Country-specific rigidities and investment decisions: quantity competition and demand uncertainty

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Abstract

This paper develops a two-stage quantity competition game between two firms in two different countries. Under demand uncertainty, firms invest in capital by installing physical capacity and then produce output. Introducing country-specific institutions in the credit and labour market as a source of cost asymmetry, I assess how different settings impact firms’ investment decisions. Higher or lower investment costs reflect more or less rigid credit markets, whereas production costs capture the country labour market rigidity. By concentrating on one asymmetry at a time, this model predicts a substitution effect between the two factors of production, whose magnitude and direction depend on the type of rigidity considered. The firm in the country with lower investment costs or a more rigid labour market ends up installing higher production capacity; moreover, these firms are going to gain higher expected profits.

Keywords: Industrial organization, International Economics, Trade

JEL Classification: L, F, F1.

1 Introduction

Economic literature is increasingly devoting attention to the impact of different forms of institutional rigidity on firms’ investment decisions, mainly from an empirical perspective. For instance, Autor et al. (2007) identify a positive relation between the level of Employment Protection Legislation, henceforth EPL, and the capital-labour ratio of US firms. This result is opposite to the empirical finding of Cingano et al. (2010), who notice instead a negative impact of EPL on the capital-labour ratio of financially constrained European firms. These opposed

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evidences have been recently reconciled by the theoretical framework suggested by Janiak and Wasmer (2012): in their scenarios the link between these two elements is, generally speaking, negative. However, it can turn positive for a sufficient complementarity between the two factors of production considered. To contextualize their analysis, Janiak and Wasmer (2012) relate the cross-country EPL to the capital-labour ratio index provided by Caselli (2005).

![Figure 1: EPL and capital-labour ratio](image)

From Figure (1), a U-shaped relation between EPL and the capital-labour ratio emerges: low EPL levels correspond to a positive linkage, which turns negative when the labour market becomes more rigid.

Despite the framework provided by Janiak and Wasmer (2012), the economic literature, to the best of my knowledge, fails to theoretically illustrate the role played by more or less rigid institutions in the determination of firms’ investment decision. In fact, according to Cingano et al. (2010), ”while theoretical models offer clear predictions regarding the effects of adjustment costs on job turnover, they provide no guidance on the expected effects of employment protection laws on capital investment, the capital-labour ratio and productivity”.

Through this paper, I attempt to fill this gap by modeling the role of country-specific rigidities on the determination of firms’ investment decisions, meant as the installation of production capacity. To do this, I set up a two-stage model of quantity competition between two firms in two different countries. The two stages of the game reflect an initial investment stage, followed by a production phase. Under demand uncertainty, firms simultaneously install their optimal level of capacity; firms can costly adjust their output targets only once demand has realized\(^1\), by producing above or below the level of capacity previously cho-

\(^1\)Demand realization occurs between the two stages of the game, when an exogenous shock,
In this context, cost asymmetries are precisely related to the different institutional setting in which firms are located and can be traced back to either the credit or the labour market. A more rigid credit market implies higher investment costs in the first stage\(^2\), whereas a higher level of EPL leads to an increase in the second stage adjustment costs. As we will see later on, adjustment costs can be interpreted as overtime costs, whenever final demand is higher than what initially expected and firms have to pay overtime wages to produce above capacity\(^3\). If final demand is instead lower than the initial prediction\(^4\), the production target is below the capacity level and firms have to pay temporary lay-off wages to keep the plant partially idle\(^5\).

By treating one source of institutional asymmetry at a time, I compare the optimal level of capacity installs by applying backward induction. The results are somehow surprising since, regardless of the type of asymmetry considered, the firm in the more rigid country\(^6\) installs a higher level of capacity. What emerges is a substitution effect between capital and labour: for higher investment costs, the rigid firm prefers to wait for the realization of demand in the second stage of the game. If instead we look at the prediction when EPL asymmetries are taken into account, we see that higher labour costs imply a higher investment in capital, regardless of the realization of demand.

An other important aspect taken into account is the the effect of uncertainty on the first stage investment. We have already seen that the realization of final demand depends on the exogenous shock \(\varepsilon\), which can be either positive or negative. Defining the probability of a positive shock as \(P(\varepsilon > 0) = \gamma\), the model predicts that, when only two firms are considered, they are going to install capacity if and only if \(\gamma > \frac{1}{2}\). That is, firms are willing to invest only if they expect the positive shock to occur with a sufficiently high probability. This result strongly affects the comparison of expected profits, since, on expectations, the firm with a higher level of capacity attains higher expected profits.

The paper is structured as follows: section (2) provides a useful insight to the related literature; section (3) introduces the theoretical framework I have developed, in terms of preferences and production structure; section (4) introduces a baseline model, which consists of a perfectly symmetric duopoly between two firms operating in two identical countries. Subsections (5.1) and (5.2) introduce institutional asymmetries in the first and second stage of the game, respectively. Sub subsection (5.3) extends the analysis of the second stage cost asymmetries by analyzing respectively a three country-three firms model. Last, section (6) draws the main conclusions.

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\(\varepsilon\), demand additive, has hit final demand

\(^2\)And thus reflect the returns on capital firms have to pay to lending institutions

\(^3\)Which occurs for a positive realization of the exogenous shock \(\varepsilon\).

\(^4\)And the exogenous shock \(\varepsilon\) is negative.

\(^5\)Since capacity cannot be dismantled.

\(^6\)Which thus faces higher first or second stage costs.
2 Literature review

This paper refers mainly to two different streams of economic literature. First and foremost, the seminal contributions of Spence (1977), Dixit (1979), Dixit (1980) investigated the role of strategic interaction within quantity competition games. According to Spence (1977)’s Stackelberg model, the incumbent’s quantity commitment, which is assumed to be invariant with respect to the entrant’s decision\(^7\), is an effective entry deterrence tool; still, the quantity chosen at the first stage might lead to second stage excess of capacity, with respect to final demand.

By preserving this constant commitment assumption\(^8\), Dixit (1979) further investigates the role of initial quantity pre-commitment as an effective entry-deterrence strategy chosen by the incumbent. His duopoly model mainly investigates the role of product differentiation in easing or impeding entry; still, in an extension of his model, he studies how the commitment to an excess of capacity level, prior to entry, is a reliable threat of predatory output in case of entry by the inactive firm.

In his seminal contribution, Dixit (1980) extends his previous work by assuming that the pre-entry investment decision can be ex-post adjusted\(^9\). In this way, the established firm can changing the initial conditions of the game\(^{10}\) by changing the post-entry structure of its marginal costs.

Kreps and Scheinkman (1983) extend Dixit (1980), by implementing product differentiation in a two stage game in which firms simultaneously set their capacity and, once they know the level chosen by the competitor, engage in price competition. The fundamental prediction of this model is that the equilibrium is not only unique but it corresponds exactly to the standard Cournot outcome.

The role of commitment has also be analyzed by Spencer and Brander (1992), who investigate the trade-off between quantity pre-commitment and flexibility by introducing uncertainty\(^{11}\) in alternative contexts of oligopolistic competition. They see how this source of variation affects the solution to this trade-off, by stressing the importance of pre-commitment as an entry-deterrent strategy.

The second literature branch is the international trade debate. First of all, Neary (2002) strongly contributes to the formalization of oligopolistic models by elevating the research to a general equilibrium framework. Furthermore, Neary and Tharakan (2012) brush up the main implications of Kreps and Scheinkman (1983) by identifying an endogenous competition threshold according to which the equilibrium outcome switches from Cournot to Bertrand.

\(^7\)That is, the quantity commitment is constant even in case of no-entry in the second stage.
\(^{8}\)Also known as the Sylos Postulate.
\(^{9}\)Even though the only admitted adjustment is positive, so that investments can only increase.
\(^{10}\)Which are however assumed to be exogenous.
\(^{11}\)Meant as an exogenous, demand additive, shock.
3 Model specification

In this section I describe the preferences and the cost structure of the model. Throughout the rest of the paper, I will be mainly referring to a duopoly model of free trade, with one firm per country. Country-specific institutional settings impact the cost structure of the two firms, thus generating cost asymmetries. Still, the two firms produce the same homogenous good.

3.1 Utility

Preferences in the two countries are defined over a homogenous good $x$. Since this model displays a free trade setting, consumers in the two countries can equivalently purchase the same good, regardless of its origin, according to this individual, quadratic, utility function:

$$u(x) = Ax - \frac{bx^2}{2}$$  \hspace{1cm} (1)

with $A, b > 0$. Assuming that overall population size is equal to $L$, we can derive the indirect demand function

$$p = \frac{L(A - bx)}{\lambda}$$  \hspace{1cm} (2)

where $\lambda$ is the marginal utility of income. Since the model describes a partial equilibrium setting, we can assume with no loss of generality that $\lambda = 1$. Furthermore, for simplicity, I shall assume that $L = b = 1$. Equation (2) is the demand function the two firms expect when choosing their capital investment in the first stage of the game.

However an exogenous, demand additive\(^{12}\) shock $\varepsilon$ occurs right before second stage production effectively takes place. A positive, demand increasing, shock occurs with probability $P(\varepsilon > 0) = \gamma$. The real, and final, demand function, according to which firms must make their second stage production decisions is thus

$$p = A - x + \varepsilon$$

where $\varepsilon > 0$, $E(\varepsilon) = 0$ and $E(\varepsilon) = \sigma^2$.

Market clearing in the goods market must always hold, regardless of the realization of demand:

$$x = x_i + x_j = (k_i + q_i) + (k_j + q_j)$$

where $x_i$ is the overall output of firm $i$, which is in turn given by the installed capacity, $k_i$, and the second stage output $q_i$.\(^{13}\)

\(^{12}\)As in Spencer and Brander (1992).
\(^{13}\)With $q_i < 0$ for $\varepsilon < 0$
3.2 Production

As displayed in figure (2), firms must decide their optimal level of capacity installation $k_i$ in the first stage of the game, basing their choices on the expected final demand (2). In the second stage, production eventually takes place: according to the realization of demand, firms can adjust the previously installed capacity, by producing above $k_i$, with $q_i > 0$ and $x_i = k_i + q_i$, or below capacity, with $q_i < 0$ and $x_i = k_i - |q_i|$, by keeping the plant partially idle.

From this point of view, this model departs from Dixit (1980), whose capacity commitment could only be increased. In my setting, firms can instead costly adjust capacity in either ways. Whereas in Dixit (1980) the adjustment cost is the marginal cost of installing capacity plus the marginal cost of producing the additional output\textsuperscript{14}, in my model the cost for capacity installation has already been paid for. What firms have to sustain is the mere adjustment cost, whose burden reflects the rigidity of the labour market.

![Figure 2: Timing of the model](image-url)

In this setting, there are two factors of production: capital\textsuperscript{15} and labour. Each factor of production is stage specific: the former at the first stage, and the latter at the second one. To invest in capital, firms need to borrow liquidity from the financial market, at a cost $c > 0$, which reflects the borrowing rate per unit of capacity. The overall first stage cost is assumed to be quadratic, with $ck_i^2$.

Physical labour is needed to produce output, according to the linear cost function $\theta |q_i|$. The parameter $\theta$ thus mirrors the EPL of the country of firm $i$. To produce above capacity, firms have in fact to pay overtime wages; to produce below capacity firms must instead keep the plant idle and incur temporary lay-off costs. Throughout this paper, I assume that production below or above capacity imply the same cost $\theta$. Future work could extend this setting to different adjustment costs.

The profit function of the profit maximizing firm $i$ is given by:

$$\pi_i = [A \pm \varepsilon - x](k_i \pm |q_i|) - \theta |q_i| - ck_i^2.$$  \hspace{1cm} (3)

By focusing on a country-specific rigidity at a time, the purpose of this model is to understand how alternative institutional settings impact and affect the

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\textsuperscript{14}The marginal cost is thus given by $w_i + r_i$.

\textsuperscript{15}Meant as physical capacity.
individual firms investment decisions. To better understand the mechanisms behind strategic interaction and institutional rigidity, as well as their role in the determination of capacity investment, I have set up four different models:

1. The baseline model, in which the two countries are perfectly identical, and the equilibrium outcome is obviously symmetric;

2. A first stage asymmetric model, in which the two countries display two different costs of borrowing capital\(^\text{16}\);

3. A second stage asymmetric model, with different regimes in the labour regulation framework\(^\text{17}\);

4. A three firm model with different EPL levels, to extend the results of the second stage asymmetric model.

All these models are solved by means of backward induction. Starting from the second stage of the model, I derive each firm’s optimal second stage output in the good and bad state of the world. Then, these two different optimal production choices are introduced in the expected profits to identify the optimal level of capital.

4 The baseline model

This section describes a perfectly symmetric duopoly, in which the two countries have the same institutional setting and the firms face identical cost costs: \(c_i = c_j = c\) and \(\theta_i = \theta_j = \theta\), at the first and second stage respectively. To solve the model, we start considering the optimal production level for each firm once final demand has realized. Since demand can be either higher or lower, we always have to distinguish the two different scenarios, namely the good and the bad ones. The scenario specific level of production that maximizes profits is then plugged in the expression for expected profits that is maximized at the capacity installation stage.

4.1 Second stage

In the second stage, each firm knows final demand: as a consequence, we need to derive each firm’s optimal second stage output \(q_i\) for each state of the world.

In the good state, firm \(i\) maximizes

\[
\pi_{i,g} = [\bar{A} - (k_i + q_i) - (k_j + q_j)](k_i + q_i) - \theta q_i - c k_i^2,
\]

Denoted by \(c_i\) and \(c_j\).

Denoted by \(\theta_i\) and \(\theta_j\).
where $\bar{A} \equiv A + \varepsilon$, from which we obtain the reaction function

$$q_{i,g} = \frac{\bar{A} - q_j - k_j - 2k_i - \theta}{2}.$$  

Symmetry leads to the simplified best response function

$$q_{i,g} = \frac{\bar{A} - k_j - 2k_i - \theta}{3}.$$  

The market clearing price is

$$p_g = \frac{\bar{A} + 2\theta}{3},$$  

which is increasing in the second stage cost component $\theta$ and in the shock realization. That is, the cost to increase output beyond capacity is directly transferred to the final consumer.

If the shock is instead negative, firms produce below their initial commitment by setting a negative adjustment output in the second stage: they thus choose $|q_i|$, from $x_i = k_i - |q_i|$\textsuperscript{18}, and maximizing the corresponding profits

$$\pi_{i,b} = [\bar{A} - (k_i - q_i) - (k_j - q_j)](k_i - q_i) - \theta|q_i| - c k_i^2$$

where $\bar{A} \equiv A - |\varepsilon|$, with $|\varepsilon| < A - \theta$. The reaction function is

$$q_{i,b} = \frac{2k_i + k_j - q_j - (\bar{A} + \theta)}{2}$$

which shrinks to

$$q_{i,b} = \frac{2k_i + k_j - (\bar{A} + \theta)}{3}$$

once symmetry is taken into account. The market clearing price in the bad state of the world is

$$p_b = \frac{\bar{A} - 2\theta}{3},$$  

which is decreasing in both $|\varepsilon|$ and $\theta$. Opposite to the good state of the world, during economic downturns firms will be charging lower prices because of lower demand level. Moreover, the adjustment cost will not be transferred to the final consumer.

### 4.2 First stage

Backward induction allows us to take the optimal values of the second stage back to the first stage. Recalling that a positive shock $\varepsilon$ occurs with a probability

\textsuperscript{18} Throughout the rest of the paper, I’ll be ruling out the extreme case in which $|\varepsilon| < A - \theta$ to avoid the extreme situation in which $q_{i,b} = k_{i,b} = x_{i,b} = 0$.  

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\( \gamma \), firms maximize expected profits

\[
E(\pi_i) = \gamma E \left\{ p_g x_{i,g} - \theta q_{i,g} - ck_i^2 \right\} + (1 - \gamma)E \left\{ p_b x_{i,b} - \theta q_{i,b} - ck_i^2 \right\}
\]

\[
= -ck_i^2 + \gamma \left\{ \frac{k_i(A + 2\theta)}{9} + \frac{2\theta k_i}{3} \right\} + (1 - \gamma) \left\{ \frac{k_i(A - 2\theta)}{9} - \frac{2\theta k_i}{3} \right\} + K \quad (6)
\]

with respect to \( k_i \), where \( K \) is a set of parameters not affected by the variable of interest.

The result of this problem is

\[
k_i = \frac{A + 8\theta(2\gamma - 1)}{18c} \quad (7)
\]

which is decreasing in the installation cost component, \( c \). According to the value of \( \gamma \), we can distinguish up to three different levels for the optimal level of capacity:

\[
k_i = \begin{cases} 
    \frac{A + f(\theta)}{18c}, & \text{for } \gamma \in \left( \frac{1}{2}, 1 \right] \\
    \frac{A}{18c}, & \text{for } \gamma = \frac{1}{2} \\
    \frac{A - f(\theta)}{18c}, & \text{for } \gamma \in [0, \frac{1}{2})
\end{cases}
\]

where \( f(\theta) \) is increasing in \( \theta \).

4.2.1 Comparative statics

This section describes how firms react to different levels of uncertainty volatility or to exogenous changes in the cost structure. First of all, capacity is always increasing in the probability \( \gamma \) of the positive shock, since

\[
\frac{\partial k_i}{\partial \gamma} = \frac{8\theta}{9c}
\]

If the positive shock becomes more probable, firms have a stronger incentive to accumulate more and more capacity, to better accommodate future consumption needs and to reduce adjustment costs during the production stage.

If instead we look at the impact of the two cost components on the investment decision of firms, we can come up with predictions on the factor substitution effect\(^{20}\). The effect of the mere investment cost \( c \) is generally negatively related to the optimal investment \( k \), since

\[
\frac{\partial k_i}{\partial c} = -\left[ \frac{A + 8\theta(2\gamma - 1)}{18c^2} \right] < 0 \text{ for } \gamma \geq \frac{1}{2}
\]

\(^{19}\)And, consequently, the negative shock occurs with probability \( 1 - \gamma \).

\(^{20}\)Recalling that the labour workforce is fixed.
The impact of second stage adjustment cost depends on the probability $\gamma$, too, since
\[
\frac{\partial k_i}{\partial \theta} = \frac{2\gamma - 1}{2c} = \begin{cases} 
> 0 & \text{for } \gamma > \frac{1}{2} \\
= 0 & \text{for } \gamma = \frac{1}{2} \\
< 0 & \text{for } \gamma < \frac{1}{2} 
\end{cases}
\]
Higher EPL makes firms install more capacity only if the positive shock is more likely to occur: firms prefer in fact to invest more in capital rather than incurring higher labour market rigidity at the second stage. If the negative shock is instead more probable, higher levels of EPL do not threaten enough firms, that decide to commit to a lower output\(^{21}\) and eventually adjust more in the second stage. As long as $\gamma > 1/2$, the effects of $c$ and $\theta$ move in opposite directions, but since the former is negative, we can conclude that the latter will prevail.

4.3 Performances

The optimal level of $k_i$ enables us to have an explicit expression for the good state output levels:
\[
q_{i,g} = \frac{\bar{A} - \theta}{3} - \frac{A + 8\theta(2\gamma - 1)}{18c}
\]
\[x_{i,g} = \frac{\bar{A} - \theta}{3}. \tag{8}\]

The second stage output level is increasing in $c$ and in $\varepsilon$. From the analysis of capacity, we know that $k_i$ negatively depends on $c$: higher installation costs are translated into lower capacity. If final demand is higher, firms will have to adjust more if the initial investment does not allow them to catch up with demand needs. Nonetheless, this effect might be partially offset by the impact of $\theta$ on $q_{i,g}$, which has to be declined according to the realization of $k_i$:

- For $\gamma \geq 1/2$, $q_{i,g}$ negatively depends on $\theta$;
- For $\gamma < 1/2$, $q_{i,g}$ is increasing in $\theta$ for $4|2\gamma - 1| > 3c$.

Last but not least, the overall output $x_{1,g}$ is decreasing in $\theta$: despite the decomposition of the effect $\theta$ on either $k_i$ and $q_{i,g}$, the overall effect on total, and final output, is negative.

In the bad state the performances of the two firms are given by
\[
q_{i,b} = \frac{A + 8\theta(2\gamma - 1)}{18c} - \frac{\bar{A} + \theta}{3}
\]
\[x_{i,b} = \frac{\bar{A} + \theta}{3}. \tag{9}\]

Clearly, the second stage output is now decreasing in $c$: less output is now needed and, as a consequence, the more the firm has installed, the more it will have to keep idle to accommodate for the new demand needs. Furthermore, we can decompose the effect of $\theta$ according to the probability $\gamma$: $q_{i,b}$ is increasing in

\(^{21}\)Thus investing less in capacity.
only for a very high probability of a positive shock, with \( \gamma > 7/8 \). Whenever the probability is lower than 7/8, regardless the level of initial commitment, higher second stage costs will be negatively related to the second stage output.

The profits in the good state of the world are given by

\[
\pi_{i,g} = \frac{(\overline{A} - \theta)^2}{9} + \frac{A + 8\theta(2\gamma - 1)}{18^2c}[2\theta(13 - 8\gamma) - A]
\]

whereas the bad state profits are

\[
\pi_{i,b} = \frac{(\overline{A} + \theta)^2}{9} - \frac{A + 8\theta(2\gamma - 1)}{18^2c}[A + 2\theta(5 + 8\gamma)],
\]

with

\[\pi_{i,g} > \pi_{i,b}.\]

The expected profits are

\[
E(\pi_i) = \frac{A^2 + \sigma^2 + \theta^2 + 2A\theta(1 - 2\gamma)}{9} - \frac{A + 8\theta(2\gamma - 1)}{18^2c}[A + 10\theta(2\gamma)].
\] (10)

5 The asymmetric model

In this section, cost asymmetries\(^{22}\) are gradually introduced. By focusing on one source of asymmetry at a time, this section respectively describes different access to the credit market and more or less rigid labour markets. Last, I extend the EPL analysis to a three country model.

5.1 First stage: access to the capital market

The first asymmetric duopoly involves different capital investment costs. Throughout the remaining of this section, I’ll be referring to a flexible and a rigid country according to the magnitude of this first-stage cost. More precisely, the firm in the flexible country 2 can benefit from lower investment costs, whereas the firm in the rigid country 1 faces higher costs.

Assumption A In country 1, credit and financial support are more costly than what they are in country 2, that is \( c_1 > c_2 \).

With this assumption in mind we can now look at the implications of these institutional differences on the installation of capacity. The procedure follows the steps performed in the baseline model analysis: we start from the second stage, deriving the optimal second stage output, for each firm and for each state of the world. Then, we use these optimal values to identify the optimal capital investment by looking at the expected profits.

\(^{22}\)Related to the country-specific institutional setting.
5.1.1 Second stage

In the second stage, whenever final demand is higher than what initially expected, firm $i = 1, 2$ maximizes

$$\pi_{i,g} = (A - (k_i + q_i) - (k_j + q_j))(k_i + q_i) - \theta q_i - c_i k_i^2,$$

with respect to $q_i$. The reaction function for firm $i$ is

$$q_{i,g} = \frac{\bar{A} - 2k_i - k_j - q_j - \theta}{2}$$

and, solving for $q_{i,g}$ and $q_{j,g}$ we obtain

$$q_{i,g} = \frac{\bar{A} - 3k_i - \theta}{3} \quad q_{j,g} = \frac{\bar{A} - 3k_j - \theta}{3}.$$  \hspace{1cm} (11)

The market clearing price in the good state of the world is

$$p_g = \frac{\bar{A} + 2\theta}{3}$$  \hspace{1cm} (12)

which is, coherently to the symmetric model discussed above, increasing in both $\theta$ and $\varepsilon$.

When the shock is negative, firms must instead produce below capacity; they consequently solve

$$\pi_{i,b} = (A - (k_i - q_i) - (k_j - q_j))(k_i - q_i) - \theta q_i - c_i k_i^2,$$

with respect to $q_i$: the reaction function is

$$q_{i,b} = \frac{2k_i + k_j - q_j - (\bar{A} + \theta)}{2}$$

from which we obtain the second stage output of firm $i$ and $j$ respectively:

$$q_{i,b} = \frac{3k_i - (\bar{A} + \theta)}{3} \quad q_{j,b} = \frac{3k_j - (\bar{A} + \theta)}{3}.$$  \hspace{1cm} (13)

Last, the market clearing price in the bad state is

$$p_b = \frac{\bar{A} - 2\theta}{3}$$  \hspace{1cm} (14)

which is decreasing in $\varepsilon$ and $\theta$.

5.1.2 First stage

Maximizing expected profits we obtain the optimal level for the initial investment in capacity:

$$k_i = \frac{\theta(2\gamma - 1)}{2c_i}$$  \hspace{1cm} (15)
which is positive only for a sufficiently likely good shock, that is, as long as $\gamma > \frac{1}{2}$. Since capacity cannot be negative, we can state that

$$k_i = \begin{cases} \frac{\theta(2\gamma - 1)}{2c_i}, & \gamma \in \left(\frac{1}{2}, 1\right) \\ 0, & \gamma \in \left[0, \frac{1}{2}\right] \end{cases}$$

The optimal level of capacity is increasing in the second stage cost component $\theta$ and decreasing in the installation costs $c_i$. The introduction of cost asymmetries allows us to make predictions over the different levels of capacity that the two competitors are going to choose. Recalling assumption (A), we can immediately see that the firm in the flexible country is going to have a strong incentive to install more capacity than the firm in the rigid one. That is

$$k_2 > k_1 \text{ for } c_1 > c_2$$

5.1.2.1 Comparative statics The effects of $\gamma$ and of the two cost components are very similar to the relationship described for the baseline model (4.2.1), that is

$$\frac{\partial k_i}{\partial \gamma} = \frac{\theta}{c_i} > 0$$

and

$$\frac{\partial k_i}{\partial \theta} = \frac{2\gamma - 1}{2c_i} \begin{cases} > 0 \text{ for } \gamma > 1/2 \\ = 0 \text{ for } \gamma = 1/2 \\ < 0 \text{ for } \gamma > 1/2 \end{cases}$$

What is more predictable in this scenario is the effect of the investment cost $c$ for a low level of $\gamma$, that is

$$\frac{\partial k_i}{\partial c} = -\frac{\theta(2\gamma - 1)}{2c_i^2} = \begin{cases} < 0 \text{ for } \gamma > 1/2 \\ = 0 \text{ for } \gamma = 1/2 \\ > 0 \text{ for } \gamma > 1/2 \end{cases}$$

5.1.3 Performances

The second stage and total output of firm $i = 1, 2$ in the good state of the world are

$$q_{i,g} = \frac{\bar{A} - \theta \cdot \phi}{3} - \frac{\theta(2\gamma - 1)}{2c_i} \quad \text{ and } \quad x_{i,g} = \frac{\bar{A} - \theta}{3}$$

The second stage output is increasing in $\varepsilon$ and is decreasing in $\theta$ as long as $\gamma > 1/2$. In terms of aggregate production, the two firms achieve the same final
level: therefore, the firm in the rigid country is somehow able to catch up by significantly produce above capacity:

\[ k_1 < k_2 \text{ and } x_{1,g} = x_{2,g} \implies q_{1,g} > q_{2,g} \text{ for } c_1 > c_2 \]

In the bad state firm \( i \) ends up producing

\[ q_{i,b} = \frac{\theta(2\gamma - 1)}{2c_i} - \frac{\bar{A} + \theta}{3} \quad x_{i,b} = \frac{\bar{A} + \theta}{3} \tag{17} \]

In line with the baseline model, the second stage output is increasing in \( c \) whenever the shock is negative. Moreover, overall production is equally split between the two firms, but the firm with a higher level of capacity has to keep idle a higher portion of the plant:

\[ k_1 < k_2 \text{ and } x_{1,b} = x_{2,b} \implies |q_{2,b}| > |q_{1,b}| \text{ for } c_1 > c_2 \]

Whenever \( \gamma > 1/2^{23} \), the profits of firm \( i \) in the good state are given by

\[ \pi_{i,g} = \frac{(\bar{A} - \theta)^2}{9} + \frac{\theta^2(2\gamma - 1)(3 - 2\gamma)}{4c_i} \]

we have with \( \pi_{2,g} > \pi_{1,g} \). Recalling our assumption that \( c_1 > c_2 \), the low cost firm is going to attain higher equilibrium profits when the shock is positive. Facilitated by the lower investment costs, this firm has in fact optimally chosen a higher level of capacity, which allows it to adjust less once a higher final demand has revealed. If the shock is instead negative, the corresponding profits of the high-cost firm \( i \) are given by

\[ \pi_{i,b} = \frac{(\bar{A} + \theta)^2}{9} - \frac{\theta^2(4\gamma^2 - 1)}{4c_i} \]

Because of the different investment decisions, the portion of capacity to be kept idle is smaller in the high cost country. As a consequence, the firm in the rigid country will achieve higher bad state profits than the low-cost firm, that incurs instead higher adjustment costs. In terms of profits, we have:

\[ \pi_{i,b} > \pi_{2,b} \]

Still, comparing the profits of the firm on the rigid country we do notice that good state profits are obviously greater than bad state profits:

\[ \pi_{i,g} > \pi_{i,b} \]

To correctly assess the pay-offs associated with the investment in capacity, we need to compare the expected profits each firm achieves. On expectations, firm \( i \) would get

\[ E(\pi_i) = \frac{A^2 + \sigma^2 + \theta^2 + 2A\theta(1 - 2\gamma)}{9} + \frac{\theta^2\gamma(2\gamma - 1)}{2c_i} \tag{18} \]

\(^{23}\)The results when \( \gamma \leq 1/2 \) are described in the next subsection
with

\[ E(\pi_2) > E(\pi_1) \]

That is, besides for the worst performance in the negative state of the world, the firm investing more in capital realizes higher expected profits.

5.1.3.1 \( \gamma \leq 1/2 \) If the positive shock occurs with a low probability, that is \( \gamma \leq 1/2 \), none of the two firms will be investing in capacity: \( k_i = 0, i = 1, 2 \). According to this result, firms prefer to target any possible production level as if they were always producing above capacity. This result is very similar to the setting of Neary and Tharakan (2012), in which firms are allowed to choose between the installation of some capacity and the installation of no capacity at all.

As a consequence, we can distinguish their overall output level according to the realization of the shock\(^{24}\), ruling out the limit case, \( |\varepsilon| < A - \theta \). In this very specific setting, the lack of capacity restores symmetry. The production level of firm \( i \) in the good and bad states are given by

\[ q_{i,g_{nk}} = x_{i,g_{nk}} = \frac{A - \theta}{3} \quad q_{i,b_{nk}} = x_{i,b_{nk}} = \frac{A - \theta}{3}. \]

Profits in the two states are respectively

\[ \pi_{i,g_{nk}} = \frac{(A - \theta)^2}{9} \quad \pi_{i,b_{nk}} = \frac{(A - \theta)(A - 5\theta)}{9} \]

and expected profits are

\[ E(\pi_{i,nk}) = \frac{A^2 + \sigma^2 + \theta^2(5 - 4\gamma) - 2A\theta(4\gamma - 3)}{9}. \]  \( (19) \)

5.2 Second stage: ELP

To introduce asymmetries in the second stage of the game I refer to more or less rigid labour market regulation, which is translated into higher or lower adjustment costs during the production phase. As already introduced, in this setting EPL refers to either the overtime wages that must be paid for production above capacity or to the temporary lay-off costs to keep the plant idle. For the time being, I have assumed that this cost is symmetric in the direction of the adjustment.

What really matters, in fact, is the effect of the country-specific EPL on the determination of the firm’s investment decision. In the annex section I have reported the extreme scenario in which only country is affected by EPL (section (6)). Throughout this analysis I will assume that the firm in country \( i \) is the firm facing a stiffer labour market:

**Assumption B** In country 1, EPL is more binding and the related second stage costs are higher than what they are in country 2; that is \( \theta_1 > \theta_2 \).

\(^{24}\)Though a negative shock, demand is still positive.
5.2.1 Second stage

In the good state-second stage, firm $i = 1, 2$ optimizes

$$\pi_{i,g} = [\bar{A} - (k_i + q_i) - (k_j + q_j)](k_i + q_i) - \theta_i q_i - c k_i^2,$$

with solution

$$q_{i,g} = \frac{\bar{A} - \theta_i - 2k_i - k_j - q_{j,g}}{2}.$$

Since the two firms set their optimal choice simultaneously, by replacing the corresponding value for $q_{j,g}$ we obtain an expression for $q_{i,g}$ that depends only on the installed capacity:

$$q_{i,g} = \frac{\bar{A} - 2\theta_i + \theta_j - 3k_i}{3}.$$

The market clearing price in the good state is

$$p_g = \frac{\bar{A} + \theta_i + \theta_j}{3}. \quad (20)$$

In the bad state instead the firm has to choose how much capacity to keep idle by maximizing profits $\pi_{i,b}$ with respect to $q_{i,b}$:

$$\pi_{i,b} = [\bar{A} - (k_i - q_i) - (k_j - q_j)](k_i - q_i) - \theta_i q_i - c k_i^2$$

from which

$$q_{i,b} = \frac{2k_i + k_j - q_{j,b} - (\bar{A} + \theta_i)}{2} \quad \text{and} \quad q_{i,b} = \frac{\theta_j - 2\theta_i + 3k_i - \bar{A}}{3}. \quad (21)$$

The market clearing price in the bad state of the world is

$$p_b = \frac{\bar{A} - \theta_i - \theta_j}{3}. \quad (22)$$

5.2.2 First stage

Each firm maximizes its expected profits taking into account the optimal decisions of the second stage:

$$E(\pi_i) = \gamma E[p_g(k_i + q_{i,g}) - \theta_i q_{i,g} - c k_i^2] + (1 - \gamma) E[p_b(k_i - q_{i,b}) - \theta_i q_{i,b} - c k_i^2]$$

The optimal investment level chosen at the first stage is

$$k_i = \frac{\theta_i(2\gamma - 1)}{2c} \quad (23)$$

According to the value of the probability $\gamma$, it is possible to distinguish two different cases:

$$k_i = \begin{cases} \frac{\theta_i(2\gamma - 1)}{2c} & \text{for } \gamma \in \left(\frac{1}{2}, 1\right] \\ 0 & \text{for } \gamma \in [0, \frac{1}{2}] \end{cases}$$

Intuitively, we can immediately see how the optimal capacity varies with the first and second stage costs respectively:

\[25\] The second scenario stems from the fact that capacity cannot be negative
• It is obviously decreasing in the investment cost $c$;

• It is increasing in the country specific labour market rigidity, $\theta_i$: since the firm is going to install some capacity only for a very high probability of the positive shock, higher rigidity in the second stage makes the firm increase its capital-labour ratio by choosing ex-ante a higher level of capacity $k_i$;

• What is even more surprising is that this specific substitution effect is stronger for the firm in the rigid country, which is going to set a higher capacity than the firm with a more flexible labour market regulation, that is

$$k_1 > k_2$$

for $\gamma > 1/2$ and $\theta_1 > \theta_2$

5.2.2.1 Comparative statics The impacts of $\gamma$ and the two cost components confirms the pattern identified when analyzing the first stage asymmetries:

$$\frac{\partial k_i}{\partial \gamma} = \frac{\theta_i}{c} > 0,$$

$$\frac{\partial k_i}{\partial c} = -\frac{\theta_i(2\gamma - 1)}{2c^2} = \begin{cases} < 0 & \text{for } \gamma > 1/2 \\ = 0 & \text{for } \gamma = 1/2 \\ > 0 & \text{for } \gamma < 1/2 \end{cases}$$

and

$$\frac{\partial k_i}{\partial \theta_i} = \frac{2\gamma - 1}{2c} = \begin{cases} > 0 & \text{for } \gamma > 1/2 \\ = 0 & \text{for } \gamma = 1/2 \\ < 0 & \text{for } \gamma < 1/2 \end{cases}$$

5.2.3 Performances

The equation for capacity allows us to retrieve the second stage and the overall output levels in both the state of the world:

$$q_{i,g} = \frac{(\bar{A} - 2\theta_i + \theta_j)}{3} - \frac{\theta_i(2\gamma - 1)}{2c}$$

$$x_{i,g} = \frac{(\bar{A} - 2\theta_i + \theta_j)}{3}$$

and

$$q_{i,b} = \frac{\theta_j - 2\theta_i - \bar{A}}{3} + \frac{\theta_i(2\gamma - 1)}{2c}$$

$$x_{i,b} = \frac{2\theta_i - \theta_j + \bar{A}}{3}$$

In the good scenario, the overall production target achieved by the firm in the flexible country is higher, that is $x_{j,g} > x_{i,g}$. Since firm $j$ had previously installed a lower level of capacity, it must necessary fully exploit its cost advantage in the second stage to achieve a higher second stage adjustment, thus

$$x_{2,g} > x_{1,g} \text{ and } k_2 < k_1 \implies q_{2,g} > q_{1,g}.$$
In the bad state, the scenario is reversed: since firm $i$ invested more in capacity, it needs to dismantle more in the second stage. However, since the firm in the flexible firm preferred to commit to a lower level of output, the firm with the higher installed capacity can capture a higher share of the market and attain a greater overall production volume, that is

$$x_{1,b} > x_{2,b} \text{ and } k_1 > k_2 \implies q_{1,b} > q_{2,b}.$$  

In the good states, profits of firm $i$ are given by

$$\pi_{i,g} = \left(\bar{A} - 2\theta_i + \theta_j\right)^2 + \frac{\theta_i^2(2\gamma - 1)(3 - 2\gamma)}{4c}$$

and in the bad state they are

$$\pi_{i,b} = \left(\bar{A} + 2\theta_i - \theta_j\right)^2 - \frac{\theta_i^2(4\gamma^2 - 1)}{4c}$$

Under the assumption $\theta_1 > \theta_2$, good state profits are greater than bad state ones if and only if the absolute value of the shock is such to nullify the negative effect related to its own second stage costs, that is

$$\pi_{1,g} > \pi_{1,b} \text{ for } |\varepsilon| > 2\theta_1 - \theta_2 > 0$$

Moreover, from the comparison of the overall output targets, we can conclude that

$$\pi_{2,g} > \pi_{1,g} \text{ for } \theta_1 > \theta_2.$$  

In the bad scenario, instead, it is the firm in the rigid country that is able to achieve a higher level of profits:

$$\pi_{2,b} > \pi_{1,b} \text{ for } \theta_1 > \theta_2.$$  

Last but not least, to correctly understand the trade-off between flexibility and rigidity we have to look at the expected profits:

$$E(\pi_i) = \frac{(A^2 + \sigma^2)}{81} + \frac{(2\theta_i - \theta_j)^2}{81} + \frac{2A(2\theta_i - \theta_j)(1 - 2\gamma)}{81}$$  

(26)

If we compare the two expected profits, we clearly see how the investment in capacity pays-off in terms of expected profits, since

$$E(\pi_1) > E(\pi_2) \text{ for } \theta_1 > \theta_2$$

Far from being general, this result is consistent with the outcome of the first stage asymmetries\textsuperscript{26} and shows that the firm investing more in capital is going to achieve higher expected pay-offs.

\textsuperscript{26}Despite the opposite impact of country rigidity on the investment decision.
5.3 The three firm model

Paving the way to a more generalized analysis of this specific quantity competition model, I have so far extended the asymmetries at the EPL level to a three firms model, to assess whether the relation between installed capacity and probability become less trivial. In the annex, I have also included an extreme version of EPL asymmetries, in which only one country faces some labour market regulation.

In the alternative setup I am about to describe, there are three firms, located in three different countries, with identical access, or conditions of access, to financial support. Cost asymmetries are instead introduced in the second stage of the game, reflecting the different institutional background each country is endowed with, in terms of labour market regulation and employment protection.

The three country setup implies a specific sorting of the second stage costs:

**Assumption C** Country 1 is the more rigid country, whereas country 3 is the country with the higher unskilled labour market flexibility, that is \( \theta_1 > \theta_2 > \theta_3 \).

5.3.1 Second stage

The profit functions that are going to be maximized in each state of the world are those of the generic model representation previously described. The second stage output to be produced in the good state of the world are

\[
q_{1,g} = \frac{\bar{A} - 3\theta_1 + \theta_2 + \theta_3}{4} - k_1
\]

\[
q_{2,g} = \frac{\bar{A} + \theta_1 - 3\theta_2 + \theta_3}{4} - k_2
\]

\[
q_{3,g} = \frac{\bar{A} + \theta_1 + \theta_2 - 3\theta_3}{4} - k_3
\]

and the associated market clearing price is

\[
p_g = \frac{\bar{A} + \theta_1 + \theta_2 + \theta_3}{4}
\]

(27)

The bad state displays these second stage production levels

\[
q_{1,b} = \frac{\bar{A} + \theta_1 - 3\theta_1 - \bar{A}}{4} + k_1
\]

\[
q_{2,b} = \frac{\bar{A} + \theta_1 - 3\theta_2 - \bar{A}}{4} + k_2
\]

\[
q_{3,b} = \frac{\bar{A} + \theta_2 - 3\theta_3 - \bar{A}}{4} + k_3
\]

and the corresponding market clearing price

\[
p_b = \frac{\bar{A} - (\theta_1 + \theta_2 + \theta_3)}{4}
\]
5.3.2 First stage

The maximization of the expected profits in the first stage leads to the following optimal capacity installations:

\[
\begin{align*}
  k_1 &= \frac{(1-\gamma)(A-\theta_2-\theta_3)+\theta_1(5\gamma-3)}{4c} \\
  k_2 &= \frac{(1-\gamma)(A-\theta_1-\theta_3)+\theta_2(5\gamma-3)}{4c} \\
  k_3 &= \frac{(1-\gamma)(A-\theta_1-\theta_2)+\theta_3(5\gamma-3)}{4c}
\end{align*}
\]

(28)

that vary with the probability \(\gamma\):

\[
\begin{align*}
  k_i &= \begin{cases} 
    \theta_i, & \text{if } \gamma = 1 \\
    \frac{(1-\gamma)\sum_{j\neq i} \theta_j + f(\theta_i)}{(1-\gamma)(A - 4c \sum_{j\neq i} \theta_j)}, & \text{if } \gamma \in \left(\frac{3}{5}, 1\right) \\
    \frac{(1-\gamma)\sum_{j\neq i} |\theta_j| - |f(\theta_i)|}{4c}, & \text{if } \gamma \in \left(0, \frac{3}{5}\right) \\
    \frac{(A - \sum_{j\neq i} \theta_j) - 3\theta_1}{4c}, & \text{if } \gamma = 0
  \end{cases}
\end{align*}
\]

where \(f(\theta_i) = \theta_i(5\gamma - 3)\). Whenever \(\gamma = 0\), we have to rule out negative realization of the investment in capacity thus imposing \(k_i = \max\left\{0, \frac{(A - \sum_{j\neq i} \theta_j) - 2\theta_1}{4c}\right\}\).

In the two firms model, we noticed how the firm facing the most rigid unskilled labour market is the firm going to install the highest level of capacity. In the three firms scenario instead, the relation is more subtle. The specific distribution of labour costs introduced with assumption (C) allows us to make some predictions on the investment decisions of each firm: the firm in the rigid country is the one investing the highest level of capacity for \(\gamma \in \left(\frac{1}{2}, 1\right]\). If instead we have \(\gamma \in \left[0, \frac{1}{2}\right)\), the firm in the flexible country is going to have the highest level of capacity.

\[
\begin{align*}
  k_1 > k_2 > k_3 & \text{ if } \gamma \in \left(\frac{1}{2}, 1\right] \\
  k_1 = k_2 = k_3 & \text{ if } \gamma = \frac{1}{2} \\
  k_3 > k_2 > k_1 & \text{ if } \gamma \in \left[0, \frac{1}{2}\right]
\end{align*}
\]

The second departure from the two firm model concerns the individual firm’s initial investment. Previously, we had seen how a positive demand shock not likely enough\(^{27}\) makes the firm not invest in capacity at all. In the three firms model instead, the strategic interaction among the three competitors is such that even for \(\gamma = 0\), some capacity is going to be installed\(^{28}\). This result paves

\(^{27}\)That is, for \(\gamma \leq \frac{1}{2}\). \(^{28}\)Recalling that \(k_i = \max\left\{0, \frac{(A - \sum_{j\neq i} \theta_j) - 2\theta_1}{4c}\right\}\).
the way to the analysis of comparative statics, to assess precisely how capacity varies with $\gamma$ and $\theta_i$.

5.3.2.1 Comparative statics The relation between $k_i$ and $\theta_i$ turns out to essentially depend on the probability $\gamma$. As the rigidity of the unskilled labour market becomes more and more binding, the firm is going to install more and more capacity, to avoid the higher second stage costs, if and only if the positive shock is sufficiently likely. If instead the negative shock is more probable, higher rigidity will lead to a lower initial installation of capacity.

$$\frac{\partial k_i}{\partial \theta_i} = \frac{(5\gamma - 3)}{4c} = \begin{cases} \geq 0 & \text{if } \gamma \geq \frac{3}{5} \\ < 0 & \text{if } \gamma < \frac{3}{5} \end{cases}$$

The effect played by $\gamma$ on the initial commitment $k_i$ crucially depends on the sorting of the second stage costs, that is

$$\frac{\partial k_i}{\partial \gamma} = 0 \quad \Rightarrow \quad \theta_i \equiv \hat{\theta}_i = \frac{A - \sum_{j \neq i} \theta_j}{5}$$

Since $c \neq 0$ by assumption, we have that

$$\frac{\partial k_i}{\partial \gamma} = \begin{cases} \geq 0 & \text{if } \theta_i \geq \hat{\theta}_i \\ < 0 & \text{if } \theta_i < \hat{\theta}_i \end{cases}$$

with $\hat{\theta}_1 > \hat{\theta}_2 > \hat{\theta}_3$. This relation implies that only for a high rigidity of the unskilled labour market, the level of capacity chosen by firm $i$ is increasing with probabilistic component $\gamma$. Otherwise, the chosen commitment is decreasing in it.

5.3.3 Performances

Thanks to the value of capacity obtained in the previous step, we can now retrieve the values of the second stage and overall outputs chosen by each firm. In the good state the production levels chosen by each firm correspond to

\[
q_{1,g} = \frac{\tilde{A} - 3\theta_1 + \theta_2 + \theta_3}{4} - \frac{(1 - \gamma)(A - \theta_2 - \theta_3 + \theta_1(5\gamma - 3))}{4c} \\
x_{1,g} = \frac{\tilde{A} - 3\theta_1 + \theta_2 + \theta_3}{4}
\]

\[
q_{2,g} = \frac{\tilde{A} - 3\theta_2 + \theta_1 + \theta_3}{4} - \frac{(1 - \gamma)(A - \theta_1 - \theta_3 + \theta_2(5\gamma - 3))}{4c} \\
x_{2,g} = \frac{\tilde{A} - 3\theta_2 + \theta_1 + \theta_3}{4}
\]

\[
q_{3,g} = \frac{\tilde{A} - 3\theta_3 + \theta_1 + \theta_2}{4} - \frac{(1 - \gamma)(A - \theta_1 - \theta_2 + \theta_3(5\gamma - 3))}{4c} \\
x_{3,g} = \frac{\tilde{A} - 3\theta_3 + \theta_1 + \theta_2}{4}
\]
When comparing the different level of capacity chosen by each firm, we noticed how the ranking of "overcapacity installation" varies with the probability \( \gamma \). To compare also these two additional output measures we have to bear in mind the different range implications of \( \gamma > < \frac{1}{2} \). The firm that invested less in the first stage of the game is the firm that is going to adjust proportionately less in the second stage of the game, that is

\[
\begin{align*}
q_{1,g} < q_{2,g} < q_{3,g} & \text{ if } \gamma > \frac{1}{2} - c \\
q_{1,g} > q_{2,g} > q_{3,g} & \text{ if } \gamma < \frac{1}{2} - c
\end{align*}
\]

This result not only confirms the prediction that the firm holding more capacity is the firm adjusting less once demand has realized, but it also restricts the range of \( c \) to the range \( c \in (0, \frac{1}{2}) \). In the first scenario, the flexible firm is going to generate the highest output in the second stage (good state) given the competitive advantage it has in terms of unskilled labour costs and given the fact that it has installed the lowest level of capacity. In the second scenario instead, the rigid firm is going to generate a higher adjustment output, due to the low investment in capacity and to the negative effect of \( \theta \).

For any level of \( \gamma \), the flexible firm fully exploits the more favorable working conditions and achieve the highest overall level of production, with

\[
x_{3,g} > x_{2,g} > x_{1,g}.
\]

The result so far provided only reflected the good state of the world. As long as the bad state is concerned, each firm achieves these following production targets:

\[
\begin{align*}
q_{1,b} &= \frac{\theta_2 + \theta_3 - 3\theta_1 - \bar{A}}{4} + \left[ \frac{1}{4c} \left( 1 - \gamma \right) \left( A - \theta_2 - \theta_3 \right) + \theta_1 \left( 5\gamma - 3 \right) \right] \\
x_{1,b} &= \frac{3\theta_1 - \theta_2 - \theta_3 + \bar{A}}{4} \\
q_{2,b} &= \frac{\theta_1 + \theta_3 - 3\theta_2 - \bar{A}}{4} + \left[ \frac{1}{4c} \left( 1 - \gamma \right) \left( A - \theta_1 - \theta_3 \right) + \theta_2 \left( 5\gamma - 3 \right) \right] \\
x_{2,b} &= \frac{3\theta_2 - \theta_1 - \theta_3 + \bar{A}}{4} \\
q_{3,b} &= \frac{\theta_1 + \theta_2 - 3\theta_3 - \bar{A}}{4} + \left[ \frac{1}{4c} \left( 1 - \gamma \right) \left( A - \theta_1 - \theta_2 \right) + \theta_3 \left( 5\gamma - 3 \right) \right] \\
x_{3,b} &= \frac{3\theta_3 - \theta_1 - \theta_2 + \bar{A}}{4}
\end{align*}
\]

In the bad state of the world, ex-post adjustment basically implies that some capacity is not going to be used. To understand which firms destroy more, we need to recall the different impact on the overcapacity installation as a function of \( \gamma \). Intuitively, we would expect that the firm holding more capacity is the firm that has to dismantle more ex-post, since the excessive level of capacity
chosen exceeds the real demand needs. This line of reasoning is correct since
\[
\begin{align*}
q_1 > q_2 > q_3 & \quad \text{if } \gamma > \frac{1}{2} + c \\
q_1 < q_2 < q_3 & \quad \text{if } \gamma < \frac{1}{2} + c
\end{align*}
\]
This result allows us to once more restrict the values of installation costs to the range \( c \in (0, \frac{1}{2}) \). Nonetheless, in the bad state, the rigid firm is the firm able to achieve the highest overall output level, for any level of \( \gamma \), that is
\[
x_{1,b} > x_{2,b} > x_{3,b} \quad \forall \gamma.
\]
As a last step, we are left with the analysis of profits. For sake of notation, I’ll simply focus on the generic expression for a generic firm \( i = 1, 2, 3 \). Firm \( i \) gains in the good and bad state respectively
\[
\begin{align*}
\pi_{i,g} &= \left( \bar{A} - 3\theta_i + \sum_{j \neq i} \theta_j \right)^2 + k_i \theta_i (1 - 5\gamma) + (1 - \gamma) (A - \sum_{j \neq i} \theta_j) \\
\pi_{i,b} &= \left( \bar{A} + 3\theta_i - \sum_{j \neq i} \theta_j \right)^2 - k_i \theta_i (5\gamma + 1) + (1 - \gamma) (A - \sum_{j \neq i} \theta_j)
\end{align*}
\]
which, on expectations, lead to
\[
E(\pi_i) = \left( \frac{A^2 + \sigma^2}{16} \right) + \frac{A(1 - 2\gamma)(\theta_i - \sum_{j \neq i} \theta_j)}{9} + \frac{(3\theta_i + \sum_{j \neq i} \theta_j)^2}{16} + k_i \left[ \frac{1}{4} \left( A - \sum_{j \neq i} \theta_j \right) (1 - \gamma) - \theta_i (5 - 3\gamma) \right]. \quad (29)
\]
The pairwise comparison between the expected profits of each firm leads to the following statement concerning the pay-offs from over-investing in capacity. Over investment in capacity pays-off if and only if the following inequality holds:
\[
\frac{\partial E(\pi_i)}{\partial k_i} = 0 \implies \frac{A - \sum_{j \neq i} \theta_j}{\theta_i} > \frac{5 - 3\gamma}{1 - \gamma}
\]
In this way, the firm with the higher level of capacity gets higher expected profits. If the sign of the inequality is instead reversed, the over investment in capacity does not pay off.

6 Conclusions

This quantity competition model is an initial attempt to explain how firms take into account the level of institutional rigidity when installing their production capacity. By embedding these different institutional backgrounds within a two-stage game of quantity competition, I have assumed that firms in the two
countries face different cost structures either in the investment phase, thus reflecting differences in the access to the capital market, or during the production process, when higher or lower adjustment costs reflect more or less strict EPL.

In a free trade setting, in which firms in the two countries face uncertainty over the realization of demand, the firm’s installation in capacity and, consequently, its capital-labour ratio\(^{29}\), depend on the level of institutional rigidity considered and on the probability of a positive shock.

Dealing with institutional rigidity and demand uncertainty at the same time, this model predicts that

- The firm facing higher investment costs\(^{30}\) is going to install a lower level of capacity;
- The firm in the country with a more rigid EPL framework\(^{31}\) is going to install a lower level of capacity.

On top of these outcomes, the model further predicts that firms, in both the scenarios, are willing to invest in capital only for a sufficiently high probability of the positive shock. If instead they expect the final demand to be lower with a sufficiently high probability, they prefer not to install in capacity.

In addition to the understanding of the capital investment decision making of the single firm, this model also explains that, regardless of the type of cost asymmetries considered, investing in capacity does pay-off since the firm that invested more in capital is attaining higher expected profits.

The model here described is a very simple attempt to provide an alternative explanation, with respect to the entry deterrence analysis, for capacity accumulation. Far from providing generalized predictions, this model is a first exercise to reconcile current labour protection issues with the firm’s investment decisions and to shed some light on the policy implications these two topics deserve, especially during a period of economic downturn.

**Annex - Extreme EPL asymmetries**

To introduce cost asymmetries in the second stage I shall assume that only one firm, which I will be referring to as the rigid firm, faces positive adjustment costs, that is \(\theta_1 = \theta > 0\), whereas the flexible firm can freely either increase or decrease its initial commitment without incurring any additional costs, that is \(\theta_2 = 0\). To solve the model, we know have to separately look at the decisions of each firm.

**Second stage**

Whenever the shock is positive, firm 1 maximizes profits

\[
\pi_{1,g} = [\bar{A} - (k_1 + q_1) - (k_2 + q_2)](k_1 + q_1) - \theta q_1 - c k_1^2,
\]

\(^{29}\)Which, in this model, displays a constant and given labour workforce

\(^{30}\)Thus, the firm in the rigid country.

\(^{31}\)Thus, the firm facing higher production costs.
which leads to the reaction function

\[ q_{1,g} = \frac{\bar{A} - 2k_1 - k_2 - q_2 - \theta}{2}. \]

The flexible firm chooses its \( q_{2,g} \) to maximize

\[ \pi_{2,g} = [\bar{A} - (k_1 + q_1) - (k_2 + q_2)](k_2 + q_2) - c k_2^2, \]

from which we obtain its reaction function

\[ q_{2,g} = \frac{\bar{A} - k_1 - 2k_2 - q_1}{2}. \]

Solving for \( q_{1,g} \) and \( q_{2,g} \) we obtain

\[ q_{1,g} = \frac{\bar{A} - 2\theta - 3k_1}{3}, \quad q_{2,g} = \frac{\bar{A} + \theta - 3k_2}{3}. \tag{30} \]

The market clearing price in the good state of the world is

\[ p_g = \frac{\bar{A} + \theta}{3}, \]

increasing in both \( \varepsilon \) and \( \theta \).

With a negative shock, still with \( |\varepsilon| < A - \theta \), the rigid firm solves

\[ \pi_{1,b} = [\bar{A} - (k_1 - q_1) - (k_2 - q_2)](k_1 - q_1) - \theta q_1 - c k_1^2 \]

and the flexible firm solves

\[ \pi_{2,b} = [\bar{A} - (k_1 - q_1) - (k_2 - q_2)](k_2 - q_2) - c k_2^2. \]

The reaction functions are respectively

\[ q_{1,b} = \frac{2k_1 + k_2 - (\bar{A} + \theta)}{2}, \quad q_{2,b} = \frac{k_1 + 2k_2 - q_1 - \bar{A}}{2}. \tag{31} \]

which lead to

\[ q_{1,b} = \frac{3k_1 - (\bar{A} + 2\theta)}{3}, \quad q_{2,b} = \frac{3k_2 + \theta - \bar{A}}{3}. \tag{32} \]

The market clearing price in the bad state is

\[ p_b = \frac{\bar{A} - \theta}{3}, \]

decreasing in both \( \theta \) and \( \varepsilon \).
First stage

In the first stage, the rigid firm chooses the level of capacity that maximizes

\[ E(\pi_1) = -ck^2_1 + \gamma(\theta k_1) - (1 - \gamma)\theta k_1 + K \]

where \( K \) is a set of parameters that do not affect the optimal choice of \( k_1 \). The optimal investment made by the rigid firm is

\[ k_1 = \frac{\theta(2\gamma - 1)}{2c} \]

which is positive as long as \( \gamma > 1/2 \), that is, for a sufficiently likelihood of a positive shock. Recalling the results of the first stage cost asymmetries model, we can relate the choice of \( k_1 \) to the probability \( \gamma \):

\[ k_1 = \begin{cases} \frac{\theta(2\gamma - 1)}{2c}, & \gamma \in (\frac{1}{2}, 1] \\ 0, & \gamma \in [0, \frac{1}{2}] \end{cases} \]

Furthermore, the level of capacity to be installed is decreasing in the installation cost \( c \) and increasing in the second stage cost \( \theta \). The first stage problem of the flexible firm is the choice of \( k_2 \) to maximize

\[ E(\pi_2) = -ck^2_2 + K \]

where, once more, \( K \) is a set of parameters that do not enter the first stage decision. This problem is solved only by

\[ k_2 = 0. \]

Since the flexible firm faces no cost in the second stage, it is more profitable to invest nothing and wait for the realization of demand in the second stage. Whenever \( \gamma > 1/2 \), we have \( k_1 > k_2 = 0 \), because the rigid firm prefers to install some capacity now to save on the second stage costs.

Results

The choice of \( k_1 \) and \( k_2 \) allows us to write down an expression for each firm’s second stage output and overall output in the two states of the world. As far as the good state of the world is concerned, we have

\[ q_{1,g} = \frac{\bar{A} - 2\theta}{3} - \frac{\theta(2\gamma - 1)}{2c} \quad x_{1,g} = \frac{\bar{A} - 2\theta}{3} \]  

(33)

The second stage output is increasing in \( c \) but it is decreasing in \( \theta \) for any level of \( \gamma \), also when there is no capacity to overcome.

\[ q_{2,g} = x_{2,g} = \frac{\bar{A} + \theta}{3}. \]
**Profits when** $\gamma > 1/2$. As long as $\gamma > 1/2$, the overall output of the flexible firm is higher than the final output of the rigid one, that is $x_{2,g} > x_{1,g}$, which implies $q_{2,g} > q_{1,g}$. If instead $\gamma \leq 1/2$, $k_1 = k_2 = 0$ but we end up with $q_{1,g} = x_{1,g} < q_{2,g} = x_{2,g}$.

In the bad state of the world, we have instead

$$q_{1,b} = \frac{\theta(2\gamma - 1)}{2c} - \frac{(\bar{A} + 2\theta)}{3}, \quad x_{1,b} = \frac{\bar{A} + 2\theta}{3}; \quad (34)$$

the second stage output is increasing in $c$ and it is decreasing in $\theta$ for $\gamma > 1/2$.

In the bad state, the flexible firm would end with

$$q_{2,b} = x_{2,b} = \frac{\bar{A} - \theta}{3}.$$

The profits of the rigid firm in the good and bad state are, respectively,

$$\pi_{1,g} = \frac{(\bar{A} - 2\theta)^2}{9} + \frac{\theta^2(2\gamma - 1)(3 - 2\gamma)}{4c},$$

$$\pi_{1,b} = \frac{(\bar{A} + 2\theta)^2}{9} - \frac{\theta^2(2\gamma - 1)(2\gamma + 1)}{4c}.$$

The flexible firm has profits equal to

$$\pi_{2,g} = \frac{(\bar{A} + \theta)^2}{9}$$

and

$$\pi_{2,b} = \frac{(\bar{A} - \theta)^2}{9}.$$

In the good state, the flexible firm is able to produce as much as it wants at no cost. Since its overall output is greater than the overall output of the rigid firm, we can immediately conclude that $\pi_{2,g} > \pi_{1,g}$.

When the shock is instead negative, as long as $\gamma > 1/2$, $\pi_{2,b} = 0 > \pi_{1,b}$. If instead $\gamma \leq 1/2$, $\pi_{1,b} = \pi_{2,b} = 0$. On expectations, the rigid and flexible firm attain respectively

$$E(\pi_1) = \frac{A^2 + \sigma^2 + 4A\theta^2 - 4A\theta(2\gamma - 1)}{9} + \frac{\theta^2(2\gamma - 1)^2}{4c},$$

and

$$E(\pi_2) = \frac{A^2 + \sigma^2 + \theta^2 + 2A\theta(2\gamma - 1)}{9},$$

with $E(\pi_1) > E(\pi_2)$.

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32Taking into account that no production takes place in the flexible country for a negative shock.
Results when $\gamma \leq 1/2$

Whenever the positive shock is not sufficiently probable, none of the firms is going to invest in capacity, thus $k_1 = k_2 = 0$. However, final demand is still positive regardless of the sign of the shock and the two firms can indeed attain a positive production level, as long as we rule out extreme realizations of the shock: $|\varepsilon| < A - \theta$. In the good state, the rigid firm attains

$$q_{1,g_{nk}} = x_{1,g_{nk}} = \frac{A - 2\theta}{3}$$

and the flexible one

$$q_{2,g_{nk}} = x_{2,g_{nk}} = \frac{A + \theta}{3}$$

If instead the shock is negative, they produce a positive amount respectively equal to

$$q_{1,b_{nk}} = x_{1,b_{nk}} = \frac{\bar{A} - \theta}{3}$$

and the flexible one

$$q_{2,b_{nk}} = x_{2,b_{nk}} = \frac{\bar{A} + \theta}{3}$$

When no capacity is being installed, the profits of the rigid firm are

$$\pi_{1,g_{nk}} = \frac{(\bar{A} - 2\theta)^2}{9} \quad \text{and} \quad \pi_{1,b_{nk}} = \frac{(\bar{A} - \theta)(\bar{A} - 4\theta)}{9}$$

and, in expectations,

$$E(\pi_{1,nk}) = \frac{A^2 + \sigma^2 + 4\theta^2 - A\theta(3\gamma - 5)}{9}.$$  

When the rigid firm installs nothing, the performance of the flexible firm is affected, too:

$$\pi_{2,g_{nk}} = \frac{(\bar{A} + \theta)^2}{9} \quad \text{and} \quad \pi_{2,b_{nk}} = \frac{(\bar{A}^2 - \theta^2)}{9}$$

and, in expectations,

$$E(\pi_{2,nk}) = \frac{A^2 + \sigma^2 + \theta^2(2\gamma - 1) + 2A\theta\gamma}{9}.$$  

In the good state, the flexible firm gains more: $\pi_{2,g_{nk}} > \pi_{1,g_{nk}}$. However, the result in the bad state is somehow surprising: despite the positive stage costs, the rigid firm attains higher profits as long as $|\varepsilon| < A - \theta$: positive costs in the second stage make the firm save and produce a lower output, which in turns allow it to attain higher profits in the negative state.
References


