption and capital production. To finance their capital, goods producers borrow from banks. Banks receive deposits from households in both countries and lend to goods producers in both countries. I use the term “non-financial sector” to refer to households and producers of goods and capital, and the term “financial sector” to refer to banks. The problem facing each agent in the Home economy is described in this section. Foreign variables are denoted by “*”. Lower case letters denote individual variables and real prices while upper case letters denote aggregate variables and nominal prices.

A. Households

There is a unit-continuum of infinitely lived households. Households consume homogeneous goods, supply labour, and save. The menu of assets available to households includes a deposit in domestic banks, $d_{h,t}$, a deposit in foreign banks, $d_{f,t}$, and a domestic government bond, $b_t$. Without loss of generality, only domestic citizens can hold their own government’s bonds. All these assets are risk-free one-period bonds denominated in terms of the issuing country’s goods and paying gross real return, $r_t$, or, $r^*_t$. Households also hold shares in non-financial firms.

Each household consists of workers and bankers who pool consumption risk perfectly. Workers provide labour to goods producers and bring wages to the household.

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8Deposit market segmentation would change the results mildly up to a misallocation of household savings. On the other hand, the security market integration is a key channel for shock propagation. In a slightly different context, Brunnermeier and Sannikov (2015) consider the case of an integrated deposit market but an separated security market.
Bankers manage a bank and transfer the profits to the household when they exit from the business. It is convenient to assume that households do not save in their own banks. Complete consumption insurance allows me to express the problem facing the consolidated representative household. The representative household chooses consumption, \( c_t \), labour supply, \( l_t \), and end-of-period wealth, consisting of domestic bonds, \( d_{h,t} + b_t \) and foreign bonds, \( d_{f,t} \) to maximize their expected discounted life-time utility, taking the wage rate and the interest rate as given:

\[
\mathbb{E}_t \sum_{j=0}^{\infty} \beta_{t,t+j} \left[ \frac{(c_{t+j} - hc_{t+j-1})^{1-\sigma}}{1-\sigma} - \chi \frac{l_{t+j}^{1+\varphi}}{1+\varphi} \right],
\]

where \( h \in [0,1) \) is the habit parameter, \( \sigma > 0 \) is the coefficient of relative risk aversion, \( \varphi > 0 \) is the inverse of the (Frisch) elasticity of labour supply, \( \chi \geq 0 \) is the relative utility weight on labour, and \( \beta_{t,t+j} \) is the subjective discount factor from period \( t \) to \( t+j \). To induce stationarity with incomplete financial markets, the discount factor is assumed to depend on aggregate consumption relative to aggregate income, \( \beta_{t,t+1} = \bar{\beta} + \psi \beta \log \left( \frac{C_t}{Y_t} \right) \), following Kollmann (2016). Denoting \( \Pi_t \) as the net profit distribution that the household earns from its ownership of banks and non-financial firms, \( w_t \) as the real wage rate, \( \tau_{w,t} \) as a tax rate on wages, \( T_t \) as a lump-sum tax, the household faces the budget constraint

\[
c_t + d_{h,t} + b_t + d_{f,t} = (1 - \tau_{w,t}) w_t l_t + \Pi_t + (d_{h,t-1} + b_{t-1}) r_{t-1} + d_{f,t-1} r^*_t - T_t.
\]

Let \( \Xi_{t,t+1} \equiv \beta_{t,t+1} \left( \frac{(c_{t+1} - hc_{t+1})^{-\sigma} - \beta_{t+1,t+2} h (c_{t+2} - hc_{t+1})^{-\sigma}}{(c_t - hc_{t-1})^{-\sigma} - \beta_{t,t+1} h (c_{t+1} - hc_t)^{-\sigma}} \right) \) be the stochastic discount factor. The first-order conditions for the household problem are fairly standard:

\[
w_t (1 - \tau_{w,t}) = \frac{\chi l^*_t}{(c_t - hc_{t-1})^{-\sigma} - \beta_{t,t+1} h (c_{t+1} - hc_t)^{-\sigma}},
\]

(1) \[ \mathbb{E}_t [\Xi_{t,t+1} r_t] = 1, \]

(2) \[ \mathbb{E}_t [\Xi_{t,t+1} r^*_t] = 1. \]

Equations 1 and 2 imply that the risk free rate is equalized across countries thanks to deposit market integration.

B. Non-financial firms

There are two types of non-financial firms: capital producers and goods producers.
Goods producers. — Goods producers hire workers and purchase capital from capital producers to produce final goods that are homogeneous across countries. They operate in markets for goods, capital, and labour that are perfectly competitive. The production technology is a standard Cobb-Douglas function

\[ y_t = A_t (\xi_t k_{t-1})^{\alpha} l_t^{1-\alpha} \]

where \( \alpha \) is the capital share, \( A_t \) is total factor productivity, and \( k_t \) is the capital stock at the end of period \( t \). Using \( \delta \) to denote the depreciation rate, and \( \xi_t \) to govern the quality of capital, a goods producer acquires additional capital

\[ i_t = k_t (1 - \delta) \xi_t k_{t-1} \]

at a given price \( q_t \). To finance its physical investment, the goods producer borrows from banks by issuing securities

\[ q_t^s (s_t - s_{t-1}) = i_t q_t, \]

where \( s_t \) denotes the number of securities issued at the end of period \( t \) and \( q_t^s \) is the period \( T \) price of security issued at period \( t \). Each security is a state-contingent claim to the future return from one unit of investment:

\[ z_{t+1}, (1 - \delta) \xi_{t+1} z_{t+2}, (1 - \delta)^2 \xi_{t+1} \xi_{t+2} z_{t+3}, \ldots \] with \( z_t \) denoting the gross profit per unit of capital.

The problem faced by the representative goods producer is

\[
\max \{ l_{t+j}, k_{t+j}, s_{t+j} \}_{j=0}^{\infty} \mathbb{E}_t \sum_{j=0}^{\infty} \Xi_{t,t+j} \\
\times \left[ (1 - \tau_{y,t}) y_{t+j} - w_{t+j} l_{t+j} - i_{t+j} q_{t+j} + q_t^s (s_{t+j} - s_{t+j-1}) - z_{t+j} s_{t+j-1} \right],
\]

subject to equation 3, the production function, and the capital accumulation equation. \( \tau_{y,t} \) is a sales tax. Let the multiplier associated with equation 3 be \( \lambda_t^{nf} \), the first-order conditions for the goods producer’s problem are then

\[ w_t = (1 - \alpha) \frac{y_t (1 - \tau_{y,t})}{l_t},\]

\[ q_t \left( 1 + \lambda_t^{nf} \right) = \mathbb{E}_t \Xi_{t,t+1} \left[ \frac{\partial y_{t+1}}{\partial k_t} + (1 - \delta) \xi_{t+1} q_{t+1} \left( 1 + \lambda_t^{nf} \right) \right],\]

\[ q_t^s \left( 1 + \lambda_t^{nf} \right) = \mathbb{E}_t \Xi_{t,t+1} \left[ z_{t+1} + q_t^s \left( 1 + \lambda_t^{nf} \right) \right].\]

It is important to assume that investment is fully financed by securities, i.e., \( \lambda_t^{nf} \neq 0 \). Otherwise, firms can effectively borrow directly from households by paying a negative dividend, which makes the banking sector trivial. Using equations 4 and 5, it is easy to show that the time \( T \) price of security issued at time \( t \) is

\[ q_T^s = q_T (1 - \delta)^T \prod_{j=1}^{T} \xi_{t+j}, \]

and \( s_t = k_t \), given \( s_0 = k_0 \). I can define the return of holding security issued in period \( t \) from \( t \) to \( t+1 \) as
where the gross profit per unit of capital is obtained using the zero profit condition

\[ z_t = \frac{y_t - w_t l_t}{k_{t-1}} = a \frac{y_t (1 - \tau_{yt})}{k_{t-1}}. \]

**Capital producers.** — Given the demand for new capital \( i_t \) and the market price \( q_t \), capital producers maximize their expected profit discounted by the household’s stochastic discount factor

\[
\max_{\{i_{t+j}\}_{j=0}^{\infty}} \mathbb{E}_t \sum_{j=0}^{\infty} \Xi_{t+t+j} [q_{t+j} i_{t+j} - f(k_{t+j-1}, i_{t+j})],
\]

subject to the cost function

\[ f(\cdot) = i_t + \frac{\eta}{2} \left( \frac{i_t}{\delta k_{t-1}} - 1 \right)^2 \delta k_{t-1}, \]

where \( \eta \geq 0 \). The first-order condition for the production decision pins down the market price of new capital

\[ q_t = 1 + \eta \left( \frac{i_t}{\delta k_{t-1}} - 1 \right). \]

**C. Banks**

Banks receive deposits amounting to \( d_{h,t} \) and \( d_{f,t} \) from domestic and foreign depositors, respectively, and purchase \( s_{h,t} \) and \( s_{f,t} \) units of securities from domestic and foreign goods producers. The lending channel from banks to goods producers is frictionless. If banks have difficulty raising deposits, they have the option of borrowing \( d_{g,t} \) from the central bank’s discount window at the interest rate \( r_{g,t} \), or they may receive equity injections from their government. By holding long-term risky securities funded by short-term risk free deposits, banks in this model act as investment banks as well as commercial banks, which is a stylized fact of the recent financial crisis. For the same reason, the literature often refers to such banks as financial intermediaries.

The balance sheet of a representative bank is given by

\[
\omega_t \equiv q_t s_{h,t} + q_t^* s_{f,t} = d_{h,t} + d_{h,t}^* + d_{g,t} + e_{h,t} + e_{g,t},
\]

where \( \omega_t \) denotes the total assets of the bank, and \( e_{h,t} \) and \( e_{g,t} \) are equity held by
households and the government, respectively. Total bank profits, referred to as net worth $n_t$, is given by

$$e_{h,t} + e_{g,t} = n_t = q_{t-1}s_{h,t-1}r_{k,t} + q_{t-1}s_{f,t-1}r_{r,t} - d_{h,t-1}r_{l-1} - d_{h,t-1}r_{r,t-1} - d_{g,t-1}r_{g,t-1}.$$  

I refer to deposits and household equity as private funds and refer to discount window lending and government equity as public funds. Bank leverage may be defined as assets financed by deposits and private equity divided by private equity

$$\phi_t = \frac{\omega_t - d_{g,t} - e_{g,t}}{e_{h,t}}.$$  

Following Gertler and Karadi (2011), I assume that the bank shuts down with probability $r_{n,t}$ at the end of each period and will distribute its net worth evenly to all equity. The probability of shutting down can be exogenous stochastic. Then, the banker becomes a worker. In the meantime, a similar number of workers from the same household randomly become new bankers. New bankers receive “start-up” funds from their household at a proportion $\omega$ of the total assets owned by a representative incumbent plus the value of the central bank’s asset purchase program. The probability of a shutdown has two roles. First, an infinitely lived bank will sooner or later accumulate enough net worth to finance its investment without borrowing from households. In this case, the financial constraint that I will detail shortly plays no role. Second, the probability enters the bank’s stochastic discount factor, which ensures that the bank is always “less patient” than households so that funds always flow from households to banks. The notation of $r_{n,t}$ follows the suggestion of Swarbrick, Holden and Levine (2017) that the probability of shutting down can be interpreted as an exogenous dividend rate.

The bank chooses $s_{h,t}$, $s_{f,t}$, $d_{h,t}$, $d_{h,t}^*$, and $d_{g,t}$, given prices and rates of returns, to maximize the expected present value of net worth paid upon closure

$$V_t(n_t) = \max \mathbb{E} \sum_{j=0}^{\infty} r_{n,t+j,t+j} (1 - r_{n,t,t+j-1}) \Xi_{t,t+j+1} (n_{t+1+j})$$

$$= \max \mathbb{E} \Xi_{t,t+1} [r_{n,t,t} n_{t+1} + (1 - r_{n,t,t}) V_{t+1} (n_{t+1})]$$

$$= \nu_{n,t} n_t,$$

where the third equality follows a conjecture that the value function is linear in net worth and $(1 - r_{n,t,j})$ is the probability that the bank operates until the end of period

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9 As I will show in impulse responses, a central bank’s asset purchase crowds out banks. I assume the value of the government’s asset purchase program enters here so that these purchases do not lead to fewer start-up funds received by new banks. If the start-up funds can be interpreted as new equity, data shows counter-cyclical and positively skewed equity issuance. However, there is no evidence suggesting that equity issuance should be affected by, and only by, asset purchase policy. Therefore this assumption ensures that public asset purchases do not suffer from artificial disadvantage comparing to other policies.
conditional on the bank operating at the beginning of period \(i\). The bank’s ability to raise deposits is restricted up to an incentive constraint (or financial constraint)

\[
OBC_t \equiv \nu_{n,t} (n_t - e_{g,t}) - [\theta_t (\omega_t - \theta_g d_{g,t}) - \nu_{n,t} e_{g,t}] \geq 0
\]

where \(OBC_t\) stands for an occasionally binding constraint and measures the distance of this constraint from binding, \(\theta_t \in [0, 1]\) is exogenous stochastic, and \(\theta_g \in [0, 1]\) is a parameter. The intuition behind this constraint is the following. Banks are able to declare bankruptcy and exit. In this case, the banker diverts to his or her family a proportion \(\theta_t\) of the divertable assets, \(\omega_t - \theta_g d_{g,t}\), minus government equity. The creditors can reclaim only the un-diverted funds. Therefore, creditors are willing to lend to a bank only if the bank has no incentive to default, i.e., the value of the private equity is larger than the value of default. The fact that \(\theta_g d_{g,t}\) is un-divertable and government equity are fully secured indicates that the central bank has superior power to enforce repayment.

For convenience, the decision on \(s_{h,t}\) and \(s_{f,t}\) can be written in terms of the total assets \(\omega_t\) and the portfolio \(\alpha_{p,t} = \frac{q_t s_{f,t}}{\omega_t}\). Denoting the multiplier associated with inequality 9 by \(\lambda_t \geq 0\), the necessary conditions of the maximization include the slackness condition of inequality 9, and the first-order conditions with respect to the total assets \(\omega_t\), the portfolio \(\alpha_{p,t}\), and the borrowing from the government \(d_{g,t}\)

\[
\begin{align*}
\mathbb{E}_t \Xi_{t,t+1} (r_{n,t,t} + (1 - r_{n,t,t}) \nu_{n,t+1}) (r_{k,t+1} - r_t) & \equiv \frac{\nu_{\omega,t}}{\lambda_t} \theta_t, \\
\mathbb{E}_t \Xi_{t,t+1} (r_{n,t,t} + (1 - r_{n,t,t}) \nu_{n,t+1}) (r_{k,t+1} - r^*_{k,t+1}) & \equiv \nu_{\alpha_{p,t}}, \\
\mathbb{E}_t \Xi_{t,t+1} (r_{n,t,t} + (1 - r_{n,t,t}) \nu_{n,t+1}) (r_{g,t} - r_t) & \equiv \frac{\nu_{d_{g,t}}}{\lambda_t} \theta_g,
\end{align*}
\]

where in equation 10 I use the fact that \(\nu_{\alpha_{p,t}} = 0\) for all \(t\) thanks to market integration. Given \(\nu_{n,t+1} \geq 1\), the extra term \((r_{n,t,t} + (1 - r_{n,t,t}) \nu_{n,t+1})\) multiplying the stochastic discount factor suggests that banks are generally less patient than households. The unknown time-varying coefficient in the value function can be solved using the first-order conditions and the financial constraint:

\[
\nu_{n,t} = \nu_t \left( \frac{\nu_{\omega,t}}{\theta_t - \nu_{\omega,t}} + 1 \right),
\]

where \(\nu_t \equiv \mathbb{E}_t \Xi_{t,t+1} (r_{n,t,t} + (1 - r_{n,t,t}) \nu_{n,t+1}) r_t\) is defined similarly to \(\nu_{\omega,t}\), \(\nu_{\alpha_{p,t}}\), and \(\nu_{d_{g,t}}\). Because \(\nu_{n,t}\) is independent of the bank’s decision variables, equation 13
verifies the earlier conjecture that the value function is linear.

The properties of the bank’s problem have been well discussed in Gertler and Kiyotaki (2010). Here I only underline some key results related to the occasionally binding constraint. To begin, use equations 6, 7, and 8 to write the value function as

\[ v_{n,t} = v_{w,t} - \alpha_{p,t} - \nu_{d,g} d_{g,t} + \nu_t n_t. \]

Then, \( v_{n,t}, v_{w,t}, \alpha_{p,t}, \nu_{d,g}, \nu_t \) can be conveniently interpreted as the expected marginal value of net worth, total assets, portfolio, borrowing from the government, and deposits, respectively. If the financial constraint is not binding, then \( \lambda_t = 0 \) and the first-order conditions imply that having one extra unit of \( \omega_t \) by borrowing from households or the government does not raise the bank’s value. In addition, equation 13 becomes \( v_{n,t} = \nu_t \approx 1 \), meaning that net worth and deposits are equally valued at the margin. If the financial constraint is binding, \( \lambda_t > 0 \) implies \( v_{w,t} > 0, \nu_{d,g} > 0, \) and \( v_{n,t} > \nu_t \). Securities, borrowing from the government, and equity are more valuable than deposits because they also help relax the financial constraint. In addition, \( v_{w,t} > 0 \) indicates a credit spread between returns on securities and returns on deposits. The spread, \( \text{spread}_t \equiv E_t (r_{k,t+1} - r_t) \), is a convenient measure of financial friction. However, unless the model is solved with certainty equivalence, the spread is positive even when the financial constraint is not binding.

**D. Government and unconventional policies**

Following the standard approach in the public finance literature, the specific agency that implements the unconventional policies is abstracted in the model. The consolidated government budget is given by

\[ G_t + \Gamma_{i,t} + r_t B_{t-1} + AP_{i,t} = T_t + \tau_{w,t} w_t L_t + \tau g_t Y_t + B_t + L_{i,t}, \]

where \( G_t \) and \( T_t \) are lump-sum government spending and tax, respectively, \( B_t \) are government bonds, \( \Gamma_{i,t}, AP_{i,t}, \) and \( L_{i,t} \) are resource costs, aggregate spending, and the gross profit of unconventional policy. The government can provide liquidity to the economy in three ways, namely, public asset purchases, lending to banks, and equity injections, indexed by \( i = 1, 2, 3 \). The aggregate spending is therefore the total asset purchased, the total discount window lending, and the value of government equity, respectively. All policies can be financed by government bonds (or reserves, the liabilities of the central bank), a lump-sum tax, a labour income tax, or a sales tax. As argued in Del Negro and Sims (2015), in order to avoid central bank insolvency, it would be appropriate for a central bank conducting unconventional policies to receive fiscal backing from the fiscal authority. I now proceed to describe each policy.

Using public asset purchases, the government lends directly to domestic goods producers.\(^\text{10}\) Hence, the government acts like a financial intermediary but faces

\(^{10}\)Quint and Rabanal (2017) show how the model can be modified slightly such that the asset purchases
no constraint in addition to its budget. The banks only need to fulfill the goods producers’ remaining demand for funds. However, I assume that the government does not purchase foreign securities for political reasons or due to a very high cost of evaluation and monitoring.\textsuperscript{11} If the government lends to banks via the discount window, replacing one unit of deposits by one unit of government lending relaxes the financial constraint, thanks to the government’s superior power of enforcement. Consequently, the banks can expand their investment to the extent allowed by the relaxed financial constraint. Government equity stabilises the financial sector in a similar way. The constraint-relaxing effect of one unit of equity, held either by households or the government, is multiplied by the marginal value of net worth $\nu_{n,t}$, which is high in a financial crisis. While private equity can only be accumulated slowly, the stock of government equity is freely adjustable. Clearly, these three policies may work against each other. For example, banks that receive equity injections would wish to expand their asset holding. However, they may not able to do so if the government also conducts a large scale asset purchase program. In this paper, I consider one policy at a time.

Before proceeding, I briefly compare the policies discussed above to those in the literature. I model public asset purchases and lending to banks following Gertler and Karadi (2011), but I model equity injections differently. Gertler and Karadi (2011) assume that a unit of government equity has the same payout stream as a unit of security. The government is willing to pay a higher price than the prevailing market price of securities. They also assumed that government equity is non-divertable. Due to these assumptions, equity injections are effectively public asset purchases with a lump-sum transfer to banks, and hence, they have very similar effects on the economy. By contrast, I assume that government equity are identical to private equity in nature, which makes this policy similar to its counterpart in He and Krishnamurthy (2013). Negro et al. (2017) consider two types of assets. The private assets are illiquid and can be sold up to a certain fraction of holding in each period. Government bonds and money, on the other hand, are liquid and not subject to this constraint. Therefore, the unconventional policy in their paper is to sell liquid assets and buy illiquid assets, roughly in line with the evolution of the asset side of the Federal Reserve balance sheet during the crisis. Illiquid assets are similar to securities in this paper. Thus, the asset swap policy in Negro et al. (2017) can be seen as a mix of discount window lending and public asset purchases in this paper.

So far, using unconventional policy is costless to the economy. Any level of interventions between just offsetting the financial friction and fully crowding out private funds is equally optimal. In the literature, policy costs are either abstracted in the analysis (Negro et al., 2017; Quint and Rabanal, 2017) or modeled in a reduced form (Gertler and Karadi, 2011; Dedola, Karadi and Lombardo, 2013; Foerster, 2015). To the best of my knowledge, there is only one paper (Kandrac, 2014) that attempts to are applied to long-term government bonds. These modifications should not change the main implications of this paper.

\textsuperscript{11}There is a natural upper bound that the government cannot buy more assets than those available on the market. We are unlikely to hit this bound with reasonable calibration.
evaluate the potential costs of the Fed’s QE in the sense that the Federal Reserve, as a dominant buyer, may deteriorate the financial market functioning. To form policy trade-offs, I follow the literature in the main text and assume that the government must pay a reduced form resource cost on its holding of securities, equity, and its lending to banks:

$$\Gamma_{t,t} = \tau AP_{i,t}^2.$$  

This cost represents inefficient public activism in private financial markets or the cost of strengthened financial surveillance.\textsuperscript{12} To facilitate a comparison across policies, I assume the same $\tau$ for each policy. However, we should keep in mind that the intervention costs are arguably smaller for high-grade instruments like commercial papers, agency debt and mortgage backed securities (Gertler and Kiyotaki, 2010). Sensitivity of the optimal policy to this cost will be discussed in section V.

In appendix B, I consider the policy cost as a distorting effect of the tax that is necessary to finance at least a proportion of the policy spending. I conclude that the distortionary tax is too expensive to finance the unconventional policy. A comprehensive investigation of how unconventional policy is financed and the associated costs is left for future work.

\textbf{E. Aggregation and the market clearing conditions}

The law of motion for the aggregate equity held by households is given by

$$E_t = \frac{E_{t-1}}{E_{t-1} + E_{g,t-1}} (1 - r_{n,t,t}) \left( q_{t-1} S_{h,t-1} r_{h,t} + q_{t-1}^* S_{f,t-1} r_{f,t}^* - D_{h,t-1} r_{t-1}^* - D_{h,t-1}^* r_{t-1}^* - D_{g,t-1} r_{g,t-1}^* \right) + \varpi \left( \omega_{t-1} + AP_{1,t-1} \right),$$

where the last term is the start-up funds received by new banks. Finally, the model is closed by market clearing conditions on the goods and security markets

$$Y_t + Y_t^* = C_t + C_t^* + G_t + G_t^* + \Gamma_t + \Gamma_t^* + f \left( K_{t-1}, I_t \right) + f \left( K_{t-1}^*, I_t^* \right),$$

$$q_t S_t = q_t (S_{h,t} + S_{h,t}^*) + AP_{1,t},$$

$$q_t^* S_t^* = q_t^* (S_{f,t} + S_{f,t}^*) + AP_{1,t}^*.$$
III. The numerical method

A. Simulation method

Dealing with the occasionally binding constraints (OBCs). — Stochastic models with OBCs are typically simulated using global methods. However, the model I describe above and the corresponding model to solve the Ramsey policy contain too many state variables to be solved even by methods that are explicitly designed to deal with large state spaces, such as that of Maliar and Maliar (2015). Guerrieri and Iacoviello (2015) provide a fast algorithm based on piecewise linearization which, however, gives certainty equivalent results. I employ the approach proposed by Holden (2016b,a). This approach supports second-order approximation to evaluate welfare and captures the risk of the constraint binding in the future. Dynare-OBC13 created by the same author is a toolkit to implement this approach, which roughly consists of the following steps. First, the model is Taylor approximated up to a chosen order around the deterministic steady state. All inequalities are ignored during the approximation, but enter the approximated model. Then, the approximated model with OBCs can be solved under perfect foresight using Holden (2016b)’s algorithm. We can simulate a stochastic version of the model using the idea of the extended path (EP) algorithm of Fair and Taylor (1983). For a model that is linear apart from the OBCs (due to first-order approximation), the simulation is certainty equivalent. For a model that is non-linear apart from the OBCs (due to higher-order approximation), the simulation captures the risk stemming from non-OBC nonlinearity so that the slopes of variables’ responses change at the bound. To further capture the risk of hitting the bound in the future, Holden (2016a) applies a modified version of the stochastic extended path (SEP) algorithm of Adjemian and Juillard (2013). To form expectations, the SEP algorithm involves integrating the model over a certain number of periods of future uncertainty. I integrate over 50 periods and find no considerable change from integrating over longer periods. I refer to the solutions based on the EP and the SEP algorithm as EP alike and SEP alike solutions, respectively. I will compare these two solutions to show the precautionary motives to avoid the bound.

Dealing with the indeterminate portfolio. — An issue related to the perturbation based method is indeterminacy of the equilibrium portfolio $\alpha_{p,t}$. According to Devereux and Sutherland (2011), a second (third, fourth, ...) order approximation of the model is generally enough to pin down up to zero (first, second, ...) order term(s) of the portfolio, while up to the first (second, third, ...) order terms of the portfolio are relevant for the second (third, fourth, ...) order approximated model. The zero-order term is the deterministic steady state. Devereux and Sutherland (2011) propose a general solution as follows. Conjecturing $\alpha_{p,t}$ as a (N-1)th order

polynomial of the model’s state variables, we can use this conjecture to replace equation 11. Then, we can simulate the Nth order approximated model and search for parameters in the conjecture such that the (N+1)th order approximation of equation 11 is satisfied.

As in Dedola, Karadi and Lombardo (2013), I only solve the zero-order portfolio. This is sufficient when I focus on dynamics and calculate a first-order approximation to the model. In the evaluation of welfare, the first-order terms of the portfolio are neglected as it is very demanding to compute portfolio dynamics in a model with OBCs.

B. Parameterization

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady-state discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Elasticity of discount factor</td>
<td>$\psi_b$</td>
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</tr>
<tr>
<td>Habit</td>
<td>$h$</td>
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</tr>
<tr>
<td>Risk aversion</td>
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</tr>
<tr>
<td>Weight on disutility of labour</td>
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</tr>
<tr>
<td>Inverse elasticity of labour supply</td>
<td>$\varphi$</td>
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</tr>
<tr>
<td>Capital share</td>
<td>$\alpha$</td>
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</tr>
<tr>
<td>Inverse elasticity of investment to the capital price</td>
<td>$\eta$</td>
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</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>Steady-state survival probability of banks</td>
<td>$1 - \bar{r}_n$</td>
<td>0.972</td>
</tr>
<tr>
<td>Transfer rate from households to new banks</td>
<td>$\bar{w}$</td>
<td>0.0045</td>
</tr>
<tr>
<td>Steady-state fraction of divertable assets</td>
<td>$\theta$</td>
<td>0.2457</td>
</tr>
<tr>
<td>Fraction of un-divertable discount window borrowing</td>
<td>$\theta_g$</td>
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</tr>
<tr>
<td>Reduced form policy costs</td>
<td>$\tau$</td>
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</tr>
<tr>
<td>Persistence of financial shock</td>
<td>$\rho_\theta$</td>
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</tr>
<tr>
<td>Standard deviation of financial shock</td>
<td>$\sigma_\theta$</td>
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<tr>
<td>Persistence of capital quality shock</td>
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<tr>
<td>Standard deviation of capital quality shock</td>
<td>$\sigma_\xi$</td>
<td>0.05</td>
</tr>
<tr>
<td>Persistence of productivity shock</td>
<td>$\rho_A$</td>
<td>0.95</td>
</tr>
<tr>
<td>Standard deviation of productivity shock</td>
<td>$\sigma_A$</td>
<td>0.0044</td>
</tr>
</tbody>
</table>

Table 1 shows the parameterization of the economy based on the second-order approximated model where no policy is employed. The behaviour of such an economy is reported in appendix A. The quantitative results are similar to that of Dedola, Karadi and Lombardo (2013). However, I show in the next section how the OBC setting matters for the optimal policy.

Parameters concerning the non-financial sector are standard in the literature and are borrowed from Dedola, Karadi and Lombardo (2013). I depart from the litera-
ture by choosing a steady state in which the financial constraints are slack. The constraints may bind endogenously due to adverse shocks. I define a financial crisis as the occasion when the financial constraints are tight such that the credit spread is two standard deviations above its mean. Since 1983 in the U.S., this definition corresponds to the early 21st century recession and the 2007 - 2008 financial crisis. The unconditional probability of a financial crisis is 5.28% under my calibration.

There are three parameters in the financial sector. Following Gertler and Kiyotaki (2010), I choose a survival rate implying that, on average, bankers survive for around 8 years. Next, I set the steady-state leverage ratio to 4. Given that the financial constraint is slack in the steady state, this pins down \( \varpi = \left( 1 - \frac{1-r_n}{\beta} \right) / \phi \).

The steady-state proportion of divertable assets \( \theta \) is chosen such that the financial constraint is close to being binding in the steady state.

There are three exogenous variables in each country, namely productivity, \( A_t \), capital quality, \( \xi_t \), and the fraction of divertable assets, \( \theta_t \). Each of them follows an uncorrelated AR(1) process. Parameters for the productivity are taken from the estimate of Heathcote and Perri (2002). Parameters for the capital quality follow Gertler, Kiyotaki and Queralto (2012), the working paper version of which provides the microfoundations in its appendix. I choose a standard deviation of \( \theta_t \) to make the mean of the annualized spread about 2.35%. However, without features such as liquidity premia and true default risk, I inevitably overestimate the standard deviation of the spread. Or I would underestimate the spread mean if I calibrated the model to match the standard deviation.

As a robustness check, I also consider other relevant calibrations. For example, in an alternative parameterization, \( \bar{r}_n \) is set to match a dividend rate of 5.15% made by the largest 20 U.S. banks during 1965–2013. Quint and Rabanal (2017) use GMM to estimate a similar model with nominal frictions, a Taylor rule, and an always binding financial constraint. They find a much larger steady-state leverage of 16. This is probably not very surprising as Gertler and Kiyotaki (2010) consider the leverage of 4 as an average across sectors with vastly different financial structures. All alternative calibrations change my results quantitatively but do not change the main conclusions.

There are two policy-related parameters. For the proportion of un-divertable dis-

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14With integrated security markets and a steady state in which the financial constraints are not binding, there is indeterminacy between the bank’s total assets, \( \omega_t \) and \( \omega_t^* \). To pin down these variables, it is sufficient to introduce an asset adjustment cost to the bank’s balance sheet:

\[
\frac{\psi_{\omega}}{2} \left( \omega_t + AP_{t,t} - SteadyState \right)^2,
\]

where \( \psi_{\omega} = 10^{-5} \) in practice.

15This figure is calculated from quarterly data of Moody’s seasoned Bbb corporate bond yield relative to the yield on 10-year treasury constant maturity, 1983q1-2017q1, retrieved from FRED, Federal Reserve Bank of St. Louis. By contrast, Gertler and Kiyotaki (2010) and Dedola, Karadi and Lombardo (2013) use 1%, which is roughly the mean of Moody’s seasoned Aaa corporate bond yield relative to the yield on 10-year treasury constant maturity.

16Recently Gertler, Kiyotaki and Prestipino (2017) are working on a similar model where banks can default on their debts possibly due to a bank run.

17This number is calculated by Swarbrick, Holden and Levine (2017) using the Baron (2017) dataset.
count window lending, I consider two values. When \( \theta_g = 1 \), discount window lending cannot be diverted. This choice makes the three policies more comparable because the funds of the two other policies are effectively also non-divertable. However, as noted by Gertler and Kiyotaki (2010), there is likely to be a capacity constraint on the central bank’s ability to retrieve funds. Thus, the true \( \theta_g \) could be less than one and I suppose \( \theta_g = 0.8 \). The second policy parameter is the intervention cost, \( \tau \). Since it is difficult to measure the inefficiency of public activism in private financial markets, I set it to a number small enough to allow me to focus on the benefits of interventions. I consider larger intervention costs in section V.

IV. The Ramsey cooperative policy

In this section, I focus on a benchmark case in which the Home and Foreign governments conduct cooperative policy, and the policy is financed by government bonds (or equivalently a lump-sum tax) and only entails small reduced form cost.

The government in each country jointly maximizes a single objective function - the life-time utilities averaged across countries - by committing to a state-contingent plan of one of the three policies discussed previously. Policy makers solve the following problem:

\[
-\mathbb{E}_t \sum_{j=t}^{\infty} \beta_{t,t+j}^{g} \left[ \frac{(c_{t+j} - h c_{t+j-1})^{1-\sigma}}{1-\sigma} - \lambda_{t+j}^{1+\varphi} + \frac{(c_{t+j}^* - h c_{t+j-1}^*)^{1-\sigma}}{1-\sigma} - \lambda_{t+j}^*^{1+\varphi} \right]
\]

subject to all the equilibrium conditions of private agents,\(^{18}\) where \( \beta_{t,t+1}^{g} \) is identical to the households’ discount factor. In solving for the optimal policy, I follow the “timeless” perspective advocated by Woodford (2003). However, the resulting system is very difficult to simulate when I employ the SEP alike solution. Thanks to the small policy cost, I can approximate the true Ramsey problem arbitrarily well by a slightly simplified problem. Specifically, I make the following assumption

ASSUMPTION 1: The governments conduct policy subject to \( \lambda_t = \lambda_t^* = 0 \), instead of \( \lambda_t \geq 0 \) and \( \lambda_t^* \geq 0 \).

According to equation 10, the positive multiplier implies a positive spread. In the true Ramsey problem, the governments can tolerate positive spreads (roughly at \( 10^{-6} \) given my main calibration) to the extent that the marginal benefits of reducing the spreads equal the marginal costs of interventions. Hence, we have \( \lim_{\tau \to 0^+} \lambda_t = \lim_{\tau \to 0^+} \lambda_t^* = 0 \) in the Ramsey equilibrium, which justifies assumption 1 when \( \tau \) is small. The qualitative results in this section remain unchanged if I do not make assumption 1.

\(^{18}\)These conditions include two inequalities, \( \lambda_t \geq 0 \), \( OBC_t \geq 0 \), a slackness condition \( \lambda_t OBC_t = 0 \), and their Foreign counterparts. It can be verified that \( \lambda_t \geq 0 \) is a redundant constraint. Intuitively, \( \lambda_t \geq 0 \) roughly implies \( r_{t,t+1} - r_t \geq 0 \) according to equation 10, which a benevolent policy maker would never violate.
A. Impulse response analysis

Following the literature, I consider policy responses to two shocks. First, I consider a negative capital quality shock $\xi_t$ in Home. The impacts of this shock can be decomposed into two stages. In the first stage, this shock has real impacts similar to those of a productivity shock. Specifically, the return on securities is low, which reduces the banks’ net worth by a multiplier of their leverage. Consequently, the financial constraints may be binding and the second stage “financial accelerator” effects take place. In the case when the financial constraints bind, banks must fire sell their assets. Since banks take asset prices as given, it is their externality that the firesale depresses asset prices and further impairs their net worth. As a result, the financial constraint binds even tighter. In the second stage, banks are inefficient financial intermediaries. The second shock $\theta_t$, referred to as a financial shock, tightens the financial constraints directly (Perri and Quadrini, 2011; Dedola and Lombardo, 2012; Negro et al., 2017), and hence, only has the second-stage effects. How the economy responds to these shocks under no policy intervention can be found in appendix A.

![Graphs showing impulse responses](image)

**Figure 1. Cooperative policies under Home capital quality shock**

Figures 1 and 2 plot the one standard deviation impulse responses for three variables, namely banks’ assets financed by private funds, private equity, and policy spending as a fraction of domestic asset value. Other variables are not shown be-
cause they behave as if there were no financial constraint, i.e., like the black broken lines in Figures A1 and A2 in the appendix. The results obtained with public asset purchases, discount window lending, equity injections, and no policy are shown in red, blue, green, and black, respectively.

By design, all unconventional policies address the financial constraints and hence exhibit only the second-stage effects of the shocks. They share two common roles. First, policy provides public funds to the economy when banks are inefficient in intermediating private funds. The public funds support asset prices at their optimal level and hence undo any effect of the bank’s externality. Consider, for example, public asset purchases. Banks can sell more of their assets and retain a higher level of private equity (or net worth) than the no policy case. As a result, public funds effectively crowd out private funds. This crowding-out effect implies that private equity grows slowly, however. To stabilize the financial sector, the government must also exit slowly from the policy until banks accumulate enough private equity. The exit path needs to be consistent with the growth path of private equity.

Private equity grows at the same rate under all policies because public equity, discount window lending, and securities (purchased by the government as an opportunity cost for banks) are equally valued. However, banks’ assets revert much faster under equity injections than under the other two policies, which is financed by a fast growth of deposits banks receive. This implies that banks must face a much relaxed financial constraint under equity injections. The second role of policy is to relax the
financial constraint 9, which reduces the crowding-out effect. The smaller crowding-out effect allows banks to depend less on policy, and allows the government to exit faster from policy. The fact that it is hard to discriminate among policies in terms of the second role makes the analysis in this section nontrivial even under assumption 1. Since the economy must exit from interventions eventually and conducting the policy entails a non-zero cost, we would like to exit as soon as possible. Equity injections clearly constitute the most efficient policy. Discount window lending is at most as efficient as public asset purchases in the extreme case of $\theta_g = 1$. According to equation 9, discount window lending with a smaller $\theta_g$ is less efficient in relaxing the financial constraint.

On the international dimension, the policy responses are asymmetric. Following a Home capital quality shock, Foreign interventions are roughly half as strong as Home interventions. This is because banks hold a portfolio that consists of more domestic assets ($\alpha^p = 0.4$). Since Home banks are more affected by the shock, they benefit more from the purchases of Home assets. However, discount window lending and equity injections are not immune to this portfolio effect and these two policies affect the whole portfolio by design. Following a Home financial shock, however, Foreign need not intervene at all. A financial shock does not have the first-stage real impacts. Foreign banks would only be affected by depressed asset prices worldwide if there were no policy response. Home interventions fully stabilize asset prices in both countries so the Foreign country can enjoy a free ride.

B. Precautionary effects

The slow exit from interventions that we see above is partially shaped by precautionary effects arising from the future risk of binding financial constraints. In figure 3, I compare the EP alike solution (black broken lines) and the SEP alike solution (coloured solid lines) to the impulse responses to a Home financial shock. The SEP alike solution is also employed in the previous subsection. If no risk of future bonding constraints is taken into account, unconventional policy ends in the same period when the economy escapes from the financial constraint. By contrast, if this risk is predicted by both the private agents and the government, the precautionary policy is relatively stable and persistent in the longer term while the economy has escaped from the constraint even sooner than in the former case. Intuitively, the policy should give some precautionary protection to banks for a few periods after a crisis, during which the banks, though having enough net worth to escape from the financial constraints, are vulnerable to another adverse shock.

V. The Ramsey noncooperative policy

Without international cooperation, everything else in the cooperative policy problem applies, but each government now maximizes domestic welfare using domestic instruments, taking the entire path of foreign instruments as given. The equilibrium

\footnote{Without assumption 1, foreign interventions approach zero as the policy cost approaches zero.}
is an outcome of an open-loop dynamic Nash game. Following Coenen et al. (2007),
taking the entire path of foreign instruments as given is an unrealistic assumption
but a necessary simplification to the problem. Since policy cost plays an impor-
tant role in noncooperative policy, it is important not to make assumption 1, which
only provides reasonable approximation to the true Ramsey problem when the cost
approaching zero. However, without assumption 1, I can only simulate the model
reasonably fast with the EP alike solution. I confine my discussion to the most
efficient policy, equity injections. Other policies generate similar results.

As shown in figures 4 and 5, the noncooperative equilibrium is identical to the
cooperative equilibrium when the unconventional policy is very cheap to use ($\tau =
0.0001$). Increasing $\tau$ makes it favorable to share the intervention cost across coun-
tries. Given a Home shock, this means fewer interventions by the Home government
and more by the Foreign government, which results in higher credit spreads glob-
ally. However, after a certain point, further increasing $\tau$ affects the noncooperative
policy more than the cooperative policy, and the cooperation gain becomes a posi-
tive number. The tipping point is about $\tau = 0.01$ for the capital quality shock and
about $\tau = 0.001$ for the financial shock. Under a Home shock, the noncooperative
equilibrium features excessive interventions in Foreign and insufficient interventions
in Home. The consequences of noncooperation for credit spreads also depend on the
shock. Under a capital quality shock, the spreads are higher and more persistent in

Note: Coloured solid lines are SEP alike results and black broken lines are EP alike results.

Figure 3. Cooperative policies with and without precautionary effects
Note: The cooperative equilibrium is shown in red solid lines, and the noncooperative equilibrium is shown in black broken lines.

Figure 4. Noncooperative policy responses to a Home capital quality shock

the noncooperative equilibrium than in the cooperative equilibrium. Under a financial shock, however, the noncooperative policy achieves smaller spreads in the short run. My results are similar to the results of the optimal noncooperative simple rule studied in Dedola, Karadi and Lombardo (2013) but different in the following ways. In their paper, increasing $\tau$ always reduces the policy responses in both countries. After a certain point, policy responses in the noncooperative equilibrium quickly approaching zero in both countries. Further increasing the cost also makes even policy responses in cooperative equilibrium equal to zero. By contrast, in my context, policy responses are re-balanced across countries to tackle the increase in the cost.

VI. Simple Rules

It is well known that the Ramsey solution is silent with regard to policy implementation. In this section, I examine the extent to which various simple feedback rules can approximate the Ramsey policy. I focus on the cooperative case. The noncooperative simple rule is studied by Dedola, Karadi and Lombardo (2013). The benchmark rule proposed in the literature is

$$ P_{t,t} = \kappa \bar{E}_t (r_{k,t+1} - r_t) $$
Note: The cooperative equilibrium is shown in red solid lines, and the noncooperative equilibrium is shown in black broken lines.

**Figure 5. Noncooperative policy responses to a Home financial shock**

in Home and similarly in Foreign where $P_{i,t}$ is the policy spending proportional to domestic asset value, $i$ indexes the three policies and parameter $\kappa$ determines the aggressiveness of the interventions. I refer to this rule as the spread rule. Foerster (2015) proposes an improvement on equation 15 by adding an autoregressive term (AR spread rule):

$$(16) \quad P_{i,t} = \kappa (1 - \rho P) E_t (r_{k,t+1} - r_t) + \rho P_{i,t-1}. $$

Based on my discussion of the first role of policy in section IV.A, another rule that is intuitively promising is to respond to asset price gaps (price rule)

$$(17) \quad P_{i,t} = -\kappa (\ln q_t - \ln q_{t,\text{potential}}),$$

where $q_{t,\text{potential}}$ is the asset price that would occur in a world without the financial constraints, and $\ln q_t - \ln q_{t,\text{potential}}$ is the asset price gaps (in percentage). The negative sign before $\kappa$ reflects the fact that the asset price is low during a financial crisis.

The unconditional welfare loss and the optimized policy rule parameters are reported in table 2. Once more, the government needs to pay a reduced form cost
TABLE 2—Unconditional welfare loss under optimized simple rules

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Spread rule</td>
<td>6.84 (150)</td>
<td>7.43 (150)</td>
<td>4.30 (115)</td>
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<tr>
<td>AR spread rule</td>
<td>5.80 (150, 0.9)</td>
<td>6.20 (150, 0.9)</td>
<td>4.18 (115, 0.9)</td>
</tr>
<tr>
<td>Price rule</td>
<td>3.83 (450)</td>
<td>4.66 (450)</td>
<td>2.49 (450)</td>
</tr>
</tbody>
</table>

Note: The numbers outside the brackets are base points of the unconditional welfare loss under an optimized rule relative to that under the optimal allocation. The relative welfare loss of optimal policy is close to zero. The numbers inside the brackets are the parameters of the optimized rules.

with $\tau=0.01$. First note that, regardless of the rule, the most efficient policy is again equity injections, followed by public asset purchases, and discount window lending. Second, regardless of the policy, adding an autoregressive term always improves the spread rule, but both spread-based rules always generate a much larger welfare loss than the price rule.

To better understand how these rules differ, it is useful to consider the impulse response of the economy under each rule. For the purpose of illustration, I only show the responses of equity injections in figures 6 and 7. Consider first the spread rule shown in red. Subject to either shock, the spread rule is not aggressive enough to stabilize the financial sector, leaving a significant gap in asset prices and spreads. As a result, there is also a substantial fluctuation in the consumption gap. However, it exits from interventions at roughly the same speed as the Ramsey results. Adding an AR term (shown in blue) makes the interventions more persistent in response to a financial shock, but less persistent in response to a capital quality shock. Generally, the AR spread rule features stronger interventions than the naive spread rule. It allows relatively high spreads and price gaps in the first few periods, but makes them smaller thereafter. So the fluctuation in consumption is smaller and the life-time utility is improved. This improvement is particularly clear for the capital quality shock. Interestingly, the AR spread rule features hump-shaped responses to the shock, which seem to capture the observation that central banks tend to strengthen unconventional policy at the early stage of the crisis.

The price rule (shown in green) characterises the Ramsey outcomes. There is barely any fluctuation in the consumption gaps, the asset price gaps, and the spreads. This is expected because the unconventional policy works through the capital gain channel. The price rule still generates a substantial welfare loss because the government cannot customize rule parameters to shocks with different nature and sources. Consequently, facing a Home shock, interventions are relatively weak in Home but relatively strong in Foreign, and visa versa when facing a Foreign shock. In addition, the price rule is not practicable to respond to a capital quality shock, or any other real shock, because the potential asset price is not observable.

VII. Conclusions

I study the Ramsey optimal unconventional monetary policy in a two-country version of Gertler and Kiyotaki (2010), with and without cross-country policy coop-
The gap variable is the difference between the actual variable that is realized in the model and its counterpart that would be realized in a world without the financial constraints. C denotes consumption and Q denotes the asset price.

The main findings are threefold. First, I suggest that unconventional policy should be designed to address the financial constraint that banks face. Second, after giving a strong initial response, the central bank should exit slowly from the policy even after a financial crisis has passed. Third, if cross-country policy cooperation is not imposed, the interventions are too strong in one country and too weak in the other. The cooperation gain is zero if the intervention cost is small. Increasing the intervention cost to a certain point lets the cooperation gain positive. I also evaluate several simple feedback rules. I find that the policy should respond to asset prices facing a financial shock but respond to the credit spread with an autoregressive term facing other shocks.

Naturally, this paper is subject to several limitations. On one hand, this paper focuses on the capital gain channel to keep the problems at hand relatively simple. Section I discuss many other possible channels. These channels may open for some but not all unconventional policies. For instance, forward guidance works mainly through the signalling channel. One important channel missing in this paper is the bank's lending channel. In my model, there is no friction between banks and non-financial firms. Reducing credit spread automatically boosts lending and investment. This is not necessarily true in data according to Chakraborty, Goldstein and
Figure 7. Response to Home financial shock, with different policy rules

Note: The gap variable is the difference between the actual variable that is realized in the model and its counterpart that would be realized in a world without the financial constraints. C denotes consumption, Q denotes the asset price.

MacKinlay (2017) and Acharya et al. (2017). Thus we need better capture banks’ behaviours and their heterogeneity. Nonetheless, I expect that the main conclusions of this paper can be generalized to a more sophisticated model. On the other hand, following the literature, I assume a reduced form cost for the unconventional policy. The cost of the unconventional policy, while critical to the optimal policy analysis, is still little understood. A recent paper by Kandrac (2014) summarises several potential costs that have been discussed in the Federal Reserve. These costs introduce new policy trade-offs and may make new implications for optimal policy.

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**Competitive Equilibrium**

Here, I examine the quantitative behaviours of the model without policy interventions, and the size of precautionary effects that originate from the OBCs.

**A1. Impulse response analysis**

I consider responses to two shocks, namely a standard deviation negative Home capital quality shock $\xi_t$ and a standard deviation positive Home financial shock $\theta_t$. I use black broken lines to represent variables that would be realized in a financially frictionless world (potential variables) and red solid lines to represent actual variables.

As shown in figure A1, the capital quality shock creates a deep and persistent global recession. Thanks to the OBC setting, I can decompose the effects of this shock into two “stages”. In the first stage, the shock has a real impact shown by the black broken lines. When the financial constraints are binding, there are second-stage “financial accelerator” effects. In this case, banks must firesale their assets,
which depresses asset prices and further impairs their net worth. As a result, the financial constraints bind even tighter. The positive spreads suggest that banks are inefficient financial intermediaries. Overall, the second-stage effects amplify the first-stage effects. The shock propagates to Foreign via the equalization of asset returns across countries, as suggested by equation 11, and a diversified portfolio. With a portfolio featuring home bias ($\alpha_p = 0.4$), Foreign banks suffer a smaller loss on their net worth than Home banks. Figure A2 shows that the financial shock has only the second-stage impacts on the economy. The shock tightens the domestic financial constraint so domestic banks firesale their assets. Foreign banks would like to pick up those assets when the prices are low. However, Foreign banks have limited ability to do so due to their own financial constraint. Overall, the global investment drops and affects consumption and output in both countries symmetrically.

No matter whether the financial constraints bind forever or occasionally, the second-stage effects are much smaller than the first-stage effects. The literature notes at least two reasons why this is the case. Jakab and Kumhof (2015) suggest that banks in the real world provide financing through money creation but banks in most models accept pre-existing real resources from savers and then lend them to borrowers. They find that adding money creation to the model allows the same

Note: Black broken lines represent variables that would be realized in a financially frictionless world (potential variables), red solid lines represent actual variables, and the dotted line represents the steady state.
shocks to have much greater effects on the non-financial sector. Negro et al. (2017) highlight a role of the nominal rigidity and the zero lower bound without which the financial friction accounts for a drop in investment, but not in output, thanks to a rise in consumption.

A2. The precautionary effects of risk

If the financial constraints are always binding, banks always hold the maximum level of assets permitted by their net worth. However, in the OBCs setting, the amount of assets held by banks is also affected by the likelihood that financial constraints are binding in the future. This is known as the precautionary effects of risk. To visualize the effects, I simulate the model with and without integrating over future uncertainty. The difference between the SEP alike solution and the EP alike solution shows the precautionary effects.

Table A1 compares the sample mean and the standard deviation of a few variables computed from the two simulations. First, the precautionary effects reduce the probability of a financial crisis (defined in the main text) by 0.2%. To avoid being constrained, banks would like to hold fewer assets on average. If banks do not do this, they suffer from a larger volatility of asset holding. The precautionary effects...
also reduce both the mean and the standard deviation of the spread. However, the precautionary effects are small on non-financial variables. In a smaller open economy model with an occasionally binding collateral constraint, Mendoza (2010) also finds that long-run business cycle moments are largely unaffected by precautionary savings. Gertler, Kiyotaki and Prestipino (2017) show that the nonlinearity induced by the OBCs are quantitatively small relative to the nonlinearity induced by the bank run mechanism they add to their early model.

**Fiscal distortion**

While policy cost plays an important role in shaping the optimal unconventional policy, there is no hard evidence to quantify it in the reduced form. Another possible form of policy cost is a distorting effect of a tax by which the unconventional policy must be financed. To simulate the model in this case, again, I do not make assumption 1.

Suppose that the most efficient policy, i.e. equity injections, is solely financed by a sales tax, figure B1 plots the cooperative responses to a Home financial shock. The unconventional instruments are not shown because they are only used passively. The true instrument here is the sales subsidy, which is financed by a negative unconventional policy. This fiscal stimulation boosts investment and hence asset prices. Banks earn a fortune from their investment and escape from the financial constraints. In contrast to the unconventional policy, the fiscal policy addresses a financial crisis from the demand side of capital. However, the credit spreads are very large, suggesting inefficient financial markets. Since these results are obtained from maximizing welfare, varying the unconventional instrument actively must worsen welfare because the necessary sales tax to finance the active unconventional policy would have a strong distortionary effect on the economy. In other words, the distorting tax is an expensive source of fund to finance the unconventional policy. This result is robust to the nature of the shock, the chosen unconventional instrument, the chosen fiscal instrument (such as a labour tax), and an alternative reasonable parameterization of the model such as a smaller Frisch elasticity of labour supply.\(^\text{20}\) Nonetheless, this result is not very surprising because the benefits of the

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\(^{20}\)Brendon, Paustian and Yates (2011) study a similar question where the tax follows a simple rule and the government budget is balanced by government debts. It is not surprising that the distorting tax plays a minor role in their paper.
unconventional policy are bounded from above by the welfare losses caused by the second-stage effect of a shock. In this and similar models, such losses are relatively small (see, Dedola, Karadi and Lombardo 2013, and appendix A) so the benefits of the unconventional policy are dominated by the distorting effect of the tax. If a smaller proportion of unconventional policy is financed by the tax, the active unconventional policy becomes cheaper but the active sales subsidy becomes more expensive. The threshold below which the unconventional policy is active is about 15% of the unconventional policy financed by the sales tax.