Competition in the Presence of Individual Demand Uncertainty

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Abstract

This paper introduces individual demand uncertainty into Hotelling’s model of horizontal product differentiation. We show that in the unique equilibrium, firms offer advance purchase discounts, inducing consumers to trade-off an early (uninformed) purchase at a low price against a late (informed) purchase at a high price. Relative to the monopolistic benchmark, competing firms offer larger discounts, leading to an increase in the number of uninformed purchases and hence a reduction in welfare. Competition also has a negative effect on consumer surplus if the degree of product differentiation is high. However, since profits are decreasing in the level of demand uncertainty, competing firms have an incentive to inform their customers which is absent under monopoly. A ban on advance purchase discounts leads to an increase in consumer surplus and a decrease in profits under monopoly but has the opposite effect under competition.

Keywords: Individual Demand Uncertainty, Advance Purchase Discounts, Competition.

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1 Introduction

In markets for differentiated products, the introduction of competition provides consumers with three well known benefits. Competition leads to price reductions bringing production closer to its efficient level. In addition, competition stimulates the choice of product characteristics in accordance with the consumers’ preferences. Finally, competition has a positive effect on the number varieties that are introduced into the market. In this paper we show that competition may also come at a cost.

Our argument is based on the observation that in many markets consumers are uncertain about their individual preferences. For example, at the time of purchase, a consumer choosing between a Thursday flight and a Friday flight may not know (perfectly) which option he will prefer at the time of consumption (travel). Further examples include the advance booking of theater tickets, pre-orders of music albums, or the acquisition of wine for long-term storage.

In this paper we provide a tractable and intuitive description of competition in the presence of individual demand uncertainty by adding an advance purchase period to Hotelling’s model of product differentiation. In the advance purchase period consumers are uncertain about their individual preferences. Each consumer merely receives a signal about the identity of his preferred product. The signals’ precision is the same for all consumers and constitutes a key parameter of the model. As in the Hotelling model, consumers differ in the degree to which they differentiate between the two products. Firms can commit to a price schedule in advance.

We first show that in equilibrium firms will implement advance purchase discounts (APDs) to screen consumers according to their types. Consumers with weak preferences (e.g. leisure travelers) are induced to buy in advance, whereas consumers with strong preferences (e.g. business travelers) will postpone their purchase until their individual demand uncertainty has been resolved. In the presence of individual demand uncertainty, intertemporal price discrimination allows firms to optimally exploit the consumers’ trade–off between an early (uniformed) choice at a low price and a late (informed) choice at a high price.

A comparison with the monopolistic case in which both products are sold by a single firm reveals that competition leads to larger APDs and hence to a greater fraction of consumers
who are served in advance at a discount. In view of the firms’ desire to mitigate competition by differentiating themselves from their rivals, this finding comes as a surprise. Competition induces firms to serve a greater fraction of consumers in advance, even though at this stage consumers perceive their products as more homogeneous. The result is driven by the firms’ attempt to capture those consumers in advance who might become their rival’s customer in the future. This is similar to the occurrence of customer poaching in markets with switching costs (see Chen (1997) and Fudenberg and Tirole (2000)) with the difference that consumers are captured ex ante rather than ex post. Our result is in line with recent empirical evidence provided by Asplund et al. (2008) who show that Swedish regional newspapers with a local rival are about twice as likely to offer introductory discounts as regional monopolies.

In the presence of individual demand uncertainty, an increase in the fraction of advance purchases implies a greater risk of mismatch between consumer preferences and product characteristics. Since in our setting quantity effects are absent, this is the only effect on welfare. We therefore find that competition leads to a reduction in total surplus. With respect to consumer surplus, the increased mismatch due to competition might be compensated by a decrease in prices. We show that competition increases aggregate consumer surplus if and only if the degree of product differentiation is low. If the degree of product differentiation is high then competition increases the surplus of consumers with weak preferences but decreases the surplus for consumers with strong preferences.

We also investigate the effects of a ban on APDs by restricting firms to charge a uniform price across periods. Banning APDs is shown to have a positive effect on total surplus, independently of market structure. However, market structure determines whether it is firms or consumers who benefits from such a ban. In particular, banning APDs leads to an increase in consumer surplus and a decrease in profits under monopoly but has the opposite effect under competition. Under competition, the firms ability to price discriminate intertemporally puts additional downward pressure on prices, making APDs desirable from the consumers’ viewpoint.

A final contribution of the paper, is to highlight a novel relation between market structure and individual demand uncertainty. We show that in the face of competition, profits are decreasing in the degree of individual demand uncertainty whereas for a monopolist, profits
are increasing. This implies that the incentives for monopolization are stronger in markets with a higher degree of demand uncertainty. The reason for this result is that an increase in uncertainty makes products more homogeneous from the viewpoint of the uninformed consumer. As a consequence, competition becomes stronger at the advance selling stage, leading to a reduction in profits.

Related literature

The existing literature on intertemporal price discrimination with individual demand uncertainty lacks the analysis of competition: DeGraba (1995), Courty and Li (2000), Courty (2003), Möller and Watanabe (2010), and Nocke, Peitz, and Rosar (2011) all consider the monopolist’s problem. An exception is Gale (1993) who features a duopoly but assumes that products are homogeneous ex ante with the immediate consequence that price equals marginal cost in the advance purchase period. When products are differentiated not only ex post but also ex ante, the effect of competition on the sellers’ ability to screen buyers intertemporally by use of APDs is still unclear.

APDs have been derived as optimal selling mechanisms in other settings. Dana (1998) derives an APD for a perfectly competitive industry characterized by aggregate demand uncertainty. His analysis suggests that market power may not be necessary to explain the observation of an APD. Firms use APDs in order to reduce the risk of holding unutilized capacity. Similarly, Gale and Holmes (1993) show that an airline may use APDs to divert consumers from a peak period where demand exceeds capacity to an off-peak period. In our setting, aggregate demand is certain and capacity is neither restricted nor costly. The sole purpose of an APD is the extraction of consumer (information) rents.

The effect of competition on price discrimination has attracted considerable attention in the empirical literature, especially in the field of airline pricing. Using a cross section study of the 1986 US airline industry, Borenstein and Rose (1994) find that routes characterized by higher levels of competition exhibit more price dispersion. Explanations for this relation are mainly based on third degree price discrimination and have been provided by Borenstein and Rose (1994), Borenstein (1985), and Holmes (1989). According to these models, business travelers are brand royal and hence price insensitive, so that relatively high prices can be
sustained for such customers even when competition reduces the prices targeted to more price sensitive customers, e.g. tourists. This explanation finds empirical support in Borenstein (1989) who shows that competition lowers a US carrier’s low-end prices but raises its high-end prices. Dana (1999) offers an alternative explanation based on the firms’ ability to offer a restricted amount of tickets in a number of differing fare classes. He shows that, in line with the empirical finding, intra-firm price dispersion increases with the number of firms.

Recent empirical evidence contradicts the earlier findings. Gerardi and Shapiro (2009) extend the work of Borenstein and Rose (1994) using panel data from 1993 to 2006 and find a negative effect of competition on fare dispersion. Similarly, Gaggero and Piga (2011), using fares posted on the Internet at specific days before takeoff, find that competition reduces intertemporal price dispersion for routes between the UK and the Republic of Ireland.

Common to the above studies is the use of the Gini coefficient as the empirical measure of price dispersion. In order to relate our theoretical findings to the existing evidence we determine the Gini coefficients implied by our model. Since competition not only increases the spread between the advance price and the spot price but also influences the distribution of buyers across the two selling periods, the overall effect is unclear. Interestingly, in our model, the effect of competition on the Gini coefficient turns out to depend on the level of individual demand uncertainty and the degree of product differentiation. Competition decreases price dispersion if and only if demand uncertainty is low and product differentiation is high. This new relation between price dispersion, demand uncertainty, and product differentiation may help to shed light on the controversy in the empirical literature, motivating further empirical research in this direction.

Since APDs influence the number of consumers who buy in advance and hence the information that is available to consumers at the time of purchase, our model also relates to the literature on information disclosure in market settings. Lewis and Sappington (1994) consider the issue of whether a monopolist should provide buyers with information about their individual preferences. In their model, each buyer receives a signal which is informative about the buyer’s valuation of the monopolist’s good. The monopolist chooses the signal’s precision (at zero cost). Lewis and Sappington identify a class of (informational) settings in which the monopolist will choose signals to be either perfect or completely uninformative. Although
an exogenous parameter of our model, the fact that monopoly profits are increasing in the
degree of demand uncertainty whereas duopoly profits are decreasing suggests that market
structure may have a crucial influence on the amount of information a market is supplied
with.

Contrasting with these cases where the supply of either full or no information is optimal,
Bar Isaac et al. (2010) find that a monopolist may want to choose an intermediate value for
the cost at which buyers are able to assess their individual valuation for the seller’s product.
This strategy of “intermediate marketing” constitutes a non-price means of discriminating
between buyers with high expected valuations (who buy the product without assessment)
and buyers with low expected valuations (who assess the product and buy only when their
valuation turned out to be high). In our model, discrimination works via prices but the
effect is similar: some consumers buy in advance, with imperfect knowledge of their preferred
product, whereas others postpone their purchase until their preferences are known. The
implied pattern of informed purchases at a high price and uninformed purchases at a low
price is also present in the duopoly model of informative advertising by Meurer and Stahl
(1994) who show that firms may employ a mixed pricing strategy by targeting informed
consumers with high prices and competing for uninformed consumers with low prices.

2 Model

We consider a market with two differentiated products $A$ and $B$, e.g. a Thursday flight and
a Friday flight between identical destinations. The products are located at the extremes of a
Hotelling line $[0, 1]$. There is a continuum of consumers with mass 1. Consumer preferences
are characterized by a location $\hat{x} \in [0, 1]$ along the Hotelling line. Consumers have unit
demands and derive utility $s > 0$ from consuming either product. They incur a disutility
equal to $t$ times the distance between their location and the product of their choice. The
parameter $t > 0$ measures the degree of product differentiation. The unit cost of production
is constant and identical across products. In order to simplify the exposition we normalize
unit costs to zero.

There are two periods. In period 1, consumers face individual demand uncertainty. In
period 2 all uncertainty is resolved. We assume that firms can commit to a price schedule 
\((p_1, p_2)\) where \(p_1\) and \(p_2\) denote prices in period 1 and 2 respectively.

Our main assumption is that each consumer knows the intensity with which he differentiates
between products \(A\) and \(B\) but is uncertain about the identity of his preferred product. For example, a traveller may very well be able to judge the importance of flying on the correct day, but may not know the correct date in advance. We capture this by assuming that in period 1, a consumer knows that he is located at one of two possible locations \(\hat{x} \in \{\frac{1}{2} - \frac{\sigma}{2}, \frac{1}{2} + \frac{\sigma}{2}\}\). The intensity with which the consumer distinguishes between products \(A\) and \(B\) is given by \(\sigma \in [0, 1]\) and constitutes the consumer’s private knowledge. A consumer with \(\sigma = 0\) is completely indifferent between products \(A\) and \(B\) whereas for a consumer with \(\sigma = 1\), the disutility from consuming the “wrong product” is maximal and given by \(t\). For example, flying on Thursday rather than Friday may imply a considerable degree of inconvenience for a business man whereas tourists may care little.

In period 1, each consumer privately receives a signal \(S \in \{A, B\}\) about the identity
of his preferred product. The signal’s precision, i.e. the probability with which the signal
reveals the consumer’s true preference, is given by \(\gamma \in (\frac{1}{2}, 1)\). The parameter \(\gamma\) measures
the degree of individual demand uncertainty and is the same for all consumers. For \(\gamma \to \frac{1}{2}\),
consumers face complete uncertainty whereas for \(\gamma \to 1\) consumers know their preferences
with certainty. One possible interpretation of \(\gamma\) is as the length of the time horizon during
which consumers are entitled to receive the (low) first period price \(p_1\). For example, due to the
temporally restricted availability of certain (reduced) fare classes, travellers are often required
to purchase a flight long in advance, implying a rather high degree of demand uncertainty.

For each consumer, the values of \(\sigma\) and \(S\), determine a location \(x \in [0, 1]\) on the Hotelling
line where the consumer is most likely to be located. \(x\) will be denoted as the consumer’s
\textit{type} and is given by \(x = \frac{1}{2} - \frac{\sigma}{2}\) for \(S = A\) and \(x = \frac{1}{2} + \frac{\sigma}{2}\) for \(S = B\). We assume that all
\(\sigma \in [0, 1]\) are equally likely and that for each \(\sigma\), the mass of consumers who receive signal \(A\)
is the same as the mass of consumers who receive signal \(B\). These assumptions imply that
types are uniformly distributed across \([0, 1]\).

In our setting, consumers who postpone their purchase until period two insure themselves
against the possibility of a sub-optimal product choice. Alternatively, consumer could achieve
such insurance by either buying both products in period one or by purchasing the second product in period two in case of a mismatch. For simplicity, we rule out these alternative forms of insurance. In particular, we assume that each consumer can purchase at most one product. This assumption might be motivated by budgetary restrictions on the side of the consumers or capacity constraints on the side of the firms. The consequences of relaxing this assumption are discussed in Section 4 following Proposition 2.

Finally, we make the standard covered market assumption that $s$ is sufficiently large to make all consumers purchase one unit in equilibrium. We will see that this requires

\[ s > \bar{s} \equiv \frac{9\gamma - 3}{-8\gamma + 14\gamma - 2}t \text{ or equivalently } t < \bar{t} \equiv \frac{8\gamma^2 + 14\gamma - 2}{9\gamma - 3}s. \]

In the following we first consider the monopoly case in which both products are offered by one single firm. Subsequently we will analyze the duopoly case where products are offered by two different firms. The comparison of the two cases will allow us to shed light on the effect of competition on intertemporal price discrimination in the presence of individual demand uncertainty.

## 3 Monopolistic benchmark

Due to symmetry a monopolist will choose identical price schedules for both products and we can restrict our attention to one side of the Hotelling line. Suppose the monopolist commits to a price schedule $(p_1, p_2)$. If $p_1 > p_2$, all consumers prefer to purchase in period 2 rather than in period 1. Hence we can assume without loss of generality, that the monopolist sets $p_1 \leq p_2$.

In the proof of Proposition 1, contained in the Appendix, we show that the monopolist will optimally sell to all consumers and offer a discount $\Delta p = p_2 - p_1 > 0$ to those customers who purchase in advance. Here we offer a derivation of the optimal discount which will make the interpretation of the subsequent results more intuitive.

For this purpose, consider a consumer of type $x \in [0, \frac{1}{2}]$. The consumer obtains utility $U_1(x) = s - p_1 - \gamma tx - (1 - \gamma)t(1 - x)$ from buying product $A$ in period 1. If he waits until period 2 then he will purchase product $A$ when located at $\hat{x} = x$ and product $B$ when located at $\hat{x} = 1 - x$. His expected utility is therefore given by $U_2(x) = s - p_2 - tx$. The difference

$$\Delta U(x) = U_2(x) - U_1(x) = t(1 - \gamma)(1 - 2x) - \Delta p$$

(1)
can be interpreted as the consumer’s value of waiting. It is strictly decreasing in \( x \in [0, \frac{1}{2}] \) and becomes zero at
\[
x_W = \frac{1}{2} - \frac{\Delta p}{2t(1-\gamma)} < \frac{1}{2}.
\] (2)

Given a discount of size \( \Delta p \in (0, t(1-\gamma)) \), consumers in \([0, x_W)\) prefer to wait until period 2 whereas consumers in \((x_W, \frac{1}{2}]\) prefer to purchase early.

The monopolist’s optimal selling strategy maximizes total surplus minus the sum of information rents. For a late buyer, surplus is given by \( s - tx \). A late buyer obtains information rents of size \( t(x_W - x) \) from pooling with the lowest type of consumer \( x_W \) who purchases in period two. The monopolist is therefore able to extract a rent \( s - tx_W \) from each type of consumer in \([0, x_W]\). Similarly, for an early buyer surplus is \( s - \gamma tx - (1-\gamma)t(1-x) \) and information rents are \( t[\frac{1}{2} - \gamma tx - (1-\gamma)t(1-x)] \). The monopolist can extract the rent \( s - \frac{t}{2} \) from each type of consumer in \((x_W, \frac{1}{2}]\). Due to (2), the choice of a discount \( \Delta p \) is equivalent to the selection of a cutoff \( x_W \). A high \( x_W \) is good for total surplus due to the elimination of the potential product mismatch for early buyers. However, a high \( x_W \) also leads to high information rents since it enables late buyers to pool with very low types. The optimal \( x_W \) trades off the surplus gains from the elimination of potential mismatch with the losses in information rents. Formally,
\[
x^M_W = \arg \max_{x_W \in (0, \frac{1}{2})} x_W(s-tx_W) + (\frac{1}{2} - x_W)(s - \frac{t}{2}) = \frac{1}{4}
\] (3)
and (2) leads the monopolist’s optimal discount \( \Delta p^M = t(1-\gamma) \). Finally, the price level is determined to make the lowest type \( x = \frac{1}{2} \) indifferent between buying and not buying, leading to \( p^M_1 = s - \frac{t}{2} \). In summary, we have:

**Proposition 1** The profit maximizing monopolistic price schedule \((p^M_1, p^M_2)\) is given by \( p^M_1 = s - \frac{t}{2} \) and \( p^M_2 = s - \gamma \frac{t}{2} \). It offers an APD of size \( \Delta p^M = p^M_2 - p^M_1 = \frac{t}{2}(1-\gamma) > 0 \) which induces half of the consumers to buy in advance. The monopolist’s profits are \( \Pi^M = s - \frac{t}{2} + \frac{t}{4}(1-\gamma) \).

**Proof:** See Appendix.

The following comparative statics results are immediate:
Corollary 1 In markets with a higher degree of individual demand uncertainty (lower $\gamma$) a monopolist will offer a larger APD and earn greater profits.

To understand the intuition for this result, consider the effect of a decrease in $\gamma$. Since the first period price is set to extract the entire surplus from the buyers who are indifferent between both products, it is independent of $\gamma$. As consumers become more uncertain about their individual preferences, they are willing to pay a larger premium for a late purchase. Since, independently of $\gamma$, half of the consumers are induced to purchase late, the monopolist’s profits increase. As can be seen from (1), a higher degree of demand uncertainty (lower $\gamma$) makes consumers more heterogenous with respect to their intertemporal preferences. This facilitates intertemporal price discrimination, leading to higher profits for the monopolist.

4 Competition

In order to understand the effect of competition on intertemporal price discrimination in the presence of individual demand uncertainty suppose for the remainder that products $A$ and $B$ are produced by two competing firms. Given the symmetry of the setup, we focus on a symmetric equilibrium in which firms offer the same price schedule, denoted $(p_1, p_2)$. Below we derive an equilibrium in which firms offer an APD by setting $p_1 < p_2$. For this purpose, we assume that firm $B$ chooses the equilibrium price schedule $(p_1, p_2)$ and consider a potential deviation by firm $A$. This allows us to derive necessary conditions for an APD equilibrium.

In the proof of Proposition 2, contained in the Appendix, we show that these conditions are not only necessary but also sufficient.

**First period price**: Suppose firm $A$ deviates from the equilibrium price schedule by choosing $(p'_1, p_2)$. In order to derive $A$’s profit in dependence of $p'_1$ we require firm $A$’s first and second period demands, $D_{A1}(p'_1)$ and $D_{A2}(p'_1)$. If a consumer of type $x \in [0, 1]$ buys in period 1, his expected payoff is $U_{1,A}(x) = s - p'_1 - \gamma tx - (1 - \gamma)t(1-x)$ if he buys from firm $A$ and $U_{1,B}(x) = s - p_1 - \gamma t(1-x) - (1 - \gamma)tx$ if he buys from firm $B$. Note that

$$U_{1,A}(x) > U_{1,B}(x) \iff x < \frac{1}{2} - \frac{p'_1 - p_1}{2t(2\gamma - 1)} \equiv x_{AB}(p'_1).$$
If instead the consumer waits until period 2 then he learns whether his location is \( \hat{x} = x \) or \( \hat{x} = 1 - x \). Given identical second period prices he will buy from the firm at the shortest distance. The expected payoff from waiting is therefore given by \( U_2(x) = s - p_2 - t \min(x, 1 - x) \). For \( x < x_{AB} \), waiting dominates purchasing from firm A in advance if and only if
\[
U_2(x) > U_{1,A}(x) \iff x > \frac{1}{2} - \frac{p_2 - p_1'}{2t(1 - \gamma)} \equiv x_{AW}(p_1').
\]
Similarly, for \( x > x_{AB} \) purchasing from firm B in advance dominates waiting if and only if
\[
U_2(x) > U_{1,B}(x) \iff x > \frac{1}{2} + \frac{p_2 - p_1}{2t(1 - \gamma)} \equiv x_{BW}.
\]
At the equilibrium price \( p_1' = p_1 \) it holds that \( x_{AW}(p_1) < x_{AB}(p_1) = \frac{1}{2} < x_{BW} \), i.e. both firms obtain positive demand in period 1. For demand to be positive also in period 2 it has to hold that
\[
x_{AW}(p_1) > 0 \iff x_{BW} < 1 \iff p_2 - p_1 < t(1 - \gamma).
\]
Condition (4) needs to be checked once the equilibrium prices have been determined. When (4) is satisfied and \( p_1' \) is close to \( p_1 \), firm A’s first and second period demands are given by \( D_{A1}(p_1') = x_{AB}(p_1') - x_{AW}(p_1') \) and \( D_{A2}(p_1') = \gamma x_{AW}(p_1') + (1 - \gamma)(1 - x_{BW}) \) respectively.

Note that the second period demand consists of two parts: consumers with type \( x < x_{AW} \) who wait and turn out to be located at \( \hat{x} = x \); and consumers with type \( x > x_{BW} \) who wait and turn out to be located at \( \hat{x} = 1 - x \). Now consider firm A’s profit
\[
\Pi_A(p_1') = p_1'D_{A1}(p_1') + p_2D_{A2}(p_1').
\]
If an equilibrium exists, then at \( p_1' = p_1 \) it has to hold that
\[
\frac{\partial \Pi_A}{\partial p_1'} = x_{AB}(p_1') - x_{AW}(p_1') - p_1' \frac{\partial [x_{AW}(p_1') - x_{AB}(p_1')]}{\partial p_1'} + p_2 \gamma \frac{\partial x_{AW}(p_1')}{\partial p_1'} = 0.
\]
The increase in the revenue obtained from selling \( x_{AB} - x_{AW} \) infra-marginal units at a higher price in advance and the increase in revenue from selling a higher number of units in the second period has to be exactly compensated by the decreases in revenue from selling a smaller number of units in advance. Substitution leads the following necessary condition for an equilibrium:
\[
\frac{1}{2} - \left( \frac{1}{2} - \frac{p_2 - p_1}{2t(1 - \gamma)} \right) - p_1 \left( \frac{1}{2t(1 - \gamma)} + \frac{1}{2t(2\gamma - 1)} \right) + p_2 \gamma \frac{1}{2t(1 - \gamma)} = 0.
\]
**Second period price:** Suppose firm A deviates from the equilibrium price schedule by choosing \((p_1, p_2')\). When firms offer different prices in the second period, a consumer who turns out to be located at \(\hat{x}\) will choose A over B if and only if

\[
p_2' + t\hat{x} < p_2 + t(1 - \hat{x}) \iff \hat{x} < \frac{1}{2} + \frac{p_2 - p_2'}{2t} = \hat{x}_{AB}(p_2').
\]

All consumers with type \(x \in (\min(\hat{x}_{AB}, 1 - \hat{x}_{AB}), \max(\hat{x}_{AB}, 1 - \hat{x}_{AB}))\) will make the same purchase decision in period 2 no matter whether they turn out to be located at \(\hat{x} = x\) or at \(\hat{x} = 1 - x\). Hence for these consumers the value of waiting is zero and they are better off by buying at a lower price in advance. Given that firms offer identical prices in period 1, consumers with types \(x \in (\min(\hat{x}_{AB}, 1 - \hat{x}_{AB}), \frac{1}{2})\) will purchase from firm A whereas consumers with types \(x \in (\frac{1}{2}, \max(\hat{x}_{AB}, 1 - \hat{x}_{AB}))\) will purchase from firm B.

Consumers with type \(x \in (0, \min(\hat{x}_{AB}, 1 - \hat{x}_{AB}))\) have an expected payoff from waiting given by \(U_2(x) = s - \gamma p_2' - (1 - \gamma)p_2 - tx\). They prefer waiting over purchasing from firm A in advance if and only if

\[
U_{1,A}(x) < U_2(x) \iff x < \frac{1}{2} - \frac{\gamma p_2' + (1 - \gamma)p_2 - p_1}{2t(1 - \gamma)} \equiv x_{AW}(p_2').
\]

Similarly, consumers with type \(x \in (\max(\hat{x}_{AB}, 1 - \hat{x}_{AB}), 1)\) have an expected payoff from waiting given by \(U_2(x) = s - \gamma p_2' - (1 - \gamma)p_2' - t(1 - x)\). They prefer waiting over purchasing from firm B in advance if and only if

\[
U_{1,B}(x) < U_2(x) \iff x > \frac{1}{2} + \frac{\gamma p_2 + (1 - \gamma)p_2 - p_1}{2(1 - \gamma)t} \equiv x_{BW}(p_2').
\]

For \(p_2'\) close to \(p_2\) firm A’s first and second period demands are thus given by \(D_{A1}(p_2') = \frac{1}{2} - x_{AW}(p_2')\) and \(D_{A2}(p_2') = \gamma x_{AW}(p_2') + (1 - \gamma)(1 - x_{BW}(p_2'))\) respectively. Similar to before, demands are positive in both periods at \(p_2' = p_2\) if condition (4) holds. Now consider firm A’s profits:

\[
\Pi_A(p_2') = p_1 D_{A1}(p_2') + p_2' D_{A2}(p_2').
\]

If an equilibrium exists, then at \(p_2' = p_2\) it has to hold that

\[
\frac{\partial \Pi_A}{\partial p_2'} = -p_1 \frac{\partial x_{AW}(p_2')}{\partial p_2'} + \gamma x_{AW}(p_2') + (1 - \gamma)[1 - x_{BW}(p_2')]
\]

\[
-\gamma p_2'[1 - \gamma] \frac{\partial x_{BW}(p_2')}{\partial p_2'} = 0
\]

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The gain in revenue from selling a higher number of units in advance and from selling \( \gamma x_{AW}(p'_2) + (1-\gamma)(1 - x_{BW}(p'_2)) \) infra-marginal units at a higher price in period 2 has to be exactly compensated by the loss in revenue from selling a smaller number of units in period 2. Substitution leads the following necessary condition for equilibrium:

\[
p_1 \frac{\gamma}{2t(1-\gamma)} + \frac{1}{2} - \frac{p_2 - p_1}{2t(1-\gamma)} - p_2[\frac{1-\gamma}{2t} + \frac{\gamma}{2t(1-\gamma)}] = 0.
\]

(6)

**Equilibrium:** The above analysis shows that a symmetric APD equilibrium must satisfy (5) and (6). Solving both conditions simultaneously, one obtains the unique candidate price schedule \((p_1^*, p_2^*)\) for a symmetric APD equilibrium. It remains to check that condition (4) is satisfied. This is indeed the case since

\[
p_2^* - p_1^* = \frac{2\gamma (1-\gamma)t}{-4\gamma^2 + 7\gamma - 1} < t(1-\gamma) \Leftrightarrow 0 < -4\gamma^2 + 5\gamma - 1 = (1-\gamma)(4\gamma-1).
\]

(7)

Also note that the above derivation implicitly assumed that all consumers buy in some period at the prices established in equilibrium. To guarantee that this is indeed the case, the consumer with the lowest type \(x = \frac{1}{2}\) must obtain non–negative utility, i.e. \(s - \frac{t}{2} - p_1^* \geq 0\) which is equivalent to Assumption 1 being satisfied. In the Appendix we show that (5) and (6) are not only necessary but also sufficient, leading to the following:

**Proposition 2** There exists a unique symmetric equilibrium in which firms offer APDs. Each firm serves a fraction \(a^* = 1 - 2x_{AW} = 2\gamma[-4\gamma^2 + 7\gamma - 1]^{-1} \in (0, 1)\) of its customers in advance. Prices are \(p_1^* = a^*\left(\frac{1}{\gamma} + 1\right)(\gamma - \frac{3}{2})t\) and \(p_2^* = a^*\left(\frac{3}{2} - \frac{1}{2\gamma}\right)t\) implying an APD of size \(\Delta p^* = p_2^* - p_1^* = a^*(1-\gamma)t > 0\). Profits are \(\Pi^* = \frac{t(a^*)^2}{8\gamma^2}(-8\gamma^3 + 21\gamma^2 - 10\gamma + 1) > 0\).

The following comparative statics results are immediate:

**Corollary 2** In markets with a higher degree of individual demand uncertainty (lower \(\gamma\)) competing firms will charge lower prices in all periods and offer larger APDs. Firms will serve a smaller fraction of their customers in advance and earn smaller profits.

A higher degree of uncertainty (smaller \(\gamma\)) makes consumers less willing to buy in advance. As a response, firms will offer a larger APD. However, the increment chosen in equilibrium is not sufficient to offset the consumers’ increased propensity to postpone their purchase until
their demand uncertainty has been resolved. As a consequence, the number of units sold in advance goes down. This stands in sharp contrast to the monopoly case in which the number of units sold in advance is independent of $\gamma$.

The fact that profits are increasing in $\gamma$ contrasts with the decreasing profits under monopoly. This can be explained as follows. A decrease in $\gamma$ makes consumers more heterogeneous with respect to their intertemporal preferences in (1), making intertemporal price discrimination more profitable. However, in the presence of competition, a decrease in $\gamma$ has a second effect. It makes the two products appear more homogeneous from the consumers’ viewpoint in period 1. The less certain consumers are about the identity of their preferred product, the more indifferent they become between purchasing product $A$ or product $B$ in advance. As a consequence, advance competition becomes stronger, leading to a reduction in first period prices. In the limit as $\gamma$ approaches $\frac{1}{2}$ and individual demands become maximally uncertain, both products appear as perfect substitutes in period 1 and first period prices under competition approach marginal cost. Corollary 2 shows that the negative effect on advance competition is strong enough to offset the positive effect on the firms’ ability to screen customers.

Note that the consumers’ cost of purchasing a product in advance becomes negligible, $p^*_1 \to 0$, as $\gamma$ approaches $\frac{1}{2}$, whereas the cost of buying the correct product in period two, $p^*_2 \to \frac{t}{3}$, is bounded away from zero. Hence, those consumers, who postpone their purchase in equilibrium, would actually prefer to buy their favorite product in period one and the other product in period two in case of a mismatch, for a total (expected) payment of $p^*_1 + (1-\gamma)p^*_2 < p^*_2$. In our model we have ruled out this possibility by restricting consumers to purchase at most one product. Generalizing the equilibrium conditions (5) and (6) to the case where marginal costs are positive, $c > 0$, it is straightforward to show that equilibrium prices satisfy $p^*_1(c) = c + p^*_1(0)$ and $p^*_2(c) = c + p^*_2(0)$. In order to make multiple purchases suboptimal for the consumer it is therefore sufficient that $p^*_1(c) + (1-\gamma)p^*_2(c) > p^*_2(c)$ for $\gamma \to \frac{1}{2}$, which is equivalent to $c > \frac{t}{3}$. This shows that our restriction to zero marginal costs and single product purchases was made for simplicity since the same results could be obtained in a more general (but expositionally more complicated) model in which consumers are allowed to purchase multiple products and firms’ marginal costs are (sufficiently) greater than zero.
Comparison with the monopolistic benchmark

We are now ready to compare the duopolistic outcome with the monopolistic benchmark in order to highlight the effects of competition in the presence of individual demand uncertainty.

Corollary 3 Competing firms offer larger advance purchase discounts than a monopolist, \( \Delta p^* > \Delta p^M \), and serve a greater fraction of consumers in advance: \( a^* > a^M = \frac{1}{2} \). Competition reduces the price paid by early buyers, \( p^*_1 < p^M_1 \). Whether competition increases or decreases the price paid by late buyers depends on the degree of product differentiation: \( p^*_2 > (\leq) p^M_2 \) if \( t > (\leq) \hat{t} \) where \( \hat{t} \equiv \frac{-8\gamma^2 + 14\gamma - 2}{4\gamma^2 + 7\gamma + 5\gamma - 2} s \). The threshold \( \hat{t} \in (0, \bar{t}) \) is decreasing in \( \gamma \) and converges to \( \bar{t} \) for \( \gamma \to 1 \).

In Figure 1 we compare the intertemporal allocation of sales for the two market structures. Under competition, the thresholds \( x^*_{AW} \) and \( x^*_{BW} \), determining the consumer types who are indifferent with respect to the timing of their purchase, are given by \( x^*_{AW} = \frac{1}{2}(1 - a^*) < \frac{1}{4} \) and \( x^*_{BW} = 1 - x^*_{AW} = \frac{1}{2}(1 + a^*) > \frac{3}{4} \). Consumers with types \( x \in [0, x^*_{AW}] \cup [x^*_{BW}, 1] \), that is, those with high preference intensities \( \sigma \in [a^*, 1] \), postpone their purchase under both market structures. Similarly, consumers with types \( x \in \left[ \frac{1}{4}, \frac{3}{4} \right] \), that is, those with low preference intensities \( \sigma \in [0, \frac{1}{2}] \), buy in advance under both market structures. However, there exists a group of consumers with intermediate preference intensities, \( \sigma \in \left( \frac{1}{2}, a^* \right) \), who would postpone their purchase under monopoly but are induced to buy in advance under competition. To understand the intuition for this result, recall that a monopolist benefits

![Figure 1: Intertemporal allocation of sales: Comparison monopoly versus competition.](image-url)
from lowering his APD due to the elimination of potential mismatch for those consumers who switch from buying in advance to waiting until their demand uncertainty has been resolved. In the presence of competition firms fail to internalize fully the implied increase in consumer surplus. This is because only a fraction $\gamma$ of the consumers who are induced to postpone their purchase by the ADP of firm $A$, will eventually be customers of firm $A$. The remaining fraction $1 - \gamma$ will purchase from firm $B$ and the increment in these consumers’ surplus due to a better match between product and preferences will be extracted by firm $B$ rather than firm $A$. Under competition firms will induce less consumers to postpone their purchase than under monopoly since they fail to internalize the positive externality of an improved consumer–product matching on the rival firm.

To see that first period prices are smaller under competition, note that $p^M_1 - p^*_1 = s - \frac{t}{2} - p^*_1$. Since the lowest type of consumer, $x = \frac{1}{2}$, is assumed to accept the equilibrium price $p^*_1$ (Assumption 1), it has to hold that $s - \frac{t}{2} > p^*_1$. To see that, on the contrary, second period prices may be higher than under monopoly, note that for $s \to \bar{s} = \frac{t}{2} + p^*_1$ we have

$$p^M_2 - p^*_2 = s - \frac{\gamma t}{2} - p^*_2 \to \frac{t}{2} + p^*_1 - \frac{\gamma t}{2} - p^*_2 = t(1 - \gamma)(\frac{1}{2} - a^*) < 0.$$ (8)

Competition reduces second period prices if and only if the consumers’ valuations, $s$, are sufficiently high, or, equivalently, if the product differentiation parameter $t$ is sufficiently low. The reason for this result is that, in the second period, competing firms, having served a larger number of customers in advance, face consumers with preferences more in line with their product’s characteristics. This effect counter acts the competitive pressure on prices and results in higher prices when competition is sufficiently weak, that is, when products are sufficiently differentiated.

In order to relate to the empirical controversy about the effect of competition on price dispersion we now compare the Gini coefficients under monopoly and under competition. In both cases, a fraction $a \in (0, 1)$ of consumers buy in advance at the low price $p_1$ while the remaining fraction $1-a$ postpone their purchase and buy at the high price $p_2$. The calculation of the Gini coefficient is straightforward and leads:

$$G = a(1 - \frac{p_1}{ap_1 + (1-a)p_2}).$$ (9)
Figure 2 depicts the Gini coefficients in dependence of the degree of product differentiation, \( t \). As can be seen from the figure, competition has a positive effect on price dispersion only for low values of \( t \). Instead, for high values of \( t \), competition leads to a reduction in price dispersion. This result is due to the fact that under competition the advance price is paid by a larger fraction of the consumers than the spot price whereas under monopoly both prices apply with equal frequency. The result remains valid as long as the degree of individual demand uncertainty is not too high. For high values of demand uncertainty, the competition’s influence on price spreads dominates its allocational effects, leading to a positive effect on price dispersion independently of the value of \( t \).

5 Welfare effects

The model’s assumptions of unitary demands and covered markets allows us to highlight the welfare effects that originate from the uncertain nature of individual preferences. In particular, price changes affect the intertemporal allocation of sales but have no influence
on the overall quantity supplied. The potential mismatch between consumer-preferences and product-characteristics therefore constitutes the sole source of welfare loss. Consumers with preference intensities \( \sigma \in \left( \frac{1}{2}, a^* \right) \) buy in advance under competition but postpone their purchase under monopoly. \( (1 - \gamma) \) of these consumers incur a disutility of size \( t \sigma \) from purchasing the wrong product. The welfare loss due to competition is therefore given by

\[
W^* - W^M = -(1 - \gamma)t \int_{\frac{1}{2}}^{a^*} \sigma d\sigma = -\frac{1}{2}(1 - \gamma)t[(a^*)^2 - \frac{1}{4}] \tag{10}
\]

**Proposition 3** In comparison with the monopolistic benchmark, competition leads to a welfare loss:

\[
W^* - W^M = -\frac{1}{2}(1 - \gamma)t[(a^*)^2 - \frac{1}{4}] < 0.
\]
The welfare loss is increasing in the degree of individual demand uncertainty: \( \frac{d}{d\gamma}(W^* - W^M) > 0. \)

So if competition leads to a welfare loss, to what extent are consumers affected? To answer this question, we consider how the change in a consumer’s surplus depends on his preference intensity \( \sigma \in [0, 1] \):

\[
CS^*(\sigma) - CS^M(\sigma) = \begin{cases} 
  p^M_1 - p^*_1 & \text{if } \sigma \leq \frac{1}{2} \\
  p^M_2 - p^*_1 - t(1 - \gamma)\sigma & \text{if } \frac{1}{2} < \sigma < a^* \\
  p^M_2 - p^*_2 & \text{if } a^* \leq \sigma.
\end{cases}
\]

Consumers with low preference intensities, \( \sigma \in [0, \frac{1}{2}] \), purchase in advance under both market structures and therefore benefit from the lower first period prices \( p^*_1 < p^M_1 \) charged under competition. Consumers with intermediate preference intensities, \( \sigma \in \left( \frac{1}{2}, a^* \right) \), switch from buying late under monopoly to buying early under competition. Switching buyers experience an even greater price reduction, \( p^M_2 - p^*_1 > p^M_1 - p^*_1 > 0 \), at the (expected) cost of a potential product-mismatch, \( (1 - \gamma)t \sigma \). Note that for \( p^M_2 > p^*_2 \) all switching buyers benefit from competition since their preference for buying early implies that \( p^*_2 - p^*_1 - (1 - \gamma)t \sigma > 0. \) Finally, consumers with high preference intensity, \( \sigma \in [a^*, 1] \), postpone their purchase under both market structures. Whether they benefit from competition depends on the comparison of second period prices.

The influence of competition on aggregate consumer surplus is given by

\[
\Sigma CS^* - \Sigma CS^M = W^* - W^M + p^M_2 - \frac{1}{2}\Delta p^M - (p^*_2 - a^*\Delta p^*).
\]
It consists of two parts: the welfare loss due to increased consumer-product mismatch; and a price effect. Under monopoly, the spot selling price is given by $p^M_2$ and half of the consumers buy at a discount $\Delta p^M$. Under competition, the spot selling price is given by $p^*_2$ and a higher fraction $a^* > \frac{1}{2}$ of the consumers buy at a larger discount $\Delta p^* > \Delta p^M$.

Corollary 3 has shown that, under competition, second period prices are higher than under monopoly if and only if the degree of product differentiation, $t$, falls below the threshold $\hat{t}$. It therefore follows from the above that for $t \leq \hat{t}$ competition has a positive effect on the surplus of each single consumer and thus $\Sigma CS^* - \Sigma CS^M > 0$. Moreover, for $t \to \tilde{t}$, second period prices are higher under competition and first period prices converge towards the monopoly value, $p^*_1 \to p^M_1$. This implies that for $t \to \tilde{t}$, all consumers must be worse off under competition than under monopoly and thus $\Sigma CS^* - \Sigma CS^M < 0$. In the Appendix we show that the change in aggregate consumer surplus, $\Sigma CS^* - \Sigma CS^M$, is strictly decreasing in the degree of product differentiation, $t$. In particular, we show the following:

**Proposition 4** The effect of competition on consumer surplus depends on the degree of product differentiation, $t$, as follows:

1. For $t \leq \hat{t}$, competition increases each consumer’s surplus. For $t > \hat{t}$, competition increases surplus for consumers with low preference intensities, but decreases surplus for consumers with high preference intensities: $CS^*(\sigma) > (<)CS^M(\sigma)$ if $\sigma < (>){\sigma}_{CS}$. The threshold $\sigma_{CS} \equiv a^* - \frac{p^*_2 - p^M_2}{t(1 - \gamma)} \in \left(\frac{1}{2}, a^*\right)$ is decreasing in $t$.

2. Competition increases (decreases) aggregate consumer surplus when the degree of product differentiation is low (high): $\Sigma CS^* > (<)\Sigma CS^M$ if $t < (>){t}_{CS}$. The threshold $t_{CS} \in (\hat{t}, \tilde{t})$ is decreasing in $\gamma$ and converges to $\tilde{t}$ for $\gamma \to 1$.

6 Uniform pricing

In this section we compare the setting in which intertemporal price discrimination is feasible with the case where ADPs are ruled out by the requirement of uniform pricing: $p_1 = p_2 = p$. When firms are not allowed to charge different prices across periods, a sale can occur only in period two. Hence firms solve the standard Hotelling problem. The equilibrium price is
given by \( p^* = t \) and all buyers participate in the market if and only if \( t < \frac{2}{3}s \). With these parameter values, all buyers participate under monopoly as well, where a monopolistic seller charges a price equal to \( p^M = s - \frac{t}{2} > p^* \).

In contrast to the case where APDs are allowed, competition has no influence on total welfare. Under both market structures, all consumers make an informed purchase. There is no mismatch between consumer preferences and product characteristics and welfare takes its first best value. Moreover, by decreasing the price, competition raises the surplus of all consumers independently of their preference intensity and the degree of product differentiation. This shows that our finding from the previous section, that competition can be detrimental both for total welfare as well as for consumer surplus, is driven by the firms’ ability to offer APDs.

So what is the effect of intertemporal price discrimination on consumer surplus? For a monopolistic market it is straightforward to see that any consumer’s surplus is lower with APDs than without. This is because, under uniform pricing, a monopolist would offer a price, \( p^M \), that is the same as the first period price, \( p^M_1 \), of an APD. In the presence of an APD, those consumers who buy in period one therefore merely face the additional risk of obtaining the wrong product, while those who buy in period two, pay a higher price.

Interestingly, under competition, allowing for intertemporal price discrimination has the opposite effect. The possibility of intertemporal price discrimination leads to lower prices for both periods: \( p^*_1 < p^* \) and \( p^*_2 < p^* \). As a consequence, price discrimination leaves all consumers better off. We summarize these findings in the following:

**Proposition 5** A ban on advance purchase discounts leads to an increase in welfare independently of market structure. It leads to an increase in consumer surplus and a decrease in profits under monopoly but has the opposite effect under competition.

The first part of Proposition 5 is in line with the general view that for price discrimination to have a positive effect on welfare it must lead to an increase in sold quantities (see Stole (2007) and references therein). When quantities are left unchanged, the sole effect of price discrimination is the creation of consumer misallocations.

The second part of Proposition 5 is reminiscent of the findings of Thisse and Vives (1988) for the case of third degree price discrimination. Using a generalized Hotelling framework
(without demand uncertainty), Thisse and Vives consider the possibility that firms may offer (delivered) price schedules, making a consumer’s payment dependent on his location. They show that in equilibrium firms will price discriminate, although both firms would be better off by setting a uniform price and letting consumers bear their transportation costs. Price discrimination leaves consumers better off since in equilibrium delivered prices are lower than the sum of uniform price and transportation costs, independently of the consumer’s location.

7 Conclusion

Based on the standard Hotelling model of horizontal differentiation, we have proposed a tractable model of individual demand uncertainty which allows us to study the effect of competition. Competition introduces larger discounts and more advance purchases, thereby increasing the amount of mismatch between products and preferences. This welfare loss of competition has never been pointed out in the economics literature. We also found that the size of demand uncertainty has opposite effects on the firms’ profits and consumer welfare with and without competition. Hence, our theory points out that policies promoting entry and competition can be detrimental to consumers even when they achieve lower market prices.

Appendix

Proof of Proposition 1

The monopolist has two options. He can either set prices such that all consumers buy (in some period) or instead choose a price schedule under which some consumers do not buy at all. Since a consumer with type $x = \frac{1}{2}$ obtains the lowest (expected) utility from buying and always prefers to buy in advance the two cases are distinguished by whether $p_1$ is smaller or greater than $s - \frac{t}{2}$ respectively. If $p_1 \leq s - \frac{t}{2}$ then all consumers buy in some period and profits are

$$
\pi = 2(\frac{1}{2} - xW)p_1 + 2xWp_2 = p_1 \frac{p_2 - p_1}{t(1 - \gamma)} + p_2(1 - \frac{p_2 - p_1}{t(1 - \gamma)}) = p_2 - \frac{(p_2 - p_1)^2}{t(1 - \gamma)}.
$$

(12)
Since profits are increasing in \( p_1 \) it is optimal to set \( p_1 = s - \frac{t}{2} \). Moreover with respect to \( p_2 \), profits are strictly concave in \([p_1, p_1 + t(1-\gamma)]\) and
\[
\frac{\partial \pi}{\partial p_2} = 1 - \frac{2(p_2 - p_1)}{t(1-\gamma)} = 0 \iff p_2 = p_1 + \frac{1}{2} t(1-\gamma). \tag{13}
\]
Note that this price implies an APD of size \( \Delta p = \frac{1}{2} t(1-\gamma) \) and \( x_W = \frac{1}{4} \). Hence we have shown that if the monopolist sells to all consumers then he will do so by way of an APD.

It remains to show that the monopolist has no incentive to restrict his sales by charging a higher price \( p_1 > s - \frac{t}{2} \).

A consumer of type \( x \) who prefers buying in advance over waiting will refrain from buying all together iff
\[
s - p_1 - \gamma tx - (1 - \gamma) t(1-x) < 0 \iff x > x_0 = \frac{s - p_1 - (1 - \gamma) t}{(2\gamma - 1) t} . \tag{14}
\]
For \( p_1 > s - \frac{t}{2}, x_0 < \frac{1}{2} \), and profit is given by
\[
\pi = 2p_1(x_0 - x_W) + 2p_2 x_W = 2p_1 \frac{s - p_1 - (1 - \gamma) t}{(2\gamma - 1) t} + (p_2 - p_1) (1 - \frac{p_2 - p_1}{t(1-\gamma)}) . \tag{15}
\]
As before the optimal APD follows from \( \frac{\partial \pi}{\partial p_2} = 0 \), leading \( p_2 - p_1 = \frac{1}{2} t(1-\gamma) \). Substitution gives
\[
\pi = \frac{t}{4} (1-\gamma) + 2p_1 \frac{s - p_1 - (1 - \gamma) t}{(2\gamma - 1) t} \tag{16}
\]
and
\[
\frac{\partial \pi}{\partial p_1} = \frac{2[s - 2p_1 - (1 - \gamma) t]}{(2\gamma - 1) t} . \tag{17}
\]
Increasing \( p_1 \) beyond \( s - \frac{t}{2} \) raises profits, i.e. \( \frac{\partial \pi}{\partial p_1}|_{p_1 = s - \frac{t}{2}} > 0 \) if and only if \( t - s - (1 - \gamma) t > 0 \iff s < \gamma t \). Since Assumption 1 implies that \( s > \gamma t \), it follows that increasing \( p_1 \) beyond \( s - \frac{t}{2} \) cannot be profitable.

**Proof of Proposition 2**

This proof shows that the conditions (15) and (19) are necessary and sufficient for symmetric equilibrium. In the main text, we have considered a small deviation, and derived an equilibrium price \( p_i \) one by one for \( i = 1, 2 \), taking the other price \( p_j, j = 1, 2 \neq i \), as fixed at the
equilibrium. Here, we provide a complete proof: a seller can deviate to any price level, and both prices are allowed to vary simultaneously. The proof proceeds as follows. In Step 1, we identify a subset of deviating prices where its profit is the same form as given in the main text. In Figure 1, this set is depicted as $P_a$. In Step 2, we show that the profit is maximized at $(p'_1, p'_2) = (p_1, p_2)$ within this subset. We examine other relevant subsets of deviating prices in Step 3 – 6. As we mentioned what remains to be checked is subsets $\bigcup_{i=b}^c P_i$. We show that deviating to any point in these subsets can never be profit maximizing. Hence, the price schedule described by (15) and (19) is indeed an equilibrium, which exists and is unique.

Figure 3: Possible deviations from the equilibrium price schedule.
Step 1. Define
\[ \mathbf{P}_a = \left\{ (p'_1, p'_2) \in \mathbb{R}^2_+ \mid p'_1 \in [\max\{p'_1^{W}, p'_1^{WA}\}, \bar{p}_1], p'_2 \in [\bar{p}_1, \bar{p}_2] \right\}, \]
where the critical values satisfy
\[
\begin{align*}
p'_1 & \geq p'_1^{W} = \gamma p'_2 + (1 - \gamma)p_2 - t(1 - \gamma) \Leftrightarrow x_{AW}(p'_1, p'_2) = \frac{1}{2} - \frac{\gamma p'_2 + (1 - \gamma)p_2 - p'_1}{2t(1 - \gamma)} \geq 0 \ \\
p'_1 & \geq p'_1^{WA} = \frac{\gamma p_1 - (2\gamma - 1)(\gamma p_2 + (1 - \gamma)p'_2)}{1 - \gamma} \Leftrightarrow x_{BW}(p'_2) = \frac{1}{2} + \frac{\gamma p_2 + (1 - \gamma)p'_2 - p_1}{2(1 - \gamma)t} \geq x_{AB}(p'_1) \ \\
p'_1 & \leq \bar{p}_1 = \frac{(1 - \gamma)p_1 + (2\gamma - 1)(\gamma p_2 + (1 - \gamma)p_2)}{1 - \gamma} \Leftrightarrow x_{AB}(p'_1) = \frac{1}{2} + \frac{p_1 - p'_1}{2t(2\gamma - 1)} \geq x_{AW}(p'_1, p'_2) \ \\
p'_2 & \leq \bar{p}_2 = \frac{(1 - \gamma)t - \gamma p_2 + p_1}{1 - \gamma} \Leftrightarrow x_{BW}(p'_2) \leq 1.
\end{align*}
\]

Then, the profit of a deviating firm (firm A) for \((p'_1, p'_2) \in \mathbf{P}_a\) is given by
\[ \pi_A(p'_1, p'_2) = p'_1 \left( x_{AB}(p'_1) - x_{AW}(p'_1, p'_2) \right) + p'_2 \left( \gamma x_{AW}(p'_1, p'_2) + (1 - \gamma)(1 - x_{BW}(p'_2)) \right). \]

Proof of Step 1. Throughout the proof, keep in mind that for all \((p'_1, p'_2) \in \mathbf{P}_a\), it holds that \(1 - x_2 \geq x_{AW}\), since
\[ 1 - x_2 = \frac{1}{2} - \frac{p_2 - p'_2}{2t} \geq \frac{1}{2} - \frac{\gamma p'_2 + (1 - \gamma)p_2 - p'_1}{2t(1 - \gamma)} = x_{AW} \Leftrightarrow p'_2 \geq p'_1, \]
and \(x_{BW} \geq x_2\) since
\[ x_{BW} = \frac{1}{2} + \frac{\gamma p_2 + (1 - \gamma)p'_2 - p_1}{2(1 - \gamma)t} \geq \frac{1}{2} + \frac{p_2 - p'_2}{2t} = x_2 \Leftrightarrow p'_2 \geq \frac{p_1 - (2\gamma - 1)p_2}{2(1 - \gamma)}, \]
where the last inequality is implied by \(p'_2 \geq p'_1\) and \(p'_1 \geq p'_1^{WA} = \frac{\gamma p_1 - (2\gamma - 1)(\gamma p_2 + (1 - \gamma)p'_2)}{1 - \gamma}\). Note further that for any \((p'_1, p'_2) \in \mathbb{R}^2_+\), we have \(x_{BW} \geq 1 - x_2 \Leftrightarrow p_2 \geq p_1\), which is always true.

We proceed by partitioning \(\mathbf{P}_a\) (see Figure 1).

Define
\[ \mathbf{P}_a(0) = \left\{ (p'_1, p'_2) \in \mathbf{P}_a \mid p'_1 \in [\bar{p}_1^{A2}, \bar{p}_1^{A2}], p'_2 \in [\bar{p}_1, \bar{p}_2] \right\}, \]
where the critical values satisfy
\[
\begin{align*}
p'_1 & \geq \bar{p}_1^{A2} = p_1 - (2\gamma - 1)(p_2 - p'_2) \Leftrightarrow x_2(p'_2) \geq x_{AB}(p'_1) \ \\
p'_1 & \leq \bar{p}_1^{A2} = p_1 + (2\gamma - 1)(p_2 - p'_2) \Leftrightarrow x_{AB}(p'_1) \geq 1 - x_2(p'_2).
\end{align*}
\]

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Within this subset, it holds that \( x_{AW}(p'_1, p'_2) \leq 1 - x_2(p'_2) \leq x_{AB}(p'_1) \leq x_2(p'_2) \). For \((p'_1, p'_2) \in \mathbf{P}_a(0)\), all the buyers with \( x \in (1 - x_2, x_2) \) will buy from A, if they wait till period 2, and so their expected payoff of waiting is
\[
V(x) = s - p'_2 - \gamma tx - (1 - \gamma)t(1 - x).
\] (18)

Consider now a buyer \( x \in (1 - x_2, x_{AB}) \). Since this buyer will buy from A (rather than from B) in period 1, he prefers to buy in period 1 (rather than to wait) if and only if
\[
U_A(x) = s - p'_1 - \gamma tx - (1 - \gamma)t(1 - x) \geq V(x) \iff p'_2 \geq p'_1,
\]
which is true for all \((p'_1, p'_2) \in \mathbf{P}_a\). Consider next a buyer \( x \in (x_{AB}, x_2) \). Since this buyer will buy from B (rather than from A) in period 1, he prefers to buy in period 1 (rather than to wait) if and only if
\[
U_B(x) = s - p_1 - \gamma t(1 - x) - (1 - \gamma)t(1 - x) \geq V(x) \iff x \geq \frac{1}{2} + \frac{p_1 - p'_2}{2t(2\gamma - 1)} \equiv \bar{x}_{BW}(p'_2).
\]

Since \( \bar{x}_{BW}(p'_2) \leq x_{AB} = \frac{1}{2} + \frac{p_1 - p'_1}{2t(2\gamma - 1)} \iff p'_2 \geq p'_1 \), we have \( U_B(x) \geq V(x) \) for all \((p'_1, p'_2) \in \mathbf{P}_a\). All in all, buyers with \( x \in [x_{AW}, x_{AB}] \) choose to buy from A in period 1, and buyers with \( x \in [0, x_{AW}] \cup (x_{BW}, 1] \) choose to wait, and so the demand in period 1 and 2 is given by
\[
D_{A1}(p'_1, p'_2) = x_{AB}(p'_1) - x_{AW}(p'_1, p'_2), \quad D_{A2}(p'_1, p'_2) = \gamma x_{AW}(p'_1, p'_2) + (1 - \gamma)(1 - x_{BW}(p'_2)),
\] (19)
respectively, for \((p'_1, p'_2) \in \mathbf{P}_a(0)\).

Define
\[
\mathbf{P}_a(1) \equiv \left\{ (p'_1, p'_2) \in \mathbf{P}_a \mid p'_1 \leq \min\{\bar{x}_{A2}(1), \bar{x}_{A2}(p'_1)\} \right\},
\]
where it holds that \( x_{AW}(p'_1, p'_2) \leq \min\{x_2(p'_2), 1 - x_2(p'_2)\} \leq \max\{x_2(p'_2), 1 - x_2(p'_2)\} \leq x_{AB}(p'_1) \).

Consider buyers with \( x \in (\min\{x_2, 1 - x_2\}, \max\{x_2, 1 - x_2\}) < x_{AB} \). When \( p'_2 \leq p_2 \), we have \( 1 - x_2 \leq \frac{1}{2} \leq x_2 \) and the analysis is the same as above: each buyer with \( x \in (1 - x_2, x_2) \) has the expected value of waiting given by (18) and all these buyers will buy from A in period 1. When \( p'_2 > p_2 \), we have \( x_2 < \frac{1}{2} < 1 - x_2 \). In this case, buyers \( x \in (x_2, 1 - x_2) \) will always
prefer buying from B in period 2 (if they wait) and so their expected value of waiting is given by
\[ V(x) = s - p_2 - \gamma t (1 - x) - (1 - \gamma)tx. \] (20)

Then, we have \( U_A(x) \geq V(x) \Leftrightarrow x \leq \frac{1}{2} + \frac{p_2 - p_1'}{2t(2\gamma - 1)} \) where the R.H.S. of the last inequality is greater than 1 \(-x_2\) as long as \( p_1' \leq \bar{p}_1^{A2} \). Hence, buyers with \( x \in \min\{x_2, 1 - x_2\}, \max\{x_2, 1 - x_2\} \) will all buy from A in period 1 for \((p_1', p_2') \in \mathbf{P}_a(1)\).

Consider next buyers with \( x \in (\max\{x_2, 1 - x_2\}, x_{AB}) \). In period 2, these buyers prefer to buy from B with probability \( \gamma \) and from A with probability \( 1 - \gamma \) and so their expected value of waiting is given by
\[ V(x) = s - \gamma(p_2 + t(1-x)) - (1 - \gamma)(p_2' + t(1-x)). \] (21)

We have
\[ U_A(x) \geq V(x) \Leftrightarrow x \leq \frac{1}{2} - \frac{p_1 - \gamma p_2 - (1 - \gamma)p_2'}{2\gamma t} = \tilde{x}_{AW}(p_1', p_2') \]
and
\[ \tilde{x}_{AW}(p_1', p_2') \geq x_{AB}(p_1') = \frac{1}{2} + \frac{p_1 - p_1'}{2t(2\gamma - 1)} \Leftrightarrow p_1' \geq \frac{\gamma p_1 - (2\gamma - 1)(\gamma p_2 + (1 - \gamma)p_2')}{1 - \gamma} = \tilde{p}_1^{WA}. \]
The last inequality always holds true when \((p_1', p_2') \in \mathbf{P}_a(1)\). Hence, all the buyers with \( x \in (\max\{x_2, 1 - x_2\}, x_{AB}) \) will buy from A in period 1.

To sum up the analysis for \((p_1', p_2') \in \mathbf{P}_a(1)\), buyers with \( x \in [x_{AW}, x_{AB}] \) choose to buy from A in period 1, and buyers with \( x \in [0, x_{AW}] \cup [x_{BW}, 1] \) choose to wait, and so the demand functions are given by (19).

Define
\[ \mathbf{P}_a(2) = \left\{(p_1', p_2') \in \mathbf{P}_a \mid \bar{p}_1^{A2} \leq p_1' \leq \bar{p}_1^{A2}, \right\}, \]
where it holds that \( x_{AW}(p_1', p_2') \leq x_2(p_2') \leq x_{AB}(p_1') \leq 1 - x_2(p_2') \).

Consider buyers with \( x \in (x_2, x_{AB}) \). These buyers always prefer to buy from B in period 2, thus a similar analysis to the above applies: each buyer with \( x \in (x_2, x_{AB}) \) has the
expected value of waiting given by (20) and all these buyers will buy from A in period 1, i.e.,
\[ U_A(x) \geq V(x) \]
if and only if
\[ x_{AB} = \frac{1}{2} + \frac{p_1 - p_1'}{2t(2\gamma - 1)} \leq \frac{1}{2} + \frac{p_2 - p_1'}{2t(2\gamma - 1)} \Leftrightarrow p_1 \leq p_2, \]
which is always true. Consider next \( x \in (x_{AB}, 1 - x_2) \). These buyers prefer B in both periods and so \( U_B(x) \geq V(x) \) since \( p_1 \leq p_2 \). Therefore, the demand functions for \( (p_1', p_2') \in P_a(1) \) are the same as before, and are given by (19).

Define
\[ P_a(3) = \left\{ (p_1', p_2') \in P_a \mid \max\{p_1'^{A2}, \bar{p}_1'^{A2}\} \leq p_1' \right\}, \]
where it holds that \( x_{AW}(p_1', p_2') \leq x_{AB}(p_1') \leq \min\{x_2(p_2'), 1 - x_2(p_2')\} \).

Consider buyers with \( x \in \left( \min\{x_2, 1 - x_2\}, \max\{x_2, 1 - x_2\} \right) > x_{AB} \). When \( p_2' \leq p_2 \), we have \( 1 - x_2 \leq \frac{1}{2} \leq x_2 \) and the analysis is similar to the one given in \( P_a(0) \): each buyer with \( x \in (1 - x_2, x_2) \) has the expected value of waiting given by (18) and \( U_B(x) \geq V(x) \) when \( p_1' \leq p_2' \). When \( p_2' > p_2 \), we have \( x_2 < \frac{1}{2} < 1 - x_2 \). In this case, the buyers \( x \in (x_2, 1 - x_2) \) will always prefer buying from B in period 2 (if they wait) and so their expected value of waiting is given by (20). Then, we have \( U_B(x) \geq V(x) \Leftrightarrow p_1 \leq p_2 \). Hence, buyers with \( x \in \left( \min\{x_2, 1 - x_2\}, \max\{x_2, 1 - x_2\} \right) > x_{AB} \) will all buy from B in period 1 for \( (p_1', p_2') \in P_a(3) \).

Consider next buyers with \( x \in (x_{AB}, \min\{x_2, 1 - x_2\}) \). In period 2, these buyers prefer to buy from A with probability \( \gamma \) and from B with probability \( 1 - \gamma \) and so their expected value of waiting is given by
\[ V(x) = s - \gamma(p_2' + tx) - (1 - \gamma)(p_2 + tx). \tag{22} \]

We have
\[ U_B(x) = s - p_1 - \gamma t (1 - x) - (1 - \gamma)tx \geq V(x) \Leftrightarrow x \geq \frac{1}{2} - \frac{\gamma p_2' + (1 - \gamma)p_2 - p_1}{2\gamma t} \equiv \hat{x}_{BW}(p_2'), \]
where
\[ \hat{x}_{BW}(p_2') \leq x_{AB}(p_1') = \frac{1}{2} + \frac{p_1 - p_1'}{2t(2\gamma - 1)} \Leftrightarrow p_1' \leq \frac{(1 - \gamma)p_1 + (2\gamma - 1)(\gamma p_2' + (1 - \gamma)p_2)}{\gamma} \equiv \bar{p}_1. \]
The last inequality always holds true when \((p'_1, p'_2) \in P_a(3)\). Hence, all the buyers with \(x \in (x_{AB}, \min\{x_2, 1-x_2\})\) will buy from B in period 1.

Therefore, for \((p'_1, p'_2) \in P_a(3)\), buyers with \(x \in [x_{AW}, x_{AB}]\) choose to buy from A in period 1, and buyers with \(x \in [0, x_{AW}] \cup [x_{BW}, 1]\) choose to wait, and so the demand functions are given by (19).

To summarize, we have shown that the demand functions of a deviator (seller A) are given by (19) for a subset \(P_a = \bigcup_{h=0}^3 P_a(h)\). This completes the proof of Step 1. ■

**Step 2.** The profit of a deviator (seller A) is maximized at \((p'_1, p'_2) = (p_1, p_2)\) when \((p'_1, p'_2) \in P_a\).

**Proof of Step 2.** Note that \((p_1, p_2) \in \text{int} \ P_a\) and the first order conditions,

\[
\frac{\partial \pi_A}{\partial p'_1} = \frac{(1-\gamma)p_1 - 2\gamma p'_1 + (2\gamma - 1)(2\gamma p'_2 + (1-\gamma)p_2)}{2t(2\gamma - 1)(1-\gamma)} = 0,
\]

\[
\frac{\partial \pi_A}{\partial p'_2} = \frac{t(1-\gamma) - 2\gamma(1-\gamma)p_2 - 2(\gamma^2 + (1-\gamma)^2)p'_2 + (1-\gamma)p_1 + 2\gamma p'_1}{2t(1-\gamma)} = 0,
\]

are satisfied if and only if

\[
p'_1 = \frac{(1+\gamma)(2\gamma - 1)t}{-4\gamma^2 + 7\gamma - 1} = p_1 \quad (23)
\]

\[
p'_2 = \frac{(3\gamma - 1)t}{-4\gamma^2 + 7\gamma - 1} = p_2, \quad (24)
\]

that is, \((p'_1, p'_2) = (p_1, p_2)\) is a unique solution to the first order conditions. Given this result, we now show that this solution indeed achieves a global maximum for \((p'_1, p'_2) \in P_a\).

Since the profit function is twice differentiable, we use the Hessian matrix given by

\[
H \equiv \begin{pmatrix}
\frac{\partial^2 \pi_A}{\partial p'_1^2} & \frac{\partial^2 \pi_A}{\partial p'_1 \partial p'_2} \\
\frac{\partial^2 \pi_A}{\partial p'_2 \partial p'_1} & \frac{\partial^2 \pi_A}{\partial p'_2^2}
\end{pmatrix} = \begin{pmatrix}
\frac{\gamma}{((2\gamma-1)(1-\gamma))} & \frac{\gamma}{(1-\gamma)^2} \\
\frac{\gamma}{(1-\gamma)^2} & \frac{(\gamma^2 + (1-\gamma)^2)}{((1-\gamma)^2)}
\end{pmatrix}.
\]

The \(i\)-th leading principal minor of this Hessian, denoted by \(H(i)\), is \(H(1) = \frac{\partial^2 \pi_A}{\partial p'_1^2} = -\frac{\gamma}{((2\gamma-1)(1-\gamma))} < 0\) and

\[
H(2) = \det (H) = \frac{\partial^2 \pi_A}{\partial p'_1^2} \frac{\partial^2 \pi_A}{\partial p'_2^2} - \left(\frac{\partial^2 \pi_A}{\partial p'_1 \partial p'_2}\right)^2 = \frac{\gamma}{t^2(1-\gamma)(2\gamma-1)} > 0.
\]

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Hence, \( H \) is negative definite and so the profit function \( \pi_A \) is strictly concave in all \((p'_1, p'_2) \in P_a\). Therefore, the solution satisfying the first order conditions, \((p'_1, p'_2) = (p_1, p_2)\), achieves a global maximum in \( P_a \). This completes the proof of Step 2. ■

**Step 3.** Define

\[
P_b = \left\{(p'_1, p'_2) \in \mathbb{R}^2_+ \mid p'_1 \in [p'_1^{W_1}, p'_1^{W_A}], p'_2 \geq p'_1\right\},
\]

where the critical values, \( p'_1^{W_1} \) and \( p'_1^{W_A} \), are defined in Step 1. Then, the profit of a deviating firm (firm A) for \((p'_1, p'_2) \in P_b\) is given by

\[
\pi_A(p'_1, p'_2) = p'_1 \left\{\tilde{x}_{AW}(p'_1, p'_2) - x_{AW}(p'_1, p'_2)\right\} + p'_2 \left\{\gamma x_{AW}(p'_1, p'_2) + (1 - \gamma)(1 - \tilde{x}_{AW}(p'_1, p'_2))\right\},
\]

where

\[
\tilde{x}_{AW}(p'_1, p'_2) \equiv \frac{1}{2} + \frac{\gamma p_2 + (1 - \gamma)p'_2 - p'_1}{2\gamma t}.
\]

A deviation to any \((p'_1, p'_2) \in P_b\) is never profitable.

**Proof of Step 3.** When \((p'_1, p'_2) \in P_b\), it holds that \(1 - x_2 \geq x_{AW}\). Since buyers with \( x \in (1 - x_2, x_2) \) always prefer to buy from A in period 2, their expected payoff of waiting is given by (18), and these buyers have \( U_A(x) \geq V(x) \iff p'_1 \leq p'_2\).

Consider next buyers with \( x \in (x_2, x_{AB})\). In period 2, these buyers prefer to buy from B with probability \( \gamma \) and from A with probability \( 1 - \gamma \) and so the situation is the same as in \( P_a(1) \) described in Step 1: the expected value of waiting for these buyers is given by (21), and

\[
U_A(x) \geq V(x) \iff x \leq \frac{1}{2} - \frac{p'_1 - \gamma p_2 - (1 - \gamma)p'_2}{2\gamma t} \equiv \tilde{x}_{AW}(p'_1, p'_2),
\]

where

\[
\tilde{x}_{AW}(p'_1, p'_2) \leq x_{AB}(p'_1) = \frac{1}{2} + \frac{p_1 - p'_1}{2t(2\gamma - 1)} \iff p'_1 \leq \frac{\gamma p_1 - (2\gamma - 1) \left(\frac{p_2 - (1 - \gamma)p'_2}{1 - \gamma}\right)}{1 - \gamma} \equiv p'_1^{AW},
\]

and

\[
\tilde{x}_{AW}(p'_1, p'_2) \geq x_2(p'_2) = \frac{1}{2} + \frac{p_2 - p'_2}{2t} \iff p'_1 \leq p'_2.
\]
Hence, when \((p_1', p_2') \in P_b\), buyers with \(x \in [x_{AW}, \tilde{x}_{AW}]\) buy in period 1 from A. Buyers with \(x \in [0, x_{AW}) \cup (\tilde{x}_{AW}, 1]\) wait: In period 2, the demand of seller A is from the proportion \(\gamma\) of buyers \(x \in [0, x_{AW})\) and the proportion \(1 - \gamma\) of buyers \(x \in (\tilde{x}_{AW}, 1]\). To sum up,

\[
\pi_A(p_1', p_2') = p_1' \{ \tilde{x}_{AW}(p_1', p_2') - x_{AW}(p_1', p_2') \} + p_2' \{ \gamma x_{AW}(p_1', p_2') + (1 - \gamma)(1 - \tilde{x}_{AW}(p_1', p_2')) \}
\]

\[
= p_1' - p_1' + 2\gamma(1 - \gamma)p_2 + ((1 - \gamma)^2 + \gamma^2)p_2' \cdot \frac{2\gamma(1 - \gamma)t}{2\gamma(1 - \gamma)t} + p_2' \gamma(1 - \gamma)t - (\gamma^3 + (1 - \gamma)^3)p_2' - \gamma(1 - \gamma)p_2 + ((1 - \gamma)^2 + \gamma^2)p_1'
\]

Now, observe that for any \((p_1', p_2') \in P_b\), we have

\[
\frac{\partial \pi_A(p_1', p_2')}{\partial p_1'} = -p_1' + \gamma(1 - \gamma)p_2 + ((1 - \gamma)^2 + \gamma^2)p_2' \geq \frac{-\min\{p_1^{WA}, p_2\} + \gamma(1 - \gamma)p_2 + ((1 - \gamma)^2 + \gamma^2)p_2'}{\gamma(1 - \gamma)t}.
\]

To identify the sign of the R.H.S. of the last inequality, consider first the case \(p_2^{WA} \leq p_2'\) with \(p_2' \geq p_2^B \equiv \frac{p_2 - (2\gamma - 1)p_1}{2(1 - \gamma)}\). Observe that

\[
-p_1^{WA} + \gamma(1 - \gamma)p_2 + ((1 - \gamma)^2 + \gamma^2)p_2' = \frac{\gamma (\gamma^2 p_2 - p_1 + 2\gamma(1 - \gamma)p_2')}{1 - \gamma} \geq \frac{\gamma (\gamma^2 p_2 - p_1 + 2\gamma(1 - \gamma)p_2^B)}{1 - \gamma} = \gamma(\gamma p_2 - p_1) = \frac{(1 - \gamma)^2 \gamma t}{-4\gamma^2 + 7\gamma - 1} > 0.
\]

Consider next the case \(p_2^{WA} > p_2'\) with \(p_2' < p_2^B\). Observe that

\[
p_2' + \gamma(1 - \gamma)p_2 + ((1 - \gamma)^2 + \gamma^2)p_2' = \gamma(1 - \gamma)(p_2 - 2p_2') > \gamma(1 - \gamma)(p_2 - 2p_2^B) = \gamma(\gamma p_2 - p_1) = \frac{(1 - \gamma)^2 \gamma t}{-4\gamma^2 + 7\gamma - 1} > 0.
\]

Hence, the profit \(\pi_A\) is strictly increasing in \(p_1'\) when \((p_1', p_2') \in P_b\). This implies that the profit is maximized at \(p_1' = \min\{p_2', p_1^{WA}\}\) when \((p_1', p_2') \in P_b\). In the latter case with \(p_1^{WA} = \min\{p_2', p_1^{WA}\}\), the profit equals to \(\pi_A(p_1^{WA}, p_2')\), i.e., the profit with \((p_1', p_2') \in P_a(1)\)
evaluated at $p'_1 = p'_{1W}$, due to the continuity of $\pi_A(p'_1, p'_2)$ for $(p'_1, p'_2) \in P_a(1) \cup P_b$ (followed by $p'_1 = p'_{1W} \iff x_{AB} = \tilde{x}_{AW}$). As shown in Step 2, this profit is strictly smaller than $\pi(p_1, p_2)$. In the former case, where it holds that $p'_2 \in [p_2 - t, p_2] \Leftrightarrow p'_2 = \min\{p'_2, p'_{2W}\}$, observe that

$$\frac{\partial \pi_A(p'_2, p'_2)}{\partial p'_2} = \frac{-2p'_2 + p_2 + t}{2t}$$

and

$$\frac{\partial \pi_A(p'_2, p'_2)}{\partial p'_2} \big|_{p'_2 = p_2} = \frac{-p_1/(2\gamma - 1)p_2 + p_2 + t}{2t} > 0 \iff t(1 - \gamma) + \gamma p_2 - p_1 > 0$$

$$\iff \frac{2t\gamma(1 - \gamma)(3 - 2\gamma)}{-4\gamma^2 + 7\gamma - 1} > 0,$$

where the last inequality holds true for all $\gamma \in (\frac{1}{2}, 1)$. Therefore, the profit $\pi_A(p'_2, p'_2)$ is strictly increasing in $p'_2 \in [p_2 - t, p_2]$. This implies $\pi_A(p'_2, p'_2) < \pi_A(p'_2, p'_2) < \pi_A(p_1, p_2)$ and completes the proof of Step 3. ■

**Step 4.** Define

$$P_c \equiv \{(p'_1, p'_2) \in \mathbb{R}^2_+ \mid p'_1 \leq \min\{p'_{1W}, p'_{1WA}\}, p'_2 \geq p_2 - t\},$$

where the critical values, $p'_{1W}$ and $p'_{1WA}$, are defined in Step 1. Then, the profit of a deviating firm (firm A) for $(p'_1, p'_2) \in P_c$ is given by

$$\pi_A(p'_1, p'_2) = p'_1\tilde{x}_{AW}(p'_1, p'_2) + p'_2(1 - \gamma)(1 - \tilde{x}_{AW}(p'_1, p'_2)),$$

where

$$\tilde{x}_{AW}(p'_1, p'_2) \equiv \frac{1}{2} + \frac{\gamma p_2 + (1 - \gamma)p'_2 - p'_1}{2\gamma t}.$$  

A deviation to any $(p'_1, p'_2) \in P_c$ is never profitable.

**Proof of Step 4.** Consider first the case $p'_2 < p_2 \iff 1 - x_2 < \frac{1}{2} < x_2$. Buyers with $x \in (1 - x_2, x_2)$ always prefer to buy from A in period 2. Hence, their expected payoff of waiting is given by (18), and these buyers have $U_A(x) \geq V(x) \iff p'_1 \leq p'_2$. Buyers with $x \in (x_2, x_{AB})$ prefer to buy from B in period 2 with probability $\gamma$ and from A with
probability $1 - \gamma$. Hence, the expected value of waiting for these buyers is given by (21) and $U_A(x) \geq V(x) \iff x \leq \bar{x}_{AW}(p'_1, p'_2)$, where $\bar{x}_{AW}(p'_1, p'_2) \leq x_{AB}(p'_1) \iff p'_1 \leq p''_1$, and $\bar{x}_{AW}(p'_1, p'_2) \geq x_2(p'_2) \iff p'_1 \leq p'_2$.

Consider next the case $p'_2 \geq p_2 \iff x_2 \leq \frac{1}{2} \leq 1 - x_2$. Buyers $x \in (x_2, 1 - x_2)$ will always prefer buying from B in period 2 (if they wait) and so their expected value of waiting is given by (20). Then, we have $U_A(x) \geq V(x) \iff x \leq \frac{1}{2} + \frac{p_2 - p'_1}{2(2\gamma - 1)}$ where the R.H.S. of the last inequality is greater than $1 - x_2$ if and only if $p'_1 < p_2 + (2\gamma - 1)(p_2 - p'_2)$, which is always the case when $p'_1 \leq p''_1$. Buyers with $x \in (1 - x_2, x_{AB})$ prefer to buy from B in period 2 with probability $\gamma$ and from A with probability $1 - \gamma$ and so their expected value of waiting is given by (21). The proof of $P_a(1)$ in Step 1 shows that $U_A(x) \geq V(x) \iff x \leq \bar{x}_{AW}(p'_1, p'_2)$ and $\bar{x}_{AW}(p'_1, p'_2) \leq x_{AB}(p'_1) \iff p'_1 \leq p''_1$. Further,

$$\bar{x}_{AW}(p'_1, p'_2) \equiv \frac{1}{2} - \frac{p'_1 - \gamma p_2 (1 - \gamma) p'_2}{2\gamma t} > \frac{1}{2} - \frac{p_2 - p'_2}{2t} \equiv 1 - x_2 \iff p'_1 < 2\gamma p_2 - (2\gamma - 1)p'_2,$$

where the last inequality always holds true when $p'_1 \leq p''_1$.

All in all, whenever $(p'_1, p'_2) \in P_c$, buyers with $x \in [0, \bar{x}_{AW}]$ will buy from A in period 1, buyers with $x \in (\bar{x}_{AW}, 1]$ wait, and the proportion $1 - \gamma$ of the latter buyers buy from A in period 2. Hence,

$$\pi_A(p'_1, p'_2) = \frac{p'_1 \bar{x}_{AW}(p'_1, p'_2) + p'_2 (1 - \gamma)(1 - \bar{x}_{AW}(p'_1, p'_2))}{2\gamma t} + \frac{\gamma t - \gamma t + p'_1 - \gamma p_2 - (1 - \gamma) p'_2}{2\gamma t}.$$

Now, observe that for all $(p'_1, p'_2) \in P_c$, it holds that

$$\frac{\partial \pi_A(p'_1, p'_2)}{\partial p'_1} = \frac{\gamma t - 2p'_1 + \gamma p_2 + 2(1 - \gamma)p'_2}{2\gamma t} \geq \frac{\gamma t - 2 \min\{p''_1, p''_2\} + \gamma p_2 + 2(1 - \gamma) p'_2}{2\gamma t}.$$

To identify the sign of the R.H.S. of the last inequality, consider first the case with $p''_1 = \min\{p''_1, p''_2\} \iff p'_2 \geq \frac{\gamma p_1 + t (1 - \gamma)^2 - (3\gamma^2 - 3\gamma + 1)p_2}{(3\gamma - 1)(1 - \gamma)}$. Then, we have

$$\gamma t - 2p''_1 + \gamma p_2 + 2(1 - \gamma)p'_2 = \frac{2(1 - \gamma)p_2 - 2p_1 + (1 - \gamma)t + (3\gamma - 1)p_2}{1 - \gamma} > 0 \iff p'_2 > \frac{2p_1 - (1 - \gamma)t - (3\gamma - 1)p_2}{2(1 - \gamma)}.$$
and
\[
\frac{2p_1 - (1 - \gamma)t - (3\gamma - 1)p_2}{2(1 - \gamma)} < \frac{\gamma p_1 + t(1 - \gamma)^2 - (3\gamma^2 - 3\gamma + 1)p_2}{(3\gamma - 1)(1 - \gamma)}
\]
\[\iff -2\gamma^3 + \gamma^2 + 4\gamma - 1 > 0.\]

Since the last inequality holds for all \(\gamma \in (\frac{1}{2}, 1)\), this implies that the profit is strictly increasing in \(p'_1\) when \(p'_2 \geq \frac{\gamma p_1 + t(1 - \gamma)^2 - (3\gamma^2 - 3\gamma + 1)p_2}{(3\gamma - 1)(1 - \gamma)}\).

Consider next the case with \(p'_W = \min\{p'_W, p'_W\} \iff p'_2 < \frac{\gamma p_1 + t(1 - \gamma)^2 - (3\gamma^2 - 3\gamma + 1)p_2}{(3\gamma - 1)(1 - \gamma)}\). Observe that
\[
\gamma t - 2p'_W + \gamma p_2 + 2(1 - \gamma)p'_2 = (2 - \gamma)t + (3\gamma - 2)p_2 - 2(2\gamma - 1)p'_2 > 0
\]
\[\iff p'_2 < \frac{(2 - \gamma)t + (3\gamma - 2)p_2}{2(2\gamma - 1)}\]
and
\[
\frac{(2 - \gamma)t + (3\gamma - 2)p_2}{2(2\gamma - 1)} > \frac{\gamma p_1 + t(1 - \gamma)^2 - (3\gamma^2 - 3\gamma + 1)p_2}{(3\gamma - 1)(1 - \gamma)}
\]
\[\iff -2\gamma^3 + \gamma^2 + 4\gamma - 1 > 0.\]

As before, the last inequality holds for all \(\gamma \in (\frac{1}{2}, 1)\), and so the profit is strictly increasing in \(p'_1\) when \(p'_2 < \frac{\gamma p_1 + t(1 - \gamma)^2 - (3\gamma^2 - 3\gamma + 1)p_2}{(3\gamma - 1)(1 - \gamma)}\).

All in all, the profit \(\pi_A\) is strictly increasing in \(p'_1\) when \((p'_1, p'_2) \in \mathbf{P}_c\), and so the profit is maximized at \(p'_1 = \min\{p'_W, p'_W\}\). In the latter case with \(p'_W = \min\{p'_W, p'_W\}\), the profit equals to \(\pi_A(p'_W, p'_W)\), i.e., the profit with \((p'_1, p'_2) \in \mathbf{P}_b\) evaluated at \(p'_1 = p'_W\), due to the continuity of \(\pi_A(p'_1, p'_2)\) for \((p'_1, p'_2) \in \mathbf{P}_c \cup \mathbf{P}_d\) (followed by \(p'_1 = p'_W \iff x_{AW} = 0\)). This profit is strictly smaller than \(\pi(p_1, p_2)\), as shown in Step 3. The same holds true in the former case with \(p'_W = \min\{p'_W, p'_W\}\), which we will prove shortly in Step 5, noting that \(\pi_A(p'_1, p'_2)\) is continuous in \((p'_1, p'_2) \in \mathbf{P}_c \cup \mathbf{P}_d\) (followed by \(p'_1 = p'_W \iff x_{AB} = x_{AW}\)). This completes the proof of Step 4. ■

**Step 5.** Define
\[
\mathbf{P}_d = \left\{ (p'_1, p'_2) \in \mathbb{R}_+^2 \mid p'_1 \in [\min\{0, p'_W\}, p'_W], p'_2 \leq p_2 \right\}
\]
where the critical values, $\tilde{p}_2$, $L^W_1$ and $L^{WA}_1$, are defined in Step 1. Then, the profit of a deviating firm (firm A) for $(p'_1, p'_2) \in P_d$ is given by

$$\pi_A(p'_1, p'_2) = p'_1 x_{AB}(p'_1) + p'_2 (1 - \gamma)(1 - x_{BW}(p'_2)).$$

A deviation to any $(p'_1, p'_2) \in P_d$ is never profitable.

**Proof of Step 5.** The proof is similar to that of Step 4. Consider first the case $p'_2 < p_2 \Leftrightarrow 1 - x_2 < \frac{1}{2} < x_2$. Buyers with $x \in (1 - x_2, x_2)$ always prefer to buy from A in period 2. Hence, their expected payoff of waiting is given by (18), and these buyers have $U_A(x) \geq V(x) \Leftrightarrow p'_1 \leq p'_2$. Buyers with $x \in (x_2, x_{AB})$ prefer to buy from B in period 2 with probability $\gamma$ and from A with probability $1 - \gamma$. Hence, the expected value of waiting for these buyers is given by (21), and $U_A(x) \geq V(x) \Leftrightarrow x \leq \tilde{x}_{AW}(p'_1, p'_2)$ where $\tilde{x}_{AW}(p'_1, p'_2) = x_{AB}(p'_1) \Leftrightarrow p'_1 = L^{WA}_1$.

Consider next the case $p'_2 \geq p_2 \Leftrightarrow x_2 \leq \frac{1}{2} < 1 - x_2$. Buyers $x \in (x_2, 1 - x_2)$ will always prefer buying from B in period 2 and so their expected value of waiting is given by (20). Then, we have $U_A(x) \geq V(x) \Leftrightarrow x \leq \frac{1}{2} + \frac{x_2 - p'_1}{2(2\gamma - 1)}$ where the R.H.S. of the last inequality is greater than $1 - x_2$ if and only if $p'_1 < p_2 + (2\gamma - 1)(p_2 - p'_2)$, which is always the case when $p'_1 = \tilde{p}^{A2}_1$. Buyers with $x \in (1 - x_2, x_{AB})$ prefer to buy from B in period 2 with probability $\gamma$ and from A with probability $1 - \gamma$ and so their expected value of waiting is given by (21). The proof of $P_d(1)$ in Step 1 shows that $U_A(x) \geq V(x) \Leftrightarrow x \leq \tilde{x}_{AW}(p'_1, p'_2)$ and $\tilde{x}_{AW}(p'_1, p'_2) \geq x_{AB}(p'_1) \Leftrightarrow p'_1 \geq L^{WA}_1$.

All in all, whenever $(p'_1, p'_2) \in P_d$, buyers with $x \in [0, x_{AB}]$ will buy from A in period 1, buyers with $x \in (x_{BW}, 1]$ wait, and the proportion $1 - \gamma$ of the latter buyers buy from A in period 2. Hence,

$$\pi_A(p'_1, p'_2) = p'_1 x_{AB}(p'_1) + p'_2 (1 - \gamma)(1 - x_{BW}(p'_2)) = p'_1 \frac{(2\gamma - 1)t - p'_1 + p_1}{2(2\gamma - 1)t} + p'_2 \frac{(1 - \gamma)t + p_1 - \gamma p_2 - (1 - \gamma)p'_2}{2(1 - \gamma)t}.$$

For any $(p'_1, p'_2) \in P_d$, we have

$$\frac{\partial \pi_A(p'_1, p'_2)}{\partial p'_1} = \frac{(2\gamma - 1)t - 2p'_1 + p_1}{2(2\gamma - 1)t} > \frac{(2\gamma - 1)t - 2p^{AW}_1 + p_1}{2(2\gamma - 1)t}.$$
To identify the sign of the R.H.S. of the last inequality, observe that

\[
(2\gamma - 1)t - 2p^W_1 + p_1 = (2\gamma - 1)t - 2(\gamma p'_2 + (1 - \gamma)(p_2 - t)) + p_1
\]

\[
> 0
\]

\[
\Leftrightarrow p'_2 < \frac{p_1 + t - 2(1 - \gamma)p_2}{2\gamma} \equiv p^*_2 \in (p_2, \bar{p}_2).
\]

Hence, for \( p'_2 < p^*_2 \), the profit is strictly increasing in \( p'_1 \), and so

\[
\pi_A(p'_1, p'_2) < \pi_A(p^W_1, p'_2) < \pi_A(p_1, p_2).
\]

For \( p'_2 \geq p^*_2 \), observe that

\[
\frac{\partial \pi_A(p'_1, p'_2)}{\partial p'_2} = \frac{(1 - \gamma)t - \gamma p_2 - 2(1 - \gamma)p'_2 + p_1}{2t} \leq \frac{(1 - \gamma)t - \gamma p_2 - 2(1 - \gamma)p^*_2 + p_1}{2t},
\]

where the R.H.S. of the last inequality satisfies

\[
\frac{(1 - \gamma)t - \gamma p_2 - 2(1 - \gamma)p'_2 + p_1}{2t} < 0 \Leftrightarrow 2(1 - \gamma)(2\gamma^2 + 1) > 0,
\]

which holds true for all \( \gamma \in (\frac{1}{2}, 1) \). Hence, the profit is strictly decreasing in \( p'_2 \geq p^*_2 \). Further, since

\[
\frac{\partial \pi_A(p'_1, p'_2)}{\partial p'_1} \big|_{p'_1 = p_1^W} > 0 \Leftrightarrow 4\gamma(1 - \gamma)^2 > 0,
\]

this implies that for \( p'_2 > p^*_2 \) the profit is smaller than the one evaluated at

\[
(p'_1, p'_2) = (p^*_1, p^*_2) \equiv \left( \frac{(2\gamma - 1)t + p_1}{2}, \frac{t + p_1 - 2(1 - \gamma)p_2}{2\gamma} \right) > (p_1, p_2).
\]

The rest of the proof shows that this profit \( \pi_A(p'_1, p'_2) \) is strictly smaller than the equilibrium profit \( \pi_A(p_1, p_2) \):

\[
\pi_A(p'_1, p'_2) = \frac{2\gamma t(2\gamma - 1)(\gamma(2 - \gamma)^2 + (1 - \gamma)^2)}{(-4\gamma^2 + 7\gamma - 1)^2}
\]

\[
\pi_A(p_1, p_2) = \frac{t[2\gamma(1 + \gamma)(2\gamma - 1) + (4\gamma - 1)(1 - \gamma)(3\gamma - 1)]}{2(-4\gamma^2 + 7\gamma - 1)^2}
\]

and

\[
\pi_A(p_1, p_2) - \pi_A(p'_1, p'_2) = \frac{t(1 - \gamma)^2[4\gamma(2\gamma - 1)(1 - \gamma) + 1]}{2(-4\gamma^2 + 7\gamma - 1)^2} > 0.
\]
This completes the proof of Step 5. ■

**Step 6.** Define

\[ P_e = \left\{ (p'_1, p'_2) \in \mathbb{R}^2_+ \mid p'_1 \in [\bar{p}_1^W, \bar{p}_1], p'_2 \geq \bar{p}_2 \right\}, \]

where the critical values, \( \bar{p}_2, \bar{p}_1 \) and \( \bar{p}_1^W \), are defined in Step 1. Then, the profit of a deviating firm (firm A) for \((p'_1, p'_2) \in P_e\) is given by

\[ \pi_A(p'_1, p'_2) = p'_1(x_{AB}(p'_1) - x_{AW}(p'_2)) + p'_2 \gamma x_{AW}(p'_2). \]

A deviation to any \((p'_1, p'_2) \in P_e\) is never profitable.

**Proof of Step 6.** There are two cases. Consider first the case \(p'_1 \leq \bar{p}_1^{A2}\). In this case, \(x_2 \leq x_{AB}\). It holds that \(U_A(x) > V(x)\) for all the buyers \(x \in (x_2, \max\{1 - x_2, x_{AB}\})\).

Consider next the case \(p'_1 > \bar{p}_1^{A2}\). In this case, \(x_2 > x_{AB}\). In period 2, buyers \(x \in (x_{AB}, x_2)\) will prefer to buy from A with probability \(\gamma\) and from B with probability \(1 - \gamma\) and so their expected value of waiting is given by (22). As shown in the proof of \(P_e(3)\), we have \(U_B(x) \geq V(x) \Leftrightarrow x \geq x_{BW}(p'_2), \) where \(x_{BW}(p'_2) \leq x_{AB}(p'_1) \Leftrightarrow p'_1 \leq \bar{p}_1\). Buyers with \(x \in (x_2, 1 - x_2)\) prefer to buy from B in both periods and so \(U_B(x) > V(x) \Leftrightarrow p_2 > p_1\).

All in all, whenever \((p'_1, p'_2) \in P_e\), buyers with \(x \in [0, x_{AW}]\) wait, the proportion \(\gamma\) of them will buy from A in period 2, and buyers with \(x \in [x_{AW}, x_{AB}]\) will buy from A in period 1. Hence,

\[ \pi_A(p'_1, p'_2) = p'_1(x_{AB}(p'_1) - x_{AW}(p'_2)) + p'_2 \gamma x_{AW}(p'_2) \]

\[ = p'_1 (1 - \gamma)p_1 - \gamma p'_1 + (2\gamma - 1)(\gamma p'_2 + (1 - \gamma)p_2) \]

\[ = \frac{p'_1 (1 - \gamma)p_1 - \gamma p'_1 + (2\gamma - 1)(\gamma p'_2 + (1 - \gamma)p_2)}{2t(2\gamma - 1)(1 - \gamma)} + \frac{p'_2 \gamma (1 - \gamma)t - \gamma p'_2 - (1 - \gamma)p_2 + p'_1}{2t(1 - \gamma)}. \]

Differentiation yields

\[ \frac{\partial \pi_A(p'_1, p'_2)}{\partial p'_2} = \frac{\gamma [t(1 - \gamma) - 2\gamma p'_2 - (1 - \gamma)p_2 + 2p'_1]}{2t(1 - \gamma)}. \]

Observe that, with noting \(\bar{p}_1 = \frac{(1-\gamma)p_1 + (2\gamma - 1)(\gamma p'_2 + (1 - \gamma)p_2)}{\gamma}\),

\[ \frac{\partial \pi_A(p'_1, p'_2)}{\partial p'_2} \bigg|_{\bar{p}_1} < 0 \quad \Leftrightarrow \quad \frac{t\gamma + (3\gamma - 2)p_2 + 2p_1}{2\gamma} < p'_2, \]

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and, with noting $\bar{p}_W \equiv \frac{(1-\gamma)t - \gamma p_2 + p_1}{1-\gamma}$,

$$\frac{\partial \pi_A(p_1', p_2')}{\partial p_2'} |_{p_1' = \bar{p}_1, p_2' = \bar{p}_W} < 0 \iff 0 < \gamma (1-\gamma) t + (\gamma^2 - 5\gamma + 2)p_2 + 2(2\gamma - 1)p_1$$

$$\iff 4\gamma (1-\gamma)(-\gamma^2 + \gamma + 1) > 0,$$

where the last inequality holds true for all $\gamma \in (\frac{1}{2}, 1)$. Hence, when $(p_1', p_2') \in P_e$, the profit is strictly decreasing in $p_2'$, and so it is maximized at $p_2' = \bar{p}_2$, which is strictly smaller than $\pi_A(p_1, p_2)$. This completes the proof of Step 6. ■

**Proof of Corollary 2**

The equilibrium prices satisfy:

$$\frac{dp_1^*}{d\gamma} = [2\gamma^2 + (1-\gamma)^2] \frac{3t(a^*)^2}{2\gamma^2} > 0, \quad \frac{dp_2^*}{d\gamma} = [2\gamma^2 + (1-\gamma)^2] \frac{t(a^*)^2}{\gamma^2} > 0,$$

$$\frac{d(p_2^* - p_1^*)}{d\gamma} = -[2\gamma^2 + (1-\gamma)^2] \frac{t(a^*)^2}{2\gamma^2} < 0.$$

For the fraction of customers each firm serves in advance, $a^*$, we find

$$\frac{da^*}{d\gamma} = (4\gamma^2 - 1) \frac{(a^*)^2}{2\gamma^3} > 0 \quad \forall \gamma \in (\frac{1}{2}, 1).$$

The profit of a firm satisfies:

$$\frac{d\Pi^*}{d\gamma} = (56\gamma^4 + 28\gamma^3 - 24\gamma^2 + 11\gamma - 1) \frac{t(a^*)^2}{2\gamma^2}$$

$$> (4\gamma^2 + 11\gamma - 1) \frac{t(a^*)^2}{2\gamma^2} > 0 \quad \forall \gamma \in (\frac{1}{2}, 1).$$

This completes the proof of Corollary 2. ■

**Proof of Corollary 3**

Consider

$$p_2^* - p_1^* - (p_2^M - p_1^M) = (1-\gamma)t[a^* - \frac{1}{2}].$$

Corollary 2 has shown that $a^*$ is increasing in $\gamma \in (\frac{1}{2}, 1)$. It converges to $\frac{2}{3}$ for $\gamma \to \frac{1}{2}$. It therefore holds that $a^* > \frac{1}{2}$ and hence $p_2^* - p_1^* - (p_2^M - p_1^M) > 0$ for all $\gamma \in (\frac{1}{2}, 1)$. 37
The fact that \( p_1^M - p_1^* = s - \frac{t}{2} - p_1^* > 0 \) follows from Assumption 1 which guarantees that the lowest consumer type \( x = \frac{1}{2} \) obtains positive utility from purchasing at the equilibrium price \( p_1^* \).

Finally, consider
\[
p_2^M - p_2^* = s - \gamma \left( \frac{t}{2} - a^* \left( \frac{3}{2} - \frac{1}{2\gamma} \right) t \right) = s - \frac{ta^*}{4\gamma}(4\gamma^3 + 7\gamma^2 + 5\gamma - 2). \tag{25}
\]
Since \( 5\gamma > 2 \) and \( 4\gamma < 7 \) for all \( \gamma \in \left( \frac{1}{2}, 1 \right) \), \( p_2^M - p_2^* \) is linearly decreasing in \( t \). It becomes zero at
\[
\hat{t}(\gamma) \equiv \frac{-8\gamma^2 + 14\gamma - 2}{-4\gamma^3 + 7\gamma^2 + 5\gamma - 2} s. \tag{26}
\]
To see that \( \hat{t} < \bar{t} = \frac{-8\gamma^2 + 14\gamma - 2}{9\gamma - 3} s \) note that
\[
-4\gamma^3 + 7\gamma^2 + 5\gamma - 2 > 9\gamma - 3 \iff (1 - \gamma)(4\gamma^2 - 3\gamma + 1) > 0 \tag{27}
\]
for all \( \gamma \in \left( \frac{1}{2}, 1 \right) \). This also shows that \( \hat{t} \to \bar{t} \) for \( \gamma \to 1 \). It is easy to see that
\[
\frac{d(\hat{t})}{d\gamma} = \frac{-2(16\gamma^4 - 56\gamma^3 + 81\gamma^2 - 30\gamma + 9)}{(4\gamma^3 - 7\gamma^2 - 5\gamma + 2)^2} < 0 \tag{28}
\]
for all \( \gamma \in \left( \frac{1}{2}, 1 \right) \). ■

**Proof of Proposition 3**

To show that the welfare loss due to competition is increasing in \( \gamma \) consider
\[
\frac{d(W^* - W^M)}{d\gamma} = \frac{t}{8} \left( 4(a^*)^2 - 1 - 8(1 - \gamma)a^* \frac{da^*}{d\gamma} \right) = \frac{t}{8} \left( 8\gamma^3 - 2\gamma^2 - 6\gamma + 4 \right) \frac{(a^*)^3 - 1}{\gamma^2} > 0
\]
\[\Leftrightarrow f(\gamma) \equiv 64\gamma^6 - 336\gamma^5 + 700\gamma^4 - 527\gamma^3 + 111\gamma^2 + 11\gamma + 1 > 0. \]

To prove the last inequality, note that if there exists some \( \gamma \in \left( \frac{1}{2}, 1 \right) \) such that \( f(\gamma) < 0 \), then, since \( \lim_{\gamma \to \frac{1}{2}} f(\gamma) = \frac{21}{8} > 0 \) and \( \lim_{\gamma \to 1} f(\gamma) = 24 > 0 \), we must have at least two values of \( \gamma \in \left( \frac{1}{2}, 1 \right) \) that satisfies \( f(\gamma) = 0 \). Now, collecting terms, the function \( f \) takes the
form \( f = az^2 + bz + c \) with \( z \equiv 1 - \gamma \), and

\[
\begin{align*}
\quad a & \equiv 4\gamma^2(16\gamma^2 - 52\gamma + 55) > 0, \\
\quad b & \equiv -121\gamma^2 < 0, \\
\quad c & \equiv 12\gamma^2 + 11\gamma + 1 > 0.
\end{align*}
\]

Any solution \( z \in (0, \frac{1}{2}) \) of \( f = 0 \) must satisfy

\[
z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]

Assuming \( b^2 \geq 4ac \) (which is required for the existence of a solution) the larger of the solutions, denoted by \( z_+ \), must therefore satisfy

\[
z_+ = \frac{-b + \sqrt{b^2 - 4ac}}{2a} < \frac{1}{2} \Leftrightarrow (-b)^2 < \left(a - \sqrt{b^2 - 4ac}\right)^2 \Leftrightarrow 0 < a - 4c - 2\sqrt{b^2 - 4ac}. \tag{29}
\]

Observe that

\[
c - \frac{a}{4} = -16\gamma^4 + 52\gamma^3 - 43\gamma^2 + 11\gamma + 1 > -16\gamma^4 + 48\gamma^3 - 44\gamma^2 + 12\gamma + 1 > 0
\]

for all \( \gamma \in (\frac{1}{2}, 1) \). The first inequality holds since \( 4\gamma^3 + \gamma^2 - \gamma > 0 \) for all \( \gamma \in (\frac{1}{2}, 1) \). To see that the last term is positive note that its derivative is given by \( 4(3 - 4\gamma)(4\gamma^2 - 6\gamma + 1) \).

Since \( 4\gamma^2 - 6\gamma + 1 = 2(1 - \gamma)(1 - 2\gamma) - 1 < 0 \) for all \( \gamma \in (\frac{1}{2}, 1) \) there exists a unique minimum at \( \gamma = \frac{3}{4} \). Its value is given by \( \frac{7}{16} > 0 \).

Hence, the inequality (29) cannot hold for \( \gamma \in (\frac{1}{2}, 1) \). This implies that there is no \( \gamma \in (\frac{1}{2}, 1) \) that satisfies \( f(\gamma) = 0 \). Hence we must have \( f(\gamma) > 0 \) for all \( \gamma \in (\frac{1}{2}, 1) \). \hfill \blacksquare

**Proof of Proposition 4**

**Part 1:** Corollary 3 has shown that for \( t < \hat{t} \) competition reduces prices in both periods. Hence, in this case, all consumers benefit from competition independently of their preference intensity. Suppose \( t > \hat{t} \), so that \( p_2^* > p_2^M \). Under monopoly, a consumer with \( \sigma = \frac{1}{2} \) is indifferent between buying in advance and waiting, so that \( p_2^M - p_1^M - t(1-\gamma)\sigma = 0 \) and hence \( p_2^M - p_1^* - t(1-\gamma)\sigma > 0 \). Under competition, a consumer with \( \sigma = a^* \) is indifferent between buying in advance and waiting, so that \( p_2^* - p_1^* - t(1-\gamma)\sigma = 0 \) and thus \( p_2^M - p_1^* - t(1-\gamma)\sigma < 0 \).

Since \( p_2^M - p_1^* - t(1-\gamma)\sigma \) is strictly decreasing in \( \sigma \) it follows that there exists a \( \sigma_{CS} \equiv \frac{p_2^M - p_1^*}{t(1-\gamma)} \in \).
$(\frac{1}{4}, a^*)$ such that $CS^*(\sigma) > CS^M(\sigma) > 0$ iff $\sigma < \sigma_{CS}$. $\sigma_{CS}$ is decreasing in $t$ since $p_2^M - p_1^*$ is decreasing.

**Part 2:** Substitution of prices into (11) gives

$$
\Sigma CS^* - \Sigma CS^M = s - t[\frac{1}{8} + \frac{3}{8}\gamma + (\frac{3}{2} - \frac{1}{2\gamma})a^* - \frac{1}{2}(1 - \gamma)(a^*)^2].
$$

(30)

$\Sigma CS^* - \Sigma CS^M$ is strictly decreasing in $t$ since from $a^* \in (\frac{2}{3}, 1)$ it follows that

$$
\frac{1}{8} + \frac{3}{8}\gamma + (\frac{3}{2} - \frac{1}{2\gamma})a^* - \frac{1}{2}(1 - \gamma)(a^*)^2 > \frac{1}{8} + \frac{3}{8}\gamma + (\frac{3}{2} - \frac{1}{2\gamma})\frac{2}{3} - \frac{1}{2}(1 - \gamma)
$$

$$
= \frac{7\gamma^2 + 3\gamma - 2}{8\gamma} > 0
$$

for all $\gamma \in (\frac{1}{4}, 1)$. Hence

$$
t_{CS} = s[\frac{1}{8} + \frac{3}{8}\gamma + (\frac{3}{2} - \frac{1}{2\gamma})a^* - \frac{1}{2}(1 - \gamma)(a^*)^2]^{-1}
$$

(32)

is such that $\Sigma CS^* - \Sigma CS^M > (\vartriangleleft)0$ iff $t < (\triangleright)t_{CS}$. $t_{CS} < \hat{t}$ since for $t \to \hat{t}$ we have $p_1^* \to p_1^M$ and $p_2^* > p_2^M$ which implies that $\Sigma CS^* < \Sigma CS^M$. Similarly, $t_{CS} > \hat{t}$, since for $t \leq \hat{t}$ we have $p_1^* < p_1^M$ and $p_2^* < p_2^M$ which implies that $\Sigma CS^* > \Sigma CS^M$. Finally, to see that $t_{CS}$ is decreasing in $\gamma$ note that $\frac{da^*}{d\gamma} > 0$ by Corollary 2 and

$$
\frac{d}{d\gamma}[(1 - \gamma)(a^*)^2] = \frac{(a^*)^3}{2\gamma^2}(-4\gamma^3 + \gamma^2 + 3\gamma - 2) < 0
$$

(33)

for all $\gamma \in (\frac{1}{4}, 1)$.

\[\blacksquare\]

**References**


