Price Competition with Consumer Confusion*

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Abstract

This paper proposes a model in which identical sellers of a homogenous product compete in both prices and price frames (i.e., ways to present price information). Frame choices affect the comparability of price offers, and may lead to consumer confusion. In the symmetric equilibrium the firms randomize over both price frames and prices, and make positive profits. This result is consistent with the observed coexistence of price and price frame dispersion in some markets. We show that the nature of equilibrium depends on whether frame differentiation or frame complexity is more confusing, and an increase in the number of firms can raise industry profits and harm consumers.

Keywords: bounded rationality, framing, frame dispersion, incomplete preferences, price competition, price dispersion

JEL classification: D03, D43, L13

1 Introduction

Sellers use various ways to convey price information to consumers. Price promotions are often framed differently using, for instance, a direct price reduction, a percentage discount, a multi-unit discount, or a voucher.¹ Some restaurants and online booksellers offer a single price, while others divide the price by quoting the table service, the shipping fee, or the VAT separately. A search for a popular textbook at a book price comparison website led to the following results:

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¹For example, to buy a 50 ml whitening toothpaste in a grocery store one can choose between Macleans which is sold at £2.31 with a “buy one get one free” offer and an Aquafresh which “was £1.93 now is £1.28 saves 65p”.

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<tr>
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<td>Quartermelon</td>
<td>£32.97</td>
<td>£4.32</td>
</tr>
<tr>
<td>Blackwell Online</td>
<td>£38.95</td>
<td>Free</td>
</tr>
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Prices of Jean Tirole’s “The Theory of Industrial Organization”
www.abebooks.co.uk (November 15, 2009)

Similarly, airlines and travel agencies present differently the fees they charge for card payment. For example, Wizz charges a £4 flat fee per person, while Virgin Atlantic charges 1.3% of the total booking.\(^2\) Retailers offer store cards with diverse terms such as “10% off first shop if opened online or 10% for first week if opened in store”, “500 bonus points on first order”, or “£5 voucher after first purchase”. Financial product prices are also often framed distinctively: mortgages might have the arrangement fees rolled in the interest rate or not; some loans may specify the “repayment amount”, while others the “APR”.

Despite the prevalence of price framing, the practice has received little attention in the economic literature. There is no explanation why different firms employ different price frames or why the same firm changes its price frame over time (as in the case of supermarket discounts). If firms use different price presentation modes to compete better for consumers, industry-specific pricing schemes whose terms allow for better comparisons should emerge. In contrast, the persistence of much variation in the price frames seems more likely to confuse consumers and harm competition.\(^3\) As the above examples suggest, markets with price frame dispersion may also exhibit price dispersion.

To address this issue, we develop a model in which firms compete to supply a homogeneous product by simultaneously choosing both price frames and prices. We assume that price framing can confuse consumers and as a result they fail to compare some prices in the market.\(^4\) We consider two possible categories of frames—one simple and the other one relatively complex. Consumers might be confused either by frame differentiation (i.e., they may fail to compare prices in different frames) or by frame complexity (i.e., they may fail to compare prices in a common complex frame).\(^5\)

\(^2\)See “Calls for airline charges clean-up” on BBC News on July 17, 2009 at http://news.bbc.co.uk.

\(^3\)Gabaix and Laibson (2006) and Brown et al. (2009) amongst others studied a related price framing technique—partitioning and concealing, in which part of the relevant price information is hidden and so may be ignored by consumers. In contrast, in our model consumer confusion is not driven by the existence of shrouded information, but rather by market complexity (i.e., the relevant information is available, but involved).

\(^4\)Research in psychology and behavioral economics has long recognized the significance of framing effects in decision making (see Tversky and Kahneman, 1981, for instance). Often, people’s responses to essentially the same decision problem are systematically different when the problem is framed in different ways. In this paper, we focus on frames as price presentation modes and on their ability to cause confusion by limiting price comparability.

\(^5\)The marketing literature acknowledges the fact that consumers have difficulties in comparing prices in different frames (prices which are presented differently) or prices in complex frames (prices which are complicated). See, for instance, Estelami (1997), Morwitz et al. (1998), and Thomas and Morwitz (2009).
Our model studies both sources of confusion in a unified framework.

Sometimes, frame differentiation is a main source of confusion. For example, comparing two prices both including the VAT or both excluding the VAT (given that the same percentage tax applies) is easy, but comparing a price excluding VAT with an all-inclusive one might be more difficult. Similarly, variation in grocery price promotions might make it harder for the consumers to compare the actual prices, although each discount method is not particularly involved. Other times, frame complexity is also a source of consumer confusion, and it may even be the dominant one. Comparing offers which quote separately the shipping fees might be confusing if the applicable fees differ across sellers. The same is true in the mortgage markets where deals with the service fee quoted separately are usually harder to compare than deals with the service fees rolled in the interest rate. Much evidence suggests that consumers do not understand well the prices when the sellers use complex price frames (which involve many elements or pieces of information) in markets such as financial services or electricity and gas.\(^6\)

We address the following questions: Can frame dispersion and price dispersion coexist in an oligopoly market? Do market outcomes depend on whether frame differentiation or frame complexity is more confusing? In the presence of price framing, does an increase in the number of competitors reduce market complexity and enhance consumer welfare?

We first study a duopoly example in Section 2, to illustrate the coexistence of frame and price dispersion in a simple way. Let us discuss here a version in which only frame differentiation causes confusion (i.e., if the firms use the same frame all consumers can accurately compare prices, and otherwise all consumers get confused and shop randomly). If both firms used the same frame in equilibrium, then they could only price at marginal cost. But, a unilateral deviation to the other frame will confuse all consumers and lead to a positive profit. If the firms used different frames, they could only choose the monopoly price. But, a unilateral deviation to use the rival’s frame and a slightly lower price will then attract all consumers. So, in any equilibrium the firms must mix on frames. As the firms face both price aware buyers (who compare prices perfectly) and confused buyers (who shop at random) with positive probability, they must also mix on prices in equilibrium.

In general, the nature of the equilibrium depends on which source of confusion dominates. If frame complexity is more confusing than frame differentiation, then the more complex frame is always associated with higher prices. In contrast, if frame differentiation is more confusing than complexity, there is no clear monotonic relationship between the prices associated with different

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\(^6\)An EU study of mortgage markets states that “even if consumers do have the relevant information [to make a decision] they do not necessarily understand it”. (See the “White paper on the integration of EU mortgage credit markets”, 2007.) Research on the gas and electricity market in UK by a consumer organization called Which? says that “complex tariff structures made it very difficult for consumers to understand what type of deal they were on and how to reduce energy use and costs”. (See “Customers confused by energy tariffs” at http://www.which.co.uk/news on May 7, 2009.)
frames. (With two firms, the pricing strategy is actually independent of the framing strategy at equilibrium, though this result does not hold in general.)

Section 3 presents the oligopoly model and investigates the impact of an increase in the number of firms on market outcomes. To do so, we first generalize the price and frame dispersion result. We find that when competition becomes fiercer the ability of frame differentiation to reduce price competition decreases, and firms rely more on frame complexity. In particular, in fragmented oligopolies, firms use the more complex frame almost surely. A consequence of this is that industry profits are bounded away from zero regardless of the number of competitors. In addition, an increase in the number of firms might boost industry profits and harm the consumers under certain market conditions. Therefore, when firms compete in both prices and frames, a naive competition policy which simply increases the number of firms might have undesired effects.

Section 4 extends our model by considering more than two frames, but focuses on the tractable case in which all frames are symmetric and only frame differentiation causes consumer confusion. We show that the availability of more frames softens price competition and improves profits. Section 5 discusses the robustness of our results to alternative modelling approaches and the empirical relevance of our findings. It also explores a possible interpretation of our model in which confusion stems from product framing such as labeling, packaging or presentation rather than price framing.

An emerging economic literature documents and investigates price complexity and firms’ intentional attempts to degrade the quality of information to the consumers. Ellison and Ellison (2009) provide empirical evidence on retailers’ use of obfuscation strategies in online markets. They show, for instance, that retailers deliberately create more confusing websites to make it harder for the consumers to figure out the total price. On the theoretical side, one stream of literature adopts the standard information search framework (Carlin, 2009 and Ellison and Wolitzky, 2008) and builds on the fact that it is more costly for consumers to assess complex prices. An increase in price complexity reduces consumers’ incentive to gather information and so weakens price competition.\footnote{The underlying channels in these two papers are, however, very different. Ellison and Wolitzky (2008) introduce a convex search cost function in a sequential search model à la Stahl (1989). If a firm increases its in-store search cost (say, by making its price more complex), it makes further search more costly and, therefore, less likely. Carlin (2009) takes a more reduced-form approach. He assumes that if any individual firm makes its price more complex, then consumers will regard the market environment as being more complex and the information gathering process as more costly, and will be more likely to remain uninformed and shop randomly. See Wilson (2008) for an alternative two-stage model in which firms differentiate their price complexities in the first stage (e.g., one firm obfuscates and the other does not) in order to soften price competition in the second stage.}

Another stream of literature regards price complexity as a device to exploit boundedly rational consumers. In Spiegler (2006), consumers who face complex (multi-dimensional) prices (e.g., insurance schemes) sample just one random dimension and buy from the firm with the lowest sampled fee. As a result the firms have incentives to introduce variation across different price dimensions.
Our model also considers price complexity. However, unlike the aforementioned studies, it provides a unified framework which combines the effects of price frame differentiation and price frame complexity. In our setting, frame differentiation is also a source of “market complexity”, albeit different from frame complexity. In effect, this study disentangles the relative effects of frame differentiation and complexity on the market outcomes. The inability of boundedly rational consumers to compare framed prices leads to equilibrium frame dispersion in our model. As such, our work also contributes to a growing literature on bounded rationality in industrial organization (see Ellison, 2006 for a review).

A feature of our model is that some consumers have to choose from a “partially ordered set” since some offers in the market are incomparable due to price framing. To deal with this consumer choice issue, we draw on the literature on incomplete preferences (see, for example, Aumann, 1962 and Eliaz and Ok, 2006) and use a dominance-based choice rule. Whenever there is confusion, consumers first rule out offers which are dominated by other comparable offers in the market, and then buy from undominated ones according to a stochastic rule. In this sense, our setting incorporates consumer incomplete preferences in an oligopoly pricing model.

In a closely related article, Piccione and Spiegler (2009) also examine frame-price competition. They focus on a duopoly model, but allow for a general frame structure. The relationship between equilibrium properties and the frame structure is central to their analysis. For instance, under some frame-structure regularity condition, they prove existence of a symmetric equilibrium in which pricing is frame independent. Our duopoly example in Section 2 can be regarded as a special case of their model. However, there are two main differences between our work and theirs. First, we study frame-price competition in an oligopoly setting, which allows us to examine the impact of greater competition on firms’ framing strategies and market outcomes. Second, Piccione and Spiegler (2009) take a default-bias interpretation of the consumer choice rule. That is, consumers are initially randomly assigned to the firms, and they switch suppliers only if they find a comparable and better deal. Although our dominance-based choice rule and their default-bias choice rule are consistent in the duopoly case, they diverge when there are more than two firms. We discuss this difference further in Subsection 5.1.

Finally, our study is related to a vast literature on price dispersion (see Baye et al., 2006 for a survey). However, in our model, firms randomize over two dimensions, and price dispersion is rather a by-product of price frame dispersion. Carlin (2009) characterizes a two-dimensional equilibrium similar to ours when frame complexity dominates, though his modelling approach is different (see

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8 If the two firms use the same frame, consumers are able to compare their offers perfectly; and if they use different frames, then with a frame profile dependent probability consumers are unable to compare their offers.

9 Gaudeul and Sugden (2009) consider a similar issue of price and “standard” choices. In particular, they emphasize that, if consumers have strong preferences for firms that are using the same standard and refuse to consider all other firms which are using “individuated” standards, then a common-standard competitive equilibrium may emerge.
Section 5.1 for more details). In a model with quality choice and inattentive consumers (who care for the price, but ignore the quality), Armstrong and Chen (2009) also derive a similar equilibrium to ours. In their setting, firms randomize over offering a high and a low quality product, and the high quality product is always associated with higher prices.

2 A Duopoly Example

This section introduces the model and presents some of the main insights in a two-firm example. Consider a market for a homogeneous product with two identical sellers, firms 1 and 2. Suppose that there are two possible price presentation modes, referred to as frames A and B. The constant marginal cost of production is normalized to zero. There is a unit mass of consumers, each demanding at most one unit of the product and willing to pay at most $v = 1$. Both firms and consumers are risk neutral. The firms simultaneously and noncooperatively choose price frames and prices $p_1$ and $p_2$. Each firm can choose just one of the two frames. The timing reflects the fact that both frames and prices can be changed relatively easily.

Price framing is assumed to affect consumer choice in the following way: (i) If both firms choose frame $A$, all consumers can perfectly compare the two prices and buy the cheaper product as long as it offers positive net surplus. Formally, in this case firm $i$’s demand is

$$q_i(p_i,p_j) = \begin{cases} 1, & \text{if } p_i < p_j \text{ and } p_i \leq 1 \\ 1/2, & \text{if } p_i = p_j \leq 1 \\ 0, & \text{if } p_i > p_j \text{ or } p_i > 1 \end{cases} \quad \text{for } i,j \in \{1,2\} \text{ and } i \neq j. \quad (1)$$

(ii) If the two firms adopt different frames, a fraction $\alpha_1 > 0$ of consumers get confused and are unable to compare the two prices. In this case, we assume that they shop at random (whenever $p_i \leq v, \forall i$). The remaining $1 - \alpha_1$ fraction of consumers are still able to accurately compare prices. (iii) If both firms choose frame $B$, a fraction $\alpha_2 \geq 0$ of the consumers get confused and shop at random (whenever $p_i \leq v, \forall i$). The following table presents the fraction of confused consumers for all possible frame profiles, where $z_i$ is the frame chosen by firm $i$ and $z_j$ is the frame chosen by firm $j$.

<table>
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<tr>
<th>$z_i \setminus z_j$</th>
<th>A</th>
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<tr>
<td>A</td>
<td>$\alpha_0 = 0$</td>
<td>$\alpha_1$</td>
</tr>
<tr>
<td>B</td>
<td>$\alpha_1$</td>
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$^{10}$We also assume that even confused consumers are still able to judge if it is worth to buy the product. Alternatively, we could assume that the consumers have a budget constraint at one. Hence, in equilibrium no firm charges a price above $v = 1$. 

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Then, firm $i$’s profit is

$$
\pi_i(p_i, p_j, z_i, z_j) = p_i \cdot \left[ \frac{1}{2} \alpha_{z_i, z_j} + q_i(p_i, p_j)(1 - \alpha_{z_i, z_j}) \right],
$$

where $\alpha_{z_i, z_j}$ is presented in Table 1 and $q_i(p_i, p_j)$ is given by (1).

Without loss of generality, we assume that frame $A$ is the relatively simpler frame. The supposition that nobody gets confused when both firms use frame $A$ is only for expositional simplicity. Our main results hold qualitatively for a positive $\alpha_0$ if $\alpha_0 \leq \alpha_2$ and $\alpha_0 \neq \alpha_1$.

When consumers get confused, we assume that their choices are entirely independent of the two firms’ prices. This is a tractable way to model the idea that confusion in price comparison induces consumers to make unsystematic errors and so reduces their price sensitivity. For simplicity, in this section we also assume that confused consumers shop (uniformly) randomly. However, the oligopoly model in Section 3 allows for a more general stochastic purchase rule.

Note that there are two sources of confusion in our model: one is frame differentiation (measured by $\alpha_1$) and the other is the complexity of frame $B$ (measured by $\alpha_2$). When $\alpha_1 > \alpha_2$, frame differentiation is more confusing than frame complexity. In particular, if $\alpha_0 = \alpha_2 = 0$, the two frames are symmetric. In this case, consumers are confused only by frame differentiation (for instance, frame $A$ is “Price incl. VAT” and frame $B$ is “Price excl. VAT”). Conversely, when $\alpha_1 < \alpha_2$, frame complexity dominates frame differentiation in confusing consumers. For example, frame $A$ is an all-inclusive price and frame $B$ is a more complex price such as price plus shipping fee.

Let us characterize now the equilibrium in the duopoly case.\footnote{Our duopoly example can be regarded as a reduced-form model of the bi-symmetric graph case in Piccione and Spiegler (2009). All their results apply to our model, except that in our setting it is subtler to exclude the possibility of firms adopting deterministic frames. In their model, consumers are always able to perfectly compare prices in the same frame (i.e., frame differentiation is the only confusion source), so it is easy to see that firms will never adopt deterministic frames.} We first show that there is no pure strategy framing equilibrium. All proofs missing from this section are relegated to Appendix A.

**Lemma 1** If $\alpha_1 \neq \alpha_2$, there is no equilibrium in which both firms choose deterministic price frames.

**Proof.** (a) Suppose both firms choose frame $A$ for sure. Then, the unique candidate equilibrium entails marginal-cost pricing and zero profit. But, if firm $i$ unilaterally deviates to frame $B$ and a positive price (no greater than one), it makes a positive profit. A contradiction.

(b) Suppose both firms choose frame $B$ for sure. For clarity, consider two cases. (b1) If $\alpha_2 = 1$ (and so $\alpha_1 < \alpha_2$), at the unique candidate equilibrium $p_i = 1$ and $\pi_i = 1/2$ for all $i$. But, if firm $i$ unilaterally deviates to frame $A$ and price $p_i = 1 - \varepsilon$, it earns $(1 - \varepsilon) \left[ \alpha_1/2 + (1 - \alpha_1) \right] > 1/2$
for $\varepsilon$ small enough. (b2) If $\alpha_2 < 1$, the unique candidate equilibrium dictates mixed strategy pricing according to a cdf on $[p_0, 1]$ as in Varian (1980), and each firm’s expected profit is $\alpha_2/2 = p_0 (1 - \alpha_2/2)$.

If $\alpha_1 > \alpha_2$, firm $i$ can make a higher profit $\alpha_1/2 > \alpha_2/2$ by deviating to frame $A$ and price $p_i = 1$. If $\alpha_1 < \alpha_2$, firm $i$ can make a higher profit $p_0 (1 - \alpha_1/2) > p_0 (1 - \alpha_2/2)$ by deviating to frame $A$ and price $p_i = p_0$. Both (b1) and (b2) lead to a contradiction.

(c) Suppose firm $i$ chooses frame $A$ and firm $j$ chooses $B$. Again consider two cases. (c1) If $\alpha_1 = 1$, the unique candidate equilibrium entails $p_i = 1$ and $\pi_i = 1/2$ for all $i$. But, then, firm $j$ is better off deviating to frame $A$ and $p_j = 1 - \varepsilon$, in which case its profit is $1 - \varepsilon > 1/2$ for any $\varepsilon < 1/2$. (c2) If $\alpha_1 < 1$, then the unique candidate equilibrium is again of Varian type and dictates mixed strategy pricing according to a cdf on $[p_0, 1]$, with each firm earning $\alpha_1/2 = p_0 (1 - \alpha_1/2)$. But if firm $j$ deviates to frame $A$ and price $p_j = p_0$, it makes a higher profit $p_0$. Both (c1) and (c2) lead to a contradiction. This completes the proof.

If both firms use the same simple frame (that is, $A$ or, when $\alpha_2 = 0$, could also be $B$), they compete à la Bertrand and make zero profits. A unilateral deviation to a different frame supports positive profits as some consumers are confused by “frame differentiation” and shop at random. For $\alpha_2 > 0$, Lemma 1 also shows that at equilibrium, the firms cannot rely on only one source of confusion. Otherwise, a firm which uses frame $B$ would have a unilateral incentive to deviate to the simpler frame $A$ to attract some price aware consumers.

However, if $\alpha_1 = \alpha_2 > 0$, there is an equilibrium in which both firms use frame $B$ (see further details in the end of this section). The rest of this section focuses on the general case with $\alpha_1 \neq \alpha_2$.

By Lemma 1, in any candidate equilibrium at least one firm will randomize its frame choice. Therefore, there is a positive probability that firms have bases of fully aware consumers, and a positive probability that they have bases of confused consumers who cannot compare prices at all. The conflict between the incentives to fully exploit confused consumers and to vigorously compete for the aware consumers leads to the inexistence of pure strategy pricing equilibria. The proof of the following result is standard and therefore omitted.

**Lemma 2** There is no equilibrium in which both firms charge deterministic prices.

Lemmas 1 and 2 show that in the duopoly model there are only equilibria which exhibit dispersion in both price frames and prices.

In continuation, we focus on the *symmetric mixed-strategy equilibrium* $(\lambda, F_A, F_B)$ in which each firm assigns probability $\lambda \in (0, 1)$ to frame $A$ and $1 - \lambda$ to frame $B$ and, when a firm uses frame

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12 See Baye et al. (1992) for the uniqueness proof in the two-firm case.

13 Although parts (a) and (c) used the fact that consumers can compare prices perfectly when both firms use frame $A$, our result still holds even if $\alpha_0 > 0$ provided that $\alpha_0 \neq \alpha_1$ (the logic in (b) applies).
$z \in \{A, B\}$, it chooses its price randomly according to a cdf $F_z$ which is strictly increasing on its connected support $S_z = [p^z_0, p^z_1]$.\(^{14}\) We first show that $F_z$ is continuous (except when $\alpha_2 = 1$).

**Lemma 3** In the symmetric mixed-strategy equilibrium $(\lambda, F_A, F_B)$, the price distribution associated with frame $A$ ($F_A$) is always atomless and that associated with frame $B$ ($F_B$) is atomless whenever $\alpha_2 < 1$.

Denote by

$$x_z(p) \equiv 1 - F_z(p)$$

the probability that a firm using frame $z$ charges a price higher than $p$. Suppose firm $j$ is employing the equilibrium strategy. Then, if firm $i$ uses frame $A$ and charges $p \in [p^A_0, p^A_1]$, its expected profit is

$$\pi(A, p) = p\{\lambda x_A(p) + (1 - \lambda) [\alpha_1/2 + (1 - \alpha_1) x_B(p)]\}. \quad (2)$$

With probability $\lambda$, the rival is also using $A$ such that the firms compete à la Bertrand. With probability $1 - \lambda$, the rival is using $B$, such that a fraction $\alpha_1$ of the consumers are confused (by frame differentiation) and shop at random, and the firms compete à la Bertrand for the remaining $1 - \alpha_1$ fully aware consumers.

If instead firm $i$ uses $B$ and charges $p \in [p^B_0, p^B_1]$, its expected profit is

$$\pi(B, p) = p\{\lambda [\alpha_1/2 + (1 - \alpha_1) x_A(p)] + (1 - \lambda) [\alpha_2/2 + (1 - \alpha_2) x_B(p)]\}. \quad (3)$$

With probability $\lambda$, the rival is using $A$ such that a fraction $\alpha_1$ of the consumers are confused (by frame differentiation) and shop at random, and the firms compete à la Bertrand for the remaining $1 - \alpha_1$ fully aware consumers. With probability $1 - \lambda$, the rival is using $B$ such that a fraction $\alpha_2$ of the consumers are confused (by frame complexity) and shop at random, and the firms compete à la Bertrand for the remaining $1 - \alpha_2$ fully aware consumers.\(^{15}\)

The nature of the equilibrium depends on which source of confusion dominates. When $\alpha_1 < \alpha_2$, if a firm switches from frame $A$ to $B$, then more consumers get confused regardless of its rival’s frame. In this case, each firm charges higher prices when it uses frame $B$ than when it uses frame $A$. For $\alpha_1 > \alpha_2$, when a firm switches from frame $A$ to $B$, more consumers get confused if its rival is using $A$, while fewer consumers get confused if its rival is using $B$. In this case, there is no obvious monotonic relationship between the prices associated with $A$ and $B$. The remainder of this section analyses these two cases separately.

- Frame differentiation dominates frame complexity: $0 \leq \alpha_2 < \alpha_1$

\(^{14}\)A symmetric mixed-strategy equilibrium can also be expressed as $(F(p), \lambda(p))$ in which $F(p)$ is the price distribution and $\lambda(p)$ is the probability of adopting frame $A$ conditional on price $p$. These two expressions are equivalent to each other.

\(^{15}\)Note that the profit functions apply for any price $p$ as $F_z(p) = 0$ for $p < p^z_0$ and $F_z(p) = 1$ for $p > p^z_1$. 9
The unique symmetric equilibrium in this case dictates \( F_A(p) = F_B(p) \) and \( S_A = S_B = [p_0, 1] \). That is, a firm’s price is independent of its frame. The proof of this result is relegated to Appendix A.2. To characterize the equilibrium, we use the profit functions (2) and (3). Let \( F(p) \) be the common price distribution and \( x(p) \equiv 1 - F(p) \). From the indifference condition \( \pi(A, p) = \pi(B, p) \), we obtain \( \lambda \alpha_1 = (1 - \lambda) (\alpha_1 - \alpha_2) \), or

\[
\lambda = 1 - \frac{\alpha_1}{2 \alpha_1 - \alpha_2}.
\]  

(4)

If the two frames are symmetric (\( \alpha_2 = 0 \)), then firms are equally likely to adopt each frame (i.e., \( \lambda = 1/2 \)). If frame \( B \) is more complex (\( \alpha_2 > 0 \)), then firms are more likely to adopt frame \( B \) (i.e., \( \lambda < 1/2 \)).

Let \( \pi \) be a firm’s equilibrium profit. Since all prices on \([p_0, 1]\) should lead to the same profit, we have

\[
\pi = \pi(A, 1) = (1 - \lambda) \alpha_1/2 = \frac{\alpha_1^2}{2(2 \alpha_1 - \alpha_2)}.
\]  

(5)

Then \( F(p) \) solves \( \pi(A, p) = \pi \), or

\[
\lambda x(p) + (1 - \lambda) [\alpha_1/2 + (1 - \alpha_1) x(p)] = \frac{\pi}{p}.
\]  

(6)

Finally, \( p_0 \) solves \( x(p_0) = 1 \), so \( p_0 = \pi/(1 - \pi) \in (0, 1) \).

**Proposition 1** When \( n = 2 \) and \( 0 \leq \alpha_2 < \alpha_1 \), there is a unique symmetric mixed-strategy equilibrium in which each firm adopts frame \( A \) with probability \( \lambda \) given by (4) and frame \( B \) with probability \( 1 - \lambda \). Regardless of its frame choice, each firm chooses its price randomly according to a cdf \( F \) which is defined by (6) on \([p_0, 1]\). Each firm’s equilibrium profit \( \pi \) is given by (5).

The economic intuition of the price-frame independence result lies in the equilibrium condition for \( \lambda \). Rewritten as \( (1 - \lambda) \alpha_1 = \lambda \alpha_1 + (1 - \lambda) \alpha_2 \), (4) actually requires the expected number of confused consumers to be the same when a firm uses frame \( A \) (the left-hand side) and when it uses frame \( B \) (the right-hand side). Given that in duopoly there are only two types of consumers (the confused and the fully aware), it also implies that the expected number of fully-aware consumers is the same. Therefore, the expected market composition along the equilibrium path does not depend on a firm’s frame choice. Then, the pricing strategy is also independent of the frame choice. This is because the pricing balances the incentives to extract all surplus from the confused and to compete for the fully aware and so is determined by the market composition.

Let us analyze the impact of \( \alpha_1 \) and \( \alpha_2 \) on the equilibrium outcome. (i) When the confusion caused by frame complexity becomes more important, firms use the complex frame \( B \) more often (i.e., \( 1 - \lambda \) increases with \( \alpha_2 \)). (ii) When the confusion caused by frame differentiation becomes more important, firms use the simple frame \( A \) more often in order to increase the probability of frame
differentiation (which is $2\lambda(1-\lambda)$). (iii) Equilibrium profit $\pi$ increases with both $\alpha_1$ and $\alpha_2$. That is, confusion (regardless of its source) always boosts firms’ payoffs and harms consumers. In effect, one can check that the price distributions for higher $\alpha_1$ ($\alpha_2$) first-order stochastically dominate those for lower $\alpha_1$ ($\alpha_2$).

Finally, notice that the equilibrium price dispersion is driven by firms’ obfuscation effort through random framing but not necessarily by the coexistence of price aware and confused consumers. This is best seen in the polar case with $\alpha_1 = 1$ and $\alpha_2 = 0$, where consumers are always homogeneous both ex-ante and ex-post (i.e., once a pair of frames is realized, either all consumers are confused or all of them are fully aware), but price dispersion still persists.

- Frame complexity dominates frame differentiation: $0 < \alpha_1 < \alpha_2$

In this case, Appendix A.3 shows that the unique symmetric equilibrium dictates adjacent supports $S_A = [\hat{p}_0^A, \hat{p}]$ and $S_B = [\hat{p}, 1]$. (In particular, if $\alpha_2 = 1$, then $S_A = [\hat{p}_0^A, 1]$ and $S_B = \{1\}$). That is, frame $B$ is always associated with higher prices than frame $A$. We can characterize the equilibrium by substituting $x_B (p) = 1$ in $\pi (A, p)$ (see (2)) and $x_A (p) = 0$ in $\pi (B, p)$ (see (3)).

First, from the indifference condition $\pi (A, \hat{p}) = \pi (B, \hat{p})$, we can derive

$$\lambda = 1 - \frac{\alpha_1}{\alpha_2}. \quad (7)$$

Second, $F_A (p)$ solves $\pi (A, p) = \pi$ where $\pi$ is a firm’s equilibrium profit, or

$$\lambda x_A (p) + (1-\lambda)(1-\alpha_1/2) = \frac{\pi}{p}. \quad (8)$$

and $F_B (p)$ solves $\pi (B, p) = \pi$, or

$$\lambda\alpha_1/2 + (1-\lambda)[\alpha_2/2 + (1-\alpha_2)x_B (p)] = \frac{\pi}{p}. \quad (9)$$

Equilibrium profit $\pi$ can be calculated as

$$\pi = \pi (B, 1) = \frac{1}{2}[\lambda\alpha_1 + (1-\lambda)\alpha_2] = \alpha_1(1 - \frac{\alpha_1}{2\alpha_2}). \quad (10)$$

Finally, the boundary points $p_0^A$ and $\hat{p}$ are determined by $x_A (p_0^A) = 1$ and $x_A (\hat{p}) = 0$, respectively.

**Proposition 2** (i) When $n = 2$ and $\alpha_1 < \alpha_2 < 1$, there is a unique symmetric mixed-strategy equilibrium in which each firm adopts frame $A$ with probability $\lambda$ given by (7) and frame $B$ with probability $1-\lambda$. When a firm uses frame $z \in \{A, B\}$, it chooses its price randomly according to a cdf $F_z$: $F_A (p)$ is defined on $S_A = [p_0^A, \hat{p}]$ and solves (8), and $F_B (p)$ is defined on $S_B = [\hat{p}, 1]$ and solves (9). Each firm’s equilibrium profit is given by (10).

(ii) When $\alpha_1 < \alpha_2 = 1$, the equilibrium has the same form except that $F_B$ is a degenerate distribution on $S_B = \{1\}$, and $F_A$ is defined on $S_A = [p_0^A, 1]$. 

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Recall that when a firm shifts from frame $A$ to frame $B$, regardless of rival’s frame, more consumers get confused. Then, a firm has incentives to charge higher prices when using frame $B$.

As in the previous case, equilibrium profit increases (and so consumer surplus decreases) with both $\alpha_1$ and $\alpha_2$. However, unlike the previous case, when $\alpha_2 > \alpha_1$ the probability of using the complex frame $(1 - \lambda)$ decreases with $\alpha_2$. This happens because, when confusion from frame complexity dominates, the prices associated with frame $B$ rise (so a rival who uses frame $B$ is a softer competitor). This makes more attractive the use of frame $A$ together with a relatively high price (though still lower than $\hat{p}$).

Hence, for fixed $\alpha_1$, the overall relationship between $1 - \lambda$ and $\alpha_2$ is non-monotonic: for $\alpha_2 < \alpha_1$, the probability of using frame $B$ goes up with $\alpha_2$; while for $\alpha_2 > \alpha_1$, the opposite is true.

When $\alpha_2 \to \alpha_1$, it follows from both Propositions 1 and 2, that the firms use frame $B$ almost surely (i.e., $\lambda \to 0$), and the price distributions associated with frame $B$ in the two cases tend to coincide. Therefore, when $\alpha_1 = \alpha_2 > 0$, there is a unique symmetric equilibrium in which both firms use frame $B$.

3 The Oligopoly Model

In this section we analyze an oligopoly model in which firms simultaneously choose frames and prices. Our main objective is to investigate the impact of an increase in the number of firms on market outcomes.

Consider a homogeneous product market with $n \geq 2$ identical sellers and, as before, two categories of frames, $A$ and $B$. Let $A$ be a simple frame such that all prices in this frame are comparable. Frame $B$ may involve some complexity so that with probability $\alpha_2 \geq 0$ the consumers are unable to compare prices in this frame. Consumers can also be confused by frame differentiation and therefore unable to compare prices in different frames with probability $\alpha_1 > 0$. We assume that confusion from frame differentiation and confusion from frame complexity are independent. Consequently, there are up to four types of consumers: $(1 - \alpha_1)(1 - \alpha_2)$ fully aware consumers, $(1 - \alpha_1)\alpha_2$ consumers confused only by frame complexity, $\alpha_1(1 - \alpha_2)$ consumers confused only by frame differentiation, and $\alpha_1\alpha_2$ consumers confused by both confusion sources.\textsuperscript{16}

In the duopoly case, for any realized frame profile, there is at most one confusion source, and so there are at most two types of consumers: fully aware (who always buy the cheaper product) and totally confused (who shop randomly). However, when $n \geq 3$, for a realized frame profile (e.g., $(A, B, B)$), both confusion sources might coexist so that the more general taxonomy above

\textsuperscript{16}Note that in our model consumer confusion occurs at frame level. For example, across all pairs of an $A$ and a $B$ offer, a consumer is either able to compare all or none. This is similar to Varian (1980) where a consumer is either informed of all prices or none (and picks one firm randomly).
applies. Even if there is only one confusion source, a consumer may be only partially confused as the following example shows.

**Example 1** Let \( n = 3 \). Suppose firm 1 uses frame A and charges price \( p_1 \), and firms 2 and 3 use frame B and charge prices \( p_2 \) and \( p_3 \), respectively. If \( \alpha_1 = 1 \) and \( \alpha_2 = 0 \) (i.e., frame B is also simple), then only frame differentiation causes confusion. All consumers can accurately compare \( p_2 \) with \( p_3 \), but cannot compare \( p_1 \) with either \( p_2 \) or \( p_3 \). So consumers are neither fully aware nor totally confused.

The above discussion raises the issue of how consumers choose from a “partially ordered” set in which some pairs of alternatives are comparable, but others are not. This is also the key difference between the duopoly model and the more general oligopoly model. Building on the literature of incomplete preferences (Aumann, 1962 and Eliaz and Ok, 2006, for instance), we adopt a dominance-based consumer choice rule. (See Section 5.1 for a discussion of alternative choice rules and the robustness of our results.) The basic idea is that consumers will only choose, according to some stochastic rule, from the “maximal” alternatives which are not dominated by any other comparable alternative.

**Definition 1** Firm \( i \) which offers alternative \((z_i, p_i)\) \(\in\{A, B\}\times[0,1]\) is **dominated** if there exists firm \( j \neq i \) which offers alternative \((z_j, p_j < p_i)\) and the two offers are comparable.

Notice that the set of maximal or undominated alternatives is well-defined and non-empty (for example, the firm which charges the lowest price in the market is never dominated), and it can be constructed, for example, by conducting pairwise comparisons among all alternatives.\(^{17}\)

Then we can formally state our dominance-based consumer choice rule as follows:

1. Consumers first eliminate all dominated firms in the market.

2. They then purchase from the undominated firms according to the following stochastic purchase rule (which is independent of prices): (i) if all these firms use the same frame, they share the market equally; (ii) if among them \( n_A \geq 1 \) firms use frame A and \( n_B \geq 1 \) firms use frame B, then each undominated A firm is chosen with probability \( \phi(n_A, n_B)/n_A \) and each undominated B firm is chosen with probability \( [1 - \phi(n_A, n_B)]/n_B \), where \( \phi(\cdot) \in (0,1) \) is non-decreasing in \( n_A \) and non-increasing in \( n_B \) and \( \phi(n_A, n_B) \geq n_A/(n_A + n_B) \).

\(^{17}\)In our model, the comparability of two offers is independent of their comparability with other available offers. This excludes transitivity of comparability. Consider a consumer who can compare offers in different frames, but cannot compare offers in frame B. Then the presence of an offer in frame A (which is comparable with any of the B offers) does not help the consumer compare offers in frame B directly.
Note that, from consumers’ perspective, there is no difference among undominated firms which use the same frame. For this reason, both 2(i) and 2(ii) assume that the consumers do not discriminate among them. However, 2(ii) allows the consumers to favor one frame over the other. Specifically, \( \phi(n_A, n_B) \geq n_A / (n_A + n_B) \) means that undominated firms which use the simple frame A might be favored. The monotonicity assumption in 2(ii) requires the presence of more undominated firms with one frame to increase the overall probability that consumers buy from them.

Notice that the uniformly random purchase rule \( \phi(n_A, n_B) = n_A / (n_A + n_B) \), used in our duopoly example in Section 2, satisfies all the conditions.

The following example illustrates our consumer choice rule.

**Example 2** Consider Example 1 and let \( p_2 < p_3 \). As \( \alpha_2 = 0 \), all prices in frame B are comparable, and so firm 3 is dominated by firm 2. But, as consumers cannot compare prices in different frames (\( \alpha_1 = 1 \)), both firm 1 (with frame A) and firm 2 (with frame B) survive. Hence, consumers buy from firm 1 with probability \( \phi(1, 1) \) and from firm 2 with probability \( 1 - \phi(1, 1) \).

For the rest of the paper, let \( \phi_k \equiv \phi(1, k) \)

\( \phi_k \) denote the probability that a consumer buys from the A firm when there are \( k \) undominated B firms and one undominated A firm to choose from. Then, 2(ii) implies that \( \{\phi_k\}_{k=1}^{n-1} \) is a non-increasing sequence: when more B firms survive, the undominated A firm has less demand, and \( \phi_k \geq 1/(1+k) \).

Recall that in the duopoly case the type of market equilibrium depends on whether frame differentiation or frame complexity is more confusing. The same is true in the general case. Subsections 3.1 and 3.2 analyze the corresponding symmetric equilibrium and the impact of greater competition for \( \alpha_1 < \alpha_2 \) and \( \alpha_1 > \alpha_2 \), respectively.

Before we proceed with the analysis, let us summarize two main findings. First, when \( \alpha_2 > 0 \) (i.e., when frame B is complex), greater competition tends to induce firms to use frame B more often. In particular, when there is a large number of firms, they use frame B almost surely. Second, when \( \alpha_2 > 0 \), industry profit is bounded away from zero even when \( n \) converges to infinity, and greater competition can even increase industry profit and harm consumers (i.e., consumers may actually pay more in a more competitive market).

### 3.1 Frame differentiation dominates frame complexity (\( \alpha_1 > \alpha_2 \))

In this part, consumers are more likely to be confused by frame differentiation than by the complexity of frame B (that is, \( \alpha_1 > \alpha_2 \)). For simplicity, we mainly focus on the polar case in which prices in

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18 There is evidence that people have preferences for simpler options, especially when they face many options. See, for instance, Iyengar and Kamenica (2008) and the references therein.
different frames are always incomparable (i.e., \( \alpha_1 = 1 \)). We then discuss how the main results can be extended to the case with \( \alpha_1 < 1 \). All proofs missing from the text are relegated to Appendix B.

By applying a similar logic as in Lemma 1, we can show that there is no pure-strategy equilibrium whenever \( \alpha_2 > 0 \).

**Lemma 4** In the oligopoly model with \( 0 < \alpha_2 < \alpha_1 = 1 \), there is no equilibrium in which all firms adopt deterministic frames.

If \( \alpha_2 = 0 \) (i.e., if both frames are simple) and \( n \geq 4 \), there are always asymmetric pure-strategy equilibria in which each frame is used by more than one firm and all firms charge a price equal to the marginal cost. (See Section 4 for a more thorough treatment of the case with symmetric frames.) Nevertheless, the symmetric mixed-strategy equilibrium presented below also applies to this case.

**A symmetric mixed-strategy equilibrium.** We characterize a symmetric mixed-strategy equilibrium \((\lambda, F_A, F_B)\), where \( \lambda \) is the probability of using frame \( A \) and \( F_z \) is a price cdf associated with frame \( z \in \{A, B\} \). Let \([p_0^z, p_1^z]\) be the support of \( F_z \). As in Lemma 3, it is straightforward to show that \( F_z \) is atomless everywhere (as now \( \alpha_2 < 1 \)). For the rest of the paper, 

\[
P_{n-1}^k \equiv C_{n-1}^k \lambda^k (1-\lambda)^{n-k-1}
\]
denotes the probability that \( k \) firms among \( n-1 \) ones adopt frame \( A \) at equilibrium, where \( C_{n-1}^k \) stands for combinations of \( n-1 \) taken \( k \). Recall that \( x_z(p) = 1 - F_z(p) \).

Along the equilibrium path, if firm \( i \) uses frame \( A \) and charges price \( p \), its profit is:

\[
\pi(A,p) = p\lambda^{n-1}x_A(p)^{n-1} + \sum_{k=0}^{n-2} P_{n-1}^k x_A(p)^k \left[ \alpha_2 \phi_{n-k-1} + (1-\alpha_2) \phi_1 \right]
\]

If \( k \) other firms also use frame \( A \), firm \( i \) has a positive demand only if all other firms charge prices higher than \( p \). This happens with probability \( x_A(p)^k \). Conditional on that, if there are no \( B \) firms in the market (i.e., if \( k = n-1 \)), then firm \( i \) serves the whole market. The first term in \( \pi(A,p) \) follows from this. Otherwise, firm \( i \)'s demand depends on whether the consumer can compare offers from the \( B \) firms. If she is confused by frame complexity and unable to compare (which happens with probability \( \alpha_2 \)), all \( B \) firms are undominated (since no comparison between \( A \) and \( B \) is possible), and so firm \( i \)'s demand is \( \phi_{n-k-1} \). If she is not confused by frame complexity and, therefore, able to compare (which happens with probability \( 1-\alpha_2 \)), only one \( B \) firm is undominated and so firm \( i \)'s demand is \( \phi_1 \).

\[\text{If instead, along the equilibrium path, firm } i \text{ uses } B \text{ and charges price } p, \text{ its profit is:}
\]

\[
\pi(B,p) = p(1-\lambda)^{n-1} \left\{ \frac{\alpha_2}{n} + (1-\alpha_2) x_B(p)^{n-1} \right\}
\]

\[+p \sum_{k=1}^{n-1} P_{n-1}^k \left[ \frac{\alpha_2}{n} \frac{1-\phi_{n-k}}{n-k} + (1-\alpha_2) (1-\phi_1) x_B(p)^{n-k-1} \right].
\]

\[\text{Notice that as } F_z(p) \text{ is continuous, the probability of a tie at a price } p \text{ is zero.}\]
The first term gives the expected profit when there are no $A$ firms in the market: the consumers who are confused by frame complexity purchase randomly among all $B$ firms, while those who are not confused buy from firm $i$ only if it offers the lowest price. When $k \geq 1$ firms use frame $A$ (note that only one of them will be undominated), if the consumer is confused by frame complexity (i.e., unable to compare prices in frame $B$), all $B$ firms are undominated and have demand $1 - \phi_{n-k}$ in total. Firm $i$ shares equally this residual demand with the other $B$ firms. If the consumer is not confused by frame complexity, to face a positive demand, firm $i$ must charge the lowest price in group $B$ (this happens with probability $x_B(p)^{n-k-1}$), in which case it gets the residual demand $1 - \phi_1$.

Since price competition can only take place among firms which use the same frame (as $\alpha_1 = 1$), $x_A(p)$ ($x_B(p)$) does not appear in $\pi(B,p)$ ($\pi(A,p)$). So both profit functions also hold even if firm $i$ charges an out-of-equilibrium price. Then, in the symmetric mixed-strategy equilibrium, the upper bound of the price cdf’s is frame-independent and $p_1^A = p_1^B = 1$. Otherwise any price greater than $p_1^A$ would lead to a higher profit. We can pin down $\lambda$ from the indifference condition $\pi(A,1) = \pi(B,1)$ at $p = 1$ (note that $F_z(1) = 1$ or $x_z(1) = 0$). Dividing each side by $(1 - \lambda)^{n-1}$ and rearranging the equation we obtain

$$\alpha_2 \left( \phi_{n-1} - \frac{1}{n} \right) + (1 - \alpha_2) \phi_1 = \alpha_2 \sum_{k=1}^{n-2} C_{n-1}^k \left( 1 - \phi_{n-k} \right) \left( \frac{\lambda}{1 - \lambda} \right)^k + (1 - \phi_1) \left( \frac{\lambda}{1 - \lambda} \right)^{n-1}. \tag{13}$$

The right-hand side of (13) increases in $\lambda \in [0,1]$ from zero to infinity, and the left-hand side is positive for any $\alpha_2 \in [0,1]$ as $\phi_{n-1} \geq 1/n$. Hence, (13) has a unique solution in $(0,1)$. Each firm’s equilibrium profit is

$$\pi = \pi(A,1) = (1 - \lambda)^{n-1} [\alpha_2 \phi_{n-1} + (1 - \alpha_2) \phi_1]. \tag{14}$$

The price distributions $F_A$ and $F_B$ are implicitly given by $\pi(z,p) = \pi$ and are well defined. The boundary prices $p_0^A < 1$ are determined by $\pi(z, p_0^A) = \pi$. Deviations to $(z,p < p_0^A)$ are not profitable since they only result in a price loss and do not increase demand. We can now summarize our findings.

**Proposition 3** For $n \geq 2$ and $\alpha_2 < \alpha_1 = 1$, there is a symmetric mixed-strategy equilibrium in which each firm adopts frame $A$ with probability $\lambda$ and frame $B$ with probability $1 - \lambda$. When a firm uses frame $z \in \{A,B\}$, it chooses its price randomly according to a cdf $F_z$. Specifically, $\lambda$ solves (13), and $F_z$ is defined on $[p_0^A, 1]$ and implicitly determined by $\pi(z,p) = \pi$, where $\pi(z,p)$, $z \in \{A,B\}$, are given by (11) and (12) and $\pi$ is each firm’s equilibrium profit given by (14).

When $n = 2$ and $\phi_1 = 1/2$, this equilibrium coincides with the one in the duopoly case for $\alpha_1 = 1$. Recall that the duopoly equilibrium in Proposition 1 is characterized by price-frame independence (i.e., $F_A(p) = F_B(p)$). We discuss below the robustness of this independence result in the symmetric oligopoly equilibrium presented above.
The (im)possibility of price-frame independence. Does Proposition 3 dictate $F_A = F_B$ in general? The following graph depicts the equilibrium price distributions $F_A(p)$ (the solid line) and $F_B(p)$ (the dashed line) in the case with $n = 3$, $\alpha_2 = 0.5$, and $\phi_k = 1/(1 + k)$.

![Price distributions with $n = 3$, $\alpha_1 = 1$ and $\alpha_2 = 0.5$](image)

Figure 1: Price distributions with $n = 3$, $\alpha_1 = 1$ and $\alpha_2 = 0.5$

Clearly, $F_A \neq F_B$ in this example. The following result shows that, for $n \geq 3$, equilibrium price-frame independence holds only in special cases.

**Proposition 4** In the oligopoly model with $\alpha_2 < \alpha_1 = 1$,

(i) for $n = 2$, the symmetric equilibrium in Proposition 3 dictates $F_A = F_B$ only if $\phi_1 = 1/2$.

(ii) for $n \geq 3$, the symmetric equilibrium in Proposition 3 dictates $F_A = F_B$ only if $\phi_1 = 1/2$ and $\alpha_2 = 0$, or for a particular non-uniformly random purchase rule $\{\phi_k\}_{k=1}^{n-1}$.

In the duopoly case with $\alpha_2 < \alpha_1$, as we have shown, equilibrium $\lambda$ ensures that, regardless of its frame choice, a firm faces the same expected number of totally confused consumers. This also means that it faces the same expected number of fully aware consumers since there are only two types of consumers in the duopoly case. Then, if consumers have no exogenous bias toward either frame, the firms will adopt the same price distribution for each frame. With more than two firms, however, there are in general more than two types of consumers. Although equilibrium $\lambda$ ensures that the expected number of consumers who are totally insensitive to a firm’s price is the same regardless of this firm’s frame choice (i.e., $\pi(A, 1) = \pi(B, 1)$), this no longer guarantees that this firm also faces the same expected number of other types of consumers. In general, it is impossible for a firm to face the same market composition when it switches from one frame to the other, and so its pricing needs to adjust to different environments. Note that when $\phi_1 = 1/2$ and $\alpha_2 = 0$, the two frames are totally symmetric and the firms adopt the same price distribution for each frame in the symmetric equilibrium.

When $F_A \neq F_B$, we could not rank the expected prices associated with the two frames. Numerical simulations tend to suggest that the simple frame $A$ might have a lower price on average, but the price distribution for frame $B$ seems to be more dispersed (see Figure 1).
The impact of greater competition. We now study the impact of an increase in the number of firms on the equilibrium framing strategies, and on profits and consumer surplus. Our analysis is based on the equilibrium characterized in Proposition 3. We first consider a market with many sellers.

**Proposition 5** When there are a large number of firms in the market,

$$\lim_{n \to \infty} \lambda = \begin{cases} 1/2, & \text{if } \alpha_2 = 0 \\ 0, & \text{if } \alpha_2 > 0 \end{cases} \quad \text{and} \quad \lim_{n \to \infty} n\pi = \begin{cases} 0, & \text{if } \alpha_2 = 0 \\ > 0, & \text{if } \alpha_2 > 0 \end{cases}.$$

When frame B is also a simple frame, the only way to reduce price competition is through frame differentiation. This is why in a sufficiently competitive market \( \lambda \) tends to 1/2, which maximizes frame differentiation. However, the ability of frame differentiation alone to weaken price competition is limited. When there are a large number of firms in the market, each frame is adopted by more than one firm almost surely (as long as \( \lambda \) is bounded away from zero and one), so price competition becomes extremely intense and the market price tends to marginal cost.

When frame B is complex, the impact of greater competition on firms’ framing strategies changes completely. In a sufficiently competitive market, firms use frame B almost surely: they rely heavily on frame complexity to soften price competition. (This is true even if frame B is only slightly more complex than frame A.) The reason is that, in a large market, the effect of frame differentiation on reducing price competition becomes negligible, but the effect of frame complexity is still significant. For example, if all firms employ frame B for sure, industry profit is always \( \alpha_2 \), regardless of the number of firms in the market. Hence, when frame B is complex, competition does not drive the market price to marginal cost.

The analysis for large \( n \) suggests that, when the number of firms increases, frame B’s complexity tends to become a relatively more important anti-competitive device. In effect, as we will show shortly, \( \lambda \) tends to be decreasing in the number of firms. That is, greater competition tends to induce firms to use the complex frame more frequently. Is it then possible that, in the presence of a complex frame B, greater competition can even raise market prices by increasing market complexity? The answer, in general, depends on the parameter values. But, we show below that, at least for sufficiently large \( \alpha_2 \), greater competition can actually increase industry profit and harm consumers. Therefore, in the market with price framing, competition policy which focuses exclusively on an increase in the number of competitors, might have undesired effects.

For tractability, we focus on the uniformly random purchase rule \( \phi_k = 1/(1 + k) \). Then, (13) becomes

$$1 - \alpha_2 = 2\alpha_2 \sum_{k=1}^{n-2} \frac{C_k}{n-k+1} \left( \frac{\lambda}{1 - \lambda} \right)^k \left( \frac{\lambda}{1 - \lambda} \right)^{n-1},$$

and industry profit is

$$n\pi = n (1 - \lambda)^{n-1} \left( \frac{\alpha_2}{n} + \frac{1 - \alpha_2}{2} \right).$$
Proposition 6 With $0 < \alpha_2 < \alpha_1 = 1$ and the random purchase rule $\phi_k = 1/(1+k)$, 
(i) when $n$ increases from 2 to 3, both $\lambda$ and industry profit $n\pi$ decrease;  
(ii) for any $n \geq 3$, there exists $\hat{\alpha} \in (0,1)$ such that for $\alpha_2 > \hat{\alpha}$, $\lambda$ decreases but industry profit $n\pi$ increases from $n$ to $n+1$. 

Beyond the limit results, numerical simulations (based on the uniformly random purchase rule) suggest that $\lambda$ tends to decrease in $n$, and industry profit can increase in $n$ for a relatively large $\alpha_2$.\textsuperscript{20} The graph below describes how industry profit varies with $n$ when $\alpha_2 = 0.9$.

![Graph showing industry profit variation with n when alpha_2 = 0.9]

Figure 2: Industry profit and $n$ when $\alpha_1 = 1$ and $\alpha_2 = 0.9$

The case with $\alpha_2 < \alpha_1 < 1$. Now price competition can also take place between firms using different frames. Then both $x_A(p)$ and $x_B(p)$ will appear in the profit functions. The related analysis becomes more involved and its details are presented in Appendix D.1. We find that if a symmetric mixed-strategy equilibrium exists, then it still satisfies $p_1^A = p_1^B = 1$. Equilibrium price-frame independence requires even more stringent conditions than in the polar case $\alpha_1 = 1$. Finally, numerical simulations show that greater competition can still have undesired effects (for example, when $\alpha_1$ is large and $\alpha_2$ is close to $\alpha_1$).

3.2 Frame complexity dominates frame differentiation ($\alpha_2 > \alpha_1$)

In this part, consumers are more likely to be confused by the complexity of frame $B$ than by frame differentiation (that is, $\alpha_2 > \alpha_1$). For simplicity, we first focus on the polar case in which prices in frame $B$ are always incomparable (i.e., $\alpha_2 = 1$). We then discuss the robustness of our main results to the case with $\alpha_2 < 1$. All the proofs missing from the text are relegated to Appendix B.

There is no pure-strategy equilibrium in this case either.

Lemma 5 In the oligopoly model with $0 < \alpha_1 < \alpha_2 = 1$, there is no equilibrium in which all firms use deterministic frames.

\textsuperscript{20}If $\alpha_2 = 0$, then $\lambda = 1/2$ (for any $n$) and industry profit is $n/2^n$, which decreases in $n$. Hence, for $\alpha_2$ sufficiently close to zero, industry profit should decrease in $n$. 

A **symmetric mixed-strategy equilibrium.** We characterize a symmetric mixed-strategy equilibrium \((\lambda, F_A, F_B)\) in which \(\lambda\) is the probability of using frame \(A\), \(F_A\) is defined on \(S_A = [p_0^A, 1]\) and is atomless, and \(F_B\) is degenerate on \(S_B = \{1\}\).

Along the equilibrium path, if firm \(i\) uses frame \(A\) and charges \(p \in [p_0^A, 1]\), its profit is given by

\[
\pi(A, p) = p \sum_{k=0}^{n-1} P_{n-1}^k x_A(p)^k (\alpha_1 \phi_{n-k-1} + 1 - \alpha_1).
\]  

(17)

This expression follows from the fact that, when \(k\) other firms also use frame \(A\), firm \(i\) has a positive demand only if all other \(A\) firms charge prices higher than \(p\). Conditional on that, with probability \(\alpha_1\), the consumer is confused by frame differentiation and buys from firm \(i\) with probability \(\phi_{n-k-1}\) (since all \(n-k-1\) firms which use \(B\) are undominated); with probability \(1 - \alpha_1\), the consumer can compare \(A\) and \(B\) and, because all \(B\) firms charge price \(p_B = 1 > p\) and consequently are dominated, she only buys from firm \(i\).

A firm’s equilibrium profit is equal to

\[
\pi = \lim_{p \to 1} \pi(A, p) = (1 - \lambda)^{n-1} (\alpha_1 \phi_{n-1} + 1 - \alpha_1).
\]  

(18)

Then the expression for \(F_A(p)\) follows from \(\pi(A, p) = \pi\), and \(p_0^A\) satisfies \(\pi(A, p_0^A) = \pi\). Both of them are well defined.

If firm \(i\) uses \(B\) and charges \(p = 1\), then its profit is

\[
\pi(B, 1) = \frac{(1 - \lambda)^{n-1}}{n} + \alpha_1 \sum_{k=1}^{n-1} P_{n-1}^k \frac{1 - \phi_{n-k}}{n - k}.
\]  

(19)

Notice that firm \(i\) has a positive demand only if all other firms also use frame \(B\), or there are \(A\) firms but the consumer is unable to compare prices in different frames.

The equilibrium condition \(\pi(B, 1) = \lim_{p \to 1} \pi(A, p)\) pins down \(\lambda\):

\[
\frac{1 - 1/n}{\alpha_1} + \phi_{n-1} - 1 = \sum_{k=1}^{n-1} \frac{C_{n-1}^k (1 - \phi_{n-k})}{n - k} \left(\frac{\lambda}{1-\lambda}\right)^k.
\]  

(20)

The left-hand side of (20) is positive given that \(\phi_{n-1} \geq 1/n\), and the right-hand side is increasing in \(\lambda\) from zero to infinity. Hence, for any given \(n \geq 2\) and \(\alpha_1 \in (0, 1)\), equation (20) has a unique solution in \((0, 1)\). In the Appendix, we further show that there is no profitable deviation.

**Proposition 7** For \(n \geq 2\) and \(0 < \alpha_1 < \alpha_2 = 1\), there is a symmetric mixed-strategy equilibrium in which each firm adopts frame \(A\) with probability \(\lambda\) and frame \(B\) with probability \(1 - \lambda\). When a firm uses frame \(z \in \{A, B\}\), it chooses its price randomly according to a cdf \(F_z\). Specifically, \(\lambda\) solves (20), \(F_A\) is defined on \(S_A = [p_0^A, 1]\) and implicitly given by \(\pi(A, p) = \pi\), where \(\pi\) is each firm’s equilibrium profit given by (18), and \(F_B\) is a degenerate cdf on \(S_B = \{1\}\).
It is straightforward to check that, when \( n = 2 \) and \( \phi_1 = 1/2 \), this equilibrium coincides to that in the duopoly model with \( \alpha_2 = 1 \) (see Proposition 2). In continuation, we analyze the impact of greater competition on the market outcome using the equilibrium in Proposition 7.

**The impact of greater competition.** When there are many sellers in the market, the results in Proposition 5 for \( \alpha_2 > 0 \) carry over to this case. That is, \( \lim_{n \to \infty} \lambda = 0 \) and \( \lim_{n \to \infty} n\pi > 0 \). The previous intuition applies: in a sufficiently competitive market, firms resort to the complexity of frame \( B \) to soften price competition since the ability of frame differentiation to do so is negligible.

The following result shows that greater competition can improve industry profit and so decrease consumer surplus when \( \alpha_1 \) is sufficiently small. The reason is that, when \( \alpha_1 \) is small, the complexity of frame \( B \) is more effective in reducing price competition, which makes the frequency of using frame \( B \) increase very fast with the number of firms.

**Proposition 8** *In the case with \( 0 < \alpha_1 < \alpha_2 = 1 \), for any \( n \geq 2 \), there exists \( \hat{\alpha} \in (0, 1) \) such that for \( \alpha_1 < \hat{\alpha} \), \( \lambda \) decreases while industry profit \( n\pi \) increases from \( n \) to \( n + 1 \).*

Beyond this limit case, numerical simulations (based on the uniformly random purchase rule) suggest that \( \lambda \) tends to decrease with \( n \), and for a relatively small \( \alpha_1 \) industry profit can increase in \( n \) when \( n \) is not too large.

The following graph describes how industry profit varies with \( n \) when \( \alpha_1 = 0.05 \).

![Figure 3: Industry profit and \( n \) when \( \alpha_1 = 0.05 \) and \( \alpha_2 = 1 \)](image)

**The case with \( \alpha_1 < \alpha_2 < 1 \).** Now the analysis is more involved, and we relegate the details to Appendix D.2. We report here two main findings. First, a symmetric separating equilibrium with \( S_A = [p_A^0, \hat{p}] \) and \( S_B = [\hat{p}, p_1^B] \), resembling the one in Proposition 7, still exists under some parameter restrictions (when \( \alpha_1 \) is not too close to \( \alpha_2 < 1 \)). Second, for fixed \( \alpha_2 < 1 \), if \( \alpha_1 \) is sufficiently small, greater competition can still increase industry profit and harm consumers.

\(^21\)Given the random purchase rule, we can actually show that industry profit always decreases with \( n \) for sufficiently large \( \alpha_1 \). The details are available upon request.
4 More frames

The oligopoly model with a general frame structure is less tractable. In this section, we explore the relatively simpler case with $m \geq 2$ completely symmetric frames \{A_1, \cdots, A_m\}. Our main purpose is to study how the number of frames could affect market outcomes. Specifically, we assume that (i) consumers are able to perfectly compare prices in the same frame but are totally confused between different frames, and (ii) they use the dominance-based choice rule with the uniformly random purchase rule among undominated firms. (This is a generalization of the two-frame case with $\alpha_0 = \alpha_2 = 0$ and $\alpha_1 = 1$.) Note that in this case an offer can only be dominated by other offers in the same frame.

We first explore pure-strategy equilibria (see Appendix C.2 for the proof of the next result).

**Lemma 6** In the oligopoly model with $m$ completely symmetric frames,

(i) if $n \geq 2m$, there exist asymmetric pure-strategy equilibria where each frame is used by more than one firm and all firms charge zero price;

(ii) if $n < 2m$, there is no equilibrium in which all firms adopt deterministic frames.

Therefore, if the set of available frames is large enough, there can only be equilibria in which firms randomize over frames. However, even when fewer frames are available ($n \geq 2m$), as we show below, there also exists a mixed-strategy equilibrium in which firms randomize over frames and make positive profits.

Let us characterize the symmetric mixed-strategy equilibrium in which each firm adopts each price frame with probability $1/m$ and charges a random price according to a continuous cdf $F(p)$ defined on $[p_0, p_1]$. Notice that, in such equilibrium, $p_1 = 1$.

Along the equilibrium path, if firm $i$ adopts frame $A_j$ and charges a price $p \in [p_0, 1]$, its profit depends on the number of firms (including itself) using frame $A_j$ and the number of distinct frames in the market. If there are $k$ firms in group $A_j$ and $l \geq 1$ distinct frames in total in the market, then firm $i$’s expected demand is

$$\frac{1}{l} [1 - F(p)]^{k-1}.$$ 

Firm $i$ has a positive demand only if it offers the lowest price in group $A_j$ and, when it does so, it shares the market equally with all winners from other groups. Notice that, when $k$ firms use frame $A_j$, the number of other distinct frames in the market cannot exceed $m - 1$ and $n - k$. Then,

$$l \leq \min\{m, n - k + 1\} \equiv J(k).$$

Let $\Pr(k, l)$ be the probability that there are $k$ firms in group $A_j$ and $l$ distinct frames in total in the market conditional on the fact that firm $i$ has chosen $A_j$. (See Appendix C.2 for details on the calculation of $\Pr(k, l)$.) Then firm $i$’s expected profit is
At equilibrium, each firm earns \( \pi(A_j, 1) = a_1 \). The expression for \( F(p) \) is then implicitly given by the equation \( \pi(A_j, p) = a_1 \). Clearly, \( F(p) \) is well defined. From \( \pi(A_j, p_0) = a_1 \), we can solve \( p_0 = a_1 / \sum_{k=1}^n a_k < 1 \). It is also clear that any deviation to a price below \( p_0 \) is not profitable. This establishes the following result.

**Proposition 9** In the oligopoly model with \( m \) completely symmetric frames, there is a symmetric mixed-strategy equilibrium in which firms assign probability \( \lambda = 1/m \) to each available frame, and price according to a common cdf \( F \) which is defined on \([p_0, 1]\) and solves \( \pi(A_j, p) = \pi \), where \( \pi(A_j, p) \) is given in (21) and \( \pi = a_1 \) is each firm’s equilibrium profit.

We now investigate how profits vary with \( n \) and \( m \) at the mixed-strategy equilibrium in Proposition 9. We first consider two simple cases. (i) With only two frames, if firm \( i \) chooses one frame and charges \( p = 1 \), it has a positive demand only if all other firms use the other frame. This happens with probability \((1/2)^{n-1}\) and, in this case, firm \( i \)'s market share is \( 1/2 \). So its profit is \( a_1 = (1/2)^n \). Hence, when \( m = 2 \), both individual and industry profit decrease with \( n \). (ii) When there are only two firms, if firm \( i \) chooses one frame and price \( p = 1 \), it has a positive demand only if the other firm chooses a different frame. The probability of this event is \( 1 - \frac{1}{m} \). Hence, when \( n = 2 \), we have \( a_1 = (1 - \frac{1}{m})/2 \). Clearly, both individual and industry profits increase with \( m \).

In general,

\[
a_1 = \sum_{l=1}^{\min(m, n)} \frac{\Pr(1, l)}{l}
\]

does not have a concise expression. Numerical simulations suggest that industry profit \( (na_1) \) decreases with \( n \) for fixed \( m \) and increases with \( m \) for fixed \( n \). Intuitively, when there are more firms, it becomes more difficult for each firm to frame differentiate itself from rivals, such that firms compete more aggressively in prices. In contrast, when more frames become available, it becomes easier for firms to frame differentiate and avoid price competition. In particular, we can show that \( \lim_{n \to \infty} na_1 = 0 \) for fixed \( m \), and \( \lim_{m \to \infty} na_1 = 1 \) for fixed \( n \).

However, if each new entrant brings a new frame to the market (i.e., \( m = n \)), then simulations show that industry profit always increases with \( n \) (but lies below about 0.6). This result suggests that in some sense the frame differentiation effect is stronger than the competition effect.

The consideration of a general frame structure for \( m \geq 3 \) brings about significant technical complications. Although we do not deal here with this general setting, let us briefly comment on some possibilities for future work. An oligopoly model with a general frame structure could (i)
assign to each frame a complexity index—the probability that the consumer gets confused among prices in this frame, and (ii) assign to each pair of frames a differentiation index—the probability that the consumer gets confused between the two frames. The dominance-based choice rule with an appropriately modified stochastic purchase rule (e.g., the uniformly random one) can apply. We conjecture that our main insights would still apply in this framework.

5 Conclusion and Discussion

This paper has presented a model of competition in both prices and price frames where price framing can obstruct consumers’ price comparisons. We characterized the symmetric mixed-strategy equilibrium in which firms randomize over both frames and prices, and examined how the degree of competition affects firms’ strategies, profits, and consumer welfare.

In the remainder of this section, we discuss alternative consumer choice rules and interpretations of consumer confusion, and argue that our results match observed market outcomes.

5.1 Alternative consumer choice rules

(1) More restrictive consideration sets. In our dominance-based choice rule, consumers’ “consideration set” includes all available options. The consumers make correct comparisons among all pairs of comparable alternatives, rule out the dominated alternatives, and then select from the set of undominated ones. Alternatively, consumers may restrict their consideration set at the outset (to save time and effort, for instance). The following example illustrates such choice heuristics.

Example 3 When \( n_A \geq 1 \) firms use frame A and \( n_B \geq 1 \) firms use frame B, a consumer who cannot compare B options, restricts attention to a consideration set which consists of all A firms and \( k \leq n_B \) randomly chosen B firms. She then applies the dominance-based choice rule to this restricted consideration set.

Our benchmark choice procedure corresponds to \( k = n_B \) and is the most sophisticated one in this class. When \( k < n_B \), though consumers face a simpler choice problem, they may eventually choose some dominated options. For example, when a consumer can compare A to B, she would fail to eliminate the A option(s) if the B option(s) which dominate them had been excluded from her consideration set. It can be shown that (at least) for \( k = 1 \), our main results hold qualitatively.

(2) A default-bias choice rule. Our dominance-based choice rule embeds a simultaneous assessment of competing offers, and a consumer’s choice outcome is not affected by the particular sequence of pairwise comparisons. This “simultaneous search” feature is more suitable in a market where the consumers are not influenced by their previous experiences (or are newcomers). Piccione
and Spiegler (2009) consider a default-bias duopoly model in which consumers are initially randomly attached to one brand (their default option), and they switch to another brand only if that is comparable to their default and better than it. In this case, due to the sequential comparison, a consumer’s final choice will depend on her default option.

In the duopoly case, the default-biased model is actually equivalent to our simultaneous assessment model (with the random purchase rule for confused consumers).\(^{22}\) This is because, if the two firms’ offers are comparable, in both models the better offer attracts all consumers, whereas if they are incomparable, in both models the firms share the market equally. However, when there are more than two firms, the two approaches diverge. In fact, with more than two firms, the default-bias model calls for further structure on the choice rule. To see why, consider the following example.

**Example 4** There are three firms in the market. Let \(\alpha_2 = 1\) and \(\alpha_1 = 0\) (i.e., the only confusion source is frame complexity). Suppose that firm 1 adopts frame A and charges a price \(p_1\), while firms 2 and 3 adopt frame B and charge prices \(p_2\) and \(p_3\), respectively, with \(p_2 < p_1 < p_3\).

The dominance-based rule implies that consumers will purchase only from firm 2 since firm 3 is dominated by firm 1 and firm 1 is dominated by firm 2. Now consider the default-bias model. If a consumer is initially attached to firm 2, she will not switch. If she is initially attached to firm 1, she will switch to firm 2. However, if she is initially attached to firm 3, she will switch to firm 1, but whether she will further switch to firm 2 depends on what the choice rule of the default-biased consumer dictates. Such rule should specify if the consumer will assess firm 2’s offer from the perspective of her default option (i.e., firm 3) or from the perspective of her new choice (i.e., firm 2). In contrast, the dominance-based rule applies equally well regardless of the number of firms in the market.\(^{23}\)

Both a more restrictive consideration set and a default bias add another dimension of bounded rationality on top of consumer confusion caused by framing. In this sense, our framework is a minimal deviation from the standard Bertrand competition model.

(3) **Noisy price comparisons.** For the sake of tractability, we have assumed in our consumer choice rule that confused consumers’ choice from the set of undominated alternatives is entirely independent of the prices. Alternatively, confusion might only lead to noisy price comparisons, so that consumers’ choice still depends somewhat on prices. For instance, in the duopoly case, when the price difference between firms 1 and 2 is \(p_1 - p_2\), the consumer might misperceive it as

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\(^{22}\) More precisely, the equivalence requires the probability of being confused by two frames to be independent of which one is the default option.

\(^{23}\) The fact that these two choice rules may lead to different choice outcomes can also be seen from the following example: consider the frame choices in example 4, but let \(\alpha_2 = 0\) and \(\alpha_1 = 1\), and \(p_1 < p_2 < p_3\). Our approach (with the uniform purchase rule) predicts that firms 1 and 2 will share the market equally; while the default-bias rule predicts that firm 1 has demand \(\frac{1}{3}\) and firm 2 has demand \(\frac{2}{3}\).
being $p_1 - p_2 + \delta$, where $\delta$ is a frame profile dependent random variable. If all $\delta$'s have symmetric distributions around zero, then our result that in the symmetric equilibrium the firms randomize in both frames and prices carries over. However, unless we restrict attention to a duopoly case where confusion stems only from frame differentiation, we are unable to characterize the symmetric mixed-strategy equilibrium in this setting.

5.2 Alternative interpretations

Product framing. Although our study has been motivated by price framing, it also applies to situations where product framing reduces the comparability of products. For instance, the way in which nutritional information is presented might frame differently essentially identical food products. A label indicating an “improved recipe” or a “British meal” might spuriously differentiate a ready meal from its close substitutes. Differences in package size or quantity premia could also make it harder to compare products. On the same shelf toothpastes come in tubes of 50, 75 or 100 ml, and refreshments, cleaning products, tea boxes occasionally come in larger containers offering, say, “extra 25% free”. In addition, Betrand et al. (2009) and Choi et al. (2008) document evidence that in the personal finance market, providing some payoff irrelevant information (e.g., a female photo in the loan advertising letter or information concerning mutual fund historical returns) can significantly influence consumers’ choices. Our main insights also apply to the product framing case as long as framing is not a long-run decision or one observable to the rivals before price competition. This interpretation also relates our paper to the literature on endogenous product differentiation (see, for instance, Chapter 7 in Tirole, 1988). One main difference is that in our model firms make differentiation and price choices simultaneously.

Costly information processing. Price comparisons in the presence of framing might require costly information processing. Then, consumer confusion could be the result of consumers’ rational decision to opt out of information processing when its cost is too high or its expected benefit is too low. Therefore, our model could be interpreted as one of costly information processing with rational consumers, and not only as one of bounded rationality.

However, rational consumers should eventually be able to infer prices from frames (if they can distinguish between frames). In this case, a separating equilibrium where the complex frame is associated with higher prices (as the ones in Propositions 2 and 7) would not survive, since the consumers should then always choose simple-frame products. (This is not an issue in Propositions 24 The reportage “What’s really in our food?” broadcast on BBC One on July 14, 2009 stressed this point. For instance, interviewed customers confessed to being misled by a ready food made with imported meat and labeled as “British meal”. Also, buyers seem to have a poor understanding of what labels such as “free range” really mean.
25 For example, the first paper shows that the effect of including a female photo in the loan advertising letter on increasing customers’ loan take-up is as strong as a 25% reduction in the interest rate.
26 In this sense, our assumption that consumers weakly favor the simple frame (i.e., $\phi(n_A, n_B) \geq n_A/(n_A + n_B)$)
1 and 3, where the complex frame is not necessarily associated with higher prices.)

Our separating equilibrium still makes sense if (i) consumers are yet to understand the market equilibrium or they purchase the product infrequently such that they do not have enough opportunities to learn the market equilibrium, or (ii) there is always a non-trivial mass of naive consumers who choose randomly when they get confused.\textsuperscript{27}

Carlin (2009) considers a model similar to our case with $\alpha_2 > \alpha_1$. In his model, each firm chooses its price complexity level and consumers decide whether to learn \textit{all} prices in the market by incurring a search cost. In equilibrium, higher complexity is also associated with higher prices. Carlin avoids the inference problem by exogenously assuming that consumers can only observe the aggregate market complexity index, but not each firm’s price complexity.

5.3 Empirical relevance

The predictions of our model depend on which source of confusion dominates (frame differentiation or frame complexity). We have given in the introduction several examples in which frame differentiation is the main source of confusion. In those cases, each price presentation mode is not particularly involved, but market-wide price frame differentiation makes it more difficult (or costly) for the consumers to compare the prices of close substitutes. Our model predicts that, if frame differentiation is the dominant confusion source, there is no clear ranking (on average) among prices associated with different frames. For example, there should be no significant price differences across different discounting methods. This is an empirically testable result and seems consistent with casual observations in the markets we discussed. In addition, notice that frame differentiation seems to prevail in markets where the consumers purchase with relatively high frequency. If some frames were associated with higher prices, even consumers with high cognitive costs might be able to figure it out over time and avoid buying these products.

In markets where frame complexity is the dominant source of confusion, our model predicts that the more complex frame is always associated with higher prices. Woodward (2003) provides evidence that, in the mortgage market, the deals with the arrangement fees rolled in the interest rate are on average better than the deals with separate fees. Our model also predicts that, in markets with some frame complexity (even if it does not dominate frame differentiation), an increase in the number of firms can increase prices and harm consumers. Hortaçsu and Syverson (2004) show that in the S&P index fund market, a decrease in concentration between 1995-99 triggered an increase in the average price. Notice that price complexity is mostly common in markets where the consumers partially reflect such sophistication.

\textsuperscript{27}In the case of $\alpha_2 > \alpha_1$, suppose $\gamma < 1$ consumers are rational and understand the market equilibrium, and $1 - \gamma$ are naive (like in our model) and choose randomly from the undominated alternatives. Then, the separating equilibrium still exists with $\phi_k = \gamma + (1 - \gamma) \frac{k}{1+k}$.
participate infrequently and therefore do not have the opportunity to infer prices from presentation modes.

References


A Appendix: Proofs in the Duopoly Case

A.1 Proof of Lemma 3
Suppose that equilibrium \( F_z \) has a mass point at some price \( p \in S_z \). Then, in the symmetric equilibrium, there is a positive probability that both firms use frame \( z \) and tie at \( p \). Given \( \alpha_2 < 1 \), there is always a positive measure of price aware consumers regardless of \( z \), such that for any firm it is more profitable to offer \((z, p - \varepsilon)\) (for some small \( \varepsilon > 0 \)) than \((z, p)\). This leads to a contradiction.

A.2 Proof of Proposition 1
The proposed configuration is indeed an equilibrium since no deviation to \( p < p_0 \) is profitable. We show now that it is the unique symmetric mixed-strategy equilibrium with \( F_z \) strictly increasing on its support. Recall that, by Lemma 3, when \( \alpha_2 < 1 \), in any symmetric mixed-strategy equilibrium \( F_z \) is continuous on \( S_z \). The proof entails several steps.

Step 1: \( S_A \cap S_B \neq \emptyset \). Suppose \( p_i^A < p_i^B \). Then if a firm uses frame \( A \) and charges \( p_i^A \), its profit is

\[
\pi(A, p_i^A) = p_i^A (1 - \lambda) [(1 - \alpha_1) + \alpha_1/2].
\]

The firm has positive demand only if the rival is using frame \( B \), in which case it sells to all price aware consumers and to half of the confused ones. Clearly, this firm can do better by charging a price slightly higher than \( p_i^A \). A contradiction. Similarly, we can rule out the possibility of \( p_i^B < p_i^A \).

Step 2: \( \max\{p_i^A, p_i^B\} = 1 \). Suppose \( p_i^\ast = \max\{p_i^A, p_i^B\} < 1 \). Then, \( p_i^\ast \) is dominated by \( p_i^\ast + \varepsilon \) (for some small \( \varepsilon > 0 \)).

Step 3: \( S_A = S_B = [p_0, 1] \). Suppose \( p_i^A < p_i^B = 1 \). Then, along the equilibrium path, if firm \( i \) uses frame \( A \) and charges \( p \in [p_i^A, 1] \), its profit is

\[
\pi(A, p) = p (1 - \lambda) [(1 - \alpha_1) x_B (p) + \alpha_1/2],
\]

since it faces a positive demand only if firm \( j \) uses frame \( B \). If firm \( i \) uses frame \( B \) and charges the same price \( p \), its profit is

\[
\pi(B, p) = p \{\lambda \alpha_1/2 + (1 - \lambda) [(1 - \alpha_2) x_B (p) + \alpha_2/2]\},
\]

which should be equal to the candidate equilibrium profit. Since the supposition \( p_i^A < p_i^B = 1 \) and Step 1 imply \( p_i^A \in S_B \), the indifference condition requires \( \pi(A, p_i^A) = \pi(B, p_i^A) \) or

\[
(1 - \lambda) (\alpha_1 - \alpha_2) - \lambda \alpha_1 = 2 (1 - \lambda) (\alpha_1 - \alpha_2) x_B (p_i^A).
\]

However, if this equation holds, \( \pi(A, p) > \pi(B, p) \) for \( p \in (p_i^A, 1] \) as \( \alpha_1 > \alpha_2 \) and \( x_B \) is strictly decreasing on \( S_B \). This is a contradiction. Similarly, we can exclude the possibility of \( p_i^B < p_i^A = 1 \). Therefore, it must be that \( p_i^A = p_i^B = 1 \).
Then, from \( \pi(A, 1) = \pi(B, 1) \), it follows that
\[
\lambda \alpha_1 = (1 - \lambda) (\alpha_1 - \alpha_2).
\]

Now suppose \( p_0^A < p_0^B \). Then
\[
\pi(A, p_0^B) = p_0^B \{ \lambda x_A(p_0^B) + (1 - \lambda) (1 - \alpha_1/2) \}
\]
and
\[
\pi(B, p_0^B) = p_0^B \{ \lambda (1 - \alpha_1) x_A(p_0^B) + \alpha_1/2 + (1 - \lambda) (1 - \alpha_2/2) \}.
\]
Since the supposition \( p_0^A < p_0^B \) and Step 1 imply \( p_0^B \in S_A \), we need \( \pi(A, p_0^B) = \pi(B, p_0^B) \), or
\[
2x_A(p_0^B) = 1 + \frac{1 - \lambda \alpha_1 - \alpha_2}{\alpha_1}.
\]
The left-hand side is strictly lower than 2 given that \( x_A \) is strictly decreasing on \( S_A \) and \( p_0^A < p_0^B \). But (22) implies that the right-hand side is equal to 2. A contradiction. Similarly, we can exclude the possibility of \( p_0^A < p_0^B \). Therefore, it must be that \( p_0^A = p_0^B \).

**Step 4: \( F_A = F_B \).** For any \( p \in [p_0, 1] \), the indifference condition requires \( \pi(A, p) = \pi(B, p) \). Using (2) and (3), we get
\[
\lambda \alpha_1 [x_A(p) - 1/2] = (1 - \lambda) (\alpha_1 - \alpha_2) [x_B(p) - 1/2]
\]
for all \( p \in [p_0, 1] \). Then (22) implies \( x_A = x_B \) (or \( F_A = F_B \)).

### A.3 Proof of Proposition 2

1. Let us first prove the result for \( \alpha_2 < 1 \).

   1-1 A deviation to \((A, p < p_0^A)\) is obviously not profitable. A deviation to \((A, p > \hat{p})\) generates a profit equal to
   \[
   p \left(1 - \lambda \right) \left[ (1 - \alpha_1) x_B(p) + \alpha_1/2 \right].
   \]
   By using (7), one can easily check that this deviation profit is lower than \( \pi(B, p) \) in (3) with \( x_A(p) = 0 \). The last possible deviation is \((B, p < \hat{p})\) which results in a profit equal to
   \[
   p \left\{ \lambda \left[ (1 - \alpha_1) x_A(p) + \alpha_1/2 \right] + (1 - \lambda) (1 - \alpha_2/2) \right\}.
   \]
   Again, by using (7), one can check that this deviation profit is lower than \( \pi(A, p) \) in (2) with \( x_B(p) = 1 \).

   1-2 Let us now prove uniqueness. As in the proof of Proposition 1, we can show that \( S_A \cap S_B \neq \emptyset \) and \( \max \{ p_1^A, p_1^B \} = 1 \). Then the following two steps complete the proof.

   **Step 1:** \( S_A \cap S_B = \{ \hat{p} \} \) for some \( \hat{p} \). Suppose to the contrary that \( S_A \cap S_B = [p', p''] \) with \( p' < p'' \). Then for any \( p \in [p', p''] \), it must be that \( \pi(A, p) = \pi(B, p) \), where the profit functions are given by (2) and (3). This indifference condition requires
   \[
   \lambda \alpha_1 [x_A(p) - 1/2] = (1 - \lambda) (\alpha_1 - \alpha_2) [x_B(p) - 1/2]
   \]
   for all \( p \in [p', p''] \).
for all $p \in [p', p'']$. Since $\alpha_1 < \alpha_2$ and $F_z$ is strictly increasing on $S_z$, the left-hand side of $p$, while the right-hand side is an increasing function. So this condition cannot hold for all $p \in [p', p'']$. A contradiction.

**Step 2:** $p^B_1 = 1$. Suppose $p^B_1 < 1$. Then Step 1 and $\max\{p^A_1, p^B_1\} = 1$ imply that $p^A_1 = 1$ and $p^B_1 = p^A_0 = \hat{p} < 1$. Then each firm’s equilibrium profit should be equal to $\pi(A, 1) = (1 - \lambda) \alpha_1/2$ since the prices associated with $B$ are lower than one. However, if a firm chooses frame $B$ and $p = 1$, its profit is $|\lambda \alpha_1 + (1 - \lambda) \alpha_2|/2$ since it sells to half of the confused consumers. This deviation profit is greater than $\pi(A, 1)$ given that $\alpha_2 > \alpha_1$. A contradiction.

Therefore, in equilibrium, it must be that $S_A = [p^A_0, \hat{p}]$ and $S_B = [\hat{p}, 1]$.

(2) The equilibrium when $\alpha_2 = 1$ is just the limit of the equilibrium in (1) as $\alpha_2 \to 1$. However, now $S_A = [p^A_0, 1)$ and $S_B = \{1\}$.

### B Appendix: Proofs in the Oligopoly Model

#### B.1 Proof of Lemma 4

We prove this lemma in three steps:

(a) In any possible equilibrium in which firms use deterministic frames, at most one firm uses frame $A$. Suppose to the contrary that at least two firms use frame $A$. Then they must all earn zero profit at any putative equilibrium. But then any of them has a unilateral incentive to deviate to frame $B$ and a positive price, which results in a positive profit as $\alpha_2 > 0$. A contradiction.

(b) In any possible equilibrium in which firms use deterministic frames, at least one firm uses frame $A$. Suppose to the contrary that all firms use frame $B$. Then with probability $\alpha_2$ consumers shop randomly, and with probability $1 - \alpha_2$ they buy from the cheapest firm. This is a version of Varian (1980), and each firm earns $\alpha_2/n$. But then any firm can earn more by deviating to frame $A$ and price $p = 1$, which generates a profit of at least $\phi_{n-1} \geq 1/n$. This is because at most $n - 1$ firms can survive and the deviator would never be dominated as $\alpha_1 = 1$.

(c) Consider a candidate equilibrium in which one firm uses $A$ and all other firms use $B$. First, the $A$ firm must charge price $p = 1$ given that $\alpha_1 = 1$ and make a profit at least equal to $\phi_{n-1}$. Second, each $B$ firm must also earn at least $\phi_{n-1}$. Otherwise, any $B$ firm which earns $\pi_B < \phi_{n-1}$ can improve its profit by deviating to frame $A$ and a price $1 - \varepsilon$ for small $\varepsilon$. (The deviator would make a profit at least equal to $(1 - \varepsilon) \phi_{n-2}$ which is greater than $\pi_B$ for a sufficiently small $\varepsilon$ given that $\phi_{n-2} \geq \phi_{n-1}$.) Then, if $\phi_{n-1} > 1/n$, the sum of all firms’ profits exceeds one, and we reached a contradiction since industry profit is bounded by one. The only remaining possibility is

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28For $n \geq 3$, there are both symmetric and asymmetric mixed-strategy equilibria in the Varian model, but all of them are outcome equivalent (Baye et al., 1992).

29This part of the proof is different from that in the duopoly case since it is hard to directly characterize the pricing equilibrium when one firm uses frame $A$ and other $n - 1 \geq 2$ firms use frame $B$. 

32
that \( \phi_{n-1} = 1/n \) and each firm earns exactly 1/n. However, this means that all firms charge the monopoly price \( p = 1 \). But then any B firm has an incentive to deviate to a price slightly below one given that \( \alpha_2 < 1 \). A contradiction.

### B.2 Proof of Proposition 4

At equilibrium, each firm’s demand can be decomposed in two parts: the consumers who are insensitive to its price, and the consumers who are price-sensitive. Explicitly, we have

\[
\pi(A, p) / p = \pi(A, 1) + \{\lambda^{n-1} x_A(p)^{n-1} + \sum_{k=1}^{n-2} P_{n-1}^k x_A(p)^k [\alpha_2 \phi_{n-k-1} + (1 - \alpha_2) \phi_1] \},
\]

and

\[
\pi(B, p) / p = \pi(B, 1) + \{(1 - \alpha_2)(1 - \lambda)^{n-1} x_B(p)^{n-1} + (1 - \alpha_2)(1 - \phi_1) \sum_{k=1}^{n-2} P_{n-1}^k x_B(p)^{n-k-1} \}.
\]

Suppose now \( x_A(p) = x_B(p) = x(p) \), and the common support is \([p_0, 1] \). At equilibrium, \( \pi(A, p) = \pi(B, p) \) must hold for any \( p \in [p_0, 1] \).

(i) For \( n = 2 \), the last term in each demand function disappears. To have \( \pi(A, p) = \pi(B, p) \) for any \( p \in [p_0, 1] \), we need \( \pi(A, 1) = \pi(B, 1) \), or equivalently

\[
\frac{\lambda}{1 - \lambda} = \frac{\phi_1 - \alpha_2/2}{1 - \phi_1},
\]

and \( \lambda = (1 - \alpha_2)(1 - \lambda) \), or equivalently

\[
\frac{\lambda}{1 - \lambda} = 1 - \alpha_2.
\]

It follows that these two conditions hold simultaneously if and only if \( \phi_1 = 1/2 \).

(ii) For \( n \geq 3 \), to have \( \pi(A, p) = \pi(B, p) \) for any \( p \in [p_0, 1] \), we need \( \pi(A, 1) = \pi(B, 1) \) (see (13)), and

\[
\lambda^{n-1} + \sum_{k=1}^{n-2} P_{n-1}^k x(p)^{-k} \{\alpha_2 \phi_k + (1 - \alpha_2) \phi_1 \} = (1 - \alpha_2)(1 - \lambda)^{n-1} + (1 - \alpha_2)(1 - \phi_1) \sum_{k=1}^{n-2} P_{n-1}^k x(p)^{-k}.
\]

(To derive the latter, we divided each side by \( px(p)^{-n-1} \) and relabelled \( k \) in \( \pi(A, p) \) by \( n - k - 1 \).)

Then

\[
\sum_{k=1}^{n-2} b_k x(p)^{-k} = (1 - \alpha_2)(1 - \lambda)^{n-1} - \lambda^{n-1}
\]

\[\text{If some firm charged a price lower than one with a positive probability, then at these prices its demand would be positive (otherwise its equilibrium profit would be zero, which contradicts the fact that each firm earns 1/n). But then consumer surplus would be positive. A contradiction.}\]
where

\[ b_k \equiv P_{n-1}^{n-k-1} \left[ \alpha_2 \phi_k + (1 - \alpha_2) \phi_1 \right] - P_{n-1}^{k} (1 - \alpha_2) (1 - \phi_1). \]

Since the left-hand side of (23) is a polynomial of \(1/x(p)\) and \(x(p)\) is a decreasing function, (23) holds for all \(p \in [p_0, 1]\) only if \(b_k = 0\) for \(k = 1, \cdots, n - 2\) and the right-hand side is also zero. That is,

\[ \left( \frac{\lambda}{1 - \lambda} \right)^{n-1} = 1 - \alpha_2 \tag{24} \]

and

\[ \left( \frac{\lambda}{1 - \lambda} \right)^{n-2k-1} = \frac{(1 - \alpha_2) (1 - \phi_1)}{\alpha_2 \phi_k + (1 - \alpha_2) \phi_1} \quad \text{for } k = 1, \cdots, n - 2. \tag{25} \]

If \(\alpha_2 = 0\), both of them and (13) hold for \(\phi_1 = 1/2\) (in which case, \(\lambda = 1/2\)). Beyond this special case, (25) pins down a decreasing sequence \(\{\phi_k\}_{k=1}^{n-2}\) uniquely. Substituting (24) and (25) into (13), we can solve for \(\phi_{n-1}\). This means that, if \(n \geq 3\) and \(\alpha_2 > 0\), price-frame independence can hold only for a particular sequence of \(\phi_k\).\(^{31}\) It is easy to verify that \(\phi_k = 1/(1+k)\) does not satisfy these conditions.

### B.3 Proof of Proposition 5

When frame \(B\) is also a simple frame (i.e., when \(\alpha_2 = 0\)), (13) becomes

\[ \frac{\lambda}{1 - \lambda} = \left( \frac{\phi_1}{1 - \phi_1} \right)^{1/(n-1)}. \]

It follows that \(\lambda\) tends to \(1/2\) as \(n \to \infty.\)\(^{32}\) Then industry profit \(n\pi = n\phi_1 (1 - \lambda)^{n-1}\) must converge to zero.\(^{33}\)

Now consider \(\alpha_2 > 0\). Since the left-hand side of (13) is bounded, it must be that \(\lim_{n \to \infty} \lambda \leq 1/2\) (otherwise the right-hand side would tend to infinity). Since \(\{\phi_k\}_{k=1}^{n-1}\) is a non-increasing sequence, the right-hand side of (13) is greater than

\[ \frac{\alpha_2 (1 - \phi_1)}{n} \sum_{k=1}^{n-2} C_{n-1}^k \left( \frac{\lambda}{1 - \lambda} \right)^k = \frac{\alpha_2 (1 - \phi_1)}{n} \left[ \frac{1 - \lambda^{n-1}}{(1 - \lambda)^{n-1}} - 1 \right]. \]

\(^{31}\)Note that, although \(\{\phi_k\}_{k=1}^{n-2}\) solved from (25) is a decreasing sequence, still \(\phi_{n-1}\), which is solved from (13), may not be lower than \(\phi_{n-2}\). For example, when \(n = 3\), one can check that

\[ \phi_1 = \frac{1 - \alpha_2}{2 - \alpha_2} < \phi_2 = \frac{\phi_1 + 1/3 + \sqrt{1 - \alpha_2}}{1 + \sqrt{1 - \alpha_2}}, \]

which violates the requirement that \(\phi_k\) is non-increasing in \(k\).

\(^{32}\)How \(\lambda\) varies with \(n\) also depends on the value of \(\phi_1\). If \(\phi_1 > 1/2\), \(\lambda\) decreases to \(1/2\) with \(n\); if \(\phi_1 = 1/2\), \(\lambda\) is a constant \(1/2\); and if \(\phi_1 < 1/2\), \(\lambda\) increases to \(1/2\) with \(n\).

\(^{33}\)However, industry profit \(n\pi\) can rise with \(n\) when \(n\) is small and \(\phi_1\) takes relatively extreme values. For example, when \(\phi_1 = 0.95\) or \(0.05\), from \(n = 2\) to \(3\), industry profit \(n\pi\) increases from \(0.095\) to about \(0.099\).
So it must be that \(\lim_{n \to \infty} n (1 - \lambda)^{n-1} > 0\), otherwise the right-hand side of (13) would tend to infinity (given that \(\lim_{n \to \infty} \lambda \leq 1/2\) and so \(\lim_{n \to \infty} (1 - \lambda^{n-1}) = 1\)). This result implies that \(\lambda\) must converge to zero and industry profit \(n\pi = n (1 - \lambda)^{n-1} [\alpha_2 \phi_{n-1} + (1 - \alpha_2) \phi_1]\) must be bounded away from zero as \(n \to \infty\).

### B.4 Proof of Proposition 6

(i) For \(n = 2\), we have \(\lambda = \frac{1 - \alpha_2}{1 + (1 - \alpha_2)}\); and for \(n = 3\), we have \(\lambda = \frac{x}{1 + x}\) with \(x = \sqrt{4\alpha_2^2/9 + 1 - \alpha_2 - 2\alpha_2/3}\). The latter is smaller if \(x < 1 - \alpha_2\), which can be easily verified given that \(\alpha_2 < 1\). The industry profit result follows from straightforward algebra calculation by using (16).

(ii) We consider the limit case of \(\alpha_2 \to 1\). The equilibrium condition (15) implies that \(\lambda\) should then tend to zero. As \(\lambda \approx 0\), we have \(\lambda/ (1 - \lambda) \approx \lambda + \lambda^2\). For \(n \geq 4\), the right-hand side of (15) can be approximated as

\[
2\alpha_2 \left[ \frac{n-1}{n} (\lambda + \lambda^2) + \frac{n-2}{2} (\lambda + \lambda^2)^2 \right]
\]

by discarding all higher-order terms. (For \(n = 3\), the right-hand side of (15) is approximated by \(2\alpha_2 (\lambda + \lambda^2) + (\lambda + \lambda^2)^2\). One can check that the approximation result below still applies.)

Let \(\alpha_2 = 1 - \varepsilon\) with \(\varepsilon \approx 0\), and use the second-order (linear) approximation \(\lambda \approx k_1 \varepsilon + k_2 \varepsilon^2\). (As we will see, the first-order approximation is not sufficient for our purpose.) Substituting them into (26) and discarding all terms of order higher than \(\varepsilon^2\), we obtain

\[
\frac{2(n-1)}{n} k_1 \varepsilon + \left( \frac{2(n-1)}{n} (k_2 - k_1) + \frac{n^2 - 2}{n} k_1^2 \right) \varepsilon^2.
\]

Since the left-hand side of (15) is \(\varepsilon\), we can solve

\[
k_1 = \frac{n}{2(n-1)}; \quad k_2 = k_1 - \frac{n^2 - 2}{2(n-1)} k_1^2.
\]

As \(k_1\) decreases with \(n\), \(\lambda\) must decrease with \(n\).

As \(\varepsilon \approx 0\) (so that \(\lambda \approx 0\)), industry profit (for \(n \geq 3\)) can be approximated as

\[
\begin{align*}
n\pi &= (1 - \lambda)^{n-1} \left[ 1 + \frac{n}{2} - 1 \right] \varepsilon \\
&\approx [1 - (n - 1) \lambda + C_{n-1}^2 \lambda^2] [1 + \frac{n}{2} - 1] \varepsilon \\
&\approx 1 - \varepsilon + \frac{(n-2) n^2}{8(n-1)^2} \varepsilon^2.\end{align*}
\]

The second step follows from discarding all terms of order higher than \(\lambda^2\), and the third step is from substituting \(\lambda \approx k_1 \varepsilon + k_2 \varepsilon^2\) and discarding all terms of order higher than \(\varepsilon^2\). It is ready to see that the approximated industry profit increases with \(n\). (Notice that the first-order approximation of \(\lambda\) is not sufficient to tell how \(n\pi\) varies with \(n\).)
B.5 Proof of Lemma 5

We prove this lemma in three steps:

(a) In any pure strategy framing equilibrium, at most one firm uses frame $A$. Suppose to the contrary that at least two firms use frame $A$. Then, they must all earn zero profit at any putative equilibrium. But then any of them has a unilateral incentive to deviate to frame $B$ and a positive price. A contradiction.

(b) In any pure strategy framing equilibrium, at least one firm uses frame $A$. Suppose to the contrary that all firms use frame $B$. The only candidate equilibrium entails monopoly pricing $p = 1$ and each firm earns $1/n$. But then if one firm deviates to frame $A$ and price $1 - \varepsilon$, it will earn $(1 - \varepsilon)(\alpha_1\phi_{n-1} + 1 - \alpha_1)$. The reason is that, if the consumer is unable to compare prices in different frames (which happens with probability $\alpha_1$), the deviator’s demand is $\phi_{n-1}$; if the consumer is able to compare prices in different frames (which happens with probability $1 - \alpha_1$), the deviator serves the whole market (because all other firms charge $p = 1$ and so are dominated options). As $\phi_{n-1} \geq 1/n$, the deviation profit is greater than $1/n$ for a sufficiently small $\varepsilon$ and any $\alpha_1 \in (0, 1)$.

(c) The final possibility is that one firm uses $A$ and all other firms use $B$. Suppose such an equilibrium exists. Let $\pi_A$ be $A$ firm’s profit and $\pi_B^j$ be the profit of a $B$ firm indexed by $j$. (Notice that the $B$ firms may use different pricing strategies and make different profits). Let $p_A$ be the lowest price on which the $A$ firm puts positive probability (it might be a deterministic price). (i) Suppose that, at equilibrium, $\pi_A > \min\{\pi_B^j\}$. Then, if the $B$ firm which earns the least deviates to frame $A$ and a price $p_A - \varepsilon$, it will replace the original $A$ firm and have a demand at least equal to the original $A$ firm’s demand since it now charges a lower price and faces fewer competitors. So, this deviation is profitable at least when $\varepsilon$ is close to zero. A contradiction. (ii) Suppose now that, at equilibrium, $\pi_A \leq \min\{\pi_B^j\}$. Notice that $\pi_A \geq 1/n$, otherwise the $A$ firm would deviate to frame $B$ and a price $p = 1$, and make profit $1/n$. As industry profit cannot exceed one, all firms must earn $1/n$ at the candidate equilibrium and consumer surplus is zero. This also implies that all firms must be charging the monopoly price. But then any $B$ firm has an incentive to deviate to frame $A$ and price $1 - \varepsilon$, in which case it makes profit $(1 - \varepsilon)(\alpha_1\phi_{n-2} + 1 - \alpha_1) > 1/n$ for a sufficiently small $\varepsilon$. A contradiction.

B.6 Proof of Proposition 7

We only need to rule out profitable deviations from the proposed equilibrium. First, consider two possible deviations with frame $A$: (i) a deviation to $(A, p < p_A^1)$ is not profitable as the firm does

\footnote{When the consumer is unable to compare prices in different frames, the deviator’s demand is $\phi_{n-2}$ which is (weakly) greater than $\phi_{n-1}$, the original $A$ firm’s demand in this case. When the consumer is able to compare prices in different frames, the deviator is more likely to dominate the remaining $B$ firms (and so to have a higher expected demand) than the original $A$ firm.}
not gain market share, but loses on prices; (ii) a deviation \((A, p = 1)\) is not profitable either, since the deviator’s profit is \((1 - \lambda)^{n-1} \phi_{n-1} < \pi\).

Let us now consider a deviation to \((B, p \in [p_0^A, 1])\). Deviator’s profit is

\[
\hat{\pi}(B, p) = p \pi(B, 1) + p(1 - \alpha_1) \sum_{k=1}^{n-1} P_{n-1}^k x_A(p)^k.
\]

This expression captures the fact that when \(n - 1\) other firms also use \(B\), or when \(k \geq 1\) firms use \(A\) and the consumer is confused between \(A\) and \(B\), firm \(i\)’s demand does not depend on its price so that it is equal to \(\pi(B, 1)\). When \(k \geq 1\) firms use \(A\) and the consumer is not confused between \(A\) and \(B\), all other \(B\) firms (which charge price \(p = 1\)) are dominated by the cheapest \(A\) firm, and the consumer buys from firm \(i\) only if the cheapest \(A\) firm charges a price greater than \(p\). Notice that, from \(\pi(A, p) = \pi\) for \(p \in [p_0^A, 1]\), the second term in \(\hat{\pi}(B, p)\) is equal to

\[
\pi - p \pi - p \alpha_1 \sum_{k=1}^{n-1} P_{n-1}^k x_A(p)^k \phi_{n-k-1}.
\]

Then, \(\hat{\pi}(B, p) < p \pi + \pi - p \pi = \pi\). The deviation to \((B, p < p_0^A)\) will result in a lower profit. This completes the proof.

### B.7 Proof of Proposition 8

From (20), it follows that \(\lambda \to 1\) as \(\alpha_1 \to 0\). Let \(\alpha_1 = \varepsilon\) with \(\varepsilon \approx 0\), and \(\lambda = 1 - \delta\) with \(\delta \approx 0\). Then the right-hand side of (20) can be approximated as

\[
(1 - \phi_1) \left(1 - \frac{\delta}{\phi}\right)^{n-1} \approx \frac{1 - \phi_1}{\delta^{n-1}},
\]

since only the term with \(k = n - 1\) matters when \(\delta \approx 0\). Hence, from (20), we can solve

\[
\delta \approx \left(\frac{1}{\varepsilon} \left(1 - \frac{1 - \phi_1}{n} + \phi_{n-1} - 1\right)\right)^{1/(n-1)} \approx \left(\frac{n(1 - \phi_1)\varepsilon}{n - 1}\right)^{1/(n-1)}.
\]

The second step follows from the fact that \(\phi_{n-1} - 1\) is negligible compared to \(\frac{1}{\varepsilon}(1 - \frac{1 - \phi_1}{n})\). Given that \(\varepsilon \approx 0\), it is not difficult to see that \(\delta\) increases with \(n\) (e.g., one can show that \(\ln \delta\) increases with \(n\)). Hence, \(\lambda\) decreases with \(n\). As \(\varepsilon \approx 0\), industry profit is

\[
n\pi = n \delta^{n-1}[1 + (\phi_{n-1} - 1)\varepsilon] \approx \frac{n^2(1 - \phi_1)\varepsilon}{n - 1}
\]

by discarding the term of \(\varepsilon^2\). Clearly, \(n\pi\) increases with \(n\).
C Appendix: Proofs in the More-Frame Case

C.1 Proof of Lemma 6

The proof of (i) is straightforward and so omitted. We prove (ii) by discussing the following two cases:

(a) If some frame, say, $A_k$ is chosen by more than one firm, then these firms must earn zero profit in any possible equilibrium. Since $n < 2m$, there must exist another frame $A_l \neq A_k$ which has been chosen by at most one firm. But then, it is profitable for any firm which is using $A_k$ to deviate to $A_l$ and some positive price. (If no firm uses $A_l$ then any positive price $p \leq 1$ supports the deviation; if one firm already uses $A_l$, this firm must be charging $p = 1$ in any possible equilibrium, so that any price below one works.)

(b) If no frame is chosen by more than one firm (i.e., each firm chooses a distinct frame), the only possible equilibrium entails monopoly pricing $p = 1$ and each firm earns $1/n$. Then, any firm can earn a higher profit close to $1/(n-1)$ by deviating to one rival’s frame and a price slightly below one.

C.2 The formula for $Pr(k, l)$

Notice that

$$Pr(k, l) = \binom{n-1}{m-1} \left( \frac{1}{m} \right)^{k-1} \left( 1 - \frac{1}{m} \right)^{n-k} \mu(n-k, l-1),$$

where the product of the first three terms is the probability that $k-1$ firms among $n-1$ ones are also using frame $A_j$ given that firm $i$ has already chosen $A_j$, and $\mu(n-k, l-1)$ is the conditional probability that $n-k$ firms outside group $A_j$ adopt $l-1$ other distinct frames in total. In fact, $\mu(n-k, l-1)$ is the probability that $n-k$ balls are thrown at random into $l-1$ boxes among $m-1$ ones. (In particular, we let $\mu(n-k, 0) = 0$ for $n-k > 0$, and $\mu(0, 0) = 1$).

Now suppose $l \geq 2$ and let us derive the formula for $\mu(n-k, l-1)$. Without loss of generality, let 1 to $l-1$ be the targeted “boxes”. Denote by $E$ the event that the remaining $m-l$ boxes are empty, and by $E_i$ the event that box $i \in \{1, \cdots, l-1\}$ is empty conditional on $E$ (i.e., conditional on that all $n-k$ balls are thrown at random towards the targeted $l-1$ boxes). Then we have

$$\mu(n-k, l-1) = C_{m-1}^{l-1} \Pr(E) [1 - \Pr(\cup_{i=1}^{j-1} E_i)],$$

where $\Pr(E) = \left( \frac{l-1}{m-1} \right)^{n-k}$ and $\Pr(\cup_{i=1}^{j-1} E_i) = \sum_{h=1}^{l-1} (-1)^{h-1} C_l^{h-1} \Pr(E_1 \cdots E_h)$ with

$$\Pr(E_1 \cdots E_h) = (1 - \frac{h}{l-1})^{n-k}.$$
D Appendix: The Oligopoly Model with $\alpha_i < 1$

The oligopoly model in the main text focuses on two polar cases: $\alpha_2 < \alpha_1 = 1$ and $\alpha_1 < \alpha_2 = 1$. In this Appendix, we discuss the general case with $\alpha_1$ and $\alpha_2$ strictly smaller than one. When frame differentiation is more confusing than frame complexity ($\alpha_2 < \alpha_1 < 1$), we show that, if a symmetric equilibrium exists, it resembles the one in the polar case with $\alpha_1 = 1$. When frame complexity is more confusing than frame differentiation ($\alpha_1 < \alpha_2 < 1$), we derive a condition under which, like in the polar case with $\alpha_2 = 1$, there is an equilibrium where the complex frame is always associated with higher prices. In both cases, an increase in the number of firms can still harm consumers.

D.1 The case with $\alpha_2 < \alpha_1 < 1$

This part deals with the oligopoly model with $\alpha_2 < \alpha_1 < 1$. We focus on the symmetric equilibrium $(\lambda, F_A, F_B)$, in which $\lambda$ is the likelihood that each firm employs frame $A$ and $F_z$ is the continuous price distribution associated with frame $z \in \{A, B\}$. Let $S_z = [p_0, p_1]$ be the support of $F_z$. As before, let $P_{n-1}^k \equiv C_{n-1}^k \lambda^k (1 - \lambda)^{n-k-1}$ and $x_z(p) \equiv 1 - F_z(p)$.

We first derive a firm’s profit given that the other firms use the equilibrium strategy. If firm $i$ employs frame $A$ and prices at $p$, its profit is equal to

$$\pi(A, p) = p\lambda^{n-1}x_A(p)^{n-1} + p \sum_{k=0}^{n-2} P_{n-1}^k x_A(p)^k [(1 - \alpha_1)x_B(p)^{n-k-1} + \alpha_1(\alpha_2\phi_{n-k-1} + (1 - \alpha_2)\phi_1)].$$

Notice that when $k$ other firms also use frame $A$, firm $i$ has a positive market share only if it is undominated in group $A$, which happens with probability $x_A(p)^k$. The first term in $\pi(A, p)$ captures the fact that if $k = n - 1$, then firm $i$ serves the whole market. The second term deals with $k < n - 1$.

(i) If the consumer is able to compare $A$ and $B$, firm $i$ serves the whole market whenever it prices below all the $B$ firms (see the first term in the square bracket). (ii) If the consumer is unable to compare $A$ and $B$, then firm $i$’s demand depends on consumer’s ability to compare prices in frame $B$. If she cannot compare prices in frame $B$, then no $B$ firm is dominated so that firm $i$’s demand is $\phi_{n-k-1}$. If the consumer can compare prices in frame $B$, only one $B$ firm is selected from the $B$ group and so firm $i$’s demand is $\phi_1$.

If firm $i$ uses frame $B$ and charges price $p$, its profit is

$$\pi(B, p) = p(1 - \lambda)^{n-1}[\alpha_2/n + (1 - \alpha_2)x_B(p)^{n-1}] + p \sum_{k=1}^{n-1} P_{n-1}^k \left\{ (1 - \alpha_2)x_B(p)^{n-k-1}[\alpha_1(1 - \phi_1) + (1 - \alpha_1)x_A(p)^k] + \alpha_2[\alpha_1(1 - \phi_{n-k})/(n-k) + (1 - \alpha_1)H_k(p)] \right\}.$$

The first term captures the situation in which all the other firms also use frame $B$. Then, if the consumer is confused, she shops at random and chooses firm $i$ with probability $1/n$, and if the consumer can compare prices, she chooses firm $i$ only if it offers the best deal. The summation
term captures the case in which $k \geq 1$ firms use frame $A$. (i) If the consumer can compare prices in frame $B$ (which happens with probability $1 - \alpha_2$), firm $i$ has a positive demand only if it offers the lowest price in group $B$, the probability of which is $x_B(p)^{n-k-1}$. If the consumer is unable to compare $A$ and $B$, firm $i$’s demand is $1 - \phi_1$ since only one firm is undominated in group $A$; if the consumer is able to compare $A$ and $B$, firm $i$ serves the whole market when all $A$ firms charge prices higher than $p$ (that is, with probability $x_A(p)A$). (ii) If the consumer is unable to compare prices in frame $B$ or prices in different frames (which happens with probability $\alpha_1\alpha_2$), firm $i$ has a demand $(1 - \phi_{n-k})/(n - k)$ since all $B$ firms are undominated. (iii) If the consumer is unable to compare prices in frame $B$ but is able to compare prices in different frames (that is, with probability $\alpha_2(1 - \alpha_1))$, firm $i$’s demand is

$$H_k(p) = \sum_{l=1}^{n-k} \frac{C_l^{n-1}}{l} \int_{p_A}^{p_B} [1 - x_B(\omega)]^{l-1} x_A(\omega)^{n-k-l} dG_k(\omega)$$

$$= \frac{1}{n-k} \int_{p_A}^{p_B} \frac{1 - x_B(\omega)^{n-k}}{1 - x_B(\omega)} dG_k(\omega),$$

where $G_k(\omega) \equiv 1 - x_A(\omega)^k$ is the distribution function of the minimum price in group $A$ of cardinality $k$. In this case, to have a positive demand, firm $i$ must price below the minimum price (let it be $\omega$) in group $A$. (That is why we integrate over $\omega$ from $p$ to $p_A$.) Conditional on that, firm $i$’s market share depends on how many other $B$ firms survive. Given the minimum price $\omega$ in group $A$, the probability that exactly $l - 1$ other $B$ firms survive is $C_{n-k-1}^l \frac{1 - x_B(\omega)^{l-1}}{x_A(\omega)^{n-k-l}}$. When $l$ firms from group $B$ (including firm $i$) survive, firm $i$’s market share is $1/l$. The second step follows from noticing $C_{n-k-1}^l/l = C_{n-k}^d(n-k)$ and using the binomial formula.

If a symmetric equilibrium with continuous price distributions exists, under our assumption that $\phi_k \geq 1/(1 + k)$ (i.e., frame $A$ is always weakly favored), we still have $p_A^\lambda = p_B^\lambda = 1$ as in the polar case in Section 3.1. Then it follows that each firm’s equilibrium profit $\pi$ should be equal to $\pi(A, 1) = \pi(B, 1)$. Specifically, we have

$$\pi(A, 1) = \alpha_1(1 - \lambda)^{n-1}[\alpha_2\phi_{n-1} + (1 - \alpha_2)\phi_1],$$

and

$$\pi(B, 1) = (1 - \lambda)^{n-1}\frac{\alpha_2}{n} + \lambda^{n-1}\alpha_1(1 - \phi_1) + \alpha_1\alpha_2 \sum_{k=1}^{n-2} p_A^k \frac{1 - \phi_{n-k}}{n-k}.$$

From $\pi(A, 1) = \pi(B, 1)$, we can pin down $\lambda$:

$$\alpha_2\left(\phi_{n-1} - \frac{1}{n\alpha_1}\right) + (1 - \alpha_2)\phi_1 = \alpha_2 \sum_{k=1}^{n-2} \omega_{n-1}^k \frac{1 - \phi_{n-k}}{n-k} \left(\frac{\lambda}{1 - \lambda}\right)^k + (1 - \phi_1) \left(\frac{\lambda}{1 - \lambda}\right)^{n-1}.$$  

\(^{35}\)In the polar case with $\alpha_1 = 1$, price competition occurs only among firms using the same frame, so for $z = A, B$, $\pi(z, p)$ only includes $x_z(p)$. Hence, in a symmetric equilibrium, if $p_A^1 < 1$, it would be dominated by $p_B^1 + \epsilon \leq 1$. However, with $\alpha_1 < 1$, even firms using different frames compete in prices such that $\pi(z, p)$ includes both $x_A(p)$ and $x_B(p)$. Then it becomes less trivial to prove $p_A^1 = 1$. The details are available from the authors.
As $\phi_1 \geq \phi_{n-1} \geq 1/n$ and $\alpha_1 > \alpha_2$, the left-hand side is positive. The right-hand side rises with $\lambda$ from zero to infinity. Then, (28) has a unique solution in $(0,1)$. Therefore, if there exists a symmetric equilibrium with continuous $F_z$‘s, the expression for the equilibrium profit and the condition which determines $\lambda$ resemble those in the polar case with $\alpha_1 = 1$.

A symmetric mixed-strategy equilibrium with continuous $F_z$ exists if the system of equations

$\pi(z, p) = \pi$ for $z = A, B$ has a well defined solution $(F_A, F_B)$. Notice that for $\alpha_2 > 0$, $\pi(B, p) = \pi$ is a nonlinear integral equation due to the presence of $H_k(p)$. Proving existence in general is difficult. However, when $\alpha_2 = 0$, $\pi(B, p) = \pi$ degenerates to an ordinary polynomial equation, and the existence of equilibrium is not hard to prove. In continuation, we assume that a symmetric equilibrium with continuous $F_z$ exists for arbitrary $\alpha_2 < \alpha_1 < 1$.

**Equilibrium price-frame (in)dependence.** We explore the possibility of a symmetric equilibrium with $F_A = F_B = F$. Let $[p_0, 1]$ be the support of $F$. Then for any $p \in [p_0, 1]$, it should hold that $\pi(A, p) = \pi(B, p)$. Using the procedure in the proof of Proposition 4, we can rewrite this condition as

$$
\sum_{k=1}^{n-2} b_k x(p)^{-k} - \alpha_2 (1 - \alpha_1) \sum_{k=1}^{n-2} F^k_{n-1} H_k(p)/x(p)^{n-1} = \alpha_1 (1 - \alpha_2) (1 - \lambda)^{n-1} - (\alpha_1 - \alpha_2 + \alpha_1 \alpha_2) \lambda^{n-1} - \alpha_2 (1 - \alpha_1),
$$

(29)

where

$$
b_k \equiv \alpha_1 \{P_{n-1}^{n-k-1}[\alpha_2 \phi_k + (1 - \alpha_2) \phi_1] - P_{n-1}^k (1 - \alpha_2)(1 - \phi_1)\}.
$$

For $n = 2$, it is ready to check that conditions (28) and (29) hold only if $\phi_1 = 1/2$. For $n \geq 3$, they both hold if $\phi_1 = 1/2$ and $\alpha_2 = 0$. Except for these two cases, (29) cannot hold when $\alpha_1 < 1$ since the $H_k(p)$ term is nonzero.

**The impact of greater competition.** This part of the analysis relies only on (27) and (28). First, the results in Proposition 5 still hold. In particular, when $\alpha_2 > 0$, firms will almost surely use frame $B$ as $n \to \infty$, and industry profit does not converge to zero. The proofs are similar.

Second, numerical simulations suggest that for a sufficiently high $\alpha_1$, an increase in the number of firms still has a perverse effect on consumer welfare when $\alpha_2$ is close to $\alpha_1$. However, we do not have an analytical proof. This is because the limit analysis developed in Proposition 6 for $\alpha_2 \to \alpha_1 = 1$ does not work any more for $\alpha_2 \to \alpha_1 < 1$. Specifically, when $\alpha_2$ converges to $\alpha_1$, the left-hand side of (28) tends to

$$
\alpha_1 \phi_{n-1} + (1 - \alpha_1) \phi_1 - 1/n.
$$

With the random purchase rule, this becomes $(1 - \alpha_1) (1/2 - 1/n)$, which is not equal to zero unless $n = 2$. That is, $\lambda$ does not tend to zero as $\alpha_2 \to \alpha_1 < 1$. Then the approximation procedure in

36 Notice that, when $n \geq 3$, even if $\alpha_1 = \alpha_2 = \alpha < 1$, it cannot be true that all firms use frame $B$. Otherwise, each firm would earn $\alpha/n$, and then the deviation to frame $A$ and price equal to one would bring a higher profit $\alpha^2/n + \alpha(1-\alpha)/2$.  

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the proof of Proposition 6 cannot apply.

D.2 The case with $\alpha_1 < \alpha_2 < 1$

This part extends the oligopoly model to the case with $\alpha_2 < 1$. We show that a symmetric separating equilibrium $(\lambda, F_A, F_B)$ with $S_A = [p_0^A, \hat{p}]$ and $S_B = [\hat{p}, 1]$ exists only under certain conditions.

Consider a candidate separating equilibrium. Given that the other firms use the equilibrium strategy, if firm $i$ chooses frame $A$ and charges $p \in [p_0^A, \hat{p}]$, its profit is

$$\pi(A, p) = p \sum_{k=0}^{n-2} P^k p A(p)^k [1 - \alpha_1 + \alpha_1 \alpha_2 \phi_{n-k-1} + \alpha_1 (1 - \alpha_2) \phi_1] + p \lambda^{n-1} x_A(p)^{n-1}.$$ 

Note first that firm $i$ has a positive market share if it charges the lowest price in group $A$, which happens with probability $x_A(p)^k$ if there are $k$ other $A$ firms. The last term is firm $i$’s profit when $k = n - 1$, and the summation term is firm $i$’s profit when $k < n - 1$. If the consumer can compare prices in different frames, then firm $i$ serves the whole market since all $B$ firms’ equilibrium prices are higher than $p$. This explains the term $1 - \alpha_1$ in the square bracket. If the consumer cannot compare $A$ and $B$, firm $i$’s demand depends on whether the consumer can compare prices in frame $B$. If she cannot compare them, all $B$ firms survive and firm $i$’s demand is $\phi_{n-k-1}$; otherwise, only one $B$ firm wins in group $B$ and firm $i$’s demand is $\phi_1$. (All subsequent profit functions are constructed similarly and, therefore, we omit further explanations.) In particular, if firm $i$ charges $p = \hat{p}$, it has a positive market share only if all other firms use frame $B$ (i.e., only if $k = 0$), so its profit is

$$\pi(A, \hat{p}) = \hat{p} (1 - \lambda)^{n-1} \{1 - \alpha_1 + \alpha_1 [\alpha_2 \phi_{n-1} + (1 - \alpha_2) \phi_1]\}.$$ 

Given that the other firms use the equilibrium strategy, if firm $i$ chooses frame $B$ and charges $p \in [\hat{p}, 1]$, its profit is

$$\pi(B, p) = p \sum_{k=1}^{n-1} P^k p B(p)^k n-k \{1 - \alpha_1 \alpha_2 (1 - \phi_1)x_B(p)^{n-k-1} + (1 - \alpha_2)(1 - \phi_1)x_B(p)^{n-k-1}\}$$

$$+ p (1 - \lambda)^{n-1} [\alpha_2/n + (1 - \alpha_2)x_B(p)^{n-1}].$$

Notice that, if there are $A$ firms in the market (which are charging prices lower than the $B$ firms at equilibrium), then firm $i$ makes sales only if the consumer cannot compare prices in different frames. In particular, when firm $i$ charges $p = \hat{p}$, $\pi(B, \hat{p})$ is just $\pi(B, p)$ with $x_B(p)$ replaced by $x_B(\hat{p}) = 1$.

At equilibrium, it should hold that $\pi(A, \hat{p}) = \pi(B, \hat{p})$. Dividing each side by $\alpha_1 (1 - \lambda)^{n-1}$, we
obtain the equation which determines $\lambda$:

$$
\alpha_2 \phi_{n-1} + (1 - \alpha_2) \phi_1 + \frac{\alpha_2}{\alpha_1} \left(1 - \frac{1}{n}\right) - 1
$$

$$
= \sum_{k=1}^{n-1} C_n^k \left[\alpha_2 \frac{1 - \phi_{n-k}}{n-k} + (1 - \alpha_2)(1 - \phi_1)\right] \left(\lambda \frac{1}{1 - \lambda}\right)^k. \tag{30}
$$

(One can check that, for $\alpha_2 = 1$, this equation degenerates to (20) in Proposition 7.) Since $\phi_k$ is non-increasing in $k$, the left-hand side of (30) is (weakly) greater than

$$
\phi_{n-1} + \frac{\alpha_2}{\alpha_1} \left(1 - \frac{1}{n}\right) - 1,
$$

which is positive as $\phi_{n-1} \geq 1/n$ and $\alpha_2 > \alpha_1$. Therefore, (30) has a unique solution in $(0,1)$.

To determine $\hat{p} < 1$, we can use $\pi(B, \hat{p}) = \pi(B, 1)$, where

$$
\pi(B, 1) = \lambda^{n-1} \alpha_1 (1 - \phi_1) + (1 - \lambda)^{n-1} \frac{\alpha_2}{n} + \alpha_1 \alpha_2 \sum_{k=1}^{n-2} P_k^{n-1} \frac{1 - \phi_{n-k}}{n-k} \equiv \pi. \tag{31}
$$

(In continuation, we refer to $\pi$ as each firm’s equilibrium profit.) In addition, the expressions for $F_2, z \in \{A, B\}$, can be solved from $\pi(z, p) = \pi$, and $p_0^A$ follows from $\pi(A, p_0^A) = \pi$. All of them are well defined.

The remaining step is to check whether firms have unilateral profitable deviations. The following result gives the conditions under which profitable deviations do not exist.

**Claim 1** In the oligopoly model with $0 < \alpha_1 < \alpha_2 < 1$, there is a symmetric equilibrium $(\lambda, F_A, F_B)$ with $S_A = [p_0^A, \hat{p}]$ and $S_B = [\hat{p}, 1]$ if and only if

$$
\frac{\alpha_2 - \alpha_1}{\alpha_1 (1 - \alpha_2)} \frac{1}{1 - \phi_1} > \frac{1 - \lambda^{n-1}}{(1 - \lambda)^{n-1}} - 1, \tag{32}
$$

where $\lambda \in (0,1)$ solves (30).

**Proof.** We first show that the deviation to $(B, p \in [p_0^A, \hat{p}])$ is always unprofitable. If a firm deviates, it makes profit

$$
\hat{\pi}(B, p) = p \sum_{k=1}^{n-1} P_k^{n-1} \left[\alpha_1 \alpha_2 \frac{1 - \phi_{n-k}}{n-k} + \alpha_1 (1 - \alpha_2)(1 - \phi_1) + (1 - \alpha_1)x_A(p)^k\right]
$$

$$
+ p (1 - \lambda)^{n-1} (\alpha_2/n + 1 - \alpha_2)
$$

$$
= \frac{p \pi}{\hat{p}} + p (1 - \alpha_1) \sum_{k=1}^{n-1} P_k^{n-1} x_A(p)^k.
$$

The second equality follows from $\pi(B, \hat{p}) = \pi$. Notice that from $\pi(A, p) = \pi$ for $p \in [p_0^A, \hat{p}]$, one can check that the second term is actually equal to $\pi - p\pi/\hat{p} - M$, where

$$
M \equiv p\alpha_1 \lambda^{n-1} x_A(p)^{n-1} + p\alpha_1 \sum_{k=1}^{n-2} P_k^{n-1} x_A(p)^k [\alpha_2 \phi_{n-k-1} + (1 - \alpha_2)\phi_1].
$$

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Since $M > 0$, it is clear that $\hat{\pi}(B,p) < \pi$, so that the deviation to $(B,p \in [\hat{p}_0^1,\hat{p}_0^0])$ is not profitable. Clearly, a deviation to $(B, p < \hat{p}_0^0)$ results in even lower profits.

However, the deviation to $(A, p \in (\hat{p}, 1])$ might be profitable. Deviator’s profit is

$$\hat{\pi}(A,p) = p(1-\lambda)^{n-1} \alpha_1\alpha_2\phi_{n-1} + \alpha_1(1-\alpha_2)\phi_1 + (1-\alpha_1)x_B(p)^{n-1}.$$ 

This deviation is not profitable if $\hat{\pi}(A,p) < \pi(B,p)$, where the right-hand side is the equilibrium profit. Dividing each side of this inequality by $p\alpha_1(1-\lambda)^{n-1}$ and using (30), it follows that the condition holds if and only if

$$\frac{\alpha_2 - \alpha_1}{\alpha_1(1-\alpha_2)} \frac{1}{1-\phi_1} [1 - x_B(p)^{n-1}] > \sum_{k=1}^{n-1} \frac{C_n^k}{n-1} \left(\frac{\lambda}{1-\lambda}\right)^k [1 - x_B(p)^{n-k-1}].$$

(Notice that the term with $k = n - 1$ in the right-hand side is actually zero.) A necessary condition for the above inequality to hold for $p = 1$ is

$$\frac{\alpha_2 - \alpha_1}{\alpha_1(1-\alpha_2)} \frac{1}{1-\phi_1} > \sum_{k=1}^{n-2} \frac{C_n^k}{n-1} \left(\frac{\lambda}{1-\lambda}\right)^k = \frac{1 - \lambda^{n-1}}{(1-\lambda)^{n-1}} - 1.$$ 

Note that this is just (32). Moreover, since $x_B(p)^{n-k-1}$ increases with $k$, (32) is also a sufficient condition. Therefore, the deviation to $(A, p \in (\hat{p}, 1])$ is not profitable if and only if (32) holds. This completes the proof. 

For $n = 2$, the right-hand side of (32) is zero and the condition always holds. For $n \geq 3$, however, it may fail to hold. For example, for given $n \geq 3$ and $\alpha_2 < 1$, if $\alpha_1$ is sufficiently close to $\alpha_2$, the condition fails. This happens because, as $\alpha_1 \to \alpha_2$, $\lambda$ (derived from (30)) is bounded away from zero so that the right-hand side of (32) is also bounded away from zero, but the left-hand side tends to zero. (Notice that this argument does not apply if $\alpha_2 = 1$.)

We now report two limit conditions under which (32) must hold. (i) For fixed $n$ and $\alpha_1 < 1$, (32) holds if $\alpha_2$ is sufficiently close to one. This is not difficult to understand since we must have a separating equilibrium for $\alpha_2 = 1$. (ii) For fixed $n$ and $\alpha_2 < 1$, (32) holds if $\alpha_1$ is sufficiently close to zero. We prove this in Claim 2 below.

Notice that condition (32) is necessary and sufficient for a symmetric mixed-strategy equilibrium with adjacent supports (like the one identified in the polar case with $\alpha_2 = 1$) to exist. When (32) is violated, a symmetric mixed-strategy equilibrium might still exist, but the supports of the equilibrium price distributions will eventually overlap. Then the equilibrium characterization would be similar to the case discussed in D.1. However, recall that with overlapping supports, existence of equilibrium is hard to prove due to the fact that the price distributions are defined by a system involving functional equations.

The impact of greater competition. We now show that greater competition can still harm consumers if $\alpha_1$ is sufficiently small.
Claim 2 In the oligopoly model with $0 < \alpha_1 < \alpha_2 < 1$, for given $n$ and $\alpha_2$, there exists $\hat{\alpha} > 0$ such that for $\alpha_1 < \hat{\alpha}$, (i) condition (32) holds and so a separating equilibrium exists, and (ii) in this equilibrium industry profit $n \pi$ increases from $n$ to $n + 1$.

Proof. For fixed $n$ and $\alpha_2 < 1$, if $\alpha_1$ tends to zero, then from (30) it follows that $\lambda$ tends to one. Let $\alpha_1 = \varepsilon \approx 0$ and $\lambda = 1 - \delta$ with $\delta \approx 0$. Then the right-hand side of (30) can be approximated as

$$\left(\frac{1 - \delta}{\delta}\right)^{n-1} \frac{1 - \phi_1}{\delta^{n-1}},$$

since only the term with $k = n - 1$ matters when $\delta \approx 0$. In addition, the left-hand side can be approximated as $\frac{\alpha_2 \varepsilon}{(1 - \frac{1}{n})}$. Hence, from (30), we can solve

$$\delta \approx \left(\frac{n (1 - \phi_1) \varepsilon}{\alpha_2 (n - 1)}\right)^{1/(n-1)}.$$

(33)

We now show that (32) holds in this limit case. The left-hand side of (32) becomes

$$\frac{\alpha_2 / \varepsilon - 1}{(1 - \alpha_2) (1 - \phi_1)} \approx \frac{\alpha_2 / \varepsilon}{(1 - \alpha_2) (1 - \phi_1)}$$

(34)

as $\varepsilon \approx 0$. The right-hand side of (32) is now

$$\frac{1 - (1 - \delta)^{n-1}}{\delta^{n-1}} - 1 \approx \frac{n - 1}{\delta^{n-2}} \approx (n - 1) \left(\frac{\alpha_2 (n - 1)}{(1 - \phi_1) n \varepsilon}\right)^{(n-2)/(n-1)}.$$

The first step follows from discarding all terms for $k = 1, \ldots, n - 2$ in (31) and the constant term $-1$, while the second step used (33). Then the right-hand side of (32) is lower than the left-hand side if

$$[(1 - \alpha_2) (n - 1)]^{n-1} \left(\frac{n - 1}{n}\right)^{n-2} \frac{\alpha_2}{(1 - \phi_1) \varepsilon} < \frac{\alpha_2}{(1 - \phi_1) \varepsilon}.$$  

For fixed $n$ and $\alpha_2$, this is always true for $\varepsilon \to 0$.

In this limit case, each firm’s profit given in (31) can be approximated as

$$\pi \approx (1 - \delta)^{n-1} (1 - \phi_1) \varepsilon + \frac{\alpha_2}{n} \delta^{n-1}$$

$$\approx (1 - \phi_1) \varepsilon + \frac{1 - \phi_1}{n - 1} \varepsilon$$

$$= \frac{n (1 - \phi_1)}{n - 1} \varepsilon.$$  

The first step follows from the fact that all terms for $k = 1, \ldots, n - 2$ in (31) are of order higher than $\varepsilon$. The second step follows from discarding all terms of order higher than $\varepsilon$. It is now ready to see that $n \pi$ increases in $n$. ■